

Measurement of the top quark mass using leptonic observables in $t\bar{t}$ events
produced in pp collisions at $\sqrt{s} = 13$ TeV with the CMS Experiment at the LHC

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Measurement of the top quark mass using leptonic observables in $t\bar{t}$ events
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ABSTRACT

Title:

Measurement of the top quark mass using leptonic observables in $t\bar{t}$ events produced in pp collisions at $\sqrt{s} = 13$ TeV with the CMS Experiment at the LHC

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A novel method is presented for the measurement of the top-quark mass using only leptonic observables in $t\bar{t}$ events produced in pp collisions at $\sqrt{s} = 13$ TeV. Data, corresponding to an integrated luminosity of 35.9 fb^{-1} , were collected with the CMS detector during LHC Run II in 2016. The measurement uses the top-quark mass dependence of multiple kinematic distributions comprised from the dilepton $e\mu$ final state of $t\bar{t}$ events. The top-quark mass dependence of each kinematic distribution is calibrated using events generated with a NLO Monte Carlo simulation, and for the first time, with a second simulation that includes the exact NNLO corrections to the top quark decay subprocesses. A profile likelihood scan is utilized to simultaneously compare the shape of predicted and measured kinematic distributions and extract the value of the top-quark mass from the reconstructed leptonic observables. A second method uses the top-quark mass dependence of first and second Mellin moments of the kinematic distributions to obtain an alternative determination of the top-quark mass.

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Abbreviations

AK4	Anti- k_t jet with cone $\Delta R = 0.4$
ALICE	A Large Ion Collider Experiment
ATLAS	A Toroidal LHC ApparatuS
CMS	Compact Muon Solenoid
CERN	European Organization for Nuclear Research
CHS	Charged Hadron Subtraction
CKM	Cabibbo-Kobayashi-Maskawa
CSC	Cathode Strip Chamber
DT	Drift Tube
EB	Electromagnetic Calorimeter Barrel
ECAL	Electromagnetic Calorimeter
EE	Electromagnetic Calorimeter Endcap
EM	Electromagnetic
EW	Electroweak
FSR	Final State Radiation
HB	Hadronic Calorimeter Barrel
HCAL	Hadronic Calorimeter
HE	Hadronic Calorimeter Endcap

HF	Hadronic Calorimeter Forward
HLT	High Level Trigger
HPD	Hybrid Photodiode
IRC	Infrared and Collinear
ISR	Initial State Radiation
JEC	Jet Energy Corrections
JER	Jet Energy Resolution
L1	Level-1
L1A	Level-1 Accept
L1T	Level-1 Trigger
LHC	Large Hadron Collider
LHCb	Large Hadron Collider beauty
LINAC	Linear Accelerator
LO	Leading order
MC	Monte Carlo
ME	Matrix element
NLO	Next-to-leading order
NNLL	Next-to-next-to-leading log order
NNLO	Next-to-next-to-leading order
PDF	Parton Distribution Function
PF	Particle Flow
PMNS	Pontecorvo–Maki–Nakagawa–Sakata
PMT	Photomultiplier Tube
PS	Proton Synchrotron
PU	Pileup

PV	Primary Vertex
QCD	Quantum Chromodynamics
QED	Quantum Electrodynamics
RCP	Resistive Plate Chamber
SF	Scale Factor
SiPM	Silicon Photomultiplier
SM	Standard Model
SPS	Super Proton Synchrotron
WP	Working Point

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Dedication

This thesis is dedicated to the memory of Prof. Semen Köksal. Her kind spirit and passion for teaching were instrumental in kindling my interests in applied mathematics and inspiring my undergraduate and graduate pursuits as a mathematician.

Chapter 1

Introduction

Physics is the study of the natural world written in the language of mathematics. High energy physics is concerned with the interactions of particles and the fundamental forces at the highest achievable energy scales. Centuries of study and experimentation have culminated in the Standard Model (SM) of particle physics, a theory which accurately predicts the behavior of ordinary matter.

This thesis presents a precision measurement of the top quark mass, m_t . The top quark is the heaviest known elementary particle and its mass is a fundamental parameter of the SM. Precision studies of top quark physics ensure the self-consistency of the theory. Top quarks also constitute a primary background for other measurements, such as the newly discovered Higgs Boson. Accurate modeling of top quark processes will complement future Higgs studies and searches for new physics beyond the standard model.

1.1 The Standard Model

The Standard Model represents the most complete theory of particle physics and the forces that mediate particle interactions to date. The underlying description is based on Quantum Field Theory where "particles" are actually excitations in quantum fields. Figure 1.1 summarizes all elementary particles that comprise the SM. They can be broadly subdivided into two categories: fermions and bosons.

Fermions are particles which have half-integer spin and follow the laws of Fermi-Dirac statistics, including the Pauli Exclusion Principle which forbids any two fermions in a quantum system from simultaneously occupying the same quantum state. The SM fermions, quarks and leptons, are spin $\frac{1}{2}$ particles which come in three generations of increasing mass. These are the constituent particles of ordinary matter. There are six flavors of quarks (up, down, charm, strange, top, bottom) grouped into "uplike" (u,c,t) and "downlike" (d,s,b) varieties. Leptons include the electron-like particles (electrons, muons, taus) and the associated electron, muon, and tau neutrinos. Each fermion has a corresponding antiparticle with identical mass but opposite electric charge. All fermions beyond the first generation (excluding neutrinos) or composites thereof are unstable and will promptly decay into first generation particles.

Bosons are integer spin particles which follow the laws of Bose-Einstein statistics and are not subject to the Pauli Exclusion Principle. The SM bosons include the spin 1 vector (or gauge) bosons, which mediate the fundamental force interactions, and the scalar Higgs Boson.

There are four fundamental forces of nature: the gravitational force, the weak force, the electromagnetic (EM) force, and the strong force.

Gravity is the oldest known fundamental force. The gravitation interaction

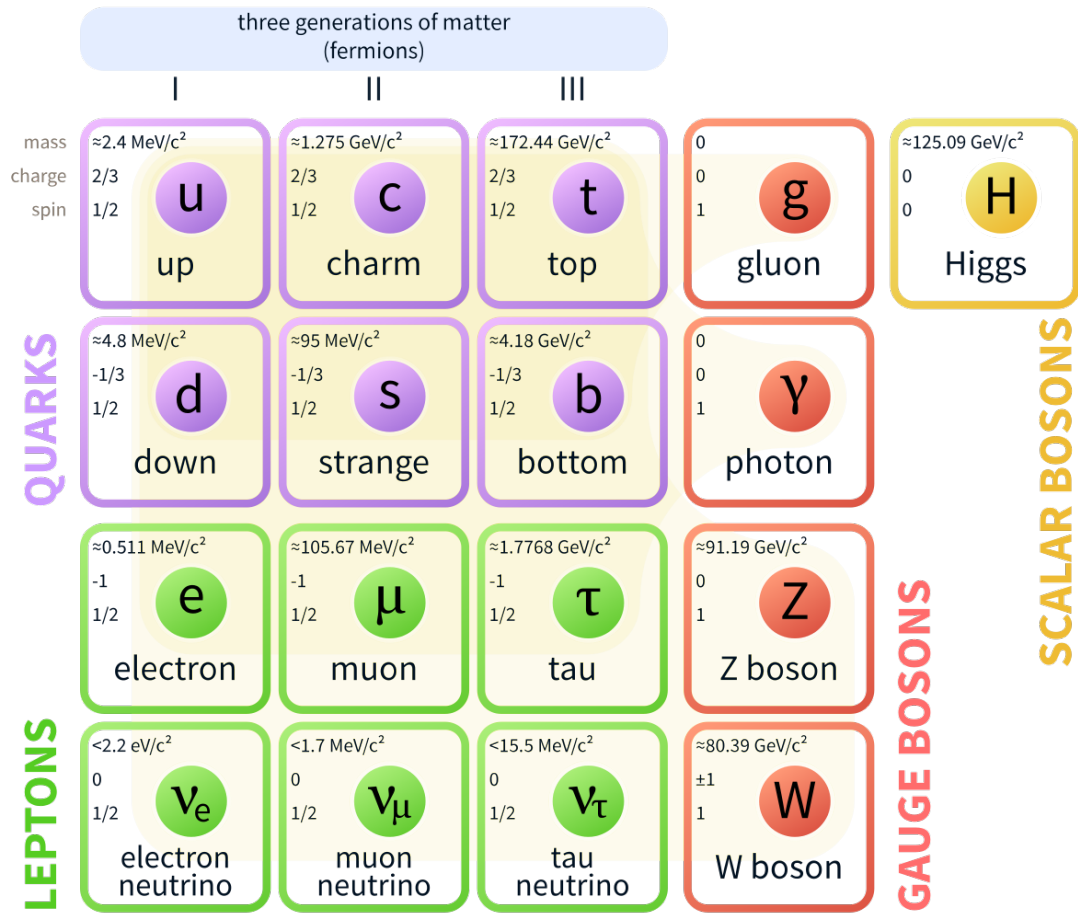


Figure 1.1: The Standard Model of particle physics. There are three generations of fermions that make up ordinary matter, subdivided into families of quarks and leptons. The gauge bosons are mediators of the electromagnetic, weak, and strong forces. Particles acquire their mass through interactions with the Higgs Field. Excitations of this field appear as the scalar Higgs boson.

was first quantified by Isaac Newton in the 1687 text, the *Principia*. The modern description of gravity, albeit still based on classical physics, is given by Albert Einstein's Theory of General Relativity which was first published in 1915. A fully quantum mechanical description of the gravitational force remains elusive. The gravitational interaction would be mediated by the hypothetical spin 2 boson

called the *graviton*. Gravity is insignificantly weak compared to the other three forces and is not described by the SM.

The theory of electromagnetic interactions was first described by James Clerk Maxwell in the nineteenth century with the unification of the disparate electric and magnetic forces. Photons, massless quanta of the EM field, were discovered in the early twentieth century. In the SM, photons mediate the EM force described by quantum electrodynamics (QED) and can interact with any electrically charged particles. Since photons themselves are electrically neutral, they cannot directly couple with other photons. The charged leptons have units of $-e$ charge for particles and $+e$ charge for antiparticles, where $e = 1.602 \times 10^{-19}$ Coulombs. Uplike quarks have an electric charge of $+\frac{2}{3}e$ and downlike quarks have electric charge $-\frac{1}{3}e$.

The idea of the weak force was first proposed by Enrico Fermi as an explanation for beta decay: the process now understood as $n \rightarrow p + e^- + \bar{\nu}_e$ where a neutron decays into a proton through the emission of an electron and an electron antineutrino. The weak and electromagnetic interactions were unified into the electroweak force by Glashow, Weinberg, and Salam in 1968. The neutral Z boson and electrically charged W^+ and W^- are mediators of the electroweak interaction. The weak force has a short effective range $\sim \mathcal{O}(10^{-17}\text{m})$ due to the mass of these bosons ($m_W = 80.379 \pm 0.012$ GeV, $m_Z = 91.1876 \pm 0.0021$ GeV [55]). Weak isospin is the analogous charge for weak force interactions and is a conserved quantity. The charged leptons and downlike quarks have weak isospin $-\frac{1}{2}$ while neutrinos and uplike quarks have weak isospin $+\frac{1}{2}$.

The strong force describes the interactions of quarks and their composite particles, called hadrons, via the theory of quantum chromodynamics (QCD). This

force is mediated by the massless gluon and has an associated charge called *color* which comes in three varieties: red, blue, green, and the anticolors antired, antiblue, and antigreen (Fig. 1.2). Quarks carry one of the three color charges and antiquarks similarly have one of the anticolor charges. Gluons contain one of 8 combinations of color-anticolor charges. Due to a property of QCD called *color confinement*, observable particles form color singlet states. A quark-antiquark pair forms a *meson* and triplet configurations are known as *baryons*. The corresponding quark color charges add to form color singlets.

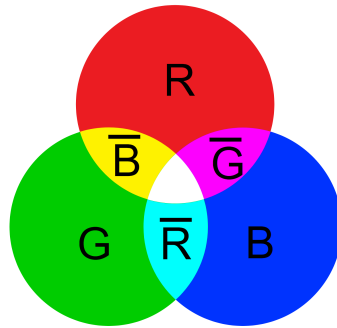


Figure 1.2: Illustration of QCD color and anticolor states. All observable particles are color singlets formed by combinations of color-charged constituent quarks.

The final constituent particle of the SM is the Higgs boson, first discovered by the CMS and ATLAS experiments at CERN in 2012. The Higgs boson is a neutral spin 0 particle with mass $m_H = 125.10 \pm 0.14$ GeV. Its production is a result of excitations in the Higgs Field, a quantum field which produces the effective mass of quarks, leptons, and the W/Z bosons that interact with it.

All possible interactions between particles and the fundamental forces in the Standard Model are governed by the SM lagrangian density (a detailed description can be found in ref [32]). Terms obey the global Poincaré symmetry of special relativity which includes invariance under translations, rotations, and changes in

inertial reference frames. The SM is a renormalizable non-Abelian gauge theory based on the $SU(3) \otimes SU(2) \otimes U(1)$ gauge group.

$U(n)$ is a Lie group of dimension n^2 consisting of all unitary $n \times n$ matrices with the group operation of matrix multiplication. A matrix $\mathbf{A} \in U(n)$ is unitary if the conjugate transpose, $\mathbf{A}^\dagger \equiv (\mathbf{A}^*)^\top$, is equal to its inverse: $\mathbf{A}^\dagger \mathbf{A} = \mathbb{I}$, where $\mathbb{I}$ is the $(n \times n)$ identity matrix. The special unitary group $SU(n)$ is a subset of $U(n)$ with determinant $+1$ and dimension $n^2 - 1$.

$U(1)$, called the *circle group*, consists of all complex numbers $e^{i\theta}$ with magnitude 1. This is an Abelian group since scalar multiplication is commutative. In the SM, the generator of $U(1)$ is the weak hypercharge Y , related to the electric charge Q and weak isospin T_3 by Equation 1.1.

$$Q = T_3 + \frac{1}{2}Y \tag{1.1}$$

The non-Abelian $SU(2) \otimes U(1)$ group describes electroweak interactions. The set of Pauli matrices form the infinitesimal generators of the $SU(2)$ and the eigenvalues give the weak isospin.

$SU(3)$ is the final symmetry group which describes strong interactions in QCD. It is generated by the set of 8 Gell-Mann matrices.

There are a total of 26 free parameters in the SM which must be determined experimentally:

- 12 fermion masses (6 quarks and 6 leptons)
- 3 coupling constants, one for each symmetry group
- 2 Higgs parameters (Higgs boson mass and vacuum expectation value)

- 3 mixing angles + 1 phase angle for Cabibbo–Kobayashi–Maskawa (CKM) matrix elements describing quark mixing (see section 1.2)
- 3 mixing angles + 1 phase angle for Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix elements describing neutrino oscillations, the changing of neutrino lepton flavor over time
- 1 charge-parity (CP) symmetry violating phase for strong interactions

1.1.1 Feynman Diagrams and Perturbative QCD

Interactions between particles and forces in the SM, such as scattering and decays, can be represented by Feynman diagrams. These are constructed from combinations of the basic interaction vertices shown in Fig 1.3. The fewest number of vertices required to represent a process is known as the tree level, or leading order (LO), diagram. This is the first term in a perturbative expansion of the scattering amplitude $\mathcal{M}$ used to calculate the probability of obtaining the given final state. Higher order terms can be constructed from tree-level diagrams by adding additional vertices and are represented by loops in the Feynman diagram.

The strength of interactions between particles and the forces mediated by gauge bosons is governed by the coupling constants: α for EM, α_W for weak, and α_S for strong interactions. The term "coupling constant" is a misnomer since their values actually depend on the interaction energy scale Q^2 (see Fig. 1.4), and as such are known as *running coupling constants*.

The EM coupling constant α is approximately $\frac{1}{137}$ at low energy and $\frac{1}{127}$ at the Z boson energy scale ($\sim 90\text{GeV}$), increasing as a function of energy. The weak coupling constant α_W is a factor of 10^{-5} smaller than α at low energies. The strong

Standard Model Interactions
(Forces Mediated by Gauge Bosons)

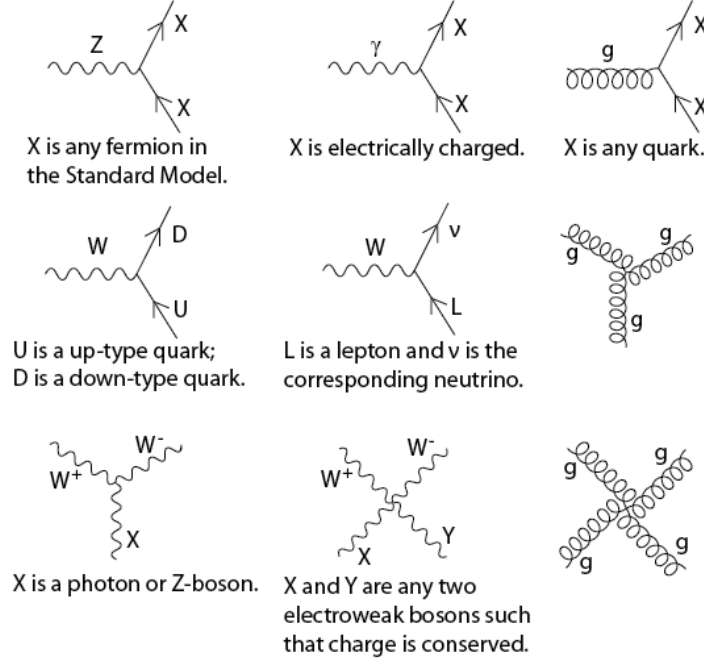


Figure 1.3: SM gauge boson interaction vertices. Feynman diagrams representing various SM processes are constructed from combinations of basic vertices (Higgs couplings not shown).

coupling constant α_S has a value at the Z mass scale of 0.1186 ± 0.0007 [55] and can be parametrized as Eq. 1.2, where n_f is the number of approximately massless quark flavors and Λ_{QCD} is the QCD scale (for $n_f = 5$, $\Lambda_{\text{QCD}} \approx 210$ MeV). Due to the non-Abelian nature of QCD gauge fields, the value of α_S increases at low energies.

$$\alpha_S(Q^2) = \frac{2\pi}{(11 - \frac{2}{3}n_f) \ln \frac{Q^2}{\Lambda_{\text{QCD}}^2}} \quad (1.2)$$

QCD processes, such as top quark production, can be expanded perturbatively

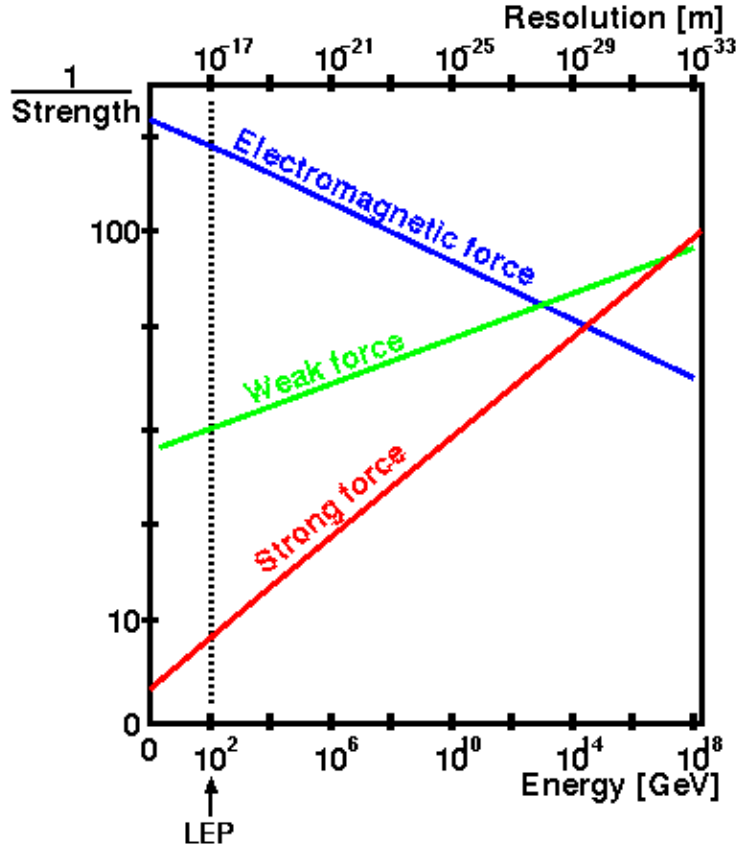


Figure 1.4: SM coupling constants as a function of interaction energy.

in α_S . Contributions from 1-loop and 2-loop diagrams are known as next-to-leading order (NLO) and next-to-next-to-leading order (NNLO). Each loop contributes an additional factor of α_S and the full expansion is an infinite sum of all possible orders. At low energies where $\alpha_S > 1$, this expansion breaks down and the process must be described by non-perturbative QCD.

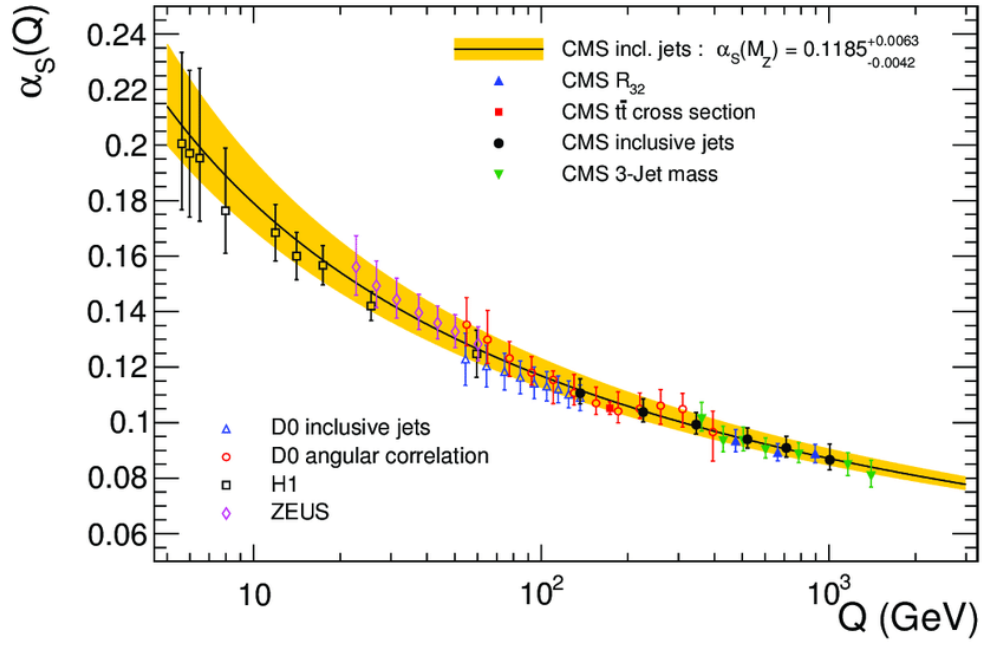


Figure 1.5: Running of the strong coupling constant α_s [39].

1.2 Top Quark Physics

The top quark is the heaviest known fundamental particle, comparable to an entire gold atom. Its mass as determined from the last world combination measurement is $m_t = 173.34 \pm 0.76$ GeV [14](Fig. 1.6). The more recent combination of measurements from the CMS experiment gives a value of $m_t = 172.44 \pm 0.48$ GeV [46].

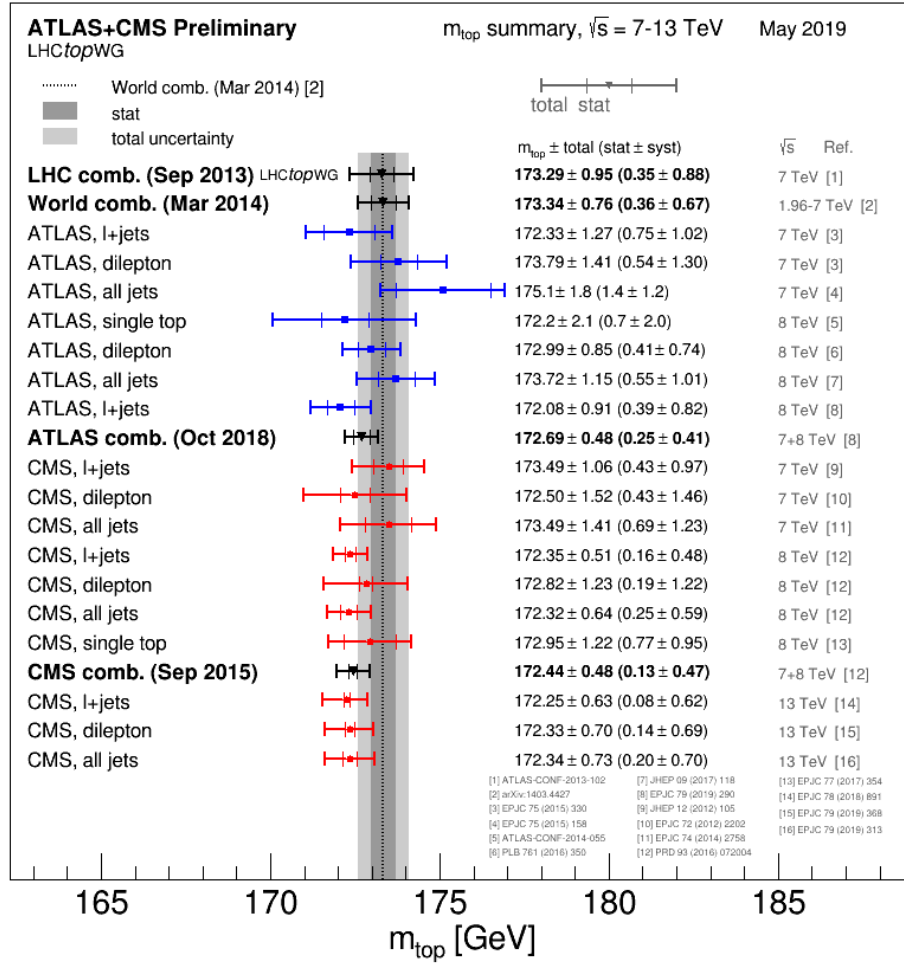


Figure 1.6: Recent measurements of the top quark mass by the CMS and ATLAS experiments. The world average combination is represented by the gray band.

The top quark has a mean lifetime of $5 \times 10^{-25}\text{s}$, shorter than the hadronization timescale ($\sim 10^{-24}\text{s}$), so it does not form resonances or composite particles like the other quark flavors. This presents an opportunity to investigate the properties of a "bare" quark.

At the LHC, the primary mode of top quark-antiquark pair production is gluon-gluon fusion, making up almost 90% of the $t\bar{t}$ production cross section. The process of quark-antiquark fusion additionally contributes to leading order top pair production. Feynman diagrams for these processes are shown in Figure 1.7.

Top quarks can also be produced singly through the s, t, and tW channel processes. LO diagrams for single top production are shown in Figure 1.8. The tW-channel process produces two W bosons coupled with top quarks which can decay into dilepton final states and at NLO interferes with $t\bar{t}$ event topology (discussed further in Section 4.5.1.9). As a result, the single top tW channel is included as a signal process. Single top s and t channels can only produce single lepton final states from a W boson coupled to the top quark, so these are considered background processes for this analysis.

The decay of the top quark into a lighter downlike quark is mediated by the electroweak force through the emission of a W boson. To conserve electric charge, the top quark (antiquark) couples to the W^+ (W^-). Quark flavor transitions are described by elements of the unitary Cabibbo–Kobayashi–Maskawa (CKM) matrix (Eq. 1.3). The top quark decays almost exclusively into a bottom quark (Figure 1.9) with transition probability proportional to $|V_{tb}|^2 \approx 1$.

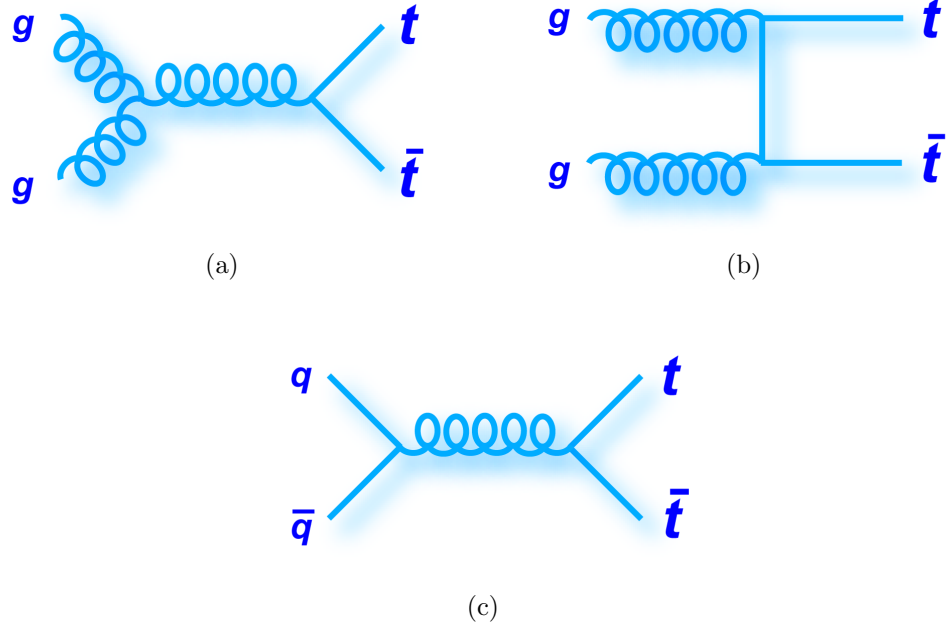


Figure 1.7: Tree level diagrams of $t\bar{t}$ production through gluon-gluon fusion (a),(b) and quark-antiquark fusion (c).

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \quad (1.3)$$

$$= \begin{pmatrix} 0.97446 \pm 0.00010 & 0.22452 \pm 0.00044 & 0.00365 \pm 0.00012 \\ 0.22438 \pm 0.00044 & 0.97359^{+0.00010}_{-0.00011} & 0.04214 \pm 0.00076 \\ 0.00896^{+0.00024}_{-0.00023} & 0.04133 \pm 0.00074 & 0.999105 \pm 0.000032 \end{pmatrix} \quad (1.4)$$

The W bosons are also unstable particles which can decay leptonically ($W \rightarrow l\bar{\nu}_l$) to produce a lepton and corresponding antineutrino or hadronically ($W \rightarrow q\bar{q}'$) into quarks (Fig. 1.10).

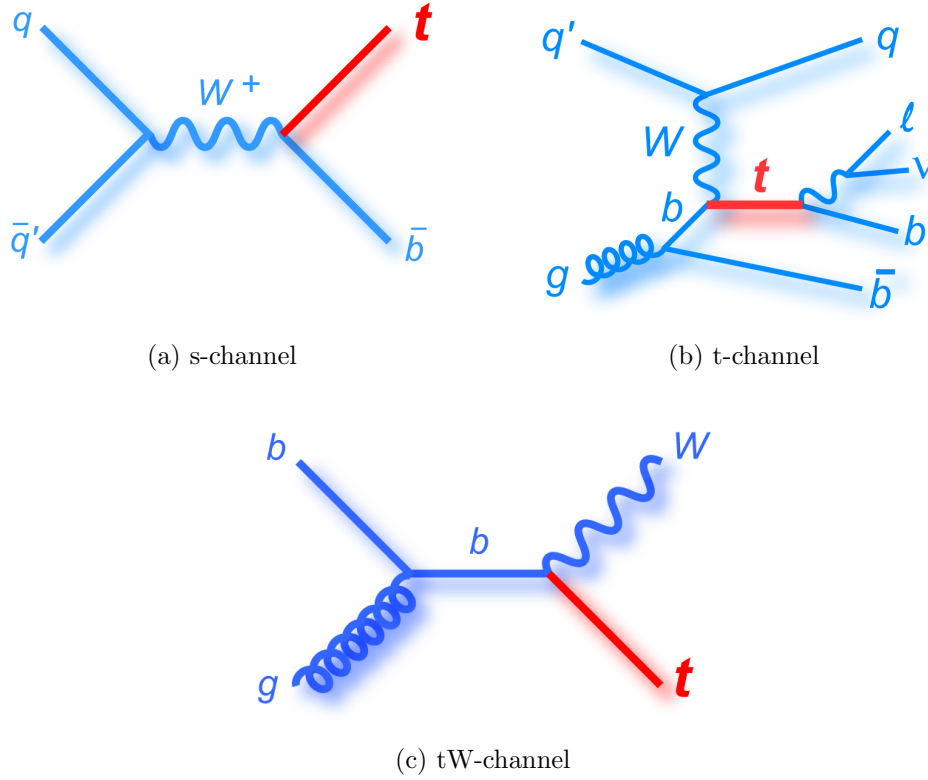


Figure 1.8: Tree level diagrams of single top production through s-channel (a), t-channel (b), and tW-channel (c) processes.

The $t\bar{t}$ final state can be described by the lepton content produced from the W decays:

- **Dileptonic** Both W bosons decay leptonically
- **Semileptonic** One W boson decays leptonically and one decays hadronically
- **All-hadronic** Both W bosons decay hadronically

Figure 1.11 shows the possible final states of $t\bar{t}$ with size proportional to the branching ratio, the probability of a particular decay mode.

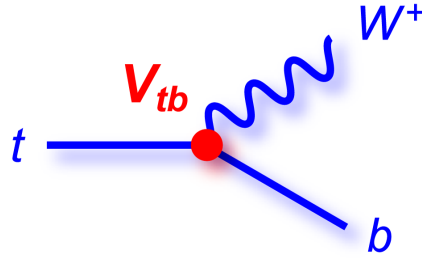


Figure 1.9: The electroweak decay of the top quark into a bottom quark through the emission of a W boson.

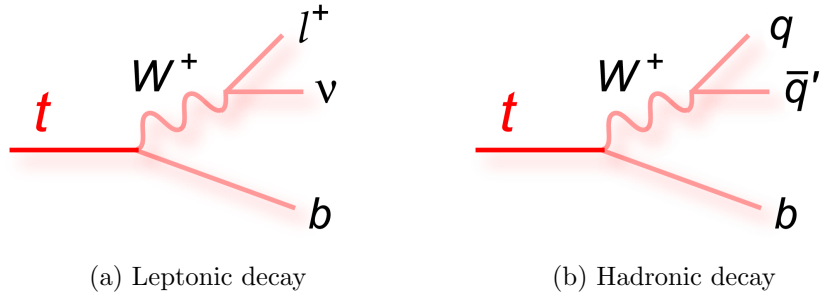


Figure 1.10: Leptonic (a) and hadronic (b) decay modes of the W boson.

This analysis considers signal events from the dilepton $e\mu$ final state of $t\bar{t}$ and tW decays. The leptons are produced from decays of W bosons that couple directly with the top quark. Other processes can produce a similar final state but with one or both leptons originating from a source other than top quark decay and are considered backgrounds. In addition to the aforementioned single top s and t channels, the following processes contribute to background:

- **$t\bar{t}+V$** A top quark-antiquark pair produced in association with a W or Z boson. The $t\bar{t}$ pair decays semileptonically and the associated vector boson decays leptonically.
- **Drell-Yan** Quarks from the interacting hadrons produce a virtual photon

Top Pair Decay Channels

$\bar{c}s$	electron+jets	muon+jets	tau+jets	all-hadronic	
$\bar{u}d$					
τ^-	$e\tau$	$\mu\tau$	$\tau\tau$	tau+jets	
μ^-	$e\mu$	$\mu\mu$	$\mu\tau$	muon+jets	
e^-	ee	$e\mu$	$e\tau$	electron+jets	
W decay	e^+	μ^+	τ^+	$u\bar{d}$	$c\bar{s}$

Figure 1.11: Final states of top quark pair decay with size proportional to the branching ratio of the corresponding channels.

or Z boson that decays into a pair of oppositely charged leptons. The primary decay mode for obtaining the $e\mu$ final state is $Z \rightarrow \tau^+\tau^- \rightarrow e^\pm\nu_e \mu^\mp\nu_\mu \nu_\tau\bar{\nu}_\tau$.

- **Diboson** A process $pp \rightarrow VV' + X$ producing two vector bosons (WW, WZ, or ZZ) which subsequently decay leptonically into an $e\mu$ pair.
- **W+jets** The production of a W boson with jets. Assuming leptonic W decay, the second lepton originates from the decay of a hadron formed during jet fragmentation or some another rare process, making this a very minor contribution to the overall background.

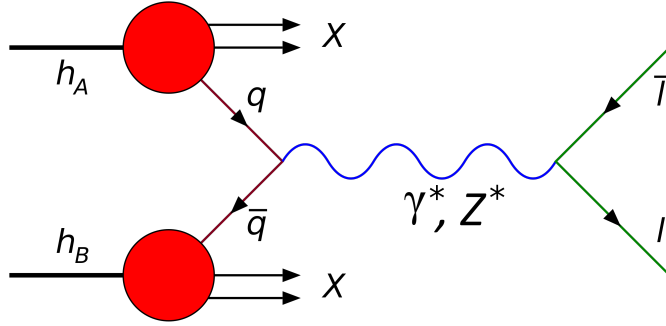


Figure 1.12: The Drell-Yan process, a primary background for $t\bar{t}$ dilepton channels. Quarks from the interacting hadrons produce a virtual photon or Z boson which can decay into a pair of oppositely charged leptons.

1.2.1 Top Quark Mass Definitions

A theoretical subtlety exists with in the definition of the top quark mass. Like other quarks, the top quark is not a directly observable particle, so its mass must be inferred from other observations [31] and is dependent on the choice of renormalization scheme. There are several definitions of the top quark mass in common use. The two that will be discussed in this analysis are the pole mass and Monte Carlo mass.

The pole mass, m_t^{pole} , is the pole of the top quark Feynman propagator. This is the analogous quantity to a "real" mass for an observable particle. Since it is not a physical observable, the mass term acquires an imaginary component. When calculating higher order perturbative corrections to the top quark self energy, this leads to infrared renormalons: terms of order α_s^n in the expansion.

The Monte Carlo mass, m_t^{MC} , is the name given to the input top quark mass for Monte Carlo generators. This is the quantity used to enforce momentum conservation for the decay products and is the mass extracted from comparisons to

MC generators.

There is ambiguity in how these two masses are related and the topic is still an active area of phenomenological research. The deviation of the two mass definitions is of order $\mathcal{O}(500 \text{ MeV})$. Some theorists believe that m_t^{MC} is same mass as m_t^{pole} with additional systematic uncertainty added, while others posture that the two masses describe distinct properties of the top quark. Reduction of the total measured uncertainty on the top quark mass below 500 MeV should help resolve some of these theoretical ambiguities.

1.3 Simulation

Collision data is compared with theoretical models of the various signal and background processes to extract relevant physics parameters. Two methods of simulation are used for different aspects of this analysis. The first is Monte Carlo (MC) simulation, a random sampling algorithm for generating events that could result from pp collisions. This can include approximations of higher-order and non-perturbative effects in addition to smearing effects caused by the detector. The second is fixed-order QCD, which calculates the cross section of a particular process using a limited set of Feynman diagrams.

1.3.1 MC Event Simulation

Monte Carlo simulation is a random sampling algorithm for obtaining numerical results of a discrete process that might otherwise be difficult to directly solve. MC event generation is split into three phases:

- A matrix element (ME) generator simulates the initial pp interaction, called the hard-scattering, up to a certain order (Fig. 1.13).
- The hadronizer/showering step produces and decays hadrons from quarks/gluons in the event, which will be reconstructed as jets (see Section 3.4), and approximates non-perturbative QCD effects.
- Particles in the event are sent through a full simulation of the detector to mimic the smearing effects that would be seen in real collision data.

The primary signal process, top quark pair production ($pp \rightarrow t\bar{t}j$), is simulated using the POWHEG ME generator at NLO [4]. The hard scattering transfers

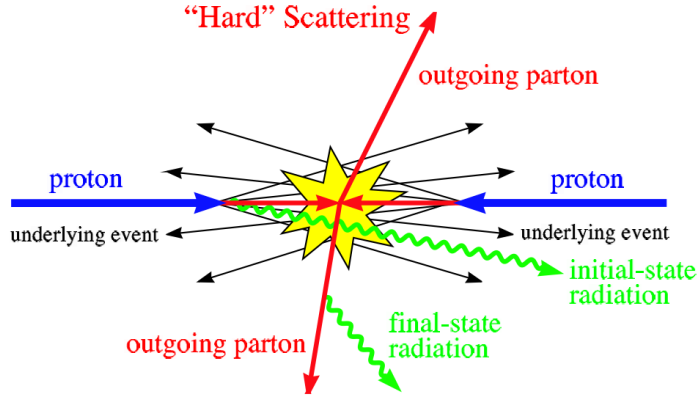


Figure 1.13: The hard scattering event, representing the initial pp interaction point.

momentum from the colliding protons to the constituent quarks and gluons, called *partons*, which will create the top quark and antiquark pair. Due to momentum conservation, the hardest jet will reside in the opposite hemisphere with respect to the $t\bar{t}$ pair momentum. Top quarks are decayed via the electroweak process $t \rightarrow Wb$ with W bosons further decayed into leptonic ($W \rightarrow l\nu$) or hadronic ($W \rightarrow q\bar{q}'$) final states.

The proton is composed of three valence quarks (two up quarks and one down quark) bound by gluons. There are also virtual quark pairs, called sea quarks, continually being produced and annihilated by the splitting of the color field. With enough momentum imparted to them, the sea quarks can become real quarks that are emitted as part of the hard scattering.

The cross section, $\sigma_{pp \rightarrow t\bar{t}j}$, gives the probability of producing a $t\bar{t}$ pair from the initial proton-proton interaction. This requires knowledge of how the momentum from the hard scattering is imparted to the quarks and gluons comprising the proton. This cannot be calculated purely from perturbative QCD but must instead

be inferred from data observations and is given in the form of Parton Distribution Functions (PDF) (Figure 1.14).

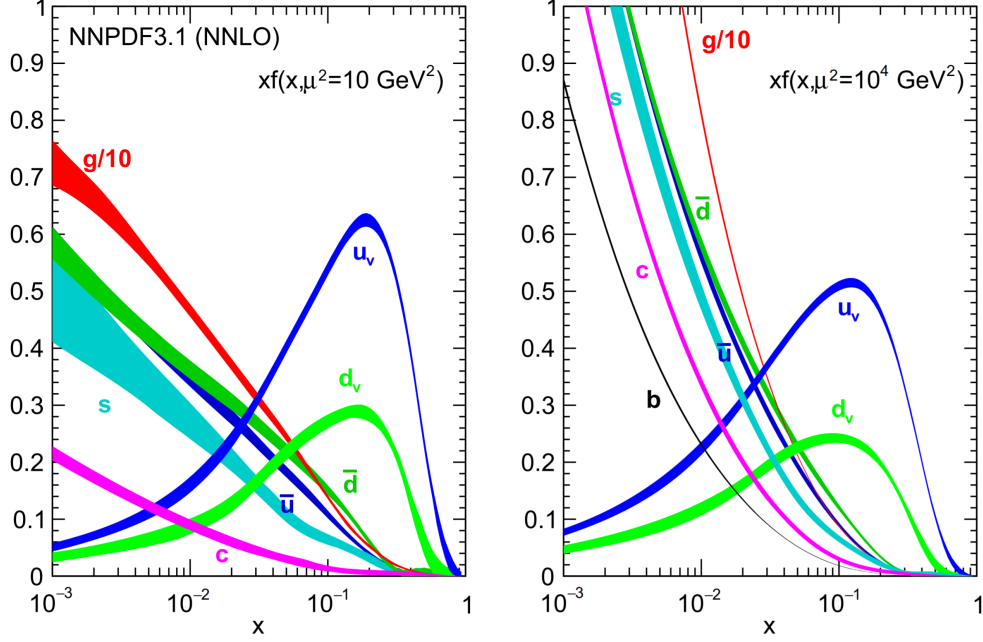


Figure 1.14: Parton distribution functions from the NNPDF3.1 set, showing the probability density for finding partons with a specific longitudinal momentum fraction x at interaction scale Q^2 [16].

MC events are weighted based on a total event weight (Eq. 1.5) and individual corrections applied on a per-event basis (see Section 4.4). Here N is the total number of MC entries generated, $\mathcal{L}$ is the total integrated luminosity of the collision data, and σ is the production cross section for the given process. For $t\bar{t}$, the cross section is a function of the center-of-mass energy of the colliding protons ($\sqrt{s} = 13$ TeV) and the top quark mass. Using an assumed value of $m_t = 172.5$ GeV, the 13 TeV inclusive $t\bar{t}$ cross section is calculated to be 831.76 pb.

$$w = \frac{\sigma \mathcal{L}}{N} \quad (1.5)$$

The next stage is performed by a secondary program called a hadronizer. The nominal $t\bar{t}$ sample uses PYTHIA8 [13] for this step. All partons produced in the hard scattering or subsequent decay products will undergo hadronization to form color singlet states due to QCD color confinement. The shower of particles produced by these short-lived hadrons will be reconstructed as jets.

Parton showering effects are added based on the value of α_s , the strong coupling constant, to simulate the effects of missing higher-order corrections not available in the ME generator. At low Q , the perturbative expansion breaks down and non-perturbative QCD effects dominate.

Highly boosted initial-state partons can emit QCD radiation known as initial-state radiation (ISR). The analogous concept for outgoing partons from the hard process is called final-state radiation (FSR). To ensure momentum conservation after radiating, the outgoing parton momenta are rescaled through momentum shuffling.

Interactions with the remnants of protons after hard scattering or other soft multi-parton interactions in a given event are described by the Underlying Event tune. The effects of multiple pp interaction vertices in an event, called pileup, are also taken into account (see Section 4.4.1 for more details).

The final stage is the detector simulation. Showered events are run through a full simulation of the CMS detector using GEANT4 [3] to simulate the smearing effects seen in data. Afterwards, the MC simulated events can be reconstructed similar to collision data using the methods outlined in Chapter 3.

1.3.2 Fixed-Order QCD Calculations

An alternative to showered MC simulation is fixed-order QCD calculations, which only include processes up to a given order in perturbative QCD. In particular this extends to orders beyond NLO, the limit of current ME generators. A recent result [17] computes top quark pair production and decay using exact NNLO corrections. This is implemented as a calculation of the differential cross section as a function of observables of the decay products (see Section 4.10 for more details).

Chapter 2

The CMS Experiment

The top quark was first discovered in 1995 at the Fermilab Tevatron accelerator with nearly simultaneous publications by the two primary experiments: DØ [36] and CDF [37]. The initial CDF top quark mass measurement from hadronic $t\bar{t}$ decay was $176 \pm 8(\text{stat}) \pm 10(\text{syst})$ GeV from 67 pb^{-1} of $\bar{p}p$ collision data at $\sqrt{s} = 1.8$ TeV. Once the Large Hadron Collider (LHC) at CERN began operational runs in 2009, precision top quark physics with the CMS and ATLAS experiments was possible with higher center-of-mass energies than ever before. Initial Run I data was collected at $\sqrt{s} = 7$ and 8 TeV before Run II operations at $\sqrt{s} = 13$ TeV. A total of 35.9 fb^{-1} was collected by CMS in 2016, with the entire Run II dataset (2016-2018) comprising 137.2 fb^{-1} . The LHC had become a top factory and precision top physics reaped the rewards.

The CERN accelerator complex *Complexe des accélérateurs du CERN*

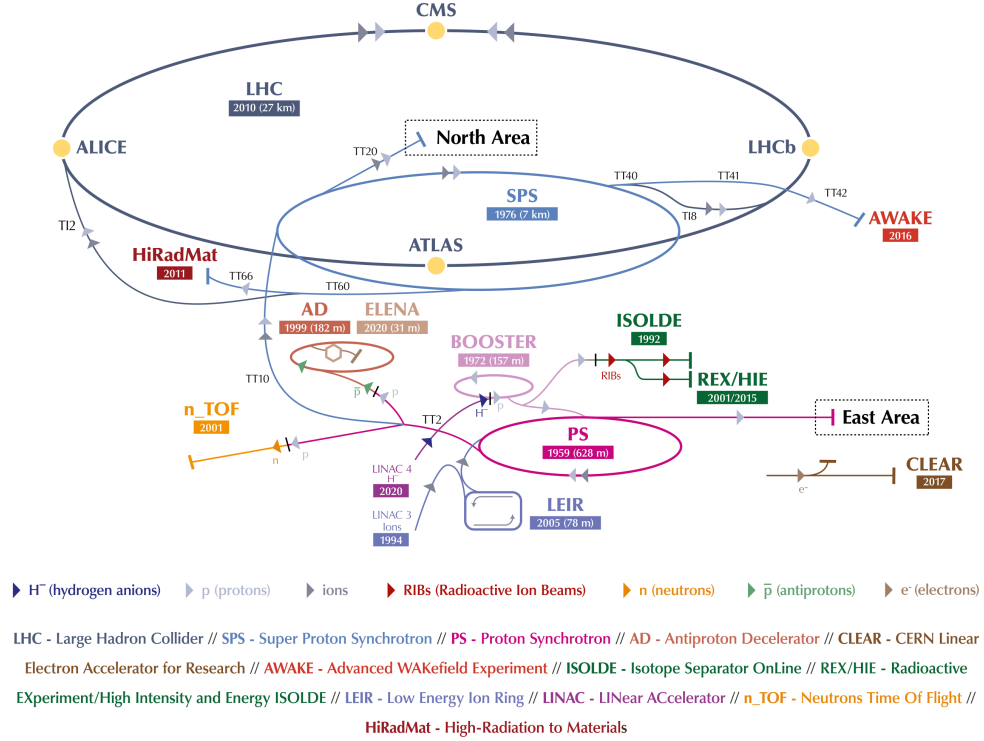


Figure 2.1: The CERN accelerator complex. Protons are accelerated by LINAC 4 (formerly LINAC 2) into the Proton Synchrotron (PS) Booster. They are then injected into the PS, SPS, and finally the LHC at increasingly higher beam energies. The LHC accelerates each beam to 6.5 TeV for collisions at a 13 TeV center-of-mass energy.

2.1 The Large Hadron Collider

The CERN Large Hadron Collider [19] is a circular accelerator, 27 km in circumference and 100m underground, straddling the border of France and Switzerland. Constructed from 1998-2008 by re-purposing the tunnel used by the Large Electron-Positron (LEP) Collider, the LHC consists of two adjacent beam pipes surrounded by 470 metric tons of Nb-Ti superconducting magnets cooled to under

2 K. It can collide protons at a center-of-mass energy $\sqrt{s} = 13$ TeV (with planned upgrades to 14 TeV) or lead ions in p-Pb and Pb-Pb runs.

There are four main experiments operating at the LHC: ALICE (A Large Ion Collider Experiment), designed to study heavy ion collisions; LHCb (Large Hadron Collider Beauty), specializing in b-physics; and the two general purpose detectors ATLAS (A Toroidal LHC ApparatuS) and CMS (Compact Muon Solenoid).

The LHC accelerates protons to nearly the speed of light in bunches of 1.15×10^{11} protons each. A total of 2808 bunches are circulated with an orbital speed of $88.9 \mu s$ and orbital frequency of 11.2 kHz. Bunch crossings, the intersection of two beam lines coinciding with one of the experiments, occur every 25 ns at a collision rate of 40 MHz.

Protons, initially separated from H_2 gas, are accelerated by LINAC 4 (previously LINAC 2), into the Proton Synchrotron (PS) Booster. They are subsequently filled into the PS, Super Proton Synchrotron (SPS), and finally LHC at progressively higher energies. The LHC accelerates each beam to 6.5 TeV each before colliding them at 13 TeV (Figure 2.1).

2.2 The CMS Detector

Located 100m below the picturesque French village of Cessy, the CMS detector is positioned opposite ATLAS (which is located in Geneva, Switzerland) in the LHC ring at Point 5 (P5). CMS is cylindrical in shape, 21.6 m long, 14.6 m in diameter, and weighing 14,000 metric tons. It contains a superconducting solenoidal magnet, weighing 12,500 metric tons, comprised of Niobium Titanium coils operating at 18,000 A to produce a 3.8 T magnetic field.

CMS is subdivided into a barrel region ($|\eta| \leq 1.479$) and two endcaps ($|\eta| > 1.479$) (the coordinate system used by CMS is shown in Figure 2.2). Figure 2.3 shows the various subdetectors that comprise the CMS experiment. The detector can be opened periodically (Figure 2.4) to allow for maintenance and hardware upgrades.

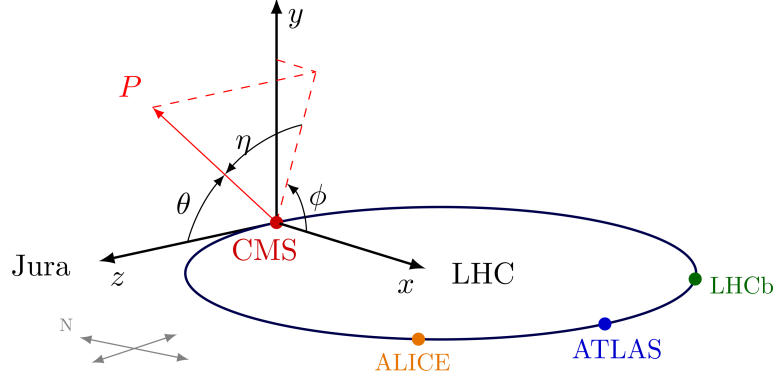


Figure 2.2: CMS coordinate system relative to LHC and surrounding region.

2.2.1 Subdetectors

The tracking system forms the innermost layer of CMS. It is comprised of 3 silicon pixel layers and 10 silicon microstrip layers. Information about the charge and position of particles can be ascertained by interactions with the tracker, which is designed to disturb incoming particles as little as possible. Measured positions are accurate to within $10 \mu m$.

The Electromagnetic Calorimeter (ECAL) is composed of lead tungstate ($PbWO_4$) crystals for both the absorber and scintillator. This material is transparent with high density, resulting in a short radiation length and rapid response time. Unlike the trackers which allow particles to pass through, the ECAL is de-

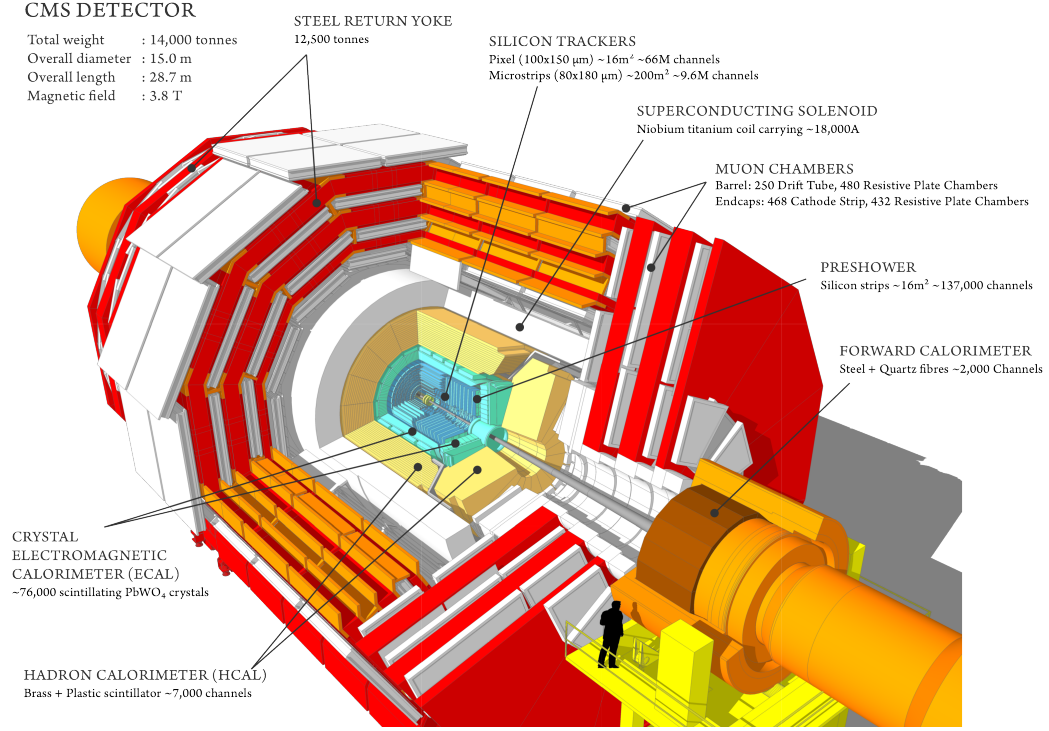


Figure 2.3: The CMS detector and various subdetector systems.

signed to fully absorb the energy from incoming electrons, positrons, and photons. Photons produced by the scintillators are detected by photodiodes and can be directly related to the energy of the incoming particle. There are separate ECAL systems for the barrel (EB, $|\eta| \leq 1.479$) and endcaps (EE, $1.479 \leq |\eta| \leq 3$).

The next layer is the Hadron Calorimeter (HCAL), designed to detect particles arising from quarks and gluons (e.g., protons, pions, jets produced by quark fragmentation). The HCAL is a sampling calorimeter composed of alternating layers of absorbers (made of brass or steel) and fluorescing PbWO_4 scintillators. These are used to ascertain a particle's energy and position. The HCAL layer is hermetic, meaning that all possible direct paths are accounted for. In addition to ensuring all hadron shower energy is accurately measured, this also helps measure

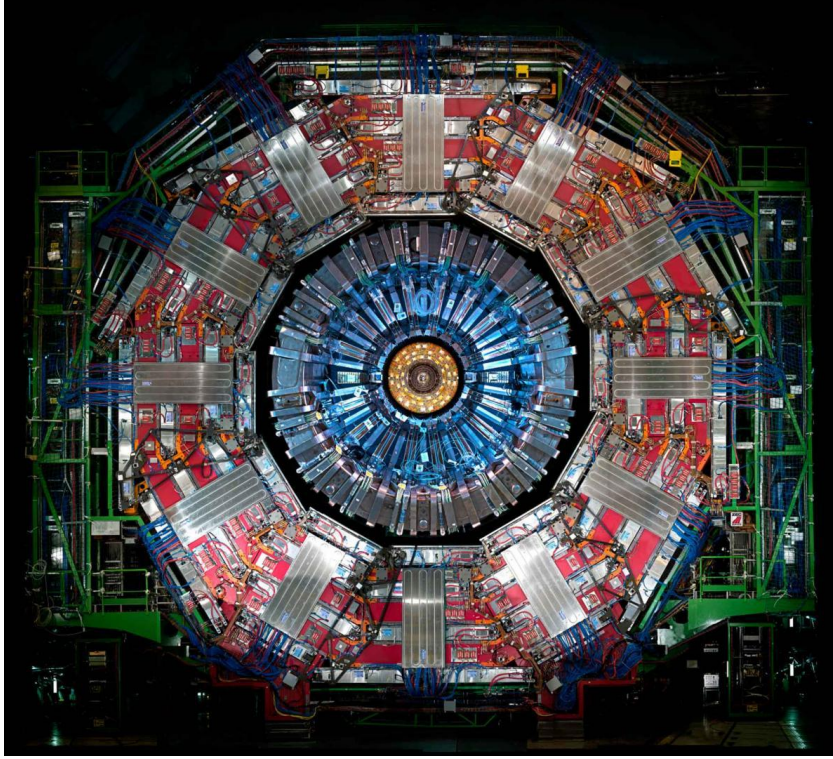


Figure 2.4: One endcap of the CMS detector.

the imbalance in transverse energy due to neutral particles such as neutrinos.

There are separate HCAL subdetectors for the barrel (HB), endcaps (HE), and forward (HF, $3 < |\eta| < 5$) regions. Much of the brass used in the HE scintillators was recycled from Russian WWII artillery shells. Hybrid photodiodes (HPDs) were originally used as photodetectors, but these were highly susceptible to radiation damage and were replaced with silicon photomultipliers (SiPMs) in HE/HB and multi-anode photomultiplier tubes (PMT) in HF as part of the Phase I Upgrade [30].

The final layer is the muon system, designed to detect the namesake particles which are not stopped by any of the calorimeter layers. A total of 1400 muon chambers are interspersed between iron return yoke plates. The barrel portion is

composed of 250 Drift Tubes (DTs) and 480 Resistive Plate Chambers (RPCs), while the endcaps are comprised of 468 Cathode Strip Chambers (CSCs) and 432 RPCs. The DTs and CSCs can precisely track a muon's position as it passes through multiple layers of the muon system and are designed to self-trigger on good muon candidates. The muon momentum can be ascertained based on how the track is bent by the magnetic field. The RPCs serve as an independent triggering system with a preference for higher p_T muons.

A transverse slice of CMS is shown Figure 2.5, demonstrating how different particle types interact with the detector.

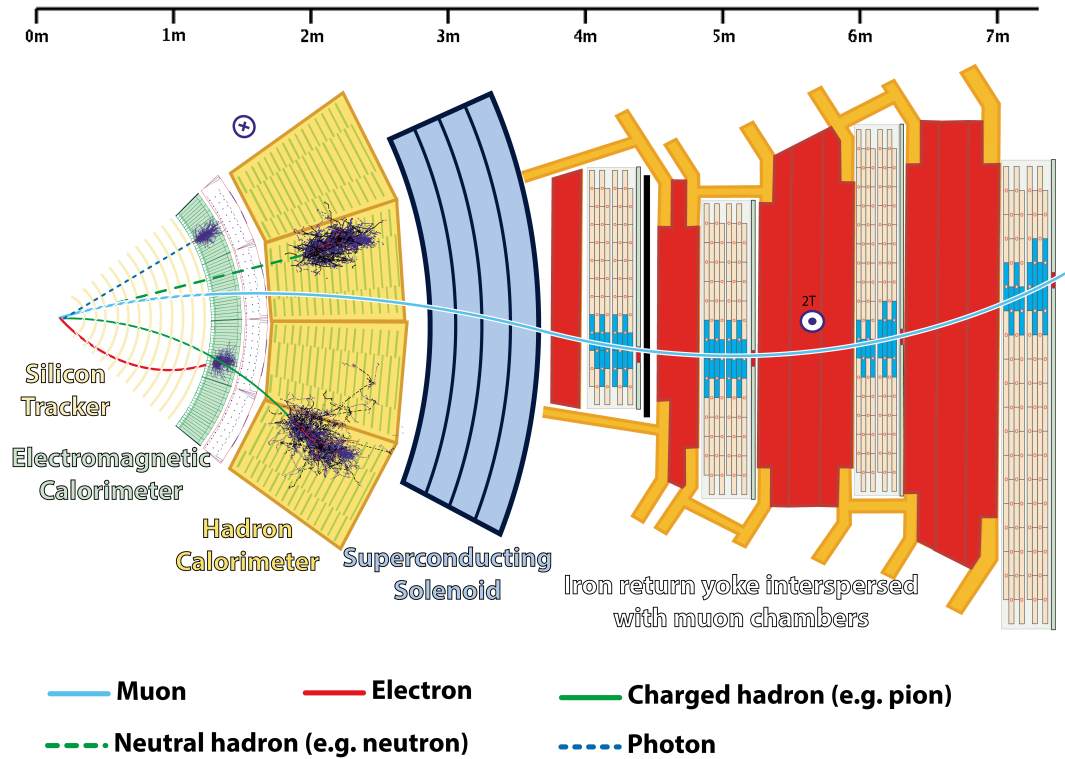


Figure 2.5: Transverse slice of the CMS detector. Particle types and trajectories are ascertained based on energy depositions in one or more subdetectors.

2.2.2 Triggering

The wealth of raw data collected by CMS is reduced to a manageable size through the implementation of hardware-level L1 triggers and software-level High Level Triggers (HLT).

L1 triggers [61] reduce the event rate from 40 MHz (the bunch crossing rate) down to 100 kHz. Data from the front-end (i.e., on-detector) electronics is sent to an underground service cavern for processing while the event data is buffered. If the event is accepted, an L1A (L1 accept) is initiated and the event data is read out from the entire detector.

High Level Triggers [50] are a software-based method for identifying interesting events (e.g., $e\mu$ candidates) using information from the full detector. The HLT system reduces the event rate from the 100 kHz L1 trigger output to 100 Hz for long term storage on tape or disk.

Chapter 3

Event Reconstruction

Proton-proton collisions produce a myriad of possible final states. The wealth of data collected by CMS must be reconstructed into particle objects with a definite charge, trajectory, and momentum before any physics analysis can be performed. Figure 3.1 shows one reconstructed event from 2016 collision data. CMS uses the Particle Flow (PF) algorithm [62] to incorporate information from all subdetectors to reconstruct the constituent particles. The method is validated on simulation before being applied to real data events.

3.1 Particle Flow

The Particle Flow algorithm relies on information from multiple subdetectors to reconstruct all stable particles produced during a collision event. Tracks from the silicon tracker and clusters from the calorimeters are referred to as *elements*. Individual elements from different subdetectors are linked to form *blocks*, which can be interpreted as particles. These particles can be used to build higher-level objects

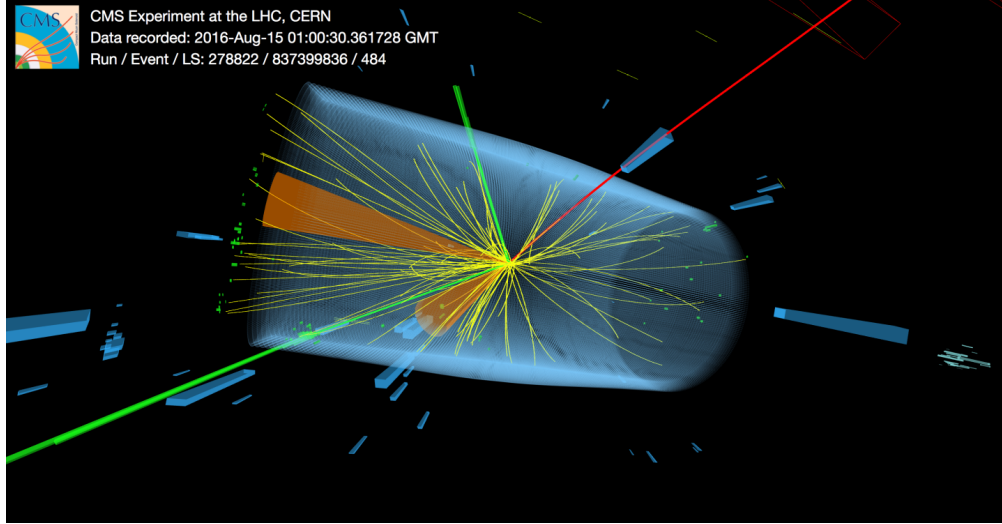


Figure 3.1: CMS event display showing the products of a pp collision. Calorimeter clusters are denoted by rectangular blocks (Green: ECAL, Blue: HCAL) with size proportional to the energy deposited. Reconstructed tracks of the decay products are color coded: green for electrons, red for muons, and yellow for various charged and neutral hadrons. Jet clusters are shown as yellow cones.

such as jets or tau leptons. A single particle is expected to produce multiple elements during reconstruction which must be accounted for to avoid reconstructing spurious objects.

Track elements for charged particles are found using an iterative process based on Kalman Filtering: an algorithm that estimates a joint probability distribution for several unknown variables using several measurements over time. An initial seed compatible with a charged particle trajectory is found, then hits from all track layers along the trajectory are added, and a final fitting step is performed to ascertain the direction, transverse momentum, and origin. This is repeated several times to maximize purity and minimize the misidentification rate.

Part of the track reconstruction process involves testing the compatibility of the track to have originated from the primary vertex. The vertex with the highest

summed transverse momentum is assumed to be the primary pp interaction vertex, with others classified as pileup vertices [25].

Calorimeter energy depositions are clustered in each ECAL and HCAL subdetector separately. A topological cluster is formed from a seed crystal, representing the local maximum energy above a certain threshold (0.23 GeV for ECAL and 0.8 GeV for HCAL). Adjacent crystals to the seed are clustered if they have energy in excess of twice the noise level.

Elements are geometrically linked in $\eta - \phi$ to form blocks that are interpreted as particles. As each object in the event is reconstructed, those blocks are then removed. Particles such as photons and neutral hadrons have links in the ECAL and HCAL, but not the tracker. Electrons would be linked in the tracker and ECAL, but not HCAL. (Global) muons are linked in the inner tracker and the muon system.

3.2 Muons

Muons have a distinct signature in the detector compared to other particle types. Their relatively large mass (105.7 MeV, 200x the electron mass) allows them to easily pass through the calorimeter system while depositing minimal energy, an example of a minimally ionizing particle (mip). Since they are charged, muons can interact with the inner tracker before registering hits in one or more muon stations.

The muons used for this analysis are tagged as Global Muons, for which the muon system hits are associated with tracker hits which allows the full path of the muon to be reconstructed [58]. Muons are also required to be PF isolated within a cone of $\Delta R = 0.4$. In case there are any calorimeter clusters overlapping with the

muon track which could be misidentified as charged hadrons, these are removed from consideration by PF.

3.3 Electrons and Photons

Electrons [48] are identified by matching inner tracker hits to ECAL clusters without corresponding HCAL clusters. The incident electron shower deposits energy in several neighboring crystals (Figure 3.2). The seed crystal, the local energy maximum above a specified threshold, forms the center of the cluster. Nearby crystals are scanned in η and ϕ to form 3x3 and 5x5 clusters which contain 94% and 97% of the incident energy, respectively. This is known as the Island Algorithm. Nearby non-overlapping Island clusters can be grouped into superclusters in a narrow η and broader ϕ window.

For electrons a distinction is made between η_{SC} , the supercluster η measured from the origin of the CMS detector coordinate system to the center of the supercluster, and the particle pseudorapidity η measuring the electron direction relative to the primary vertex. The former is used in electron selection efficiencies and for evaluating corrections to the electron energy, while the latter is used to compute four-momentum vectors used in the leptonic observables.

Photons [49] are also identified by their ECAL shower clusters, but since they are electrically neutral they leave no signature in the tracking system.

3.4 Jets

High energy inelastic hadron collisions will result in parton scattering and Gluonsstrahlung radiation. QCD color confinement requires color-charged particles

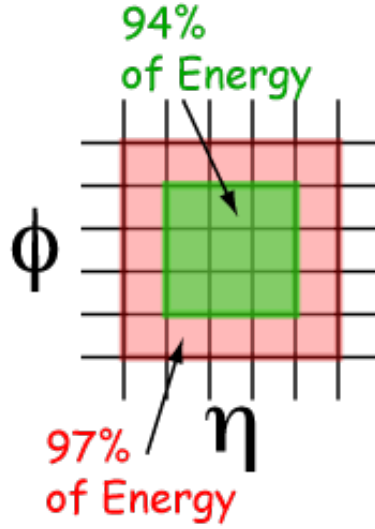


Figure 3.2: ECAL 3x3 and 5x5 cluster energies for an incident electron or photon. The seed crystal, the local energy maximum, is located in the center.

