



Universität Karlsruhe (TH)

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ENTWICKLUNG VON ANALYSE-SOFTWARE  
UND BESTIMMUNG VON PARAMETERN  
DES  $W$ -BOSONS AM LHC DURCH  
VERGLEICH MIT  $Z$ -BOSONEN

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DIPLOMARBEIT

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# Chapter 1

## Introduction

### 1.1 The Large Hadron Collider

At CERN, the “European Organization for Particle Physics Research” in Geneva, Switzerland, a new hadron collider experiment (LHC<sup>1</sup>) is under construction. It is a proton-proton collider that reaches a center of mass energy of  $\sqrt{s} = 14$  TeV. Particle detectors are placed at four interaction points. Two of these detectors (ALICE<sup>2</sup> and LHCb<sup>3</sup>) are designed for special purposes (heavy ion and b-physics), while ATLAS<sup>4</sup> and CMS<sup>5</sup> are general purpose detectors. These detectors are well suited for the search for new physics.

The LHC is installed in the 27 km long LEP<sup>6</sup> tunnel (see figure 1.1). The proton beams circle in opposite directions in two separate beamlines that are filled with 2835 bunches of  $10^{11}$  particles. The design value of the luminosity at the interaction points is  $L = 10^{34}\text{cm}^{-2}\text{s}^{-1}$  for high luminosity and  $L = 10^{33}\text{cm}^{-2}\text{s}^{-1}$  for low luminosity. To focus the beams and to force them into the right trajectories, about 5000 superconducting niobium-titanium magnets are installed. These magnets produce fields up to 8.36 Tesla. The LHC is expected to start operation in 2007.

One of the main purposes of this project is to find answers to some of the most urgent questions of modern physics. For example, the mechanism that is responsible for the mass of elementary particles is still unexplained. One of the most promising theories to answer this question is the “Higgs-Mechanism” (see section 2.4). This theory requires the existence of a new elementary particle, the “Higgs-Boson”. If this boson exists, the experiments at LHC are very likely to find it.

### 1.2 The CMS Detector

The CMS detector project is one of the largest international scientific collaborations in history. There are about 2000 people from 159 institutes in 36 countries working for CMS. An overview of the detector and the countries involved is shown in figure 1.2. The detector consists of several layers that are arranged cylindrical around the collision point. The basic components

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<sup>1</sup>Large Hadron Collider

<sup>2</sup>A Large Ion Collider Experiment

<sup>3</sup>Large Hadron Collider beauty experiment

<sup>4</sup>A Toroidal LHC Apparatus

<sup>5</sup>Compact Muon Solenoid

<sup>6</sup>Large Electron Positron collider

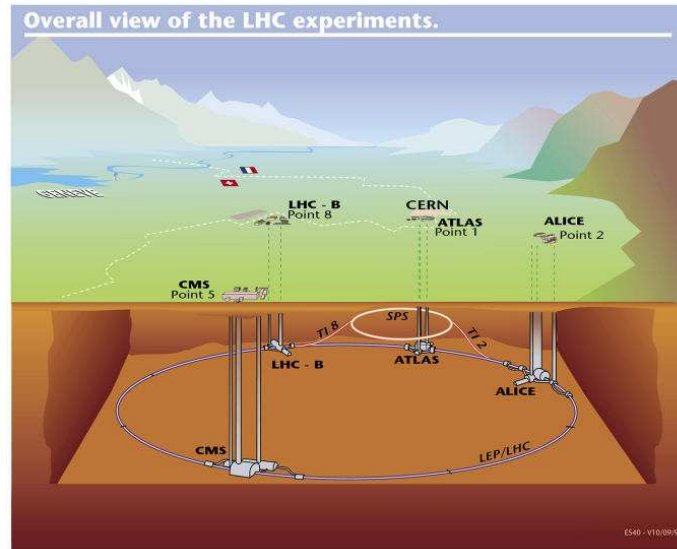


Figure 1.1: Geographic situation at the LHC tunnel.

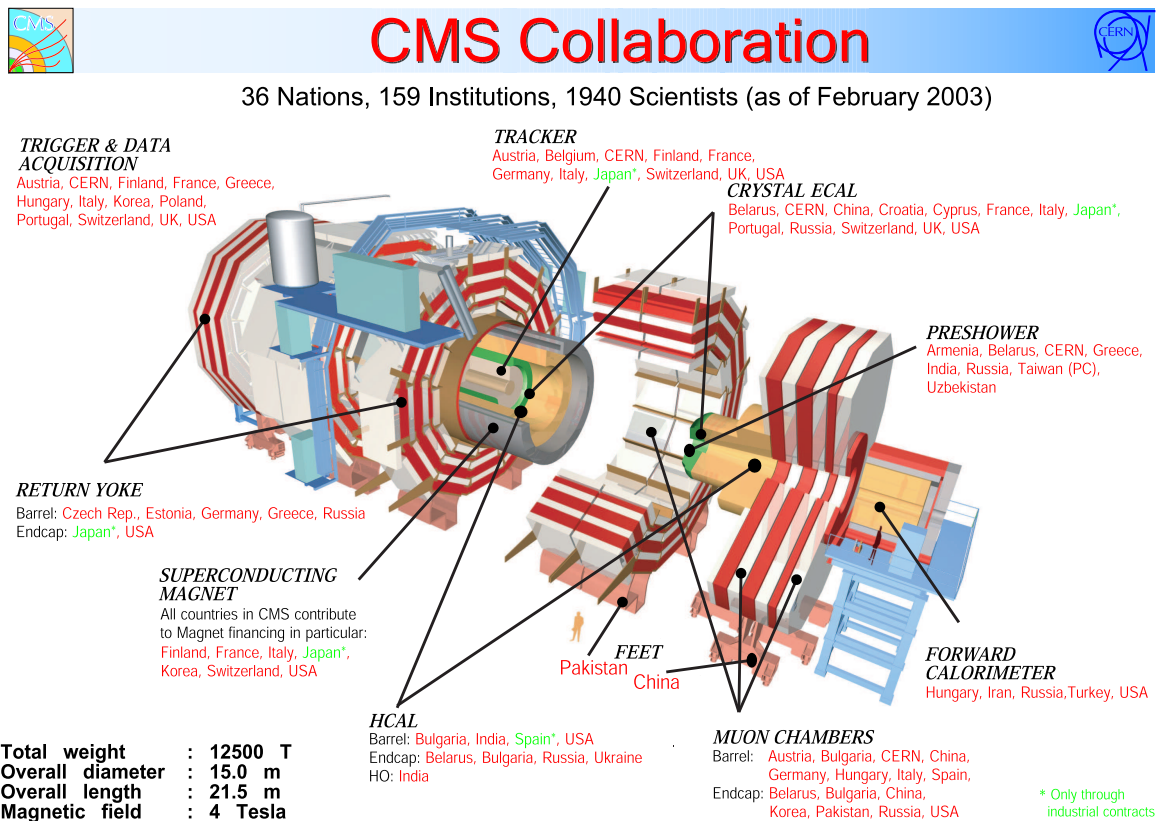


Figure 1.2: The CMS detector and countries involved

are:

- The tracking system: It implements 25,000 silicon strip sensors, covering an area of  $210 \text{ m}^2$ , two barrel and two endcap layers of silicon pixel detectors. The tracker will measure the positions of charged particles. Their momentum can be calculated from the curvature of the particle's track in the magnetic field.
- The electromagnetic calorimeter (ECAL): This is the first calorimeter layer. It is designed to measure the energies of electrons, positrons and photons with high precision. It consists of over 80,000 lead-tungstate ( $\text{PbWO}_4$ ) crystals, a scintillating material. The measurement is done by avalanche photodiodes or vacuum phototriodes.
- The hadron calorimeter (HCAL): The second calorimeter layer consists of 50 mm thick copper absorber plates interleaved with 4 mm thick scintillator sheets. It plays an important role in the measurement of quarks, gluons, and neutrinos by measuring the energy and direction of jets and of missing transverse energy.
- The very forward calorimeter (VFCAL): These calorimeters are located at each end of the detector. Their purpose is to complete the  $\eta$  coverage which is very important for the measurement of missing energy. It has a similar structure as the HCAL.
- The superconducting solenoid: It is 13 m long with a free inner diameter of 5.9 m, thus it will be the biggest and most powerful superconducting magnet ever. It will generate a magnetic field of 4 Tesla and store 2.5 Giga Joules of energy.
- The iron return yoke: The yoke is interleaved with the muon chambers. It returns the magnetic field, thus the muon momentum can be measured from the curvature of the track.
- The muon chambers: Muon measurement and identification is one of the most important tasks of CMS. Muons signalize a wide range of interesting physics processes. The identification of the muons is ensured by the thickness of the absorber material (iron), which cannot be traversed by other particles except neutrinos. Three types of muon detectors will be used: Drift Tubes (DT) in the central barrel region, Cathode Strip Chambers (CSC) in the endcap region and Resistive Parallel Plate Chambers (RPC) in both the barrel and endcaps. The RPC is very fast and will be used for the Level-1 trigger. The CSC and DT provide a high precision of the position measurement.

All in all the system will be nearly 22 m long, will have a width of 14.6 m and a total weight of about 12,500 tons. There will be 15,000,000 channels that have to be read out by sophisticated computing facilities. The data acquisition and trigger concept is briefly outlined in the following section.

### 1.3 Data Acquisition and Trigger of the CMS Detector

At the LHC the proton beam crossing frequency will be 40 MHz. At high luminosity of  $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$  there will be approximately 25 proton-proton collisions for each bunch crossing. These  $10^9$  interactions per second have to be filtered by a trigger system to reduce the amount of data and to fit the capabilities of mass storage devices and offline computing facilities.

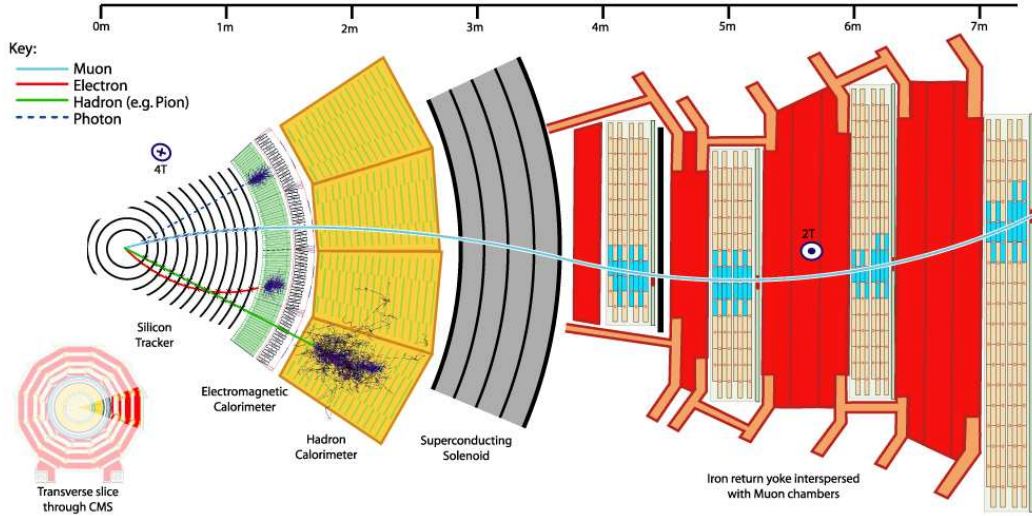


Figure 1.3: Profile of the CMS detector with some particle tracks.

The CMS Trigger and Data Acquisition System (TriDAS) is designed to select events at a maximum rate of  $O(10^2)\text{Hz}$ .

This will be achieved by a 3-level trigger system in order to find out and store only the most interesting events. Of course, this is a challenge because one could easily lose important data if the trigger is not precisely adjusted.

Within about 3 microseconds after a collision, the level-1 trigger decides the acceptance or rejection of the event. The L1 trigger consists of custom electronics and is part of the detector hardware. It is designed to reduce the rate to 100 kHz. The L1 system uses data from fast muon detectors (RPC) and coarsely segmented calorimeter data. The high-resolution data is kept in the memory pipelines of the front-end electronics while the L1 trigger decides to accept or reject the event.

The data is then passed to the High Level Trigger system (HLT), an on-line processor farm with a total processing time of max.  $\sim 1$  s. It is designed to output an event rate of 100 Hz. It implements reconstruction algorithms that are almost as sophisticated as the offline reconstruction algorithms, therefore it is necessary to use fully programmable processors for this step.

## Chapter 2

# Theory of Electroweak Interactions

In the year 1930, the first evidence for a “weak” interaction was found in the nuclear beta decay. The observed energy spectrum of the electron was continuous in contrast to nuclear  $\gamma$ -emission. If a two-body decay is assumed, then this observation contradicts energy and momentum conservation. To resolve this problem, WOLFGANG PAULI proposed an additional neutral particle, the neutrino, that is emitted with the electron. The first approach to describe this process was a four-fermion-point interaction. Today this is properly explained by virtual  $W$  exchange as in figure 2.1.

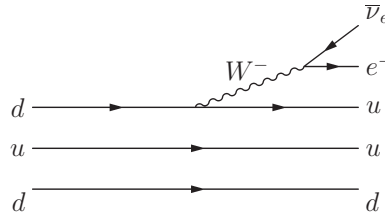


Figure 2.1: Feynman diagram of the neutron decay. Two of the quarks are “spectators” and do not participate directly.

In 1973, experiments at CERN revealed the existence of uncharged weak interactions. The first reaction observed was  $\bar{\nu}_\mu + e \rightarrow \bar{\nu}_\mu + e$ . At this time, the theory was already established and the reaction was supposed to be mediated by an uncharged partner of the  $W$ , the  $Z$ -boson.

### 2.1 Parity Nonconservation and V–A Theory

One of the most stunning properties of the weak interaction is parity nonconservation. This was first observed by Wu *et al.* in nuclear beta decay of polarized  $^{60}\text{Co}$  nuclei in a magnetic field. The relative electron intensities along and against the field direction show a forward-backward asymmetry, which implies that the reaction violates parity conservation.

It was found that neutrinos occur in left-handed helicity states only and anti-neutrinos in right-handed states <sup>1</sup>. The charged leptons produced in weak interactions are left-handed

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<sup>1</sup>The discovery of neutrino-oscillations shows that this is an approximation. The approximation is good

with a degree of polarization of  $\beta = -v/c$ . This behavior can be explained with V-A theory (vector minus axial-vector).

In this theory any Dirac-spinor  $u$  can be divided into two chirality components:

$$u = \frac{1}{2}(1 + \gamma^5)u + \frac{1}{2}(1 - \gamma^5)u = u_R + u_L$$

$P_R = \frac{1}{2}(1 + \gamma^5)$  and  $P_L = \frac{1}{2}(1 - \gamma^5)$  are the chirality-projection-operators and the spinor  $u_R$  is called right-handed and  $u_L$  left-handed. For  $E \gg m$ ,  $P_L$  and  $P_R$  become the projection-operators for negative and positive helicity.

In the according Feynman rules for the calculation of the invariant amplitude, only the appropriate chirality components are to be used. For example, at the  $\mu\nu$ -vertex, the associated factor becomes

$$\bar{u}(\mu)\gamma_\mu u_L(\nu) = \bar{u}(\mu)\gamma_\mu \frac{1 - \gamma^5}{2} u(\nu) .$$

Now there are two terms at the neutrino-muon-vertex:

- The vector-current

$$V^\mu = \bar{\psi}(\mu)\gamma^\mu\psi(\nu) ,$$

that transforms like a four-vector.

- The axial-vector-current

$$A^\mu = \bar{\psi}(\mu)\gamma^\mu\gamma^5\psi(\nu) ,$$

that transforms like an axial-vector. This vector behaves like a four-vector under Lorentz-transformations, but it keeps its sign under parity transformation.

The V-A construct therefore violates parity conservation. Because of  $P_L^2 = P_L$  the matrix element contains only the left-handed components of the spinors and the right-handed components of the anti-spinors:

$$\bar{u}_e\gamma_\mu \frac{1 - \gamma^5}{2} u_\nu = \bar{u}_e\gamma_\mu \left( \frac{1 - \gamma^5}{2} \right)^2 u_\nu = \overline{(u_e)}_L \gamma_\mu (u_\nu)_L .$$

This means that only left-handed components participate in this kind of weak interactions.

However, the coupling of the  $Z$ -boson is not so simple. Instead of the purely V-A vertex factor of the  $W$  boson, it is necessary to use a mixed form:

$$\gamma^\mu (c_V^f - c_A^f \gamma^5) ,$$

where the coefficients depend on the particular quark or lepton (f) involved. These numbers and also the coupling constants and masses of the vector-bosons are determined by one fundamental parameter, the “Weinberg angle” or “weak mixing angle”. This results from electroweak unification (section 2.3).