(quarks and gluons) be paired with other colored particles to form color singlet states (hadrons). These hadrons can also carry a significant fraction of the initial momentum. Unlike the electric potential that decreases like $1/r^2$, the QCD potential between two quarks increases linearly. As shown in Figure 3.3, the flux tube connecting two quarks is stretched until it is energetically favorable to produce a quark-antiquark pair between them. This process is called *hadronization* or *fragmentation*. The collection of unstable hadrons produced by this process is called a jet.

Jet constituents leave signatures in the tracker and primarily shower in the HCAL, though some energy can be deposited in ECAL and even reach the muon system. The hadronization products must be clustered to form a single jet object. CMS uses the sequential anti- k_t algorithm [22] to aggregate the individual partons into jet clusters. This method is infrared and collinear (IRC) safe, meaning soft emissions or splitting of the hardest parton into a nearby pair will not alter the

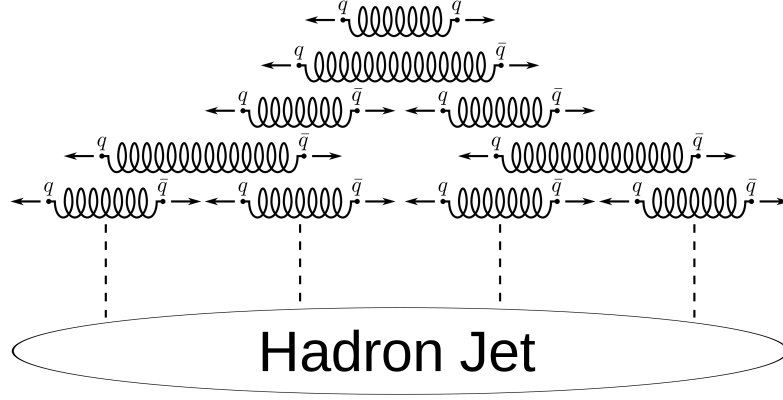


Figure 3.3: Confinement between two quarks of an energized unstable hadron leads to the production of multiple quark-antiquark pairs. The shower of particles produced from this process is called a jet.

outcome of the hard jet clustering.

The algorithm iterates over all constituents. For particle i it computes d_{ij} , the distance between entities (particles, pseudojets) i and j , and d_{iB} , the distance between i and the beam (Equation 3.1). Here k_{ti} , y_i , and ϕ_i are the transverse momentum, rapidity, and azimuthal angle of constituent i , respectively, and R specifies the clustering radius.

$$\begin{aligned}
 d_{ij} &= \min\left(\frac{1}{k_{ti}^2}, \frac{1}{k_{tj}^2}\right) \frac{\Delta_{ij}^2}{R^2}, \\
 d_{iB} &= \frac{1}{k_{ti}^2}, \\
 \Delta_{ij}^2 &= (y_i - y_j)^2 + (\phi_i - \phi_j)^2
 \end{aligned} \tag{3.1}$$

If $d_{ij} < d_{iB}$, then entities i and j are combined. Otherwise, entity i is classified as a jet and removed from the collection. The process is repeated until no entities remain.

This analysis uses AK4 jets, which are clustered using anti- k_t with a cone of

$R = 0.4$. An example of the anti- k_t algorithm (using $R = 1$) is shown in Figure 3.4.

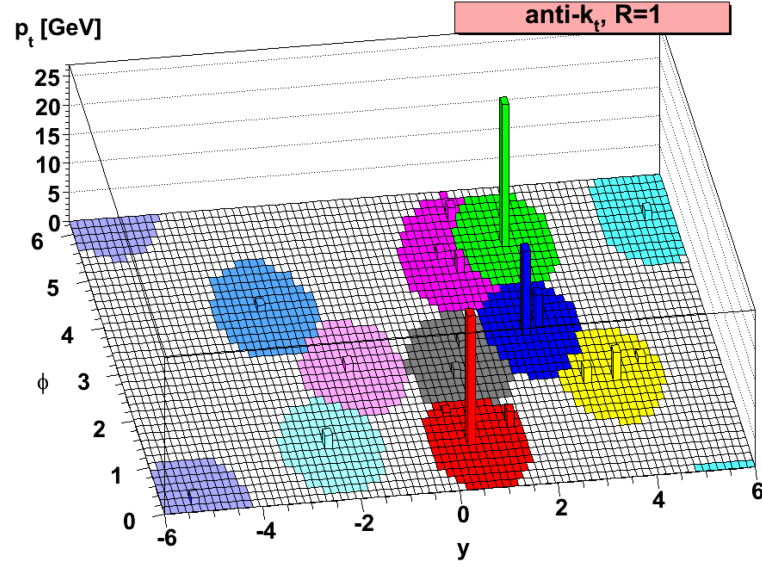


Figure 3.4: Example application of anti- k_t clustering algorithm with $\Delta R = 1$.

Chapter 4

Top Quark Mass Extraction

Many standard methods of top quark mass measurement rely on a full reconstruction of the $t\bar{t}$ system. By imposing four-momentum conservation, the sum of all decay product momenta can be set equal to the mother $t\bar{t}$ momentum. This approach, although widely used to achieve great m_t precision, has some drawbacks.

Firstly, the reconstruction relies heavily on precise knowledge of the energies of jets produced during the event as result of the hadronization of bottom quarks from $t \rightarrow Wb$ decay or hadronic decays of the W boson ($W \rightarrow q\bar{q}$). Additional jets from the underlying event or the non-primary interaction in a high pileup environment could also be mistakenly associated with the $t\bar{t}$ event. This results in a large dependence on the jet energy uncertainties as the dominant systematic uncertainty for these methods.

An additional complication arises for leptonically decaying W bosons ($W \rightarrow l\nu_l$). Since (anti)neutrinos cannot be detected by CMS, their presence is inferred through their absence in the form of Missing Transverse Energy (MET) $\cancel{E}_T$. For semi-leptonic decay modes, a single neutrino four-momentum can be resolved. However,

dilepton channels contain two neutrinos and, as such, this becomes an under-constrained problem. In practice, a matching procedure using the visible particles in the event can be used to assign the correct neutrino momenta with reasonably high efficiency, but this still relies on an accurate measurement of the b-jet energy.

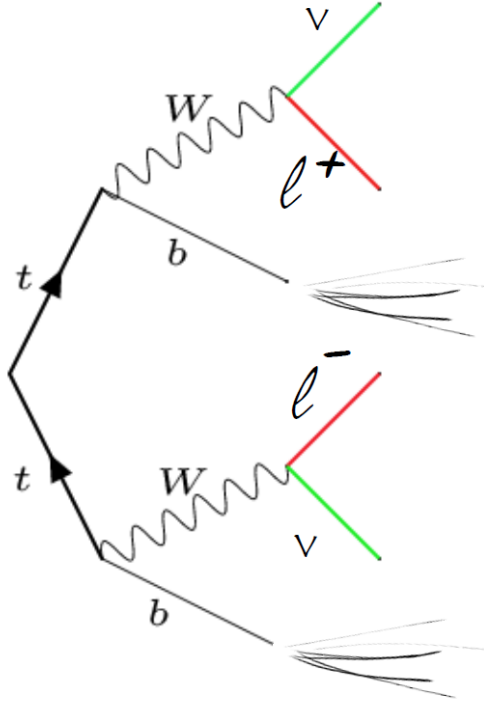


Figure 4.1: Dilepton final state of $t\bar{t}$ decay.

4.1 Leptonic Observables Method

The method of leptonic observables [41] relates the top quark mass, m_t , to the kinematic properties of the final state leptons produced by top quark decay. Figure 4.1 shows the leading order Feynman diagram for dileptonic top quark pair decay. One significant advantage of this alternate method is that the top quark

reconstruction is unnecessary.

A previous measurement using the leptonic observables method was performed using Run I data at $\sqrt{s} = 8$ TeV [2] which measured a top quark mass of $m_t = 171.7 \pm 1.1(\text{stat.}) \pm 0.5(\text{exp.})_{-3.1}^{+2.5}(\text{th.})_{-0.0}^{+0.8}(p_T(t))$ GeV. The measured uncertainties were significantly larger than the CMS Run I Combination result of $m_t = 172.44 \pm 0.13(\text{stat.}) \pm 0.47(\text{syst.})$ GeV[47]. This was largely due to the sensitivity to certain systematics such as mismodeling of the top quark p_T (see section 4.5.1.1) and QCD scale (see section 4.5.1.3), the latter exacerbated by the use of MC in Run I that was only accurate to LO. The use of NLO MC simulation in this analysis along with the implementation of a profile likelihood scan will be shown to significantly improve the m_t sensitivity compared to the 8 TeV measurement.

The top quark has a mean lifetime of $5 \times 10^{-25}\text{s}$, shorter than the hadronization timescale ($\Lambda_{\text{QCD}}^{-1} \sim 10^{-24}\text{s}$), and will immediately decay into a lighter quark through an electroweak process mitigated by a W boson. The mass of the intermediate W boson is known to within 0.02% (80.379 ± 0.012 GeV [55]) and the kinematics of leptons originating from W decays exhibit sensitivity to m_t .

The CMS detector has excellent lepton tracking and energy resolution, on the order of 1%, so the leptons can be reconstructed with extremely high accuracy. The b jets, produced by hadronization of the emitted b quarks, only serve to identify likely top decay candidate events and the MET due to the two neutrinos can be disregarded. The result is a method which can capitalize on high lepton resolution while mitigating the large uncertainties due to jet kinematics.

To optimize the signal purity and m_t sensitivity, an offline selection is applied to events reconstructed in the dilepton $e\mu$ channel. In principle, events from the ee and $\mu\mu$ final states could also be studied but these suffer significant contamination

from the primary background, the Drell-Yan process, particularly in the invariant mass range $M(l\bar{l}) \in [76, 106]$ GeV. This corresponds to a spread around the peak of Z boson mass $m_Z \pm 15$ GeV where $m_Z = 91.1876 \pm 0.0021$ GeV[55], as demonstrated in Figure 4.2.

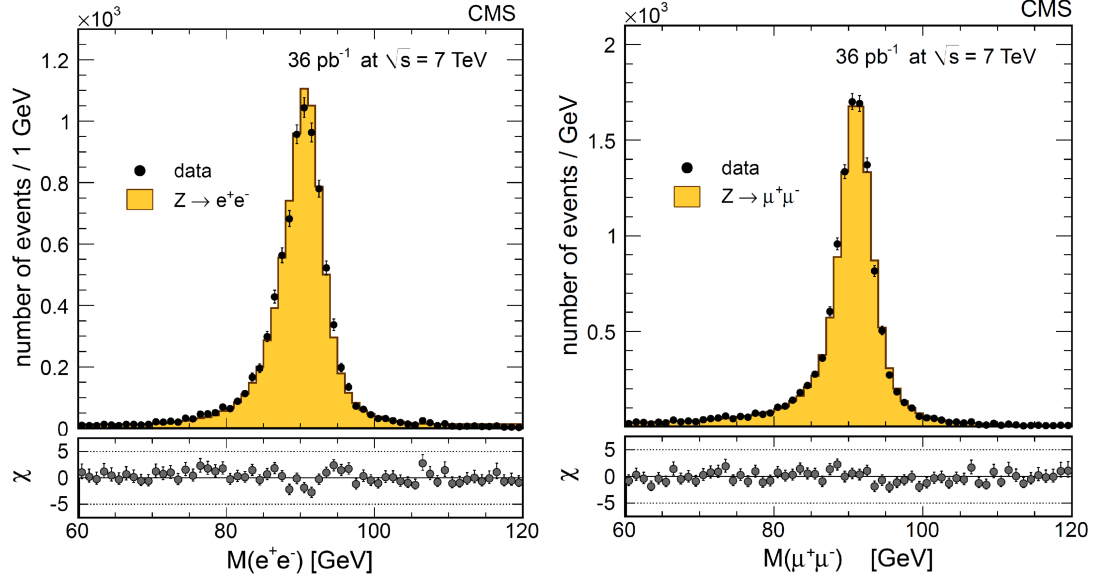


Figure 4.2: Invariant mass of dielectron and dimuon pairs produced from Z boson decay, measured by CMS at $\sqrt{s} = 7$ TeV [26].

4.2 Data and Simulation Samples

4.2.1 Data

This analysis uses data from the 2016 RunII period collected by the CMS experiment [24] at center-of-mass energy $\sqrt{s} = 13$ TeV, corresponding to a total integrated luminosity $\mathcal{L} = 35.9 \text{ fb}^{-1}$ [65]. The breakdown of datasets and integrated luminosity are given in Table 4.1.

Table 4.1: Data samples used in the analysis from the 2016 MuonEG primary datasets and the integrated luminosity collected during each run period, corresponding to a total integrated luminosity of 35.9 fb^{-1} .

Channel	Dataset	$\mathcal{L} \text{ (fb}^{-1}\text{)}$
Dilepton $e\mu$	Run2016B	5.75
	Run2016C	2.57
	Run2016D	4.24
	Run2016E	4.03
	Run2016F	3.10
	Run2016G	7.58
	Run2016H	8.65
Total		35.9

4.2.2 Monte Carlo Simulation

Signal and background processes are simulated using Monte Carlo (MC) generators to calculate S-matrix elements with QCD corrections accurate to next-to-leading order (NLO). These are then interfaced with a hadronizer and showering program to compute the quark fragmentation and approximate the effects of missing higher-order corrections. Finally, all samples are processed through a full simulation of

CMS in GEANT4 [3] to recreate the smearing effects caused by the detector.

Signal $t\bar{t}$ is simulated using POWHEG v2 [6] at Next-To-Leading (NLO) order in $t\bar{t} + 1$ jet [4] and showered using PYTHIA8 [13] with the CP5 tune [60]. The nominal sample is generated assuming $m_t = 172.5$ GeV and two alternate samples are produced at m_t hypotheses of 169.5 and 175.5 GeV.

Single top tW [56] and t -channel [5] processes are generated with POWHEG and showered in PYTHIA8 using the CUETP8M1 tune [63]. Single top s -channel is simulated with the aMC@NLO ME generator [7] and is showered with PYTHIA8 using the CUETP8M1 tune.

Other backgrounds include Diboson (WW, WZ, ZZ), Drell-Yan (DY), $t\bar{t} + V$ (TTZ, TTW), and W +Jets processes. Diboson samples are produced with POWHEG [53, 54]; TTZ with aMC@NLO; TTW with aMC@NLO, MADSPIN for spin correlations [12], and FFX [40] jet matching; W +Jets with aMC@NLO and FFX; and DY with aMC@NLO and FFX. These samples are showered in PYTHIA8 with the CUETP8M1 tune.

Simulation event yields are normalized to the total integrated luminosity and theoretical cross sections weighted by branching ratio (BR). The signal $t\bar{t}$ cross section is calculated at next-to-next-to-leading order (NNLO) in perturbative QCD and next-to-next-to-leading-log order (NNLL) in soft-gluon resummation using Top++2.0 [34]. Single top cross sections are calculated at NLO + NNLL accuracy [51]. Background sample cross sections are calculated at NNLO for Drell-Yan and W +Jets and NLO for Diboson [23] processes.

The list of nominal MC simulation samples and their corresponding cross sections are given in Table 4.2. Alternate samples, used for different m_t hypotheses or the evaluation of certain systematic uncertainties, are given in Table 4.3.

Table 4.2: Nominal Monte Carlo simulation samples for signal and background processes with their corresponding cross sections weighted by branching ratio.

Process	Dataset	$\sigma(pb) * BR$
tt	/TTTo2L2Nu_TuneCP5_PSweights_13TeV-powheg-pythia8	87.31
tt + V	/TTZToLLNuNu_M-10_TuneCUETP8M1_13TeV-amcatnlo-pythia8	0.253
	/TTWJetsToLNu_TuneCUETP8M1_13TeV-amcatnloFXFX-madspin-pythia8	0.2043
	/WWTo2L2Nu_13TeV-powheg	12.178
Diboson	/WZTo3LNu_TuneCUETP8M1_13TeV-powheg-pythia8	2.165
	/ZZTo2L2Nu_13TeV-powheg-pythia8	0.414
	/ST_s-channel_4f_leptonDecays_13TeV-amcatnlo-pythia8_TuneCUETP8M1	3.4
Single top	/ST_tW_top_5f_NoFullyHadronicDecays_13TeV-powheg_TuneCUETP8M1	19.467
	/ST_tW_antitop_5f_NoFullyHadronicDecays_13TeV-powheg_TuneCUETP8M1	19.467
	/ST_t-channel_top_4f_inclusiveDecays_13TeV-powhegV2-madspin-pythia8_TuneCUETP8M1	136.02
	/ST_t-channel_antitop_4f_inclusiveDecays_13TeV-powhegV2-madspin-pythia8_TuneCUETP8M1	80.95
	/WJetsToLNu_TuneCUETP8M1_13TeV-amcatnloFXFX-pythia8	61526.7
W + jets	/DYJetsToLL_M-50_TuneCUETP8M1_13TeV-amcatnloFXFX-pythia8	6077.22
Drell-Yan	/DYJetsToLL_M-10to50_TuneCUETP8M1_13TeV-amcatnloFXFX-pythia8	18610

Table 4.3: Monte Carlo simulation samples of systematic variations on signal processes with their corresponding cross sections weighted by branching ratio.

Systematic	Process	Dataset	$\sigma(pb) * BR$
$m_t = 169.5$ GeV $m_t = 175.5$ GeV Underlying Event	tt	/TTTo2L2Nu_TuneCP5_PSWeights_mtop1695_13TeV-powheg-pythia8	94.79
		/TTTo2L2Nu_TuneCP5_PSWeights_mtop1755_13TeV-powheg-pythia8	80.51
	tt	/TTTo2L2Nu_TuneCP5up_PSWeights_13TeV-powheg-pythia8	87.31
		/TTTo2L2Nu_TuneCP5down_PSWeights_13TeV-powheg-pythia8	87.31
Color Reconnection	tt	/TTTo2L2Nu_TuneCP5_PSWeights_erdON_13TeV-powheg-pythia8	87.31
		/TTTo2L2Nu_TuneCP5CR2_GluonMove_PSWeights_13TeV-powheg-pythia8	87.31
	tt	/TTTo2L2Nu_TuneCP5CR1_QCDbased_PSWeights_13TeV-powheg-pythia8	87.31
		/TTTo2L2Nu_hdampUP_TuneCP5_PSWeights_13TeV-powheg-pythia8	87.31
hdamp	tt	/TTTo2L2Nu_hdampDOWN_TuneCP5_PSWeights_13TeV-powheg-pythia8	87.31
		/TTJets_TuneCUETP8M2T4_13TeV-ancatnloFFFX-pythia8	831.76
	tt	/TTJets_TuneCUETP8M1_13TeV-madgraphMLM-pythia8	831.76
		/TT_TuneEE5C_13TeV-powheg-herwigpp	831.76
Alternate MC Generators	tW top	/ST_tW_top_5f_mtop1695_NoFullyHadronicDecays_13TeV-powheg-pythia8	20.4
		/ST_tW_top_5f_mtop1755_NoFullyHadronicDecays_13TeV-powheg-pythia8	18.6
	tW antitop	/ST_tW_antitop_5f_mtop1695_NoFullyHadronicDecays_13TeV-powheg-pythia8	20.4
		/ST_tW_antitop_5f_mtop1755_NoFullyHadronicDecays_13TeV-powheg-pythia8	18.6
ISR	tW top	/ST_tW_top_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
		/ST_tW_top_isrump_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
	tW antitop	/ST_tW_antitop_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
		/ST_tW_antitop_isrump_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
FSR	tW top	/ST_tW_top_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
		/ST_tW_top_isrump_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
	tW antitop	/ST_tW_antitop_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
		/ST_tW_antitop_isrump_5f_isrump_NoFullyHadronicDecays_13TeV-powheg	19.467
tt/tW interference	tW top	/ST_tW_top_5f_DS_NoFullyHadronicDecays_13TeV-powheg-pythia8_TuneCUETP8M1	19.467
		/ST_tW_top_5f_DS_NoFullyHadronicDecays_13TeV-powheg-pythia8_TuneCUETP8M1	19.467
	tW antitop	/ST_tW_antitop_5f_DS_NoFullyHadronicDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
		/ST_tW_antitop_5f_DS_NoFullyHadronicDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
QCD Scale	tW top	/ST_tW_top_5f_scaledown_inclusiveDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
		/ST_tW_top_5f_scaledown_inclusiveDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
	tW antitop	/ST_tW_antitop_5f_scaledown_inclusiveDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
		/ST_tW_antitop_5f_scaledown_inclusiveDecays_13TeV-powheg-pythia8_TuneCUETP8M1	35.85
hdamp	tW top	/ST_tW_top_5f_hdampup_NoFullyHadronicDecays_13TeV_PSWeights-powheg-pythia8_TuneCUETP8M1	19.467
		/ST_tW_top_5f_hdampdown_NoFullyHadronicDecays_13TeV_PSWeights-powheg-pythia8_TuneCUETP8M1	19.467
	tW antitop	/ST_tW_antitop_5f_hdampup_NoFullyHadronicDecays_13TeV_PSWeights-powheg-pythia8_TuneCUETP8M1	19.467
		/ST_tW_antitop_5f_hdampdown_NoFullyHadronicDecays_13TeV_PSWeights-powheg-pythia8_TuneCUETP8M1	19.467

4.3 Selection Criteria

Data and MC samples are centrally processed into the NanoAOD [52] flat ntuple format. Events are required to have at least one primary vertex and pass one of the dilepton $e\mu$ High-Level Triggers (HLT). Additional requirements for electron and muon candidates are listed in sections 4.3.2.2 and 4.3.2.3, respectively. The two leading leptons, ordered by decreasing p_T , should be an oppositely charged $e\mu$ pair with invariant mass $M_{ll} > 20$ GeV. The event should also contain ≥ 2 jets and ≥ 1 b-tagged jets. An example $e\mu$ event candidate and corresponding reconstructed objects is shown in Figure 4.3.

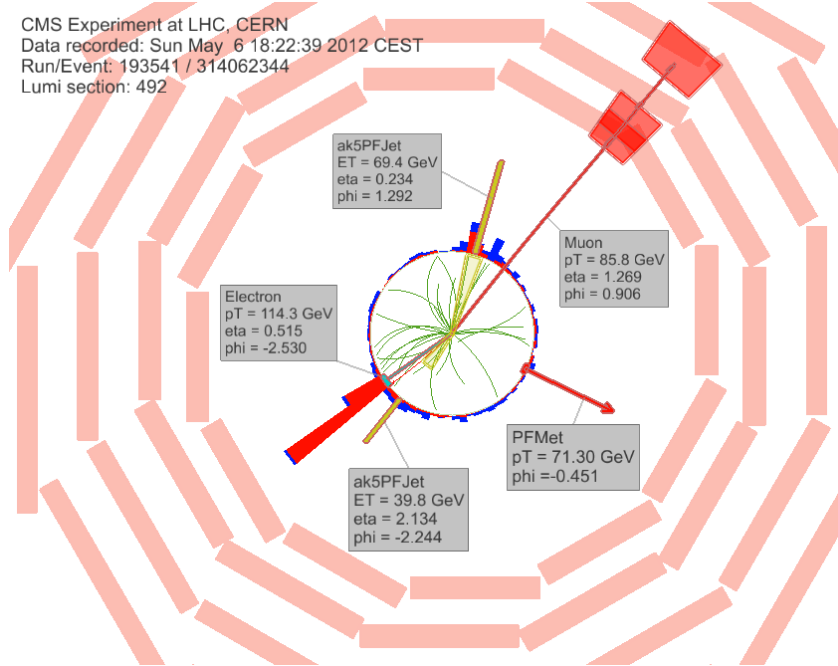


Figure 4.3: $t\bar{t}$ dilepton $e\mu$ channel event candidate.

4.3.1 Trigger Selection

High-Level Triggers to identify dilepton $e\mu$ candidate events are applied to data and simulation. A logical OR of all triggers given in Table 4.4 is used.

Table 4.4: High Level Triggers for MC and Data. The logical OR of all available triggers is used.

	Trigger Path
Data	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ_v*
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_v*
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ_v*
	HLT_Mu12_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_v*
	HLT_Mu12_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ_v*
MC	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_v9
	HLT_Mu23_TrkIsoVVL_Ele12_CaloIdL_TrackIdL_IsoVL_DZ_v4
	HLT_Mu8_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_v9
	HLT_Mu12_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_v3
	HLT_Mu12_TrkIsoVVL_Ele23_CaloIdL_TrackIdL_IsoVL_DZ_v4

4.3.2 Object Selection

4.3.2.1 Primary Vertex

The interaction point between particles in a collision is called a vertex. The CMS tracker [28] is used to reconstruct the position of the primary vertex (the pp interaction point), as well as additional secondary vertices used in the identification of jets originating from b quarks. Primary vertices are required to pass the following tracker selection:

- Number of degrees of freedom > 4
- Track transverse position $d_0 < 2 \text{ cm}$

- Longitudinal vertex position $z < 24 \text{ cm}$

Tracks are weighted from 0 to 1 based on their compatibility with the vertex. If a track is consistent with the vertex, its weight will be close to 1. The degrees of freedom are then calculated with Equation 4.1, where w_i is the weight of track i . The requirement of $\text{Ndof} > 4$ implies that at least 4 well reconstructed tracks originate from this vertex [1].

$$\text{Ndof} = 2 \sum_{i=1}^{n\text{Tracks}} w_i - 3 \quad (4.1)$$

The transverse position d_0 is the distance to the beam axis in the transverse plane and the longitudinal position. The vertex with the largest summed p_T^2 is assumed to be the primary hard-scattering vertex.

4.3.2.2 Electrons

Electrons are required to pass a cut-based ID using the TightID working point (average efficiency $\sim 70\%$). Table 4.5 outlines the selection requirements, which differ depending on whether the electron was reconstructed in the barrel ($|\eta_{\text{SC}}| \leq 1.479$) or endcap ($|\eta_{\text{SC}}| > 1.479$). Here η_{SC} is the electron supercluster η . Additional cuts on the d_0 and d_z impact parameters are imposed. A veto is applied for electrons which fall in the EE/EB gap ($1.4442 < |\eta_{\text{SC}}| < 1.566$).

The cut parameters are summarized below:

- **full 5x5 $\sigma_{\text{in}\eta\eta}$** The log energy weighted RMS of the electron shower in units of crystals (Eq 4.2). The sums are taken over all crystals in the supercluster. E_i is the energy of a single crystal, while $E_{5 \times 5}$ is the energy of the entire supercluster.

$$\sigma_{i\eta i\eta} = \sqrt{\frac{\sum_i^{5 \times 5} w_i (\eta_i - \bar{\eta}_{5 \times 5})^2}{\sum_i^{5 \times 5} w_i}} \quad (4.2)$$

$$w_i = \max(0, 4.7 + \ln \frac{E_i}{E_{5 \times 5}}) \quad (4.3)$$

- $|\Delta\eta_{\text{in}}^{\text{seed}}|$ Difference between the supercluster η of the seed crystal and the extrapolated track from the pixel tracker, accounting for the B-field.
- $|\Delta\phi_{\text{in}}|$ Difference between the azimuthal angle of the seed crystal and the extrapolated track from the pixel tracker, accounting for the B-field.
- **H/E** Ratio of hadronic energy to em (supercluster) energy. H is the total HCAL energy in a cone $\Delta R < 0.15$ from the supercluster position.
- **Rel. comb. PF iso with EA corr** Particle Flow based isolation to remove contributions from charged hadrons, neutral hadrons, photons, and pileup (Eq. 4.4) in a cone of $\Delta R < 0.3$ around the electron.

$$iso = \frac{\text{IsoCh} + \max(0, \text{IsoNh} + \text{IsoPh} - \text{PU})}{p_T} \quad (4.4)$$

$$PU = \rho * EA \quad (4.5)$$

The summed transverse energies of the charge hadrons, neutral hadrons, and photons in the cone are given by IsoCh, IsoNh, and IsoPh respectively. These are divided by the electron p_T . The pileup term is the product of ρ , the event-specific average pileup energy density per unit area in the $\eta - \phi$

plane, and the effective area constants which are computed for each dataset.

- **expected missing inner hits** Electron calorimeter deposit has a compatible track from the inner tracker
- **pass conversion veto** Electron track is compatible with the primary vertex to rule out electrons produced by photon showering
- **Impact parameters** d_0 , d_z The transverse ($d_0 = \sqrt{d_x^2 + d_y^2}$) and longitudinal (d_z) distances to the vertex at the point of closest approach in the transverse plane.

Electrons passing the TightID selection are also required to have $p_T > 20$ GeV and $|\eta| < 2.4$. Kinematic distributions for selected electrons are shown in Figure 4.4.

Table 4.5: Offline selection criteria for impact parameters and TightID working point of the 2016 Electron Cut-based ID. Different cuts are applied for electrons reconstructed in the endcap and barrel.

Cut	Barrel ($ \eta_{SC} \leq 1.479$)	Endcap ($ \eta_{SC} > 1.479$)
full 5x5 $\sigma_{i\eta i\eta} <$	0.00998	0.0292
abs(dEtaInSeed) <	0.00308	0.00605
abs(dPhiIn) <	0.0816	0.0394
H/E <	0.0414	0.0641
Rel. comb. PF iso EA corr <	0.0588	0.0571
abs(1/E - 1/p) <	0.0129	0.0129
expected missing inner hits $\leq$	1	1
pass conversion veto	yes	yes
d_0 (cm) <	0.05	0.10
d_z (cm) <	0.10	0.20

4.3.2.3 Muons

Muons are required to pass a cut-based ID with the TightID working point, detailed in Table 4.6. An additional Particle-Flow based muon relative isolation $\Delta\beta$ requirement is imposed in a cone of $\Delta R < 0.4$ where

$$\Delta\beta = \frac{\sum p_T^{ch.had.fromPV} + \max(0, \sum E_T^{neut.had.} + \sum E_T^{photon} - 0.5 \sum p_T^{ch.had.fromPU})}{p_T(\mu)} \quad (4.6)$$

This equation considers the contributions of summed p_T of charged hadrons from the Primary Vertex and Pileup interactions and the summed E_T of neutral hadrons and photons. Kinematic distributions for selected muons are shown in Figure 4.5.

Table 4.6: Offline selection criteria for PF relative isolation and TightID working point of the 2016 Muon Cut-based ID.

Cut	Value
reconstructed Global Muon	yes
Particle-Flow Muon ID	yes
$\chi^2/ndof$ of global-muon track fit	< 10
muon-chamber hit included in global-muon track fit	> 0
muon segments in at least two muon stations	> 1
transverse impact parameter d_0 (mm)	< 0.2
longitudinal impact parameter d_z (mm)	< 5
number of pixel hits	> 0
cut on number of tracker layers with hits	> 5
Particle-Flow Muon Relative Isolation $\Delta\beta$	< 0.15

4.3.2.4 Jets

At least two jets are required in the event. PF CHS jets are clustered using the anti- k_T algorithm [22] with a cone of $R = 0.4$ and referred to as AK4 jets. Here, CHS (Charged Hadron Subtraction) refers to the process of subtracting charged particles from the non-primary vertex (i.e., from pileup interactions) prior to clustering. A veto is applied for jets which coincide with a reconstructed lepton within a cone of $\Delta R < 0.4$.

Jets are required to have $p_T > 30$ GeV, $|\eta| < 2.4$, and pass the Loose ID requirements outlined in Table 4.7. Distributions of the p_T and multiplicity (number of jets per event) passing the selection are shown in Figure 4.6.

Table 4.7: Offline selection criteria for 2016 AK4 jet Loose ID.

Cut	Value
Neutral hadron fraction	< 0.99
Neutral EM fraction	< 0.99
Number of constituents	> 1
Charged hadron fraction	> 0
Charged multiplicity	> 0
Charged EM fraction	< 0.99

4.3.2.5 B-tagged Jets

The presence of b quarks indicates that a high energy interaction (such as top quark production and decay) has occurred. Hadrons produced during b quark fragmentation have a sufficient lifetime to travel some distance (up to 1cm) within the detector before decaying, as shown in Figure 4.7. This *secondary vertex* helps to identify b jets in a process known as b-tagging. Many algorithms exist for the

classification of b, c, or light (those originating from gluons or up, down, or strange quarks) jets.

The b-tagger used for this analysis, DeepCSV [10], is based on the Combined Secondary Vertex algorithm and trained on a deep neural network comprised of four hidden layers, each containing 100 nodes. The output is a single discriminator value which can be compared to established working points. This analysis requires at least one b-tagged jet using the DeepCSV Medium WP of 0.6321, which corresponds to a misidentification rate of 1% for classifying c,light as b jets.

An important note regarding terminology: a *b-tagged jet* refers to jets that have been identified by a b-tagger as originating from a b quark, whereas *b jet* is usually reserved to describe MC jets which are known to be produced from b fragmentation. In data, the parton that originated a particular jet is unknown and can only be identified with some degree of certainty. In MC simulation, the truth-level parton information is known and can be used to train b-tagging algorithms to establish efficiency and misidentification rates (see Figure 4.8).

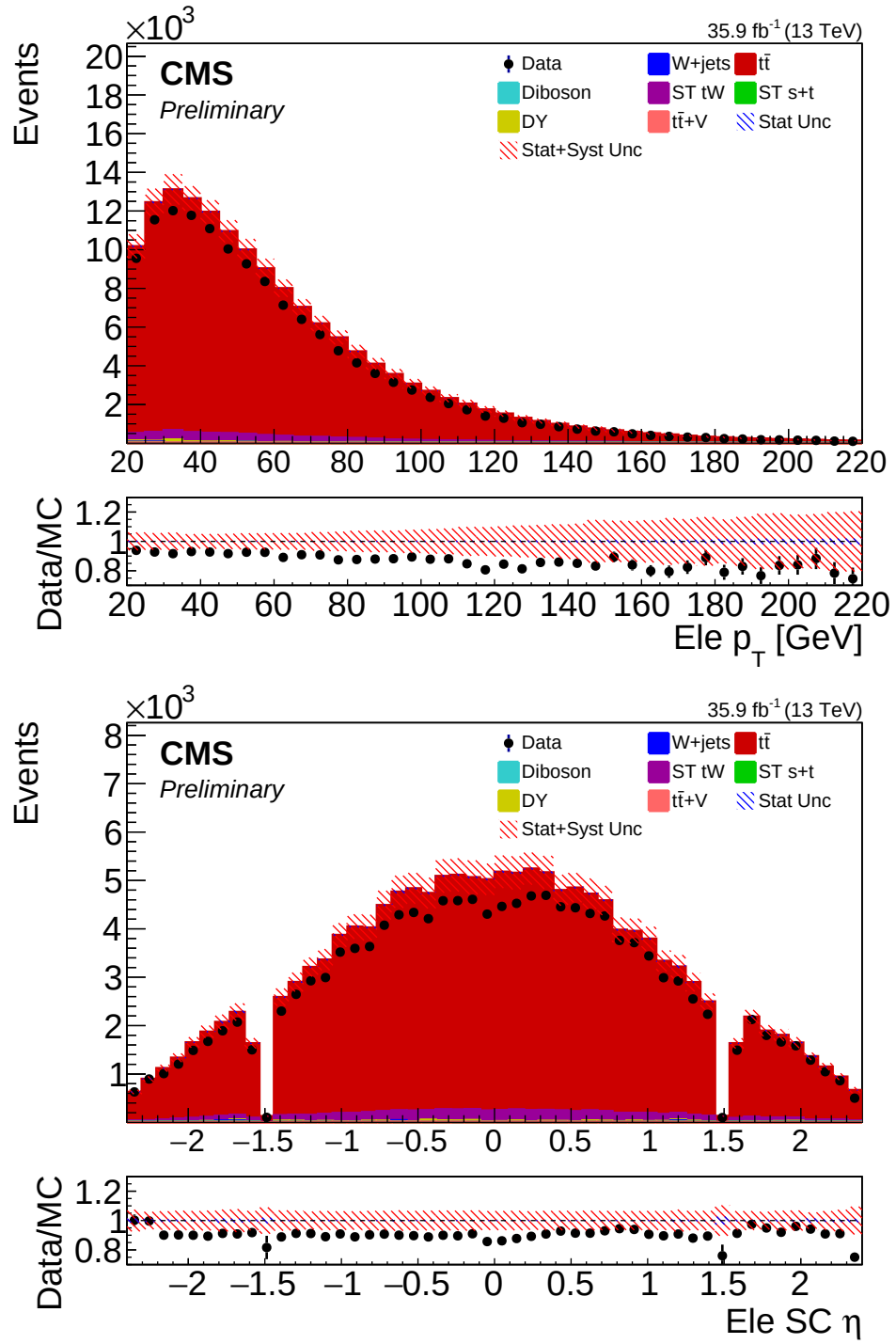


Figure 4.4: p_T and η_{SC} for selected electrons.

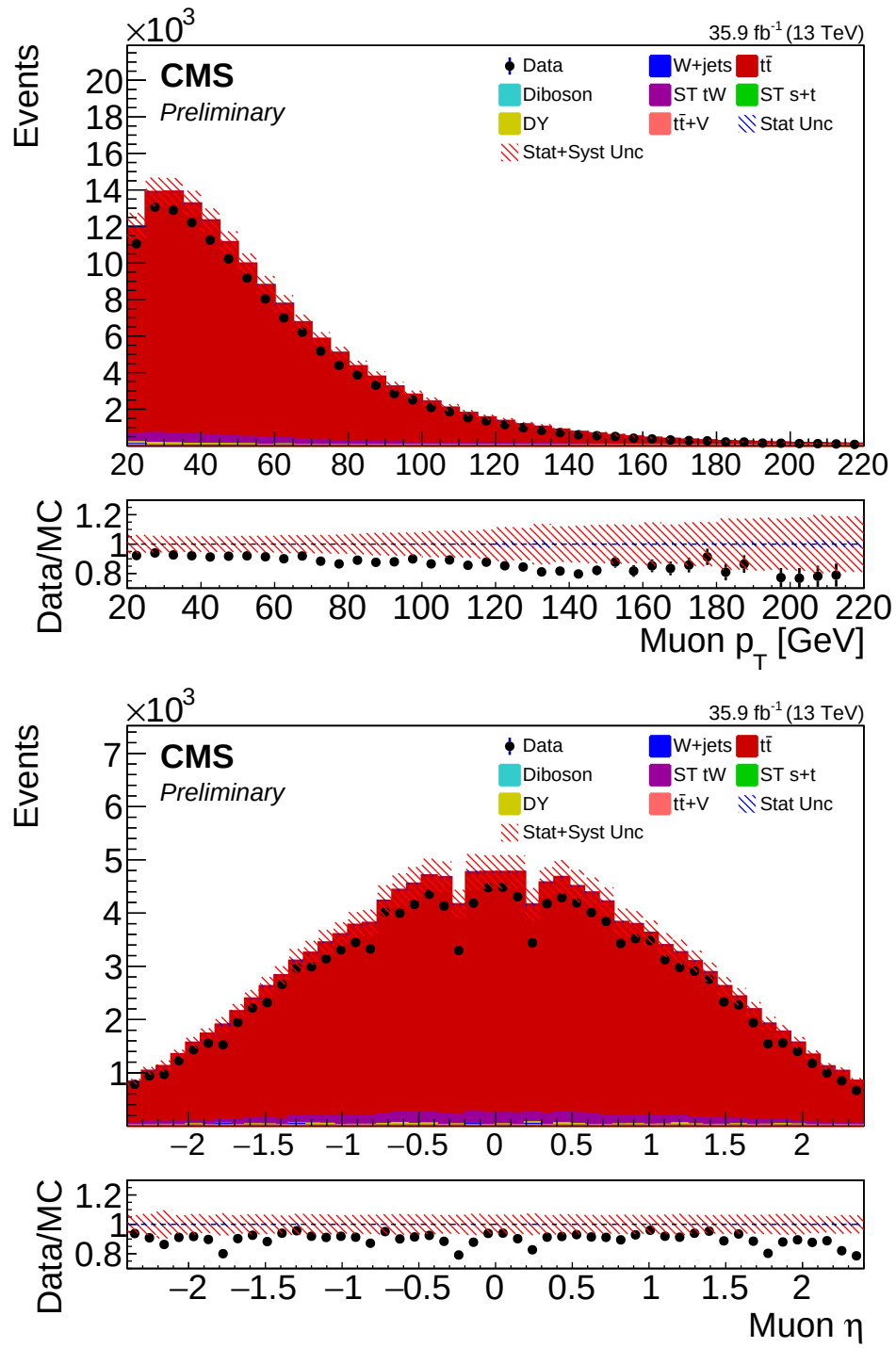


Figure 4.5: p_T and η for selected muons.

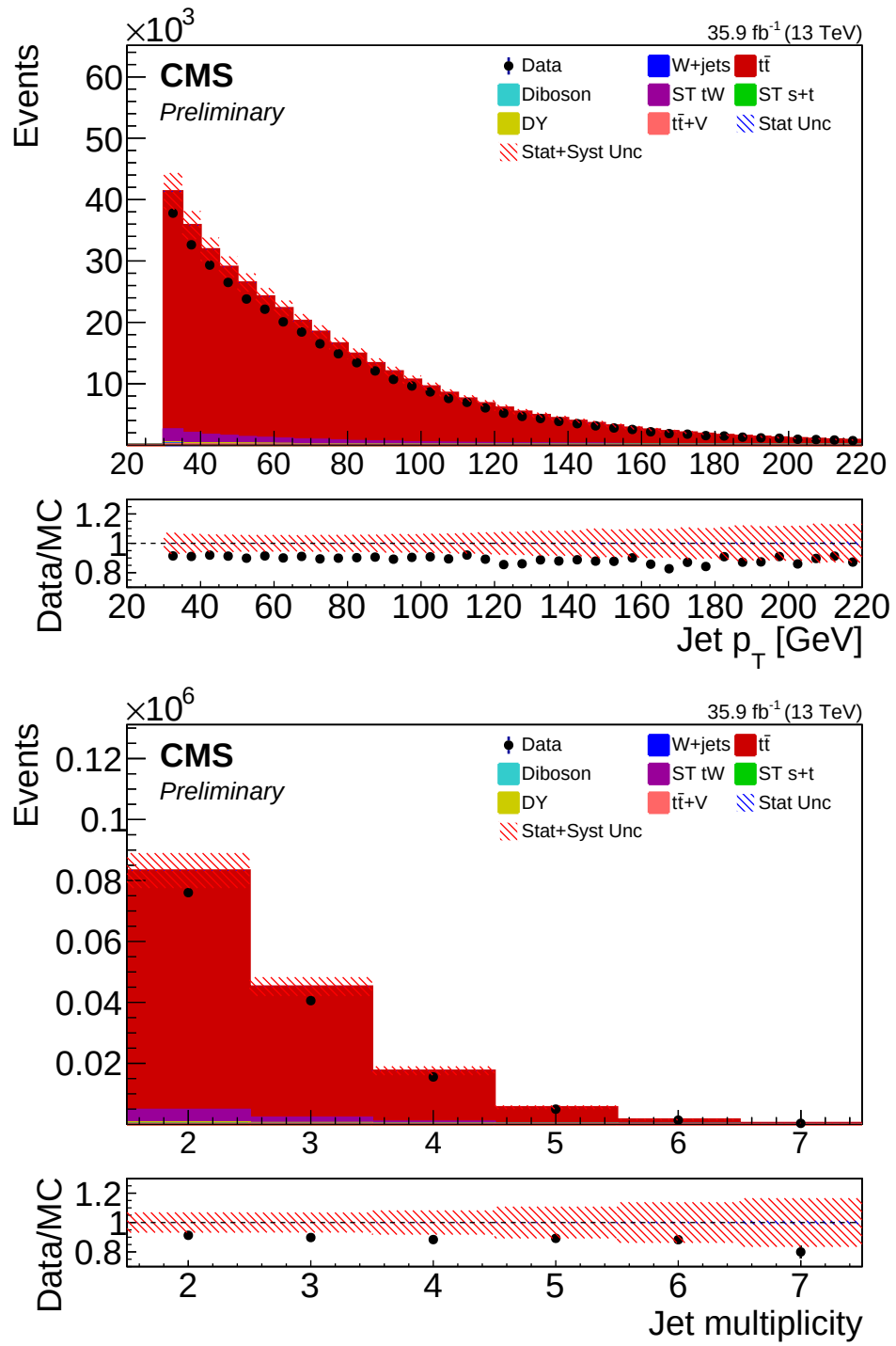


Figure 4.6: p_T and multiplicity for selected jets.

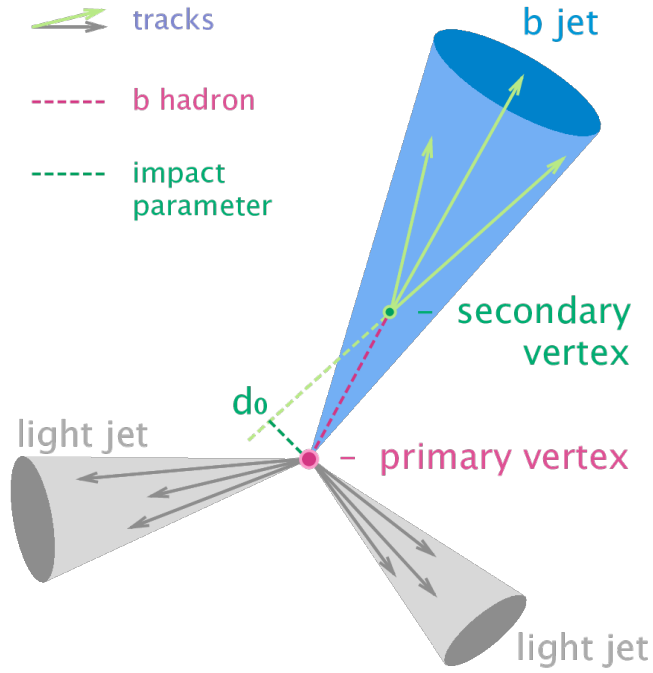


Figure 4.7: A defining feature of b jets is the secondary vertex, which is displaced several millimeters from the primary interaction vertex.

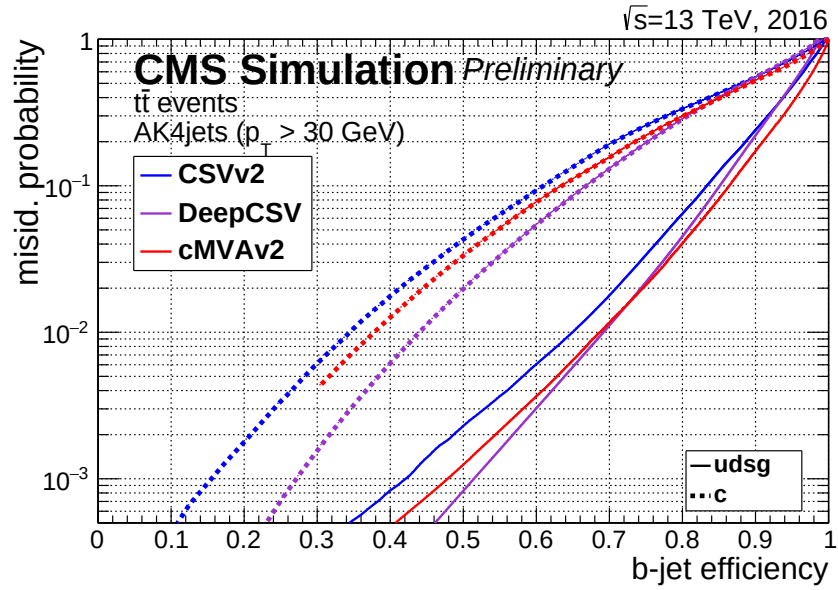


Figure 4.8: Misidentification probability and efficiency for several commonly used b-tagging algorithms in CMS.

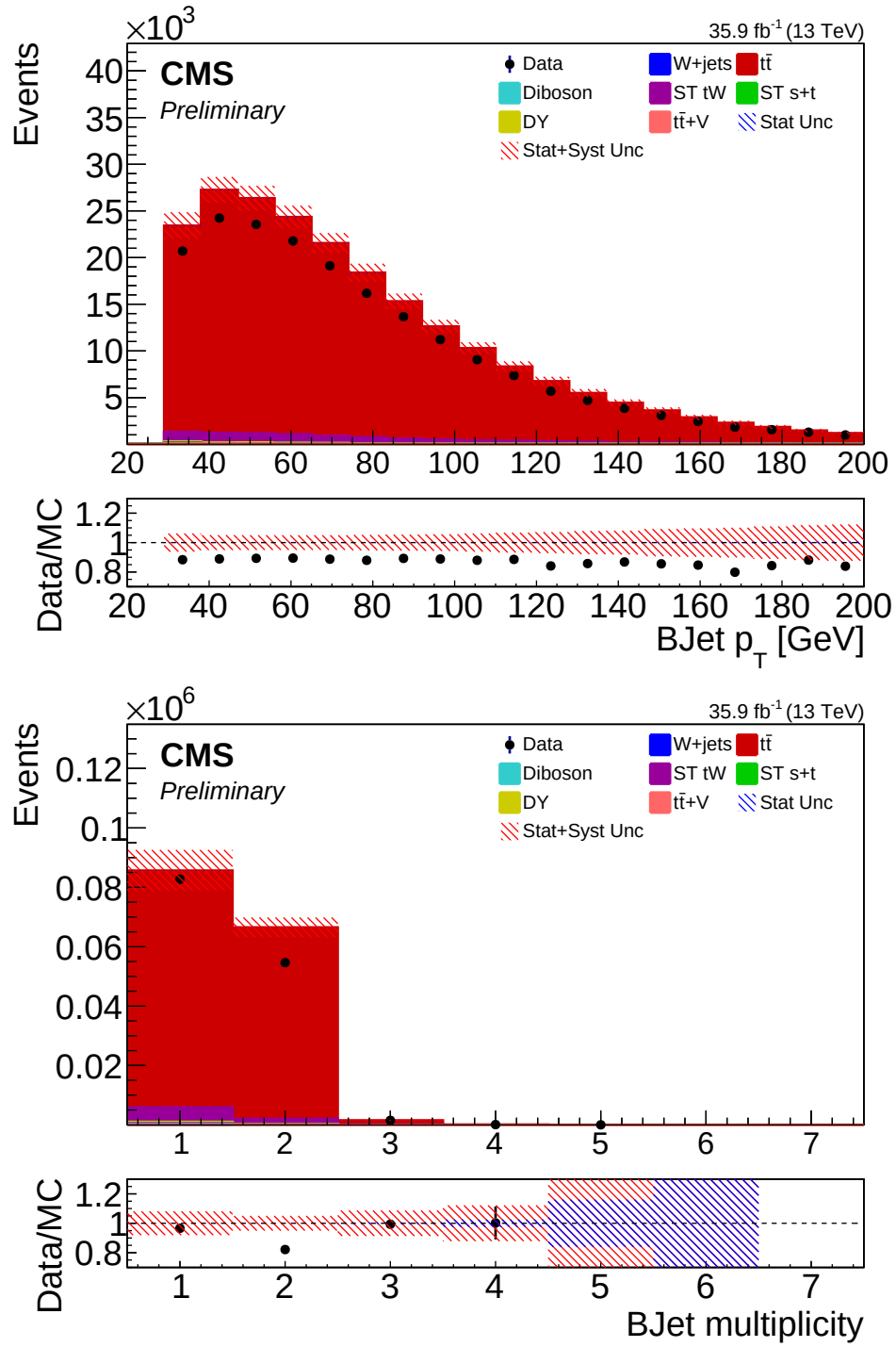


Figure 4.9: p_T and multiplicity for selected b-tagged jets.

4.3.3 Event Selection

The results at various stages of the cut flow are shown in Table 4.8 and the final selection is summarized below:

- Pass PV, HLT requirements
- At least one electron and one muon passing respective TightID cuts
- Two highest p_T leptons form oppositely charged $e\mu$ pair
 - Leading lepton $p_T > 25$ GeV, subleading lepton $p_T > 20$ GeV
 - Lepton $|\eta| < 2.4$
- Invariant mass of the lepton pair $M(l\bar{l}) > 20$ GeV
- ≥ 2 Loose ID jets with $p_T > 30$ GeV, $|\eta| < 2.4$
 - Veto jets within $\Delta R < 0.4$ cone of leptons
- ≥ 1 passing jet is b-tagged with DeepCSV Medium WP

Table 4.8: Event yields after the full dilepton selection, jet selection, and b-tag requirements. The final selection is given in the rightmost column. Uncertainties quoted are statistical in nature.

Sample	$e\mu$			≥ 2 jets			≥ 1 b-tag		
$t\bar{t}$	216424.1	$\pm$	100.2	160972.2	$\pm$	86.4	137535.5	$\pm$	79.8
Single Top tW	21183.7	$\pm$	41.1	8313.0	$\pm$	25.7	6232.1	$\pm$	22.2
Drell-Yan	65103.8	$\pm$	526.1	4923.0	$\pm$	194.9	546.0	$\pm$	65.4
$W + jets$	4204.1	$\pm$	327.9	508.1	$\pm$	110.2	103.6	$\pm$	51.8
Diboson	25283.8	$\pm$	73.5	2709.9	$\pm$	23.5	186.9	$\pm$	6.2
Single Top (s+t)	269.0	$\pm$	4.8	55.4	$\pm$	2.2	32.3	$\pm$	1.7
$t\bar{t} + V$	317.3	$\pm$	2.3	284.3	$\pm$	2.2	226.6	$\pm$	1.9
Total MC	332786	$\pm$	634	177766	$\pm$	243	144863	$\pm$	118
Data	330893			172972			135477		

4.4 Corrections Applied to Simulation

Several types of corrections are applied to improve the agreement between data and simulation which are detailed below.

4.4.1 Pileup

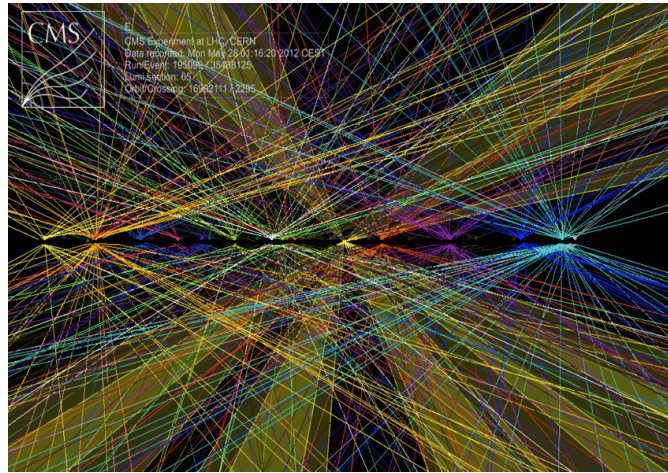


Figure 4.10: Reconstructed tracks from several primary vertices during an LHC RunI event. The high-luminosity upgrade for the LHC is expected to have an average of 200 pileup interactions per event.

During a typical LHC event, there may be dozens of pp interaction points in addition to the primary vertex of interest. These additional interactions are known as *pileup* (Fig. 4.10). The PV distribution in MC should be reweighted to match the observed values in data. The average pileup per bunch crossing is calculated from the instantaneous luminosity and a minimum bias cross section of $69.2 (\pm 4.6\%)$ mb. A scale factor is applied to each MC event to scale the simulated pileup to the value observed in data. The reweighted primary vertex distribution is shown in Figure 4.11.

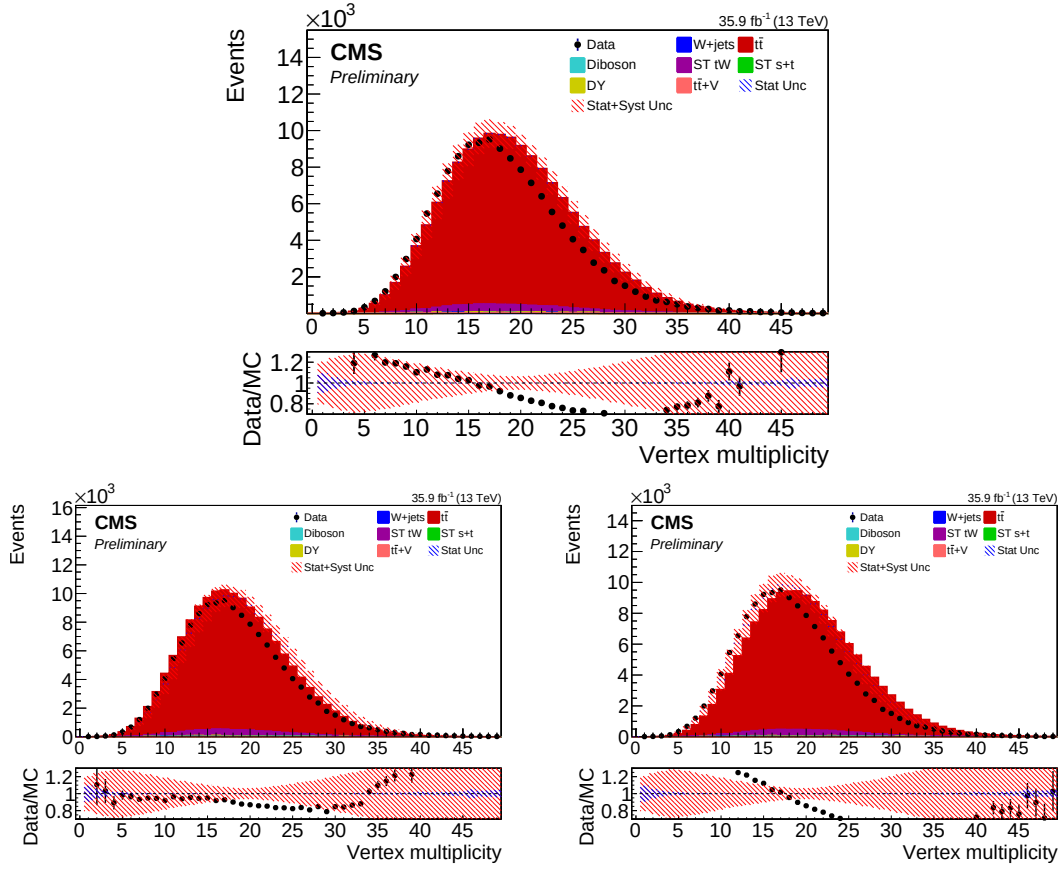


Figure 4.11: Primary Vertex multiplicity reweighted with nominal pileup corrections (top), -1σ corrections (bottom left), and $+1\sigma$ corrections (bottom right).

4.4.2 Trigger and Lepton Selection Efficiencies

The efficiency of HLT, electron ID and reconstruction, and muon ID and isolation for MC are reweighted to match the performance in data using centrally-produced corrections provided by the respective Physics Object Groups.

4.4.3 B-tag Efficiency

The b-tagging performance in MC is corrected to match the observed values in data. Scale factors are centrally provided in the form: $SF = \varepsilon_{Data}/\varepsilon_{MC}$, where ε

is a function of the jet p_T and η . For p_T bin i and η bin j the efficiency in bin (i,j) , given by Equation 4.7, is defined as the number of b-tagged jets divided by the total jet multiplicity in the given bin.

$$\varepsilon(i, j) = \frac{N^{b\text{-tagged}}(i, j)}{N^{\text{total}}(i, j)} \quad (4.7)$$

A per-event weight is computed based on b-tagged jet multiplicity and the required number of b-tagged jets: $w(i|n)$ is the weight for i b-tags in an event with n total b-tags and $\sum_{i=0}^n w(i|n) = 1$. By requiring ≥ 1 b-tagged jets in an event, this is equivalent to the expression in Equation 4.8.

$$w(\geq 1|n) = 1 - w(0|n) \quad (4.8)$$

By treating the scale factors as pseudo-probabilities, the weight for the 0 b-tag requirement is given in Equation 4.9, looping over all b-tagged jets in the event. The event weight for the ≥ 1 case can then be found by substituting this term into Equation 4.8.

$$w(0|n) = \prod_{i=1}^n (1 - SF_i) \quad (4.9)$$

4.4.4 Lepton Energy Scaling and Smearing

The energy and resolution of leptons are corrected to improve MC/data agreement. Electron energy in data is scaled to simulation and MC resolution is smeared to match the data. Muon momentum is scaled in MC using Rochester Muon Corrections [18], which are designed to correct biases in muon reconstruction from mismodeling of the magnetic field and detector alignment.

4.4.5 Jet Energy Scale and Resolution

Jet energies are scaled so that the values measured in data match the particle-level energy from simulation [9]. The p_T, η dependent Jet Energy Corrections (JEC) are centrally provided. The resolution of measured jet energies in data is worse than simulation, so MC jets have their energies smeared using Jet Energy Resolution (JER) corrections.

The JER correction factor for a reconstructed jet is calculated by Equation 4.10 if a matching gen-level jet can be found or Equation 4.11 otherwise. Matching requires the ΔR between the reconstructed and gen-level jets to be less than 0.2 and $|p_T - p_T^{\text{gen}}| < 3\sigma_{JER}p_T$. Here p_T and p_T^{gen} are the transverse momenta of the reconstructed and gen jets, respectively. s_{JER} is a data-to-simulation scaling factor and σ_{JER} is the relative p_T resolution in simulation. $\mathcal{N}(0, \sigma_{JER})$ is a random number sampled from the normal distribution with mean 0 and variance σ_{JER}^2 .

$$c_{JER} = 1 + (s_{JER} - 1) \frac{p_T - p_T^{\text{gen}}}{p_T} \quad (4.10)$$

$$c_{JER} = 1 + \mathcal{N}(0, \sigma_{JER}) \sqrt{\max(s_{JER}^2 - 1, 0)} \quad (4.11)$$

4.4.6 L1 Prefire

During 2016 and 2017, a shift in the ECAL timing caused many L1 trigger primitives to be associated with a previous bunch crossing. To mitigate this effect, a scale factor (Eq 4.12) is applied to each simulated event based on the prefiring probabilities of all offline photons and jets contained in the event. Efficiencies are

given by centrally provided prefiring maps.

$$w = 1 - P(prefiring) = \sum_{i=photons,jets} (1 - \varepsilon^{prefire}(\eta, p_T^{(EM)})) \quad (4.12)$$

4.5 Systematic Uncertainties

Assumptions made during the modeling of the underlying physics processes or reconstruction of the event data give rise to systematic uncertainties which must be quantified in the final result. The different sources are categorized into two types: modeling (theoretical) and experimental uncertainties. An additional uncertainty due to the limited precision of simulated samples, called the MC statistical uncertainty, is also investigated.

4.5.1 Modeling Uncertainties

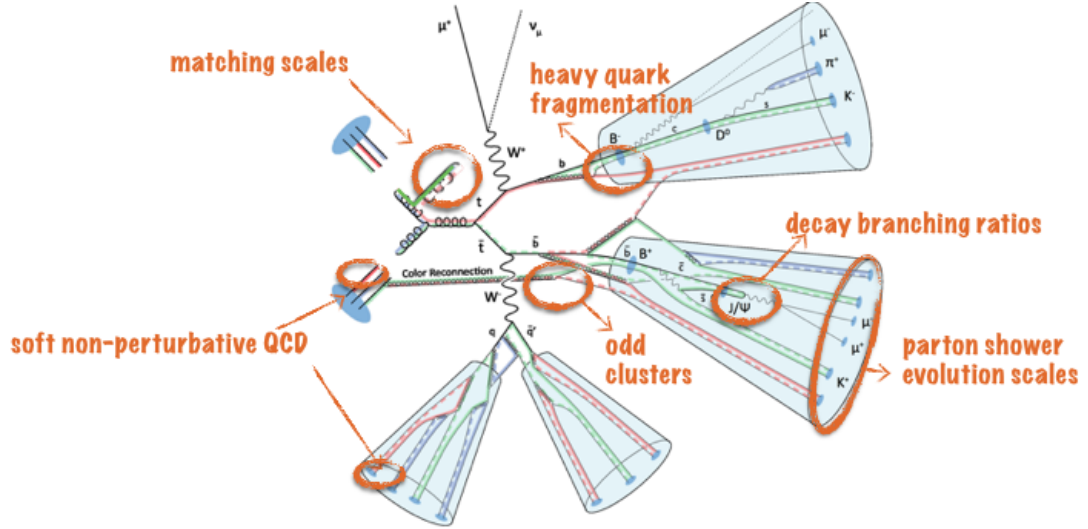


Figure 4.12: Many theory uncertainties are related to the modeling of parton showering. Non-perturbative QCD effects and multiple-parton interactions complicate the process of simulating hadron collisions.

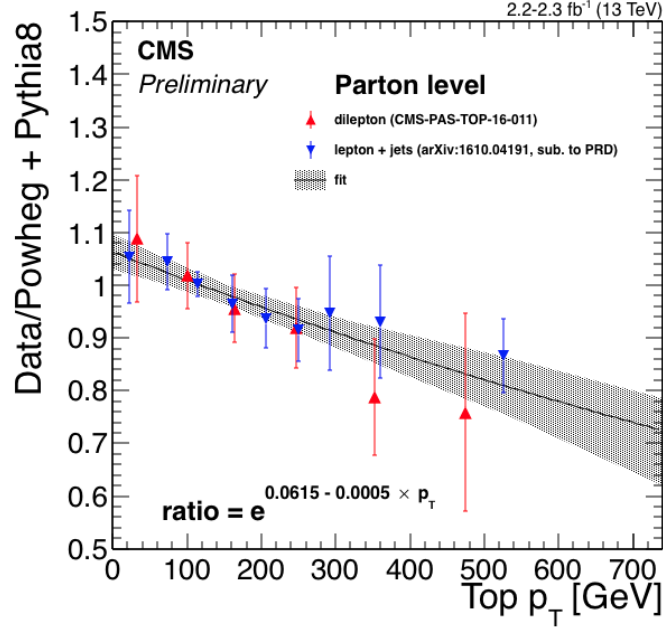


Figure 4.13: Top quark p_T re-weighting based on comparison of data to parton-level top quark p_T from simulation in the dilepton and l +jets channels.

4.5.1.1 Top quark p_T modeling

There is a disagreement between reconstructed top quark p_T observed in data and the parton-level truth p_T predicted by simulation (Figure 4.13). The current recommendation is to apply the correction to $t\bar{t}$ simulation as a systematic variation; the nominal $t\bar{t}$ sample remains uncorrected. A per-event weight (Eq 4.14) is derived based on the top quark and antiquark p_T (Eq 4.13), using the `isLastCopy()` partons (after radiation and before decay) from PYTHIA8.

$$SF(p_T) = e^{0.0615 - 0.0005 p_T} \quad (4.13)$$

$$w = \sqrt{SF(t)SF(\bar{t})} \quad (4.14)$$

4.5.1.2 ME/PS matching scale

The matching of jets at the Matrix Element (ME) and the Parton Shower (PS) levels, as well as the regulation of high- p_T radiation, is controlled by the tunable PYTHIA8 parameter h_{damp} . The default value used in the CP5 tune is: $h_{\text{damp}} = 1.379^{+0.9260}_{-0.5052} m_t$ [60]. Separate MC samples are produced for up/down variations of this parameter within its uncertainties at the nominal m_t (172.5 GeV) value only.

4.5.1.3 QCD scales

The nominal values of the factorization (μ_F) and renormalization (μ_R) scales at ME level in signal simulation are varied up (2) or down (0.5) according to the following prescription:

- μ_F fixed, μ_R varied by a factor of 2 or 0.5
- μ_R fixed, μ_F varied by a factor of 2 or 0.5
- μ_F, μ_R coherently varied by a factor of 2 or 0.5

These are given as per-event weights. The envelope of all permutations is considered, with the largest (smallest) value used as the up (down) systematic variation.

4.5.1.4 Parton distribution functions

The CP5 tune for $t\bar{t}$ MC samples uses the LO, NLO, and NNLO versions of the NNPDF3.1 Parton Distribution Function (PDF) set [16] following the PDF4LHC recommendations for RunII [21]. Alternate versions of the nominal PDF set are used to compute the PDF uncertainty. For each event, the standard deviation of

the PDF weight set is computed (Eq 4.16) and the per-event weight for up/down PDF variations is $(1 \pm \sigma_{\text{PDF}}/w_{\text{nominal}})$, where w_{nominal} is the nominal PDF weight for that event.

$$\bar{w} = \frac{1}{N} \sum_{i=1}^N w_i \quad (4.15)$$

$$\sigma_{\text{PDF}} = \sqrt{\frac{1}{N} \sum_{i=1}^N (w_i - \bar{w})^2} \quad (4.16)$$

4.5.1.5 MC generator

Nominal $t\bar{t}$ signal simulation is produced using the POWHEG ME generator and showered with PYTHIA8. Alternate $t\bar{t}$ samples are generated using aMC@NLO +PYTHIA8, MADGRAPH (LO)+PYTHIA8, and POWHEG showered with HERWIG++. These are used to test the effects of alternate generator/shower combinations. The results from alternate generators are not included as part of the total reported systematic uncertainty, but the uncertainties are noted for subsequent comparisons to other experiments.

4.5.1.6 Underlying event

The underlying event (UE) is a description of the beam-beam remnants (BBR): the remainder after the initial partons are scattered from the colliding beam hadrons; and multiple-parton interactions (MPI): softer parton-parton scatterings from the same pp collision. Up and down UE variations in the PYTHIA8 tune are available as separate samples at the nominal top quark mass point.

4.5.1.7 Color reconnection

In simulating hadron collisions, the variety of color-charged partons (gluons, quarks) produced must hadronize before decaying to form jets. The QCD color charge interaction between them can be modeled using color strings. This process is complicated by MPI and non-perturbative QCD effects. Color reconnection rearranges color strings after showering to predict which partons will interact to form hadrons (Figure 4.14).

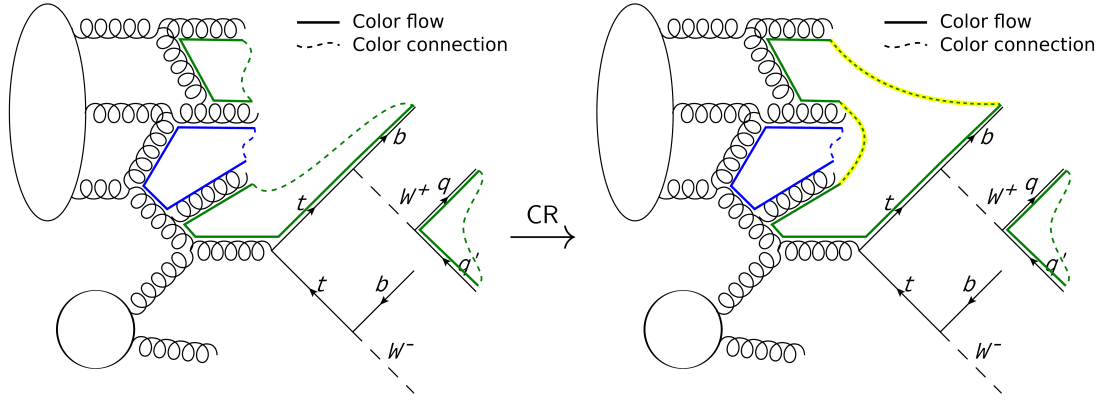


Figure 4.14: Color reconnection models attempt to predict the color string connections between charged partons.

Three different models of color reconnection are explored:

- **Erd On** Color reconnection enabled inside resonances
- **Gluon-move** Gluons allowed to move to different strings to minimize the total string length [11]
- **QCD-based** QCD color rules incorporated into color reconnection to determine the probability a connection is allowed [27]

These models are produced at the nominal top quark mass point in separate samples. The envelope of all three uncertainties is quoted as the CR systematic uncertainty.

4.5.1.8 Parton shower scales

The strong coupling constant α_S governs the rate of QCD radiation emissions from initial and final-state partons. The uncertainty on α_S in initial-state radiation (ISR) and final-state radiation (FSR) can be probed by varying μ_F in the respective emissions by a factor of 2 (up) or 0.5 (down). These variations are given in the form of per-event weights to be applied to signal simulation.

4.5.1.9 $t\bar{t}/tW$ interference

The single top tW -channel at NLO interferes with $t\bar{t}$ production due to some diagrams sharing the same initial/final states.

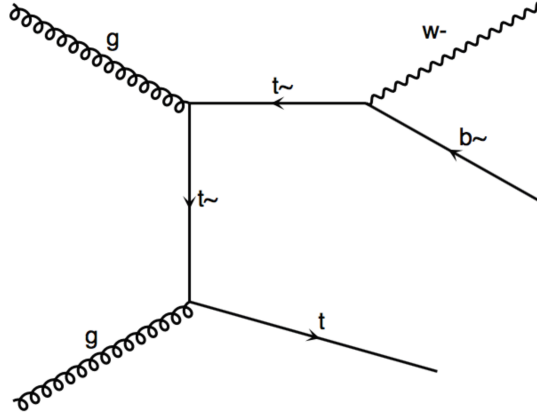


Figure 4.15: Example NLO tW diagram that interferes with $t\bar{t}$.

Two methods of handling the interference are commonly used. The first, Diagram Removal (DR), simply removes all doubly-resonant tW diagrams from the

NLO amplitudes. The DR method is used for nominal tW samples. A second method, Diagram Subtraction (DS), modifies the NLO tW cross section by locally canceling the $t\bar{t}$ contributions. The difference between masses extracted from the DS and DR models is quoted as a systematic uncertainty.

4.5.2 Experimental Uncertainties

4.5.2.1 Luminosity

The uncertainty on the measured luminosity for Run II 2016 is $\pm 2.5\%$ [65]. All signal and background processes are simultaneously scaled up and down by this factor.

4.5.2.2 Pileup

The minimum bias cross section used to determine the pileup re-weighting corrections for simulation has an uncertainty of $\pm 4.6\%$. Alternate event weights are calculated for the up/down pileup variations based on varying this cross section within its uncertainty.

4.5.2.3 Jet energy scale and resolution

JEC and JER systematics are evaluated by varying the recommended scale factors by their one sigma uncertainties. The JEC is a combination of 27 individual uncorrelated terms which are varied coherently and added in quadrature. Due to the minimal effect of jet energies on this method (only affecting event acceptance), variations on the individual JEC components are not studied.

4.5.2.4 Lepton trigger and selection efficiency

The (p_T, η) -dependent efficiencies used in trigger and lepton selection scale factors are varied within their measured uncertainties to produce systematic variations on the event weights. The variations on lepton selection efficiencies (electron ID, electron Reconstruction, muon ID, muon isolation) are each evaluated separately.

4.5.2.5 Lepton energy scale and smear

The electron energy scale correction is composed of statistical, systematic, and ECAL gain-switch components added in quadrature. A $\pm 1\sigma$ variation is coherently applied to these components to derive the up/down systematic uncertainties in simulation. The ρ and ϕ components of the Gaussian used to smear electron energy resolutions in simulation are varied to produce $\pm 1\sigma$ systematic variations.

The Rochester muon momentum corrections also provide the $\pm 1\sigma$ uncertainties on the muon p_T scaling in simulation. These are used for up/down systematic variations.

4.5.2.6 B-tagging efficiency

Efficiencies used to determine the DeepCSV b-tagging scale factors are varied within their uncertainties to give systematic variations of the per-event b-tagging weights.

4.5.2.7 L1 Prefire

The (p_T, η) -dependent efficiencies from prefire maps used to determine the correction factor are varied up and down within their measured uncertainties to derive alternate scale factors which are applied to the simulation.

4.5.2.8 Background normalization

Background contributions are small compared to signal $t\bar{t}$ and tW processes, so the cross sections for all backgrounds (DY, Diboson, $t\bar{t}+V$, single top s/t channels, and $W+jets$) are scaled coherently by $\pm 30\%$ to evaluate the background normalization uncertainty.

4.5.3 MC Statistical Uncertainty

MC simulated samples are generated with a limited number of events, leading to an uncertainty in each histogram bin called the MC statistical uncertainty. The effect of limited simulation precision is assessed through pseudoexperiments.

Toys are generated from the input signal templates by Poisson smearing each bin within its statistical uncertainty. All templates and systematic variations are smeared simultaneously. For systematics which derive from weight-based corrections to the nominal sample, their distributions are fluctuated coherently with the nominal templates. Systematics coming from alternate simulation samples are fluctuated independently.

The distribution of masses measured from the toys follows an approximately Gaussian profile. The sigma of the Gaussian fit to the toy mass distributions is taken as the MC statistical uncertainty and added as an additional uncorrelated uncertainty to the extracted top mass [59].

4.6 Leptonic Observables

The following set of leptonic observable distributions and their dependence on m_t are analyzed:

- $p_T(ll)$ Transverse momentum of the lepton pair
- $M(ll)$ Invariant mass of the lepton pair
- $p_T(l^+)$ Transverse momentum of the positively charged lepton
- $E(l^+)$ Energy of the positively charged lepton
- $p_T(l^+) + p_T(l^-)$ Scalar sum of the lepton transverse momenta
- $E(l^+) + E(l^-)$ Scalar sum of the lepton energies

The choice of positive leptons for $p_T(l^+)$ and $E(l^+)$ is arbitrary and one could readily substitute observables for the negative leptons $p_T(l^-)$ and $E(l^-)$ instead. The SM predicts that $m_t = m_{\bar{t}}$ and this method is not sensitive to differences between top quarks and antiquarks.

Figure 4.16 compares data and simulation for the six leptonic observables with Figure 4.17 plotting the same observables in log scale. All observables are dominated by signal $t\bar{t} + tW$. Figures 4.18-4.20 demonstrate the signal shape variation as a function of the top quark mass with $m_t \in [169.5, 172.5, 175.5]$ GeV for $t\bar{t}$, tW , and $t\bar{t} + tW$ simulation, respectively.

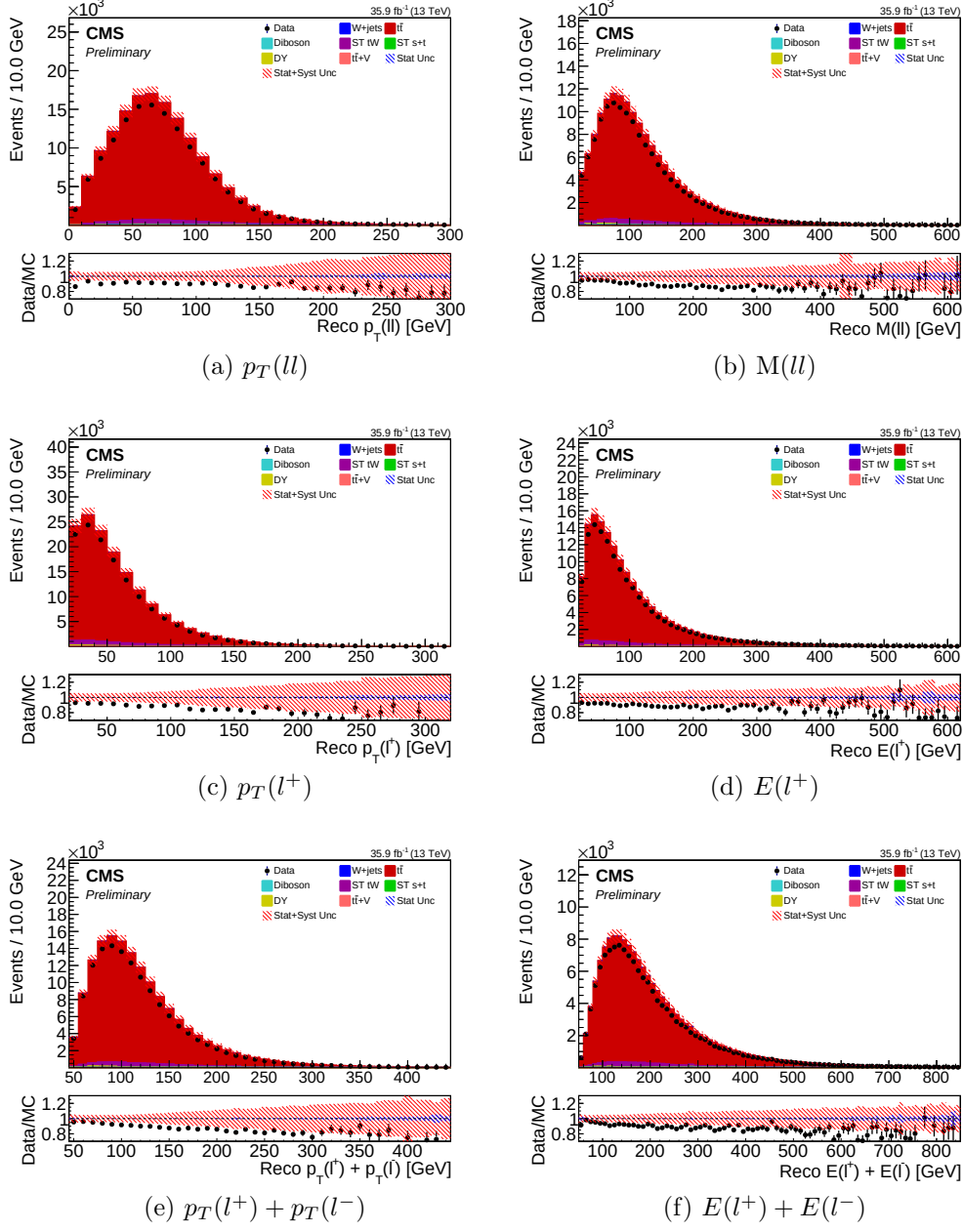


Figure 4.16: Reconstructed leptonic observables.

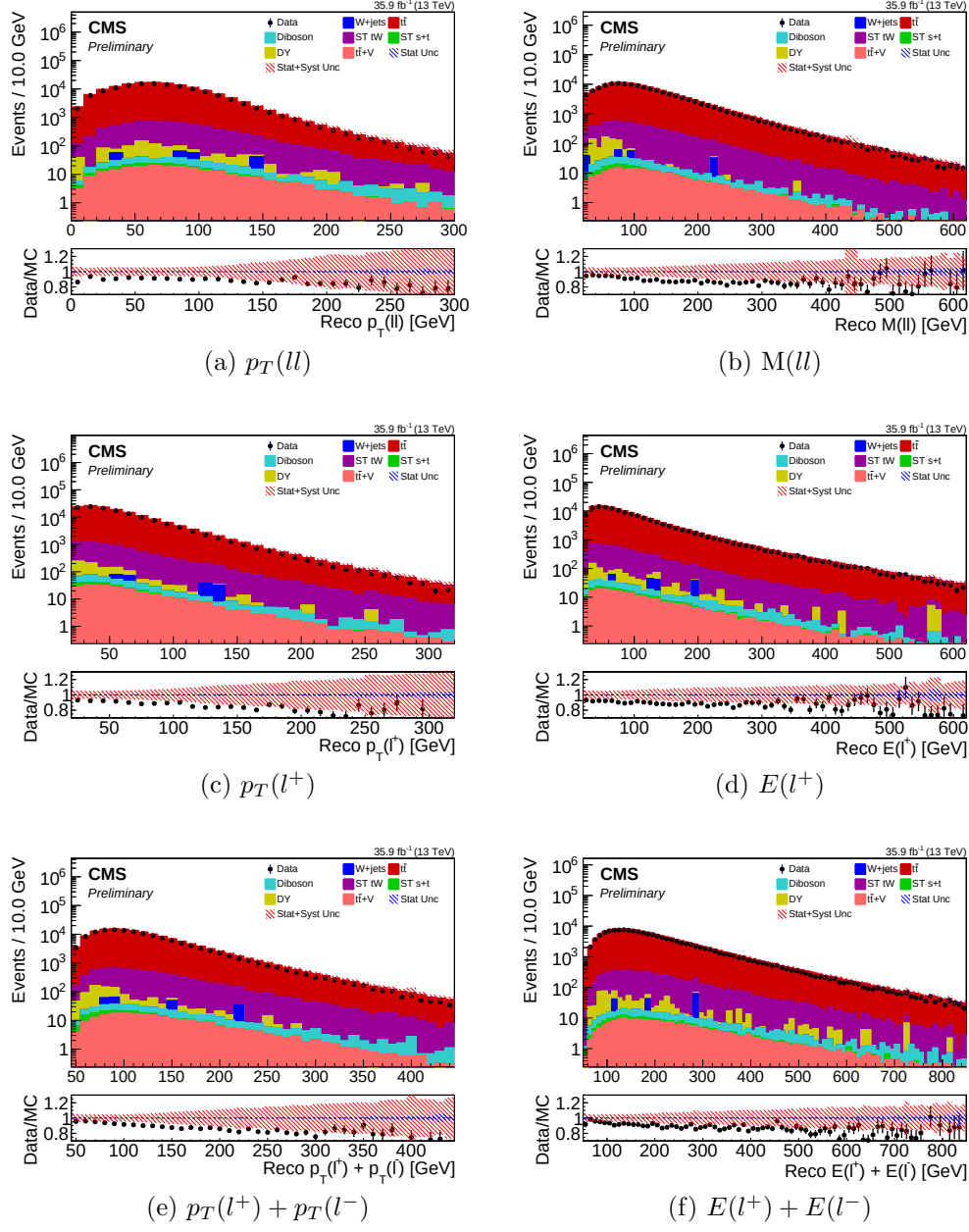


Figure 4.17: Reconstructed leptonic observables with y-axis plotted in log scale.

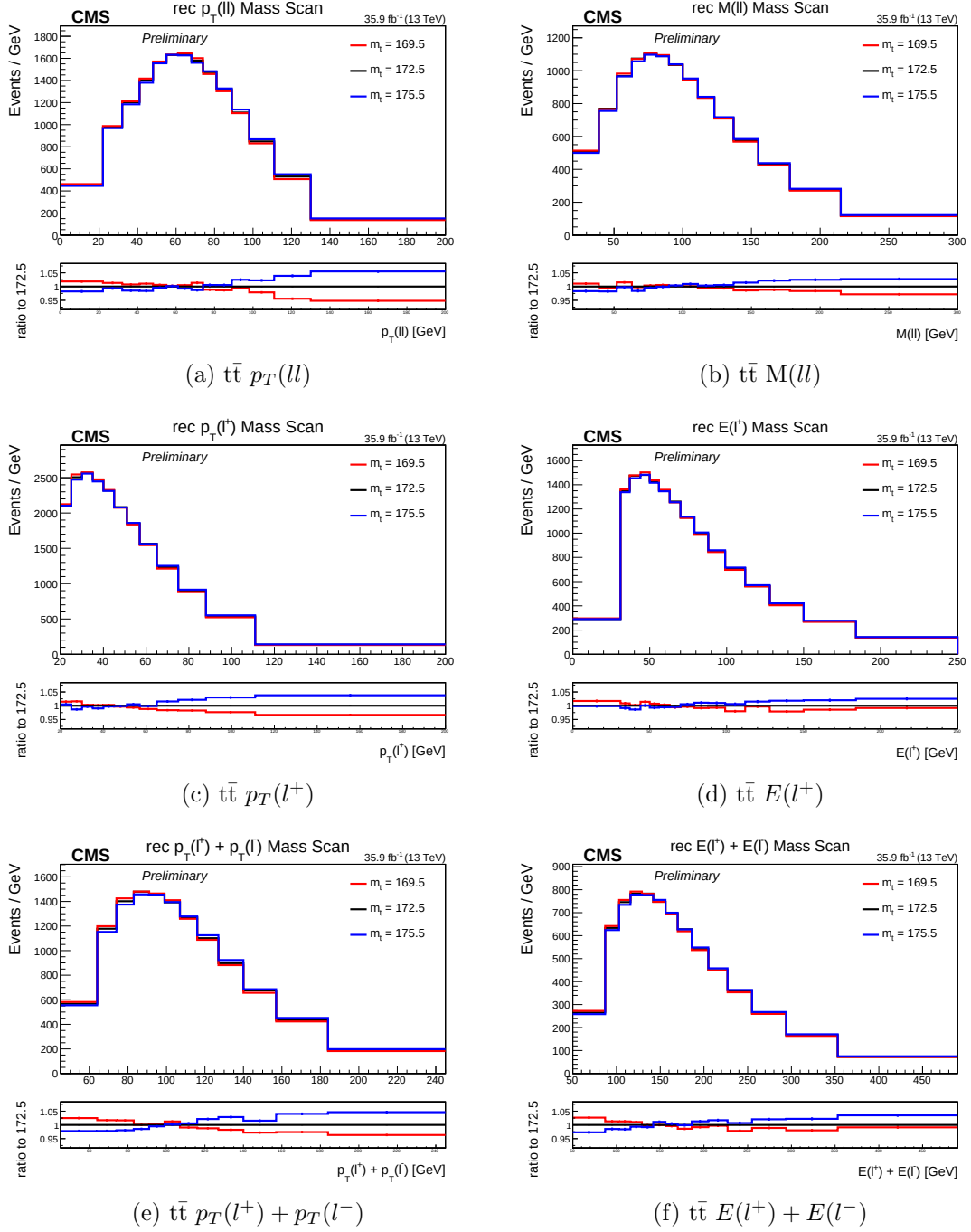


Figure 4.18: Mass scans for leptonic observables from nominal $t\bar{t}$ signal simulation. All distributions are normalized to the rate from the nominal 172.5 GeV sample.

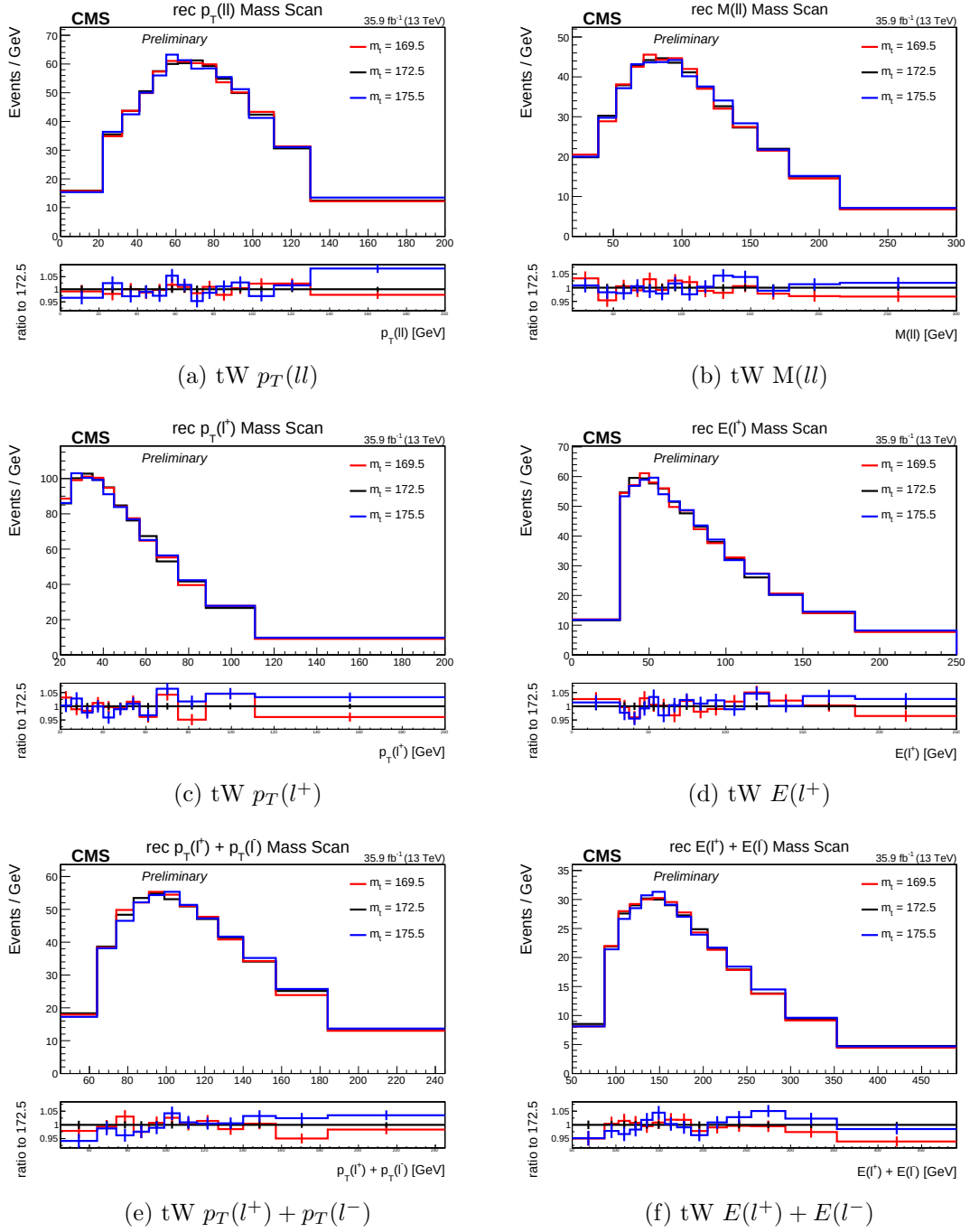


Figure 4.19: Mass scans for leptonic observables from nominal tW signal simulation. All distributions are normalized to the rate from the nominal 172.5 GeV sample.

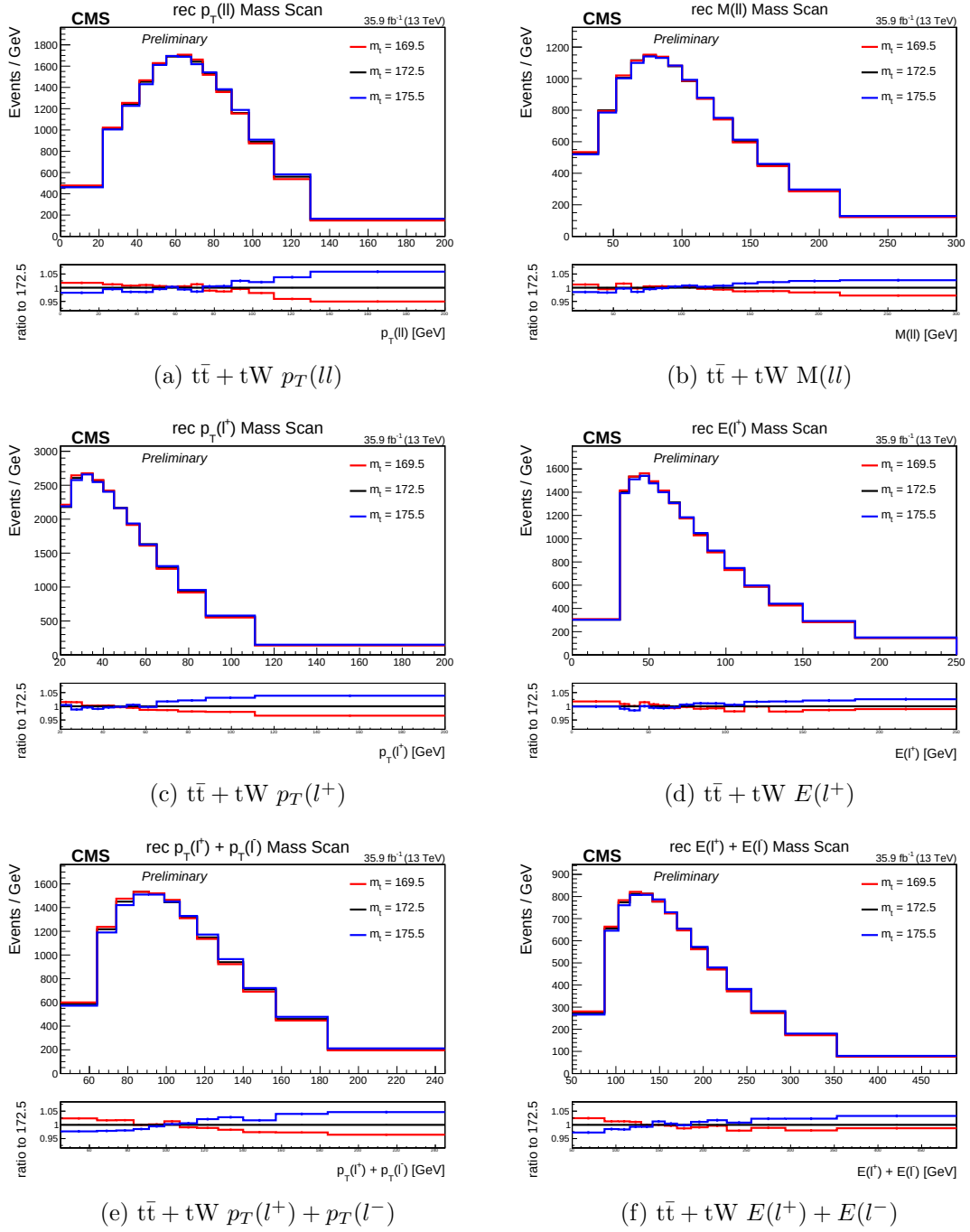


Figure 4.20: Mass scans for leptonic observables from nominal $t\bar{t} + tW$ signal simulation. All distributions are normalized to the rate from the nominal 172.5 GeV sample.

4.7 MC Truth

To compare the leptonic observables to theory predictions, the MC truth kinematic distributions are studied at three different stages in the event generation prior to detector simulation and event reconstruction. These will be called gen, dressed, and LHE levels and are described below. Only the two highest p_T leptons, corresponding to the $e\mu$ final state pair, are matched at truth levels. Additional softer leptons in an event are not included.

Gen-level leptons correspond to the final values output from PYTHIA8 after showering and radiation effects. In any given event, potential gen lepton candidates are required to have the same sign as their reconstructed (rec) counterparts. They should be prompt leptons (directly originating from W boson decay) and be flagged as `IsLastCopy()` in PYTHIA8. The ΔR between gen and reconstructed leptons should be less than 0.2 and the matching gen lepton is chosen as the one which minimizes ΔR .

Dressed-level leptons are Rivet-based [38] `ParticleLevelProducer` objects which require matching gen-level leptons. The four-momenta of radiated photons are added back into the gen-level leptons.

LHE-level leptons are those directly produced from the hard scattering in POWHEG and prior to radiation and showering by PYTHIA8. These are most directly comparable to the fixed-order QCD calculation, which is not showered. Outgoing LHE leptons are required to match the sign of the reconstructed leptons. Additionally, the event is required to have matching leptons at the gen-level.

Figure 4.21 shows the lepton p_T , η , and ϕ for events with matching leptons at the gen, dressed, and LHE levels, while figure 4.22 compares the leptonic observables. Very little difference is observed between gen and rec leptons and the p_T

is reconstructed withing 1-2%. The dressed p_T spectrum, compared to gen level, is harder for electrons than for muons. This is due to the increased propensity of electrons to radiate compared to muons. The LHE-level p_T spectrum is hardest since this occurs before additional QCD radiation effects are accounted for by PYTHIA8.

Figure 4.23 breaks down the fully reconstructed observable distributions into events matched at LHE-level with prompt leptons, events where one or more leptons originates from τ decay, and events where leptons originate from exotic decays or were not able to be reconstructed. Only events from prompt leptons are included for the comparison to the fixed-order QCD calculation, so the fraction of matching events is used to derive a bin-by-bin scale factor to be applied to data.

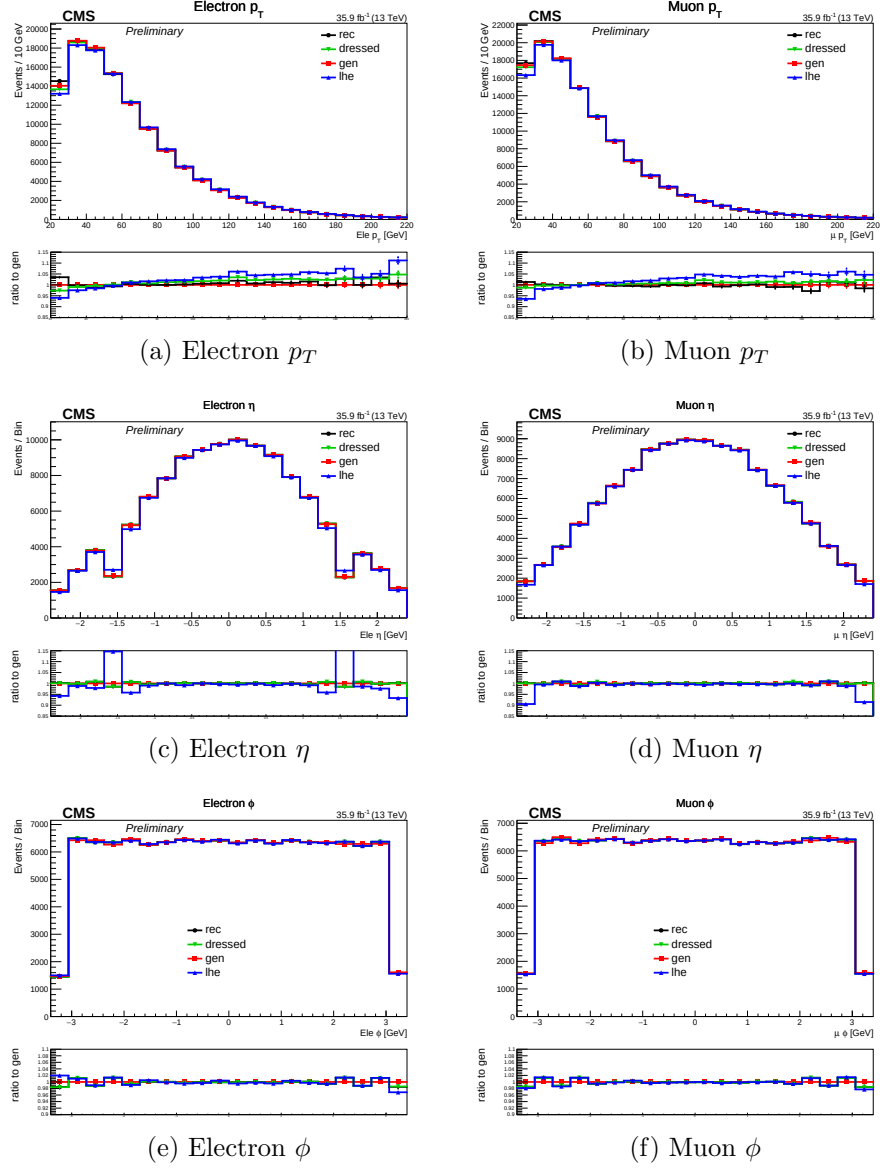


Figure 4.21: Comparison of rec, gen, dressed, and LHE level lepton kinematic distributions from nominal $t\bar{t} + tW$ simulation.

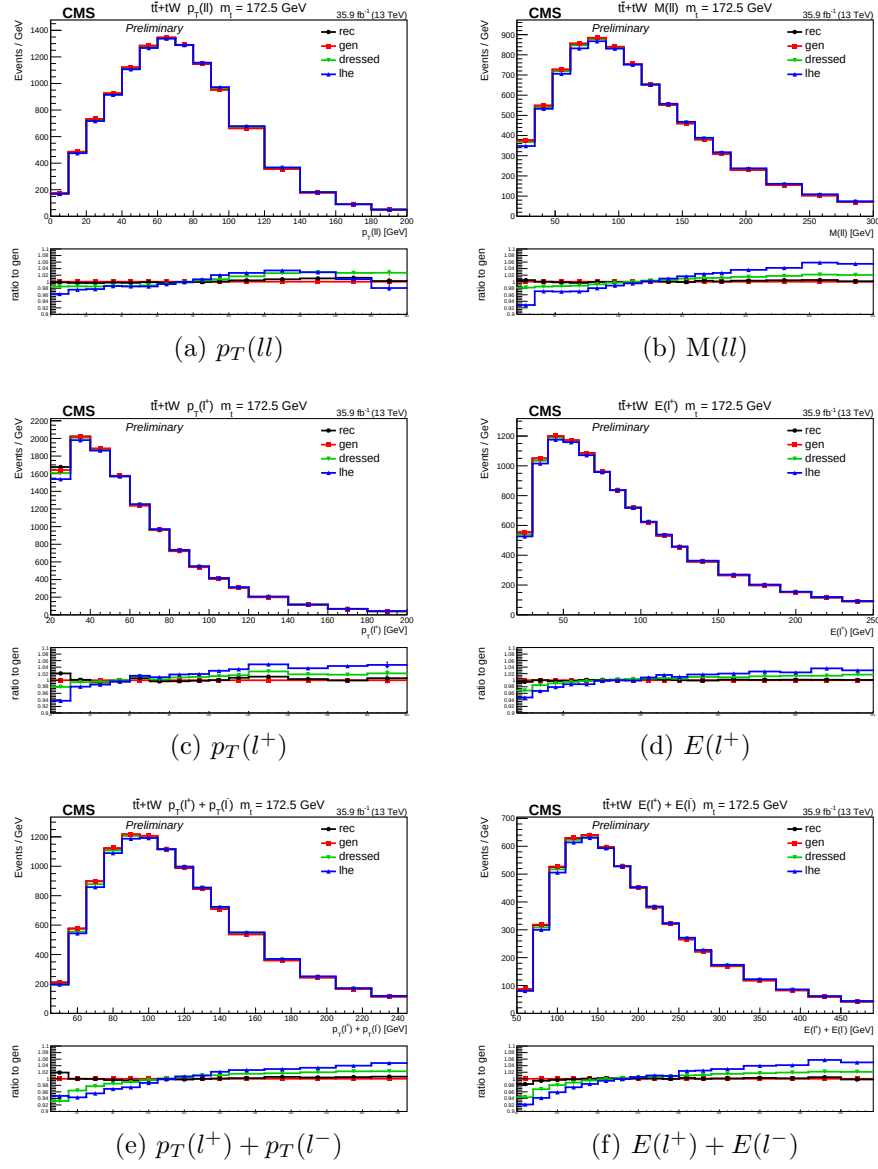


Figure 4.22: Comparison of rec, gen, dressed, and LHE level leptonic observables from nominal $t\bar{t} + tW$ simulation.

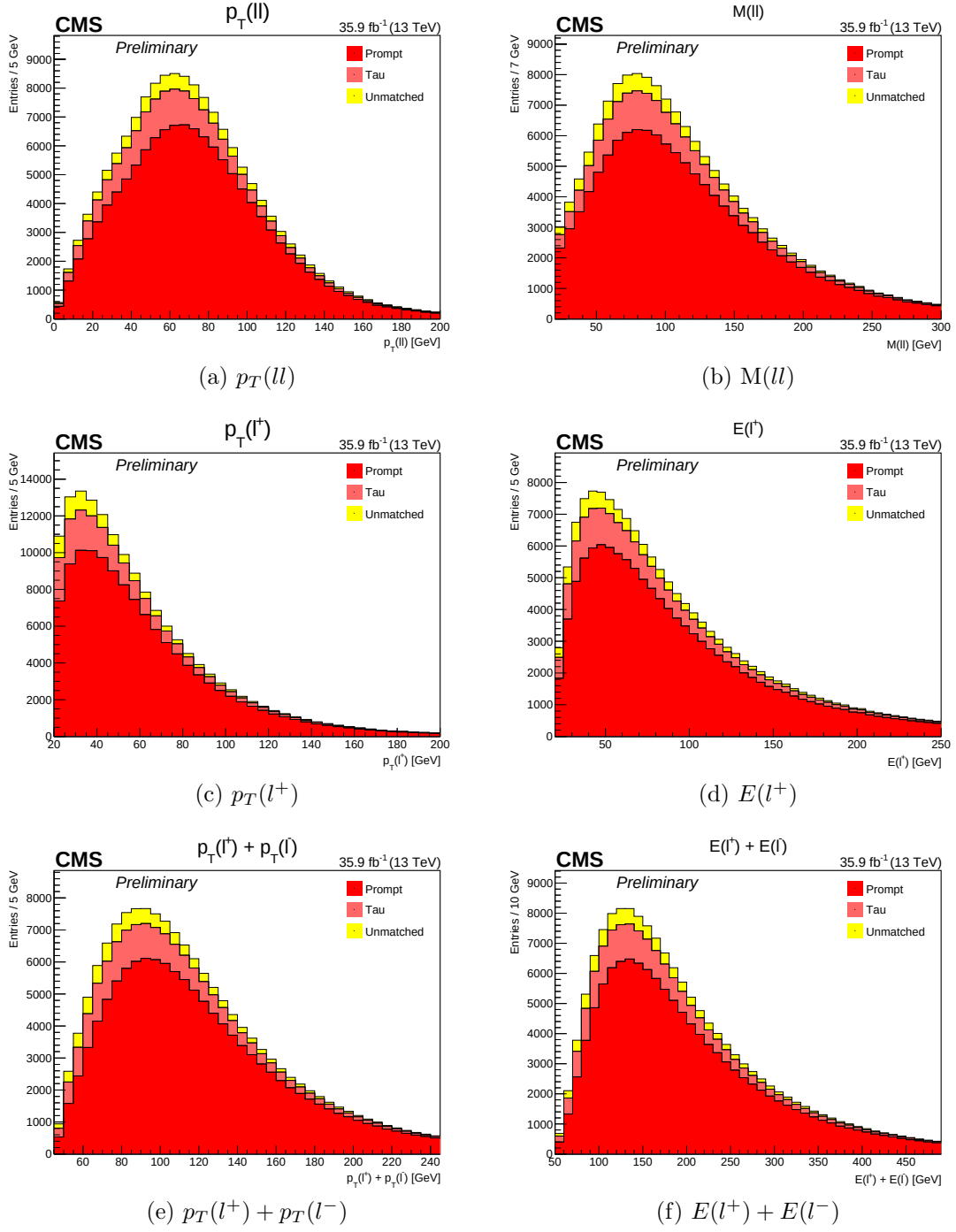


Figure 4.23: Breakdown of events with prompt leptons from W decays matched at LHE-level (Matched), one or more leptons originating from tau decays (Tau), or non-prompt leptons (Unmatched) from nominal $t\bar{t} + tW$ simulation.

4.8 Unfolding

Unfolding attempts to remove smearing from reconstructed distributions, caused by detector effects and parton showering, to recover the truth-level distribution (Figure 4.24). It is used for comparing results to theory (e.g., fixed order QCD calculations) or to other experiments, but at the expense of decreased sensitivity and increased uncertainties. This analysis uses the TUnfold [57] package to unfold the reconstructed data to LHE parton level to compare with the NNLO QCD calculation. Since the true distribution in data is unknown, the algorithm is first trained on MC simulated events before being applied to data.

The true distribution $\tilde{\mathbf{x}}$ (containing m bins) is related to the average reconstructed (reco) distribution $\tilde{\mathbf{y}}$ (containing n bins) by Equation 4.17. The matrix element A_{ij} represents the probability that a truth-level event in bin j will be reconstructed in reco bin i . Contributions from background processes are given by distribution $\mathbf{b}$.

$$\tilde{y}_i = \sum_{j=1}^m A_{ij} \tilde{x}_j + b_i, \quad 1 \leq i \leq n \quad (4.17)$$

The least square minimization algorithm used by TUnfold requires a coarser binning scheme at gen level than at reco level, so that the number of degrees of freedom ($n-m$) is greater than zero. A more detailed description of the problem can be found in Appendix B.

Additionally, the binning is chosen so that the purity (Eq 4.18) and stability (Eq 4.19) are greater than 0.5 in all bins (Figure 4.28).

$$p^k = \frac{N_{truth\&rec}^k}{N_{rec}^k} \quad (4.18)$$

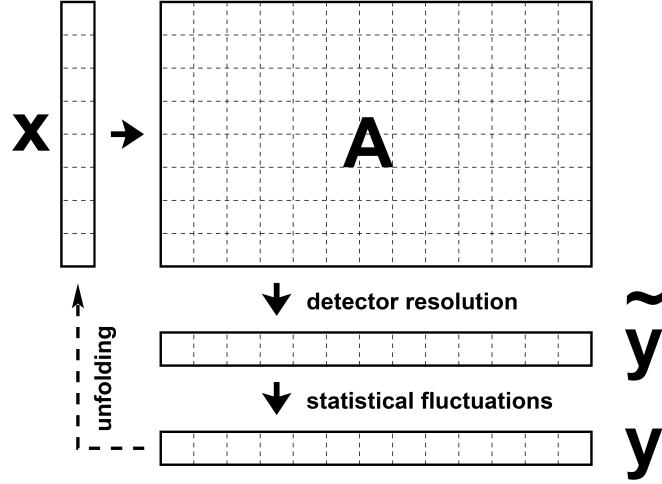
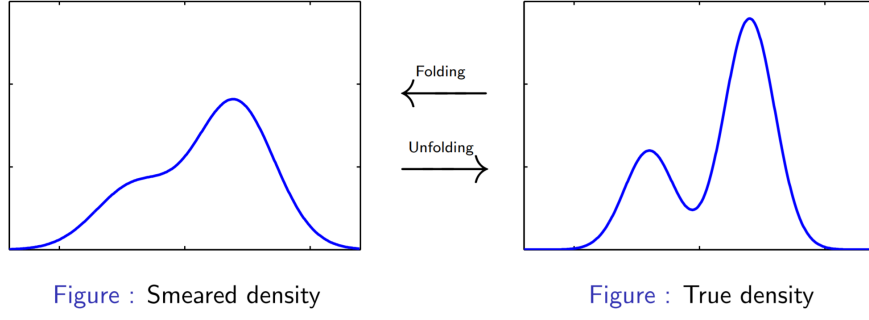


Figure 4.24: Diagrammatic overview of the unfolding problem. Unfolding is used to remove smearing effects to determine the underlying true density. The inverse problem, called folding, simulates smearing effects on the true distribution caused by an imperfect measurement device.

$$s^k = \frac{N_{truth\&rec}^k}{N_{truth}^k} \quad (4.19)$$

N_{truth}^k and N_{rec}^k are the number of truth and rec events in bin k , respectively, and $N_{truth\&rec}^k$ are the number of truth events in bin k that are correctly reconstructed in bin k . Purity thus quantifies the migration of events from neighboring bins in to bin k , while stability quantifies the reconstruction of events from bin k to other nearby bins.

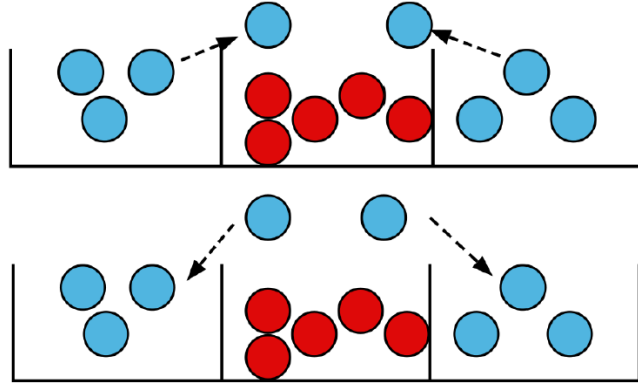


Figure 4.25: Purity (top, migration into a bin) and stability (bottom, migration out of a bin) are used to describe how events migrate from truth to reconstructed levels.

Input matrices for the six leptonic observables are shown in Figure 4.26 with bin contents representing the total number of simulated events. Truth (LHE) bins are given on the x-axis with reco bins on the y-axis. Figure 4.27 shows the (normalized) response matrices (Equation B.8) with bin contents representing the probability a truth-level event is reconstructed in the corresponding reco bin. The comparison of unfolded distributions from nominal rec-level MC simulation to LHE-level truth, with the corresponding unfolding uncertainties, are shown in Figure 4.30.

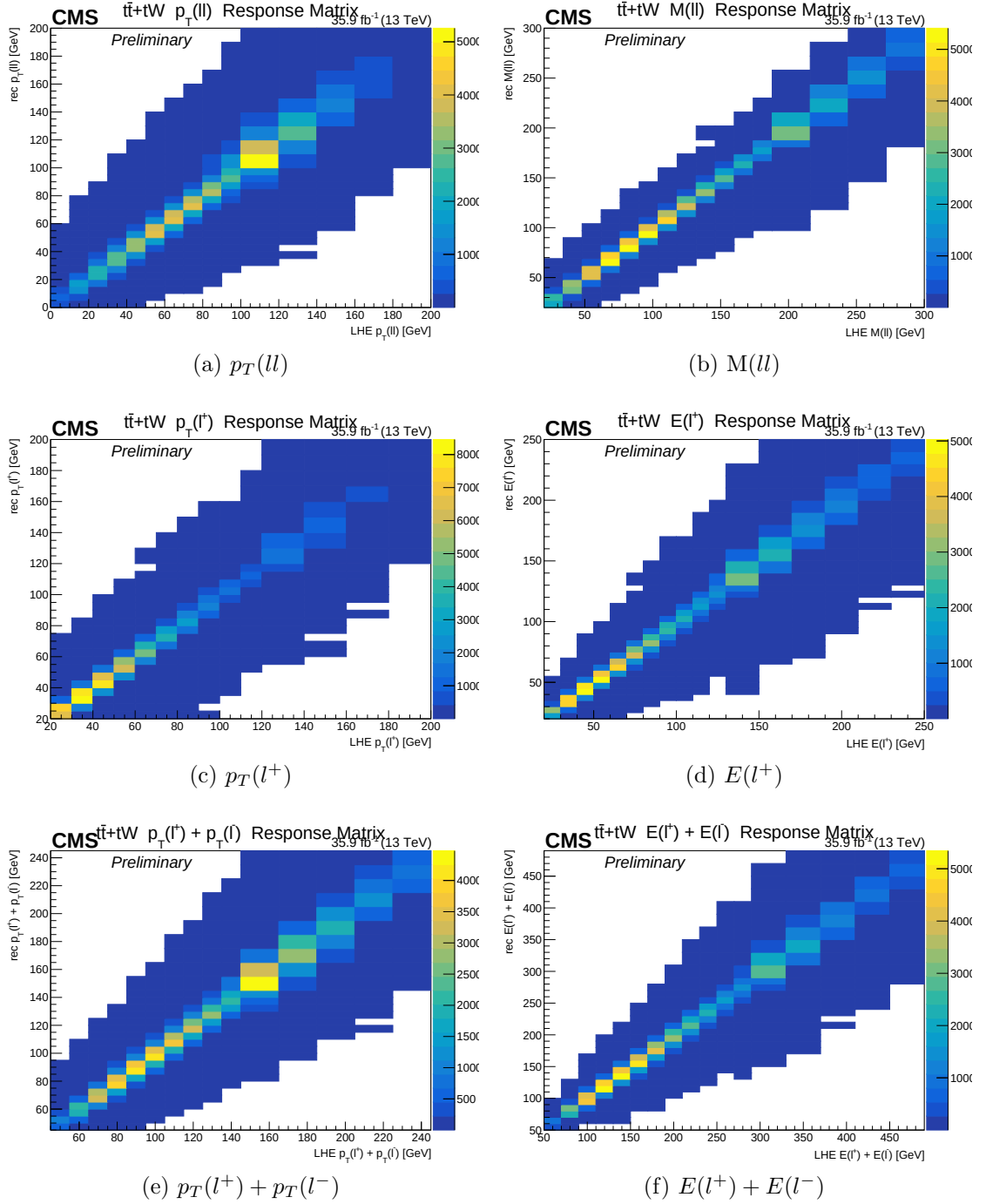


Figure 4.26: Input matrices for leptonic observables from nominal $t\bar{t} + tW$ simulation using LHE-level as MC truth.

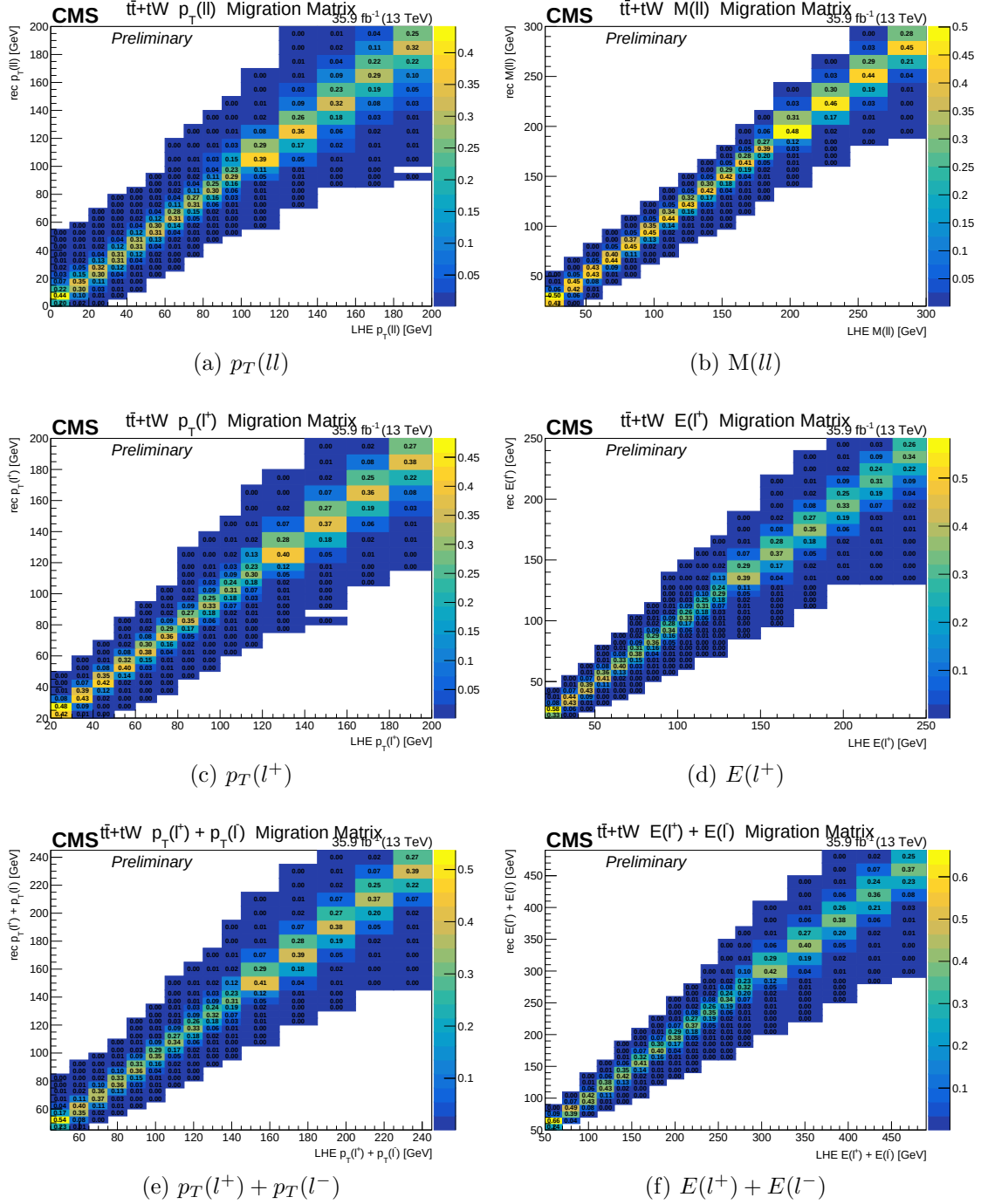
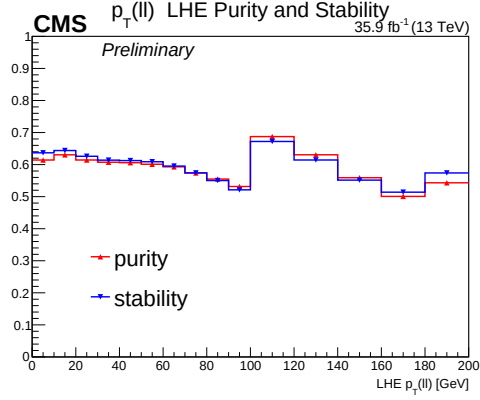
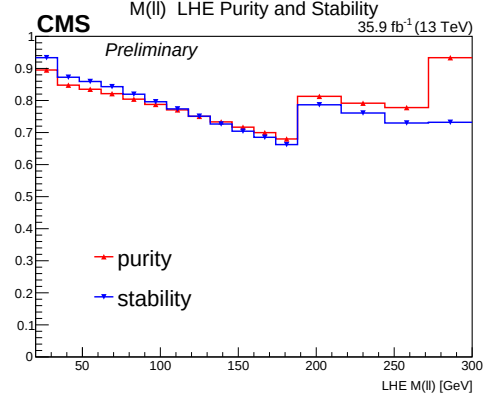


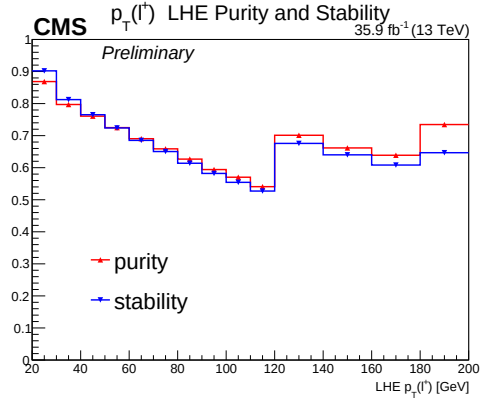
Figure 4.27: Normalized response matrices for leptonic observables from nominal $t\bar{t} + tW$ simulation using LHE-level as MC truth.



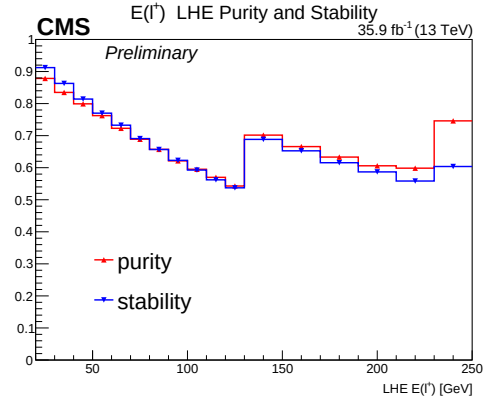
(a) $p_T(l)$



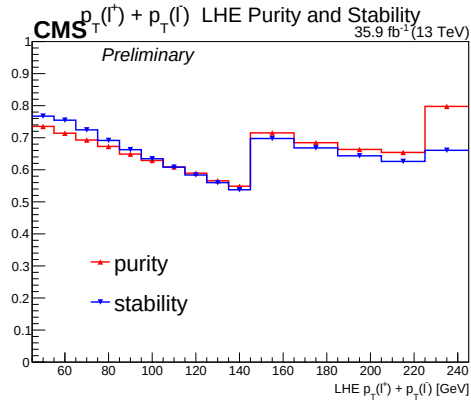
(b) $M(l)$



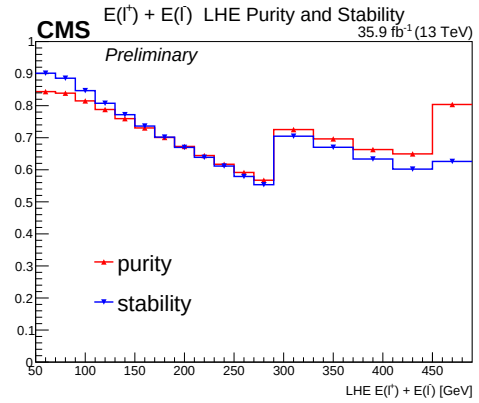
(c) $p_T(l^+)$



(d) $E(l^+)$



(e) $p_T(l^+) + p_T(l^-)$



(f) $E(l^+) + E(l^-)$

Figure 4.28: Purity and stability for leptonic observables from nominal $t\bar{t} + tW$ simulation using LHE-level as MC truth.

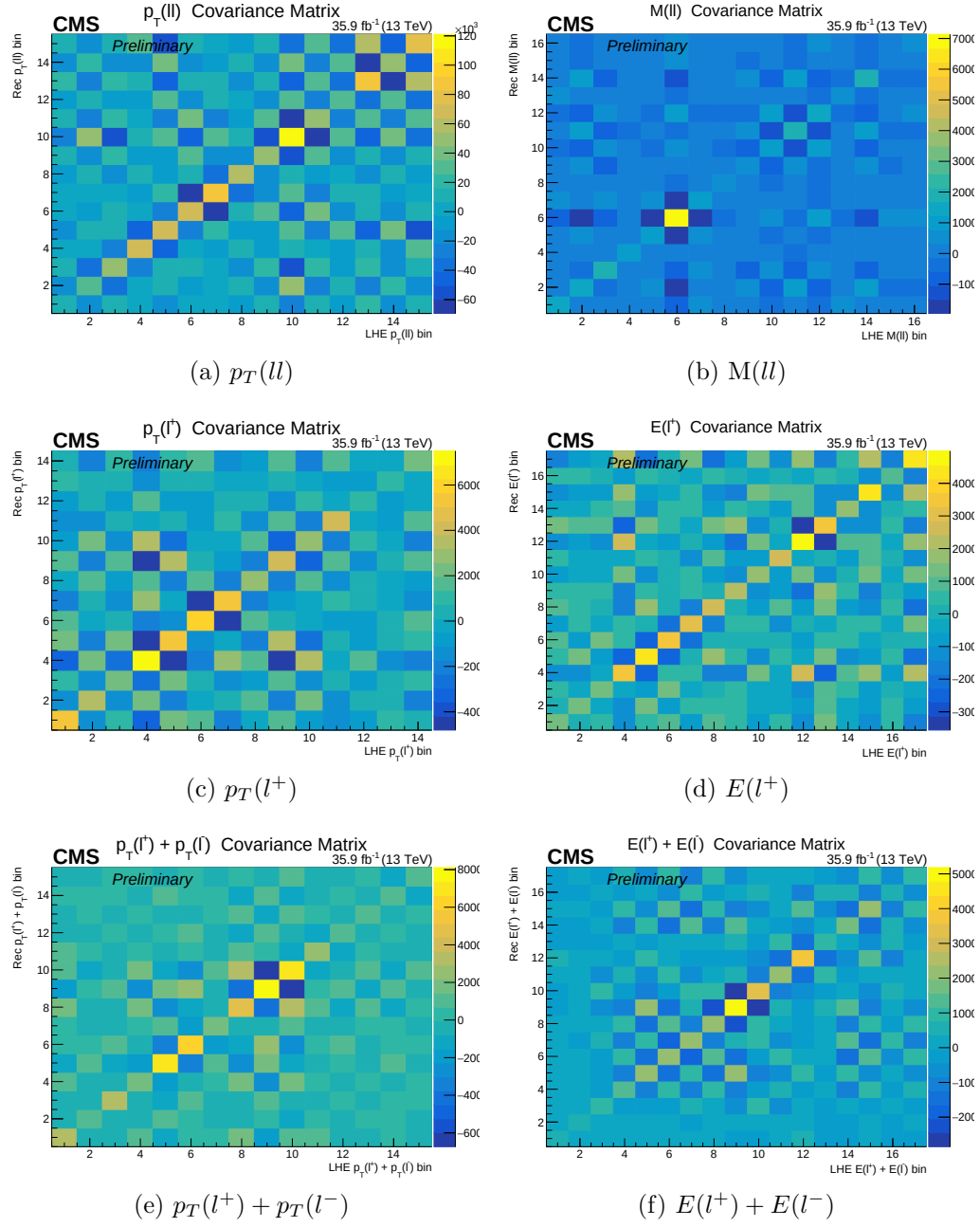


Figure 4.29: Covariance matrices representing the total uncertainties after unfolding.

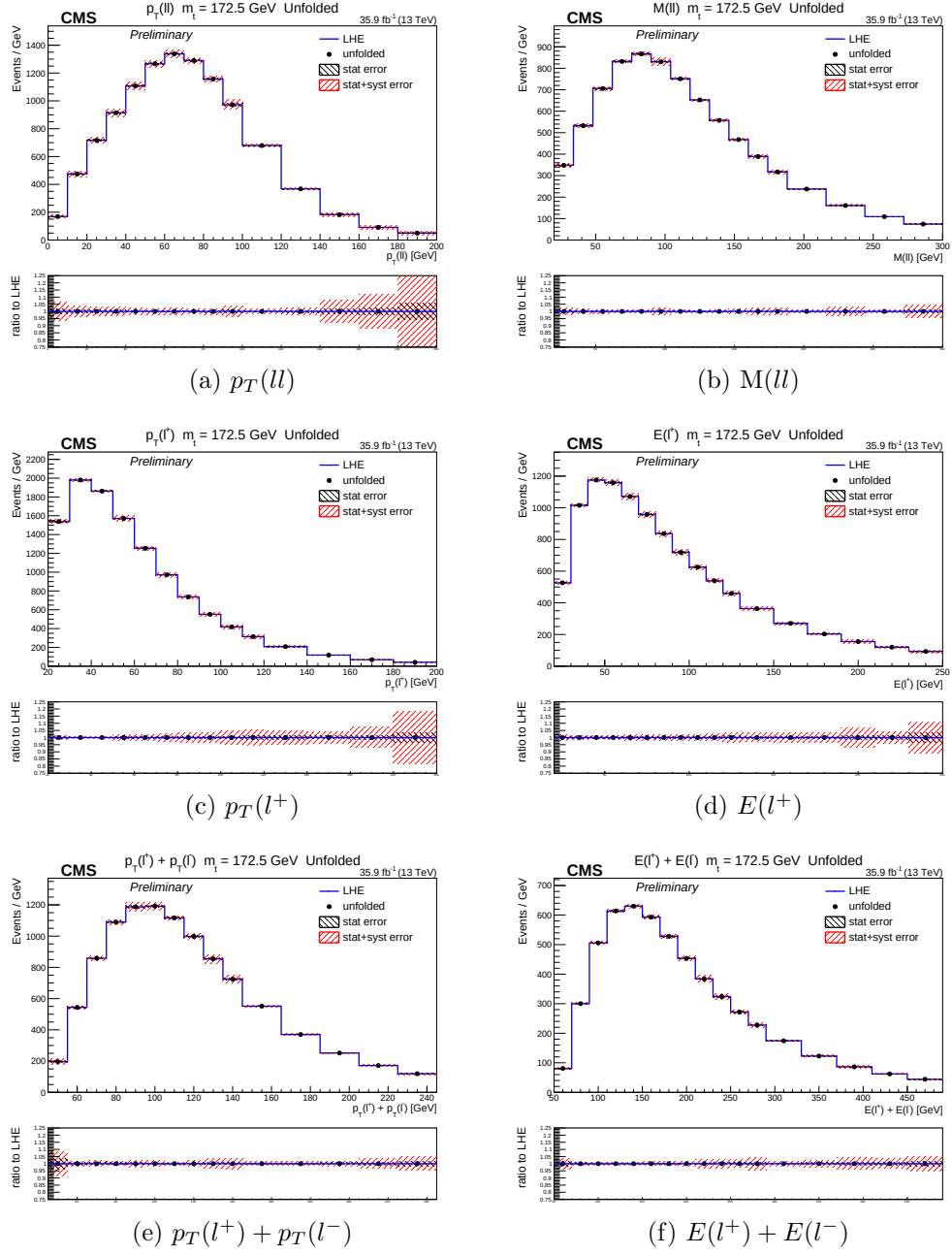


Figure 4.30: Unfolding of $t\bar{t} + tW$ distributions from nominal rec MC compared to LHE-level truth. Error bands are unfolding errors from input distribution (stat) and total unfolding error including systematics (stat+syst).

4.8.1 Regularization conditions

The naïve approach to solve Equation 4.17 is to substitute $\tilde{\mathbf{y}} \rightarrow \mathbf{y}$ and $\tilde{\mathbf{x}} \rightarrow \mathbf{x}$ and invert the equation to solve for $\mathbf{x}$. Off-diagonal terms in the response matrix will amplify statistical fluctuations in the observed density $\tilde{\mathbf{y}}$, which is usually mitigated with a smoothing process called *regularization*. Fortunately, the response matrices for leptonic observables are nearly diagonal, so regularization is not required.

This is quantified through the condition number of the response matrix, $\text{cond}(\mathbf{A})$, defined (Eq. 4.20) as the ratio of the maximum to minimum singular values obtained from the singular value decomposition $\mathbf{A} = \mathbf{U}\mathbf{D}\mathbf{V}^\top$, where $\mathbf{U}_{m \times m}$ and $\mathbf{V}_{n \times n}$ are orthogonal matrices and $\mathbf{D}_{m \times n}$ is a rectangular diagonal matrix. The singular values σ are the diagonal elements of $\mathbf{D}$. If the condition number of the response matrix is $\mathcal{O}(10)$, the matrix is said to be *well conditioned* and the inversion of Equation 4.17 does not introduce the unphysical high-frequency fluctuations that would require regularization. As shown in Table 4.9, all observables have low condition numbers and therefore do not require regularization.

$$\text{cond}(\mathbf{A}) = \frac{\sigma_{\max}}{\max(0, \sigma_{\min})} \quad (4.20)$$

Table 4.9: Condition numbers for LHE-level response matrices.

Observable	Condition Number
$p_T(l)$	35
$M(l)$	7
$p_T(l^+)$	28
$E(l^+)$	15
$p_T(l^+) + p_T(l^-)$	8
$E(l^+) + E(l^-)$	13

4.9 Top Quark MC Mass Extraction

The top quark MC mass m_t^{MC} and uncertainties are extracted from the reconstructed distributions using two approaches. The primary method uses a profile likelihood scan based on the Maximum Likelihood Estimator (MLE) principle. An alternative method uses the linearity of the Mellin Moments in m_t to extract the top quark mass.

After the CMS thesis endorsement, the MC mass results from the likelihood scan have been endorsed as CMS results by the collaboration. Masses extracted from the Mellin Moments are not to be construed as official CMS results.

4.9.1 Profile likelihood scan

The Higgs Combine tool is used to construct a likelihood function from binned signal and background templates. Details of the profile likelihood method can be found in Appendix A. The blinded analysis is performed using the Asimov dataset at $m_t = 172.5$ as pseudodata. The Asimov set is constructed from the nominal signal and background samples. Impact plots, showing postfit and systematic uncertainties for each observable, are shown in Figures 4.31 - 4.36 and all uncertainties are summarized in Table 4.10.

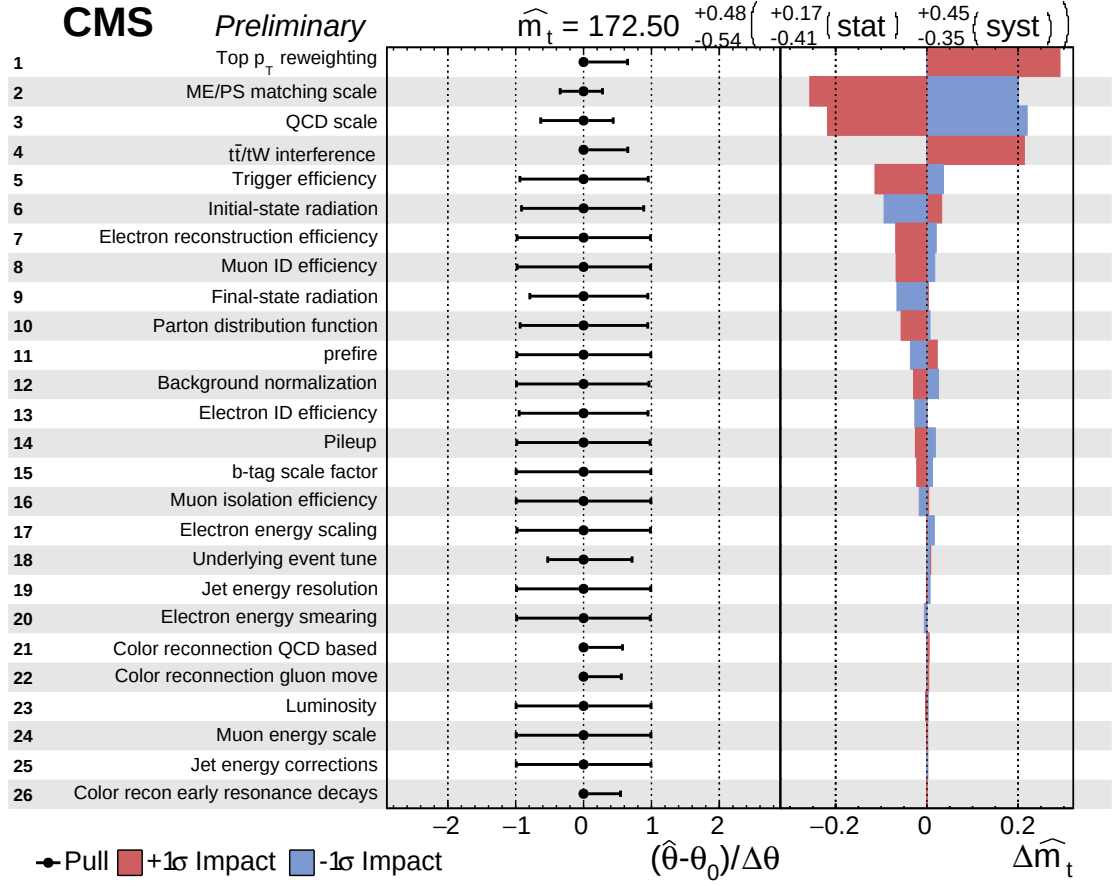


Figure 4.31: $p_T(l)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.