---

enough for the concerns of this thesis.

## 2.2 $U(1)$ Local Gauge Invariance and QED

In classical electrodynamics a global gauge transformation of the vector field  $A^\mu = (\phi, \vec{A})$  with

$$A^\mu \rightarrow A'^\mu = A^\mu - \partial^\mu \chi$$

leaves the fields  $\vec{E}$  and  $\vec{B}$  invariant since

$$\vec{B} = \nabla \times \vec{A}, \quad \vec{E} = -\nabla \phi - \frac{\partial \vec{A}}{\partial t}.$$

If the principle of gauge invariance is applied to quantum mechanics, the combined transformation turns out to be

$$\begin{aligned} \vec{A} &\rightarrow \vec{A}' = \vec{A} + \nabla \chi, \\ \phi &\rightarrow \phi' = \phi - \frac{\partial \chi}{\partial t}, \\ \psi &\rightarrow \psi' = e^{iq\chi} \psi, \end{aligned}$$

to fulfill the Schrödinger equation<sup>2</sup>. If the principle is extended to *local* invariance, one gets the fascinating result that this leads to the interaction of particles with fields. To see this, we start with the Lagrangian of a free Dirac particle

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \quad (2.1)$$

which is not invariant under local phase transformations of the form

$$\psi(x) \rightarrow e^{i\alpha(x)}\psi(x).$$

If  $\bar{\psi}$  transforms like

$$\bar{\psi} \rightarrow e^{-i\alpha(x)}\bar{\psi}$$

then the last term in  $\mathcal{L}$  is invariant, but not the derivative:

$$\partial_\mu\psi \rightarrow e^{i\alpha(x)}\partial_\mu\psi + ie^{i\alpha(x)}\psi\partial_\mu\alpha \quad (2.2)$$

The  $\partial_\mu\alpha$  term is the cause for the break of the invariance. If one postulates local gauge invariance, then a modified derivative  $D_\mu$ , that transforms like  $\psi$  itself, is necessary.

$$D_\mu\psi \rightarrow e^{i\alpha(x)}D_\mu\psi.$$

If this covariant derivative is used instead of  $\partial_\mu$  in (2.1), the Lagrangian becomes invariant. Now, a derivative that cancels the additional  $\partial_\mu\alpha$  term in (2.2) has to be found. To do this, it is necessary to introduce a vector field  $A_\mu$  with appropriate transformation properties:

$$D_\mu \equiv \partial_\mu - ieA_\mu,$$

---

<sup>2</sup>The Schrödinger equation of a particle with the charge  $q$  in an electromagnetic field is

$$\frac{1}{2m}(-i\nabla - q\vec{A})^2\psi(t, \vec{x}) = i\frac{\partial\psi}{\partial t}$$

with

$$A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha .$$

If the new field is regarded as the photon field and if an invariant term corresponding to its kinetic energy is added, we get the Lagrangian of QED:

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu A_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

This means that the interacting field theory QED is deduced by postulating local gauge invariance on the free fermion Lagrangian .

### 2.3 $SU(2)_L \times U(1)$ -Symmetry

The attempt to extend the  $U(1)$  local gauge invariance to  $SU(2)$  leads to electroweak unification, but with lots of additional requirements, e.g. the masses of the bosons have to be included. This is outlined in the following discussion.

The particles that experience electroweak transitions by emission of field bosons can be arranged in multiplets of a “weak isospin” in analogy to the spin-formalism. The left-handed fermions constitute doublets with  $I = 1/2$ :

|          |                                                |                                                    |                                                      |        |
|----------|------------------------------------------------|----------------------------------------------------|------------------------------------------------------|--------|
| Leptons: | $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ | $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$ | $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$ | $I_3$  |
|          |                                                |                                                    |                                                      | $1/2$  |
|          |                                                |                                                    |                                                      | $-1/2$ |

|         |                                           |                                           |                                           |        |
|---------|-------------------------------------------|-------------------------------------------|-------------------------------------------|--------|
| Quarks: | $\begin{pmatrix} u \\ d' \end{pmatrix}_L$ | $\begin{pmatrix} c \\ s' \end{pmatrix}_L$ | $\begin{pmatrix} t \\ b' \end{pmatrix}_L$ | $I_3$  |
|         |                                           |                                           |                                           | $1/2$  |
|         |                                           |                                           |                                           | $-1/2$ |

The right-handed leptons and quarks do not couple to the charged weak currents, therefore they are arranged in singlets:

$$I = 0 : \quad e_R^-, \mu_R^-, \tau_R^-, u_R, d_R, s_R, c_R, b_R, t_R$$

The Dirac-wavefunction of the left-handed leptons can then be expressed as a product of a left-handed Dirac-spinor  $\psi_L(t, \vec{x})$  and a weak isospinor  $\chi$ :

$$\nu_L = \psi_L(t, \vec{x}) \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad e_L = \psi_L(t, \vec{x}) \begin{pmatrix} 0 \\ 1 \end{pmatrix} .$$

The transition  $e^- \rightarrow \nu_e$  proceeds by emission of a  $W^-$ -boson and is mediated by the “step-up” operator  $\tau_+$ , the transition  $\nu_e \rightarrow e^-$  by the “step-down” operator  $\tau_-$ . The matrices  $\tau_\pm$  are linear combinations of the first two Pauli spin matrices:

$$\tau_\pm = \frac{1}{2}(\tau_1 \pm i\tau_2)$$

There should be another operator  $\tau_3$ , that leaves  $I_3$  unchanged, which is later identified with the neutral weak current.

In analogy to the  $U(1)$  case, where  $q$  is the coupling strength and  $\alpha$  the transformation angle, a phase transformation in the weak isospin space is defined. The  $SU(2)_L$  group describes transformations of the left-handed weak isospin multiplets, e.g.

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}'_L = \exp\left(i\frac{g}{2}\vec{\tau} \cdot \vec{\beta}(x)\right) \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L ,$$

where  $g$  is the coupling strength and  $\vec{\tau}$  represents the pauli matrices  $(\tau_1, \tau_2, \tau_3)$ . Now there are three angles  $\beta_j$ .

To get invariance under a local transformation it is necessary to introduce a triplet of vector-fields  $W_1^\mu, W_2^\mu, W_3^\mu$  for the  $SU(2)_L$  group. The covariant derivative for  $(\nu_e, e_L^-)$ ,  $(\nu_\mu, \mu_L^-)$ ,  $(\nu_\tau, \tau_L^-)$  is

$$D^\mu = \partial^\mu + i\frac{g}{2}\vec{\tau} \cdot \vec{W}^\mu .$$

The right-handed fermions have to be included as well since the neutral weak interaction couples right-handed states. This is accomplished by introducing the “weak hypercharge”  $Y$ :

$$Q = I_3 + \frac{1}{2}Y .$$

$Q$  is the charge and  $I_3$  the third component of weak isospin. The associated weak hypercharge current then involves left-handed and right-handed chirality states. The weak hypercharge can be associated with phase transformations as well:

$$\begin{aligned} \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}'_L &= \exp\left(i\left(\frac{g'}{2}Y_L\right)\chi(x)\right) \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \\ e'_R &= \exp\left(i\left(\frac{g'}{2}Y_R\right)\chi(x)\right) e_R . \end{aligned}$$

$\frac{g'}{2}Y$  is the coupling constant instead of the charge  $q$  in the electromagnetic case. These transformations form a  $U(1)$  group. A single vector field is necessary for local gauge invariance in the  $U(1)$  group as derived earlier. If this is combined with the three vector fields of the  $SU(2)_L$  group one gets the covariant derivative of  $SU(2)_L \times U(1)$ :

$$D^\mu = \partial^\mu + ig\vec{T} \cdot \vec{W}^\mu + i\frac{g'}{2}YB^\mu$$

For left-handed leptons it is

$$\vec{T} = \vec{\tau}/2 , Y = -1$$

and for right-handed leptons

$$\vec{T} = 0 , Y = -2 .$$

This means that for  $(\nu_e, e_L^-)$ ,  $(\nu_\mu, \mu_L^-)$ ,  $(\nu_\tau, \tau_L^-)$

$$D^\mu = \partial^\mu + i\frac{g}{2}\vec{\tau} \cdot \vec{W}^\mu - i\frac{g'}{2}B^\mu$$

and for  $e_R^-, \mu_R^-, \tau_R^-$

$$D^\mu = \partial^\mu - ig'B^\mu .$$

For the left-handed leptons this can also be expressed as

$$D^\mu = \partial^\mu + i\frac{g}{\sqrt{2}} \left( \tau_+ W^{(-)\mu} + \tau_- W^{(+)\mu} \right) + i\frac{g}{2} \tau_3 W_3^\mu - i\frac{g'}{2} B^\mu .$$

To get the transition matrix-elements, the covariant derivative has to be substituted in the Dirac-equation. For the process  $e^- \rightarrow \nu_e$  we get

$$\mathcal{M} \propto -i\frac{g}{\sqrt{s}} \bar{\nu}_L \gamma_\mu \tau_+ e_L W^{(-)\mu} .$$

The transition  $\nu_e \rightarrow \nu_e$  is mediated by

$$i\frac{g}{2} \tau_3 W_3^\mu \text{ and } -i\frac{g'}{2} B^\mu$$

and contributes to the matrix-element

$$-i\frac{g}{2} \bar{\nu}_L \gamma_\mu \tau_3 \nu_L W_3^\mu \text{ and } +i\frac{g'}{2} \bar{\nu}_L \gamma_\mu \nu_L B^\mu .$$

The electromagnetic field  $A^\mu$  can not be identified with  $W_3^\mu$  or  $B^\mu$  since the coupling to the neutrino does not disappear. It is possible to construct a linear combination of these two fields to get a vanishing coupling to the neutrino:

$$A^\mu = aW_3^\mu + bB^\mu \text{ with coupling } \sim a\left(-\frac{g}{2}\right) + b\frac{g'}{2} = 0$$

so it is

$$a = \frac{g'}{\sqrt{g^2 + g'^2}} , \quad b = \frac{g}{\sqrt{g^2 + g'^2}} .$$

This defines the “weak mixing angle” or “Weinberg angle”  $\theta_w$ :

$$\cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}} , \quad \sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}$$

It follows that

$$A^\mu = B^\mu \cos \theta_w + W_3^\mu \sin \theta_w$$

If the field  $Z^\mu$  of the neutral weak current is supposed to be orthogonal to  $A^\mu$  it is

$$Z^\mu = -B^\mu \sin \theta_w + W_3^\mu \cos \theta_w .$$

The fundamental relation between the charge  $e$  and the coupling constants  $g$  and  $g'$  is

$$e = g' \cos \theta_w = g \sin \theta_w .$$

This follows if right-handed electrons that couple to  $B^\mu$  only are considered:

$$ig' \bar{u}_R \gamma_\mu u_R B^\mu = ig' \cos \theta_w \bar{u}_R \gamma_\mu u_R A^\mu - ig' \sin \theta_w \bar{u}_R \gamma_\mu u_R Z^\mu$$

In QED the coupling of electrons is equal for right- and left-handed states:

$$ie \bar{u} \gamma_\mu u A^\mu$$

So one can see that  $e = g' \cos \theta_w$ . A similar argumentation leads to the coupling of the electron to the  $Z$ :

$$-\frac{ig}{2 \cos \theta_w} \bar{u} \gamma_\mu (v_e - a_e \gamma^5) u Z^\mu ,$$

where the vector- and axial-vector couplings are

$$v_e = 2 \sin^2 \theta_w - 1/2 , \quad a_e = -1/2$$

## 2.4 Mass of the Vector Bosons

In gauge theories, the interacting bosons are required to be massless. This is no problem for the photon and gluons, but if this is applied to weak interactions with massive bosons, we run into trouble. If mass terms of the form  $M^2 W_\mu W^\mu$  are introduced into the Lagrangian, it is no longer gauge-invariant. One possibility to explain the masses of the particles is the introduction of a background field, the Higgs field, in analogy to the theory of superconduction.

### 2.4.1 Spontaneous Symmetry Breaking

The Lagrangian of a scalar field is for example

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \left(\frac{1}{2}\mu^2 \phi^2 + \frac{1}{4}\lambda \phi^4\right),$$

where only the first two terms in the expansion of the potential  $V(\phi)$  are kept. If  $\lambda > 0$  and  $\mu^2 > 0$  the ground state is  $\phi = 0$  and the Lagrangian is symmetric. But if  $\mu^2 < 0$  the Lagrangian has a mass term with the wrong sign and two minima at

$$\phi = \pm \sqrt{-\mu^2/\lambda}$$

If the field is translated to  $\sqrt{-\mu^2/\lambda}$  by

$$\phi(x) = \sqrt{-\mu^2/\lambda} + \eta(x)$$

it is

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 - \lambda v^2 \eta^2 - \lambda v \eta^3 - \frac{1}{4}\lambda \eta^4 + \text{const.}$$

with  $v = \sqrt{-\mu^2/\lambda}$ . Now there is a mass term of the correct sign. The mass is

$$m_\mu = \sqrt{2\lambda v^2} = \sqrt{-2\mu^2}$$

and the higher order terms in  $\eta$  represent the interaction of the field with itself.

It is necessary to note that the Lagrangians  $\mathcal{L}$  and  $\mathcal{L}'$  are equivalent. Surprisingly they yield different masses. The fact that perturbation theory is always expanded around the minimum of a potential resolves this ambiguity. The Feynman calculus is a perturbation theory and it would not converge if it would be expanded around  $\phi = 0$  because this is not a stable minimum. This means that the mass was “generated” by a “spontaneous symmetry breaking”.

### 2.4.2 Spontaneous Breaking of a Local SU(2) Gauge Symmetry

It is necessary to extend the results of the previous chapter to SU(2) gauge symmetry. The Lagrangian

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2,$$

where  $\phi$  is a SU(2) doublet of complex scalar fields

$$\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \sqrt{1/2} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}$$

is invariant under global transformations of the form

$$\phi \rightarrow \phi' = e^{i\vec{\alpha}\vec{\tau}/2}\phi .$$

To extend the invariance to local transformations,  $\partial_\mu$  has to be replaced by the covariant derivative

$$D_\mu = \partial_\mu + i\frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu .$$

The invariant Lagrangian is then

$$\mathcal{L} = \left( \partial_\mu \phi + i\frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu \phi \right)^\dagger \left( \partial^\mu \phi + i\frac{g}{2}\vec{\tau} \cdot \vec{W}^\mu \phi \right) - V(\phi) - \frac{1}{4} \vec{W}_{\mu\nu} \cdot \vec{W}^{\mu\nu} ,$$

with

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 .$$

Again, a kinetic energy term is added to the Lagrangian with

$$\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g \vec{W}_\mu \times \vec{W}_\nu .$$

If  $\mu^2 < 0$  and  $\lambda > 0$  one gets a potential  $V(\phi)$  with a minimum at a finite value

$$\phi^\dagger \phi = |\phi|^2 = -\frac{\mu^2}{2\lambda} .$$

An arbitrary point in this minimum can be chosen and  $\phi(x)$  can be expanded around this point, e.g.

$$\phi_1 = \phi_2 = \phi_4 = 0 , \quad \phi_3^2 = -\frac{\mu^2}{\lambda} = v^2 .$$

The expansion

$$\phi(x) = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$$

is then substituted into the Lagrangian. This means that of the four scalar fields only one remains, the Higgs field  $h(x)$ . It is sufficient to substitute

$$\phi_0 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

into the Lagrangian to read off the mass. The relevant term is

$$\left( i\frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu \phi \right)^\dagger \left( i\frac{g}{2}\vec{\tau} \cdot \vec{W}^\mu \phi \right) = \frac{g^2 v^2}{8} ((W_\mu^1)^2 + (W_\mu^2)^2 + (W_\mu^3)^2)$$

and the mass is  $M = \frac{1}{2}gv$ .