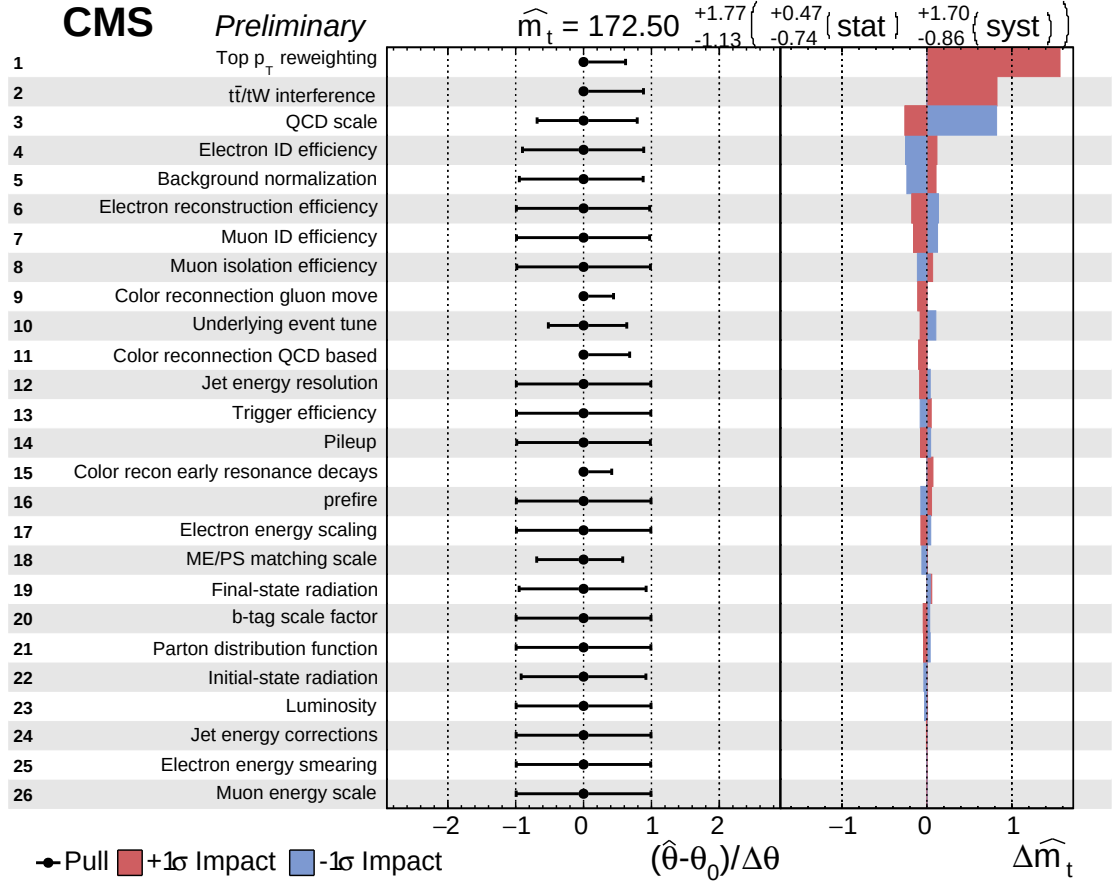


Figure 4.32: $M(ll)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.

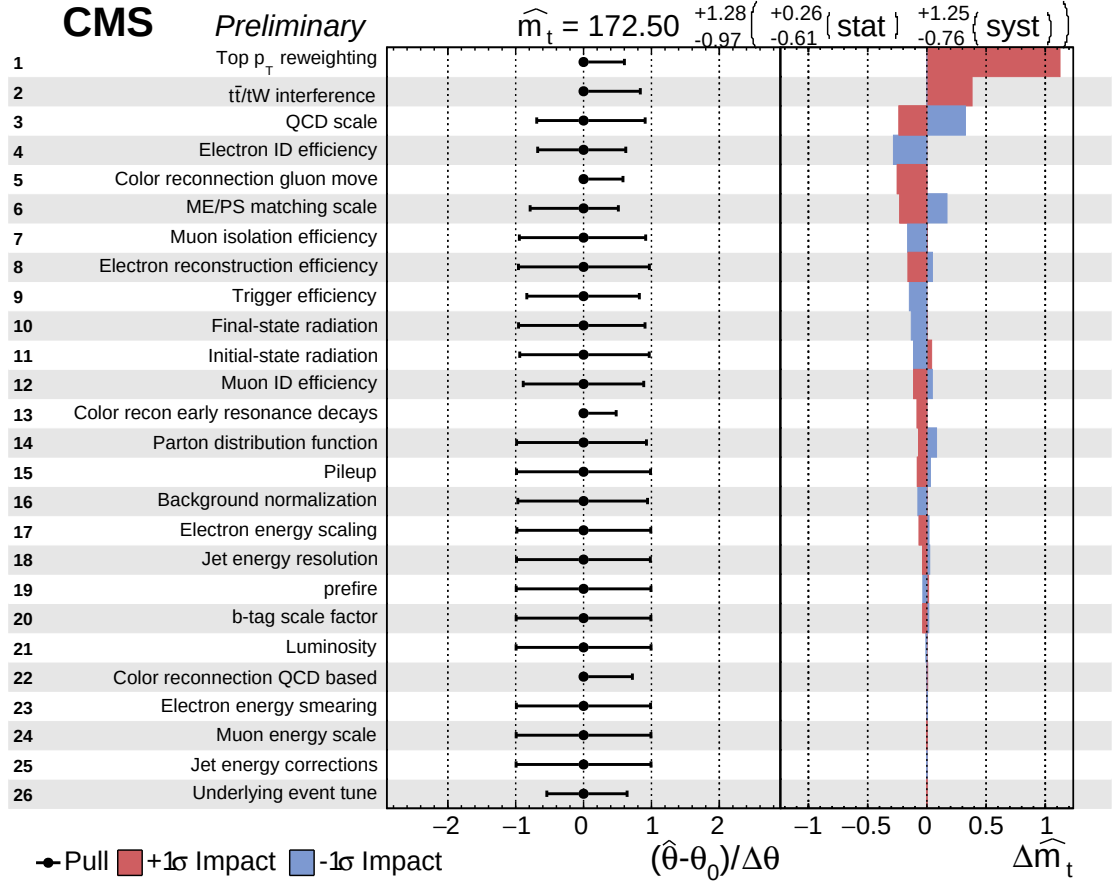


Figure 4.33: $p_T(l^+)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.

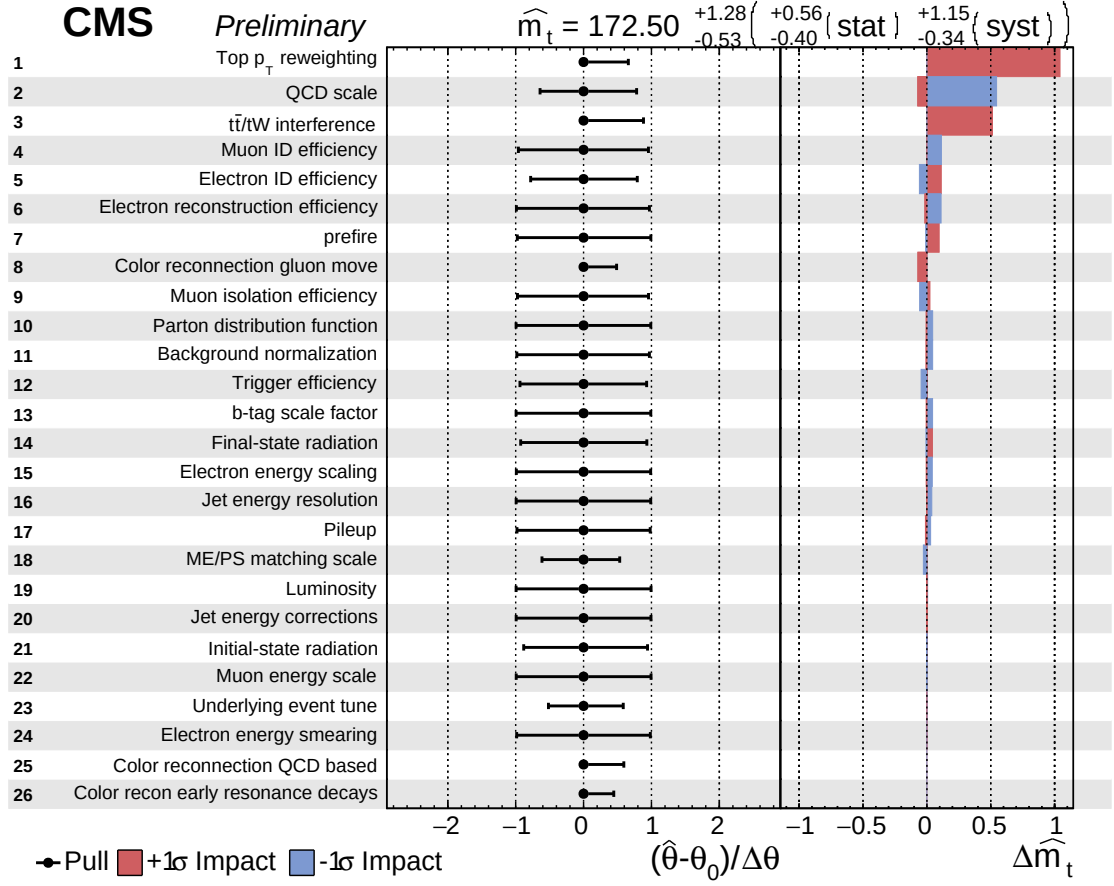


Figure 4.34: $E(l^+)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.

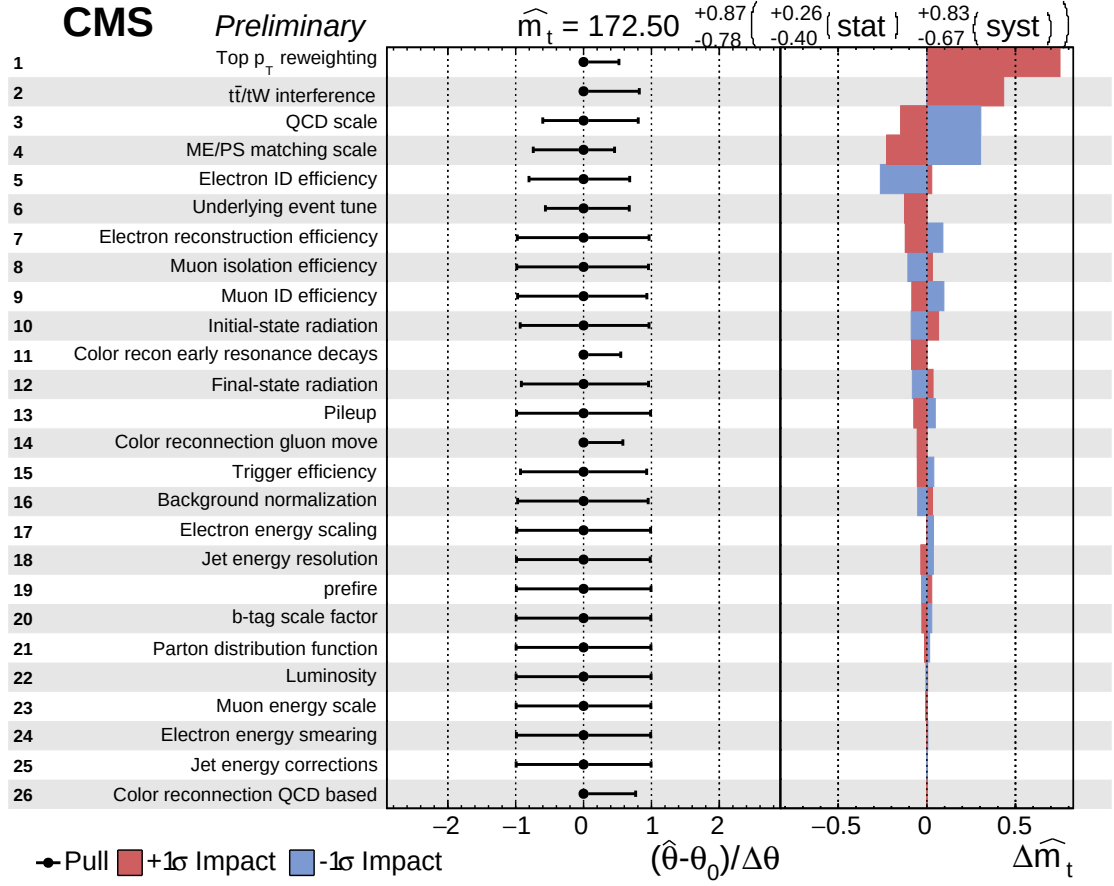


Figure 4.35: $p_T(l^+) + p_T(l^-)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.

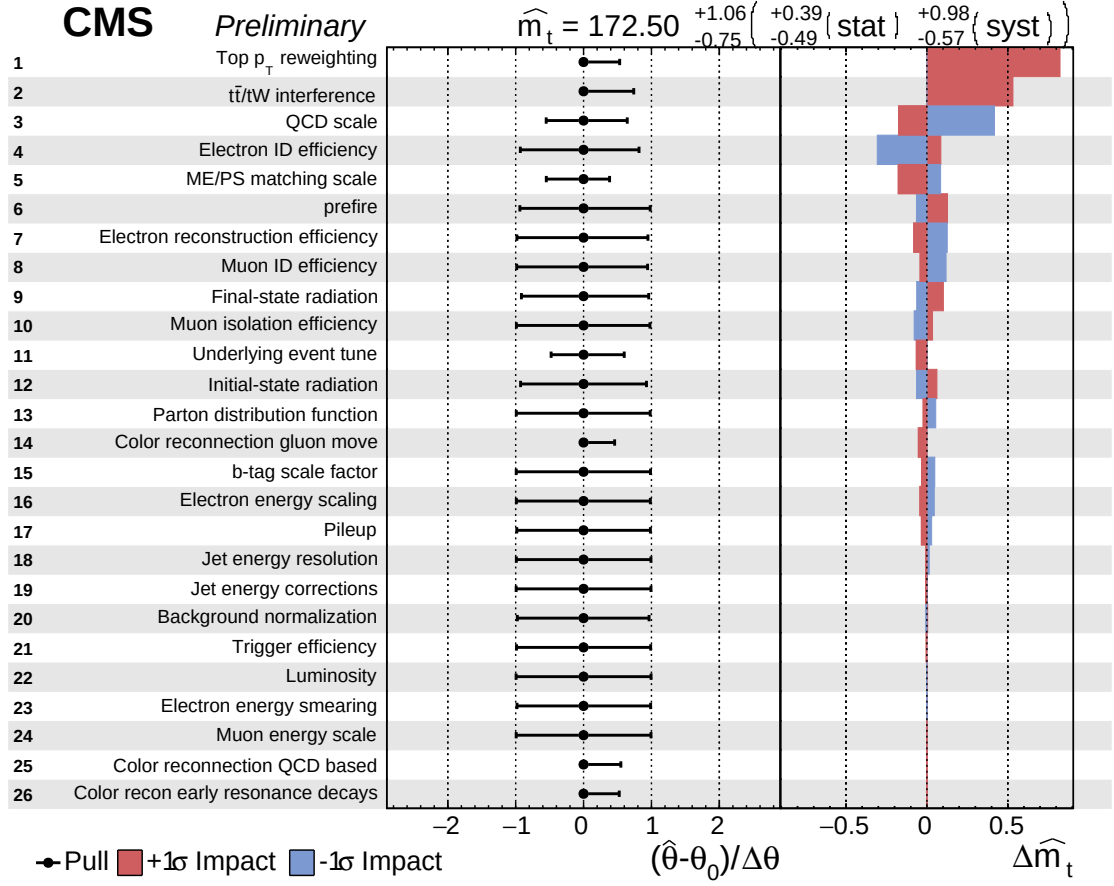
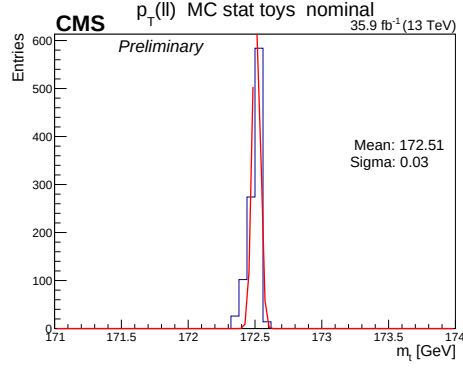
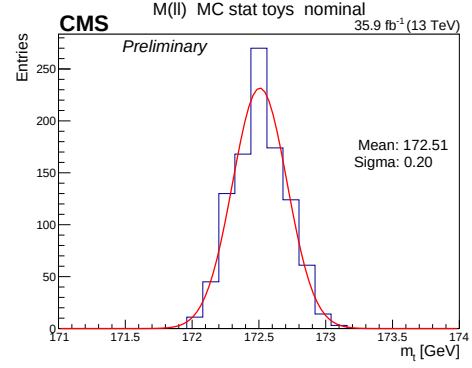


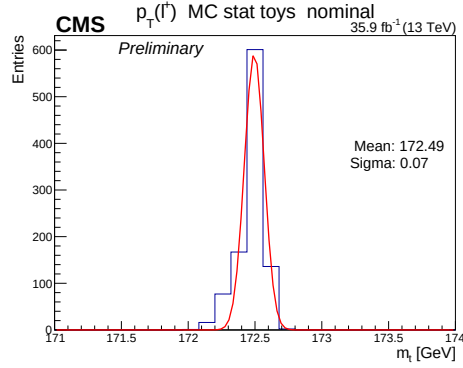
Figure 4.36: $E(l^+) + E(l^-)$ impact plot using the Asimov toy dataset based on $t\bar{t} + tW$ signal at $m_t = 172.5$ GeV and background MC.



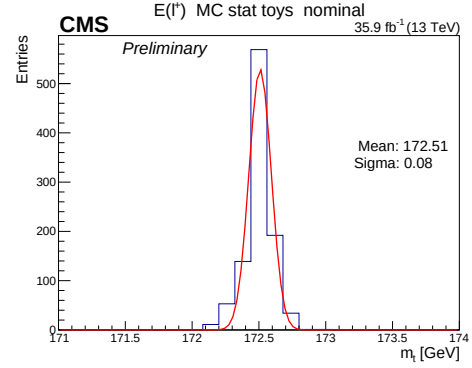
(a) $t\bar{t} + tW$ $p_T(l)$



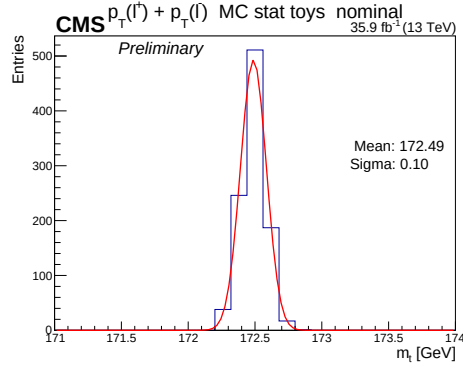
(b) $t\bar{t} + tW$ $M(l)$



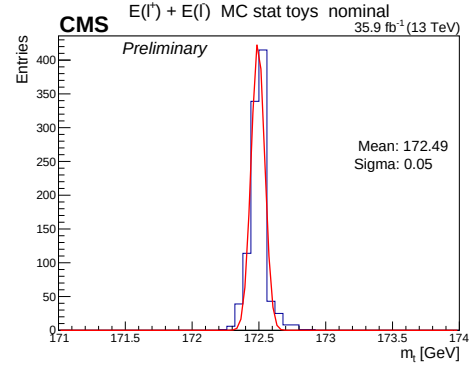
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$



(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$



(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

Figure 4.37: Distribution of masses extracted from toys for reconstructed $t\bar{t}$ and tW . The standard deviation is used as the MC statistical uncertainty.

Table 4.10: m_t and systematic uncertainties from profile likelihood scan in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	172.50	172.50	172.50	172.50	172.50	172.50
Statistical Uncertainty	-0.41/+0.17	-0.61/+0.26	-0.40/+0.56	-0.74/+0.47	-0.40/+0.26	-0.49/+0.39
Experimental Uncertainties						
Pileup	0.02 / -0.03	0.03 / -0.08	0.03 / -0.01	0.04 / -0.07	0.05 / -0.07	0.03 / -0.03
Luminosity	0.00 / -0.00	-0.01 / -0.00	0.01 / -0.00	-0.03 / 0.01	-0.01 / 0.00	-0.00 / 0.00
b-tagging	0.01 / -0.02	0.02 / -0.03	0.04 / -0.00	0.03 / -0.04	0.03 / -0.03	0.05 / -0.03
Electron ID	-0.03 / -0.00	-0.28 / 0.00	-0.06 / 0.11	-0.25 / 0.12	-0.26 / 0.03	-0.31 / 0.09
Electron reconstruction	0.02 / -0.07	0.05 / -0.16	0.11 / -0.02	0.14 / -0.18	0.09 / -0.12	0.13 / -0.08
Electron energy scale	0.02 / 0.00	0.02 / -0.07	0.04 / -0.00	0.05 / -0.07	0.04 / 0.00	0.05 / -0.04
Electron energy smear	-0.01 / 0.00	0.01 / 0.01	0.00 / 0.00	0.00 / 0.00	0.01 / 0.00	-0.00 / 0.00
Muon ID	0.02 / -0.07	0.05 / -0.11	0.11 / 0.00	0.13 / -0.16	0.10 / -0.08	0.12 / -0.04
Muon isolation	-0.02 / 0.00	-0.16 / 0.00	-0.06 / 0.02	-0.11 / 0.07	-0.11 / 0.03	-0.08 / 0.03
Muon momentum scale	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00	0.00 / 0.01	0.00 / 0.00
Trigger efficiency	0.04 / -0.11	-0.15 / 0.00	-0.05 / 0.00	-0.08 / 0.05	0.04 / -0.05	-0.00 / -0.00
Prefire	-0.04 / 0.02	-0.03 / 0.01	-0.01 / 0.10	-0.07 / 0.05	-0.03 / 0.03	-0.06 / 0.13
Jet energy scale	0.00 / 0.00	0.00 / 0.00	0.00 / 0.00	0.00 / -0.00	0.00 / 0.00	0.00 / -0.01
Jet energy resolution	0.01 / -0.00	0.02 / -0.04	0.04 / -0.00	0.04 / -0.08	0.04 / -0.03	0.01 / -0.01
Background normalization	0.03 / -0.03	-0.08 / -0.00	0.05 / -0.01	-0.24 / 0.11	-0.05 / 0.03	-0.01 / 0.00
Modeling Uncertainties						
Top p_T	0.29	1.12	1.04	1.57	0.75	0.82
μ_R/μ_F scale	0.22 / -0.22	0.33 / -0.24	0.55 / -0.07	0.82 / -0.26	0.30 / -0.15	0.42 / -0.17
ISR scale	-0.09 / 0.03	-0.11 / 0.04	-0.00 / 0.00	-0.04 / -0.00	-0.09 / 0.07	-0.06 / 0.06
FSR scale	-0.07 / 0.00	-0.13 / -0.00	-0.01 / 0.04	0.04 / 0.06	-0.08 / 0.03	-0.06 / 0.10
PDF	0.01 / -0.06	0.08 / -0.07	0.05 / -0.01	0.04 / -0.04	0.02 / -0.01	0.05 / -0.02
ME/PS matching	0.20 / -0.26	0.17 / -0.23	-0.03 / -0.00	-0.06 / 0.00	0.30 / -0.23	0.08 / -0.18
Underlying event tune	0.01 / 0.01	0.00 / -0.00	0.00 / 0.00	0.11 / -0.08	0.00 / -0.13	-0.00 / -0.07
Color reconnection	-0.00 / 0.01	-0.25 / 0.01	-0.07 / 0.00	-0.11 / 0.07	-0.09 / 0.00	-0.05 / 0.00
$t\bar{t}$ /tW interference	0.21	0.38	0.51	0.83	0.43	0.53
Systematic Uncertainty	-0.35/+0.45	-0.76/+1.25	-0.34/+1.15	-0.86/+1.70	-0.67/+0.83	-0.57/+0.98
MC Generator Uncertainties						
MADGRAPH	0.22	0.01	0.57	0.45	0.00	0.00
aMC@NLO	0.06	0.12	0.00	0.52	0.07	0.18
HERWIG++	0.21	1.69	0.44	0.99	0.84	0.51
MC Statistical Uncertainty	± 0.03	± 0.07	± 0.08	± 0.20	± 0.10	± 0.05
Total Uncertainty	-0.54/0.48	-0.98/1.28	-0.53/1.28	-1.15/1.78	-0.79/0.88	-0.75/1.06

4.9.2 Mellin moments

The lower Mellin moments of the kinematic distributions are linear in m_t and can be used to extract the top quark mass. The i^{th} moments are computed from the binned observables as

$$\mu^i = \frac{\sum_{k=1}^J x_k f(x)_k^i}{\sum_{k=1}^J f(x)_k}$$

where x_k are the observable values at the bin centers and $f(x)_k$ are the bin yields for a total of J bins. The uncertainty on the i^{th} moment is given by

$$\sigma^i(O) = \sqrt{\mu^{(2i)} - (\mu^i)^2}$$

where $\mu^{(2i)}$ is the $(2i)^{th}$ moment.

Figure 4.38 describes the procedure for m_t extraction from moments. For a given order moment and systematic, the lines f_C , f_U , and f_L represent the best fit to the moments of the central (nominal) distribution and upper and lower systematic variations, respectively. Using the nominal distribution moment μ^i , the central value of m_t and systematic variations are determined by inverting the fit lines:

$$m_{t_C} = f_C^{-1}(\mu^i) \quad m_{t_{T-}} = f_U^{-1}(\mu^i) \quad m_{t_{T+}} = f_L^{-1}(\mu^i) \quad (4.21)$$

$$\Delta m_{t_{\text{syst}}}^+ = m_{t_{T+}} - m_{t_C} \quad \Delta m_{t_{\text{syst}}}^- = m_{t_C} - m_{t_{T-}} \quad (4.22)$$

The statistical uncertainty on m_t is obtained from varying the moment by its uncertainty:

$$m_{t_{E-}} = f^{-1}(\mu^i - \delta\mu^i) \quad m_{t_{E+}} = f^{-1}(\mu^i + \delta\mu^i) \quad (4.23)$$

$$\Delta m_{t_{\text{stat}}^+} = m_{t_{E+}} - m_{t_C} \quad \Delta m_{t_{\text{stat}}^-} = m_{t_C} - m_{t_{E-}} \quad (4.24)$$

Moment fit lines for all masses and systematic variations can be found in Appendix C.1. The first two moments and calibrated masses from the nominal simulation are given in Table 4.11. A full listing of systematic impacts for the first and second moments can be found in Tables 4.12 and 4.13, respectively. This method is less sensitive to m_t than the profile likelihood scan and serves only as a cross-check.

Table 4.11: First two moments and calibrated masses of reco-level observables fit to nominal $t\bar{t} + tW$ simulation at $m_t = 172.5$ GeV. Uncertainties in extracted masses are statistical

Observable	Moment 1 [GeV]	m_t [GeV]	Moment 2 [GeV ²]	m_t [GeV]
$p_T(ll)$	65.65 ± 0.03	172.46 ± 0.25	5020.08 ± 4.18	172.46 ± 0.25
$M(ll)$	94.45 ± 0.05	172.31 ± 0.40	10629.64 ± 11.40	172.35 ± 0.42
$p_T(l^+)$	45.46 ± 0.02	172.58 ± 0.36	2472.47 ± 2.98	172.52 ± 0.39
$E(l^+)$	67.63 ± 0.04	172.46 ± 0.42	5763.49 ± 7.47	172.41 ± 0.48
$p_T(l^+) + p_T(l^-)$	103.40 ± 0.03	172.40 ± 0.20	11617.00 ± 7.92	172.40 ± 0.21
$E(l^+) + E(l^-)$	160.83 ± 0.06	172.41 ± 0.31	29534.98 ± 27.12	172.39 ± 0.36

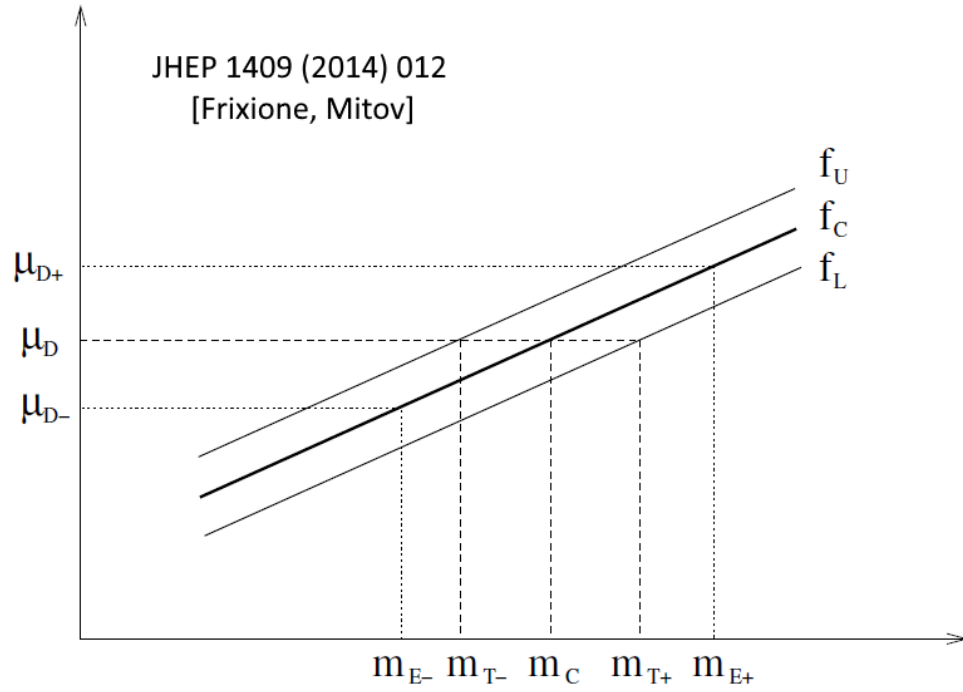


Figure 4.38: Extraction of m_t and its uncertainties using moments. For a given order moment and systematic, the lines f_C , f_U , and f_L represent the best fit to the moments of the central (nominal) distribution and upper and lower systematic variations, respectively, and are essentially parallel [41].

Table 4.12: m_t and systematic uncertainties from first moments in GeV.

Observable	$p_T(l\ell)$	$p_T(l^+)$	$E(l^+)$	$M(l\ell)$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	172.46	172.47	172.38	172.39	172.41	172.34
Statistical Uncertainty	± 0.15	± 0.21	± 0.34	± 0.26	± 0.15	± 0.24
Experimental Uncertainties						
Pileup	0.05 / -0.04	0.09 / -0.09	0.04 / -0.05	0.08 / -0.08	0.08 / -0.08	0.03 / -0.03
Luminosity	-0.00 / -0.00	-0.00 / 0.00	-0.00 / -0.00	0.00 / -0.00	-0.00 / -0.00	0.00 / 0.00
b-tagging	0.04 / -0.04	0.05 / -0.05	0.08 / -0.08	0.06 / -0.05	0.05 / -0.05	0.10 / -0.10
Electron ID	0.02 / -0.02	-0.09 / 0.09	-0.40 / 0.38	-0.26 / 0.25	-0.17 / 0.17	-0.27 / 0.26
Electron reconstruction	0.09 / -0.09	0.18 / -0.18	0.20 / -0.19	0.22 / -0.22	0.16 / -0.16	0.24 / -0.24
Electron energy scale	0.06 / -0.07	0.07 / -0.07	0.07 / -0.08	0.08 / -0.07	0.07 / -0.07	0.09 / -0.11
Electron energy smear	-0.00 / -0.01	0.01 / 0.02	-0.03 / -0.02	-0.00 / 0.01	0.00 / 0.02	-0.01 / 0.01
Muon ID	0.08 / -0.08	0.17 / -0.17	0.14 / -0.14	0.20 / -0.20	0.15 / -0.15	0.22 / -0.22
Muon isolation	-0.02 / 0.02	-0.08 / 0.08	-0.12 / 0.12	-0.12 / 0.12	-0.08 / 0.08	-0.09 / 0.09
Muon momentum scale	0.00 / -0.01	0.00 / -0.01	-0.00 / 0.01	0.00 / -0.01	-0.00 / -0.01	-0.00 / -0.01
Trigger efficiency	0.16 / -0.15	0.01 / -0.01	-0.08 / 0.08	-0.09 / 0.08	0.05 / -0.05	0.01 / -0.01
Prefire	-0.06 / 0.06	-0.05 / 0.05	-0.17 / 0.17	-0.09 / 0.09	-0.05 / 0.05	-0.25 / 0.25
Jet energy scale	0.00 / -0.00	0.00 / 0.00	0.00 / -0.01	0.00 / -0.01	-0.00 / -0.00	0.00 / -0.01
Jet energy resolution	0.01 / -0.00	0.07 / -0.08	0.06 / -0.03	0.07 / -0.08	0.06 / -0.06	0.02 / 0.00
Background normalization	-0.05 / 0.06	0.01 / -0.02	-0.07 / 0.08	0.19 / -0.25	0.05 / -0.07	-0.00 / -0.01
Modeling Uncertainties						
Top p_T	0.81	2.94	2.62	3.66	2.29	2.78
μ_R/μ_F scale	0.62 / -0.84	1.02 / -0.19	1.33 / -0.43	1.66 / -0.43	0.79 / -0.27	1.24 / -0.78
ISR scale	-0.16 / 0.15	-0.00 / 0.08	-0.15 / -0.03	-0.00 / -0.10	-0.07 / 0.10	-0.06 / 0.12
FSR scale	-0.21 / 0.03	-0.18 / -0.01	-0.19 / 0.10	0.03 / 0.09	-0.14 / 0.03	-0.09 / 0.16
PDF	0.03 / -0.05	0.15 / -0.15	0.08 / -0.08	0.01 / -0.10	0.01 / -0.04	0.10 / -0.10
ME/PS matching	1.54 / -1.37	0.40 / -0.84	0.06 / -1.21	-0.28 / -0.17	0.63 / -0.83	0.38 / -1.58
Underlying event tune	0.06 / -0.01	0.30 / -0.16	0.19 / -0.28	0.16 / -0.16	-0.08 / -0.10	-0.51 / -0.73
Color reconnection	-0.09 / 0.15	-0.49 / 0.15	-1.06 / 0.01	-0.72 / 0.01	-0.33 / 0.00	-0.87 / 0.00
$t\bar{t}$ /tW interference	0.78	0.91	0.98	1.31	0.80	1.27
Systematic Uncertainty	-1.77 / +1.14	-1.39 / +3.09	-1.84 / +2.84	-1.73 / +3.91	-1.18 / +2.44	-2.17 / +3.13
MC Generator Uncertainties						
MADGRAPH	-1.31	-0.16	-2.62	3.82	0.23	-0.79
aMC@NLO	-0.22	1.54	0.27	6.06	0.35	1.22
HERWIG++	3.95	5.83	3.78	6.25	4.65	3.26
MC Statistical Uncertainty	± 0.08	± 0.11	± 0.17	± 0.14	± 0.08	± 0.13
Total Uncertainty	-1.78 / 1.15	-1.41 / 3.10	-1.88 / 2.86	-1.76 / 3.92	-1.20 / 2.44	-2.19 / 3.14

Table 4.13: m_t and systematic uncertainties from second moments in GeV.

Observable	$p_T(l)$	$E(l^+)$	$M(l)$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	172.46	172.32	172.43	172.41	172.30
Statistical Uncertainty	± 0.15	± 0.39	± 0.28	± 0.16	± 0.28
Experimental Uncertainties					
Pileup	0.05 / -0.05	0.02 / -0.02	0.07 / -0.07	0.08 / -0.08	0.02 / -0.02
Luminosity	-0.00 / 0.00	-0.00 / -0.00	0.00 / -0.00	-0.00 / -0.00	0.00 / 0.00
b-tagging	0.04 / -0.04	0.09 / -0.09	0.06 / -0.06	0.05 / -0.05	0.11 / -0.11
Electron ID	0.05 / -0.04	-0.31 / 0.30	-0.19 / 0.11	-0.11 / 0.11	-0.21 / 0.20
Electron reconstruction	0.09 / -0.08	0.21 / -0.21	0.22 / -0.22	0.16 / -0.16	0.26 / -0.26
Electron energy scale	0.07 / -0.07	0.07 / -0.09	0.07 / -0.06	0.07 / -0.07	0.10 / -0.11
Electron energy smear	0.00 / -0.01	-0.04 / -0.03	-0.00 / 0.01	0.01 / 0.02	-0.01 / 0.01
Muon ID	0.08 / -0.08	0.18 / -0.17	0.21 / -0.21	0.16 / -0.16	0.26 / -0.25
Muon isolation	-0.02 / 0.02	-0.09 / 0.09	-0.11 / 0.11	-0.07 / 0.07	-0.07 / 0.07
Muon momentum scale	0.00 / -0.01	-0.01 / 0.01	0.00 / -0.02	-0.00 / -0.01	-0.00 / -0.01
Trigger efficiency	0.16 / -0.16	-0.03 / 0.03	-0.09 / 0.09	0.06 / -0.05	0.00 / -0.00
Prefire	-0.07 / 0.07	-0.20 / 0.20	-0.10 / 0.10	-0.05 / 0.05	-0.28 / 0.28
Jet energy scale	0.00 / -0.00	-0.00 / -0.01	0.00 / -0.01	-0.00 / -0.00	0.00 / -0.01
Jet energy resolution	0.01 / -0.00	0.06 / -0.02	0.07 / -0.07	0.07 / -0.07	0.02 / 0.01
Background normalization	-0.06 / 0.07	-0.10 / 0.11	0.14 / -0.19	0.04 / -0.06	-0.02 / 0.00
Modeling Uncertainties					
Top p_T	0.97	2.93	3.74	2.47	3.09
μ_R/μ_F scale	0.72 / -0.93	1.41 / -0.44	1.77 / -0.43	0.86 / -0.28	1.28 / -0.86
ISR scale	-0.16 / 0.15	-0.11 / -0.03	0.02 / -0.13	-0.05 / 0.10	-0.05 / 0.15
FSR scale	-0.24 / 0.03	-0.19 / 0.07	0.04 / 0.10	-0.14 / 0.04	-0.07 / 0.18
PDF	0.03 / -0.04	0.09 / -0.09	0.03 / -0.09	0.01 / -0.04	0.12 / -0.12
ME/PS matching	1.59 / -1.55	0.00 / -1.40	-0.27 / -0.12	0.65 / -0.91	0.34 / -1.74
Underlying event tune	0.01 / -0.03	-0.04 / -0.46	0.19 / -0.10	-0.11 / -0.08	-0.63 / -0.87
Color reconnection	-0.15 / 0.14	-1.38 / 0.00	-0.54 / 0.15	-0.33 / 0.00	-1.09 / 0.00
$t\bar{t}/tW$ interference	0.85	1.12	1.37	0.86	1.41
Systematic Uncertainty	-1.85 / +1.30	-2.06 / +3.16	-1.84 / +4.00	-1.28 / +2.62	-2.36 / +3.48
MC Generator Uncertainties					
MADGRAPH	-1.38	-2.53	3.62	-0.08	-1.02
aMC@NLO	0.05	0.24	6.68	0.59	1.41
HERWIG++	4.24	3.82	6.26	4.82	3.27
MC Statistical Uncertainty	± 0.08	± 0.18	± 0.13	± 0.08	± 0.15
Total Uncertainty	-1.86 / 1.31	-2.11 / 3.19	-1.86 / 4.01	-1.29 / 2.63	-2.38 / 3.50

4.9.3 Unblinded Results

The unblinded results of the likelihood fits to collision data are shown below. Figures 4.39-4.44 give the unblinded systematic impacts for each observable and a summary of the results from all observables is shown in Table 4.15. The postfit plots of data and simulation are shown in Figure 4.45. Table 4.14 gives the unblinded moments and tables 4.16 and 4.17 show the unblinded extracted masses from the first two moments. A summary of the most sensitive observable, $p_T(l\bar{l})$, is shown in Table 4.18. The MC stat uncertainty toys and likelihood scans for $p_T(l\bar{l})$ are shown in Figures 4.46 and 4.47, respectively.

Table 4.14: First two moments and calibrated masses of reco-level observables fit to data. Uncertainties in extracted masses are statistical

Observable	Moment 1 [GeV]	m_t [GeV]	Moment 2 [GeV ²]	m_t [GeV]
$p_T(l\bar{l})$	65.35 ± 0.08	171.18 ± 1.07	4948.46 ± 11.90	171.01 ± 1.14
$M(l\bar{l})$	93.16 ± 0.13	161.20 ± 3.79	10353.99 ± 30.65	162.11 ± 3.79
$p_T(l^+)$	45.14 ± 0.06	167.50 ± 2.29	2432.39 ± 8.15	167.29 ± 2.53
$E(l^+)$	68.39 ± 0.11	167.71 ± 2.61	5812.59 ± 21.68	167.48 ± 2.89
$p_T(l^+) + p_T(l^-)$	102.53 ± 0.09	167.00 ± 2.06	11418.01 ± 22.44	167.06 ± 2.19
$E(l^+) + E(l^-)$	159.74 ± 0.19	167.20 ± 2.70	29135.45 ± 78.44	167.11 ± 2.98

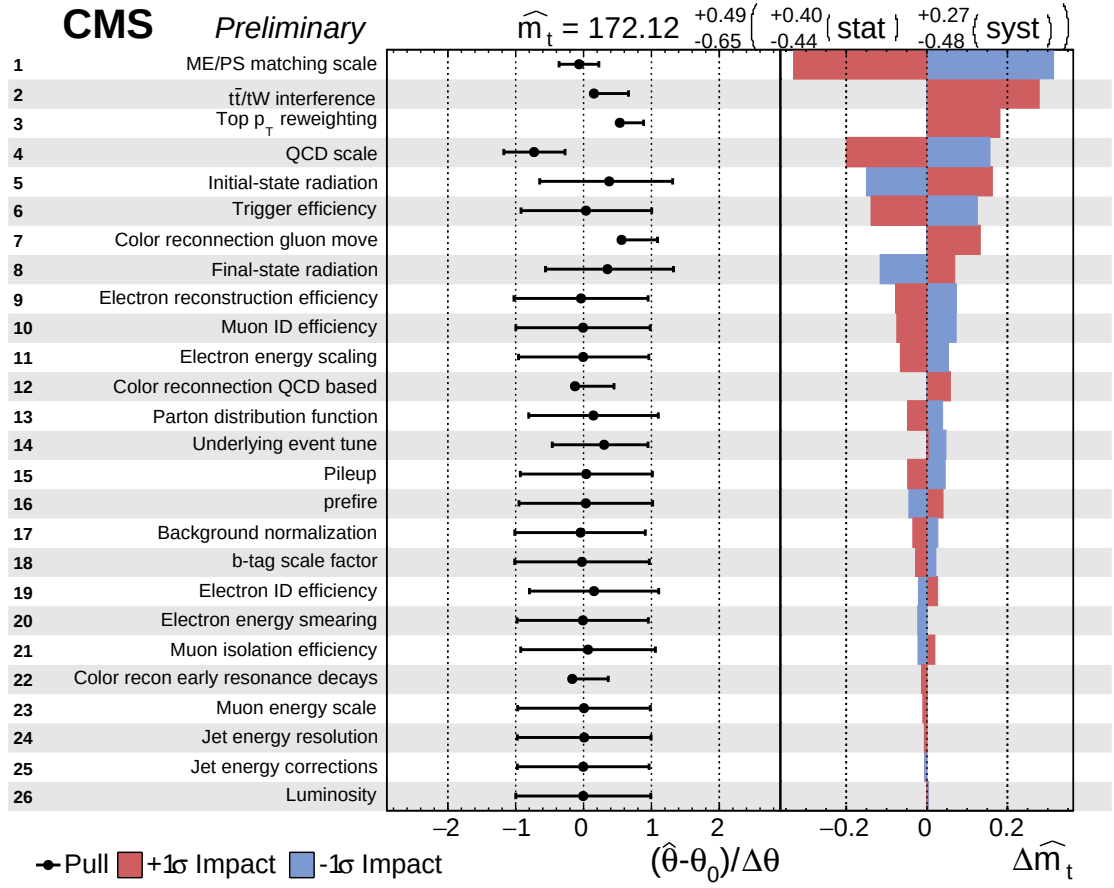


Figure 4.39: $p_T(l\bar{l})$ impact plot of likelihood fit to data.

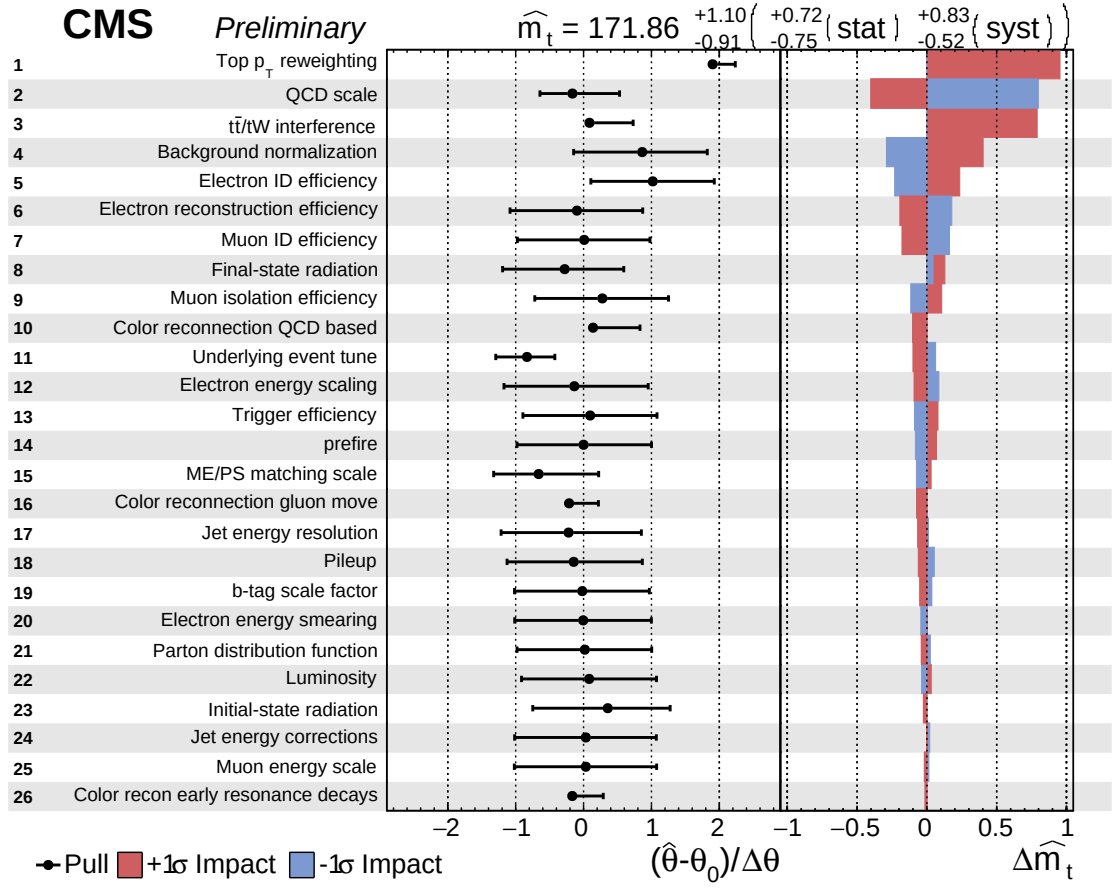


Figure 4.40: $M(l)$ impact plot of likelihood fit to data.

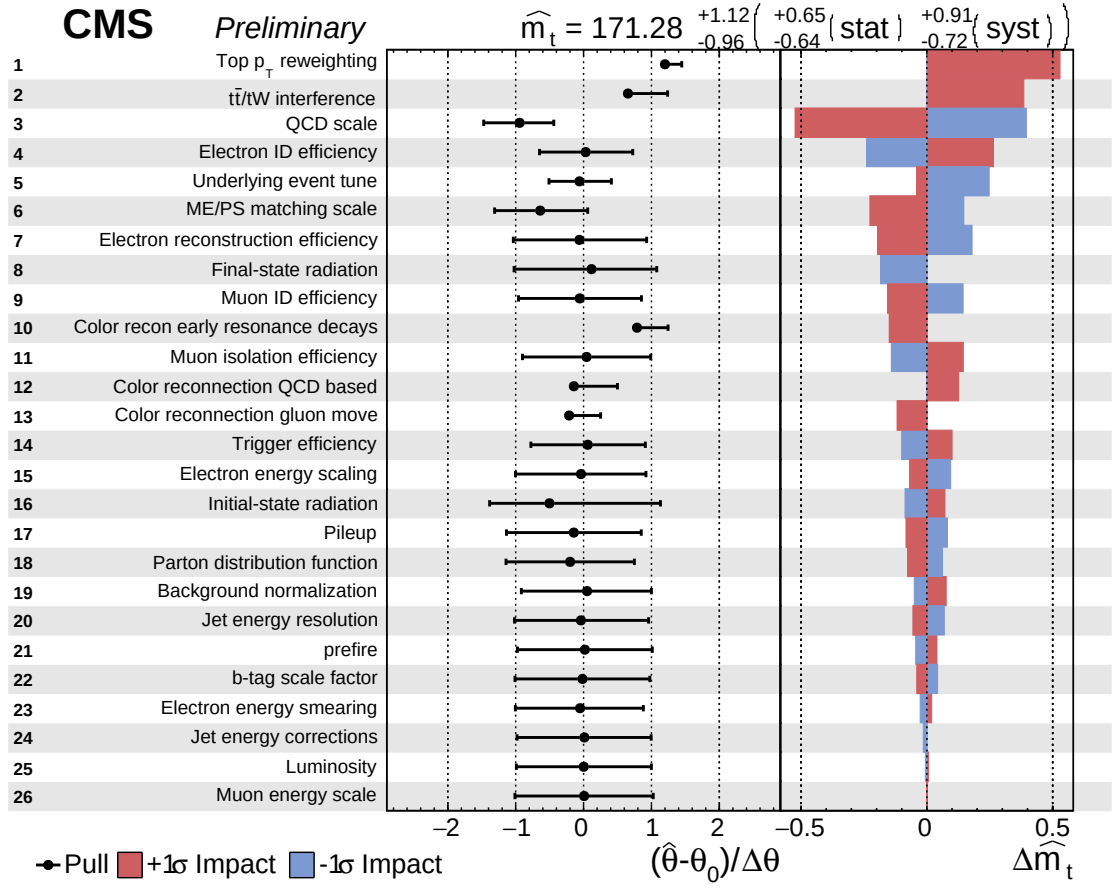


Figure 4.41: $p_T(l^+)$ impact plot of likelihood fit to data.

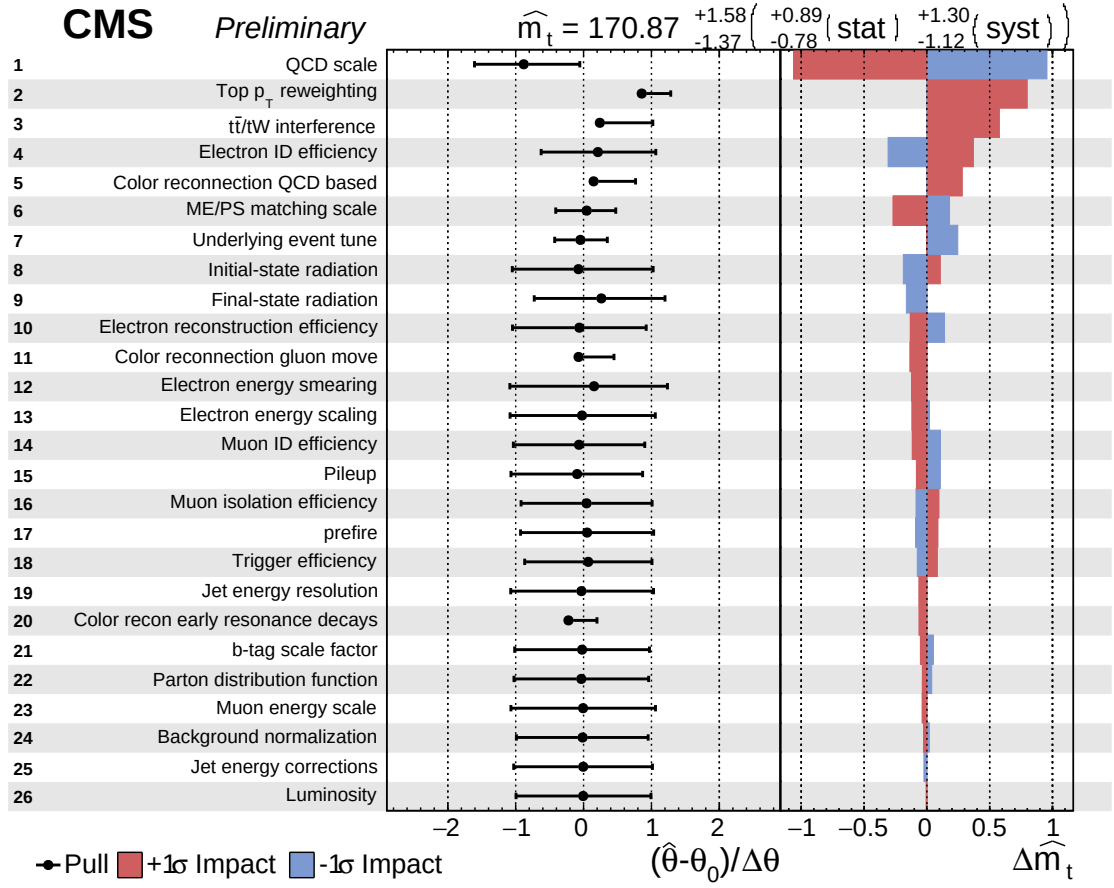


Figure 4.42: $E(l^+)$ impact plot of likelihood fit to data.

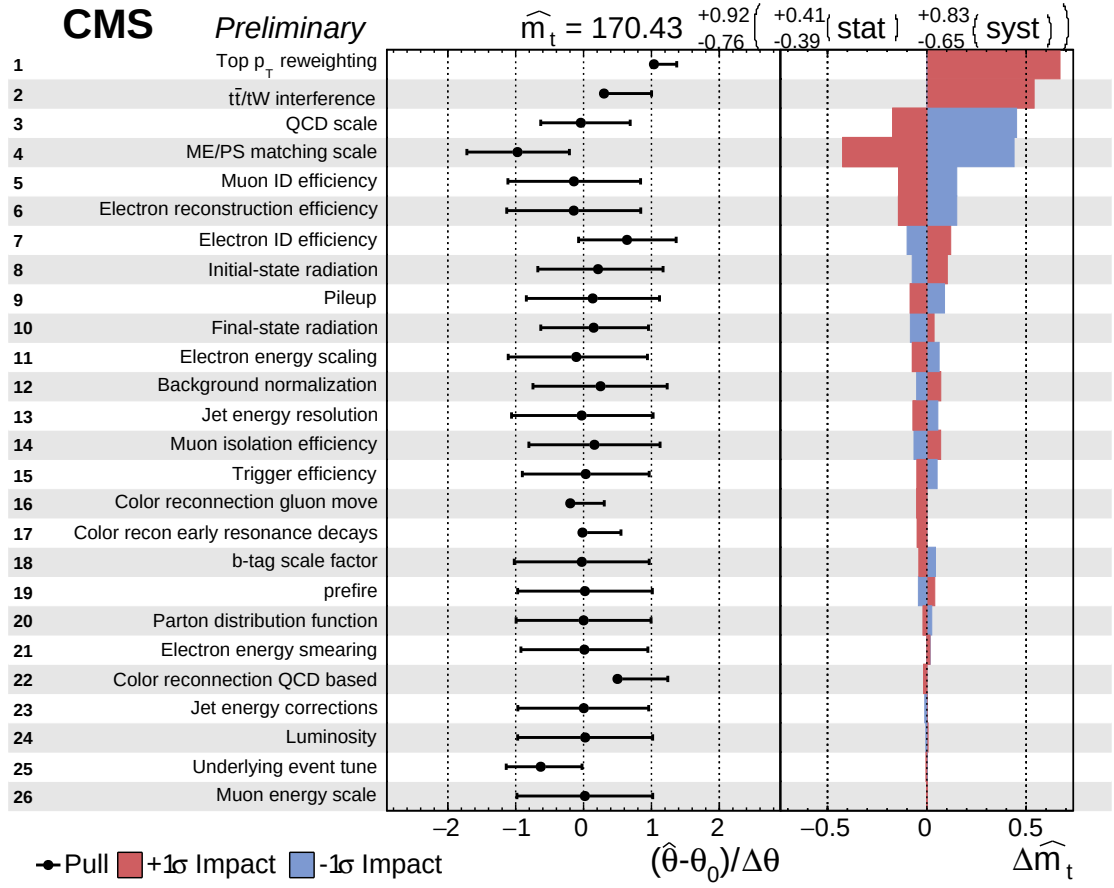


Figure 4.43: $p_T(l^+) + p_T(l^-)$ impact plot of likelihood fit to data.

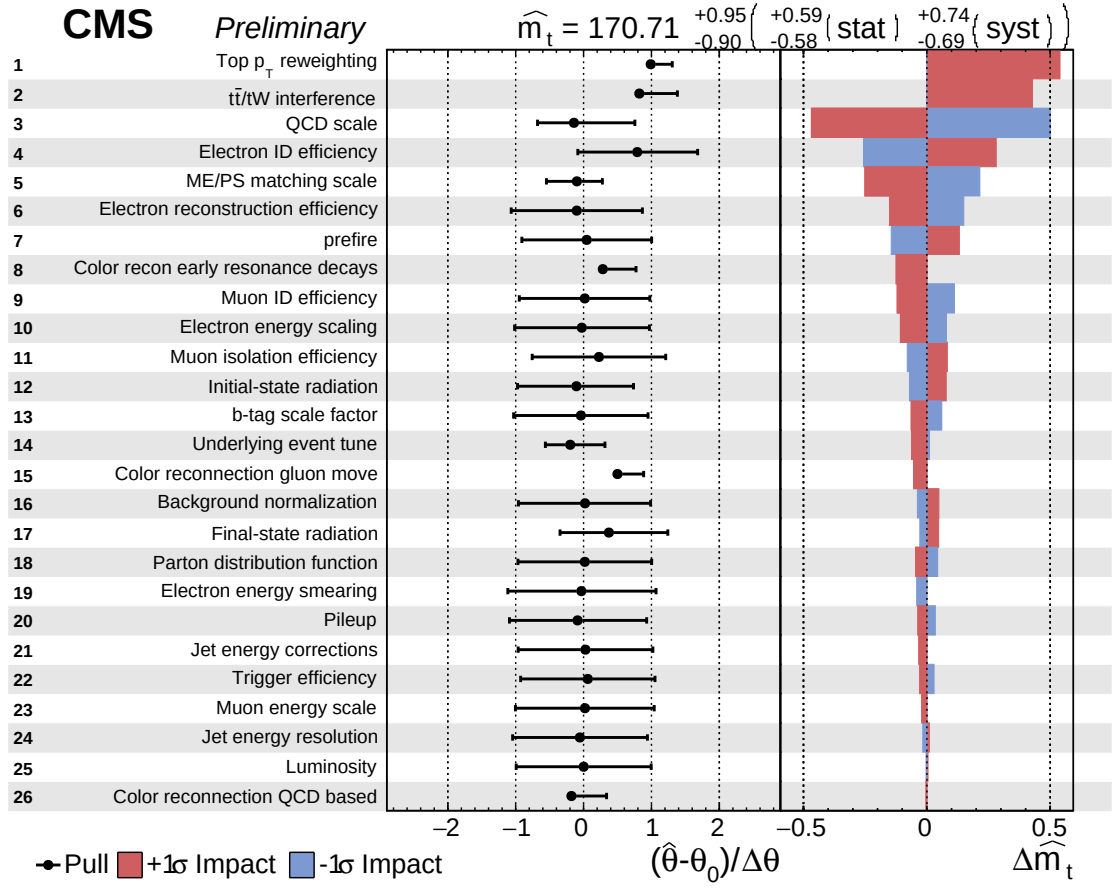


Figure 4.44: $E(l^+) + E(l^-)$ impact plot of likelihood fit to data.

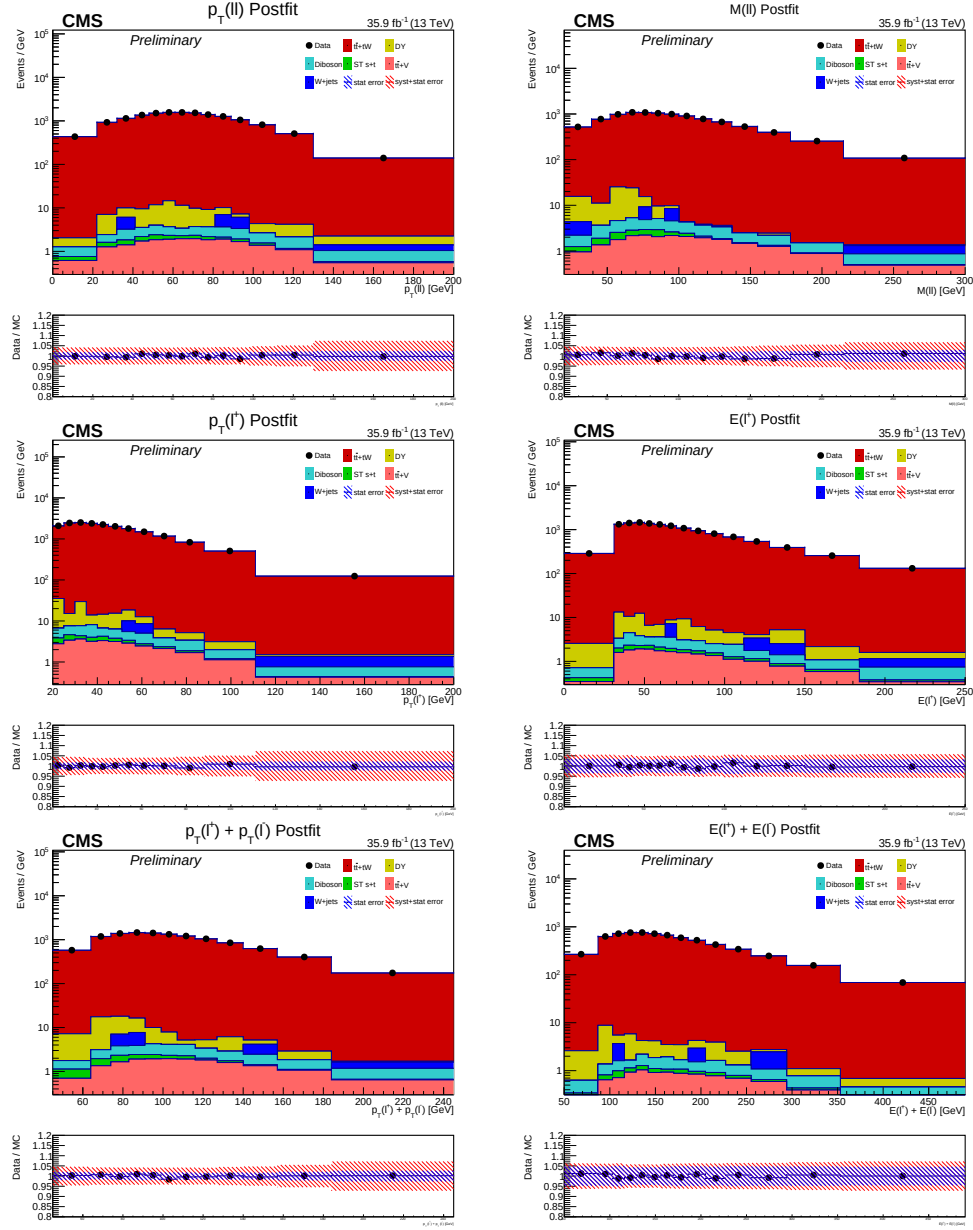


Figure 4.45: Postfit plots of likelihood fit to collision data.

Table 4.15: m_t and systematic uncertainties from profile likelihood scan fit to data in GeV.

Observable	$p_T(l\ell)$	$p_T(l^+)$	$E(l^+)$	$M(l\ell)$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	172.12	171.28	170.87	171.86	170.43	170.71
Statistical Uncertainty	-0.44/+0.40	-0.64/+0.65	-0.78/+0.89	-0.75/+0.72	-0.39/+0.41	-0.58/+0.59
Experimental Uncertainties						
Pileup	0.05 / -0.05	0.08 / -0.08	0.11 / -0.08	0.05 / -0.06	0.09 / -0.08	0.03 / -0.04
Luminosity	0.00 / 0.00	-0.01 / 0.01	-0.00 / -0.00	-0.04 / 0.03	-0.01 / 0.01	-0.00 / 0.00
b-tagging	0.02 / -0.03	0.04 / -0.04	0.05 / -0.05	0.04 / -0.05	0.04 / -0.04	0.06 / -0.06
Electron ID	-0.02 / 0.03	-0.24 / 0.26	-0.31 / 0.37	-0.23 / 0.24	-0.10 / 0.12	-0.26 / 0.28
Electron reconstruction	0.07 / -0.08	0.18 / -0.20	0.14 / -0.13	0.18 / -0.19	0.15 / -0.14	0.15 / -0.15
Electron energy scale	0.05 / -0.07	0.09 / -0.07	0.02 / -0.12	0.09 / -0.09	0.06 / -0.07	0.08 / -0.11
Electron energy smear	-0.02 / -0.01	-0.03 / 0.02	-0.12 / -0.12	-0.04 / 0.00	0.02 / 0.01	-0.04 / -0.03
Muon ID	0.07 / -0.07	0.14 / -0.16	0.11 / -0.12	0.16 / -0.18	0.15 / -0.14	0.11 / -0.12
Muon isolation	-0.02 / 0.02	-0.14 / 0.14	-0.09 / 0.10	-0.12 / 0.11	-0.06 / 0.07	-0.08 / 0.08
Muon momentum scale	0.00 / -0.01	0.00 / 0.00	-0.02 / -0.04	0.01 / -0.02	0.00 / 0.00	-0.02 / -0.02
Trigger efficiency	0.13 / -0.14	-0.10 / 0.10	-0.08 / 0.08	-0.09 / 0.08	0.05 / -0.05	0.03 / -0.03
Prefire	-0.04 / 0.04	-0.04 / 0.04	-0.09 / 0.09	-0.08 / 0.07	-0.04 / 0.04	-0.14 / 0.13
Jet energy scale	-0.01 / -0.01	-0.01 / -0.01	-0.02 / -0.02	0.02 / 0.00	-0.01 / 0.00	-0.01 / -0.03
Jet energy resolution	-0.01 / -0.01	0.07 / -0.05	-0.03 / -0.06	0.01 / -0.06	0.06 / -0.07	-0.02 / 0.01
Background normalization	0.03 / -0.03	-0.05 / 0.08	0.02 / -0.02	-0.29 / 0.40	-0.05 / 0.07	-0.04 / 0.05
Modeling Uncertainties						
Top p_T	0.18	0.53	0.80	0.95	0.67	0.54
μ_R/μ_F scale	0.16 / -0.20	0.40 / -0.52	0.96 / -1.06	0.80 / -0.40	0.45 / -0.17	0.50 / -0.47
ISR scale	-0.15 / 0.16	-0.09 / 0.07	-0.19 / 0.11	-0.01 / -0.02	-0.07 / 0.10	-0.07 / 0.08
FSR scale	-0.12 / 0.07	-0.18 / -0.01	-0.16 / -0.04	0.04 / 0.13	-0.08 / 0.04	-0.03 / 0.05
PDF	0.04 / -0.05	0.06 / -0.08	0.04 / -0.04	0.02 / -0.04	0.02 / -0.02	0.04 / -0.05
ME/PS matching	0.31 / -0.33	0.15 / -0.23	0.18 / -0.27	-0.07 / 0.03	0.44 / -0.42	0.21 / -0.25
Underlying event tune	0.05 / 0.00	0.25 / -0.04	0.25 / 0.00	0.06 / -0.10	0.00 / -0.00	0.01 / -0.06
Color reconnection	-0.01 / 0.13	-0.15 / 0.13	-0.13 / 0.28	-0.10 / 0.00	-0.05 / 0.00	-0.12 / 0.00
$t\bar{t}/tW$ interference	0.28	0.39	0.58	0.79	0.54	0.43
Systematic Uncertainty	-0.47/+0.27	-0.72/+0.91	-1.12/+1.30	-0.52/+0.83	-0.65/+0.83	-0.69/+0.74
MC Generator Uncertainties						
MADGRAPH	-0.24	0.24	-0.29	0.31	0.02	-0.07
aMC@NLO	-0.08	0.05	0.12	0.15	0.06	0.11
HERWIG++	0.30	1.94	1.00	0.47	0.74	1.06
MC Statistical Uncertainty	± 0.14	± 0.14	± 0.74	± 0.92	± 0.41	± 0.40
Total Uncertainty	-0.66/0.51	-0.97/1.13	-1.55/1.74	-1.30/1.44	-0.86/1.01	-0.99/1.03

Table 4.16: m_t and systematic uncertainties from first moments fit to data in GeV.