### 2.4.3 Masses of the Gauge Bosons

This formalism has to be applied to the weak interaction so that  $W^\pm$  and  $Z$  become massive and the photon remains massless. An  $SU(2) \times U(1)$  gauge invariant Lagrangian  $\mathcal{L}_2$  has to be added to the Lagrangian that describes the electroweak interaction:

$$\mathcal{L}_2 = \left| \left( i\partial_\mu - g\vec{T} \cdot \vec{W}_\mu - g'\frac{Y}{2}B_\mu \right) \phi \right|^2 - V(\phi) ,$$

where  $| \cdot |^2 = ( \cdot )^\dagger ( \cdot )$ . The “Weinberg-Salam model” now makes a choice for the fields so that the vacuum is invariant under U(1) transformations and the photon remains massless. Four fields are arranged in an isospin doublet with hypercharge  $Y = 1$ :

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with

$$\begin{aligned} \phi^+ &= (\phi_1 + i\phi_2)/\sqrt{2} \\ \phi^0 &= (\phi_3 + i\phi_4)/\sqrt{2} . \end{aligned}$$

The Higgs potential is chosen as in the previous section with  $\mu^2 < 0$  and  $\lambda > 0$  and the vacuum expectation value becomes

$$\phi_0 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} .$$

$\phi_0$  is then substituted into the Lagrangian  $\mathcal{L}_2$  and the masses can be read off:

$$\left| \left( -i\frac{g}{2}\vec{\tau} \cdot \vec{W}_\mu - i\frac{g'}{2}B_\mu \right) \right|^2 = \left( \frac{1}{2}vg \right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}v^2 (W_\mu^3, B_\mu) \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} .$$

The mass term  $M_W^2 W^+ W^-$  yields

$$M_W = \frac{1}{2}vg .$$

The remaining term is

$$\frac{1}{8}v^2 (g^2 (W_\mu^3)^2 - 2gg'W_\mu^3 B^\mu + g'^2 B_\mu^2) = \frac{1}{8}v^2 (gW_\mu^3 - g'B_\mu)^2 + 0 (g'W_\mu^3 + gB_\mu)^2 .$$

Since

$$\begin{aligned} A_\mu &= \frac{g'W_\mu^3 + gB_\mu}{\sqrt{g^2 + g'^2}} \\ Z_\mu &= \frac{gW_\mu^3 - g'B_\mu}{\sqrt{g^2 + g'^2}} , \end{aligned}$$

the mass terms  $\frac{1}{2}M_Z^2 Z_\mu^2 + \frac{1}{2}M_A^2 A_\mu^2$  can be identified and we get

$$\begin{aligned} M_A &= 0 \\ M_Z &= \frac{1}{2}v\sqrt{g^2 + g'^2} . \end{aligned}$$

Reexpressed in terms of the Weinberg angle:

$$\frac{M_W}{M_Z} = \cos \theta_w .$$

This is just one possibility to generate masses for the weak gauge bosons. More complicated choices for the Higgs field lead to different relations between  $M_W$  and  $M_Z$ .



## Chapter 3

# Theory of $W$ - and $Z$ -Boson Production at LHC

### 3.1 Motivation

The measurements of electroweak parameters will be improved significantly at the LHC, e.g. the top-quark mass, the electroweak mixing angle and the charged vector boson mass and width ( $M_W, \Gamma_W$ ). Dependencies in these parameters –an example is given below– facilitate consistency checks by comparing predicted values with direct measurements. Different models, in particular the standard model (SM) and its extensions like the minimal super-symmetric standard model (MSSM), will be tested and constraints to parameters of the models become available. The Higgs boson mass  $M_H$  enters the predictions only logarithmically. This means that very precise measurements and small uncertainties in the theory are necessary to get an indirect determination of  $M_H$ .

As an example, the measurements of the top-quark mass, the  $W$ -boson mass and their agreement with predictions for different Higgs masses are shown in figure 3.1. The direct and indirect measurements of  $M_W$  show a marginal agreement. The green band represents the prediction of the standard model for several Higgs masses. It intersects the confidence levels and shows that a light-weight Higgs is preferred. Figure 3.2 shows the measurements of the  $W$  mass at LEP. It also shows the theoretical prediction for the Higgs mass in dependence on the  $W$  mass. It is obvious that improvements in  $M_W$  and the top-mass  $M_t$  combined with direct measurements of  $M_H$  provide a strong consistency check for the standard model.

The current uncertainty of  $M_W$  is  $\delta M_W = \pm 0.031$  GeV [4] [2] and is expected to be improved by the LHC to  $\delta M_W = \pm 0.015$  GeV [1].

### 3.2 $W$ Production

The most important process for  $W$  production is the Drell-Yan process. It has a large cross section and is well suited for  $M_W$  measurements. In this process two quarks fuse into a  $W$ -boson which decays according to its branching ratios. The leptonic decay of the  $W$  is of special interest, because it has a clean signature:

$$u \bar{d} \rightarrow W^+ \rightarrow l^+ \nu_l$$

The associated Feynman graph is shown in Figure 3.3. This graph, of course, has to be

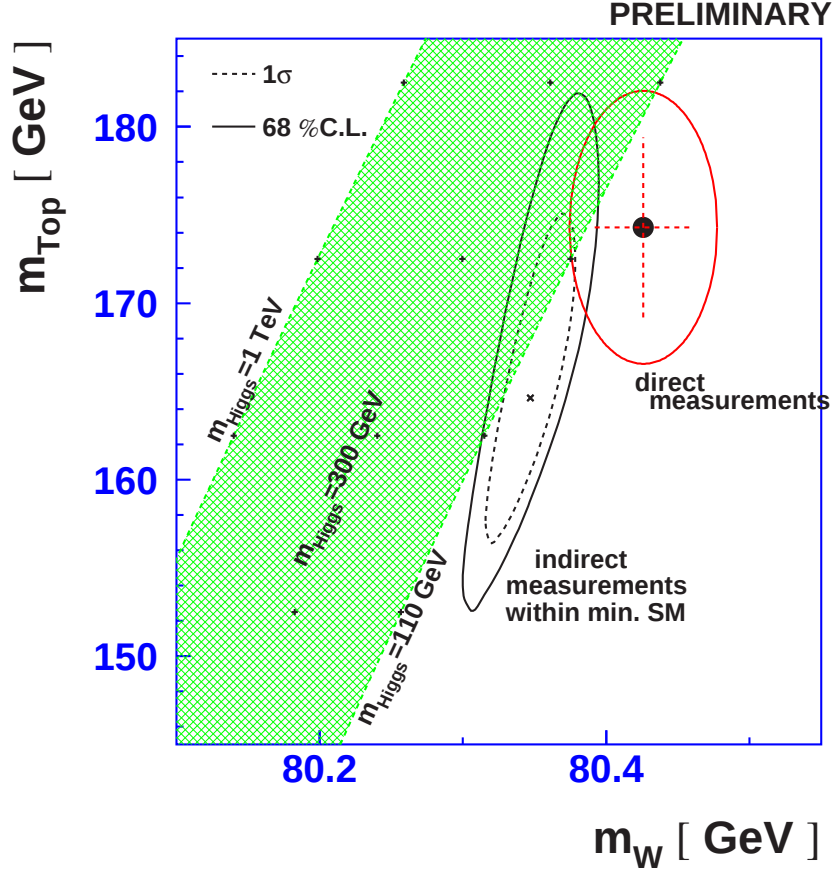


Figure 3.1: Direct and indirect measurements of  $M_W$  and  $M_t$  and the agreement with predictions for different Higgs masses. The plot shows the  $1\sigma$  and 68% confidence levels. The indirect measurement is from LEP1, SLD and the direct measurement is a combination of LEP2 and data from the Tevatron  $p\bar{p}$  collider. [4]

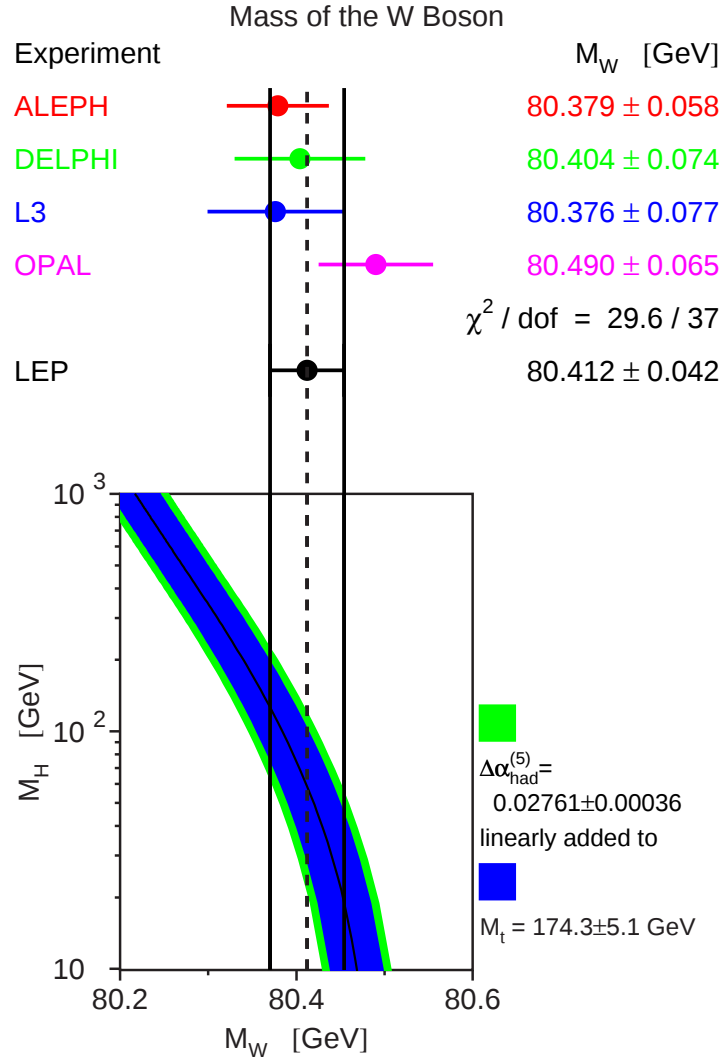


Figure 3.2: Current measurements of the  $W$  mass and the dependence of  $M_H$  on  $M_W$  as predicted by the standard model. The blue error shows the uncertainty due to the top quark mass measurement. [3]

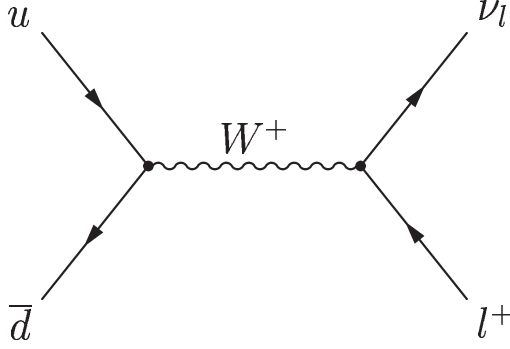


Figure 3.3: Lowest order graph of the Drell-Yan process  $u \bar{d} \rightarrow W^+ \rightarrow \nu_l l^+$

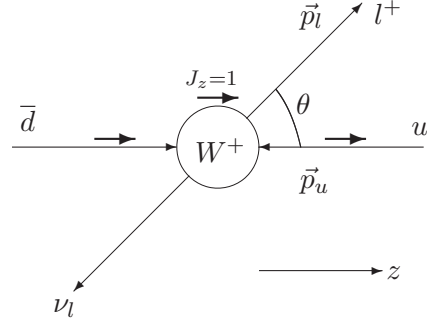


Figure 3.4: Kinematics of this process. In V-A theory the  $W$ 's are polarized with  $J_z = 1$

embedded in the proton's parton distribution functions as discussed later, and higher order corrections have to be taken into account.

In lowest order, the cross section for  $u \bar{d} \rightarrow l^+ \nu_l$  is [1]

$$\left( \frac{d\sigma}{d\Omega} \right) = \frac{\alpha^2 |V_{ud}|^2}{192 s_w^4 \hat{s}} \frac{\hat{u}^2}{|\hat{s} - M_W^2 + i M_W \Gamma_W(\hat{s})|^2}, \quad (3.1)$$

where  $\alpha$  is the electromagnetic coupling which can be a “running constant”  $\alpha(Q^2)$ ;  $\hat{s}$  is the squared center-of-mass (CM) energy and  $\hat{u} = (p_u - p_l)^2$ . The electroweak parameters are the boson mass  $M_W$  and width  $\Gamma_W$ , the mixing angle expressed as  $s_w^2 = 1 - \cos^2 \theta_w$  and the CKM matrix element for  $ud$  transitions  $V_{ud}$ .

The angular dependence of this cross section resides in  $\hat{u}$ . A glance at the kinematics of this process yields <sup>1</sup>

$$\hat{u} = (p_u - p_l)^2 \sim (1 + \cos \theta)$$

This angular distribution, of course, results from V-A theory. In this theory, the charged vector bosons couple left handed helicity states only. As shown in Figure 3.4, the  $W$ -bosons are polarized. By applying angular momentum conservation and helicity conservation, it can be worked out that a  $J_z = 1$  state decays according to a  $(1 + \cos \theta)^2$  distribution. This is quite obvious because the outgoing neutrino must be left-handed and therefore prefers small  $\theta$  angles.  $\theta$  is the angle between the outgoing lepton and the  $z$ -axis in the *Collins-Soper* rest frame [5] (see figure 3.5). Using

$$d\Omega = |\sin \theta d\theta d\phi| = |d(\cos \theta) d\phi|$$

<sup>1</sup> $p_{u,l}$  are the four-momenta of the interacting up quark and lepton.  $\hat{u} = (p_u - p_l)^2 = p_u^2 + p_l^2 - 2p_u p_l = m_u^2 + m_l^2 - 2(E_u E_l - \vec{p}_u \vec{p}_l)$ , where  $E_{u,l}$  is the CM energy and  $m_{u,l}$  are the particle masses, which can be neglected because  $m_{u,l} \ll E_{u,l}$  and  $E_{u,l} \approx |\vec{p}_{u,l}|$ . Therefore we get

$$\begin{aligned} \hat{u} &\approx 2(E_u E_l - |\vec{p}_u| |\vec{p}_l|) \cos \alpha = 2E_u E_l (1 - \cos \alpha) \\ &= 2E_u E_l (1 + \cos \theta) \end{aligned}$$

The angle  $\alpha$  between the four-momenta is  $\pi - \theta$  (see Figure 3.4).

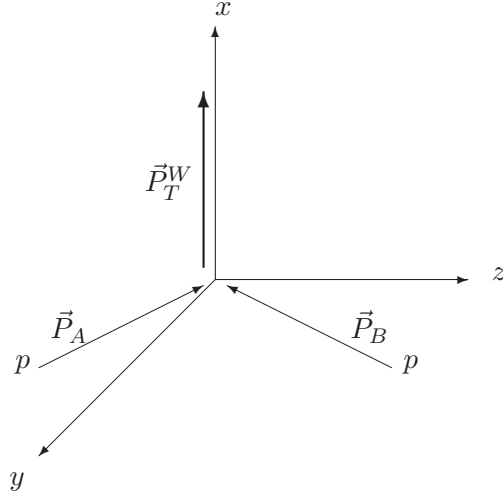


Figure 3.5: Collins-Soper frame: In the laboratory frame the proton beam is aligned along the  $z$ -axis. The transformation from the lab frame into the Collins-Soper frame is done in three steps: 1. Boost along the  $z$ -axis so the  $W$ -boson is at rest in respect to the  $z$ -axis. 2. Rotation around the  $z$ -axis so that  $\vec{P}_T^W$  is aligned along the  $x$ -axis. The  $\vec{P}_T^W$  vector shown in the figure is evaluated before the final step. 3. Boost along the  $x$ -axis so that the  $W$  is at rest.  $\vec{P}_A$  and  $\vec{P}_B$  are the proton momentum vectors in the CS frame. The  $z$ -axis bisects the angle between  $\vec{P}_A$  and  $-\vec{P}_B$ . The polar angle of the outgoing lepton in the CS frame is denominated  $\theta$ , that is the angle between the lepton momentum and the  $z$ -axis.

and integrating over  $\phi$ , the cross section (3.1) can be written as

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2|V_{ud}|^2}{24s_w^4} \frac{\hat{s}}{|\hat{s} - M_W^2 + iM_W\Gamma_W(\hat{s})|^2} (1 + \cos\theta)^2 . \quad (3.2)$$

The anti-quarks in the Drell-Yan process are sea quarks, because there are no anti-valence-quarks in the proton. This is a major difference to the Tevatron collider, where anti-protons are used and the resulting angular distribution has a forward-backward asymmetry. At the LHC the angular distribution should be symmetric, because there is no possibility to distinguish between forward and backward scattering. Therefore we get in leading order

$$\frac{d\sigma}{d(\cos\theta)} \propto (1 + \cos\theta)^2 + (1 - \cos\theta)^2 .$$

Fortunately, this makes no difference for the “transverse mass”, which is considered in more detail in section 3.5.

For a precise calculation of the final cross section, the parton distribution functions (pdf) of the proton have to be included. It is necessary to note that not only  $u\bar{d}$ - and  $\bar{u}d$ -quark fusion contribute to the cross section, but also about 15-20% of  $s\bar{c}$ - and  $\bar{s}c$ -quark fusion. The energy dependence of the decomposition of  $W$ -boson cross sections is shown in figure 3.6.

### 3.3 Higher Order Corrections

The theoretical prediction for the total cross section and branching ratio for the decay into a lepton  $[\sigma(pp \rightarrow W^+) + \sigma(pp \rightarrow W^-)] \cdot BR(W \rightarrow l\nu)$  for purely electroweak calculation is 17.9 nb [6]. Including next to next to leading order (NNLO) QCD corrections [6] this value is increased to  $20.3 \pm 1.0$  nb.

The  $W$ -bosons are produced with a transverse momentum that reaches from 0 GeV to  $\sim 100$  GeV. Leading order calculations don't describe this observation since this effect results from gluon or quark radiation in the initial state. Some of the contributing Feynman diagrams are shown in figure 3.7. The additional gluon or quark alters the helicity of the  $W$ -boson and the resulting angular distribution is predicted by next to leading order QCD to

$$\frac{d\sigma}{d(\cos\theta)} \propto 1 + \alpha_1 \cos\theta + \alpha_2 \cos^2\theta , \quad (3.3)$$

where  $\theta$  is the polar angle of the lepton in the Collins-Soper frame. In general, the parameters  $\alpha_1$  and  $\alpha_2$  are functions of the transverse momentum of the boson [7].

### 3.4 $Z$ Production

At the LHC  $Z$ -bosons are produced as well. The properties of  $Z$  bosons are very well known from experiments at LEP. This opens the possibility to use them as calibration and reference for determination of  $W$  properties, because the production mechanisms are similar. This topic is discussed in section 6.1.

The  $Z$ -boson has not a clean V-A coupling, so it couples right handed components, too. Furthermore, there are interferences with the photon. The cross section of the Drell-Yan process  $q\bar{q} \rightarrow Z \rightarrow l^+l^-$  has the form [1]

$$\left( \frac{d\sigma}{d\Omega} \right) = \frac{\alpha^2}{4s} [A_0(1 + \cos^2\theta) + A_1 \cos\theta] .$$

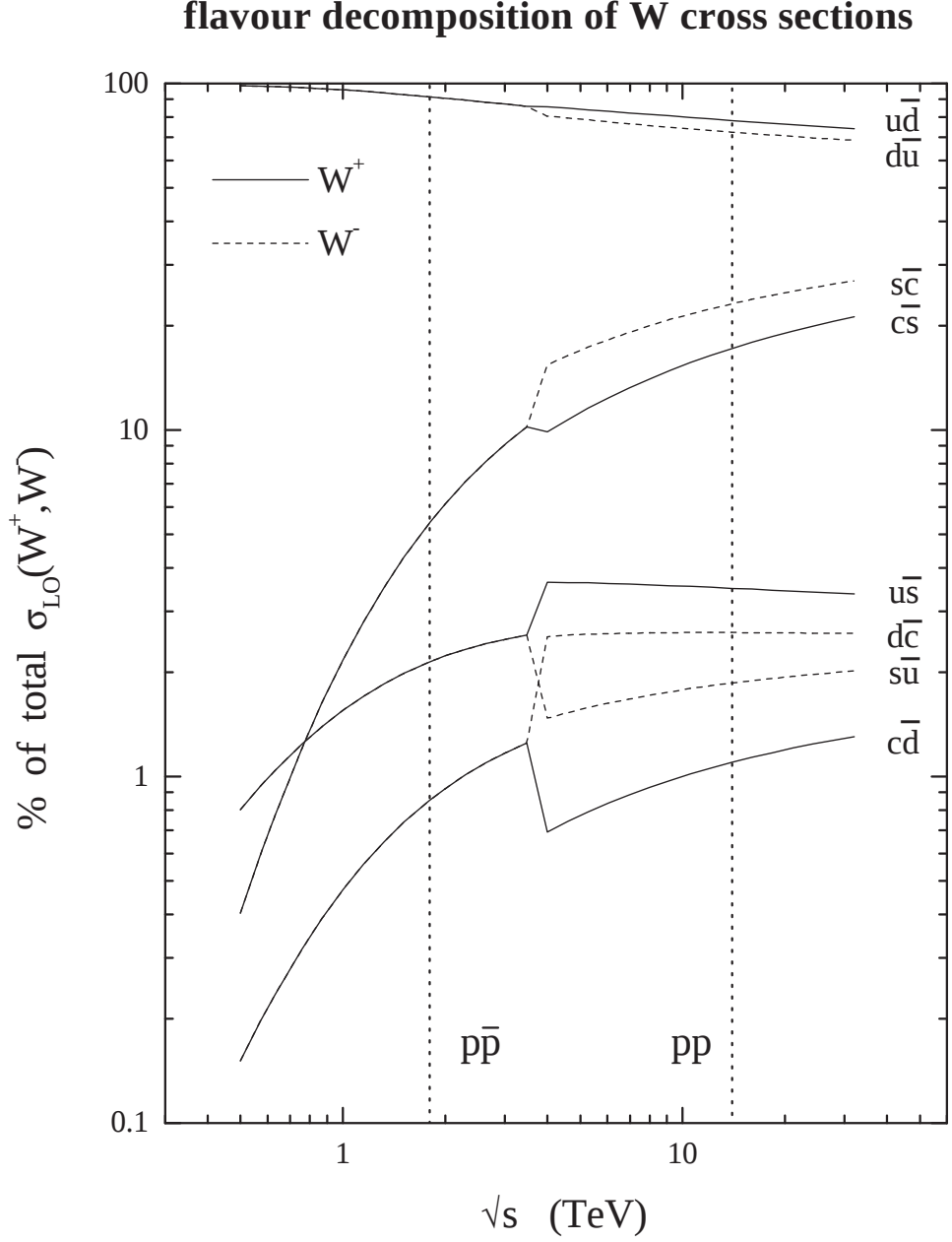


Figure 3.6: The parton decomposition of  $W^+$  and  $W^-$  bosons depending on the CM energy as percentage of the total cross section [6]. Compared is the Tevatron case at 1.96 TeV and LHC at 14 TeV. The splitting for  $W^+$  and  $W^-$  does not occur at Tevatron, because the anti-quark distribution in anti-protons equals the quark-distribution in protons. The process involving  $u$ - and  $d$ -quarks as discussed in section 3.2 contributes  $\sim 80\%$  at LHC energy.

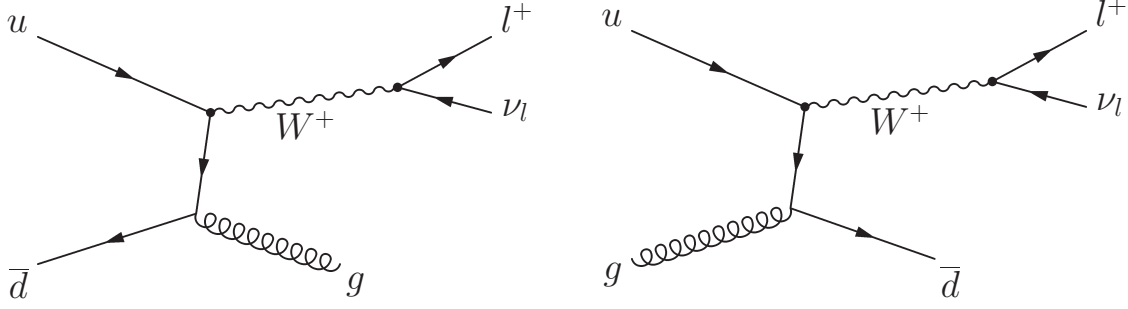


Figure 3.7: Example diagrams for  $W$ -boson production with a quark or gluon in the final state. The additional QCD contributions form the hadronic recoil  $\vec{u}$ .

The forward-backward asymmetry is denominated  $A_{FB} = \frac{3}{8} \frac{A_1}{A_0}$ . If  $A_0 = 1$  and  $A_1 = 2$ , the angular distribution would be the same as in the  $W$  case.  $A_{FB}$  depends on the initial state and on the energy. This cross section has to be folded with the parton distribution function, and since we can't distinguish between forward and backward, we have to add the same cross section with  $\theta$  exchanged by  $\pi - \theta$  so we get a symmetric angular distribution. We expect this distribution to prefer forward (backward) angles as in the  $W$  case, but not to the same degree.

The anti-quarks that participate in this Drell-Yan process are sea quarks and the quarks may be sea quarks or valence-quarks. The lowest order Feynman graph is similar to the  $W$  case in figure 3.3. The according decomposition is shown in figure 3.8.

In leading order calculations the value of  $\sigma(pp \rightarrow Z) \cdot BR(Z \rightarrow l^+ l^-)$  is 1.71 nb. In next to next to leading order QCD corrections this is increased to  $1.87 \pm 0.09$  nb. We see that this is about 1/10 of the  $W$  cross section.

### 3.5 Transverse Mass and the Jacobian Edge

In hadron colliders the z-components of initial quark momenta are a priori unknown. This is due to the composite structure of the proton, as discussed in section 3.2. To reconstruct a neutrino, the missing energy of the event has to be measured, but the proton remnants mostly go down the beam pipe and remain undetected. The longitudinal component of the neutrino can't be reconstructed. Therefore one has to rely on transverse quantities like  $E_T$  and  $P_T$ . A useful quantity, the *transverse mass*  $M_T$  [8] and the associated *Jacobian Edge* are introduced in the following. This argumentation partially follows the discussion in [9].

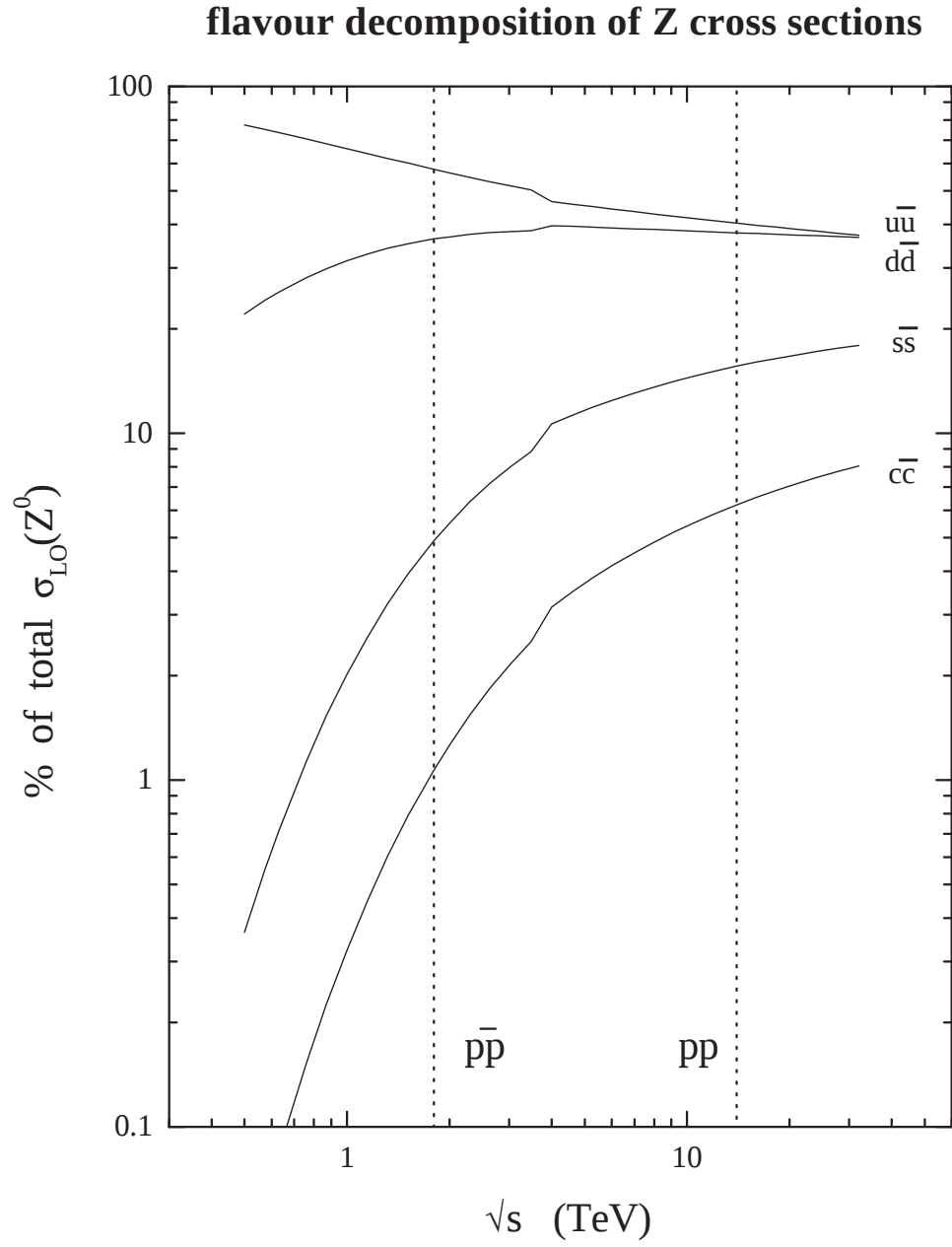
As an example the process  $W \rightarrow l \nu_l$  is considered:

As shown in section 3.2 the differential cross section for this process is

$$\frac{d\sigma}{d(\cos \theta)} = \sigma_0(\hat{s})(1 + \cos \theta)^2 ,$$

supposed the  $W$  is produced at rest and  $\sigma_0(Q^2)$  is a Breit-Wigner distribution. The center of mass energy  $\hat{s}$  is related to the transverse energy and scattering angle by

$$E_T = \frac{\sqrt{\hat{s}}}{2} \sin \theta ,$$

Figure 3.8: The decomposition plot for the  $Z$ -boson cross section [6].

because the muon takes 1/2 of the energy in a  $W$  2-body decay. The definition of the transverse energy is depicted in figure 3.9.

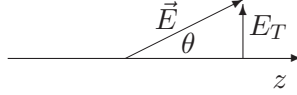


Figure 3.9: Definition of the transverse energy. The length of vector  $\vec{E}$  is the energy of the lepton. It points into the lepton direction.

The “in  $E_T$  differential” cross section is an interesting quantity because it is invariant under longitudinal boosts and leads to the transverse mass distribution as shown below. With  $dE_T = \sqrt{\hat{s}}/2 d(\sin \theta)$  it is

$$\frac{d\sigma}{dE_T} = \frac{2}{\sqrt{\hat{s}}} \frac{d\sigma}{d(\sin \theta)} = \frac{2}{\sqrt{\hat{s}}} \frac{d\sigma}{d(\cos \theta)} \frac{d(\cos \theta)}{d(\sin \theta)}$$

and with  $\frac{d(\cos \theta)}{d(\sin \theta)} = \frac{d(\cos \theta)}{d\theta} \frac{1}{\cos \theta} = \frac{\sin \theta}{\cos \theta}$

$$\frac{d\sigma}{dE_T} = \sigma_0(\hat{s}) \frac{2}{\sqrt{\hat{s}}} (1 + \cos \theta)^2 \frac{\sin \theta}{\cos \theta} .$$

Using  $\sin \theta = 2E_T/\sqrt{\hat{s}}$ :

$$\frac{d\sigma}{dE_T} = \sigma_0(\hat{s}) \frac{4E_T}{\hat{s}} \left( 2 - \frac{4E_T^2}{\hat{s}} + 2\sqrt{1 - 4E_T^2/\hat{s}} \right) \frac{1}{\sqrt{1 - 4E_T^2/\hat{s}}} . \quad (3.4)$$

$E_T$  and  $\hat{s}$  are invariant under longitudinal boosts and therefore equation (3.4) is independent of the longitudinal momentum of the  $W$ -boson.

This function has a singularity at  $E_T = \sqrt{\hat{s}}/2$ , which is called the *Jacobian Edge*. This means, for a given value of  $\hat{s}$  there is an infinitely sharp edge at  $\sqrt{\hat{s}}/2$ , which is also the maximum value. See Figure 3.10 for the plot.

In reality  $\hat{s}$  is distributed according to the parton distribution functions of the proton. This makes the edge finite. If  $\hat{s}$  is supposed to be distributed in a wide range, then the cross section will peak at the  $W$  mass, because of the Breit-Wigner part  $\sigma_0(\hat{s})$  in (3.4). This means that  $\frac{d\sigma}{dE_T}$  peaks at half the  $W$  mass, but the edge is smeared by the  $W$  width (Figure 3.10). The smeared edge can be used to determine the  $W$  width  $\Gamma_W$  [10].