Observable	$p_T(l\ell)$	$p_T(l^+)$	$E(l^+)$	$M(l\ell)$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	171.18	167.50	167.71	161.20	167.00	167.20
Statistical Uncertainty	± 0.75	± 0.99	± 1.25	± 1.10	± 0.58	± 0.90
Experimental Uncertainties						
Pileup	0.05 / -0.04	0.08 / -0.07	0.10 / -0.09	0.14 / -0.14	0.06 / -0.06	0.10 / -0.10
Luminosity	-0.00 / -0.00	-0.00 / 0.00	-0.00 / -0.00	-0.00 / -0.00	-0.00 / -0.00	0.00 / 0.00
b-tagging	0.03 / -0.03	0.04 / -0.04	0.06 / -0.06	0.05 / -0.05	0.04 / -0.04	0.08 / -0.08
Electron ID	-0.07 / 0.07	-0.62 / 0.60	-0.44 / 0.42	-0.47 / 0.46	-0.26 / 0.26	-0.35 / 0.34
Electron reconstruction	0.09 / -0.09	0.17 / -0.16	0.18 / -0.17	0.20 / -0.20	0.15 / -0.15	0.19 / -0.19
Electron energy scale	0.06 / -0.07	0.09 / -0.02	0.08 / -0.04	0.13 / -0.08	0.09 / -0.05	0.07 / -0.08
Electron energy smear	0.01 / -0.00	0.00 / 0.02	-0.01 / 0.06	0.03 / 0.01	0.04 / -0.00	0.03 / -0.01
Muon ID	0.08 / -0.08	0.12 / -0.12	0.16 / -0.16	0.16 / -0.16	0.16 / -0.16	0.16 / -0.16
Muon isolation	-0.04 / 0.04	-0.20 / 0.19	-0.09 / 0.08	-0.17 / 0.17	-0.09 / 0.09	-0.09 / 0.09
Muon momentum scale	0.00 / -0.01	0.02 / 0.02	0.01 / 0.02	-0.14 / 0.01	0.00 / 0.01	-0.04 / -0.01
Trigger efficiency	0.16 / -0.16	-0.26 / 0.25	-0.08 / 0.08	-0.07 / 0.07	0.07 / -0.07	0.04 / -0.04
Prefire	-0.04 / 0.04	-0.04 / 0.04	-0.12 / 0.13	-0.09 / 0.09	-0.05 / 0.04	-0.19 / 0.20
Jet energy scale	-0.00 / -0.01	-0.02 / 0.05	0.02 / 0.00	-0.16 / -0.01	-0.03 / 0.01	-0.04 / -0.03
Jet energy resolution	0.00 / -0.00	0.06 / -0.01	0.01 / 0.01	0.33 / 0.01	0.09 / -0.05	-0.04 / 0.05
Background normalization	-0.01 / 0.01	0.08 / -0.09	-0.03 / 0.03	0.22 / -0.30	0.06 / -0.09	0.01 / -0.02
Modeling Uncertainties						
Top p_T	0.38	1.87	2.11	3.42	1.86	2.30
μ_R/μ_F scale	0.33 / -0.57	0.67 / -0.24	1.22 / -0.30	1.27 / -0.40	0.52 / -0.23	1.21 / -0.62
ISR scale	-0.19 / 0.17	-0.16 / 0.05	-0.19 / -0.05	-0.03 / -0.01	-0.13 / 0.14	-0.09 / 0.07
FSR scale	-0.14 / 0.09	-0.15 / -0.12	-0.14 / -0.13	0.03 / 0.32	-0.15 / -0.02	-0.09 / 0.15
PDF	0.05 / -0.05	0.05 / -0.05	0.05 / -0.05	0.02 / -0.02	0.02 / -0.02	0.06 / -0.06
ME/PS matching	1.43 / -0.96	0.17 / -0.42	0.37 / -0.88	-0.11 / -0.23	0.54 / -0.66	0.40 / -1.37
Underlying event tune	0.05 / -0.01	0.73 / 0.16	0.11 / -0.11	0.21 / -0.24	-0.06 / -0.15	-0.14 / -0.37
Color reconnection	0.00 / 0.10	-0.46 / 0.26	-0.79 / 0.17	-1.06 / 0.00	-0.42 / 0.00	-0.47 / 0.00
$t\bar{t}/tW$ interference	0.62	0.50	0.70	1.04	0.60	0.98
Systematic Uncertainty	-1.55 / +0.76	-1.11 / +2.07	-1.54 / +2.29	-1.43 / +3.63	-0.90 / +1.98	-1.89 / +2.55
MC Generator Uncertainties						
MADGRAPH	-1.41	0.40	-2.99	4.11	0.73	-0.95
aMC@NLO	-1.02	0.08	-0.25	4.38	-0.13	1.09
HERWIG++	2.59	4.29	3.36	6.30	4.32	3.40
MC Statistical Uncertainty	± 0.10	± 0.36	± 0.49	± 0.72	± 0.25	± 0.42
Total Uncertainty	-1.73 / 1.07	-1.53 / 2.32	-2.04 / 2.65	-1.94 / 3.86	-1.10 / 2.08	-2.13 / 2.73

Table 4.17: m_t and systematic uncertainties from second moments fit to data in GeV.

Observable	$p_T(l)$	$E(l^+)$	$M(l)$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{MC} [GeV]	171.01	167.29	162.11	167.06	167.11
Statistical Uncertainty	± 0.72	± 1.06	± 1.14	± 0.60	± 1.04
Experimental Uncertainties					
Pileup	0.05 / -0.05	0.06 / -0.05	0.13 / -0.13	0.06 / -0.06	0.08 / -0.08
Luminosity	-0.00 / -0.00	-0.00 / -0.00	0.00 / -0.00	-0.00 / -0.00	0.00 / 0.00
b-tagging	0.03 / -0.03	0.04 / -0.04	0.05 / -0.05	0.04 / -0.04	0.09 / -0.09
Electron ID	-0.05 / 0.05	-0.41 / 0.40	-0.38 / 0.37	-0.20 / 0.19	-0.30 / 0.30
Electron reconstruction	0.09 / -0.09	0.17 / -0.17	0.21 / -0.21	0.15 / -0.15	0.21 / -0.21
Electron energy scale	0.06 / -0.07	0.08 / -0.03	0.12 / -0.06	0.08 / -0.05	0.09 / -0.08
Electron energy smear	0.01 / -0.00	-0.00 / 0.02	0.03 / 0.02	0.02 / -0.01	0.02 / -0.00
Muon ID	0.08 / -0.08	0.17 / -0.16	0.18 / -0.18	0.16 / -0.16	0.19 / -0.18
Muon isolation	-0.03 / 0.03	-0.13 / 0.13	-0.14 / 0.14	-0.08 / 0.08	-0.08 / 0.08
Muon momentum scale	0.00 / -0.01	0.02 / 0.02	-0.11 / 0.01	-0.00 / 0.01	-0.03 / -0.02
Trigger efficiency	0.16 / -0.16	-0.14 / 0.13	-0.08 / 0.08	0.07 / -0.07	0.03 / -0.03
Prefire	-0.05 / 0.05	-0.05 / 0.04	-0.09 / 0.09	-0.05 / 0.05	-0.22 / 0.22
Jet energy scale	-0.00 / -0.01	-0.03 / 0.04	-0.11 / -0.01	-0.03 / 0.01	-0.03 / -0.02
Jet energy resolution	0.01 / 0.00	0.08 / -0.01	0.29 / -0.00	0.10 / -0.06	-0.06 / 0.07
Background normalization	-0.01 / 0.01	0.05 / -0.06	0.18 / -0.24	0.05 / -0.07	-0.00 / -0.00
Modeling Uncertainties					
Top p_T	0.53	2.17	3.41	1.99	2.53
μ_R/μ_F scale	0.40 / -0.61	0.73 / -0.25	1.39 / -0.40	0.58 / -0.23	1.27 / -0.64
ISR scale	-0.18 / 0.18	-0.11 / 0.04	-0.01 / -0.00	-0.10 / 0.12	-0.08 / 0.08
FSR scale	-0.16 / 0.08	-0.16 / -0.08	0.02 / 0.38	-0.14 / -0.01	-0.08 / 0.20
PDF	0.05 / -0.06	0.06 / -0.06	0.02 / -0.02	0.02 / -0.02	0.08 / -0.07
ME/PS matching	1.46 / -1.08	0.21 / -0.57	-0.09 / -0.27	0.55 / -0.73	0.41 / -1.48
Underlying event tune	-0.06 / -0.06	0.53 / 0.04	0.13 / -0.22	-0.08 / -0.16	-0.22 / -0.47
Color reconnection	-0.03 / 0.11	-0.46 / 0.21	-0.94 / 0.00	-0.43 / 0.00	-0.63 / 0.00
$t\bar{t}$ /tW interference	0.68	0.58	1.07	0.63	1.09
Systematic Uncertainty	-1.60 / +0.89	-1.11 / +2.29	-1.52 / +3.62	-0.99 / +2.11	-2.03 / +2.79
MC Generator Uncertainties					
MADGRAPH	-1.46	0.18	3.59	0.32	-1.13
aMC@NLO	-0.85	0.35	4.92	-0.03	1.36
HERWIG++	2.85	4.66	6.05	4.48	3.30
MC Statistical Uncertainty	± 0.10	± 0.41	± 0.72	± 0.25	± 0.46
Total Uncertainty	-1.76 / 1.15	-1.59 / 2.56	-2.03 / 3.86	-1.18 / 2.21	-2.33 / 3.02

Table 4.18: m_t and systematic uncertainties for unblinded $p_T(l)$ fits.

$p_T(l)$	Likelihood	Moment 1	Moment 2
m_t^{MC} [GeV]	172.12	171.18	171.01
Statistical Uncertainty	$-0.44/+0.40$	± 0.75	± 0.72
Experimental Uncertainties			
Pileup	0.05 / -0.05	0.05 / -0.04	0.05 / -0.05
Luminosity	0.00 / 0.00	-0.00 / -0.00	-0.00 / -0.00
b-tagging	0.02 / -0.03	0.03 / -0.03	0.03 / -0.03
Electron ID	-0.02 / 0.03	-0.07 / 0.07	-0.05 / 0.05
Electron reconstruction	0.07 / -0.08	0.09 / -0.09	0.09 / -0.09
Electron energy scale	0.05 / -0.07	0.06 / -0.07	0.06 / -0.07
Electron energy smear	-0.02 / -0.01	0.01 / -0.00	0.01 / -0.00
Muon ID	0.07 / -0.07	0.08 / -0.08	0.08 / -0.08
Muon isolation	-0.02 / 0.02	-0.04 / 0.04	-0.03 / 0.03
Muon momentum scale	0.00 / -0.01	0.00 / -0.01	0.00 / -0.01
Trigger efficiency	0.13 / -0.14	0.16 / -0.16	0.16 / -0.16
Prefire	-0.04 / 0.04	-0.04 / 0.04	-0.05 / 0.05
Jet energy scale	-0.01 / -0.01	-0.00 / -0.01	-0.00 / -0.01
Jet energy resolution	-0.01 / -0.01	0.00 / -0.00	0.01 / 0.00
Background normalization	0.03 / -0.03	-0.01 / 0.01	-0.01 / 0.01
Modeling Uncertainties			
Top p_T	0.18	0.38	0.53
μ_R/μ_F scale	0.16 / -0.20	0.33 / -0.57	0.40 / -0.61
ISR scale	-0.15 / 0.16	-0.19 / 0.17	-0.18 / 0.18
FSR scale	-0.12 / 0.07	-0.14 / 0.09	-0.16 / 0.08
PDF	0.04 / -0.05	0.05 / -0.05	0.05 / -0.06
ME/PS matching	0.31 / -0.33	1.43 / -0.96	1.46 / -1.08
Underlying event tune	0.05 / 0.00	0.05 / -0.01	-0.06 / -0.06
Color reconnection	-0.01 / 0.13	0.00 / 0.10	-0.03 / 0.11
$t\bar{t}/tW$ interference	0.28	0.62	0.68
Systematic Uncertainty	$-0.48/+0.27$	$-1.55/+0.76$	$-1.60/+0.89$
MC Generator Uncertainties			
MADGRAPH	-0.24	-1.41	-1.46
aMC@NLO	-0.08	-1.02	-0.85
HERWIG++	0.30	2.59	2.85
MC Statistical Uncertainty	± 0.14	± 0.10	± 0.10
Total Uncertainty	$-0.66/0.51$	$-1.73/1.07$	$-1.76/1.15$

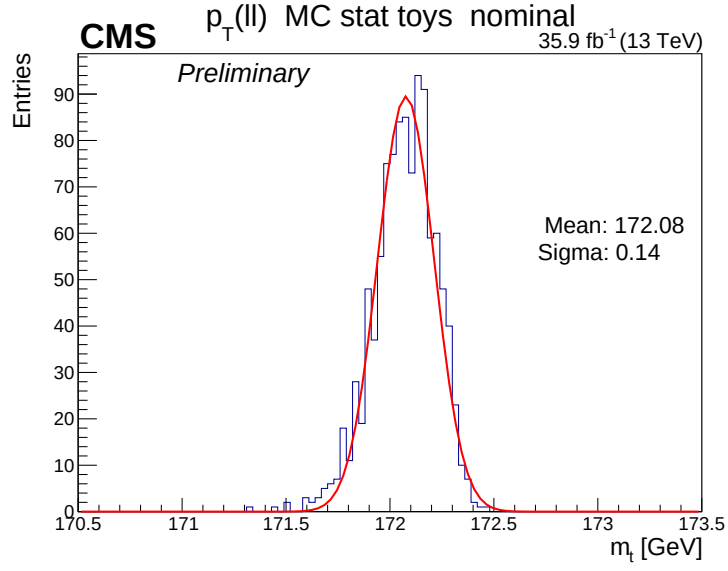


Figure 4.46: $p_T(\ell\ell)$ MC stat uncertainty toys from fit to data.

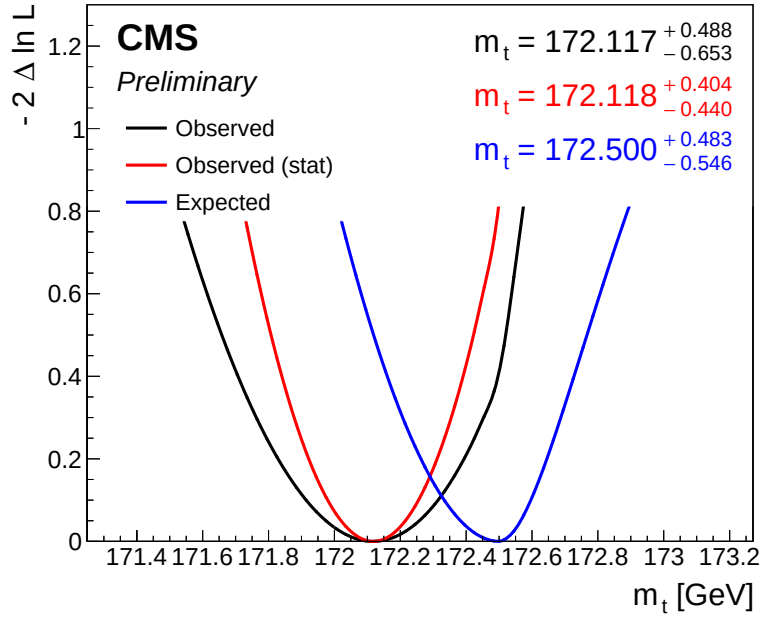


Figure 4.47: $p_T(\ell\ell)$ likelihood scans from fit to data (Observed) and Asimov (Expected).

4.10 Top Quark Pole Mass Extraction

The $t\bar{t}$ simulation used for 13TeV analyses relies on MC samples that are accurate to NLO in perturbative QCD (pQCD), whereas the Run I MC was only accurate to leading order. A new fixed-order QCD calculation [17, 42, 20, 66] is available which, for the first time, uses exact NNLO corrections for $t\bar{t}$ production and decay. The prediction is performed in the fiducial space defined in Section 4.3 assuming top quark pole mass values of [169.5, 172.5, 175.5] GeV to produce differential distributions of prompt leptons originating from W decays. No showering is performed, so to compare with MC simulation or collision data, the reconstructed distributions are unfolded to LHE-level and the expected fraction of events from non-prompt leptons are subtracted.

The calculation uses NNPDF3.1 pdf sets and the default factorization and renormalization scales are dynamically set to $\mu_F = \mu_R = \frac{1}{4}(\sqrt{m_t^2 + p_{T,t}^2} + \sqrt{m_t^2 + p_{T,\bar{t}}^2})$. Variations on the pdf sets used have less than a 1% effect on the normalized differential distributions and are negligible. Scale variations where μ_F, μ_R are simultaneously varied by a factor of 2 or 0.5 are significant and result in an additional uncertainty in the extracted pole mass.

Figure 4.51 compares the truth-level t , $\bar{t}$, and $t\bar{t}$ distributions to the current POWHEG $t\bar{t}$ MC simulation. In figure 4.52, leptonic observables calculated at LO, NLO, and NNLO are compared to POWHEG truth and unfolded distributions at $m_t = 172.5$ GeV. Figure 4.48 shows the shape dependence of the leptonic distributions on m_t .

The moment calibration method is used to extract the pole masses. The masses and uncertainties from fitting to the nominal Asimov set from 172.5 GeV NLO and NNLO distributions are listed in Tables 4.19-4.22.

The corresponding results from the unblinded data are given in Tables 4.23-4.26. The Statistical Uncertainty represents the total uncertainty after unfolding including effects from all statistical and systematic sources. Modeling uncertainties from the QCD scale and PDF variations are listed separately.

Significant shape differences are present between the POWHEG LHE level and the NLO/NNLO predictions, causing a bias on the order of 10 GeV in the pole masses extracted from unfolded data. This is likely due to incompatibility between the fiducial phase space defined by the fixed-order QCD calculation and the LHE-level space that is unfolded to.

As part of the CMS thesis endorsement procedure, the pole mass results from Mellin Moments are considered preliminary and are not to be construed as official CMS results.

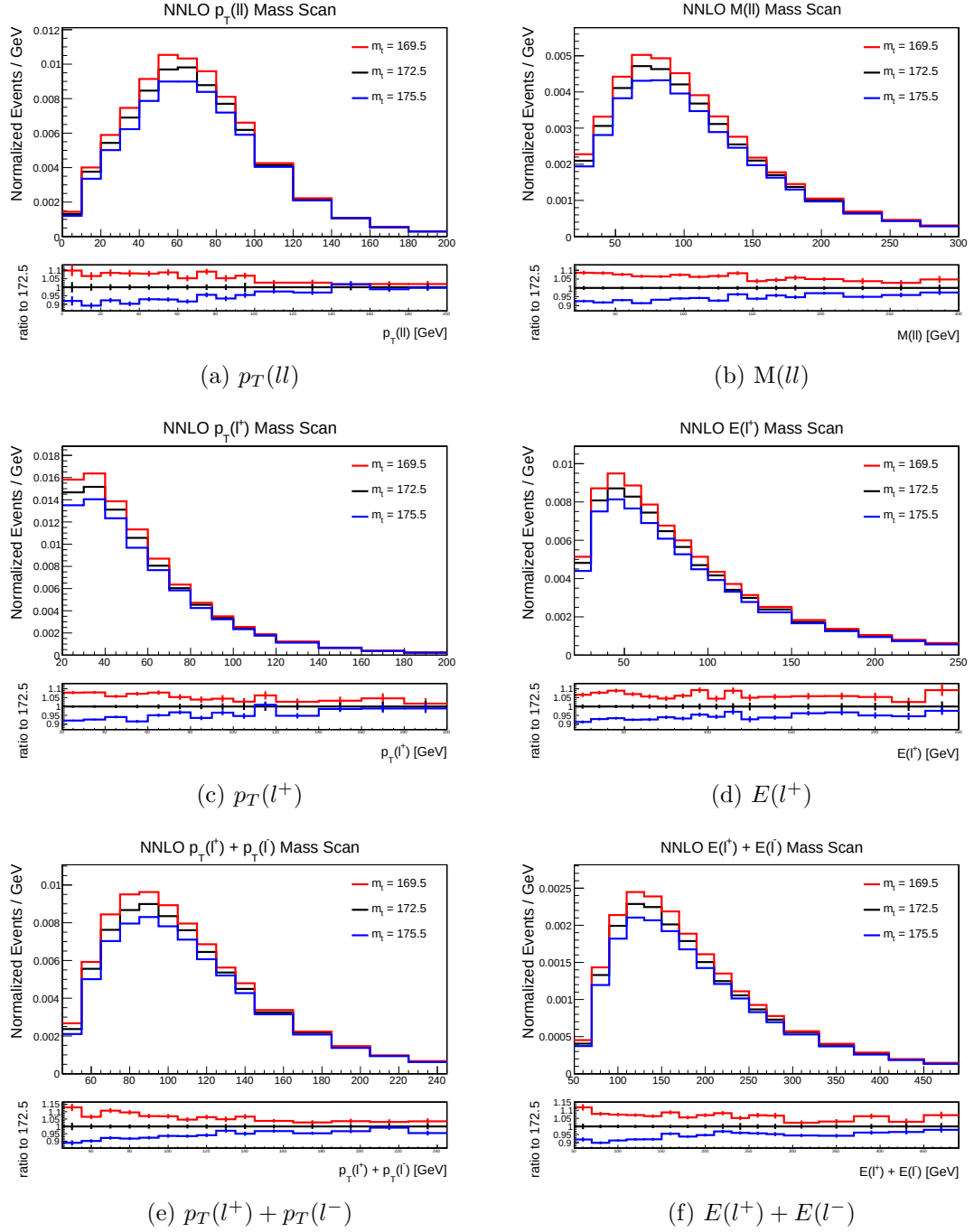


Figure 4.48: Mass scans of NNLO differential distributions. The error bars represent statistical uncertainties in the simulation.

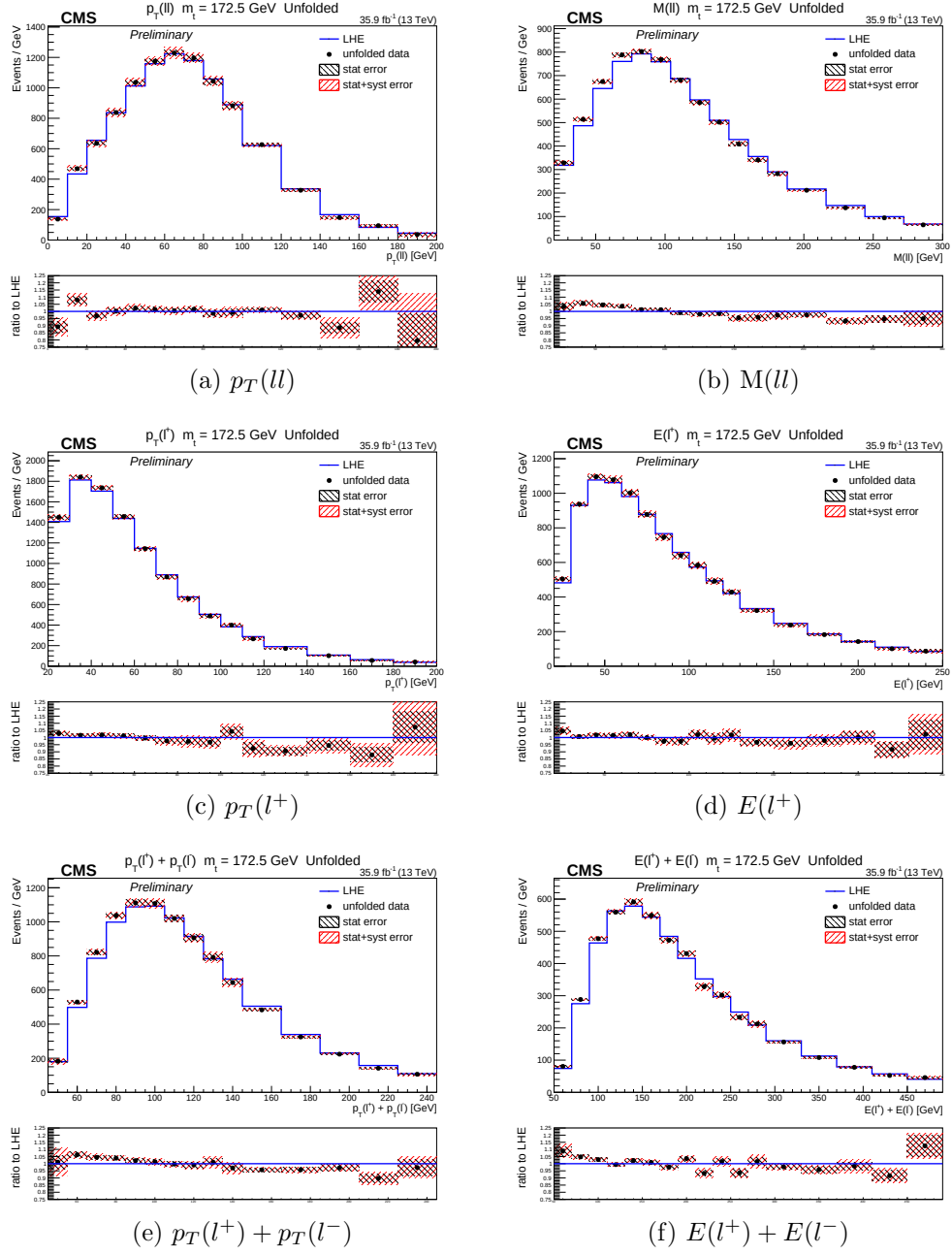


Figure 4.49: Unfolding of $t\bar{t} + tW$ distributions from nominal rec MC compared to LHE-level truth. Error bands are unfolding errors from input distribution (stat) and total unfolding error including systematics (stat+syst).

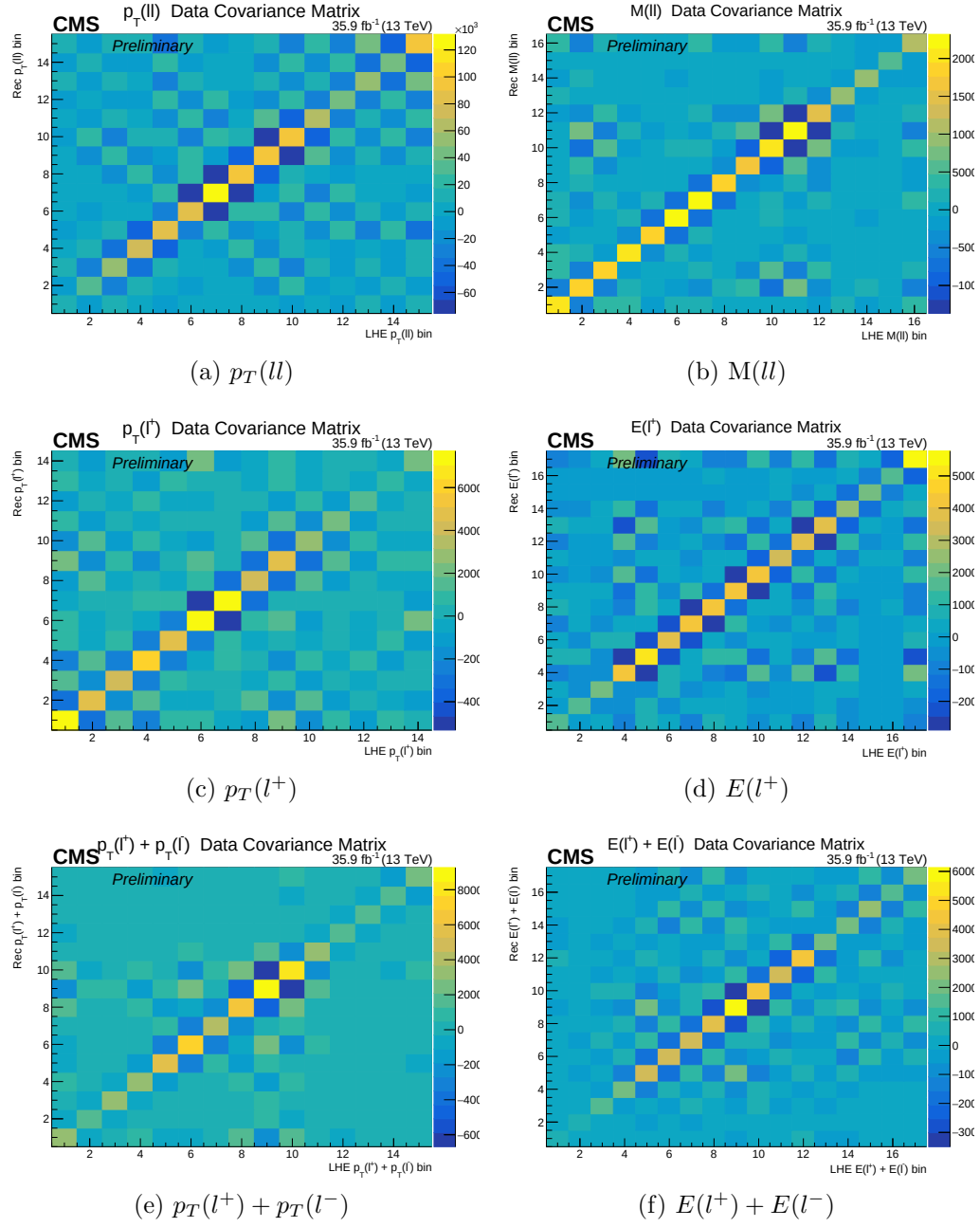


Figure 4.50: Covariance matrices representing the total uncertainties on data after unfolding.

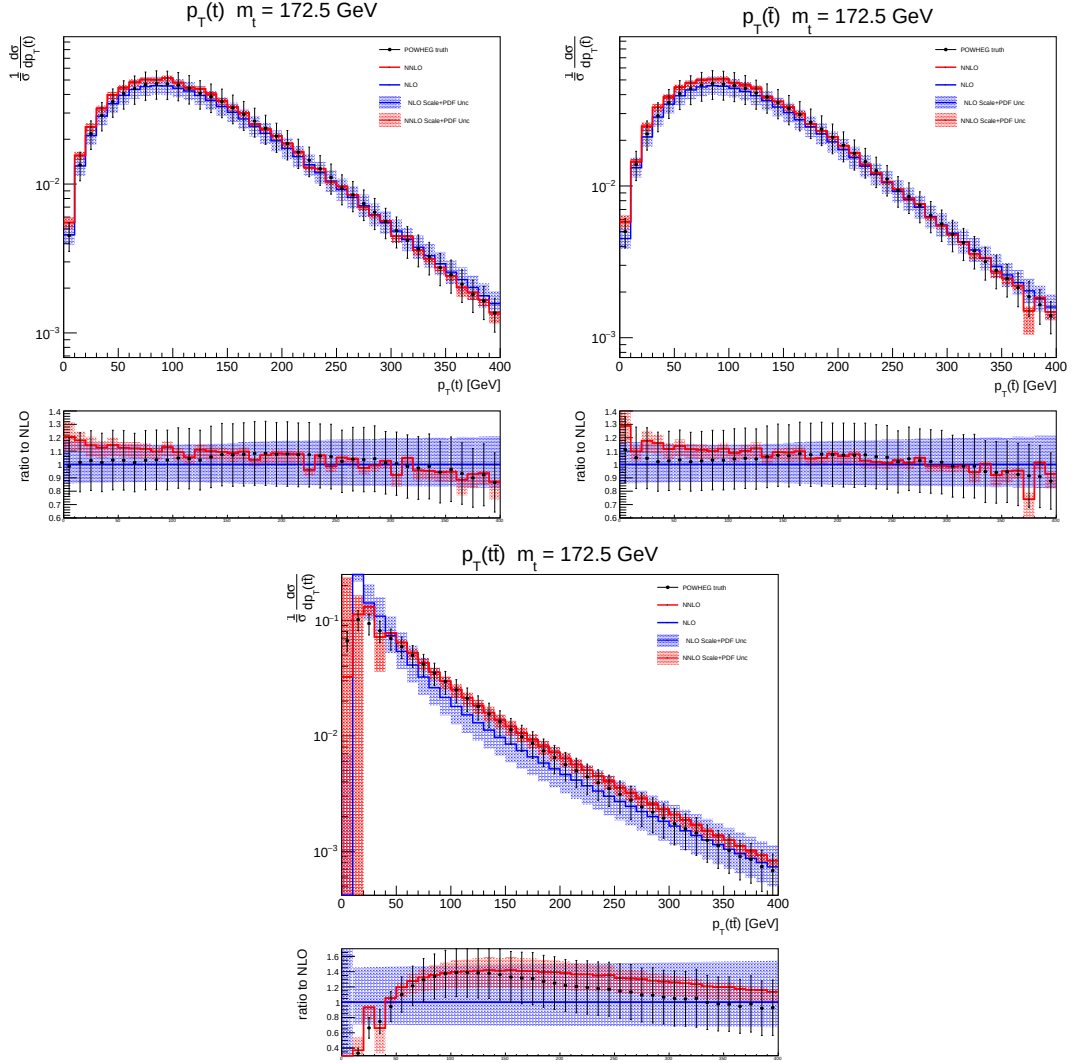


Figure 4.51: Truth top, antitop, and $t\bar{t}$ p_T at NLO and NNLO compared to POWHEG truth at $m_t = 172.5$ GeV.

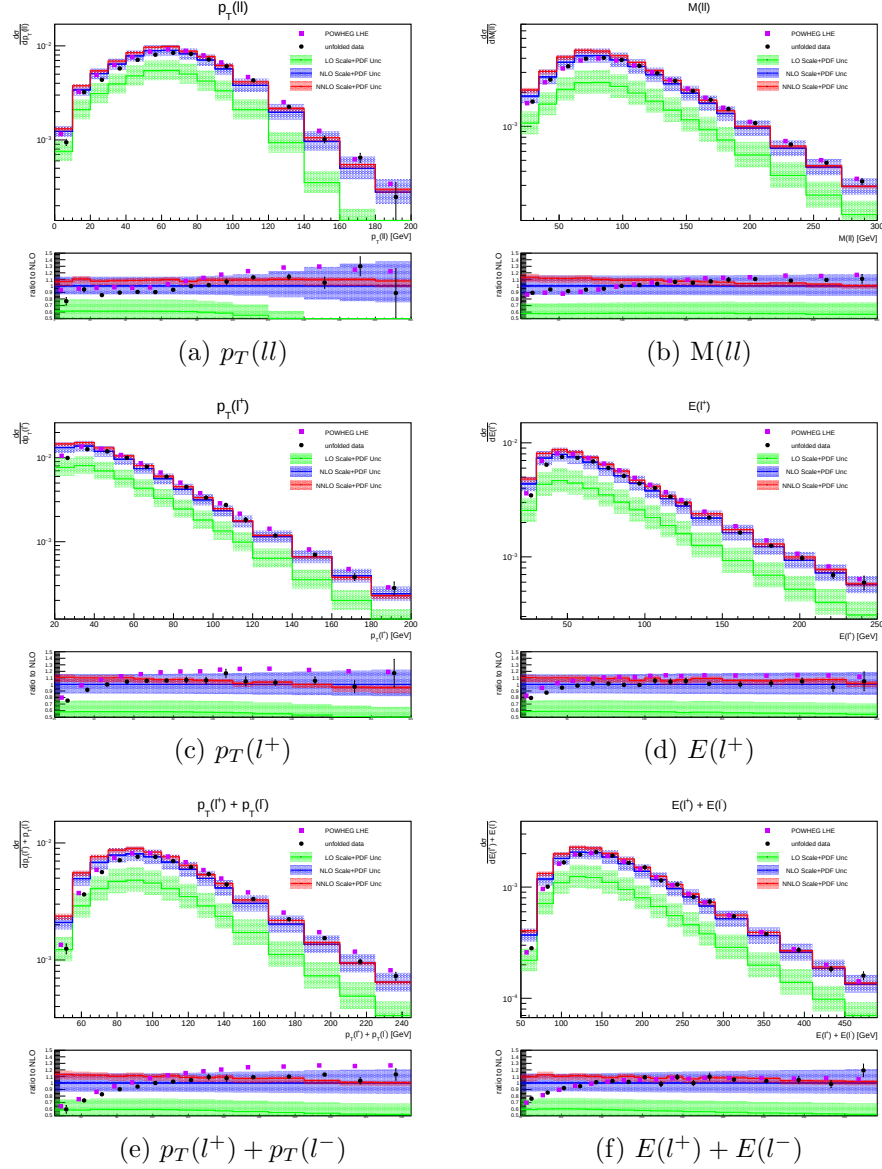


Figure 4.52: LO, NLO, and NNLO predictions at $m_t^{\text{pole}} = 172.5$ GeV compared with POWHEG LHE truth at $m_t^{\text{MC}} = 172.5$ GeV and unfolded data distributions. Uncertainties on POWHEG truth are MC statistical and uncertainties on data are the full propagated unfolded uncertainty.

Table 4.19: m_t and systematic uncertainties for NLO Asimov first moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
$m_t^{\text{pole}} [\text{GeV}]$	172.46	172.44	172.34	172.42	172.46	172.50
Statistical Uncertainty	± 0.55	± 0.79	± 1.29	± 1.01	± 0.60	± 0.84
Modeling Uncertainties						
μ_R/μ_F scale PDF	2.32 / -3.78 0.02 / -0.02	0.91 / -1.43 0.06 / -0.06	1.67 / -2.46 0.05 / -0.05	0.64 / -0.46 0.10 / -0.10	1.12 / -1.63 0.03 / -0.03	1.77 / -2.77 0.04 / -0.04
Systematic Uncertainty	-3.82/ + 2.39	-1.64/ + 1.20	-2.78/ + 2.11	-1.12/ + 1.20	-1.73/ + 1.27	-2.90/ + 1.96
MC Statistical Uncertainty	± 0.03	± 0.03	± 0.05	± 0.03	± 0.03	± 0.02
Total Uncertainty	-3.86/2.45	-1.82/1.44	-3.07/2.47	-1.51/1.57	-1.83/1.40	-3.02/2.13

Table 4.20: m_t and systematic uncertainties for NLO Asimov second moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
$m_t^{\text{pole}} [\text{GeV}]$	172.46	172.43	172.33	172.41	172.45	172.52
Statistical Uncertainty	± 0.55	± 0.85	± 1.43	± 1.07	± 0.64	± 0.95
Modeling Uncertainties						
μ_R/μ_F scale PDF	2.72 / -4.36 0.03 / -0.03	1.09 / -1.73 0.08 / -0.08	1.85 / -2.75 0.07 / -0.07	0.70 / -0.58 0.12 / -0.12	1.24 / -1.83 0.04 / -0.03	1.85 / -2.99 0.07 / -0.07
Systematic Uncertainty	-4.39/ + 2.78	-1.93/ + 1.39	-3.10/ + 2.34	-1.22/ + 1.28	-1.94/ + 1.40	-3.13/ + 2.08
MC Statistical Uncertainty	± 0.02	± 0.03	± 0.05	± 0.03	± 0.03	± 0.02
Total Uncertainty	-4.43/2.83	-2.11/1.63	-3.42/2.74	-1.62/1.67	-2.05/1.54	-3.27/2.28

Table 4.21: m_t and systematic uncertainties for NNLO Asimov first moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	172.25	172.31	172.08	172.34	172.41	172.24
Statistical Uncertainty	± 0.55	± 0.79	± 1.39	± 0.94	± 0.59	± 0.85
Modeling Uncertainties						
μ_R/μ_F scale PDF	0.59 / -0.36 0.03 / -0.03	-0.84 / 1.37 0.06 / -0.06	-0.79 / 1.24 0.05 / -0.05	-1.36 / 1.93 0.09 / -0.09	-0.57 / 1.21 0.03 / -0.03	-0.48 / 0.73 0.04 / -0.04
Systematic Uncertainty	-0.66 / +0.81	-1.16 / +1.58	-1.60 / +1.86	-1.65 / +2.14	-0.82 / +1.34	-0.97 / +1.12
MC Statistical Uncertainty	± 0.14	± 0.14	± 0.23	± 0.10	± 0.11	± 0.10
Total Uncertainty	-0.87 / 0.98	-1.41 / 1.78	-2.13 / 2.33	-1.90 / 2.34	-1.01 / 1.47	-1.29 / 1.41

Table 4.22: m_t and systematic uncertainties for NNLO Asimov second moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	172.28	172.31	172.06	172.35	172.41	172.30
Statistical Uncertainty	± 0.55	± 0.85	± 1.55	± 0.99	± 0.64	± 0.95
Modeling Uncertainties						
μ_R/μ_F scale PDF	0.72 / -0.28 0.03 / -0.03	-0.77 / 1.47 0.07 / -0.07	-0.84 / 1.38 0.08 / -0.08	-1.37 / 2.04 0.11 / -0.11	-0.63 / 1.35 0.03 / -0.03	-0.57 / 0.98 0.07 / -0.07
Systematic Uncertainty	-0.62 / +0.91	-1.15 / +1.70	-1.77 / +2.08	-1.70 / +2.27	-0.90 / +1.49	-1.11 / +1.37
MC Statistical Uncertainty	± 0.13	± 0.12	± 0.23	± 0.10	± 0.12	± 0.10
Total Uncertainty	-0.84 / 1.07	-1.43 / 1.90	-2.36 / 2.60	-1.97 / 2.48	-1.11 / 1.63	-1.47 / 1.67

Table 4.23: m_t and systematic uncertainties for unblinded NLO first moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	182.17	186.23	183.30	184.26	185.86	178.92
Statistical Uncertainty	± 1.31	± 1.41	± 2.31	± 1.24	± 1.08	± 1.35
Modeling Uncertainties						
μ_R/μ_F scale PDF	1.95 / -3.23 0.03 / -0.03	0.23 / -0.37 0.07 / -0.07	1.06 / -1.49 0.08 / -0.08	0.07 / 0.44 0.12 / -0.12	0.52 / -0.65 0.04 / -0.04	1.45 / -2.35 0.06 / -0.05
Systematic Uncertainty	-3.48 / +2.35	-1.46 / +1.43	-2.75 / +2.54	-1.25 / +1.32	-1.27 / +1.20	-2.72 / +1.98
MC Statistical Uncertainty	± 0.20	± 0.37	± 0.50	± 0.47	± 0.27	± 0.23
Total Uncertainty	-3.73 / 2.70	-2.06 / 2.04	-3.63 / 3.47	-1.82 / 1.87	-1.69 / 1.64	-3.04 / 2.41

Table 4.24: m_t and systematic uncertainties for unblinded NLO second moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	180.96	182.78	180.65	183.34	184.60	177.90
Statistical Uncertainty	± 1.33	± 1.55	± 2.59	± 1.32	± 1.19	± 1.54
Modeling Uncertainties						
μ_R/μ_F scale PDF	2.42 / -3.92 0.03 / -0.03	0.58 / -0.92 0.09 / -0.09	1.37 / -1.99 0.10 / -0.10	0.16 / 0.28 0.14 / -0.14	0.67 / -0.93 0.05 / -0.05	1.58 / -2.66 0.08 / -0.08
Systematic Uncertainty	-4.14 / +2.76	-1.81 / +1.66	-3.27 / +2.93	-1.34 / +1.36	-1.51 / +1.37	-3.07 / +2.21
MC Statistical Uncertainty	± 0.16	± 0.31	± 0.44	± 0.47	± 0.25	± 0.23
Total Uncertainty	-4.35 / 3.06	-2.40 / 2.29	-4.19 / 3.94	-1.94 / 1.95	-1.94 / 1.83	-3.44 / 2.70

Table 4.25: m_t and systematic uncertainties for unblinded NNLO first moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	181.65	191.73	188.51	189.40	189.26	182.47
Statistical Uncertainty	± 1.31	± 1.45	± 2.51	± 1.16	± 1.08	± 1.37
Modeling Uncertainties						
μ_R/μ_F scale PDF	0.51 / -0.44 0.03 / -0.03	-2.03 / 1.84 0.07 / -0.07	-1.33 / 2.02 0.10 / -0.10	-1.52 / 2.20 0.12 / -0.11	-0.89 / 1.43 0.04 / -0.04	-0.75 / 0.98 0.06 / -0.06
Systematic Uncertainty	-1.38 / +1.40	-2.50 / +2.35	-2.84 / +3.22	-1.91 / +2.49	-1.40 / +1.80	-1.56 / +1.68
MC Statistical Uncertainty	± 0.62	± 1.99	± 2.88	± 1.56	± 1.22	± 0.88
Total Uncertainty	-2.00 / 2.01	-3.51 / 3.40	-4.76 / 4.99	-2.73 / 3.16	-2.15 / 2.43	-2.25 / 2.34

Table 4.26: m_t and systematic uncertainties for unblinded NNLO second moments in GeV.

Observable	$p_T(l\bar{l})$	$p_T(l^+)$	$E(l^+)$	$M(l\bar{l})$	$p_T(l^+) + p_T(l^-)$	$E(l^+) + E(l^-)$
m_t^{pole} [GeV]	180.51	188.69	186.07	188.84	188.50	181.88
Statistical Uncertainty	± 1.32	± 1.59	± 2.84	± 1.25	± 1.20	± 1.56
Modeling Uncertainties						
μ_R/μ_F scale PDF	0.64 / -0.32 0.04 / -0.03	-1.98 / 1.91 0.09 / -0.09	-1.36 / 2.33 0.12 / -0.12	-1.59 / 2.32 0.13 / -0.13	-1.01 / 1.63 0.05 / -0.05	-0.82 / 1.18 0.09 / -0.09
Systematic Uncertainty	-1.36 / +1.47	-2.54 / +2.49	-3.15 / +3.68	-2.03 / +2.64	-1.57 / +2.03	-1.77 / +1.96
MC Statistical Uncertainty	± 0.52	± 1.60	± 2.71	± 1.60	± 1.19	± 0.57
Total Uncertainty	-1.97 / 2.05	-3.40 / 3.36	-5.03 / 5.38	-2.87 / 3.33	-2.31 / 2.64	-2.43 / 2.57

Chapter 5

Conclusion

The sensitivity of leptonic observables to the top quark mass has been demonstrated in the unblinded fits to collision data. The main results are summarized below (with the MC mass result endorsed by the CMS collaboration):

$$m_t^{\text{MC}} = 172.12^{+0.51}_{-0.66} \left({}^{+0.40}_{-0.44}(\text{stat}) {}^{+0.27}_{-0.47}(\text{syst}) \pm 0.14(\text{MC stat}) \right) \text{ GeV} \quad (5.1)$$

$$m_t^{\text{pole}} = 180.51 \pm 1.32(\text{stat}) {}^{+0.03}_{-0.04}(\text{Pdf}) {}^{+0.32}_{-0.64}(\text{scale}) \pm 0.52(\text{MC stat}) \text{ GeV} \quad (5.2)$$

With modern event simulation generated at NLO and beyond, the leptonic observables method will be a competitive method for m_t measurements in the dilepton channel. Future refinements in top quark p_T modeling should improve the known discrepancy between simulation and data, reducing the impact of this dominant source of systematic uncertainty. Multiple leptonic observables can also be studied simultaneously using multidifferential distributions. The inclusion of two or more variables, particularly combinations of single lepton and lepton pair

observables such as $E(l^+)$ and $p_T(ll)$, could improve m_t sensitivity by utilizing more of the available shape information in an event. Finally, the inclusion of additional collision data will improve the overall sensitivity further.

Bibliography

- [1] Tracking and Primary Vertex Results in First 7 TeV Collisions. Technical Report CMS-PAS-TRK-10-005, CERN, Geneva, 2010.
- [2] Determination of the top quark mass from leptonic observables using $e\mu + \text{jets}$ final states selected in proton-proton collisions at $\sqrt{s} = 8$ TeV. Technical Report CMS-PAS-TOP-16-002, CERN, Geneva, 2016.
- [3] Agostinelli, S. et al. GEANT4—a simulation toolkit. *Nucl. Instrum. Meth. A*, 506:250, 2003.
- [4] Alioli, Simone, Moch, Sven-Olaf, and Uwer, Peter. Hadronic top-quark pair-production with one jet and parton showering. *JHEP*, 01:137, 2012.
- [5] Alioli, Simone, Nason, Paolo, Oleari, Carlo, and Re, Emanuele. NLO single-top production matched with shower in POWHEG: s- and t-channel contributions. *JHEP*, 09:111, 2009. [Erratum: JHEP 02, 011 (2010)].
- [6] Alioli, Simone et al. A general framework for implementing NLO calculations in shower Monte Carlo programs: the POWHEG BOX. *JHEP*, 06:043, 2010.

- [7] Alwall, J. et al. The automated computation of tree-level and next-to-leading order differential cross sections, and their matching to parton shower simulations. *JHEP*, 07:079, 2014.
- [8] Alwall, Johan et al. Comparative study of various algorithms for the merging of parton showers and matrix elements in hadronic collisions. *Eur. Phys. J.*, C53:473–500, 2008.
- [9] A.M. Sirunyan et al. Jet energy scale and resolution in the CMS experiment in pp collisions at 8 TeV. *Journal of Instrumentation*, 12(02):P02014–P02014, Feb 2017.
- [10] A.M. Sirunyan et al. Identification of heavy-flavour jets with the CMS detector in pp collisions at 13 TeV. *Journal of Instrumentation*, 13(05):P05011–P05011, May 2018.
- [11] Argyropoulos, Spyros and Sjöstrand, Torbjörn. Effects of color reconnection on $t\bar{t}$ final states at the LHC. *Journal of High Energy Physics*, 2014(11), Nov 2014.
- [12] Artoisenet, P. et al. Automatic spin-entangled decays of heavy resonances in Monte Carlo simulations. *JHEP*, 03:015, 2013.
- [13] Ask, Stefan et al. An Introduction to PYTHIA 8.2. *Comput. Phys. Commun.*, 191:159–177, 2015.
- [14] The ATLAS, CDF, CMS, and D0 Collaborations. First combination of tevatron and lhc measurements of the top-quark mass, 2014.

- [15] Ball, Richard D. et al. Parton distributions with LHC data. *Nucl. Phys. B*, 867:244, 2013.
- [16] Ball, Richard D. et al. Parton distributions from high-precision collider data - NNPDF Collaboration. *Eur. Phys. J. C*, 77(10):663, 2017.
- [17] Arnd Behring, Michal Czakon, Alexander Mitov, Andrew S. Papanastasiou, and Rene Poncelet. Higher order corrections to spin correlations in top quark pair production at the LHC. *Phys. Rev. Lett.*, 123(8):082001, 2019.
- [18] Bodek, A. and van Dyne, A. and Han, J. Y. and Sakumoto, W. and Strel'nikov, A. Extracting Muon Momentum Scale Corrections for Hadron Collider Experiments. *Eur. Phys. J.*, C72:2194, 2012.
- [19] Brüning, Oliver Sim and Collier, Paul and Lebrun, P and Myers, Stephen and Ostojic, Ranko and Poole, John and Proudlock, Paul. *LHC Design Report*. CERN Yellow Reports: Monographs. CERN, Geneva, 2004.
- [20] Andy Buckley, James Ferrando, Stephen Lloyd, Karl Nordström, Ben Page, Martin Rüfenacht, Marek Schönherr, and Graeme Watt. LHAPDF6: parton density access in the LHC precision era. *Eur. Phys. J.*, C75:132, 2015.
- [21] Butterworth, Jon et al. PDF4lhc recommendations for LHC run II. *Journal of Physics G: Nuclear and Particle Physics*, 43(2):023001, Jan 2016.
- [22] Matteo Cacciari, Gavin P Salam, and Gregory Soyez. The anti- k_t clustering algorithm. *Journal of High Energy Physics*, 2008(04):063–063, Apr 2008.
- [23] Campbell, John M., Ellis, R. Keith, and Williams, Ciaran. Vector boson pair production at the LHC. *JHEP*, 07:018, 2011.

- [24] Chatrchyan, S. et al. The CMS Experiment at the CERN LHC. *JINST*, 3:S08004, 2008.
- [25] Chatrchyan, Serguei and others. Description and performance of track and primary-vertex reconstruction with the CMS tracker. *JINST*, 9:P10009, 2014.
- [26] Chatrchyan, Serguei et al. Measurement of the Inclusive W and Z Production Cross Sections in pp Collisions at $\sqrt{s} = 7$ TeV with the CMS experiment. *JHEP*, 10(arXiv:1107.4789. CMS-EWK-10-005. CERN-PH-EP-2011-107):132. 68 p, Jul 2011.
- [27] Christiansen, Jesper R. and Skands, Peter Z. String formation beyond leading colour. *Journal of High Energy Physics*, 2015(8), Aug 2015.
- [28] The CMS Collaboration. Description and performance of track and primary-vertex reconstruction with the CMS tracker. *Journal of Instrumentation*, 9(10):P10009–P10009, Oct 2014.
- [29] J.S. Conway. Incorporating Nuisance Parameters in Likelihoods for Multi-source Spectra. In *PHYSTAT 2011*, pages 115–120, 2011.
- [30] Cooper, Seth I. Phase I Upgrade of the CMS Hadron Calorimeter. *Nuclear and Particle Physics Proceedings*, 273-275:1002 – 1007, 2016. 37th International Conference on High Energy Physics (ICHEP).
- [31] Gennaro Corcella. The top-quark mass: Challenges in definition and determination. *Frontiers in Physics*, 7:54, 2019.
- [32] W. N. Cottingham and D. A. Greenwood. *An Introduction to the Standard Model of Particle Physics*. Cambridge University Press, 2 edition, 2007.

- [33] Czakon, Michał, Fiedler, Paul, and Mitov, Alexander. Total top-quark pair-production cross section at hadron colliders through $\mathcal{O}(\alpha_S^4)$. *Phys. Rev. Lett.*, 110:252004, Jun 2013.
- [34] Czakon, Michal and Mitov, Alexander. Top++: A Program for the Calculation of the Top-Pair Cross-Section at Hadron Colliders. *Comput.Phys.Commun.*, 185:2930, 2014.
- [35] Morris H. DeGroot and Mark J. Schervish. *Probability and Statistics, Third Edition*. Addison-Wesley, New York, NY, 2002.
- [36] Abachi, S. et al. Observation of the top quark. *Physical Review Letters*, 74(14):2632–2637, Apr 1995.
- [37] Abe, F. et al. Observation of top quark production in $p\bar{p}$ collisions with the collider detector at fermilab. *Physical Review Letters*, 74(14):2626–2631, Apr 1995.
- [38] Bierlich, Christian et al. Robust independent validation of experiment and theory: Rivet version 3. *SciPost Physics*, 8(2), Feb 2020.
- [39] d’Enterria, David et al. High-precision α_s measurements from lhcb to fcc-ee. 12 2015.
- [40] Frederix, Rikkert and Frixione, Stefano. Merging meets matching in MC@NLO. *JHEP*, 12:061, 2012.
- [41] Frixione, Stefano and Mitov, Alexander. Determination of the top quark mass from leptonic observables. *JHEP*, 09:012, 2014.

- [42] Gao, Jun and Papanastasiou, Andrew S. Top-quark pair-production and decay at high precision. *Phys. Rev. D*, 96:051501, Sep 2017.
- [43] Gavin, R., Li, Y., Petriello, F., and Quackenbush, S. FEWZ 2.0: a code for hadronic Z production at next-to-next-to-leading order. *Comput. Phys. Commun.*, 182:2388, 2011.
- [44] David J. Griffiths. *Introduction to Elementary Particles; Second Revised Edition*. Wiley, New York, NY, 2014.
- [45] M. Hatlo, F. James, P. Mato, L. Moneta, M. Winkler, and A. Zsenei. Developments of mathematical software libraries for the lhc experiments. *IEEE Transactions on Nuclear Science*, 52(6):2818–2822, Dec 2005.
- [46] Khachatryan, V et al. Measurement of the top quark mass using proton-proton data at $\sqrt{s} = 7$ and 8 TeV. *Phys. Rev. D*, 93:072004, Apr 2016.
- [47] Khachatryan, V. et al. Measurement of the top quark mass using proton-proton data at $\sqrt{s} = 7$ and 8 tev. *Phys. Rev. D*, 93:072004, Apr 2016.
- [48] Khachatryan, Vardan et al. Performance of electron reconstruction and selection with the CMS detector in proton-proton collisions at $\sqrt{s} = 8\text{TeV}$. *JINST*, 10:P06005, 2015.
- [49] Khachatryan, Vardan et al. Performance of photon reconstruction and identification with the CMS detector in proton-proton collisions at $\sqrt{s} = 8\text{TeV}$. *JINST*, 10:P08010, 2015.
- [50] Khachatryan, Vardan et al. The CMS trigger system. *JINST*, 12:P01020, 2017.

- [51] N. Kidonakis. Two-loop soft anomalous dimensions for single top quark associated production with W^- or H^- . *Phys. Rev. D*, 82:054018, 2010.
- [52] Andrea Massironi, Viktor Khristenko, and Mariarosaria D’Alfonso. The NanoAOD event data format in CMS. *Journal of Physics: Conference Series*, 1525:012038, Apr 2020.
- [53] Melia, Tom, Nason, Paolo, Rontsch, Raoul, and Zanderighi, Giulia. $W+W-$, WZ and ZZ production in the POWHEG BOX. *JHEP*, 11:078, 2011.
- [54] Nason, Paolo and Zanderighi, Giulia. W^+W^- , WZ and ZZ production in the POWHEG-BOX-V2. *Eur. Phys. J. C*, 74(1):2702, 2014.
- [55] P.A. Zyla et al. (Particle Data Group). Review of Particle Physics. *Progress of Theoretical and Experimental Physics*, 2020(8), 08 2020. 083C01.
- [56] Emanuele Re. Single-top Wt -channel production matched with parton showers using the POWHEG method. *Eur. Phys. J. C*, 71:1547, 2011.
- [57] S Schmitt. Tunfold, an algorithm for correcting migration effects in high energy physics. *Journal of Instrumentation*, 7(10):T10003–T10003, Oct 2012.
- [58] Sirunyan, A. M. et al. Performance of the CMS muon detector and muon reconstruction with proton-proton collisions at $\sqrt{s} = 13$ TeV. *JINST*, 13:P06015, 2018.
- [59] Sirunyan, Albert M et al. Measurement of the $t\bar{t}$ production cross section, the top quark mass, and the strong coupling constant using dilepton events in pp collisions at $\sqrt{s} = 13$ TeV. *Eur. Phys. J. C*, 79(5):368, 2019.

- [60] Sirunyan, Albert M et al. Extraction and validation of a new set of CMS PYTHIA8 tunes from underlying-event measurements. *Eur. Phys. J. C*, 80(1):4, 2020.
- [61] Sirunyan, Albert M et al. Performance of the CMS Level-1 trigger in proton-proton collisions at $\sqrt{s} = 13$ TeV. *JINST*, 15:P10017, 2020.
- [62] Sirunyan, A.M. et al. Particle-flow reconstruction and global event description with the CMS detector. *JINST*, 12(10):P10003, 2017.
- [63] Skands, Peter, Carrazza, Stefano, and Rojo, Juan. Tuning PYTHIA 8.1: the Monash 2013 Tune. *Eur. Phys. J.*, C74(8):3024, 2014.
- [64] The CMS Collaboration. Investigations of the impact of the parton shower tuning in Pythia 8 in the modelling of $t\bar{t}$ at $\sqrt{s} = 8$ and 13 TeV. Technical Report CMS-PAS-TOP-16-021, CERN, Geneva, 2016.
- [65] The CMS Collaboration. CMS Luminosity Measurements for the 2016 Data Taking Period. Technical Report CMS-PAS-LUM-17-001, CERN, Geneva, 2017.
- [66] Andre van Hameren. PARNI for importance sampling and density estimation. *Acta Phys. Polon.*, B40:259–272, 2009.

Appendix A

The Profile Likelihood Method

The primary result for the top quark MC mass measurement is obtained using a common technique in modern HEP analysis called the Profile Likelihood Scan, based on the Maximum Likelihood Estimator (MLE) principle [35]. Uncorrelated measurements represented by binned differential distributions are coupled with a set of nuisance parameters for the various sources of systematic uncertainties [29].

The bins of each template are described by a Poisson counting process. The probability in bin i to observed n_i events in data with the expectation μ_i from MC is given by Equation A.2. The expected event count is comprised of the sum of $t\bar{t} + tW$ signal $s_i(m_t)$ and background b_i processes (Eq A.1). For the blinded analysis, the observed distribution is derived from MC simulation and hence likely has a non-integer count. For $n_i \in \mathcal{R}^+$, a continuous analog of the Poisson p.f. (Eq. A.3) is used. The top quark mass parameter m_t is called the parameter of interest (POI).

$$\mu_i(m_t) = s_i(m_t) + b_i \tag{A.1}$$

$$\mathcal{P}(n_i|\mu_i) = \frac{\mu_i^{n_i} e^{-\mu_i}}{n_i!} \quad n_i \in \mathcal{N}, \mu_i \in \mathcal{R}^+ \quad (\text{A.2})$$

$$\mathcal{P}(n_i|\mu_i) = \frac{\mu_i^{n_i} e^{-\mu_i}}{\Gamma(n_i + 1)} \quad n_i, \mu_i \in \mathcal{R}^+ \quad (\text{A.3})$$

Each systematic, given as a separate binned distribution for up (and possibly down variations), is assigned a nuisance parameter θ_j assumed to follow a Gaussian prior distribution $\mathcal{G}(\hat{\theta}_j|\theta_j)$ (Eq A.4), with mean $\hat{\theta}_j$ the "best-fit" value of this nuisance parameter to be determined. The set of K systematic nuisance parameters is represented by the vector $\vec{\theta} = (\theta_1, \dots, \theta_K)$.

$$\mathcal{G}(\hat{\theta}_j|\theta_j) = e^{-\frac{1}{2}(\theta_j - \hat{\theta}_j)^2} \quad (\text{A.4})$$

The full likelihood function over all M bins and K nuisances is given in Equation A.5. The likelihood-ratio test (A.6) is used to assess the goodness of fit. Here $\hat{L} = L(\hat{m}_t; \hat{\theta})$ is the maximum value of the likelihood and $\hat{m}_t, \hat{\theta}$ are the corresponding best-fit values of the top quark mass and nuisance parameters.

$$L(m_t; \vec{\theta}) = \prod_{i=1}^M \mathcal{P}(n_i|\mu_i(m_t)) \prod_{j=1}^K \mathcal{G}(\hat{\theta}_j|\theta_j) \quad (\text{A.5})$$

$$\frac{L(m_t; \vec{\theta})}{L(\hat{m}_t; \hat{\theta})} \quad (\text{A.6})$$

To determine confidence intervals, the likelihood is *profiled* to eliminate nuisance parameters by scanning along the line which maximizes L , as shown in the example in Figure A.1. The MLE of this profiled function is equivalent to the MLE of the original likelihood.

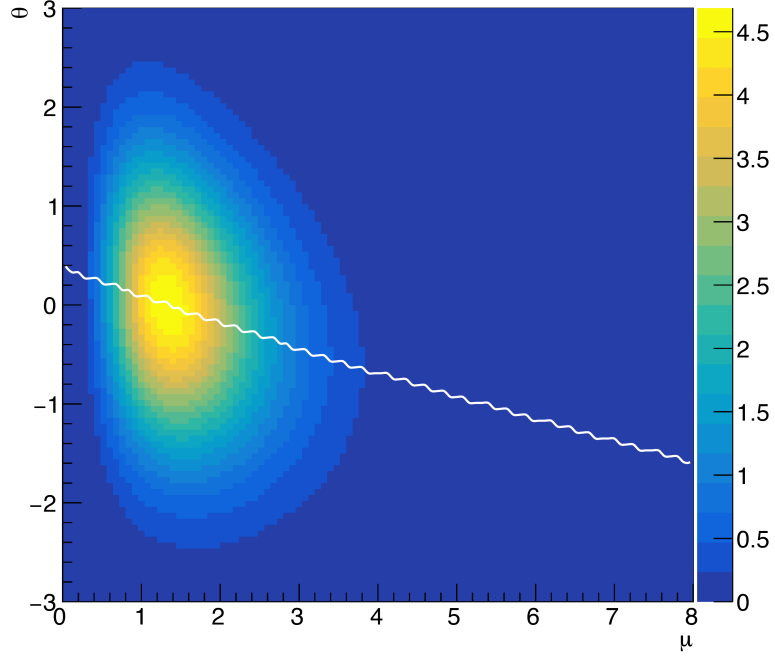


Figure A.1: The likelihood L is scanned along the line $\max_{\theta} L(\mu, \theta)$.

In practice, the negative loglikelihood is used to avoid extremely small or large likelihood values. Since the logarithm is a monotonic function, the maximum of L occurs at the minimum of $-\ln L$. Using twice the negative loglikelihood $\Lambda(m_t) \equiv -2(\ln L(m_t) - \ln \hat{L})$, where $\Lambda(\hat{m}_t) = 0$, Wilks' theorem ensures that Λ asymptotically approaches a χ^2 distribution. Moreover, the 68% confidence interval is the domain for which $\Lambda < 1$, so the values of m_t for which $\Lambda = 1$ represent the $\pm 1\sigma$ uncertainties in the extracted top mass.

Appendix B

TUnfold Algorithm

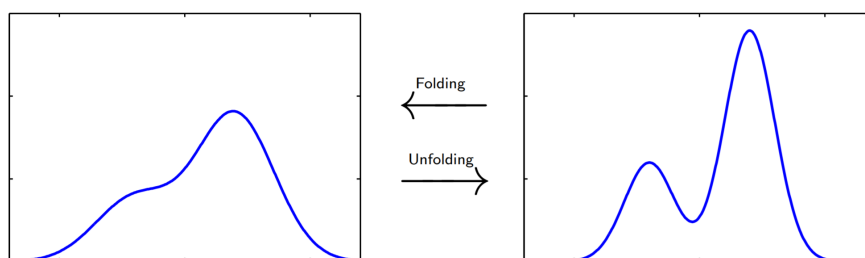


Figure : Smeared density

Figure : True density

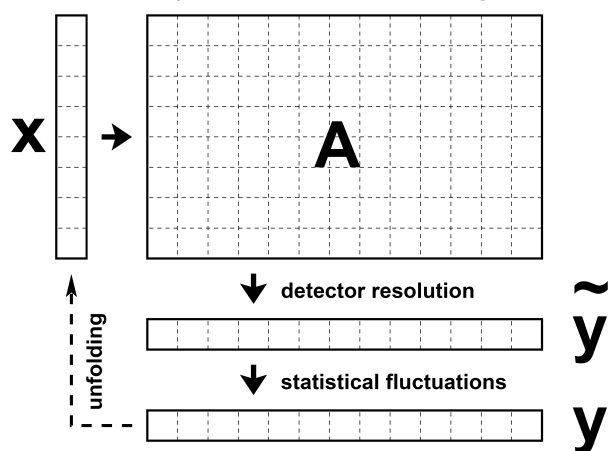


Figure B.1: Diagrammatic overview of the unfolding problem.

Unfolding is the process of recovering the true density from a smeared distribution. The smearing is caused by detector effects and various stages of the event simulation (e.g., parton showering, radiation). The true distribution $\tilde{\mathbf{x}}$ (containing m bins) is related to the average reconstructed (reco) distribution $\tilde{\mathbf{y}}$ (containing n bins) by Equation B.1. Contributions from background processes are given by $\mathbf{b}$. $\mathbf{A}$ is called the response (or migration) matrix: A_{ij} gives the probability that a truth-level event in bin j will be reconstructed in reco bin i .

$$\tilde{y}_i = \sum_{j=1}^m A_{ij} \tilde{x}_j + b_i, \quad 1 \leq i \leq n \quad (\text{B.1})$$

Observables are described by a vector of random variables $\mathbf{y}$. These are assumed to follow a Gaussian distribution with mean $\tilde{\mathbf{y}} = \mathbf{A}\tilde{\mathbf{x}}$. The output, $\mathbf{x}$, is an estimator of the truth density vector $\tilde{\mathbf{x}}$. The algorithm requires $\dim(\tilde{\mathbf{x}}) \leq \dim(\tilde{\mathbf{y}})$, implying that the density precision cannot improve after unfolding.

TUnfold uses least square minimization with Tikonov regularization. It attempts to find the stationary point of the Lagrangian $\mathcal{L}$ (Eq B.2).

$$\mathcal{L} \equiv \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 \quad (\text{B.2})$$

$$\mathcal{L}_1 = (\mathbf{y} - \mathbf{A}\mathbf{x}^\top) \mathbf{V}_{\mathbf{yy}}^{-1} (\mathbf{y} - \mathbf{A}\mathbf{x}) \quad (\text{B.3})$$

$$\mathcal{L}_2 = \tau^2 (\mathbf{x} - f_b \mathbf{x}_0)^\top (\mathbf{L}^\top \mathbf{L}) (\mathbf{x} - f_b \mathbf{x}_0) \quad (\text{B.4})$$

$$\mathcal{L}_3 = \lambda (Y - \mathbf{e}^\top \mathbf{x}) \quad (\text{B.5})$$

$$Y = \sum_{i=0}^n y_i \quad (\text{B.6})$$

$$e_j = \sum_{i=0}^n A_{ij} \quad (\text{B.7})$$

$\mathcal{L}_1$ is the expected term from least square minimization and $\mathbf{V}_{\mathbf{y}\mathbf{y}}$ is the covariance matrix of $\mathbf{y}$. $\mathcal{L}_2$ represents the regularization term which is typically employed to dampen fluctuations in $\mathbf{x}$ arising from statistical fluctuations in $\mathbf{y}$. The parameter τ is called the regularization strength and is considered a constant for determining the stationary point of the Lagrangian. $f_b \mathbf{x}_0$ is a bias vector with normalization factor f_b and vector $\mathbf{x}_0$. Since regularization is not applied for this analysis, $\tau \equiv 0$ implying $\mathcal{L}_2 \equiv 0$.

$\mathcal{L}_3$ is an optional area constraint with Lagrangian parameter λ which forces the normalization of $\mathbf{x}$, after correcting for efficiencies $\mathbf{e}$, to match the total event count Y . This is used to limit biases on the normalization assuming $\mathbf{y}$ follows Poisson statistics.