In the lab frame the  $W$  is not produced at rest, it has a transverse motion  $P_T$  as explained in section 3.3. This results in a further reduction of the sharpness of the  $E_T$  distribution. The  $P_T$  distribution of the  $W$  was simulated with PYTHIA [13] (see section 6.3.1 and figure 6.1) and included in figure 3.10.

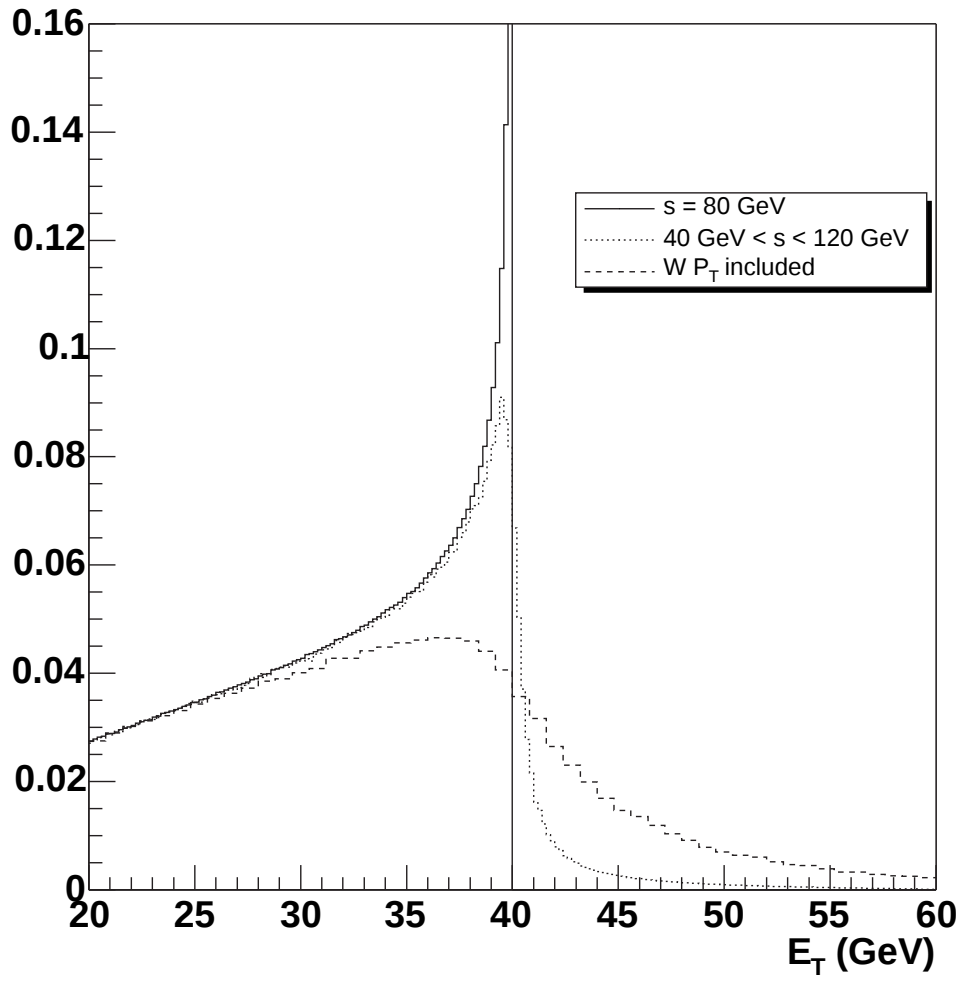


Figure 3.10: Theoretical distribution of the transverse energy  $E_T$  of the lepton for fixed CM energy  $\hat{s}$  (solid line) and for a wide  $\hat{s}$  distribution (dotted line). In the lab frame the  $W$  has a transverse momentum that reduces the sharpness of the edge (dashed line).

In analogy to the invariant mass  $M$  of two daughter particles

$$M = \sqrt{(E^l + E^\nu)^2 - (\vec{P}^l + \vec{P}^\nu)^2} ,$$

which does not depend on the momentum of the mother particle, the transverse mass  $M_T$  which is calculated from transverse quantities only is defined as:

$$M_T = \sqrt{(E_T^l + E_T^\nu)^2 - (\vec{P}_T^l + \vec{P}_T^\nu)^2} \approx \sqrt{(E_T^l + E_T^\nu)^2 - (\vec{E}_T^l + \vec{E}_T^\nu)^2} . \quad (3.5)$$

In the case where all longitudinal components are zero, the transverse mass  $M_T$  is identical to the invariant mass  $M$ . The advantage of using this quantity is that it is independent of the  $P_T$  of the boson in first order. This can be seen by using  $\vec{E}_T^\nu = \vec{P}_T^W - \vec{E}_T^l$  in (3.5) and by expanding (3.5) in first order in  $P_T^W/E_T^l$ . For a detailed calculation see appendix A. Then we get at low  $P_T^W/E_T$

$$M_T \approx 2E_T^{rest} .$$

This means that  $M_T$  depicts the Jacobian Edge of the transverse energy of the lepton in the boson's rest frame as in (3.4).

The measurement is performed in the leptonic channel using transverse components of the lepton energy  $E_T^l$  and of the neutrino energy  $E_T^\nu$ . The transverse mass is calculated according to eq. (3.5). It follows that

$$M_T = \sqrt{((E_T^l)^2 + 2E_T^l E_T^\nu + (E_T^\nu)^2) - ((E_T^l)^2 + 2\vec{E}_T^l \cdot \vec{E}_T^\nu + (E_T^\nu)^2)} = \sqrt{2E_T^l E_T^\nu (1 - \cos \Delta\phi)} , \quad (3.6)$$

where  $\Delta\phi$  is the azimuthal angle between lepton and neutrino. The neutrino  $E_T^\nu$  is obtained from the lepton  $E_T^l$  and the recoil  $\vec{u}$ , i.e. the energy of the system recoiling against the  $W$ :

$$E_T^\nu = -|\vec{E}_T^l + \vec{u}|$$

In case of a perfect measurement and if only one neutrino is present, the neutrino  $E_T^\nu$  is equal to missing energy  $E_T^{miss}$ .

The mass of the  $W$ -boson is then extracted from the shape of the  $M_T$  distribution by using a Monte Carlo model or by comparing to data from  $Z$  events. This procedure is described and performed for a simulated set of data in the subsequent sections.

## Chapter 4

# Software Environment

### 4.1 The CMS Software Framework

The CMS software is based on a standard set of tools and libraries [11]:

- Operating System: UNIX (Linux). The Linux operating system has a variety of advantages over many other systems: Security, reliability, flexibility and most important the principle of open source and the GPL license.
- Programming language: C++. This language replaces FORTRAN that was the standard programming language in high energy physics. C++ has some advantages considering object-orientation and performance.
- Script language: Perl, csh.

The CMS specific projects are:

- COBRA (Coherent Object-Oriented Base for Reconstruction, Analysis and Simulation): Implements the fundamental architecture of the CMS framework. Some of the subsystems are:
  - Mantis: GEANT4 interface to COBRA
  - DDD: Detector Description DataBase
  - Magnetic Field: Common interface and various implementations of the magnetic field map
  - GeneratorInterface: Provides access to Monte Carlo generator information of the events
  - CARF: CMS Analysis and Reconstruction Framework: This is the most fundamental part. It implements the two principles of the framework:
    - \* Event Driven Notification
    - \* Action on DemandCARF contains the abstract base for reconstruction algorithms and event objects.
- ORCA (Object Oriented Reconstruction for CMS Analysis): This is the collection of reconstruction software that enables the user to do real detector analysis. The design

of ORCA is based on CARF, the CMS Analysis and Reconstruction Framework, which was developed to prototype reconstruction methods. ORCA is intended to be used for final detector optimizations, trigger studies or global detector performance evaluation. Some of the ORCA subsystems are: Calorimetry, Tracker, Electron and Photon Reconstruction, Vertex Reconstruction, Jet Finders and the Muon Subsystem.

- SCRAM: (Software Configuration Release And Management): A configuration management and build tool to develop and test code in different physical locations while ensuring that a common configuration is available.

As mentioned above, the design, based on CARF, follows the principle of “Event Driven Notification”. This concept makes use of the “Observer” design pattern [12]. The observers are notified of the arrival of a new event and then take appropriate action, for example start the user analysis. The observers themselves are instantiated by static factories during application configuration. This means that the user analysis in ORCA consists of the implementation of an observer class `MyObserver` that is instantiated automatically by invoking

```
static PKBuilder<MyObserver> localBuilder("name");
```

The concept of “Action on Demand” is necessary for complex environments to achieve a reasonable amount of CPU usage. In the case of reconstruction this means that for instance detector hits are only reconstructed if a reconstruction algorithm asks for them. If another algorithm asks for the same data, the hits that are already reconstructed are used. This way each object is reconstructed only once and only if necessary.

## 4.2 The Detector Simulation Chain

The full detector simulation requires a large number of standalone tools. The most important ones are listed with a short description:

- PYTHIA version 6.215 [13] and CMKIN version 1.3.0 [14]: PYTHIA is the Lund Monte Carlo generator that does the theoretical physics calculations. It is interfaced by CMKIN which sets up CMS specific input parameters.
- CMSIM version 133 [14]: This is the full detector simulation based on GEANT3, a FORTRAN package that simulates passage of particles through matter. These packages are about to be replaced by OSCAR [15] and GEANT4 [16], both developed in C++.
- COBRA and ORCA versions 7.2.4 [17]: The CMS framework for detector reconstruction as described in section 4.1.

The starting point is the Monte Carlo generator that produces the HEPEVT Ntuples which contain the requested physics process. CMKIN is used to call the correct PYTHIA routines and to set up the LHC environment. One can specify numerous steering parameters, so called datacards which describe the physical process. This is the right place to set preselection cuts to improve the efficiency of the simulation.

In the next step, the HEPEVT Ntuples are read by the detector simulation CMSIM, based on GEANT3. CMSIM uses the detector geometry as another input and simulates the interaction of the generated particles with the detector. Depending on the event topology and

the number of particles, this step takes about 1 minute CPU time per event and is therefore the most CPU-intensive step of the simulation chain. The output of CMSIM is stored in “zebra” file format.

The simulated detector hits are then read with ORCA and stored in a database as so called “SimHits”. The last step is the digitization of the SimHits and storing of “RecHits”. In this step it is possible to mix pileup events into the database. The file format of the database is the ROOT streaming file format used in ORCA 7.2.4. The future file format is POOL which is much more flexible and has a lot of extended functionality.

The database can then be accessed and analyzed with ORCA as described in section 4.1.

### 4.3 PAX - Physics Analysis eXpert

PAX (Physics Analysis Expert) [18] is a new C++ toolkit, developed at University of Karlsruhe, by M. Erdmann *et al.* Its aim is to provide a new level of abstraction in particle physics analysis beyond detector reconstruction. For large teams it is necessary to become independent of the details of the underlying experiment specific software. Doing so, this enables physicists to share analysis code among different experiment environments.

Furthermore, the analysis code is protected from changes in the detector reconstruction layer, only the interface has to be changed in this case.

In the past, there have been similar projects like H1PHAN and ALPHA of the H1 and ALEPH experiments, which operated in a quite “clean”  $e^+e^-$  and  $ep$  environment. One of the future challenges is to deal with many simultaneous events ( $\sim 20$ ) and large event sizes. This results in a large number of possible interpretations of an event and implies a considerable amount of combinatorics. It is the purpose of PAX to manage this combinatorial task and reduce the amount of data to a region of interest.

Many interfaces to different HEP data sources, like monte carlo generator data (ntuples, ROOT trees) and detector reconstruction data (CMS, CDF) are already available. These interfaces can be used with identical analysis code, opening the possibility to perform quick cross comparisons.

To sum up: The main purpose of PAX is to assist in the physics analysis stage of a particle physics research project. Its proper place is between database and physics-plots. It is not a detector reconstruction tool nor a substitute for visualization or histogramming (PAW or ROOT). It rather should be considered as a supportive tool in the step from database to physics-plot. It has the potential to add a certain amount of unification and simplification to the physics analysis that very often is a sort of “black box”.

#### 4.3.1 PAX Class Structure

The basic unit in PAX is a class called *PaxEventInterpret*, the instances of which represent distinct interpretations of events. It contains physical objects, e.g. four-vectors (*PaxFourVector*), vertices (*PaxVertex*), collisions (*PaxCollision*) and the relations between them (see Figure 4.1).

In a *PaxEventInterpret* all information about the interpretation of the event is to be stored. To advance the analysis in different directions it is necessary to make copies of *PaxEventInterprets*. A copy is a physical copy in memory, so all objects and relations are duplicated and are independent of their origin. This also means that deleting a *PaxEventInterpret* deletes all its registered objects.

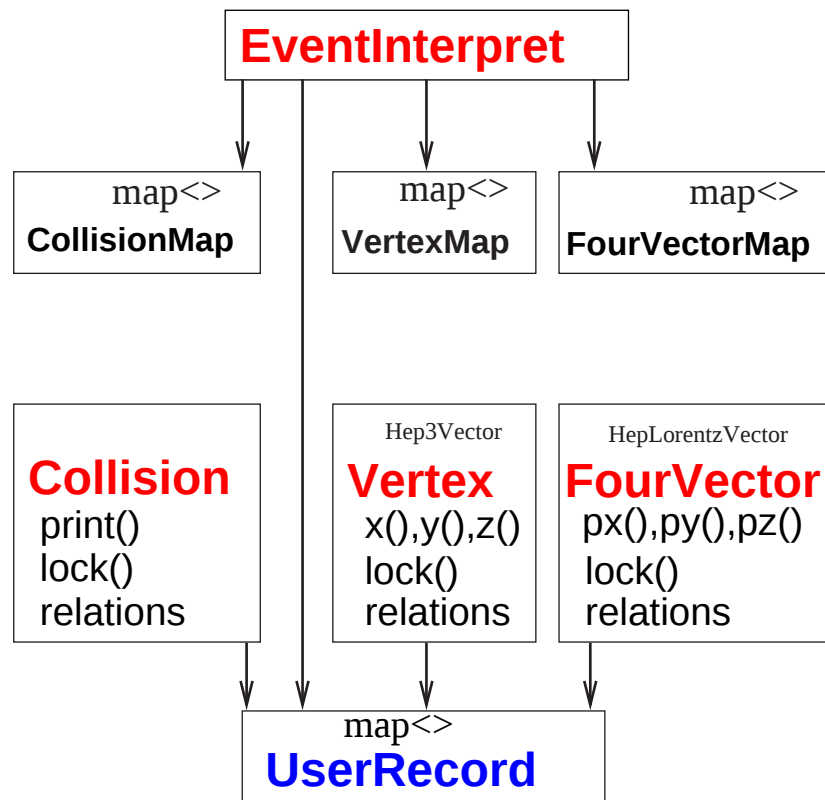


Figure 4.1: The *PaxEventInterpret* contains *PaxFourVectors*, *PaxVertices* and *PaxCollisions*. These are stored in STL maps and their relations are managed by the *PaxRelationManager*.

The *PaxFourVector* inherits from *HepLorentzVector* of CLHEP to allow vector operations, provided by CLHEP. In the latest version of PAX it is also possible to use the *TLorentzVector* of ROOT as base class. This is possible because these vector classes implement similar functionality. The *PaxFourVector* has important additional properties and methods, for example relations to begin- and end-vertices, methods to get mother and daughter particles, name, charge and so on.