The response matrix $\mathbf{A}_{(n,m)}$ is derived from MC simulated events which are inputted as a matrix $\mathbf{M}_{(n+1,m)}$, with the additional row 0 entries $\mathbf{M}_{0,j}$ describing events that are generated in bin j but not found in any reco bin. The response $\mathbf{A}$ and bias $\mathbf{x}_0$ (unless otherwise specified) are computed from $\mathbf{M}$ in (B.8) and (B.10).

$$A_{ij} = \frac{M_{ij}}{s_j}, i \in [1, n] \quad (\text{B.8})$$

$$s_j = \sum_{i=0}^n M_{ij} \quad (\text{B.9})$$

$$\mathbf{x}_0 = s_j \quad (\text{B.10})$$

The stationary point of $\mathcal{L}(\mathbf{x}, \lambda)$ is found by setting the first partial derivatives to zero (B.11, B.12) and solving for $\mathbf{x}$ (B.13) and λ (B.16) simultaneously.

$$\frac{\partial \mathcal{L}(\mathbf{x}, \lambda)}{\partial x_i} = -2(\mathbf{A}^\top \mathbf{V}_{\mathbf{yy}}^{-1}(\mathbf{y} - \mathbf{Ax}))_j - \lambda e_j \quad (\text{B.11})$$

$$\frac{\partial \mathcal{L}(\mathbf{x}, \lambda)}{\partial \lambda} = Y - \mathbf{e}^\top \mathbf{x} \quad (\text{B.12})$$

$$\mathbf{x} = \mathbf{x}|_{\lambda=0} + \frac{\lambda}{2} \mathbf{E} \mathbf{e} \quad \text{with} \quad (\text{B.13})$$

$$\mathbf{x}|_{\lambda=0} = \mathbf{EA}^\top \mathbf{V}_{\mathbf{yy}}^{-1} \mathbf{y}, \quad (\text{B.14})$$

$$\mathbf{E} = (\mathbf{A}^\top \mathbf{V}_{\mathbf{yy}}^{-1} \mathbf{A})^{-1} \quad \text{and} \quad (\text{B.15})$$

$$\lambda = \frac{2(Y - \mathbf{e}^\top \mathbf{x}|_{\lambda=0})}{\mathbf{e}^\top \mathbf{E} \mathbf{e}} \quad (\text{B.16})$$

Given the covariance matrix of $\mathbf{y}$, $\mathbf{V}_{\mathbf{yy}}$, the corresponding covariance matrix on $\mathbf{x}$, $\mathbf{V}_{\mathbf{xx}}$ (B.19), is obtained after first calculating the partial derivatives (B.17).

$$(\mathbf{D}^{\mathbf{xy}})_{ki} \equiv \frac{\partial x_k}{\partial y_i} = B_{ki} + (\mathbf{E} \mathbf{e})_k \frac{1 - (\mathbf{B}^\top \mathbf{e})_i}{\mathbf{e}^\top \mathbf{E} \mathbf{e}} \quad (\text{B.17})$$

$$\mathbf{B} = \mathbf{EA}^\top \mathbf{V}_{\mathbf{yy}}^{-1} \quad (\text{B.18})$$

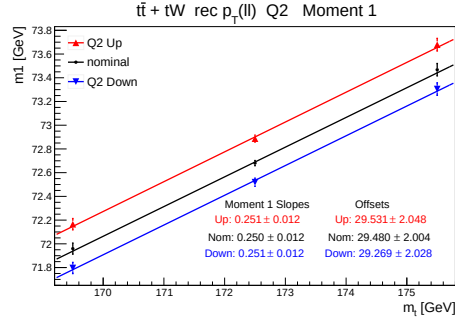
$$\mathbf{V}_{\mathbf{xx}} = \mathbf{D}^{\mathbf{xy}} \mathbf{V}_{\mathbf{yy}} (\mathbf{D}^{\mathbf{xy}})^\top \quad (\text{B.19})$$

Appendix C

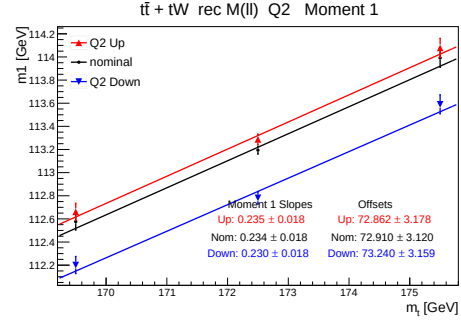
Moment Calibration

The first two moments of the leptonic observables and linear fits are shown below. Errors bars on the moments are based on the statistical uncertainty of the nominal simulation samples used to calculate the corresponding moments. The reconstructed-level observables used to extract the top quark MC mass, m_t^{MC} , are shown in Section C.1. Moments from the fixed order QCD predictions used to extract the top quark pole mass, m_t^{pole} , are shown in Section C.2.

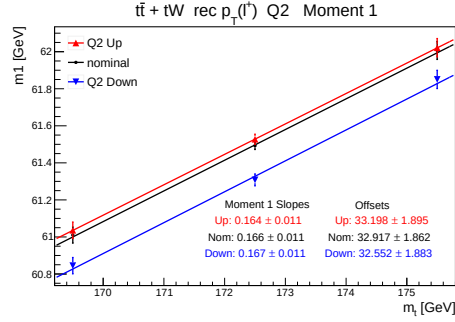
C.1 Reconstructed-Level Observables



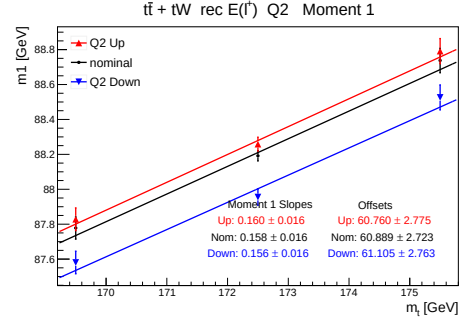
(a) $t\bar{t} + tW p_T(ll)$



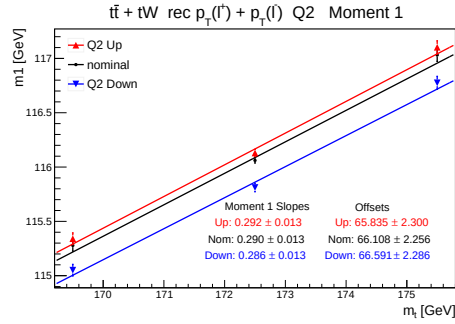
(b) $t\bar{t} + tW M(ll)$



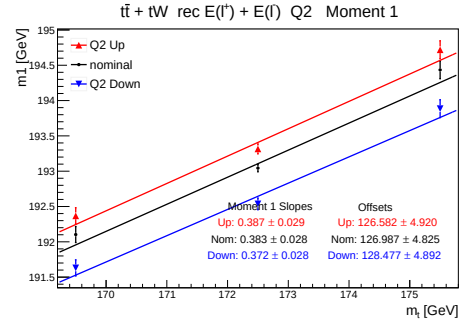
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

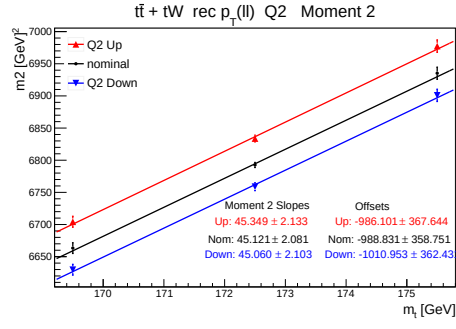


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

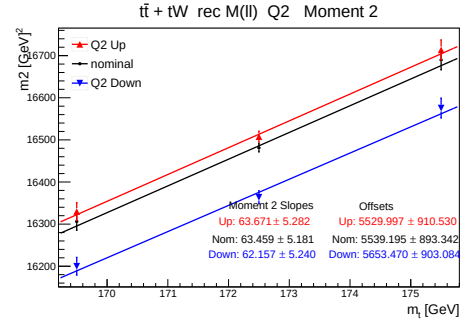


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

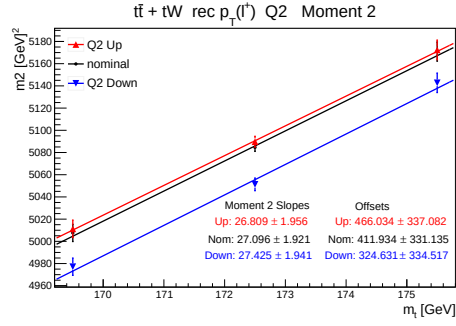
Figure C.1: QCD scale systematic first moments.



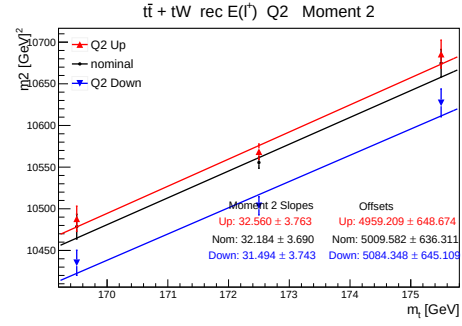
(a) $t\bar{t} + tW p_T(l\bar{l})$



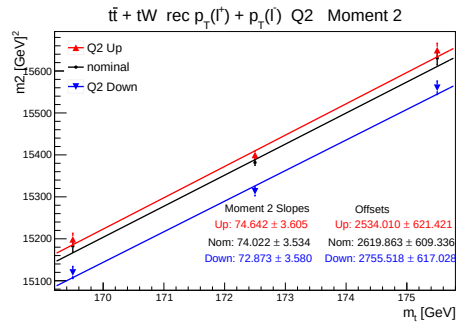
(b) $t\bar{t} + tW M(l\bar{l})$



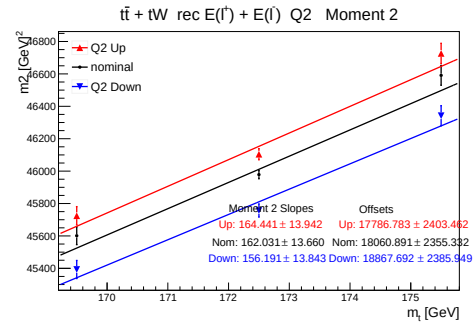
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

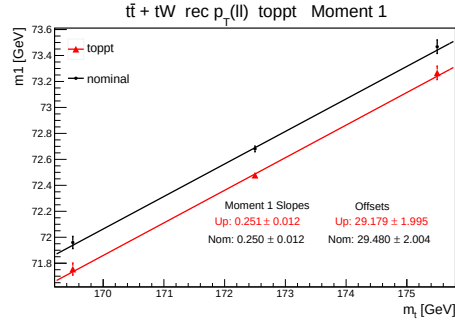


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

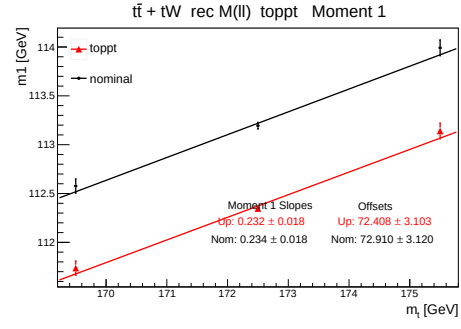


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

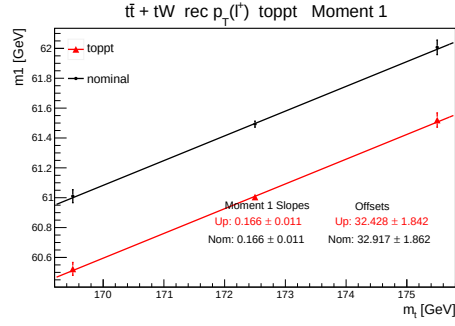
Figure C.2: QCD scale systematic second moments.



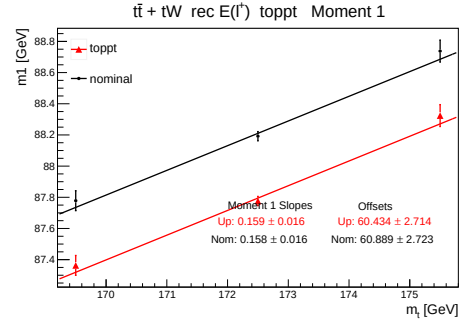
(a) $t\bar{t} + tW p_T(ll)$



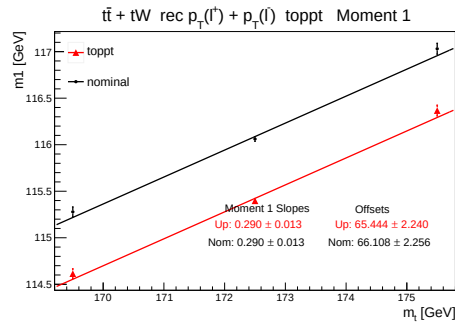
(b) $t\bar{t} + tW M(ll)$



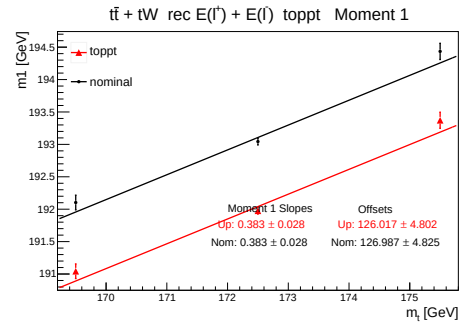
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

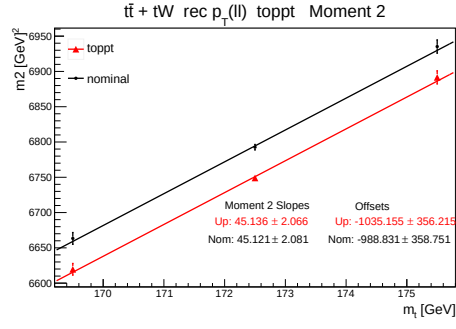


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

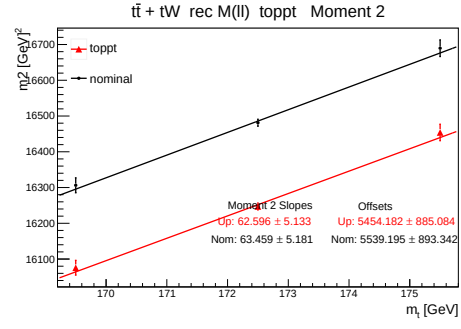


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

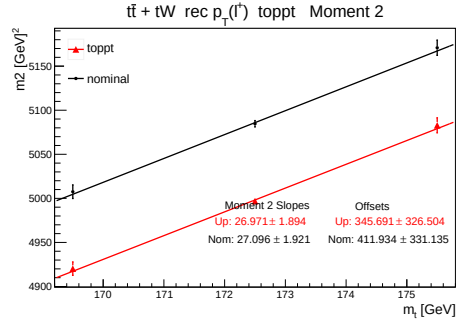
Figure C.3: Top p_T reweighting systematic first moments.



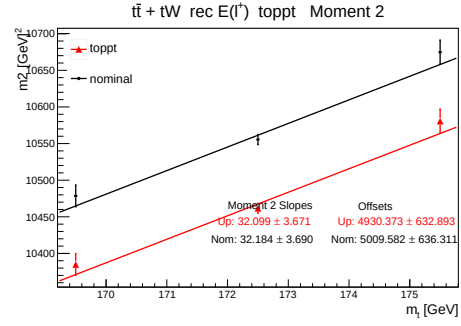
(a) $t\bar{t} + tW p_T(l)$



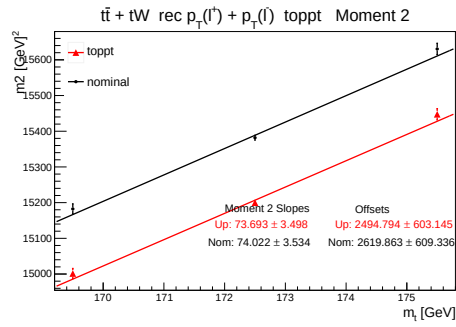
(b) $t\bar{t} + tW M(l)$



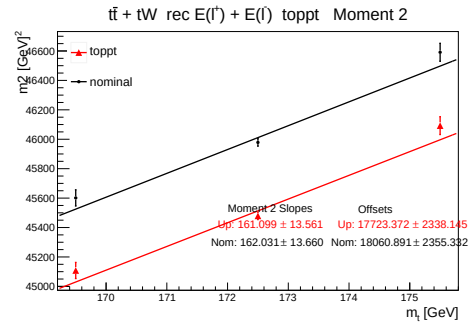
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

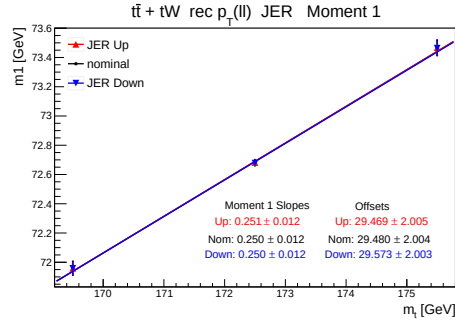


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

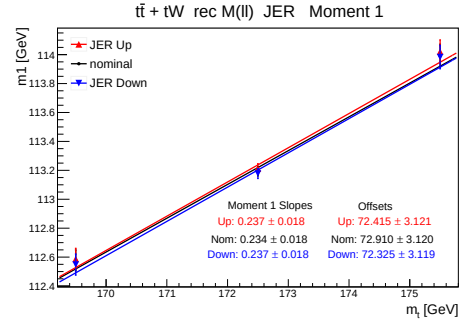


(f) $t\bar{t} + tW E(l') + E(l)$

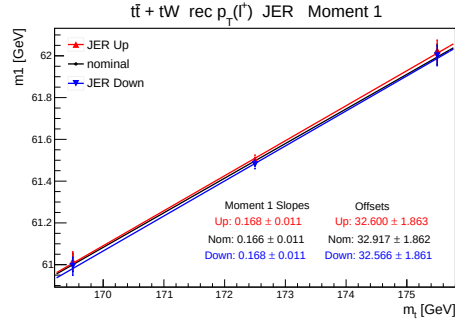
Figure C.4: Top p_T reweighting systematic second moments.



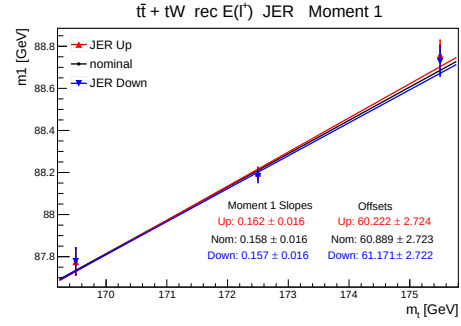
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



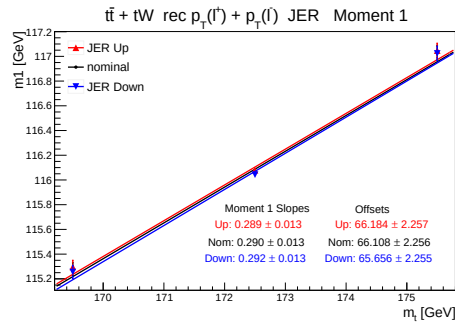
(b) $t\bar{t} + tW$ $M(l\bar{l})$



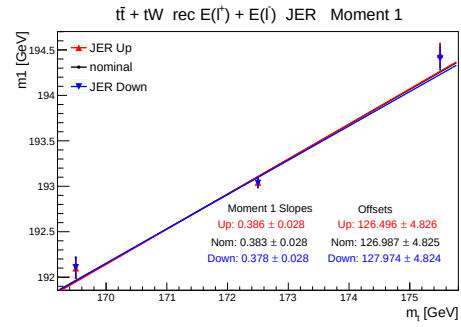
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

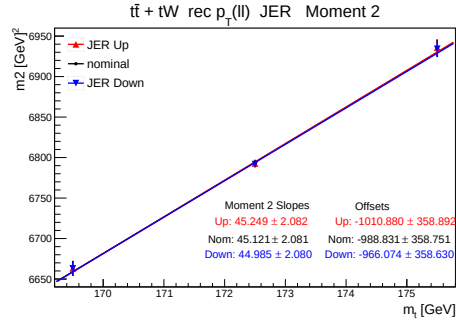


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

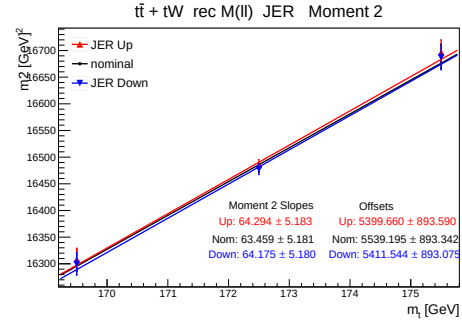


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

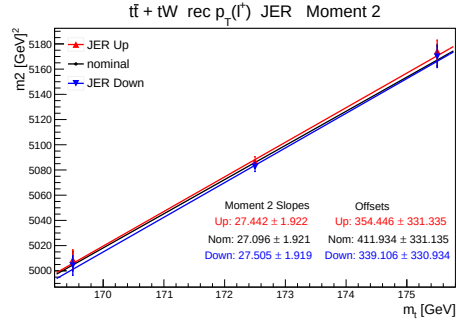
Figure C.5: JER systematic first moments.



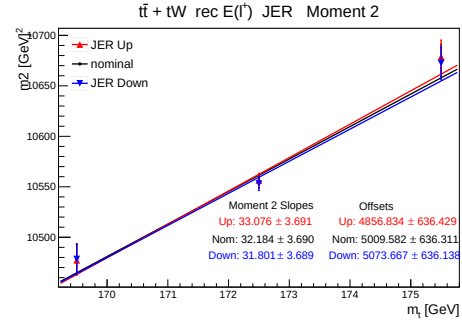
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



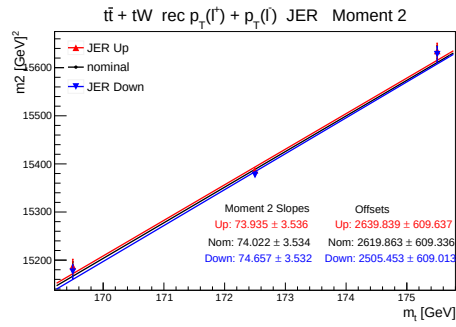
(b) $t\bar{t} + tW$ $M(l\bar{l})$



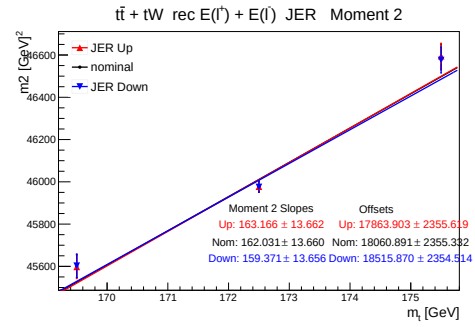
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

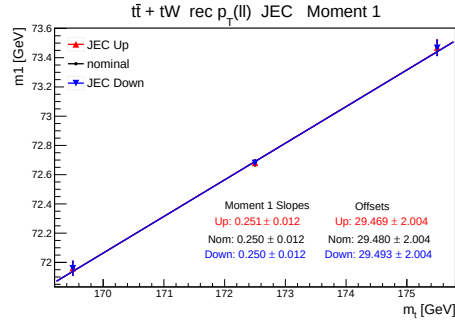


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

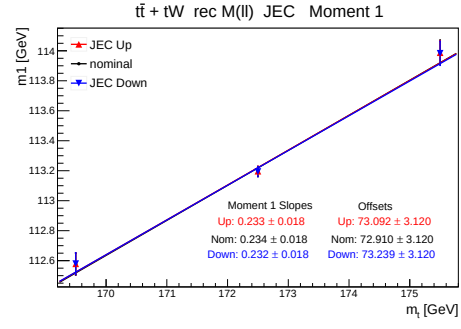


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

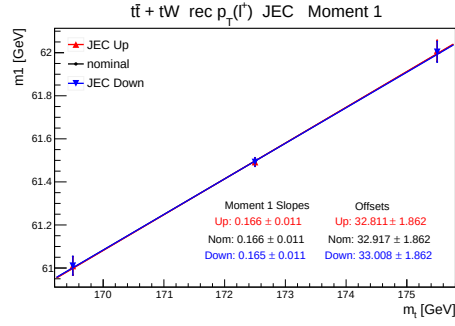
Figure C.6: JER systematic second moments.



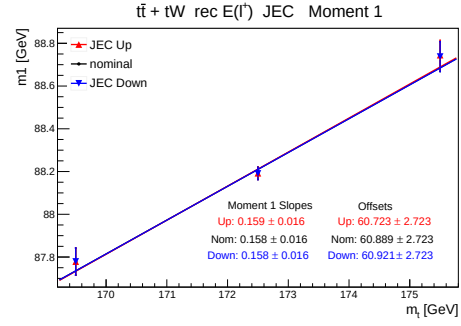
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



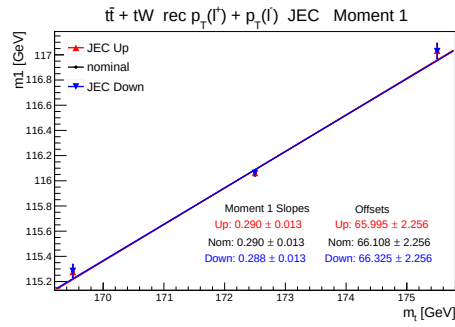
(b) $t\bar{t} + tW$ $M(l\bar{l})$



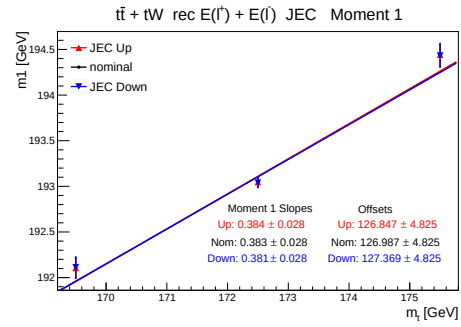
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

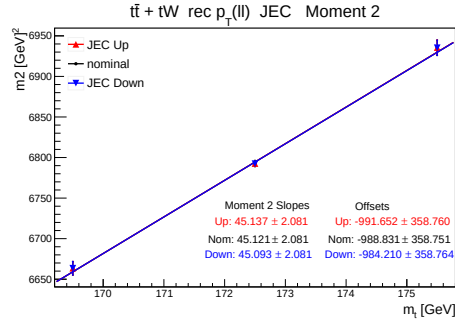


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

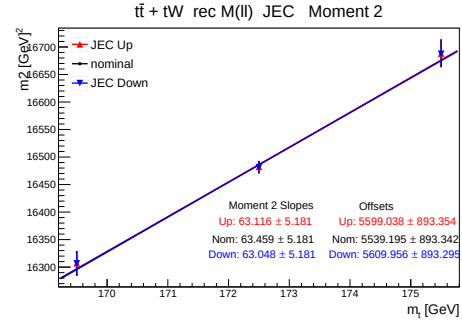


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

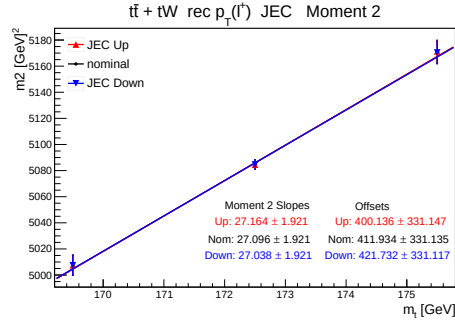
Figure C.7: JEC systematic first moments.



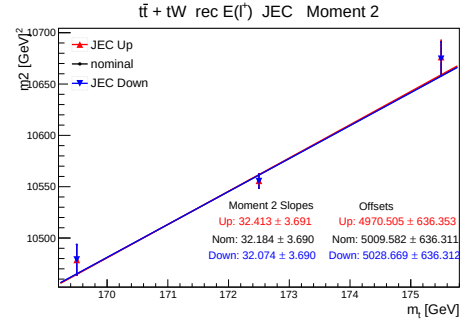
(a) $t\bar{t} + tW$ $p_T(l)$



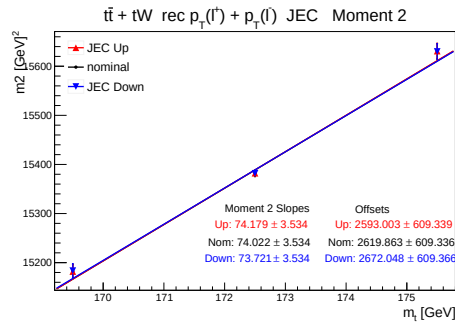
(b) $t\bar{t} + tW$ $M(l)$



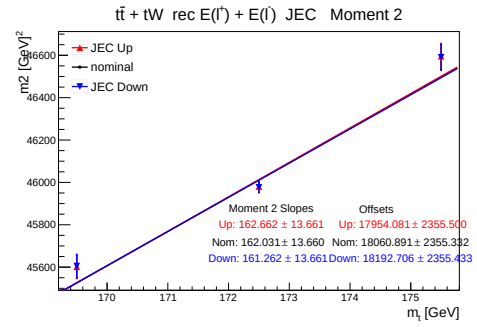
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

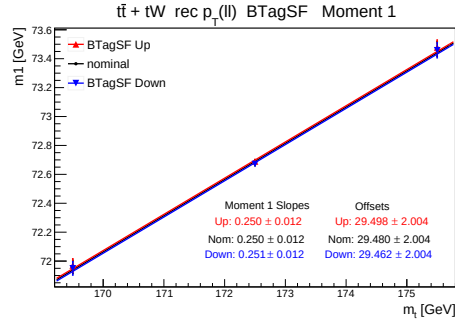


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

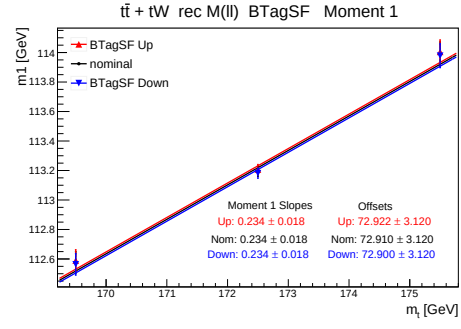


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

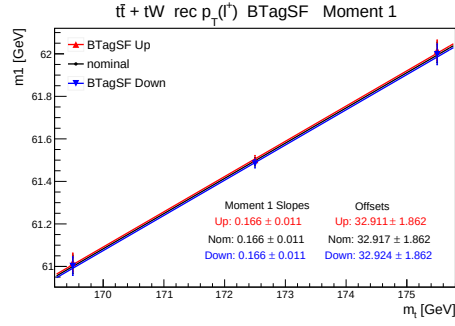
Figure C.8: JEC systematic second moments.



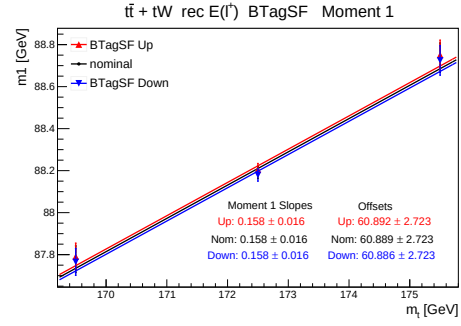
(a) $t\bar{t} + tW p_T(l\bar{l})$



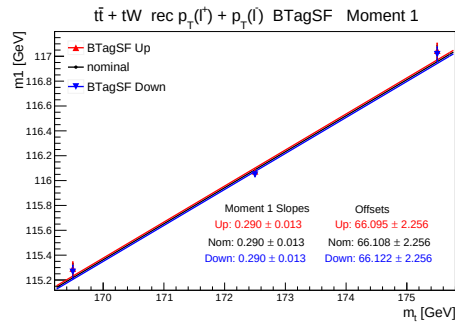
(b) $t\bar{t} + tW M(l\bar{l})$



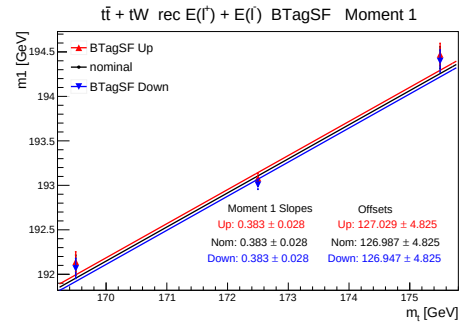
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

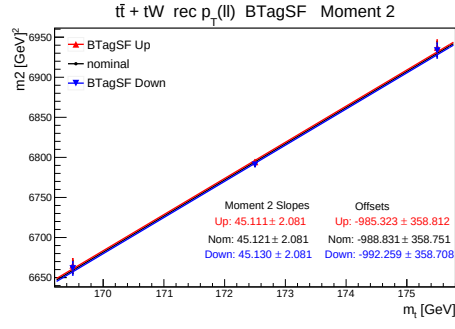


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

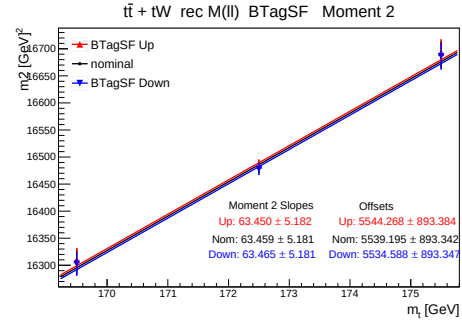


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

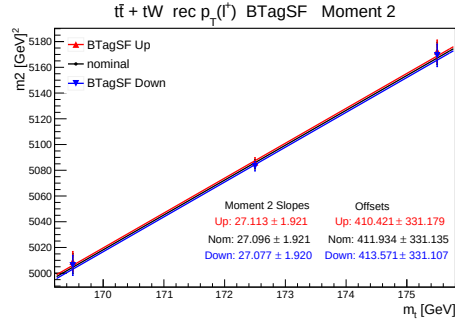
Figure C.9: b-tagging scale factor systematic first moments.



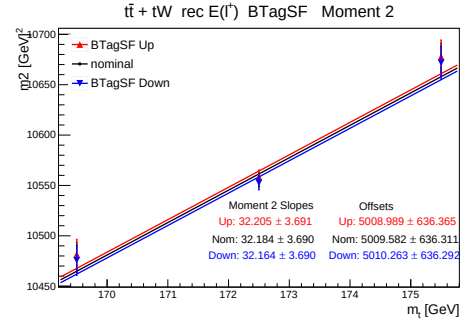
(a) $t\bar{t} + tW$ $p_T(ll)$



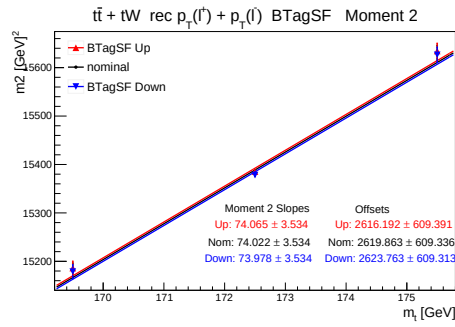
(b) $t\bar{t} + tW$ $M(ll)$



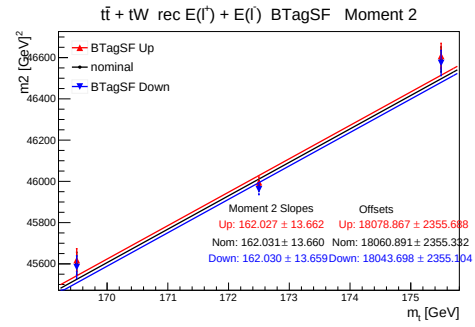
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

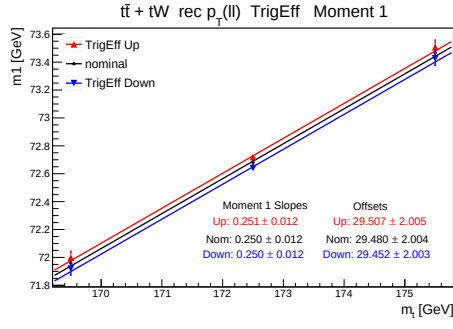


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

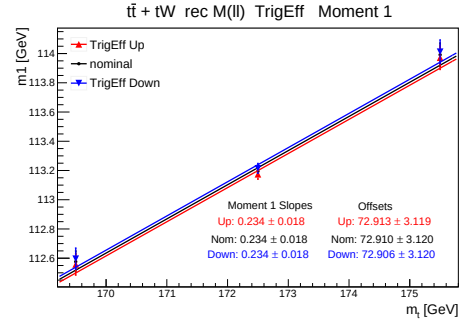


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

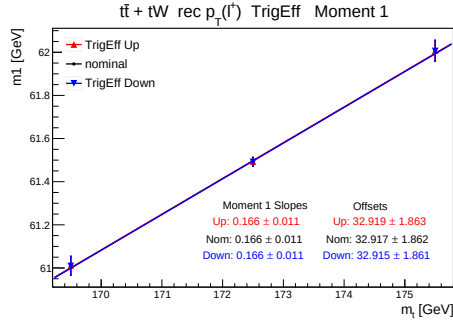
Figure C.10: b-tagging scale factor systematic second moments.



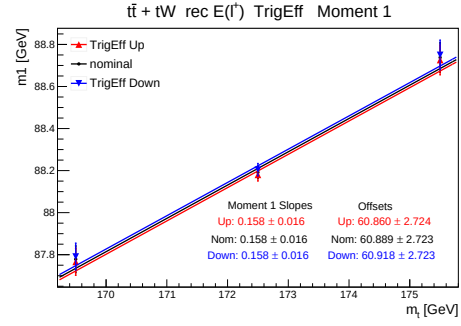
(a) $t\bar{t} + tW p_T(l)$



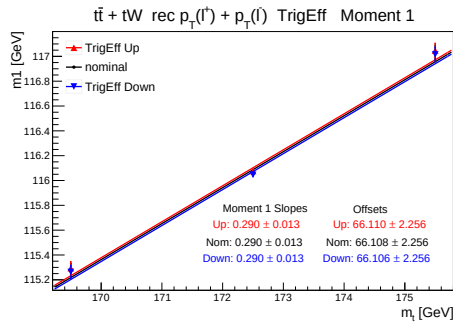
(b) $t\bar{t} + tW M(l)$



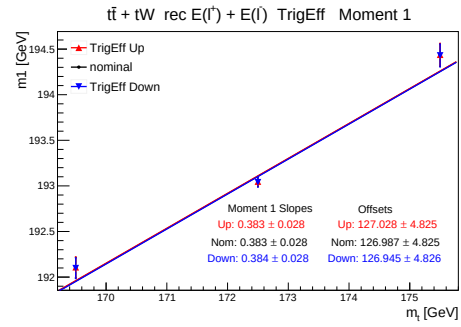
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

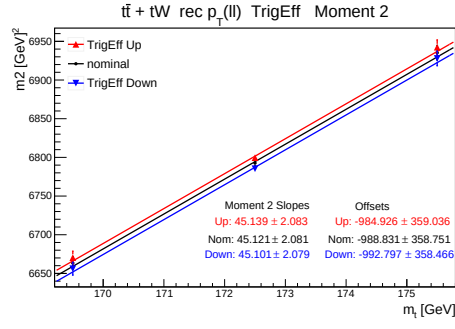


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

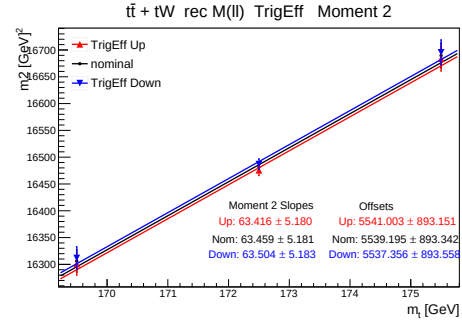


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

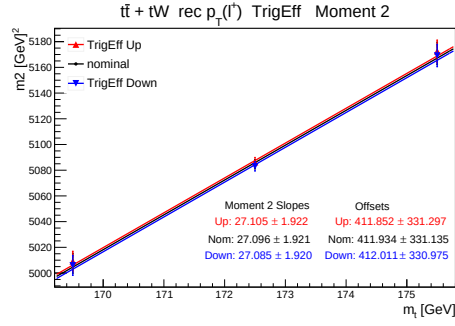
Figure C.11: HLT efficiency systematic first moments.



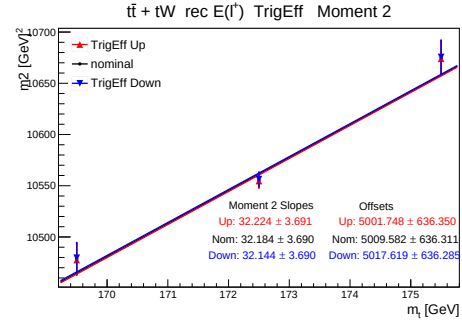
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



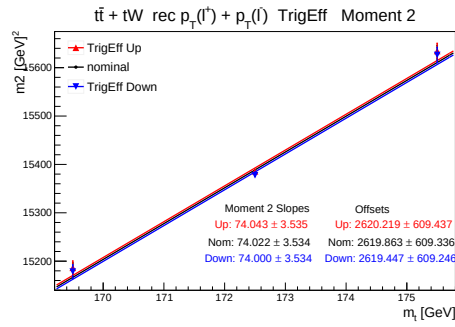
(b) $t\bar{t} + tW$ $M(l\bar{l})$



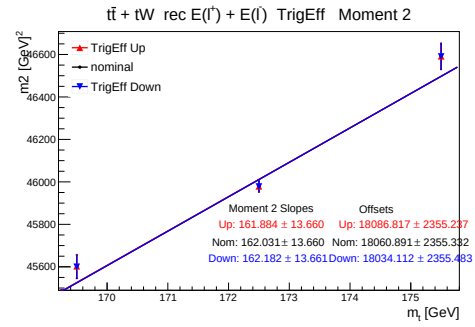
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

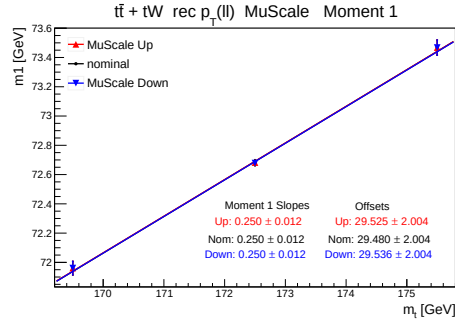


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

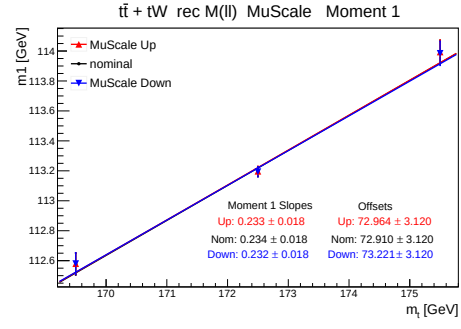


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

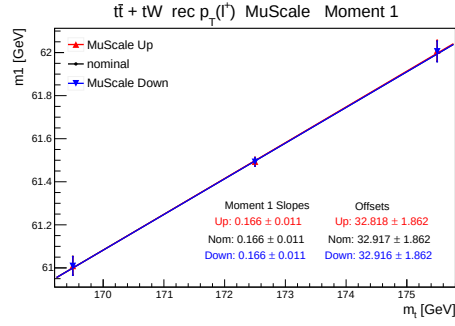
Figure C.12: HLT efficiency systematic second moments.



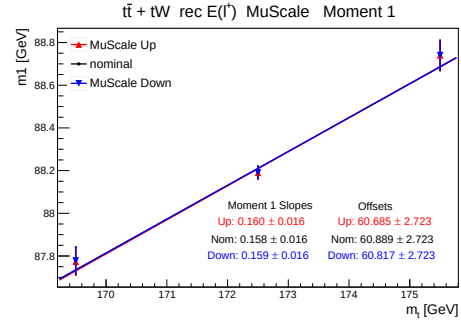
(a) $t\bar{t} + tW p_T(ll)$



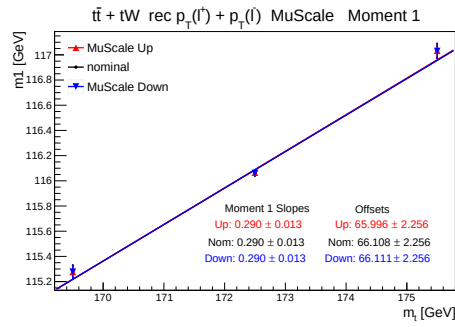
(b) $t\bar{t} + tW M(ll)$



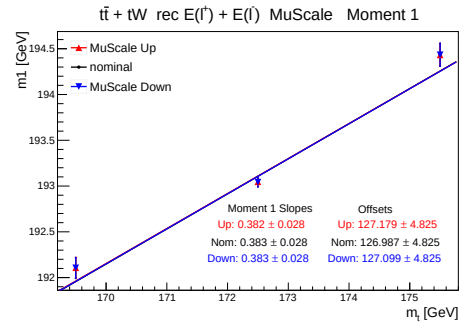
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

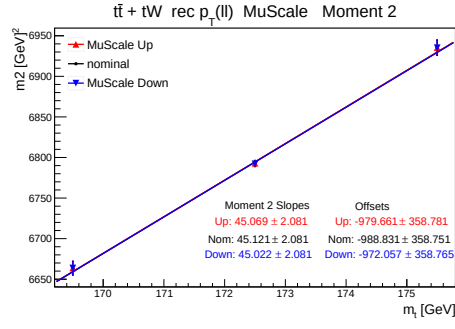


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

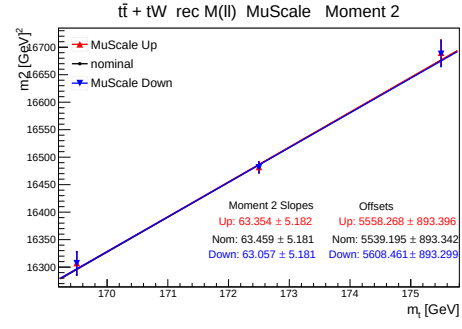


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

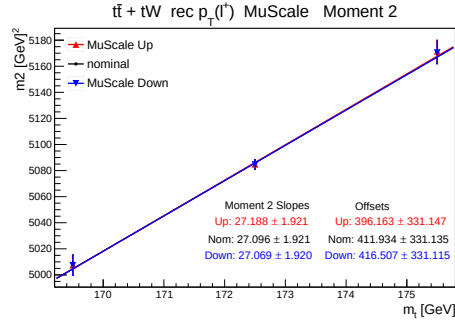
Figure C.13: Muon energy scale systematic first moments.



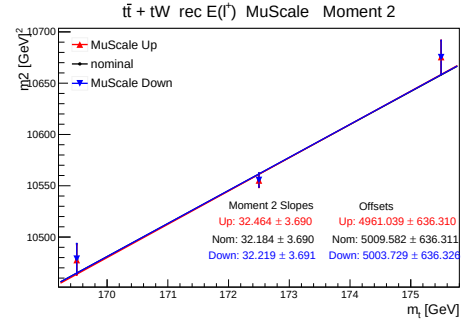
(a) $t\bar{t} + tW$ $p_T(l)$



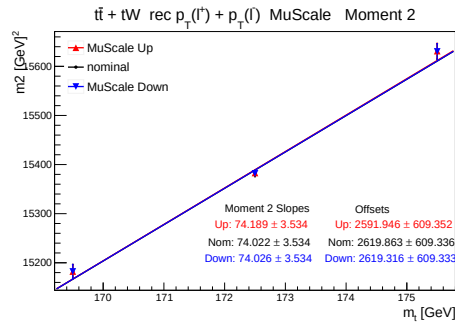
(b) $t\bar{t} + tW$ $M(l)$



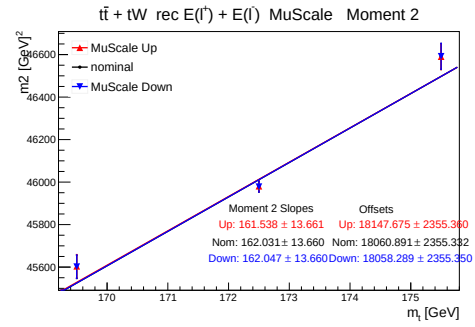
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

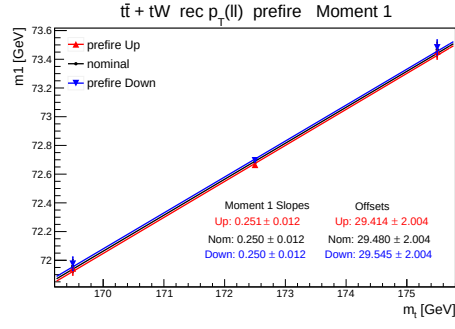


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

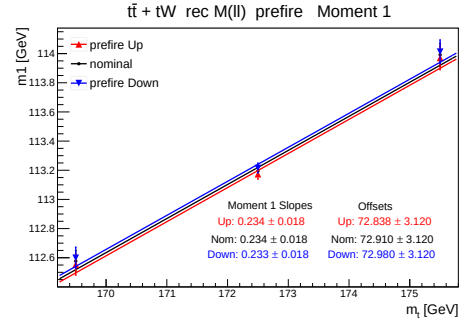


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

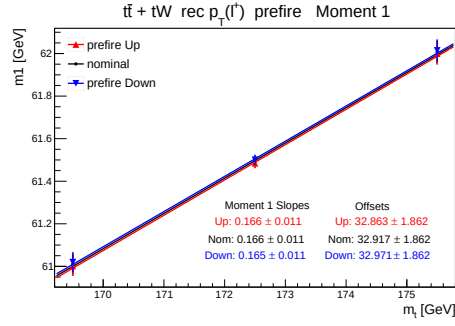
Figure C.14: Muon energy scale systematic second moments.



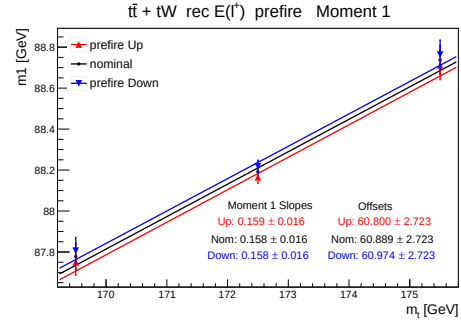
(a) $t\bar{t} + tW$ $p_T(l)$



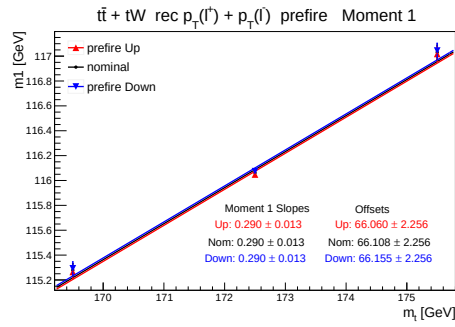
(b) $t\bar{t} + tW$ $M(l)$



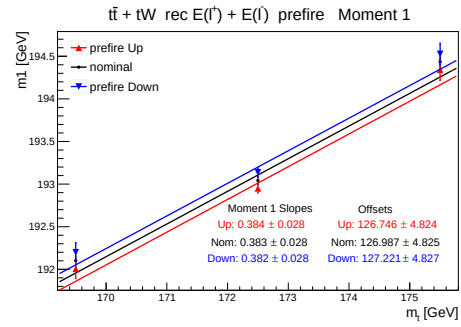
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

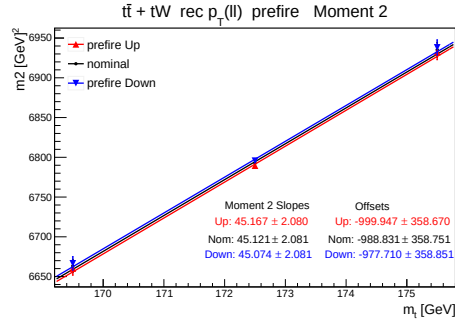


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

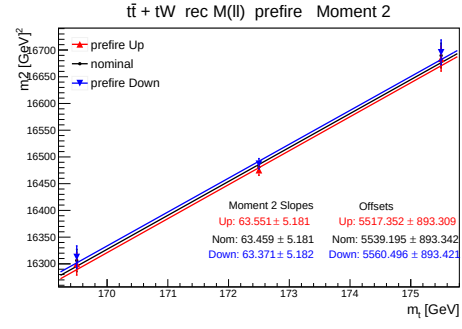


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

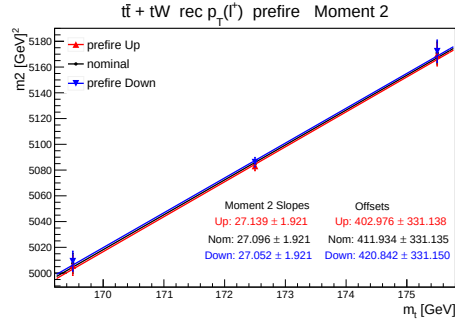
Figure C.15: Prefire scale factor systematic first moments.



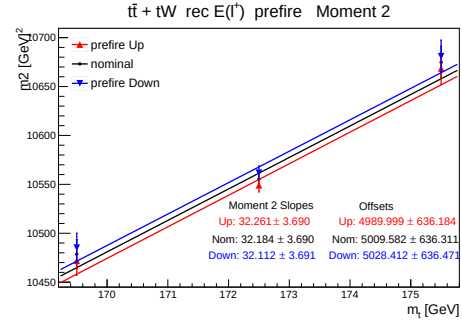
(a) $t\bar{t} + tW$ $p_T(l)$



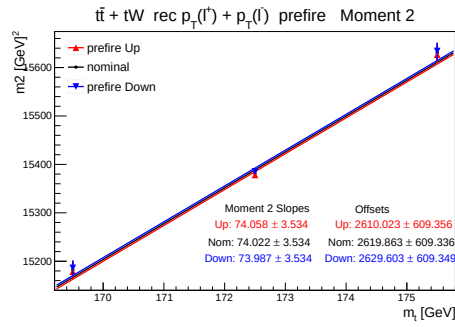
(b) $t\bar{t} + tW$ $M(l)$



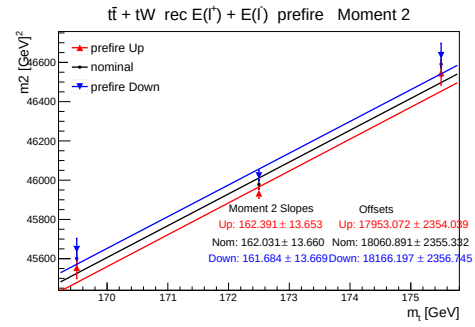
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

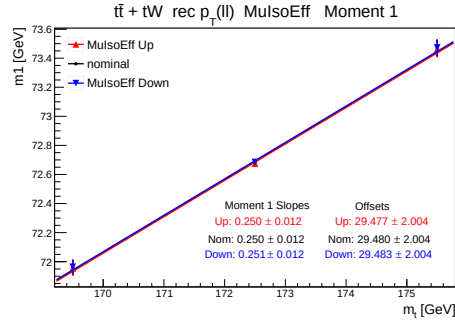


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

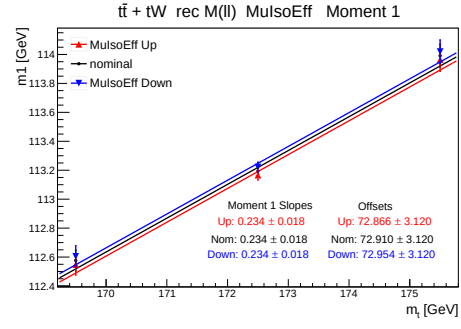


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

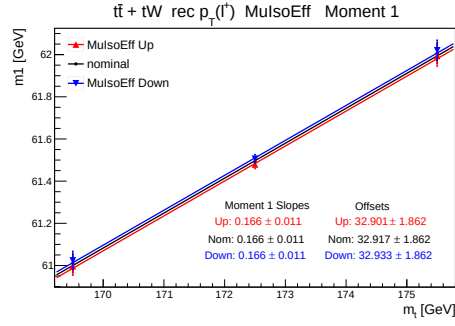
Figure C.16: Prefire scale factor systematic second moments.



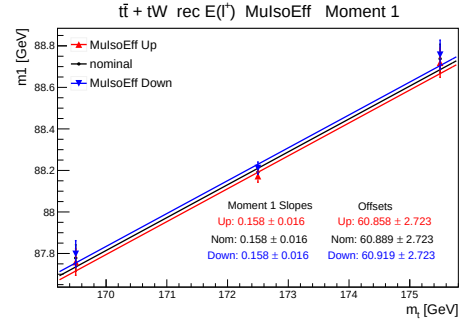
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



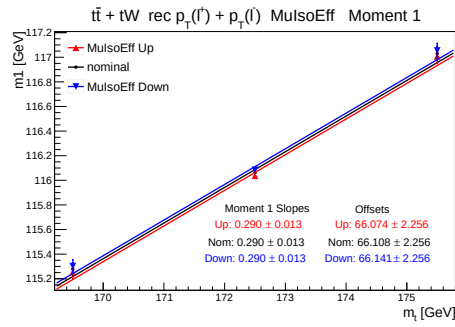
(b) $t\bar{t} + tW$ $M(l\bar{l})$



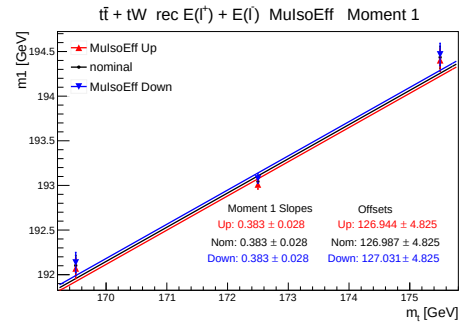
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

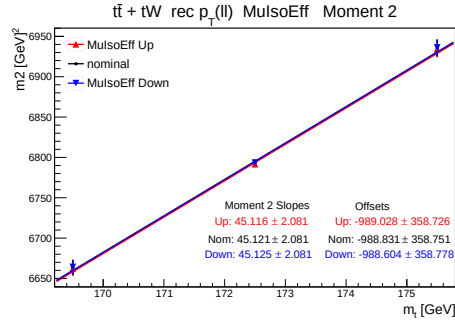


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

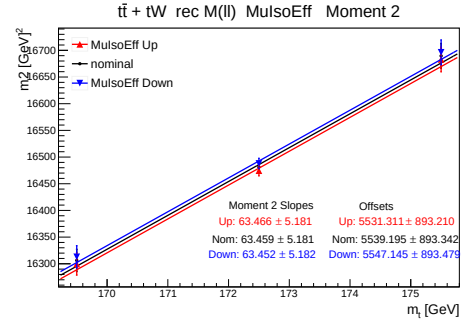


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

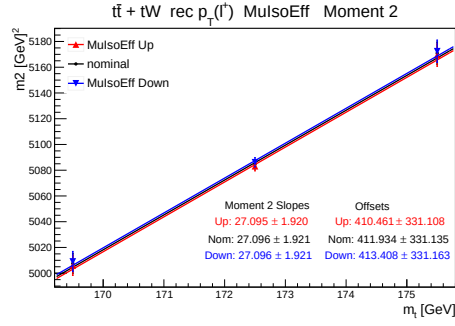
Figure C.17: Muon isolation efficiency systematic first moments.



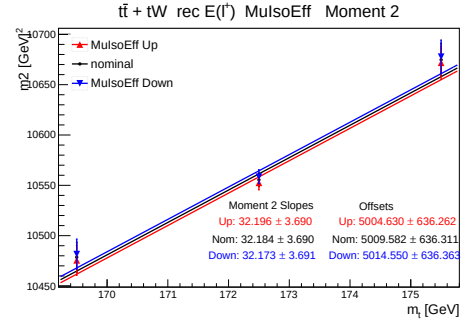
(a) $t\bar{t} + tW$ $p_T(l)$



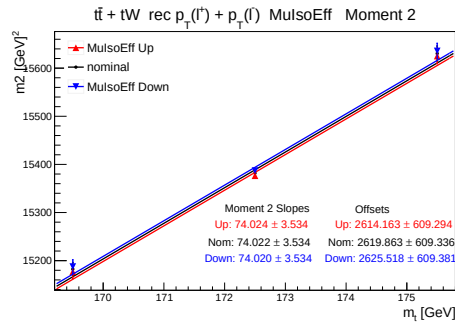
(b) $t\bar{t} + tW$ $M(l)$



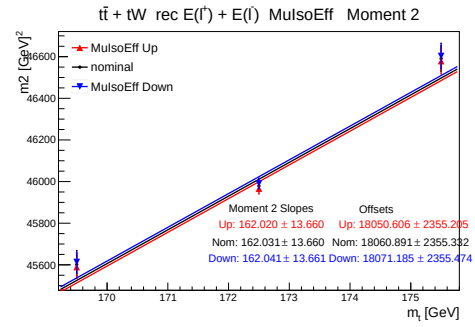
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$



(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$



(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

Figure C.18: Muon isolation efficiency systematic second moments.

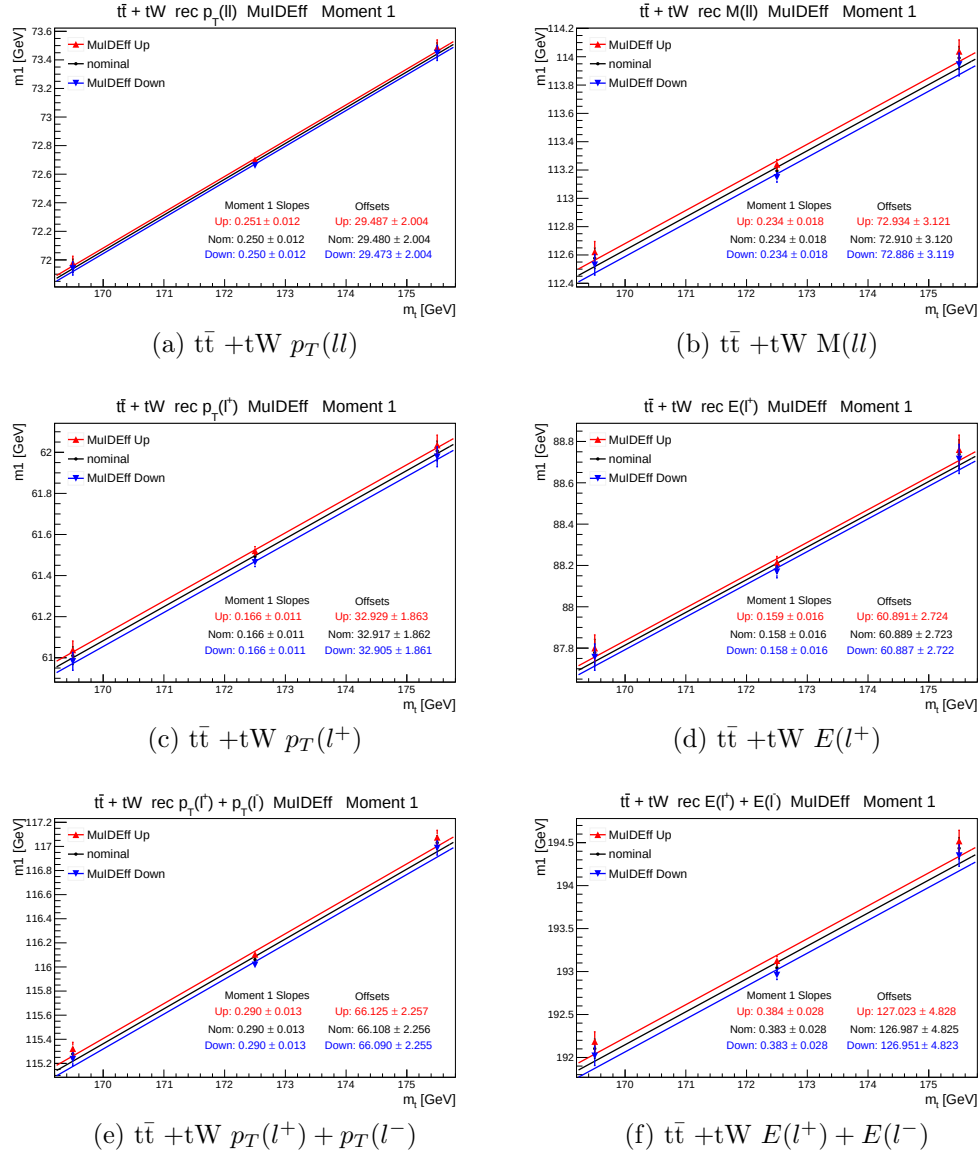
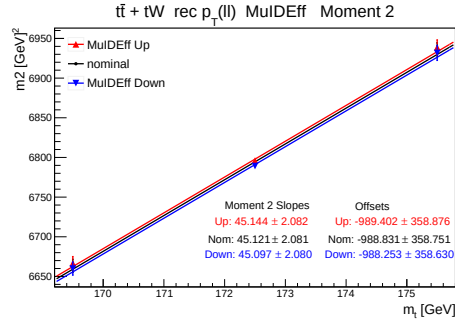
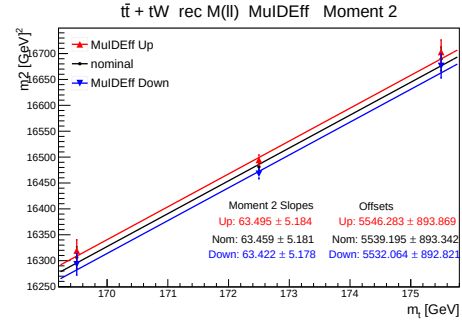


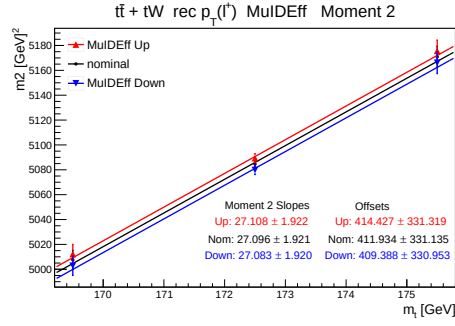
Figure C.19: Muon identification efficiency systematic first moments.



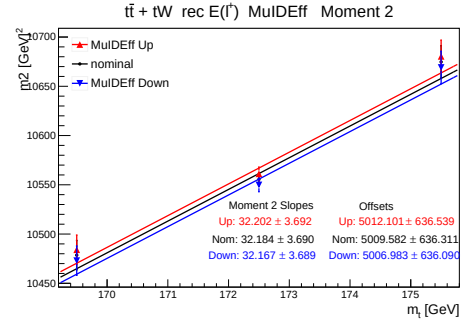
(a) $t\bar{t} + tW$ $p_T(l)$



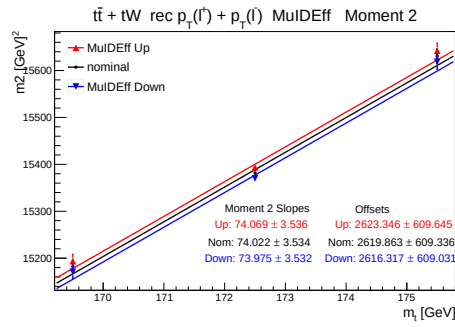
(b) $t\bar{t} + tW$ $M(l)$



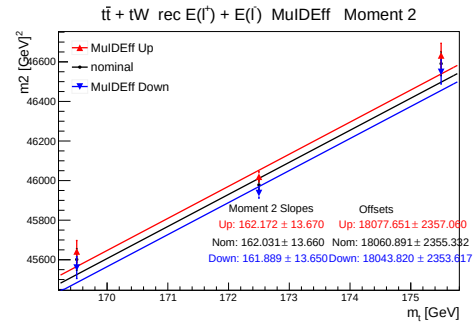
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

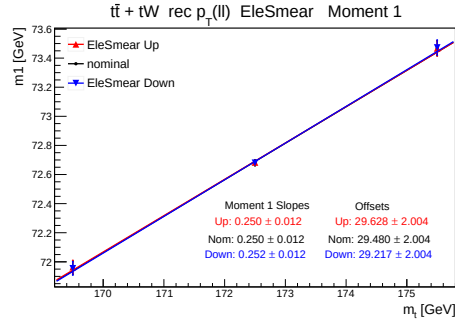


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

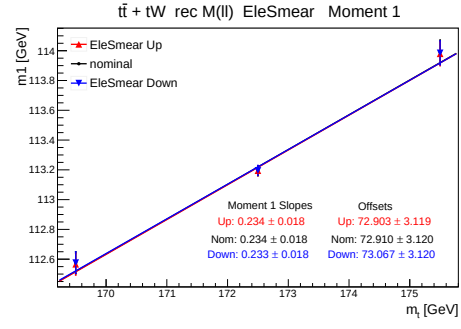


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

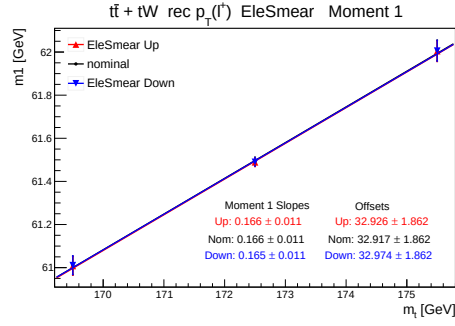
Figure C.20: Muon identification efficiency systematic second moments.



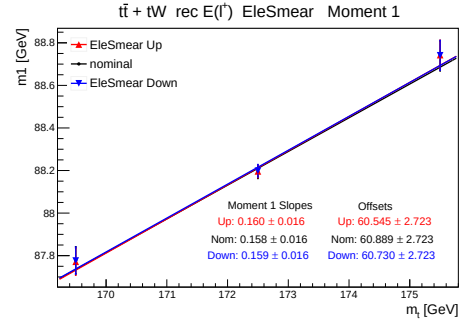
(a) $t\bar{t} + tW p_T(ll)$



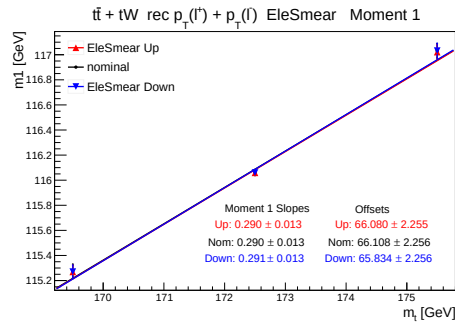
(b) $t\bar{t} + tW M(ll)$



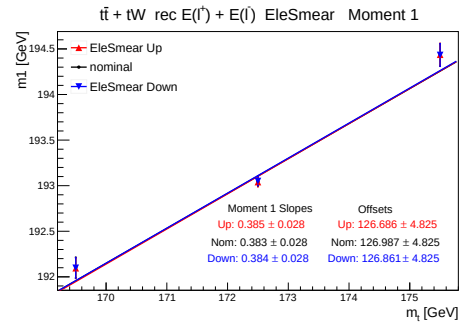
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

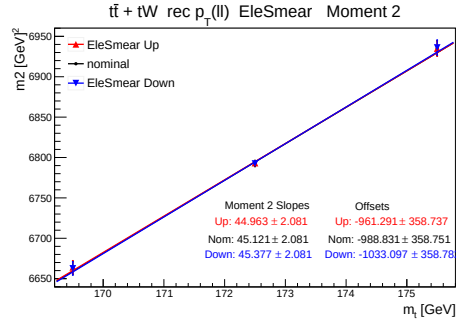


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

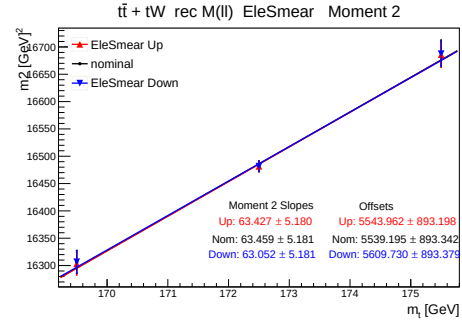


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

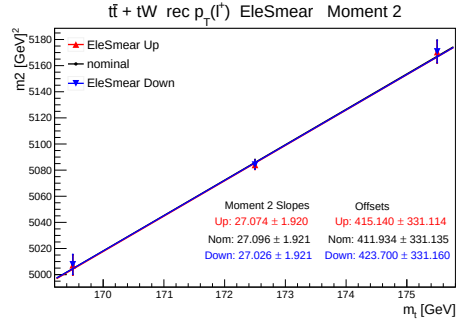
Figure C.21: Electron energy smearing systematic first moments.