The relations between the objects are managed by the *PaxRelationManager* (see Figure 4.2). This mechanism is based on the “Mediator” design pattern [12]. It takes care of estab-

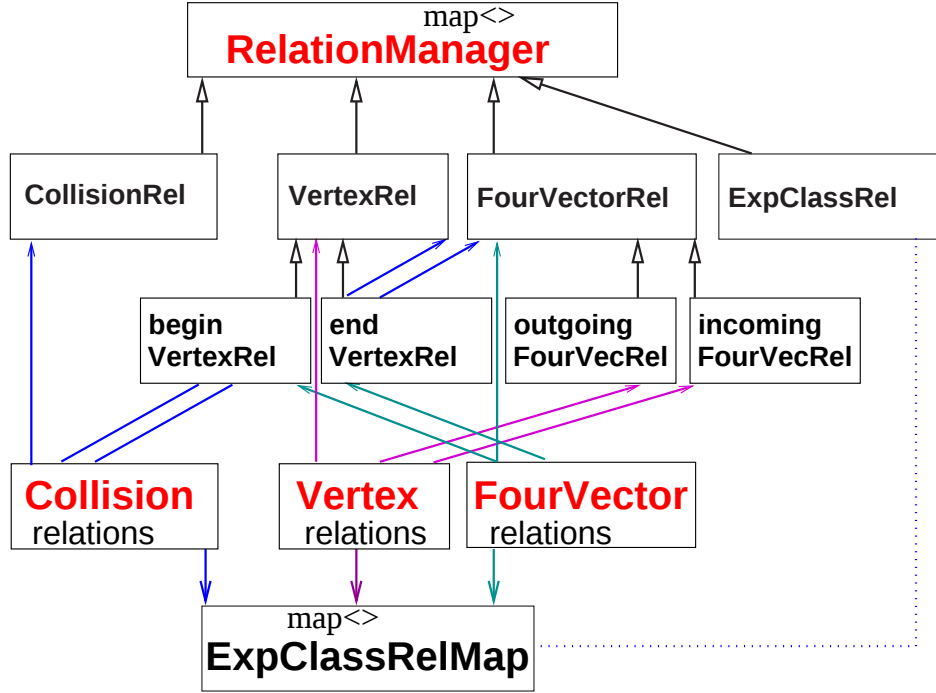


Figure 4.2: The *PaxRelationManager* establishes decay trees, records histories and locks parts of an interpretation.

lishing the right connections, e.g., it will register a begin vertex relation to a *PaxFourVector*, if the according vertex gets the vector as an outgoing four-vector relation. In case of copying an object, the relation manager stores a pointer to the previous instance in the relations of the corresponding object. That way a complete decay tree and history of the analysis is recorded. It is also possible to exclude parts of an event from the analysis by using the lock mechanism. If one object is locked, the complete decay tree of that object is locked too. This is useful, for example, to exclude a lepton from a jet finding algorithm.

The class design and documentation as well as further information and examples for the usage of PAX are available on a web page [19].

#### 4.3.2 Interface to CMS

The CMS reconstruction framework (ORCA) [17] is interfaced through a class called *PaxCMSFill*. This class is permanently under heavy construction because the CMS software

endures frequent changes. The basic reconstructed objects like muons or jets can easily be accessed by simple method calls, e.g.

```
PaxEventInterpret ei;
PaxCMSFill filler;
filler.fillMuonsViaL3(&ei);
```

fills all level 3 trigger muons as *PaxFourVectors* in a *PaxEventInterpret*. Of course, only the most basic objects can be treated like this. In a real analysis the user will soon encounter needs for his/her own interface methods, e.g. for using self defined jet finding algorithms. A *PaxCMSUserFill* class, inheriting from *PaxCMSFill* would be an appropriate solution. *PaxCMSFill* allows a quick start for the impatient user. Ideally, some of these user defined methods will be useful also for other people and then become part of *PaxCMSFill* in later release versions.

PAX has the ability to store experiment specific objects through a template class *PaxExperiment*. This way all the functionality of the experiment objects is available and can be associated with the according PAX objects. An example how this feature could be used within CMS is given in appendix B.

In the case described above, when PAX is used to interface CMS objects, the necessary PAX source code has to be embedded in the ORCA environment in the same way as every ORCA analysis has to be. This means the user has to implement an *Observer* that is registered to ORCA and gets updated for each event. This *Observer* then does the analysis and fills the requested PAX objects, which can be stored to hard disk using the PAX persistency scheme.

### 4.3.3 PAX Persistency

One of the main advantages of PAX is the possibility to store *PaxEventInterprets* on hard disk. These objects can then be analyzed with a stand alone PAX analysis. The net profit is an enormous gain of speed, because all the overhead from ORCA vanishes. Writing and reading PAX files is very fast, since the well approved ROOT I/O format is used.

With the PAX persistency scheme it is possible to store complete decay trees and analysis histories including all the relevant objects like four-vectors, vertices, collisions and the relations between them. This enables the user to focus on regions of interest, leaving out all the redundant data and therefore speeding up the analysis.

The usage of the I/O is as simple as one can imagine. To write a *PaxEventInterpret* to disk only these few lines of code are necessary:

```
PaxFile file("filename",PaxFile::Write);
PaxEventInterpret ei1 ,ei2;

ei1.store(&file);
ei2.store(&file);

file.writeEvent();
```

The `writeEvent()` method inserts a dividing rule in the *PaxFile* allowing to store an arbitrary number of sequential events.

Reading a *PaxFile* is just as simple:

```
PaxFile file("filename",PaxFile::Read);
file.readEvent();

PaxEventInterpret ei1(&file), ei2(&file);
```

#### 4.3.4 Reconstruction of an Event in PAX

After the reconstruction of the detector objects like tracks or calorimeter towers, the different possibilities of the interpretation of an event have to be taken into account. Complex events may have hundreds of different interpretation alternatives.

A class called *PaxEventClass* implements the basic functionality of the reconstruction of an event. It is a generic class that manages the combinatorics in a way that every possible and reasonable combination of objects is stored in a *PaxEventInterpret*. The different interpretations are then stored in a **map** of the Standard Template Library (STL). In fact, the *PaxEventClass* is a **map**, i.e. it inherits from the STL **map**.

An event interpretation quality is assigned to each *PaxEventInterpret*. This quality strongly depends on the demands of the user and the physics process that is analyzed. It is up to the user to implement his/her own event classes that are derived from *PaxEventClass*.

#### 4.3.5 An Existing Analysis Transferred to PAX

An existing physics analysis, which was accomplished in the CMS group Karlsruhe [20] [21], has been redone in PAX. The results of the study of the “golden channel”, a Higgs boson, produced at LHC, decaying into a 4 muon final state, are then compared to the PAX study. In the following discussion a Higgs boson produced at 130 GeV is assumed.

PAX is a very good example for a clean object oriented software project. Therefore, also this physics analysis was designed in an object oriented manner, so it would be possible to apply the analysis code to different sources of data.

The heart of the analysis are the event classes for  $ZZ^*$  and  $H$  which inherit much of their functionality from *PaxEventClass*. The quality is determined by fitting the reconstructed invariant mass of 2 muons to the  $Z$  and  $Z^*$  mass distribution, respectively. Then the best event interpretation is picked out and assumed to be the signal event interpretation.

The signal process,  $H \rightarrow ZZ^* \rightarrow 4\mu$  (Figure 4.3), has a very clean signature with four high energy muons.

The background processes considered are:

- $t\bar{t}$  background: The top quark decays into a  $W$ -boson and a  $b$  quark. The  $b$  hadronizes and in the jet a muon occurs with a probability of 10%.
- $Zb\bar{b}$  background: The  $Z$ -boson decays into two muons and the  $b$ -quarks hadronize in the same way as stated above.
- $ZZ^*$  background: This background is very similar to the signal. Fortunately it is strongly suppressed for Higgs masses lower than twice the  $Z$  mass, because in this case one of the signal  $Z$ -bosons is produced “off shell”.

To get rid of the background, some kinematical cuts and isolation cuts are applied:

- $Z$  mass cut: The reconstructed  $Z$  mass is accepted in a range of 80 - 96.7 GeV. The  $Z^*$  mass in 18 - 60 GeV.

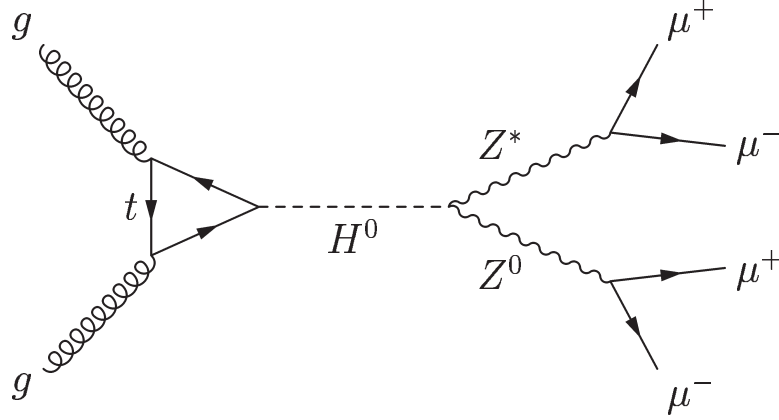


Figure 4.3: The Higgs boson is produced via gluon fusion and decays into two  $Z$ -bosons, which decay into four muons.

- $P_T$  cut: The transverse momenta  $P_T$  of the muons are sorted and accepted above

|        |          |
|--------|----------|
| 15 GeV | 1st muon |
| 12 GeV | 2nd muon |
| 12 GeV | 3rd muon |
| 8 GeV  | 4th muon |

- Isolation cut: The deposited transverse energy and momentum in a cone around the muon track is calculated and a threshold for the tracker and calorimeter is selected. The cone size is  $\Delta R = \sqrt{\Delta\eta^2 + \Delta\phi^2} = 0.45$  for the tracker and  $\Delta R = 0.24$  for the calorimeter. The  $P_T$  and  $E_T$  thresholds for tracker and calorimeter are 4 GeV and 11.5 GeV, respectively.

These cuts are derived and optimized with respect to the kinematics of the background processes [20] [21].

Table 4.1 shows the results of this cut analysis and compares them to the original results. The remaining differences could be tracked down to slightly different analysis algorithms,

| Cut             | signal |       | rejection power on |       |            |       |        |       |
|-----------------|--------|-------|--------------------|-------|------------|-------|--------|-------|
|                 | PAX    | orig. | $Zb\bar{b}$        |       | $t\bar{t}$ |       | $ZZ^*$ |       |
|                 | PAX    | orig. | PAX                | orig. | PAX        | orig. | PAX    | orig. |
| $ZZ^*$ mass cut | 32%    | 32%   | 81%                | 81%   | 89%        | 89%   | 52%    | 53%   |
| $P_T$ cut       | 28%    | 28%   | 81%                | 83%   | 53%        | 55%   | 41%    | 41%   |
| isolation cut   | 22%    | 23%   | 96%                | 98%   | 99%        | 99%   | 21%    | 20%   |

Table 4.1: Results of the PAX cut analysis compared to the results of [20] [21].

especially for events with more than four muons. These events occur very rarely and have been neglected in [20] [21]. However, PAX is naturally capable of dealing with these events and managing the additional combinatorics. The additional muons originate from pileup and

fake muons and could erroneously be identified as signal muons. Another source for differences is the number of total events considered.

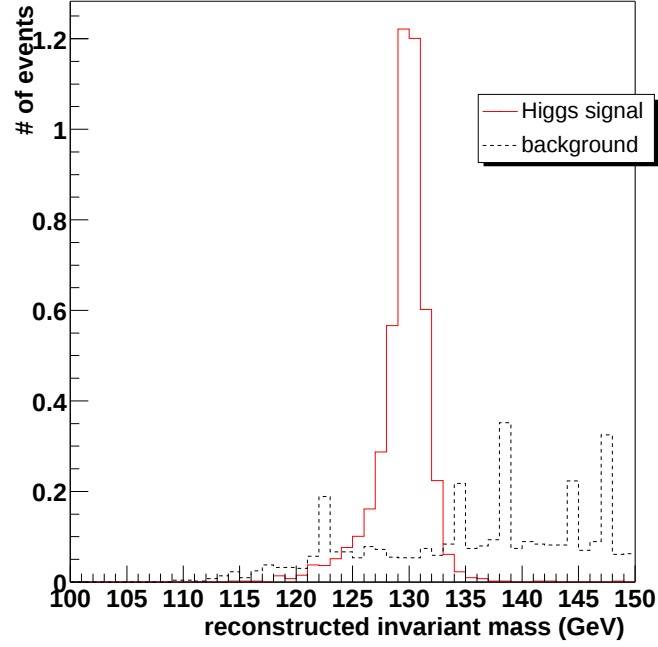


Figure 4.4: Final result of the analysis with Higgs mass of 130 GeV and an integrated luminosity of  $20 \text{ fb}^{-1}$ . The signal (solid line) can be identified clearly. The result is almost identical to the original study [20] [21].



## Chapter 5

# Detector Simulation and Resolution of Observables

### 5.1 Simulation of the $W$ Signal without Pileup

In this section a sample of 100,000  $W \rightarrow \mu\nu$  events is simulated and compared to the generator data. The simulation procedure is described in section 4.2. Pileup and minimum bias are not included. The preselection cuts are described in section 6.2, that is  $P_T > 25$  GeV and  $|\eta| < 2.3$  for the muon.

The resolution is defined as

$$\Delta X = X_{reconstructed} - X_{generated}$$

for a quantity  $X$ . For the muons, the level 3 trigger data is used. The reconstruction efficiency for the muons from  $W$  decay is 97.7% for  $|\eta| < 2.3$ .

#### 5.1.1 Resolution of Muon Properties

The reconstructed distribution of the transverse momentum  $P_T$  of the muons is shown in figure 5.1. The resolution of muon  $P_T$  and  $1/P_T$  is summarized in figures 5.2 and 5.3. The

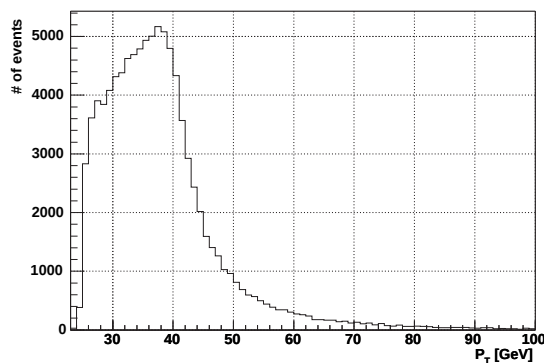


Figure 5.1: Reconstructed transverse momentum distribution of muons from  $W$  decay. The total number of events is 97,065.

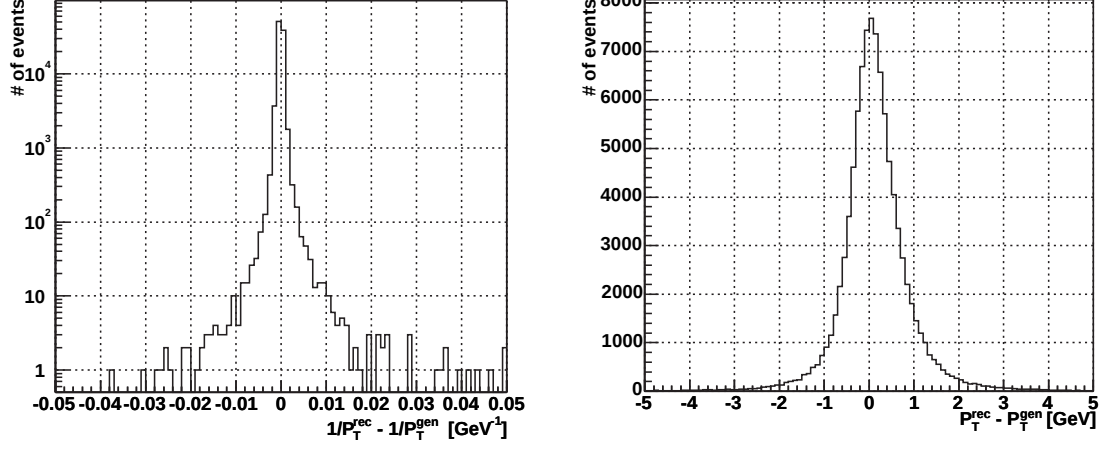


Figure 5.2: Resolution of the transverse muon momentum (right plot). The mean value is 0.1271 GeV and the RMS is 0.7926 GeV. The resolution of the inverse transverse momentum is shown on the left plot.

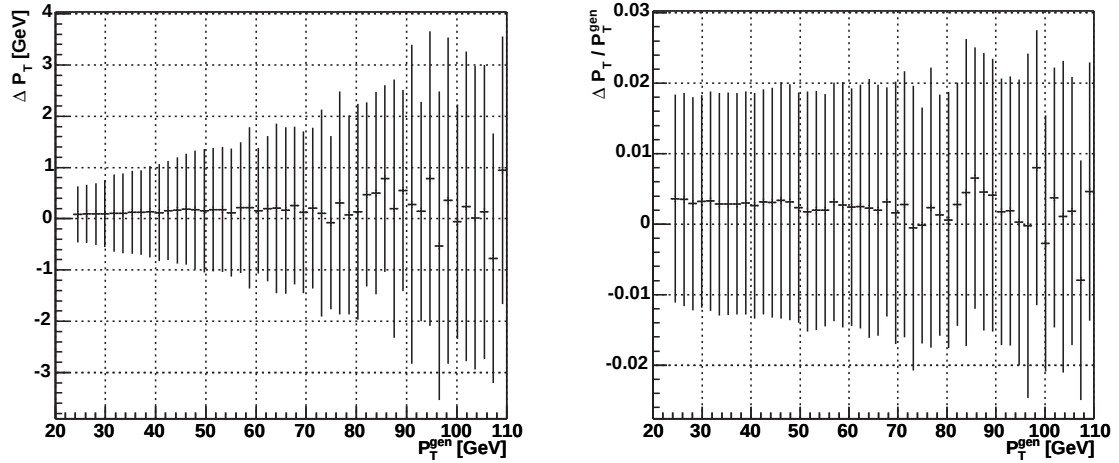


Figure 5.3: Dependence of the resolution on the generated  $P_T$ . The error bars show the RMS of  $\Delta P_T$  in each  $P_T$  bin.

distribution of the absolute resolution  $\Delta P_T$  has a mean value of 0.1271 GeV and a RMS of 0.7926 GeV. Figure 5.3 shows the dependence on generated  $P_T$ . The RMS values of  $\Delta P_T/P_T^{gen}$  correspond to the design-values stated in the Tracker Technical Design Report (TDR) [22]. In the TDR these values range between 0.007 and 0.07, depending on  $\eta$ . The resolution of the momentum  $\Delta P_T$  has an approximately linear dependence on  $P_T$  while the relative resolution  $\Delta P_T/P_T$  has a constant RMS.

In section 6.3.6, the resolution of the muon's momentum is used to perturb the Monte Carlo values. To do this, the resolution of each component of the momentum is needed (figure 5.4). The resolution is almost the same for each component.

The resolution of the azimuthal angle  $\phi$  is shown in figure 5.5. The RMS of  $\Delta\phi$  is  $2.310^{-4}$  in global context. However, the  $\Delta\phi$  resolution also depends on the transverse momentum  $P_T$

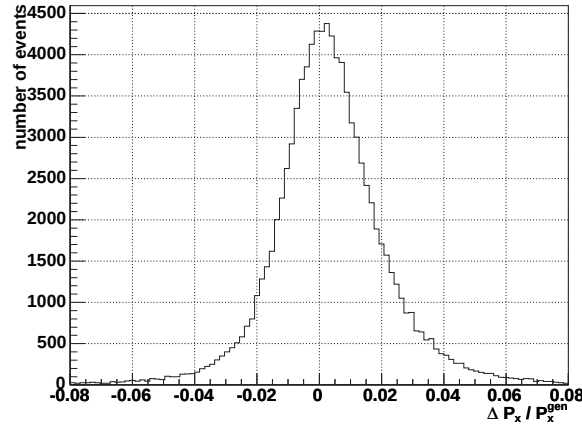


Figure 5.4: Resolution of the x-component of the muon's momentum.

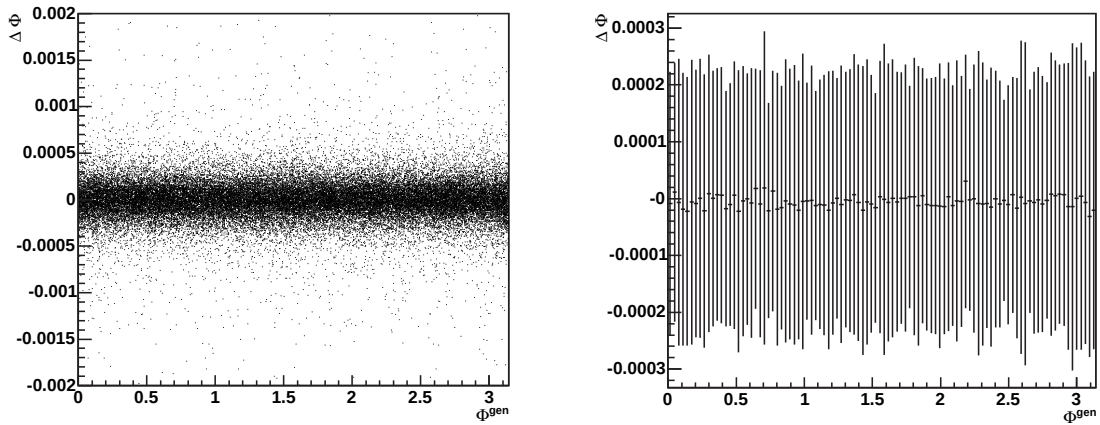


Figure 5.5: Dependence of the resolution of the azimuthal angle on generated  $\phi$ . The error bars show the RMS of  $\Delta\phi$  in each  $\phi$  bin.

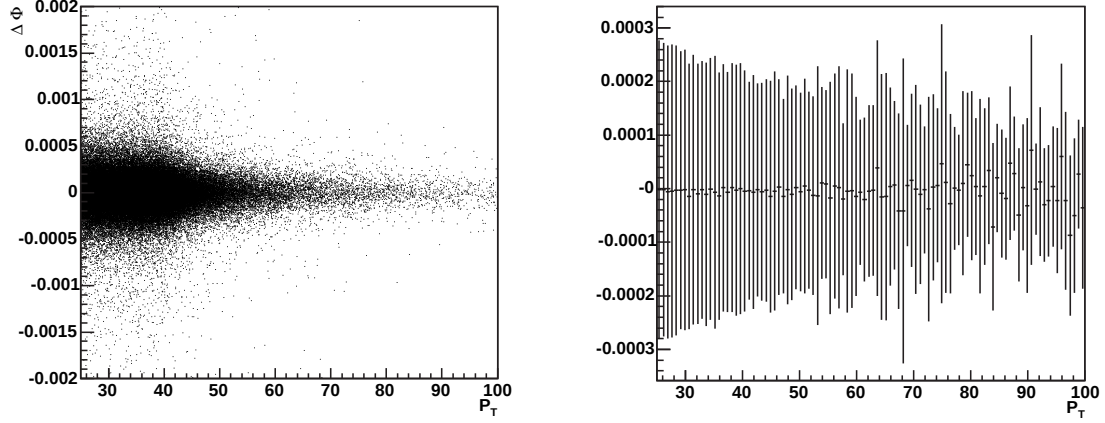


Figure 5.6: Dependence of the resolution of the azimuthal angle on generated  $P_T$ . The error bars show the RMS of  $\Delta\phi$  in each  $P_T$  bin.

of the particle, as shown in figure 5.6. Again, the values of this resolution are in agreement with the values of the TDR.

### 5.1.2 Resolution of the Hadronic Recoil

With the objective of the measurement of missing energy, the hadronic recoil against the boson is measured as a first step. As explained above, the recoil results primarily from higher order effects which are responsible for the transverse momentum of the boson. This part of the measurement is very cumbersome because the whole detector information has to be taken into account. In events with multiple collisions and pileup this could easily lead to severe mismeasurements. In this study, minimum bias events or pileup are not included.

Figure 5.7 compares the reconstructed recoil to the generated recoil. The generated recoil is approximately equal to the generated transverse momentum of the  $W$ -boson (see section 6.3.1). As expected, the amount of reconstructed energy tends to be smaller than the true value. However, this applies only to large energy values as displayed in figure 5.8. Here the relative resolution of the recoil  $\Delta Recoil/Recoil$  is shown in different energy segments. The energy range 0 GeV - 10 GeV shows some events with larger reconstructed than generated values. This effect may result from noise or low energy particles that are not part of the signal process, but it still has to be investigated in more detail. This behavior has been found earlier in [23].

Obviously, the resolution of the recoil improves at large values, but the conclusion, to analyze events with large recoil only, would be wrong. The interesting quantity is the transverse mass and this does not depend on the relative resolution of the recoil, it rather depends on the absolute value of the resolution, which is plotted in figure 5.9. The RMS of the resolution plots and its uncertainty is shown in figure 5.10.

At high recoil values we recognize a small peak at the lower edge of the resolution histograms. This peak corresponds to a total loss of reconstructed energy. This may occur if the energy disappears in dead detector areas or goes down the beam pipe. In fact we found a

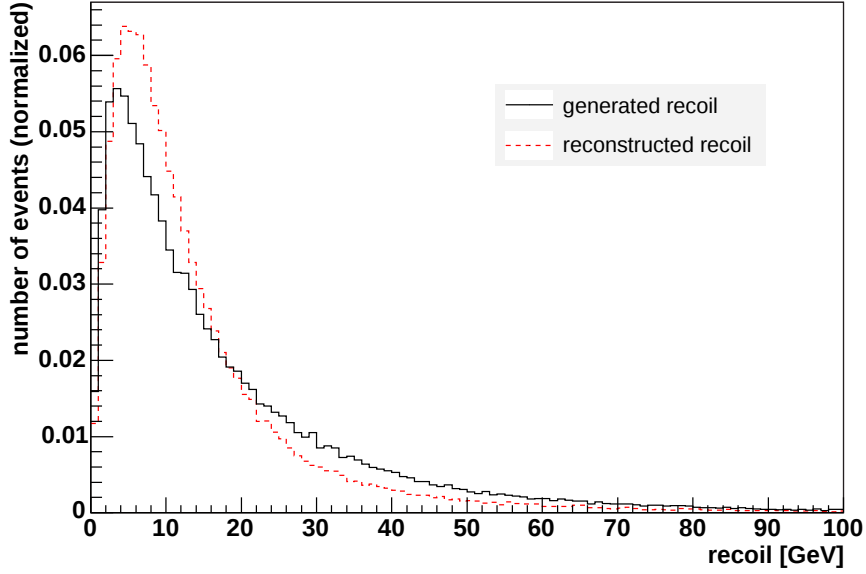


Figure 5.7: The hadronic recoil compared to generator data.

preferred value for the pseudo-rapidity of the generated recoil of these events at  $\eta \approx 3$ . This is the maximum  $\eta$  coverage of the hadronic calorimeter, for  $|\eta| > 3$  the forward calorimeter, that is placed further down the beam pipe, detects the energy. This means that we have a discontinuity that may be responsible for this effect. It is not yet possible to make a clear statement about this observation since the event numbers in the peak ( $\sim 300$ ) are too small to find a correlation to  $\eta$  with a high significance.

### 5.1.3 Resolution of the Missing Transverse Energy

A very important part of the mass measurement is the correct determination of missing transverse energy  $E_T^{miss}$ . It is defined in section 3.5 as

$$E_T^{miss} = -|\vec{E}_T^l + \vec{u}| ,$$

where  $\vec{u}$  is the hadronic recoil and  $\vec{E}_T^l$  the transverse momentum of the lepton.

Since this analysis investigates muonic decays and since the resolution of muon properties (section 5.1.1) is orders of magnitudes better than that of recoil properties, the absolute value of the resolution of  $E_T^{miss}$  depends almost exclusively on the recoil. Figure 5.11 shows the resolutions of  $E_T^{miss}$  in the recoil interval of 0 to 20 GeV. We see that the absolute resolution  $\Delta E_T^{miss}$  is approximately the same as that of the recoil in the corresponding plots in figure 5.9. The relative value  $\Delta E_T^{miss} / E_T^{miss}$  is better since  $E_T^{miss}$  represents neutrino energies which are quite large compared to the recoil that is  $< 20$  GeV.

### 5.1.4 Resolution of the Transverse Mass

After determination of  $E_T^{miss}$  it is possible to calculate the transverse mass, if  $E_T^{miss}$  is identified with the transverse energy of the neutrino  $E_T^\nu$ . This quantity is derived in section 3.5

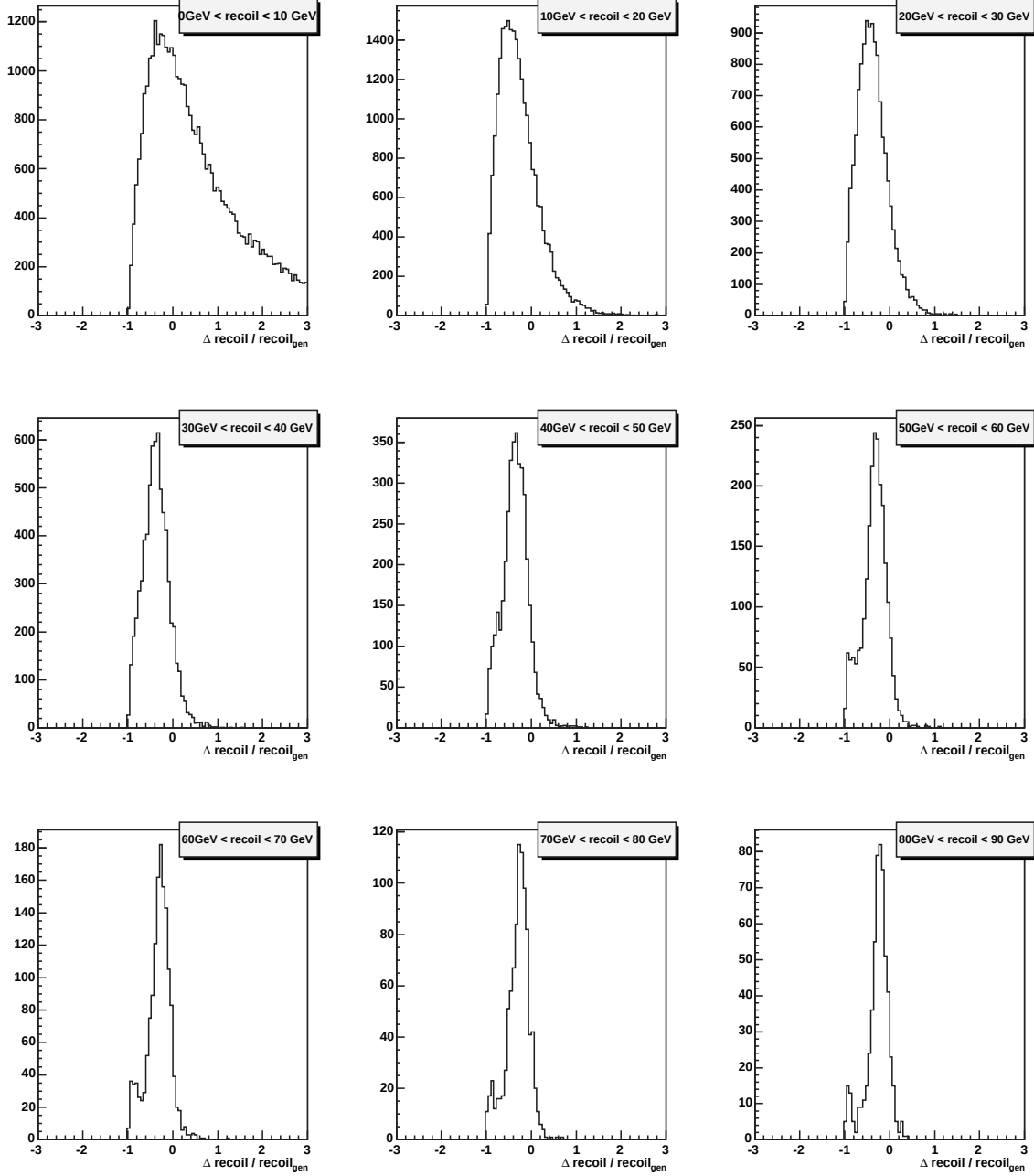
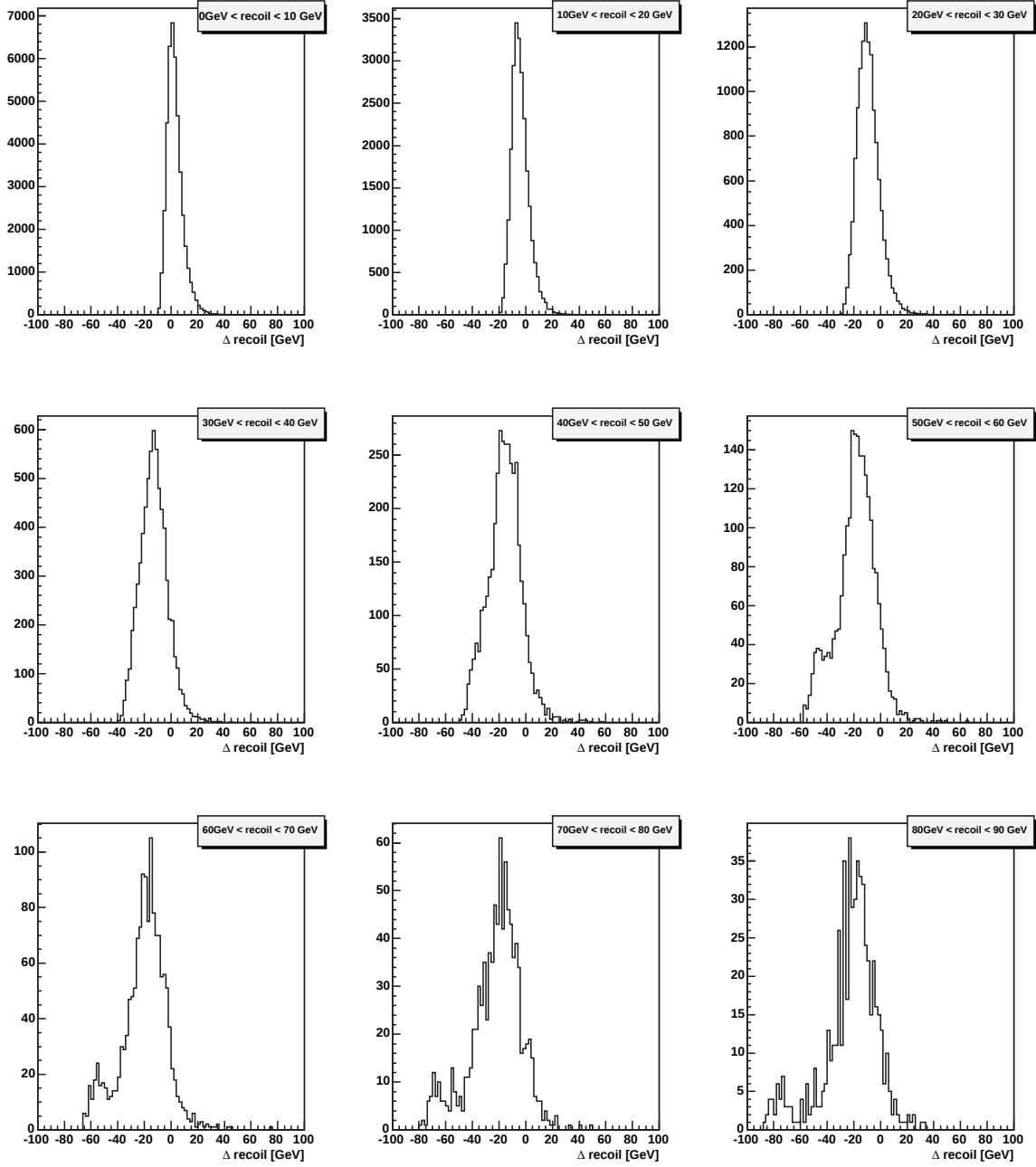