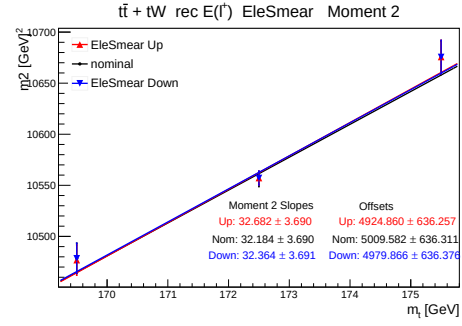
(a) $t\bar{t} + tW$ $p_T(l)$



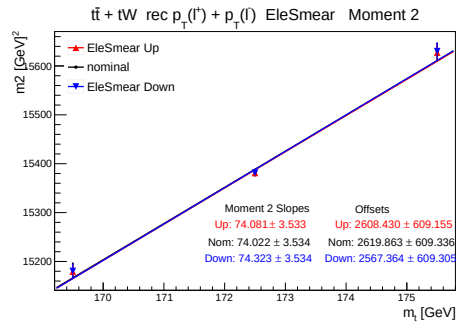
(b) $t\bar{t} + tW$ $M(l)$



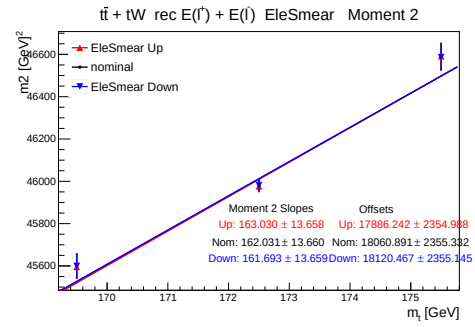
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

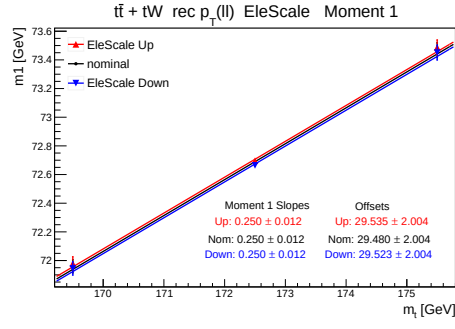


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

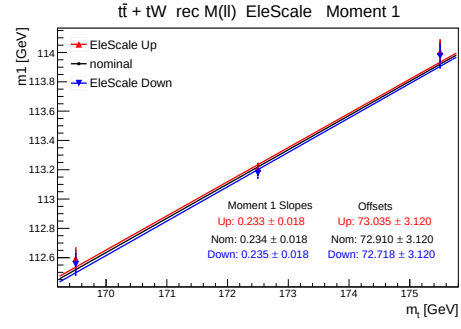


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

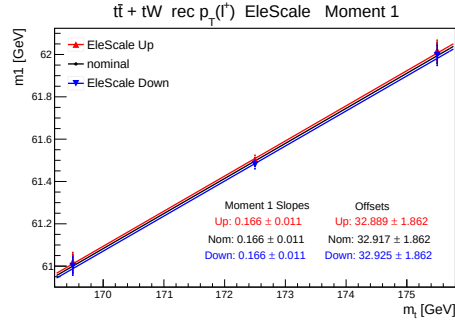
Figure C.22: Electron energy smearing systematic second moments.



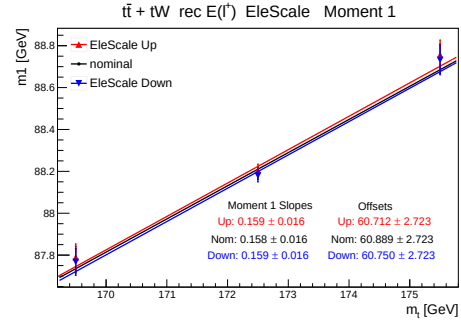
(a) $t\bar{t} + tW$ $p_T(l)$



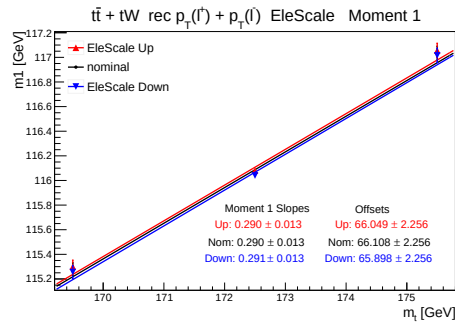
(b) $t\bar{t} + tW$ $M(l)$



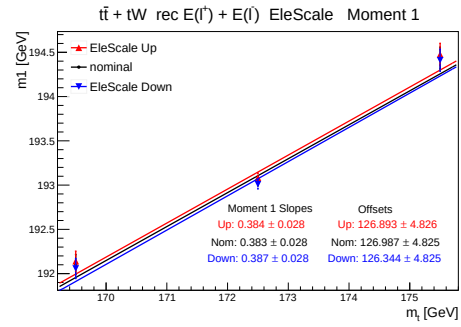
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

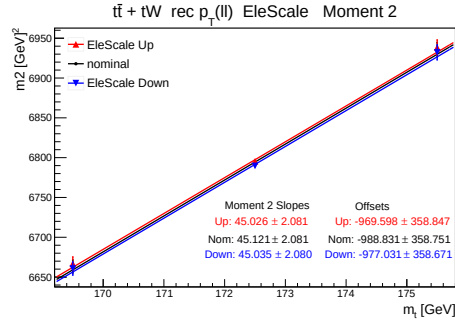


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

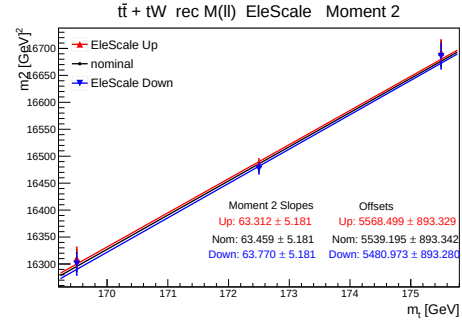


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

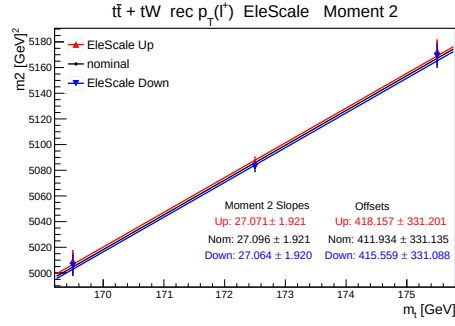
Figure C.23: Electron energy scaling systematic first moments.



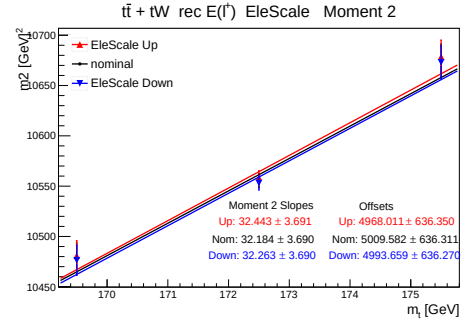
(a) $t\bar{t} + tW p_T(l)$



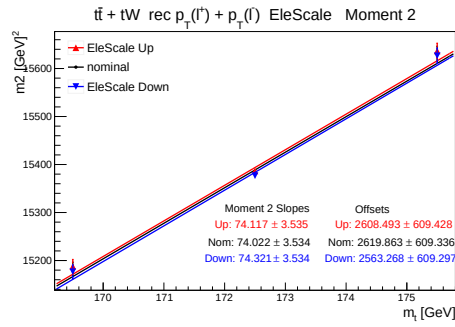
(b) $t\bar{t} + tW M(l)$



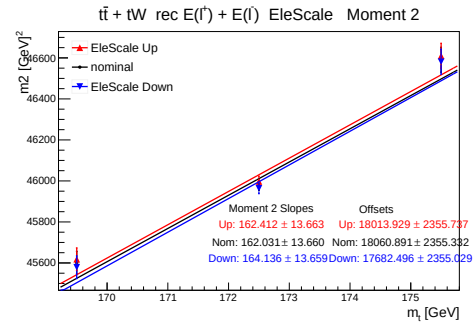
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

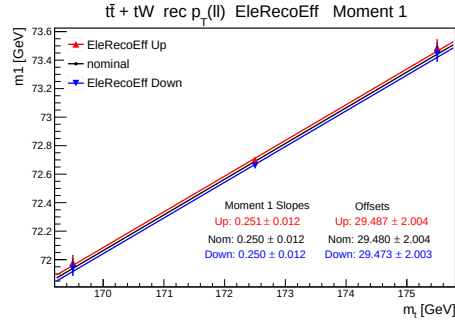


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

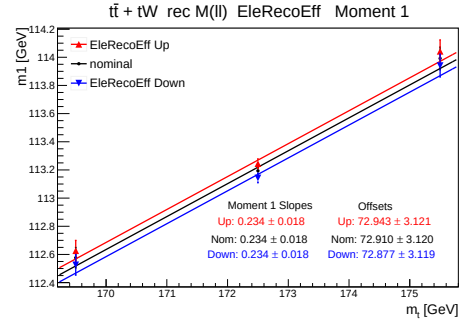


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

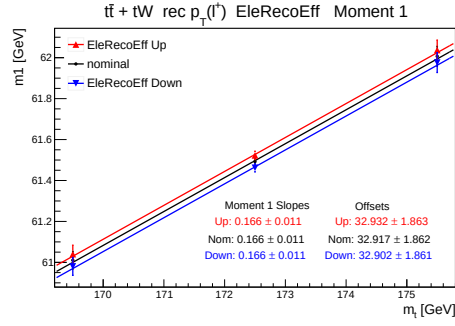
Figure C.24: Electron energy scaling systematic second moments.



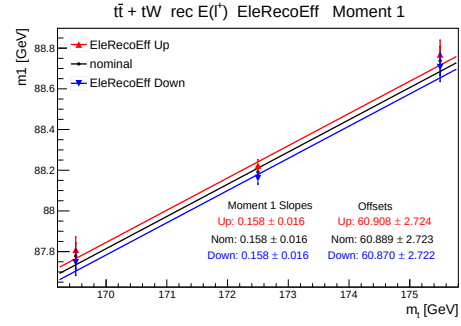
(a) $t\bar{t} + tW$ $p_T(l)$



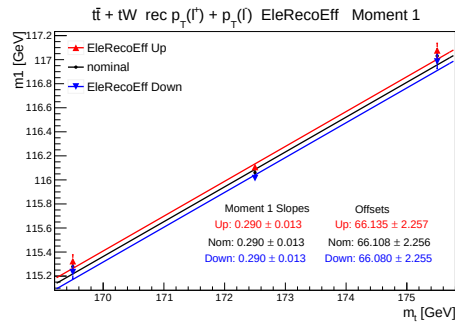
(b) $t\bar{t} + tW$ $M(l)$



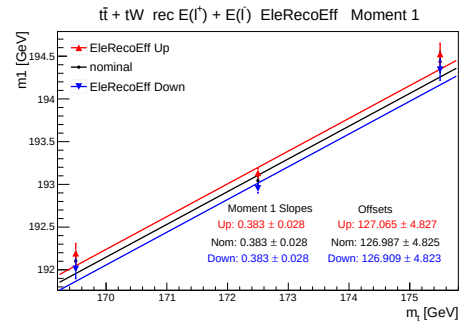
(c) $t\bar{t} + tW$ $p_T(l')$



(d) $t\bar{t} + tW$ $E(l')$

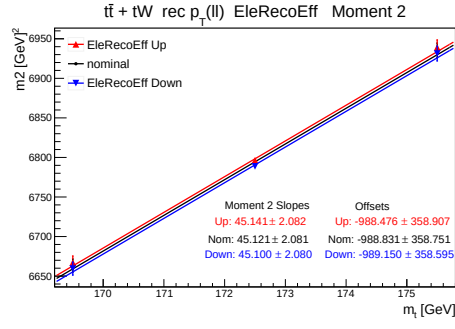


(e) $t\bar{t} + tW$ $p_T(l') + p_T(l)$

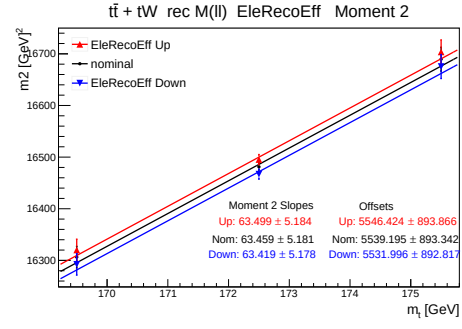


(f) $t\bar{t} + tW$ $E(l') + E(l)$

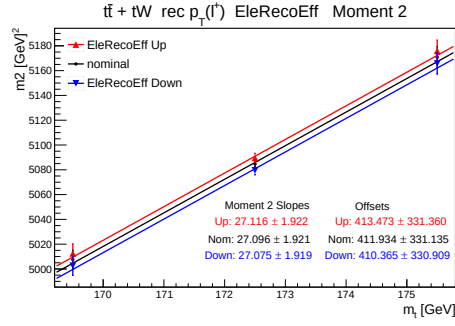
Figure C.25: Electron reconstruction efficiency systematic first moments.



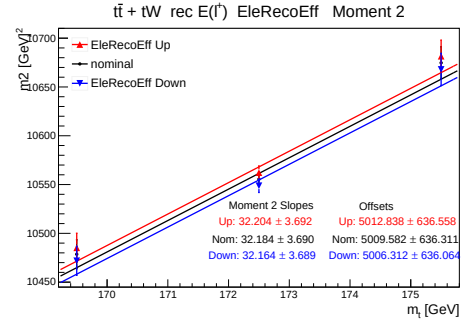
(a) $t\bar{t} + tW$ $p_T(l)$



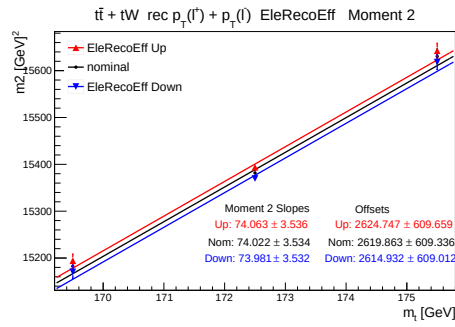
(b) $t\bar{t} + tW$ $M(l)$



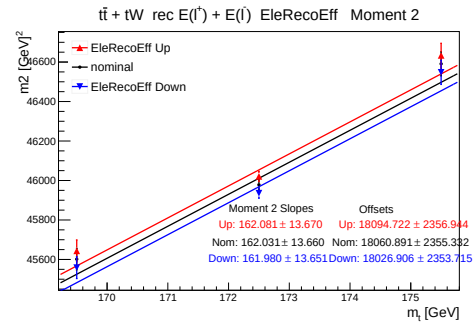
(c) $t\bar{t} + tW$ $p_T(l')$



(d) $t\bar{t} + tW$ $E(l')$

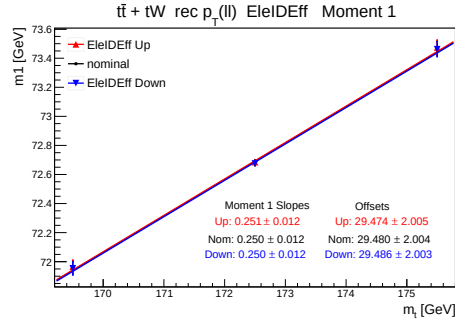


(e) $t\bar{t} + tW$ $p_T(l') + p_T(l)$

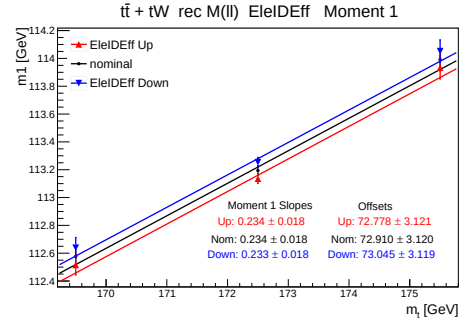


(f) $t\bar{t} + tW$ $E(l') + E(l)$

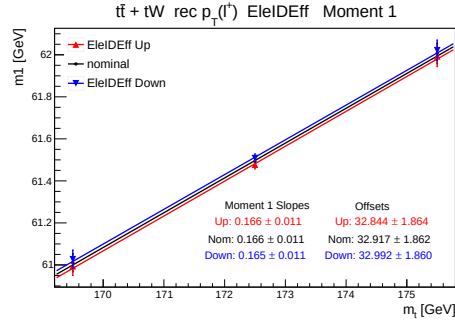
Figure C.26: Electron reconstruction efficiency systematic second moments.



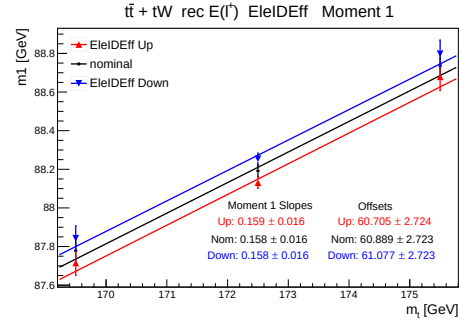
(a) $t\bar{t} + tW p_T(l\bar{l})$



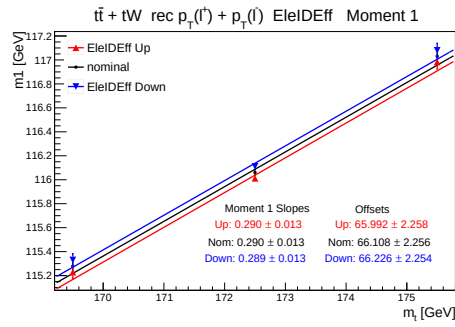
(b) $t\bar{t} + tW M(l\bar{l})$



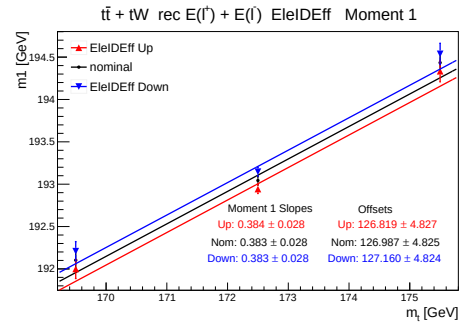
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

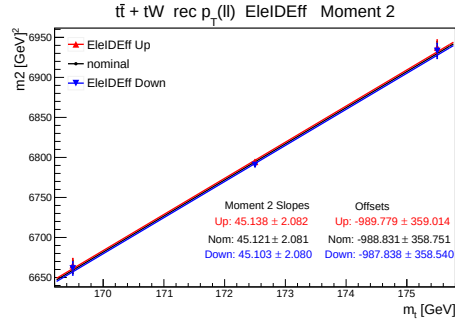


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

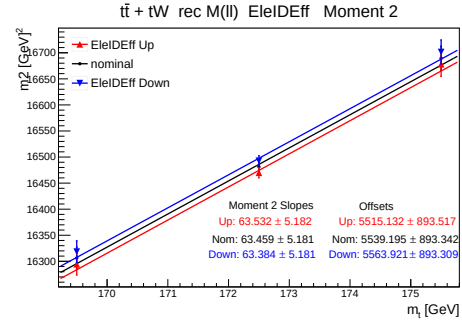


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

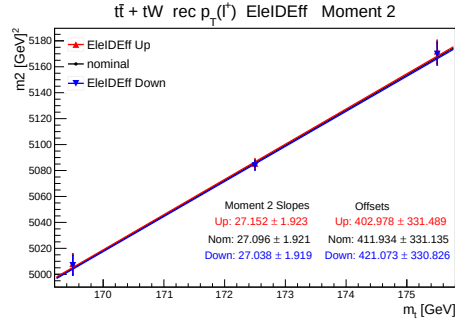
Figure C.27: Electron identification efficiency systematic first moments.



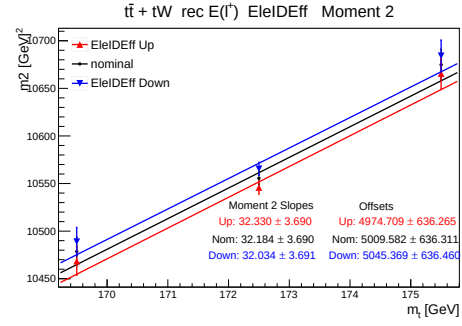
(a) $t\bar{t} + tW$ $p_T(l)$



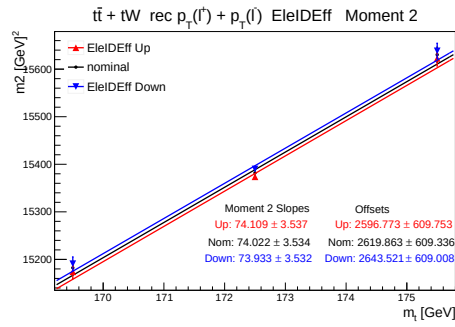
(b) $t\bar{t} + tW$ $M(l)$



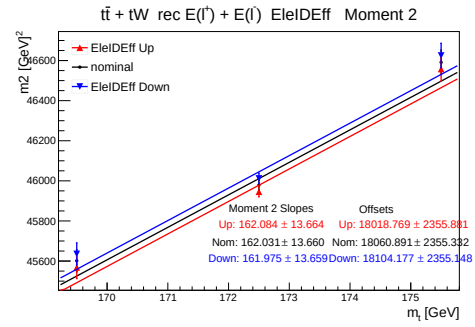
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

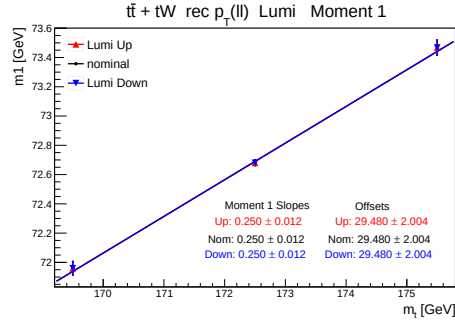


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

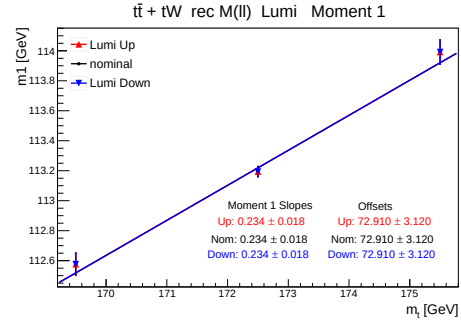


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

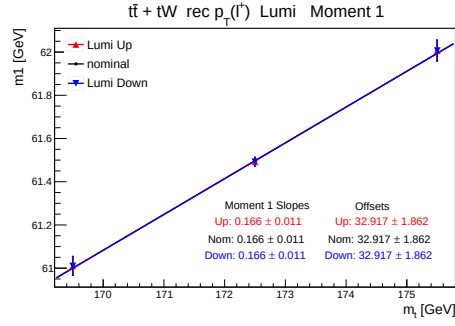
Figure C.28: Electron identification efficiency systematic second moments.



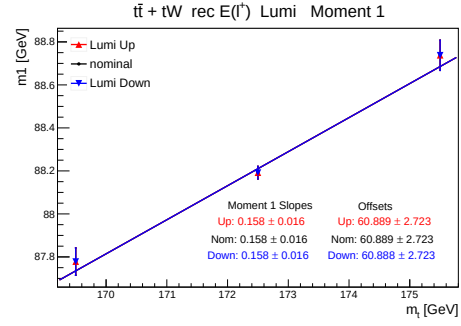
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



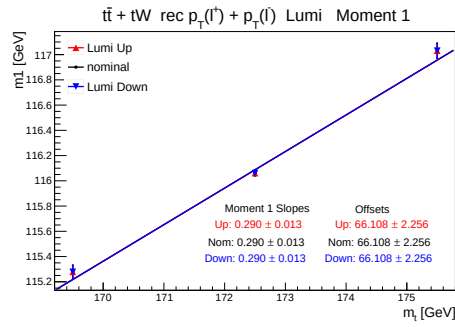
(b) $t\bar{t} + tW$ $M(l\bar{l})$



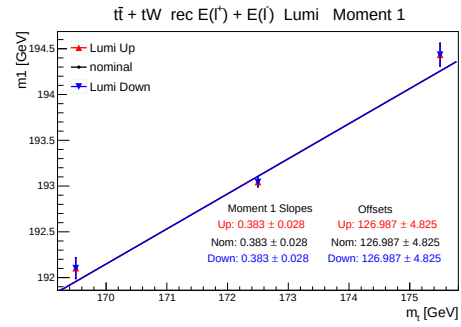
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

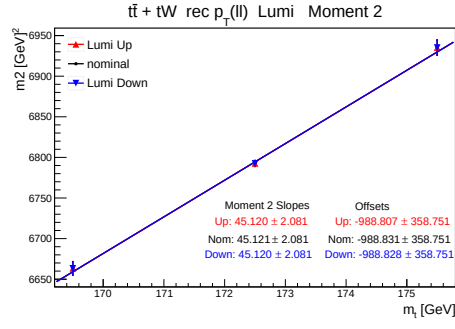


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

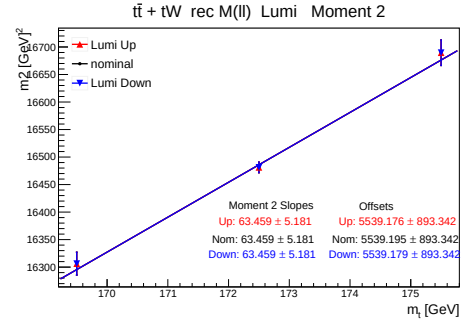


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

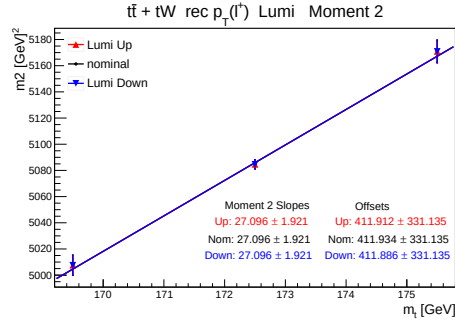
Figure C.29: Luminosity systematic first moments.



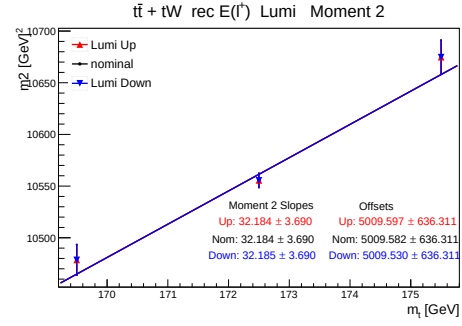
(a) $t\bar{t} + tW p_T(l)$



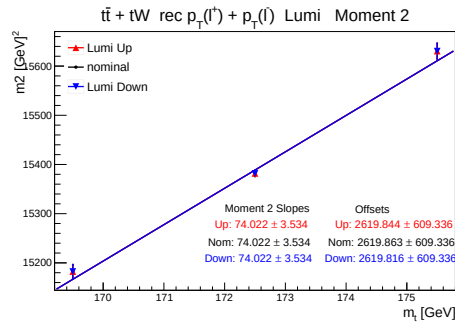
(b) $t\bar{t} + tW M(l)$



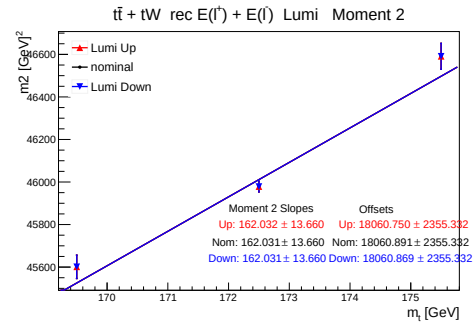
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

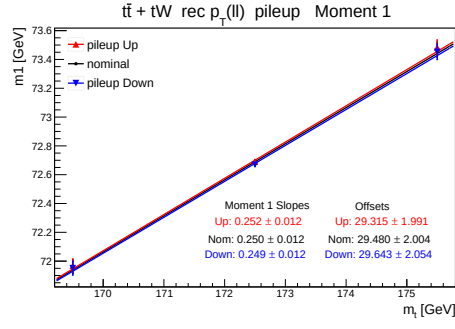


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

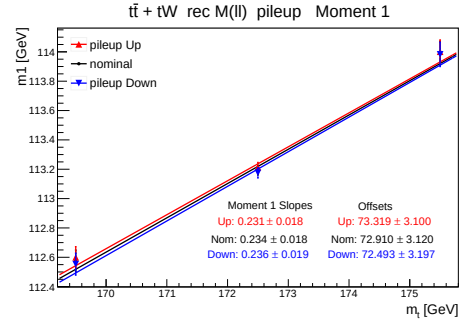


(f) $t\bar{t} + tW E(l') + E(l)$

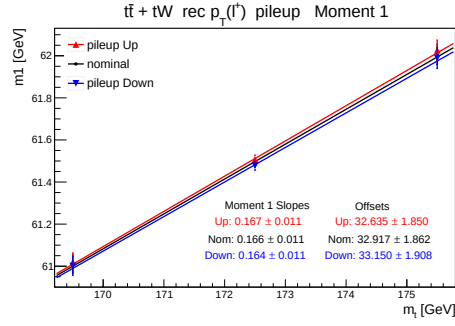
Figure C.30: Luminosity systematic second moments.



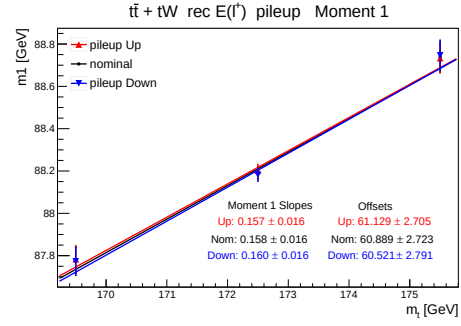
(a) $t\bar{t} + tW$ $p_T(l)$



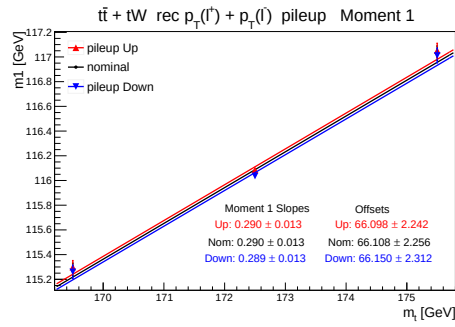
(b) $t\bar{t} + tW$ $M(l)$



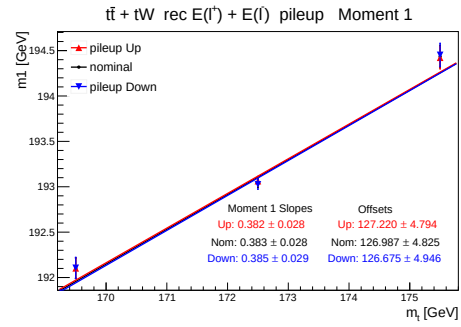
(c) $t\bar{t} + tW$ $p_T(l')$



(d) $t\bar{t} + tW$ $E(l')$

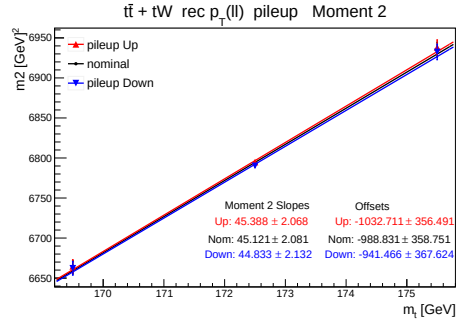


(e) $t\bar{t} + tW$ $p_T(l') + p_T(l)$

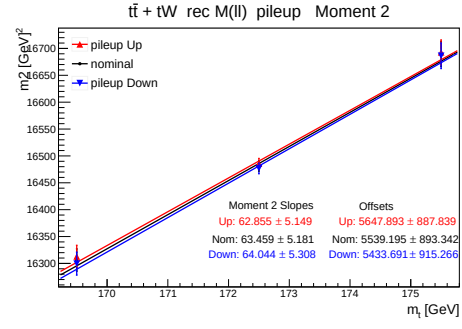


(f) $t\bar{t} + tW$ $E(l') + E(l)$

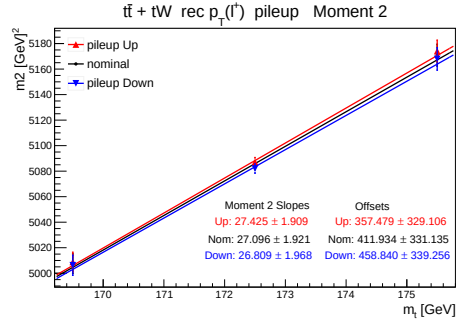
Figure C.31: Pileup systematic first moments.



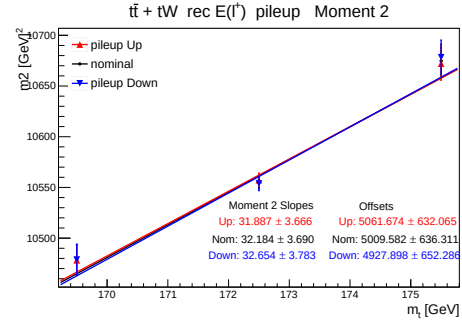
(a) $t\bar{t} + tW$ $p_T(l)$



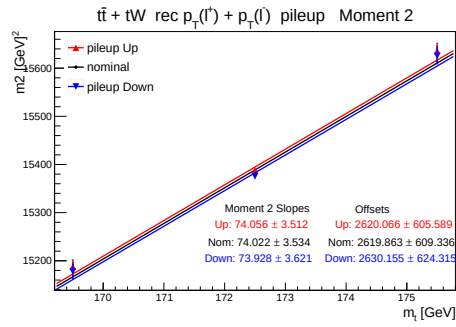
(b) $t\bar{t} + tW$ $M(l)$



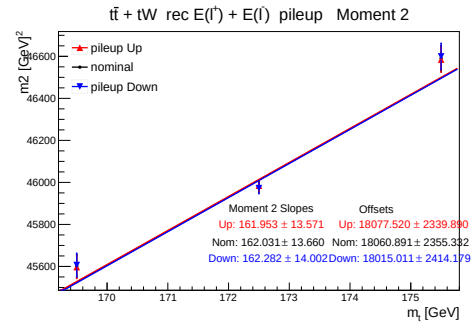
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$



(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$



(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

Figure C.32: Pileup systematic second moments.

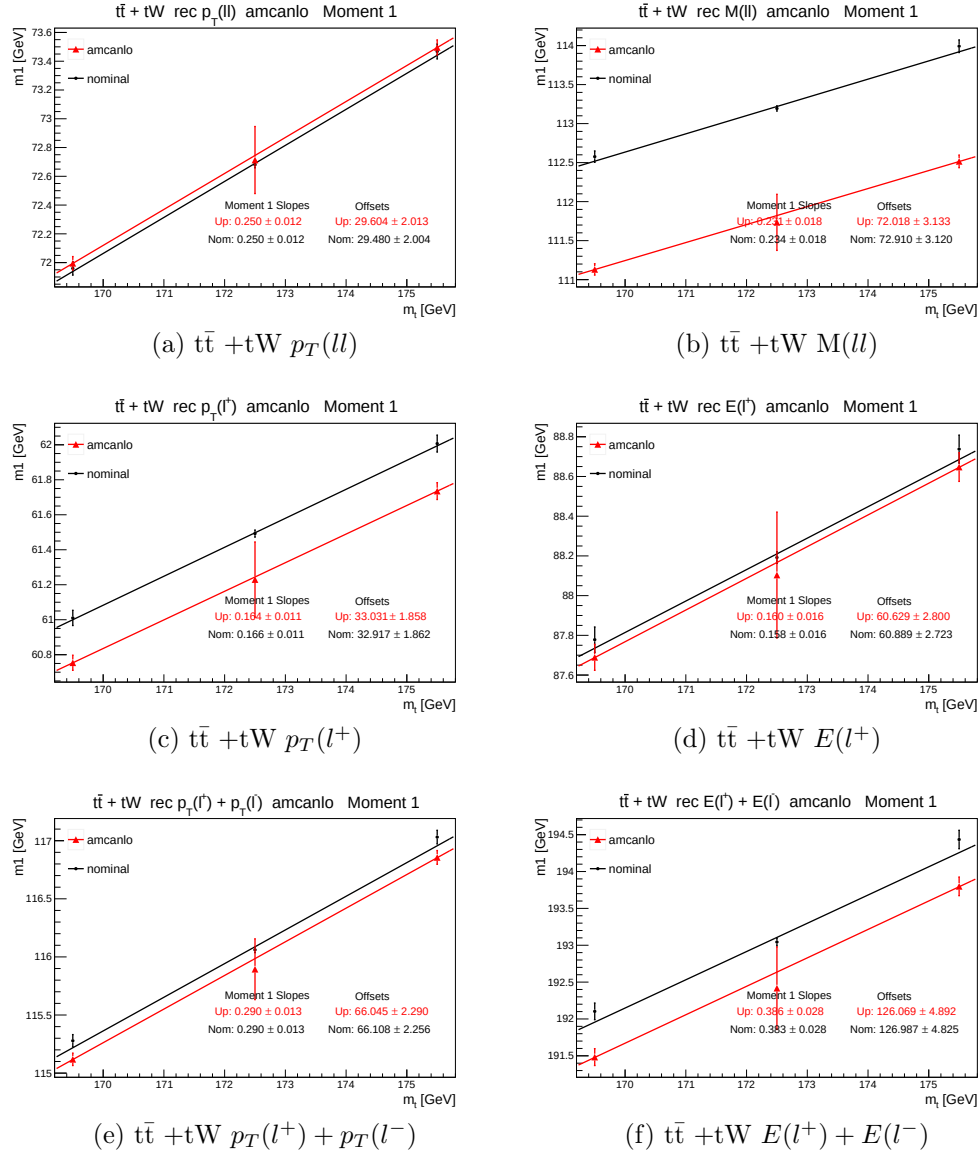
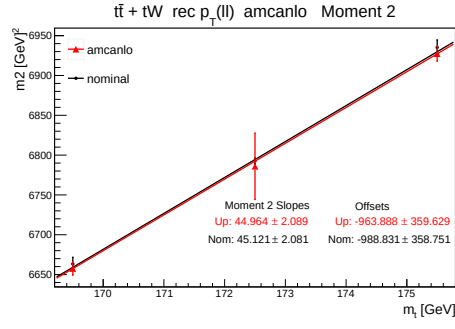
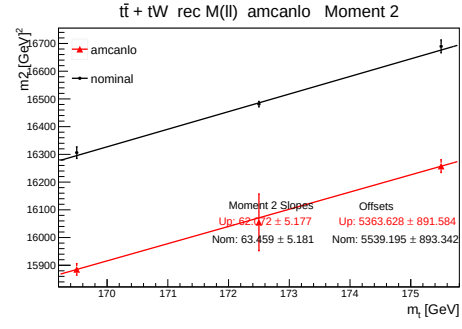


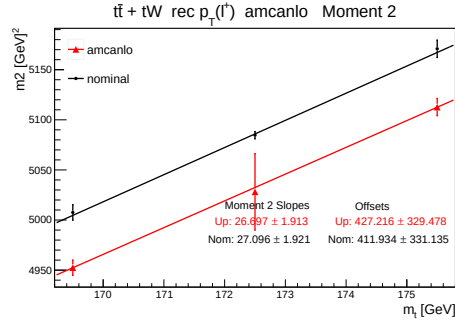
Figure C.33: aMC@NLO systematic first moments.



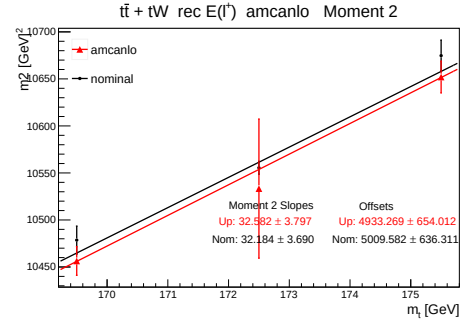
(a) $t\bar{t} + tW p_T(l)$



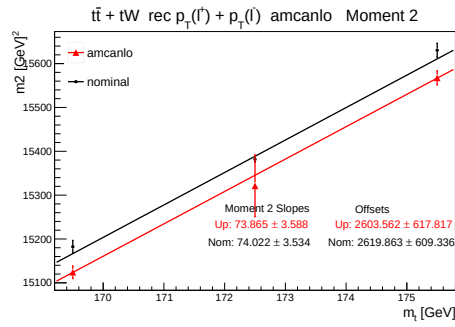
(b) $t\bar{t} + tW M(l)$



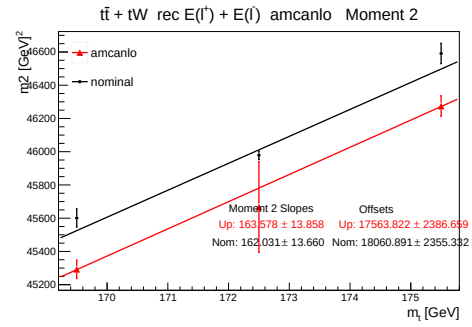
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

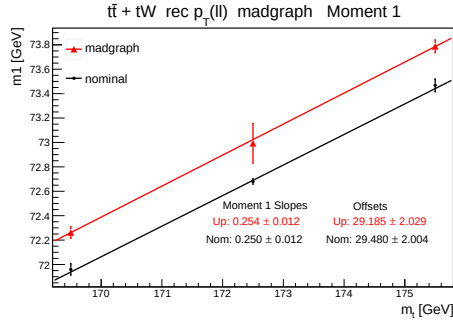


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

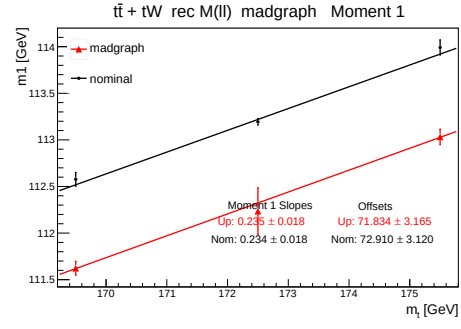


(f) $t\bar{t} + tW E(l') + E(l)$

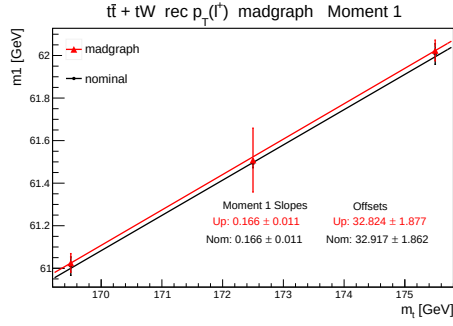
Figure C.34: aMC@NLO systematic second moments.



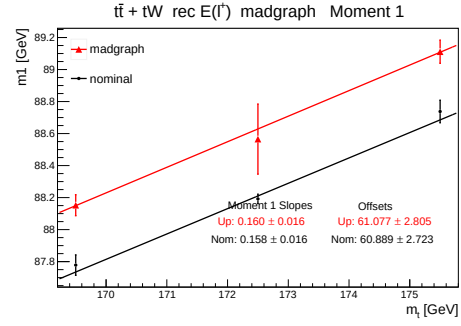
(a) $t\bar{t} + tW$ $p_T(l)$



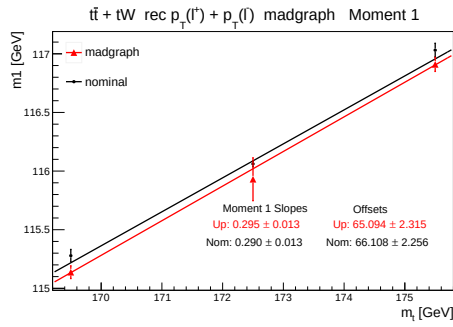
(b) $t\bar{t} + tW$ $M(l)$



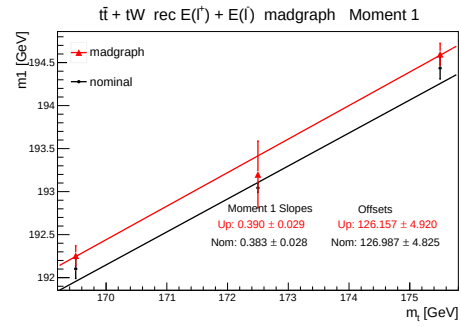
(c) $t\bar{t} + tW$ $p_T(l')$



(d) $t\bar{t} + tW$ $E(l')$

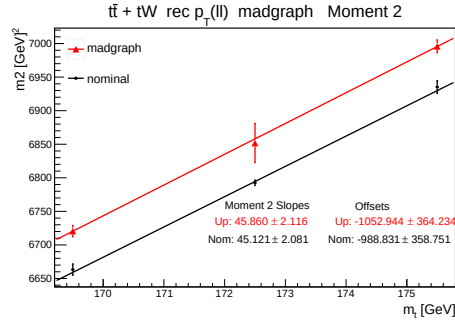


(e) $t\bar{t} + tW$ $p_T(l') + p_T(l)$

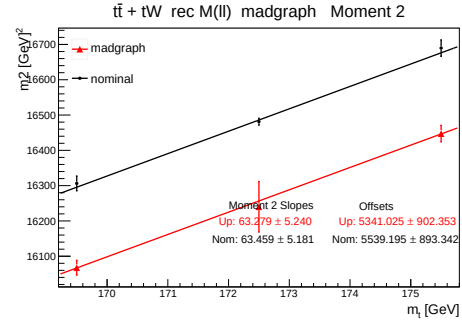


(f) $t\bar{t} + tW$ $E(l') + E(l)$

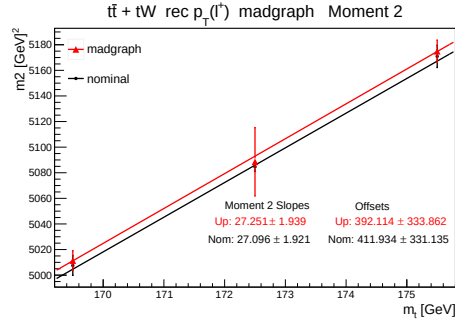
Figure C.35: MADGRAPH systematic first moments.



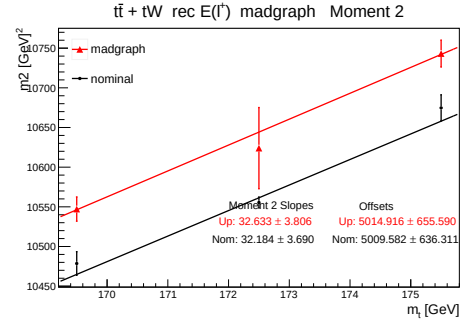
(a) $t\bar{t} + tW p_T(l)$



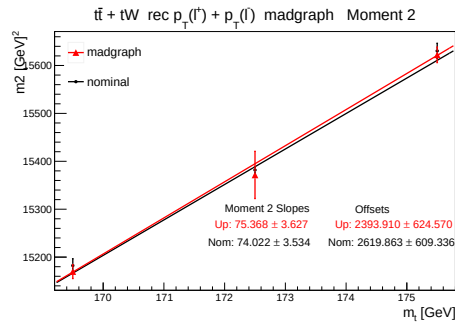
(b) $t\bar{t} + tW M(l)$



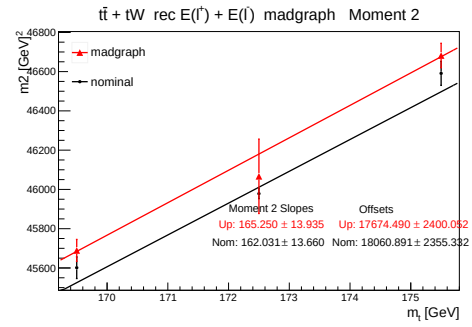
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

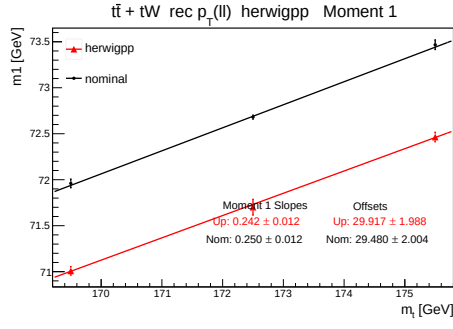


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

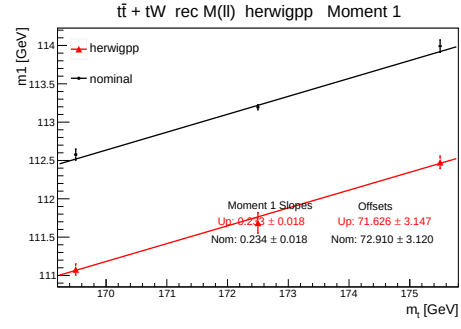


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

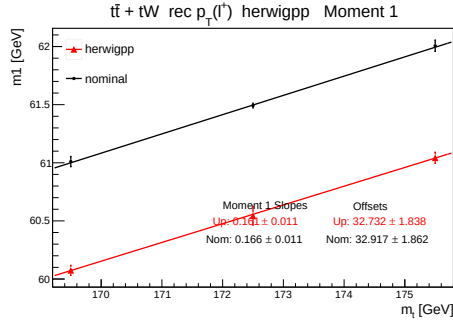
Figure C.36: MADGRAPH systematic second moments.



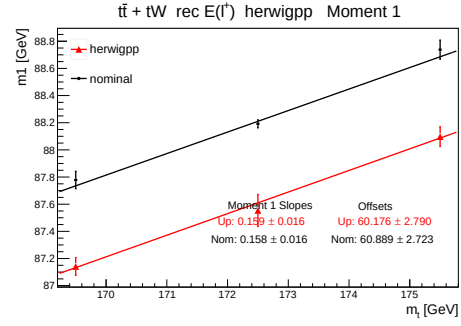
(a) $t\bar{t} + tW p_T(l\bar{l})$



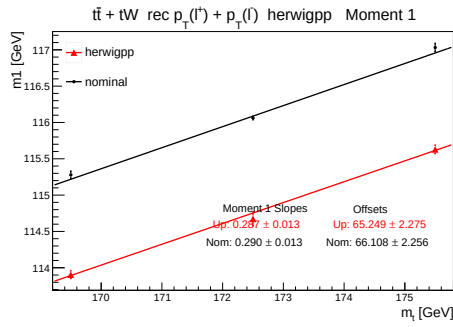
(b) $t\bar{t} + tW M(l\bar{l})$



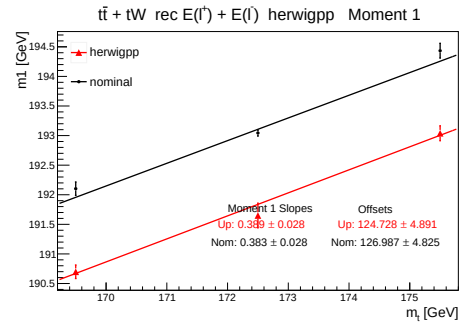
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

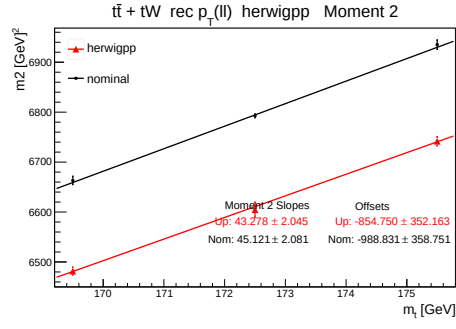


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

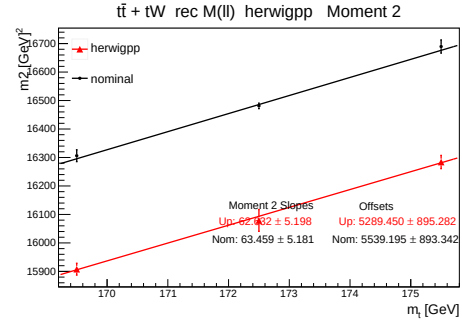


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

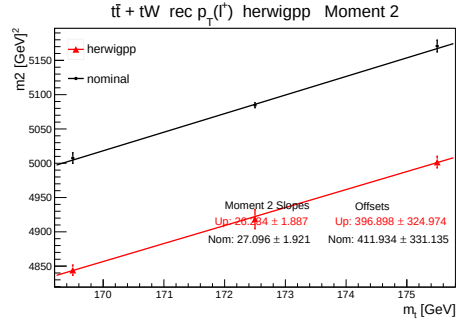
Figure C.37: HERWIG++ systematic first moments.



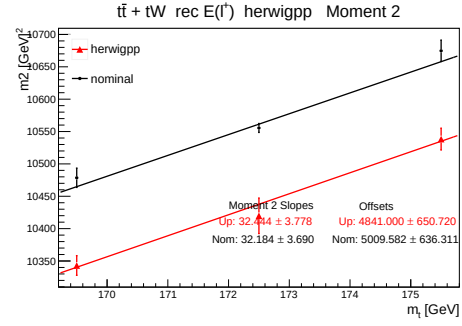
(a) $t\bar{t} + tW p_T(l)$



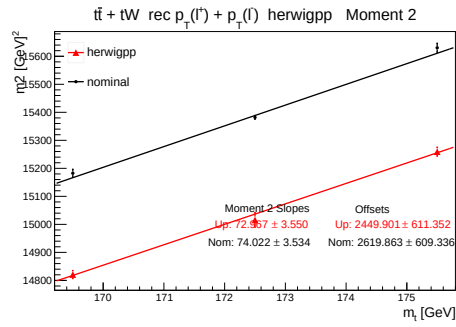
(b) $t\bar{t} + tW M(l)$



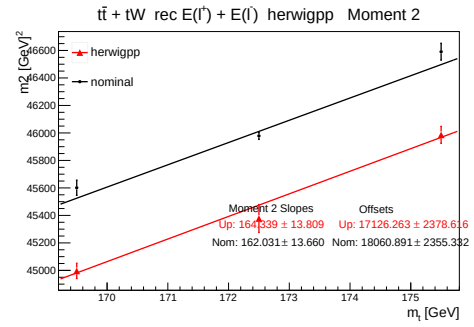
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

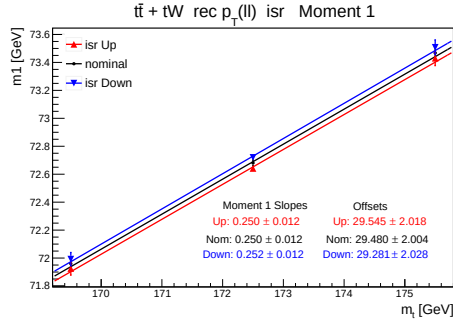


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

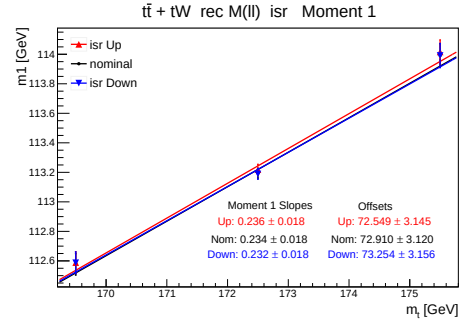


(f) $t\bar{t} + tW E(l') + E(l)$

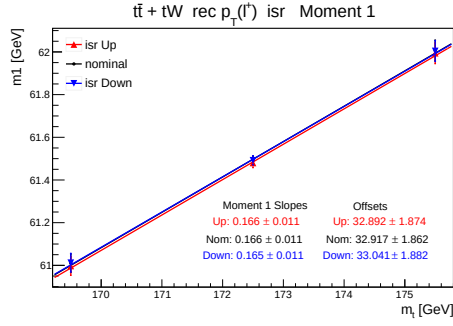
Figure C.38: HERWIG++ systematic second moments.



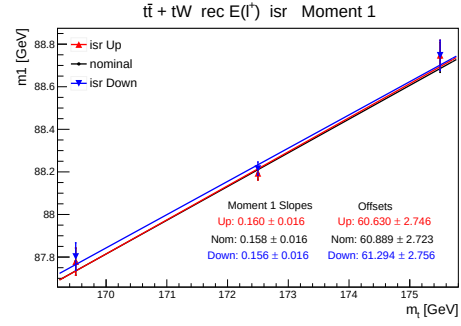
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



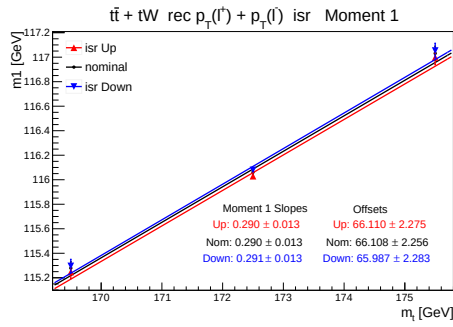
(b) $t\bar{t} + tW$ $M(l\bar{l})$



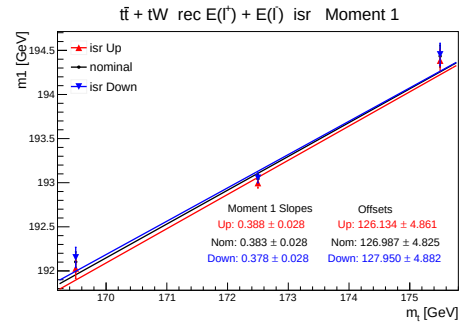
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

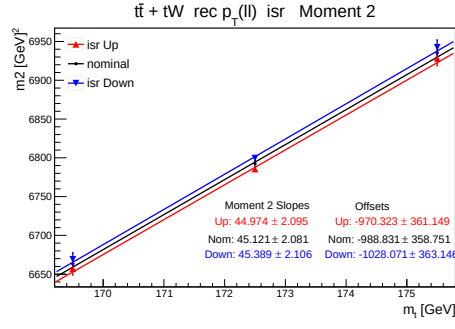


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

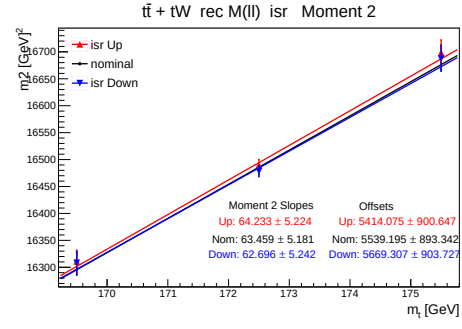


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

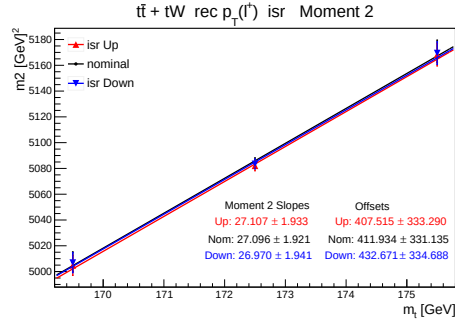
Figure C.39: Initial-state radiation systematic first moments.



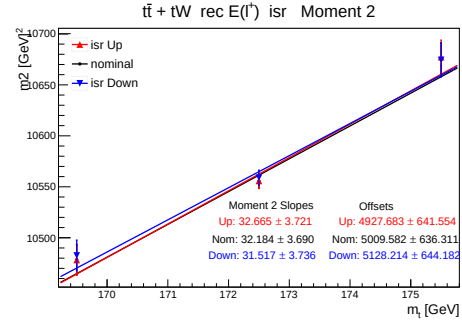
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



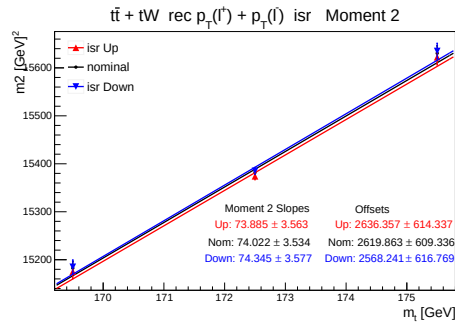
(b) $t\bar{t} + tW$ $M(l\bar{l})$



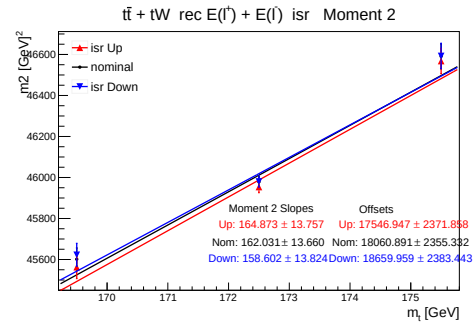
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

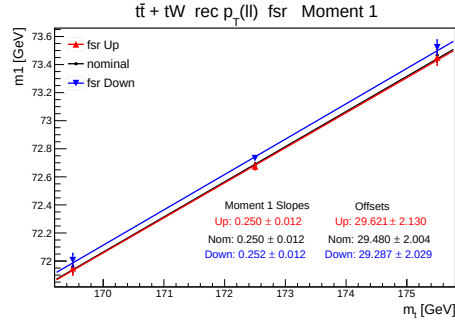


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

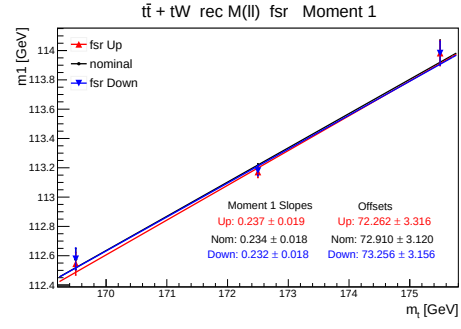


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

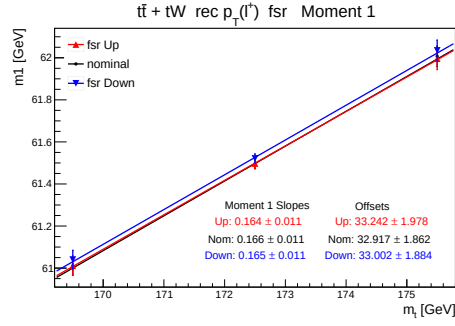
Figure C.40: Initial-state radiation systematic second moments.



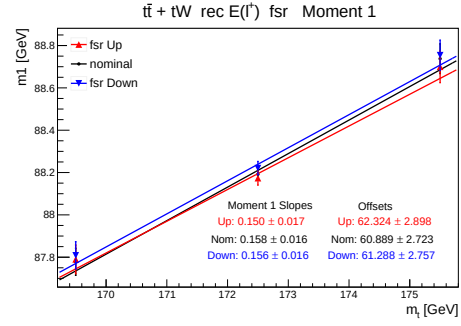
(a) $t\bar{t} + tW$ $p_T(l)$



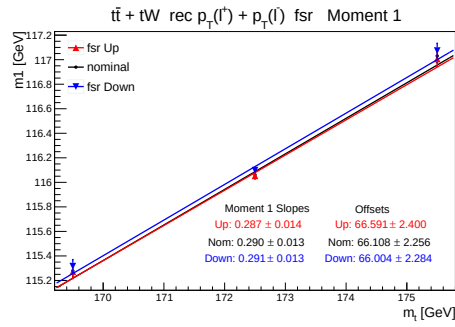
(b) $t\bar{t} + tW$ $M(l)$



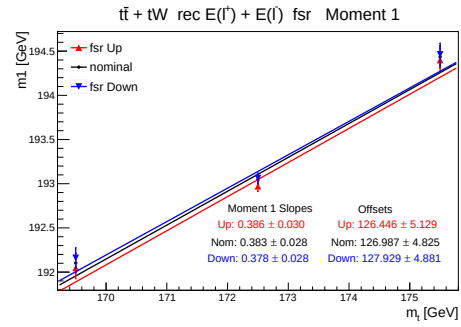
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

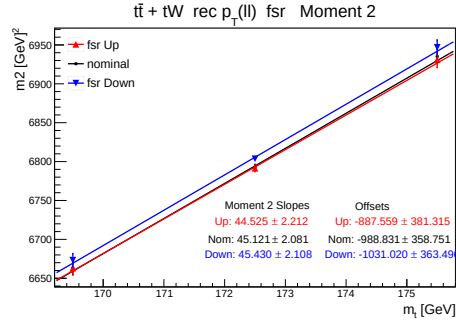


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

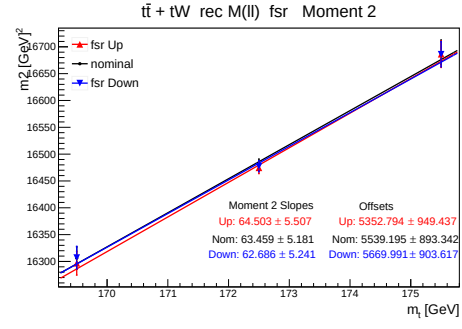


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

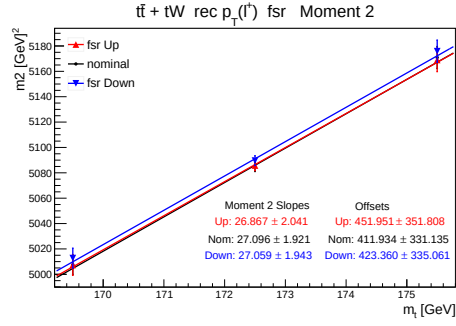
Figure C.41: Final-state radiation systematic first moments.



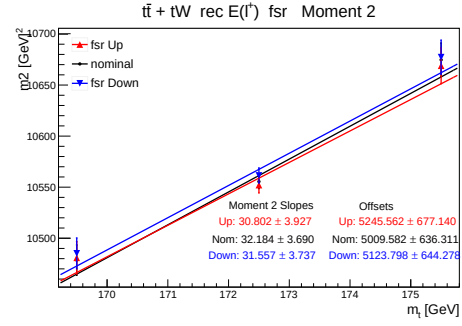
(a) $t\bar{t} + tW p_T(l\bar{l})$



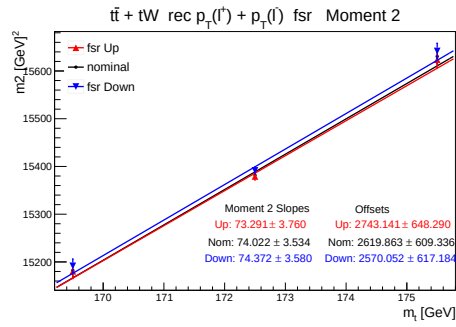
(b) $t\bar{t} + tW M(l\bar{l})$



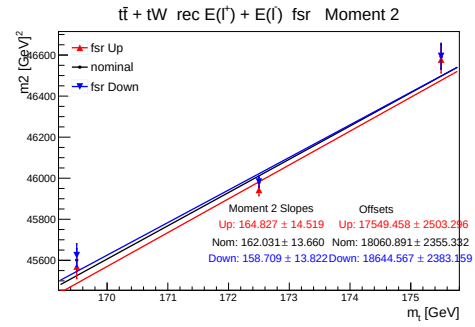
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

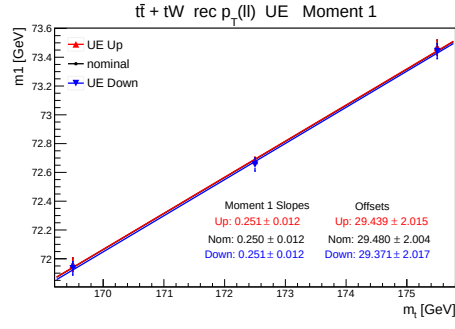


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

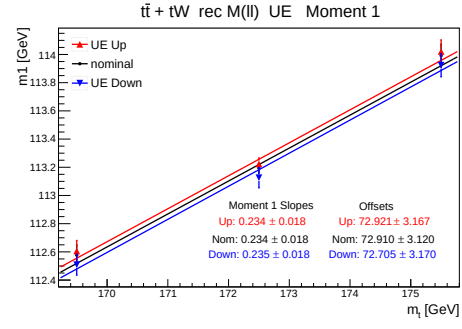


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

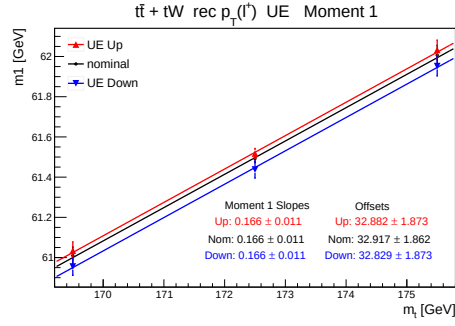
Figure C.42: Final-state radiation systematic second moments.



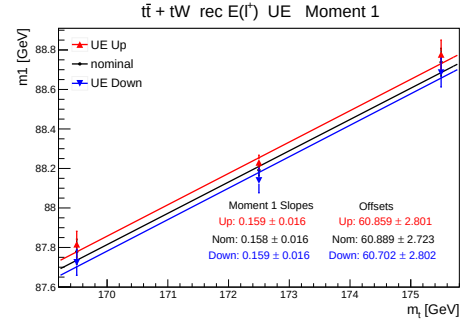
(a) $t\bar{t} + tW$ $p_T(ll)$



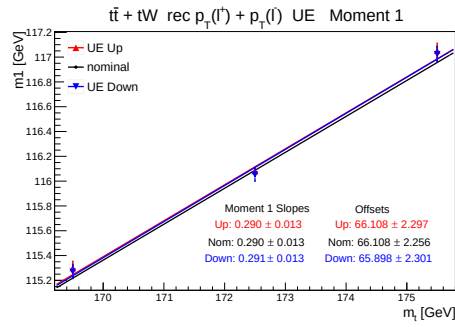
(b) $t\bar{t} + tW$ $M(ll)$



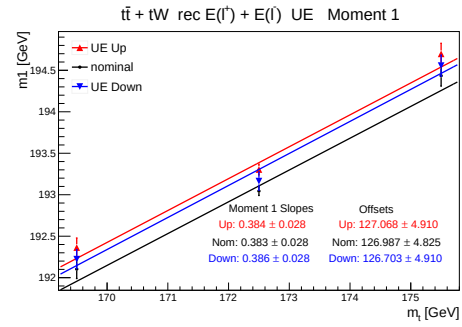
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

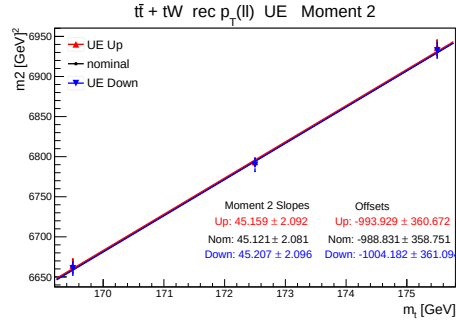


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

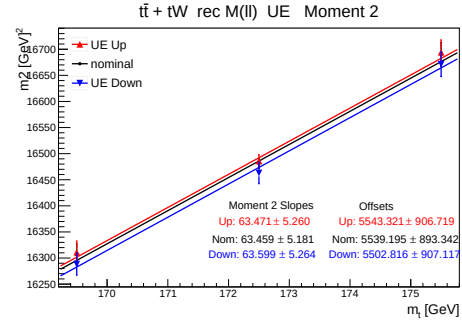


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

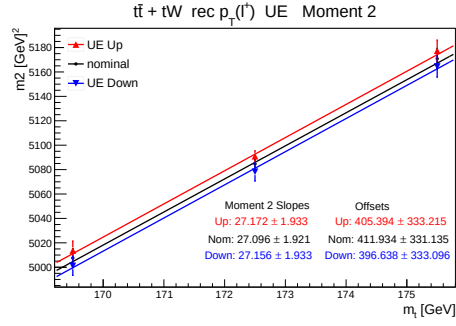
Figure C.43: Underlying event tune systematic first moments.



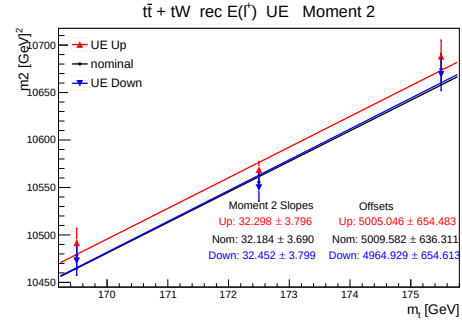
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



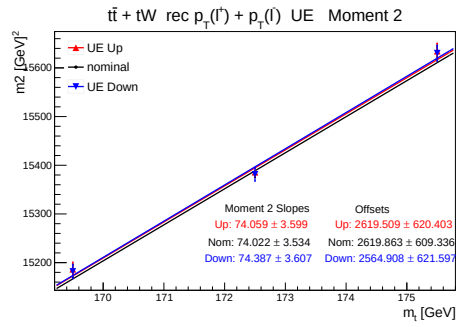
(b) $t\bar{t} + tW$ $M(l\bar{l})$



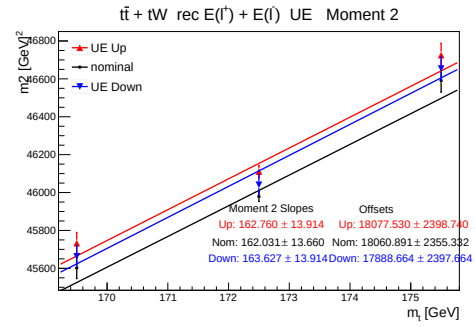
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

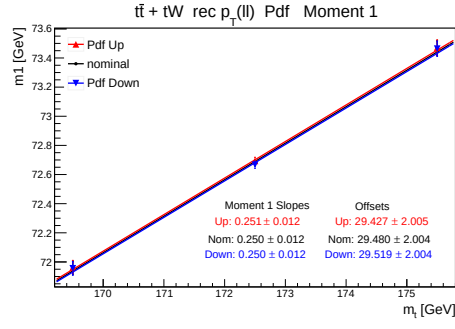


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

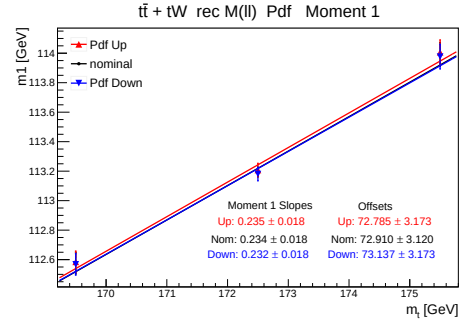


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

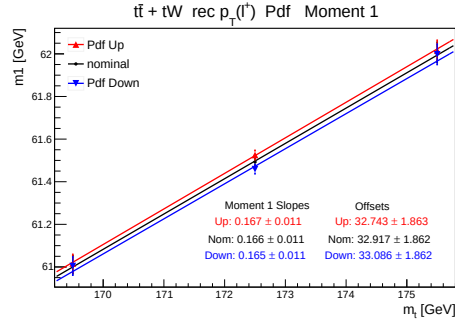
Figure C.44: Underlying event tune systematic second moments.



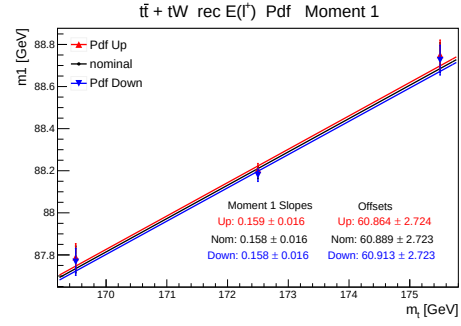
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



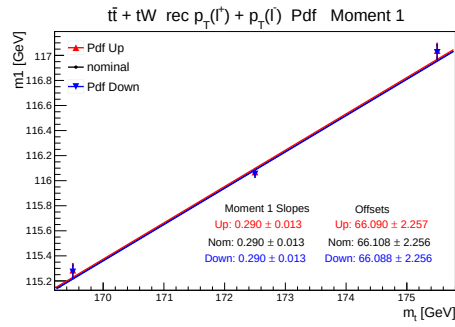
(b) $t\bar{t} + tW$ $M(l\bar{l})$



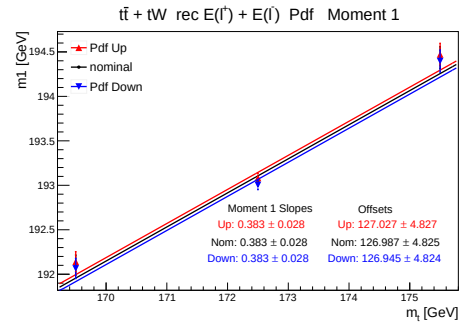
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

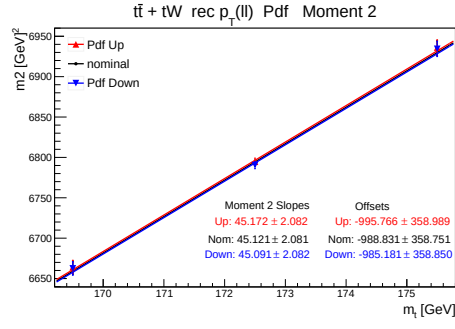


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

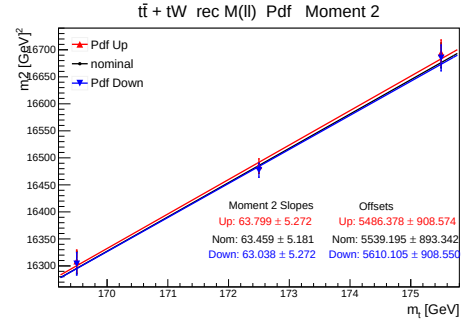


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

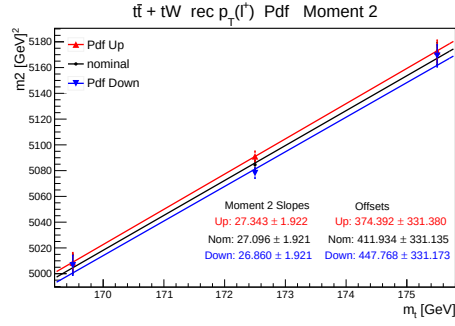
Figure C.45: Parton distribution function systematic first moments.



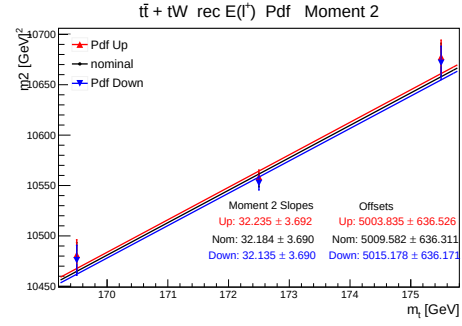
(a) $t\bar{t} + tW$ $p_T(l)$



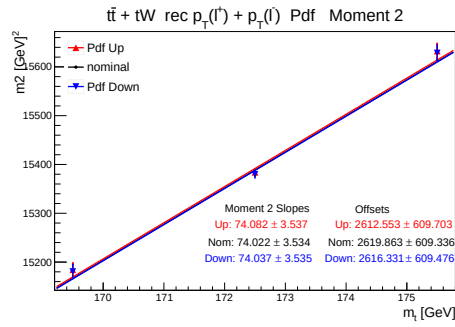
(b) $t\bar{t} + tW$ $M(l)$



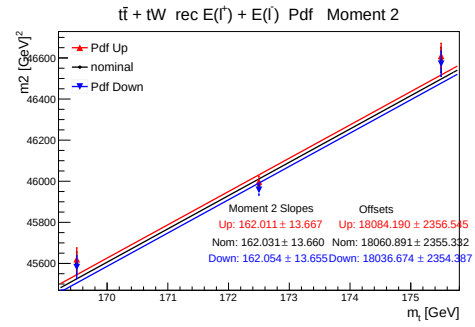
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

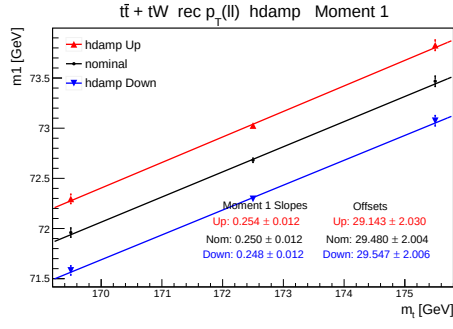


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

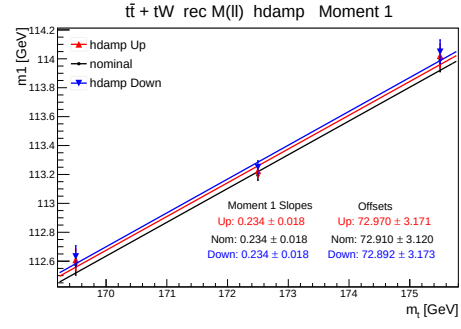


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

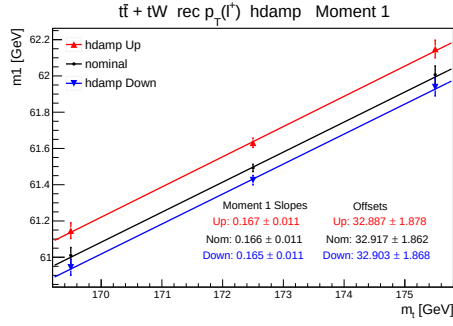
Figure C.46: Parton distribution function systematic second moments.



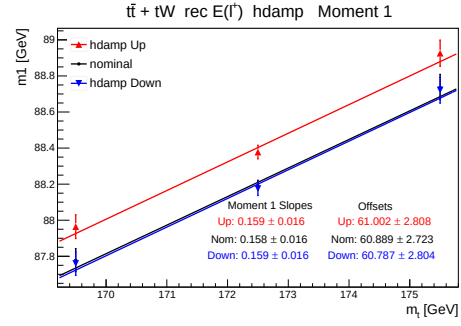
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



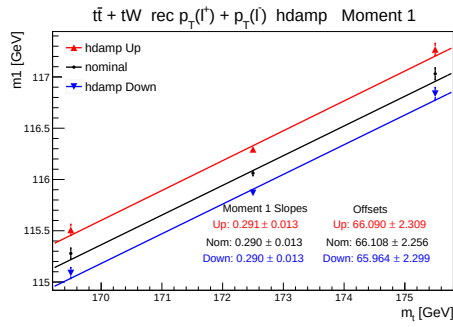
(b) $t\bar{t} + tW$ $M(l\bar{l})$



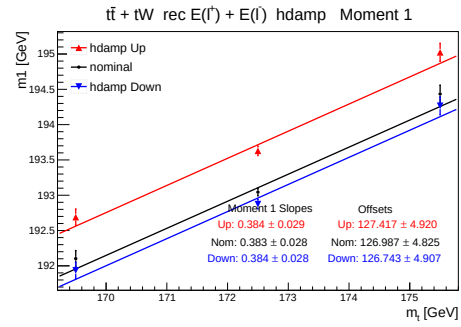
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

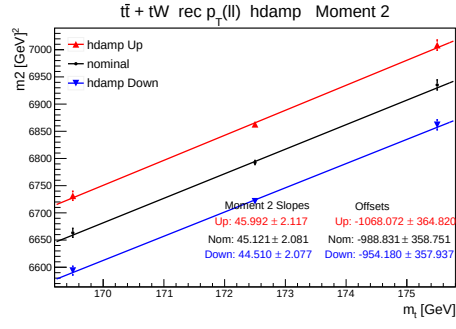


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

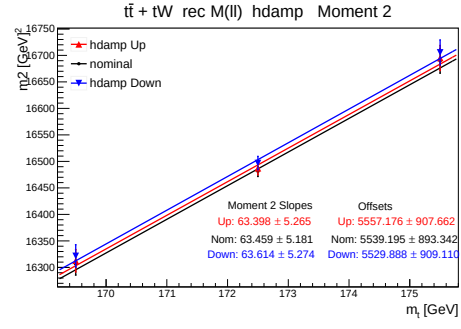


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

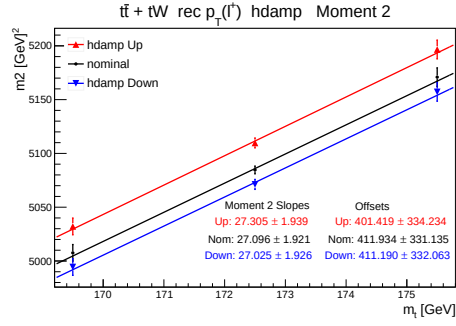
Figure C.47: ME/PS matching scale systematic first moments.



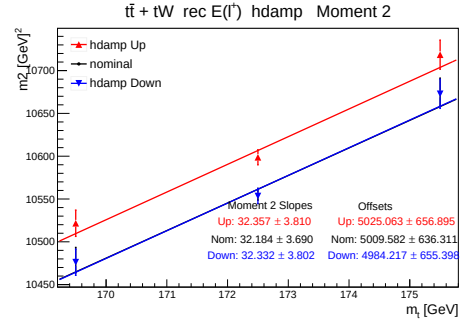
(a) $t\bar{t} + tW$ $p_T(l\bar{l})$



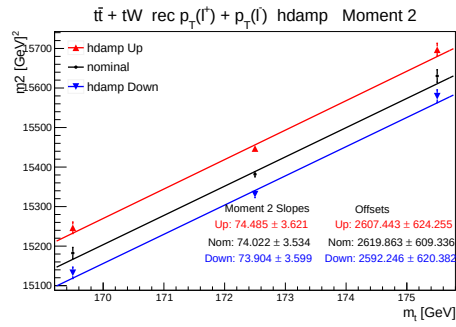
(b) $t\bar{t} + tW$ $M(l\bar{l})$



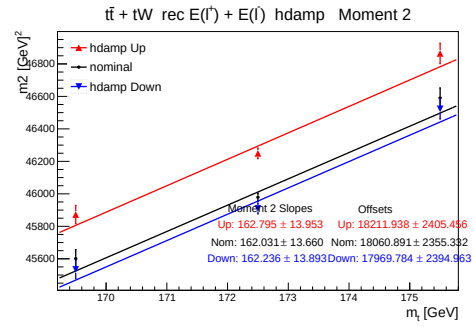
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

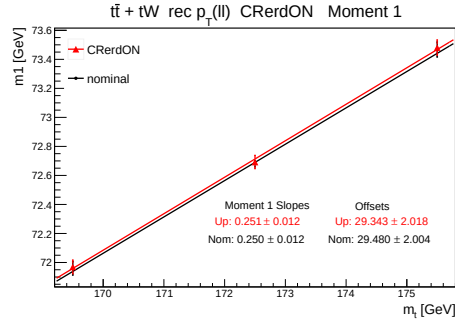


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

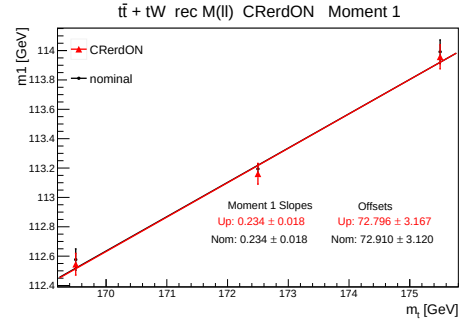


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

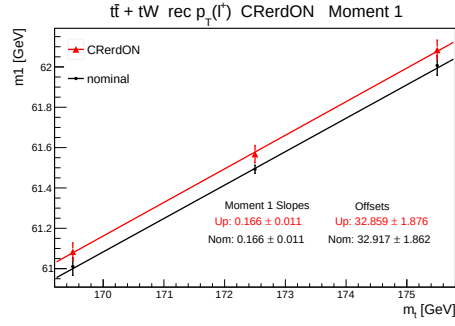
Figure C.48: ME/PS matching scale systematic second moments.



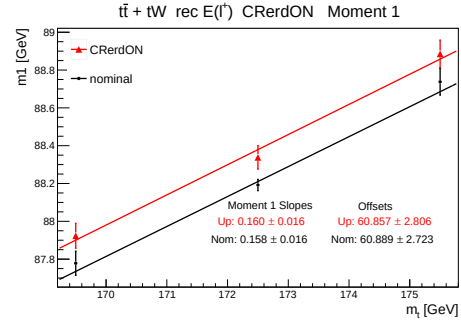
(a) $t\bar{t} + tW p_T(l\bar{l})$



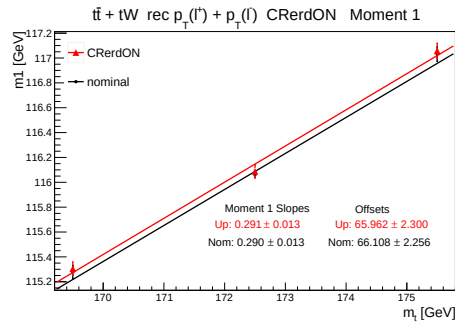
(b) $t\bar{t} + tW M(l\bar{l})$



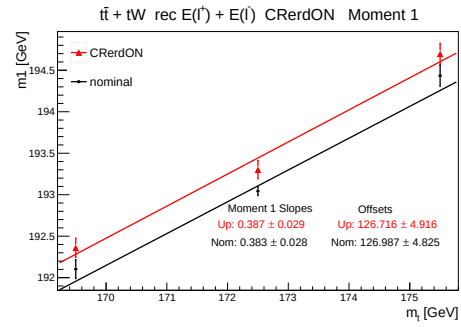
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

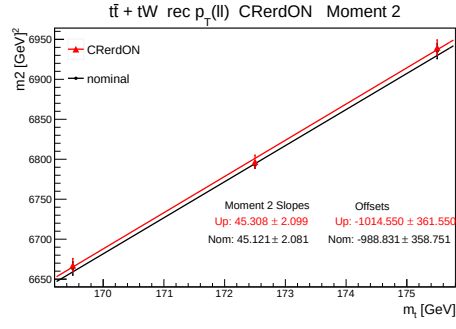


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

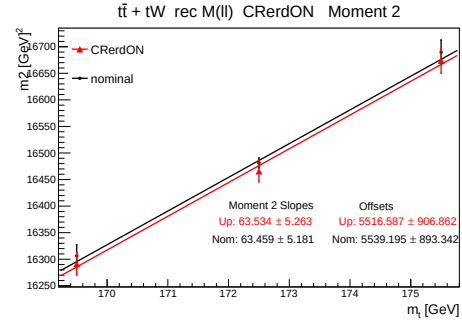


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

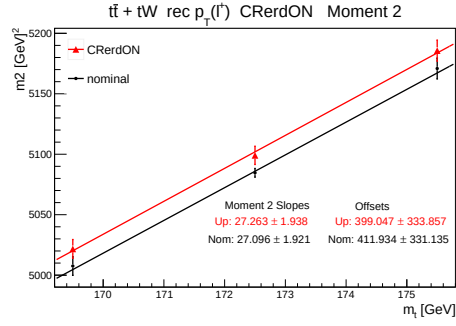
Figure C.49: Color reconnection model with early resonance decays systematic first moments.



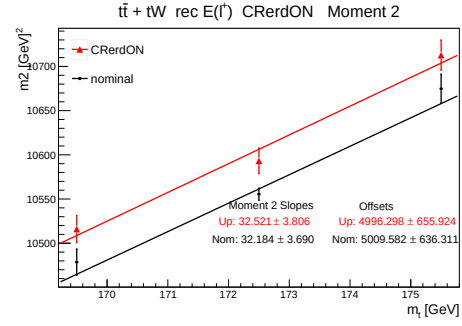
(a) $t\bar{t} + tW p_T(l)$



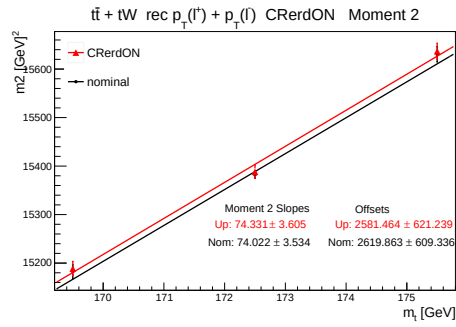
(b) $t\bar{t} + tW M(l)$



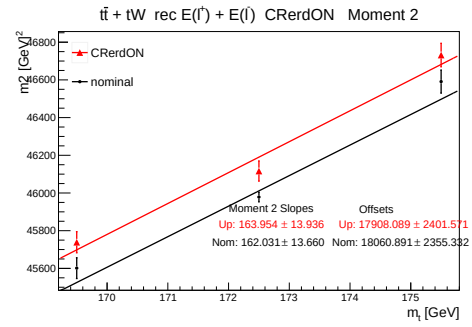
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

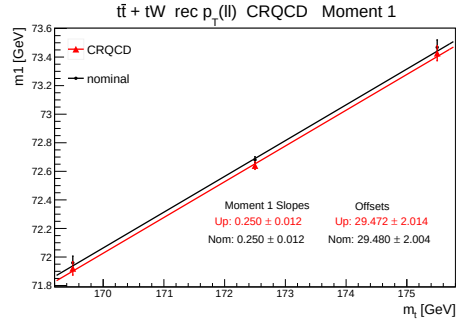


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

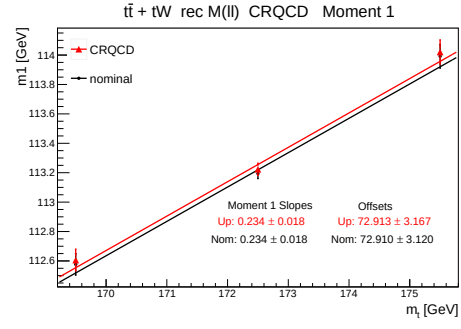


(f) $t\bar{t} + tW E(l') + E(l)$

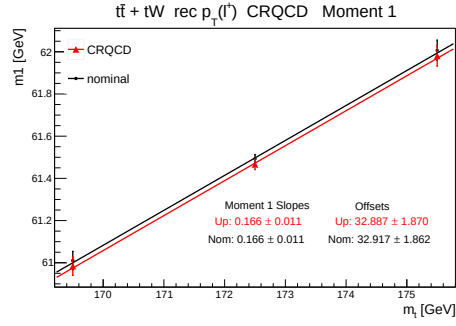
Figure C.50: Color reconnection model with early resonance decays systematic second moments.



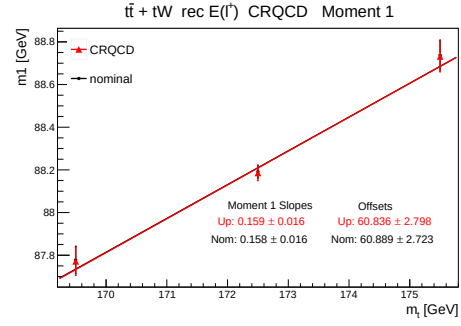
(a) $t\bar{t} + tW p_T(l\bar{l})$



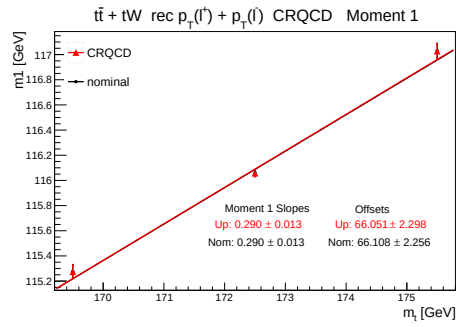
(b) $t\bar{t} + tW M(l\bar{l})$



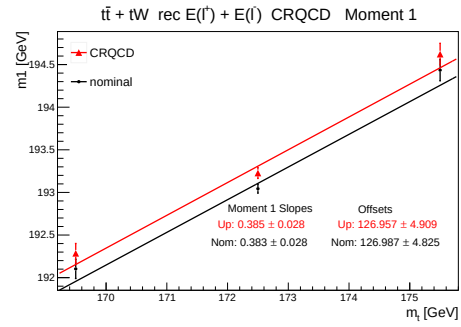
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

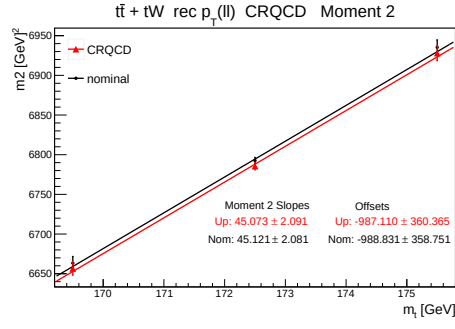


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

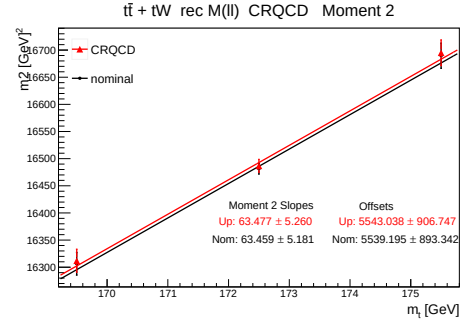


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

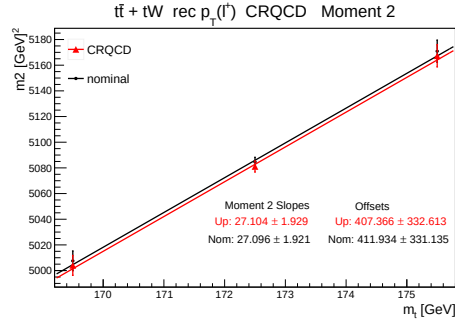
Figure C.51: Color reconnection QCD-inspired model systematic first moments.



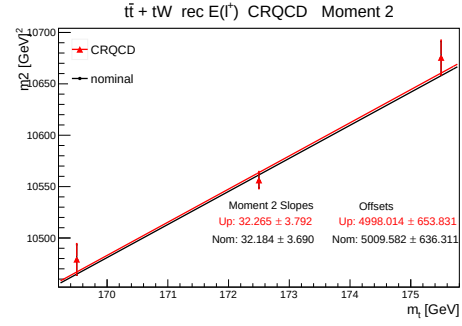
(a) $t\bar{t} + tW$ $p_T(l)$



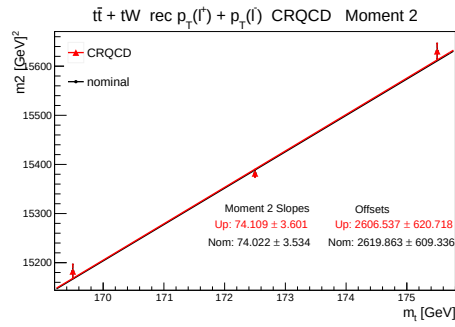
(b) $t\bar{t} + tW$ $M(l)$



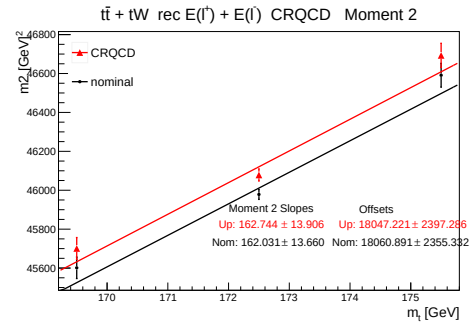
(c) $t\bar{t} + tW$ $p_T(l^+)$



(d) $t\bar{t} + tW$ $E(l^+)$

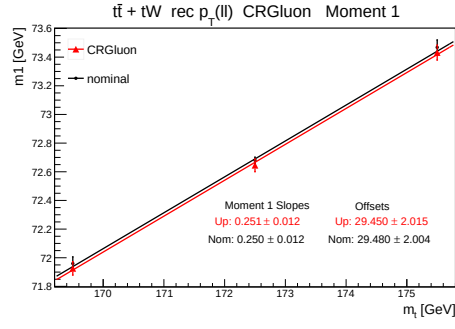


(e) $t\bar{t} + tW$ $p_T(l^+) + p_T(l^-)$

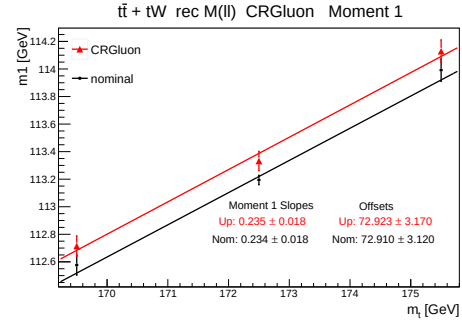


(f) $t\bar{t} + tW$ $E(l^+) + E(l^-)$

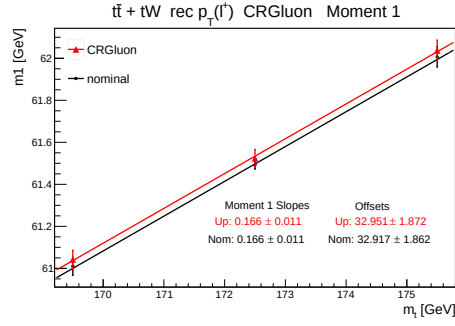
Figure C.52: Color reconnection QCD-inspired model systematic second moments.



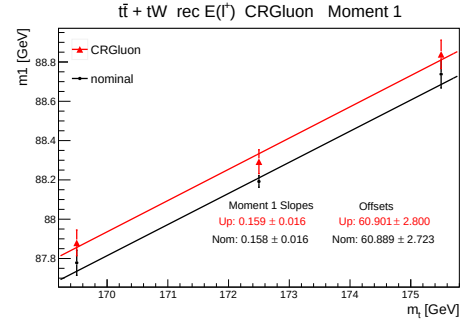
(a) $t\bar{t} + tW p_T(l)$



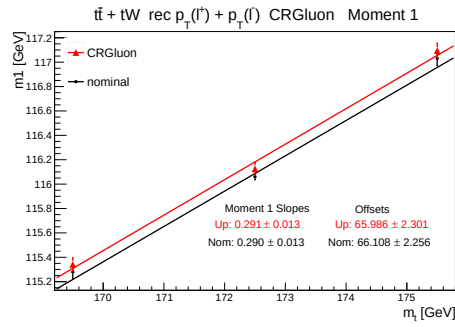
(b) $t\bar{t} + tW M(l)$



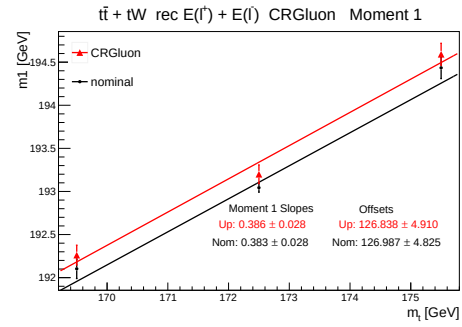
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

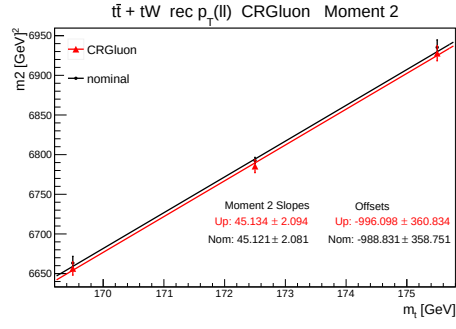


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

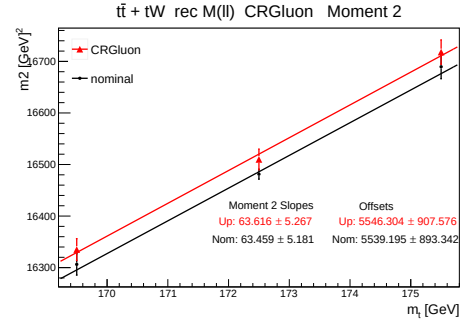


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

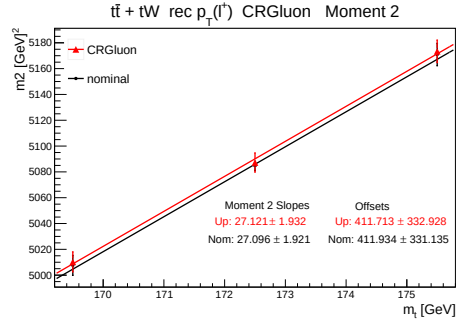
Figure C.53: Color reconnection gluon move model systematic first moments.



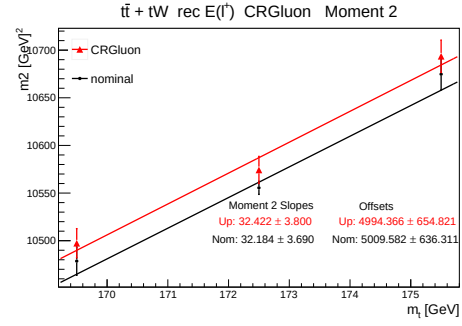
(a) $t\bar{t} + tW p_T(l)$



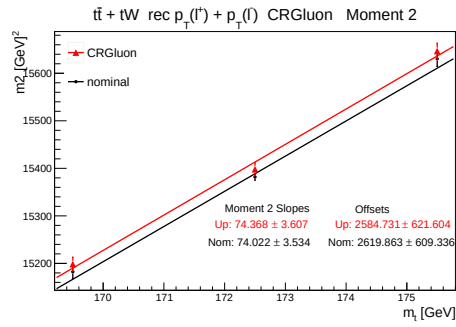
(b) $t\bar{t} + tW M(l)$



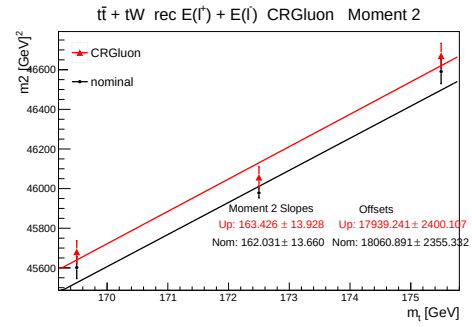
(c) $t\bar{t} + tW p_T(l')$



(d) $t\bar{t} + tW E(l')$

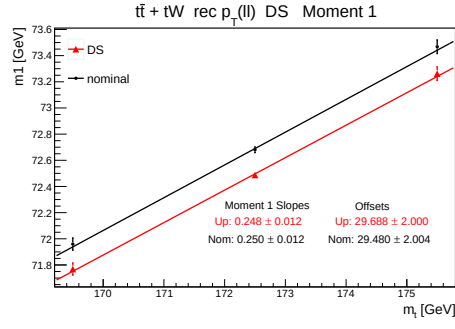


(e) $t\bar{t} + tW p_T(l') + p_T(l)$

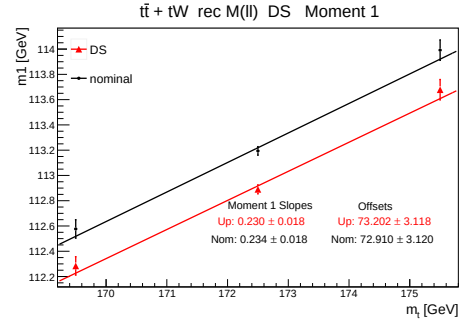


(f) $t\bar{t} + tW E(l') + E(l)$

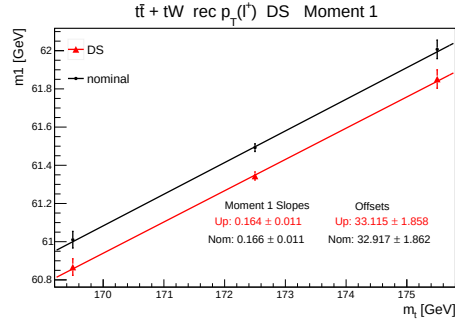
Figure C.54: Color reconnection gluon move model systematic second moments.



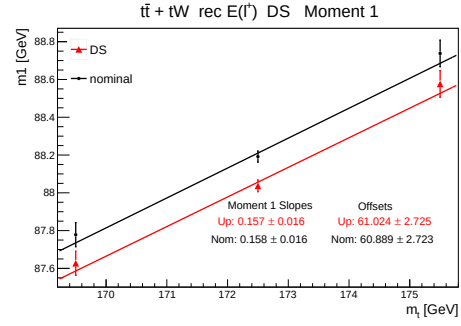
(a) $t\bar{t} + tW p_T(ll)$



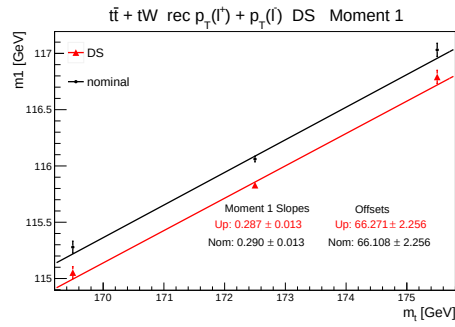
(b) $t\bar{t} + tW M(ll)$



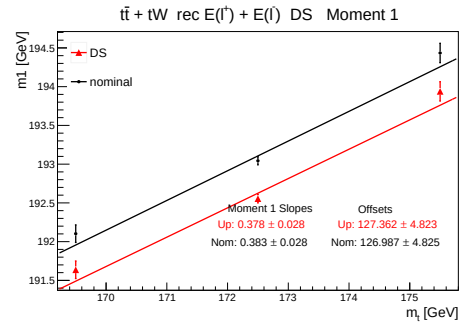
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$

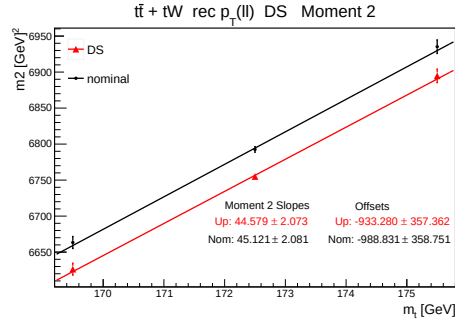


(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$

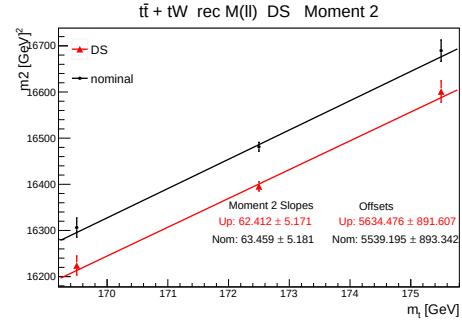


(f) $t\bar{t} + tW E(l^+) + E(l^-)$

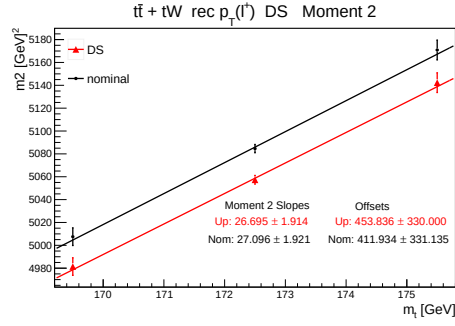
Figure C.55: $t\bar{t}/tW$ interference systematic first moments.



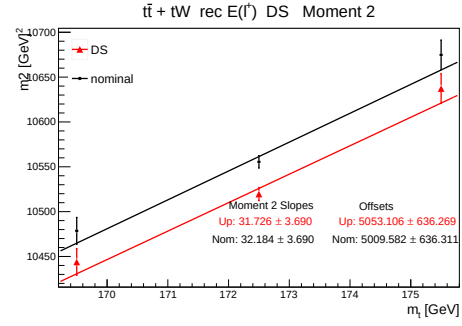
(a) $t\bar{t} + tW p_T(ll)$



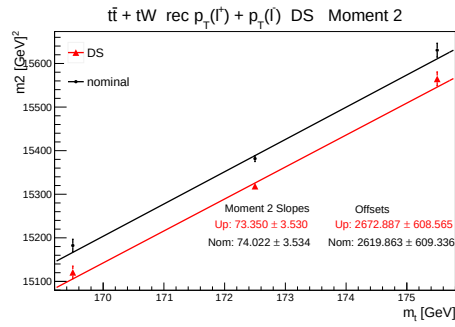
(b) $t\bar{t} + tW M(ll)$



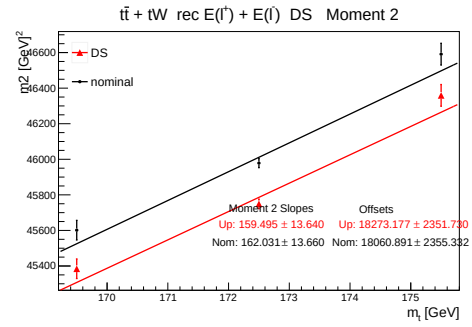
(c) $t\bar{t} + tW p_T(l^+)$



(d) $t\bar{t} + tW E(l^+)$



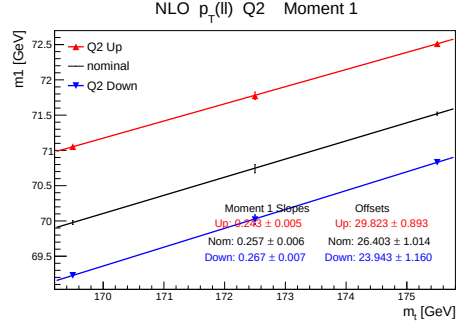
(e) $t\bar{t} + tW p_T(l^+) + p_T(l^-)$



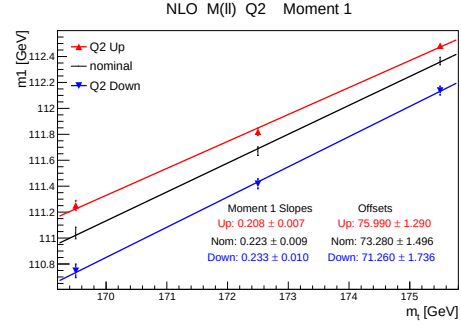
(f) $t\bar{t} + tW E(l^+) + E(l^-)$

Figure C.56: $t\bar{t}/tW$ interference systematic second moments.

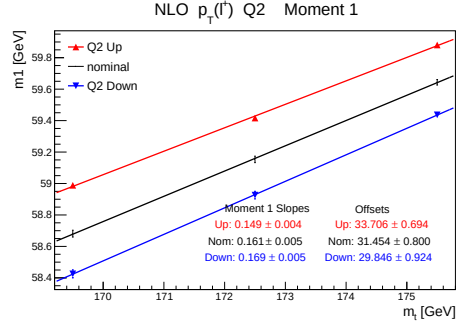
C.2 Fixed Order QCD Observables



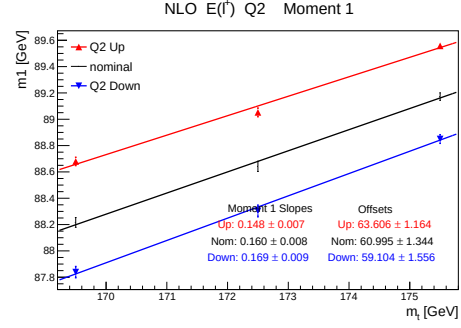
(a) $t\bar{t} p_T(l)$



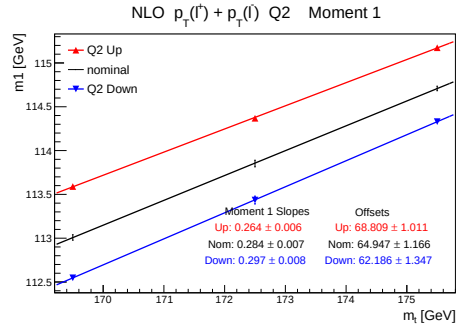
(b) $t\bar{t} M(l)$



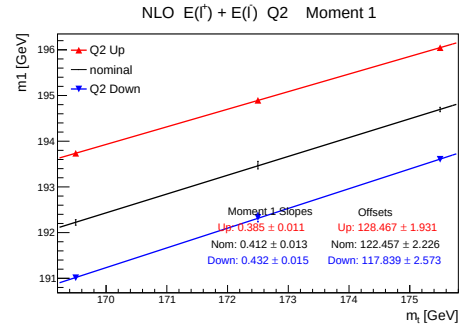
(c) $t\bar{t} p_T(l')$



(d) $t\bar{t} E(l')$

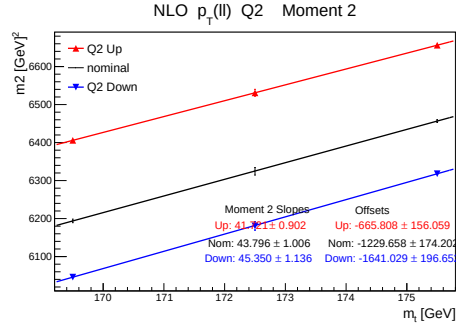


(e) $t\bar{t} p_T(l') + p_T(l)$

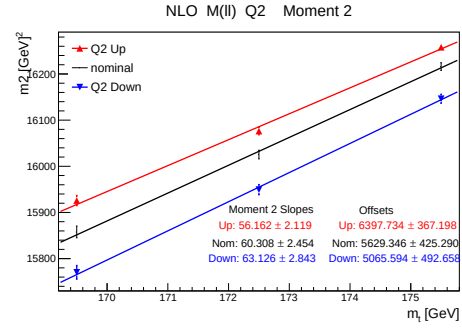


(f) $t\bar{t} E(l') + E(l)$

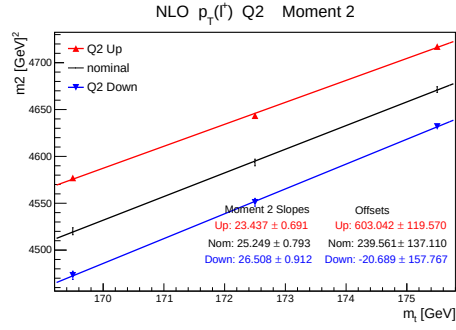
Figure C.57: NLO QCD scale systematic first moments.



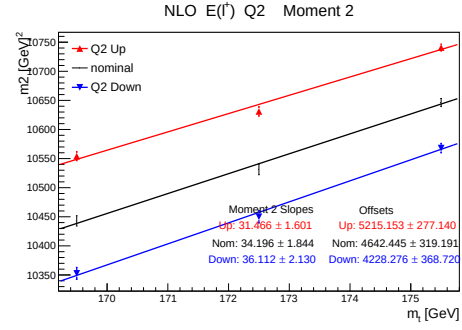
(a) $t\bar{t} p_T(l\bar{l})$



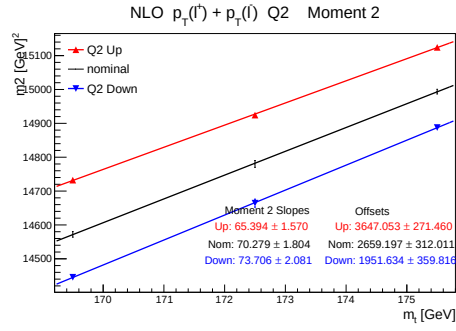
(b) $t\bar{t} M(l\bar{l})$



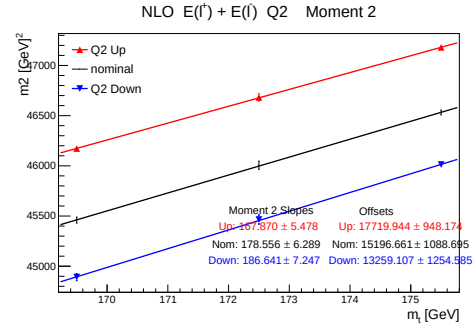
(c) $t\bar{t} p_T(l^+)$



(d) $t\bar{t} E(l^+)$

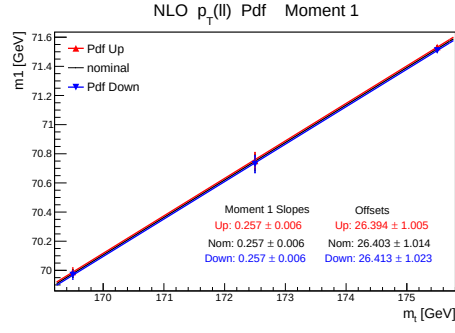


(e) $t\bar{t} p_T(l^+) + p_T(l^-)$

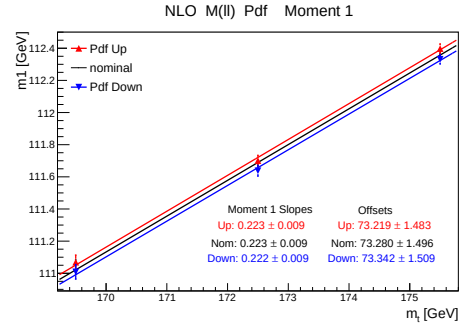


(f) $t\bar{t} E(l^+) + E(l^-)$

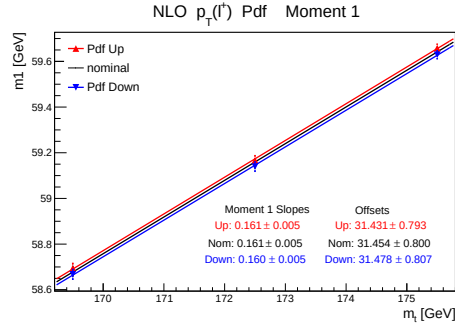
Figure C.58: NLO QCD scale systematic second moments.



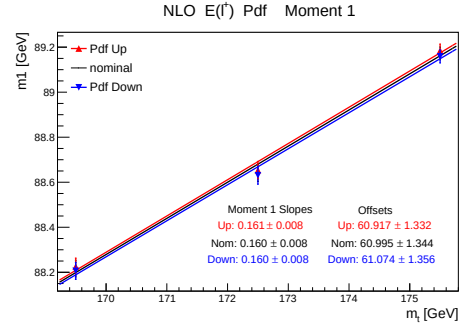
(a) $t\bar{t} p_T(l\bar{l})$



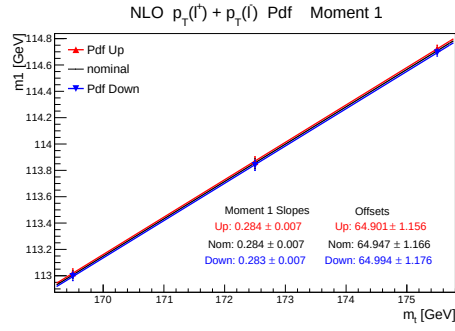
(b) $t\bar{t} M(l\bar{l})$



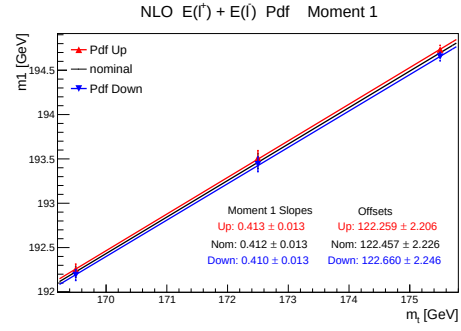
(c) $t\bar{t} p_T(l^+)$



(d) $t\bar{t} E(l^+)$



(e) $t\bar{t} p_T(l^+) + p_T(l^-)$



(f) $t\bar{t} E(l^+) + E(l^-)$

Figure C.59: NLO parton distribution function systematic first moments.

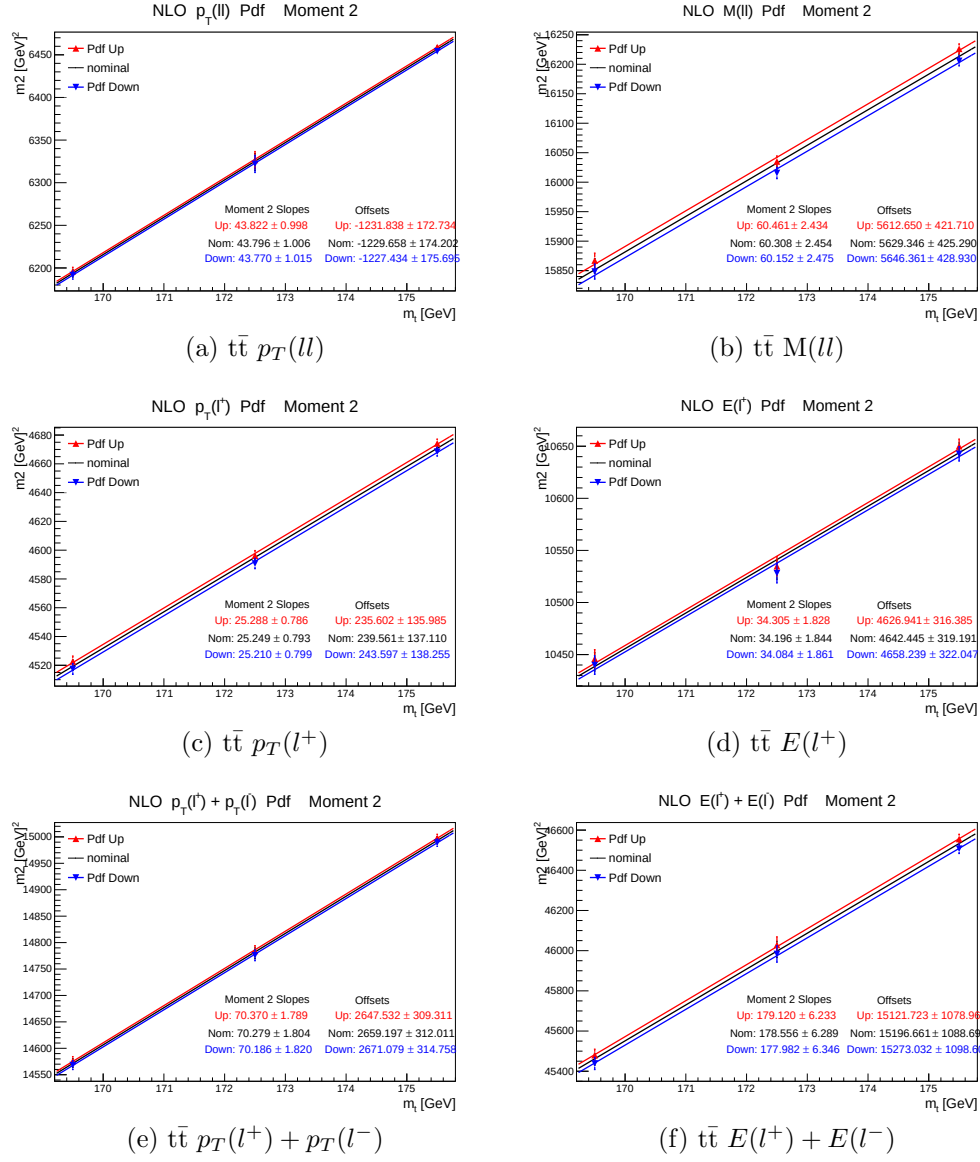
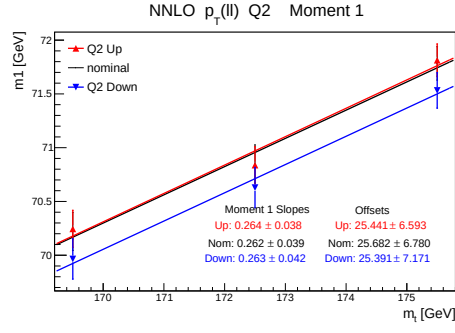
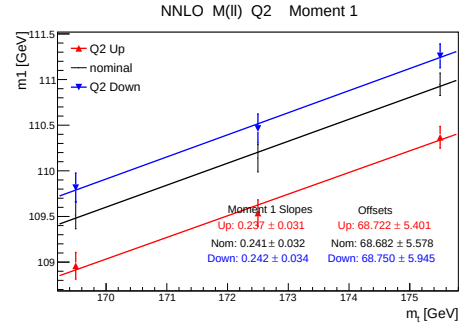


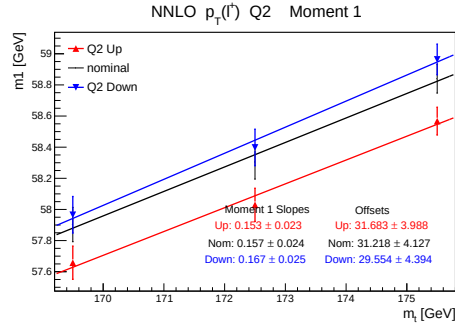
Figure C.60: NLO parton distribution function systematic second moments.



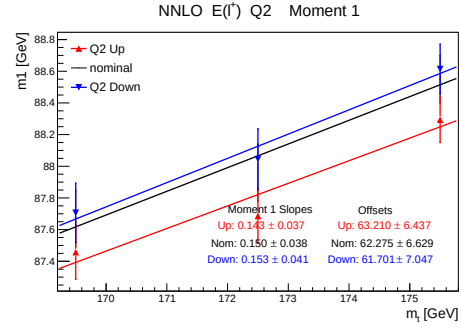
(a) $t\bar{t} p_T(l\bar{l})$



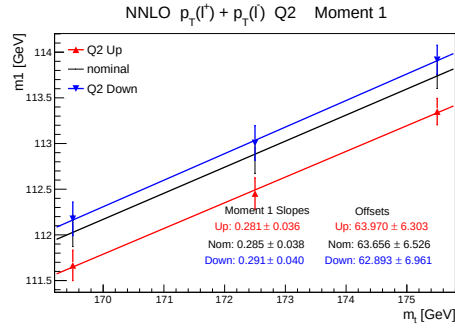
(b) $t\bar{t} M(l\bar{l})$



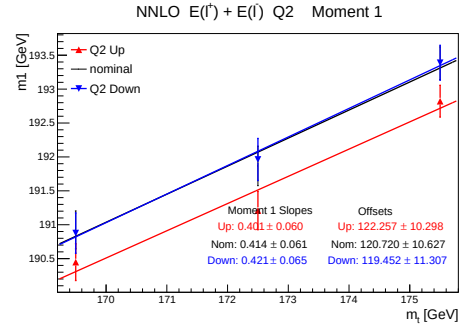
(c) $t\bar{t} p_T(l^+)$



(d) $t\bar{t} E(l^+)$



(e) $t\bar{t} p_T(l^+) + p_T(l^-)$



(f) $t\bar{t} E(l^+) + E(l^-)$

Figure C.61: NNLO QCD scale systematic first moments.

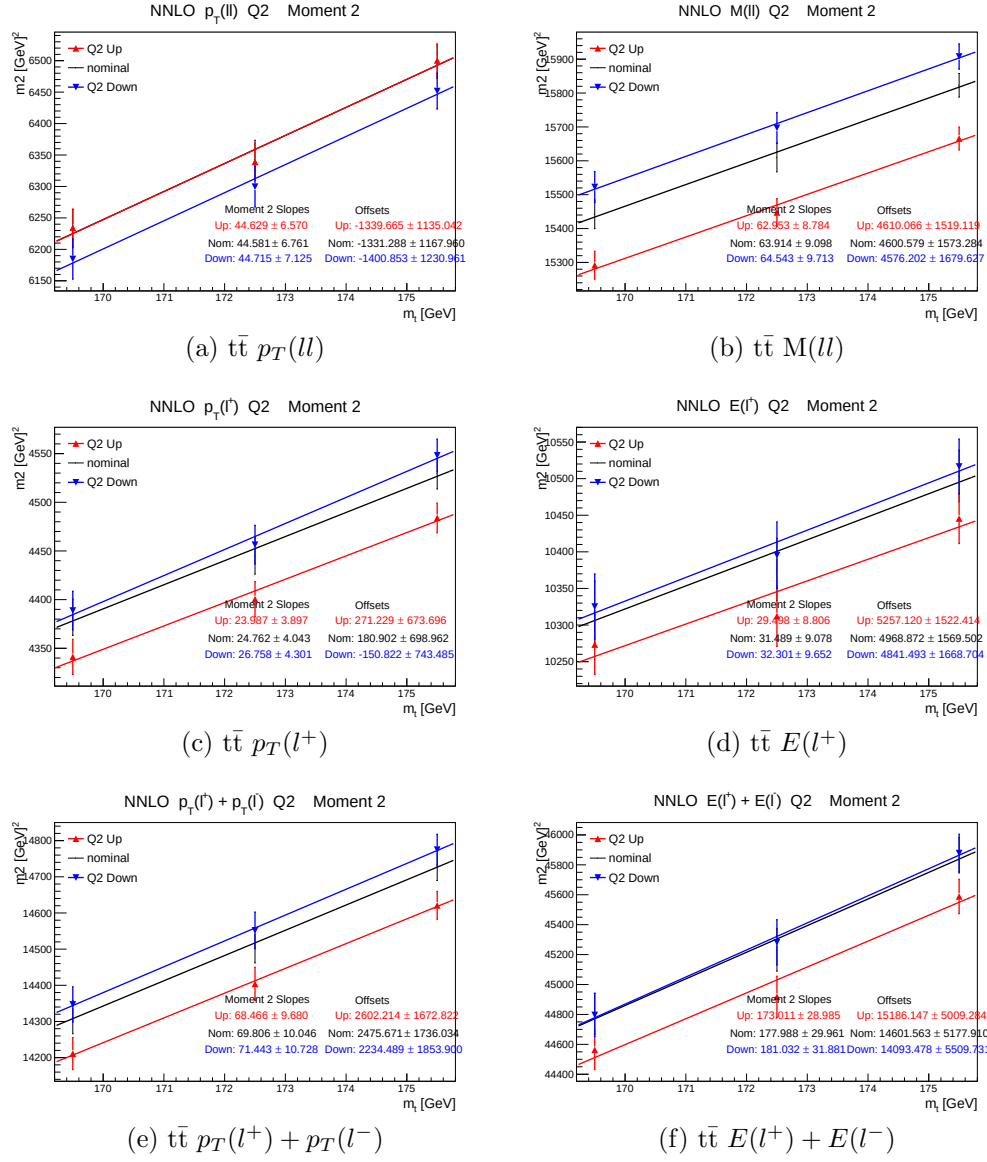


Figure C.62: NNLO QCD scale systematic second moments.

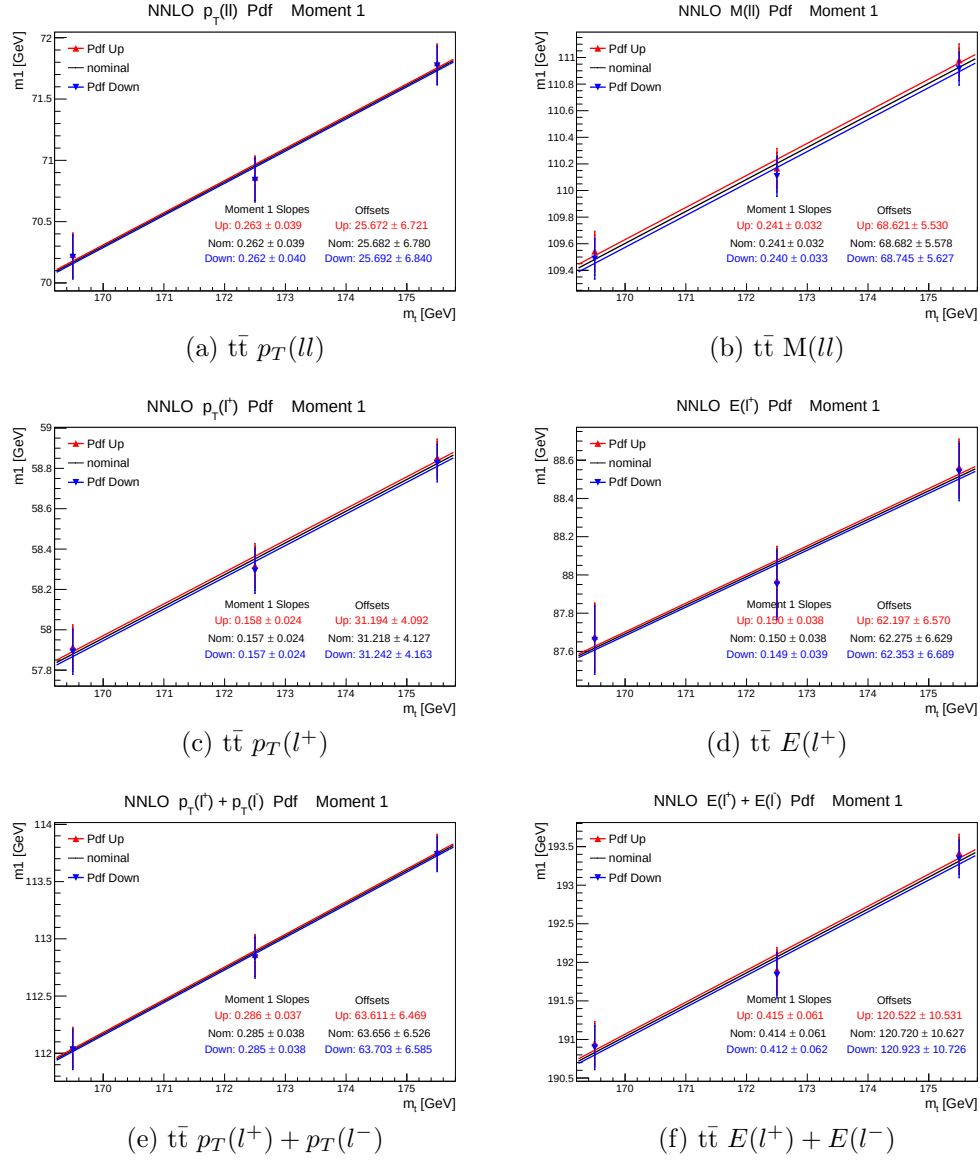


Figure C.63: NNLO parton distribution function systematic first moments.

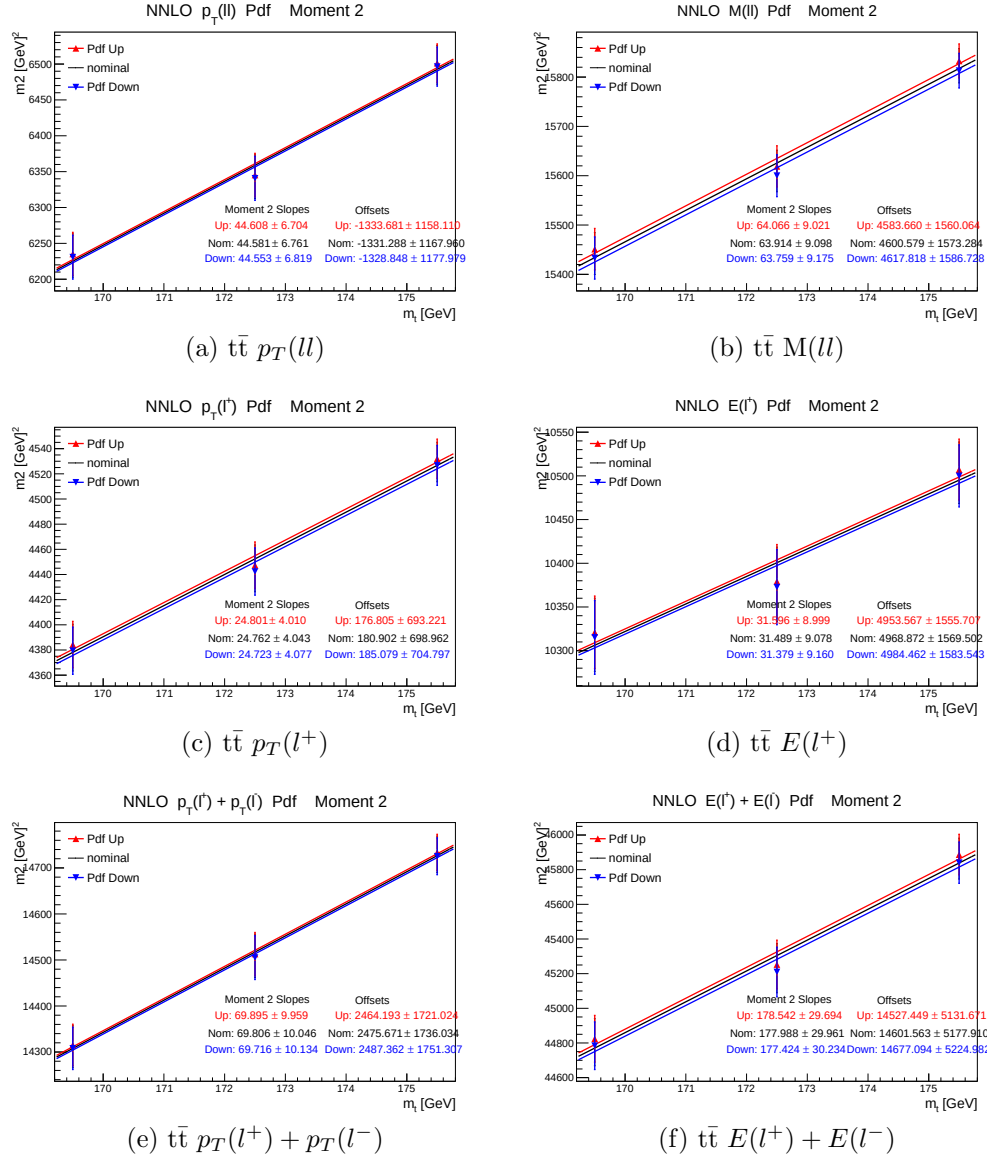


Figure C.64: NNLO parton distribution function systematic second moments.