Figure 5.8: Relative resolution of the hadronic recoil  $\Delta \text{Recoil} / \text{Recoil}$  in different energy segments. The top left plot shows an energy range of 0 GeV - 10 GeV and the bottom right 90 GeV - 100 GeV. The y-axis shows the number of events.

Figure 5.9: The absolute value of the resolution  $\text{Recoil}_{\text{rec}} - \text{Recoil}_{\text{gen}}$ .

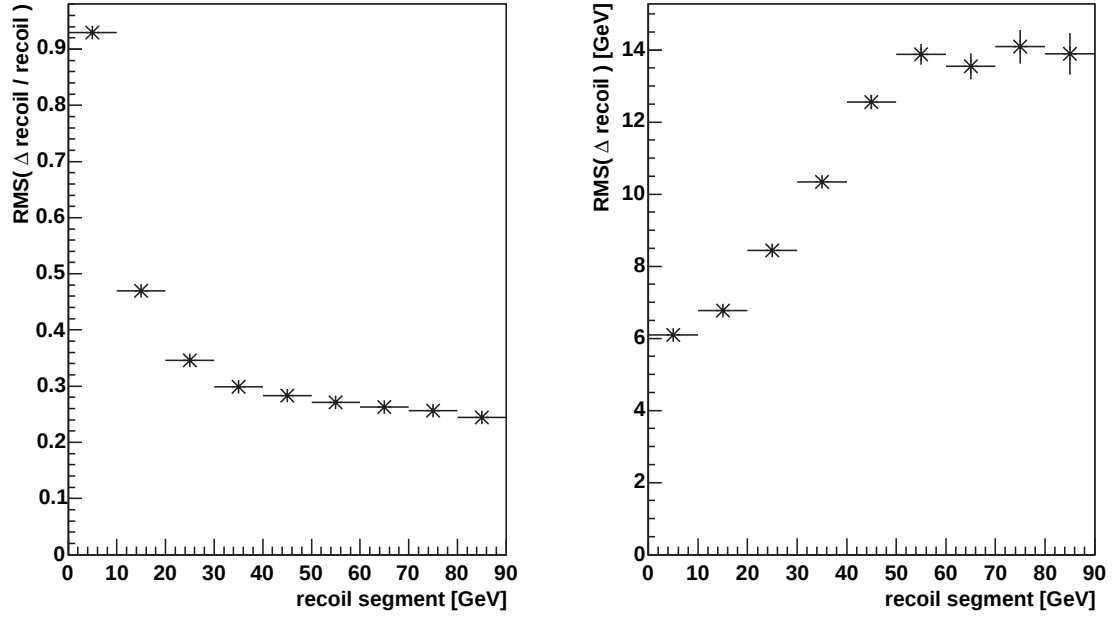


Figure 5.10: The left plot shows the RMS of the relative resolution in the according energy segments. The right plot shows the absolute values with error bars of length  $\text{value}/\sqrt{N}$ , where  $N$  is the number of events that contribute to the value.

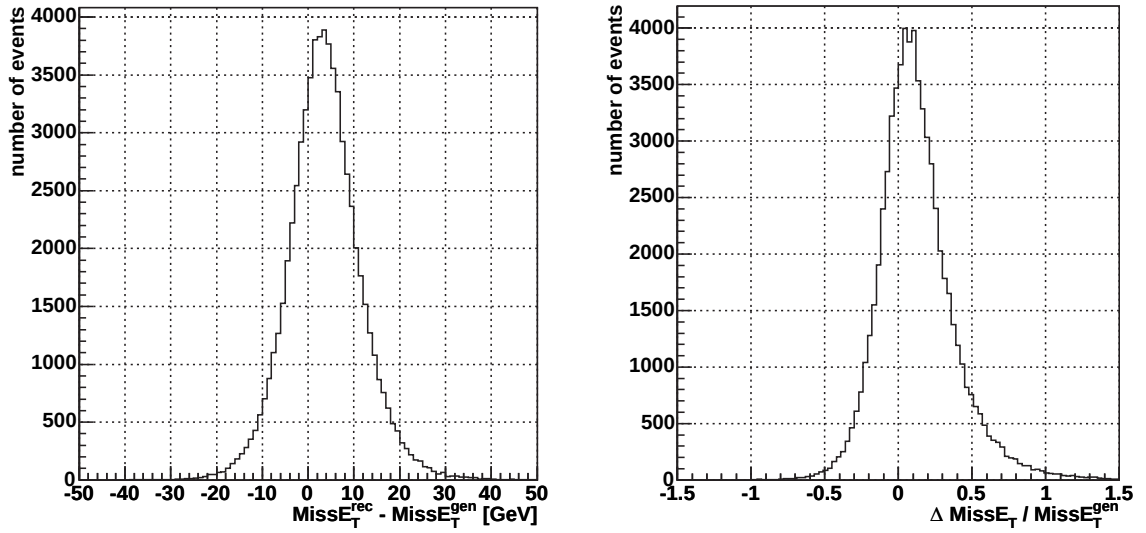


Figure 5.11: The left plot shows the absolute value of the resolution  $\Delta E_T^{\text{miss}}$  and the right plot shows  $\Delta E_T^{\text{miss}} / E_T^{\text{miss generated}}$ .

as

$$M_T = \sqrt{2E_T^l E_T^\nu (1 - \cos \Delta\phi)},$$

where  $\Delta\phi$  is the azimuthal angle between the charged lepton and neutrino.

The resolution of  $M_T$ , divided into recoil segments as before, is shown in figure 5.12. According to these plots it would be beneficial to choose events with low hadronic recoil for the mass determination. The effects of a low cut on the recoil is plotted in figure 5.13. The main effect of a cut on the recoil is a suppression of backgrounds (section 6.4).

Ideally, the boson would be produced with no transverse momentum and therefore no recoil would occur. In this case the missing energy would be exactly  $-E_T^{lepton}$ . The charged lepton properties can be measured with high precision and the resolution of the  $M_T$  measurement would only depend on the charged lepton resolution.

One has to live with the fact of higher order QCD effects which deteriorate the mass measurement. Fortunately, the majority of the events have low boson  $P_T$  and low hadronic recoil (see figure 5.7). Therefore, a cut on the recoil of  $\vec{u} < 20$  GeV, as mentioned in section 6.2, does not have great impact on the statistics of the remaining events.

## 5.2 Simulation of the Z Signal without Pileup

The Z-boson data is used to derive transverse mass distributions of the W-boson, as stated in section 6.1. It is crucial to study the measurement of the relevant Z distributions to get an estimation for the precision.

The resolution of muon properties is almost identical to the muons from W decays, because the momentum scale and the angular distributions are similar. A difference is found in the reconstruction efficiency, because reconstructing two muons is harder than reconstructing one muon. Of 100,000 simulated events, only 93,042 events are found to have two reconstructed muons.

### 5.2.1 Resolution of the Z mass

The Z-boson mass is reconstructed by calculating the invariant mass of two muons. It is expected to follow a Breit-Wigner distribution. Since the center of mass energy  $\sqrt{s}$  depends on the parton distribution functions (pdf) of the proton, the Breit-Wigner is folded with the pdfs. If a uniform distribution of  $\sqrt{s}$  is assumed, we expect a plain Breit-Wigner. Figure 5.14 compares the PYTHIA output with a Breit-Wigner distribution of mass 91.188 GeV and width 2.47813 GeV. This Z-boson mass corresponds to a Breit-Wigner resonance parameter including an s-dependent width and lies approximately 34 MeV above the real part of the position of the pole in the Z-boson propagator [24]. We therefore use  $M_Z = 91.154$  GeV in the Breit-Wigner of figure 5.14.

The Monte Carlo distribution follows the pure Breit-Wigner curve almost exactly, except for the tails. In the high energy part of the tail, the Monte Carlo distribution runs below the Breit-Wigner curve. The effect of final state radiation (FSR) distorts the symmetry and reduces the mean value.

The plot of the reconstructed Z mass shows the “smearing” of the detector. The resolution of the mass is shown in figure 5.15.

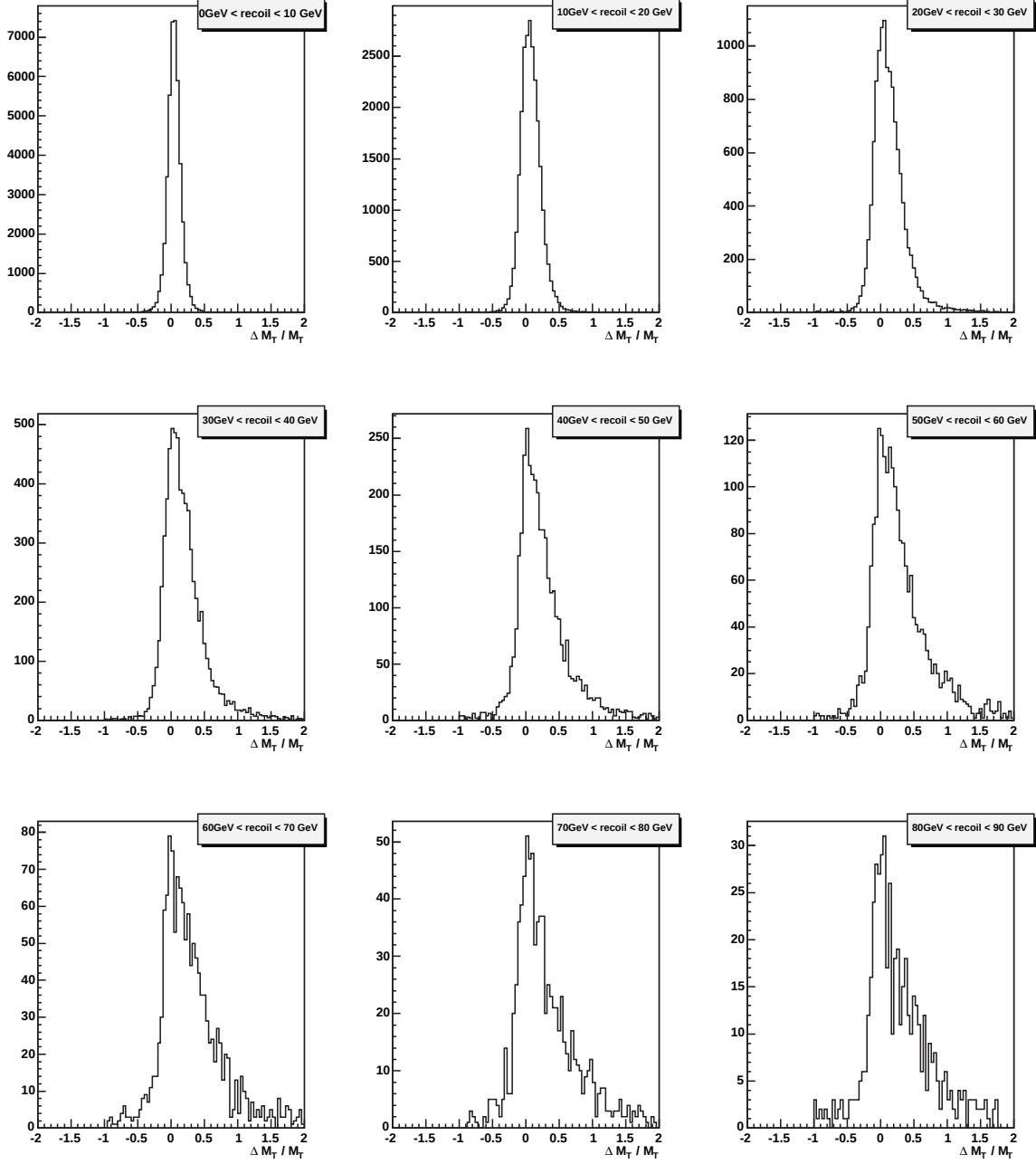


Figure 5.12: Resolution  $\Delta M_T / M_T$  of the transverse mass in different ranges of the recoil energy.

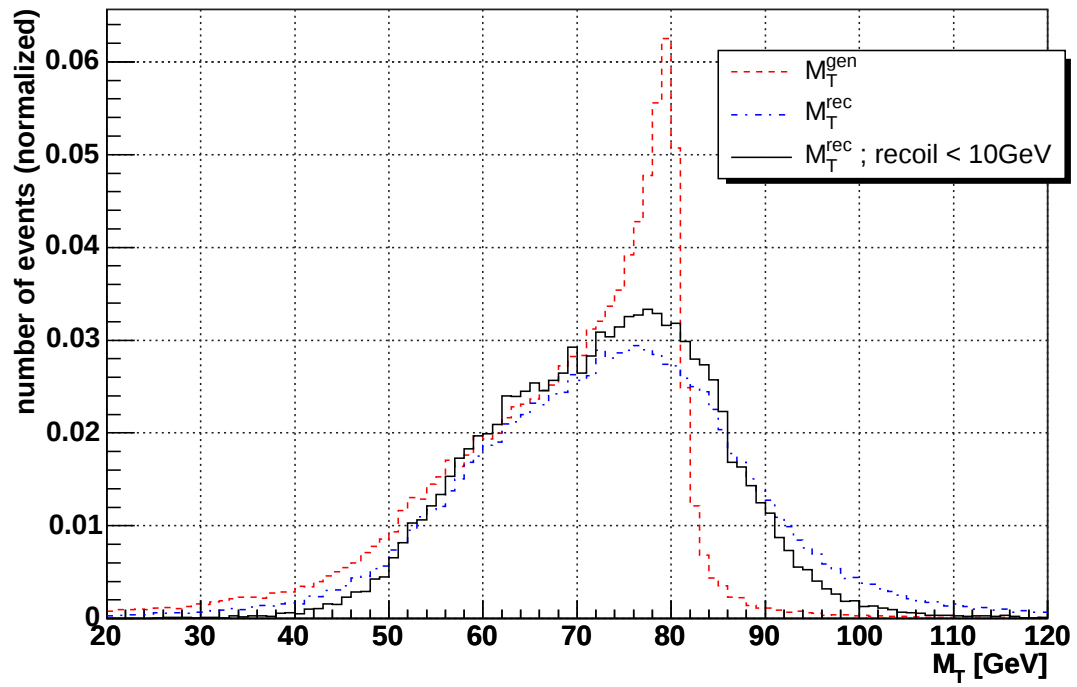


Figure 5.13: Transverse mass distribution for reconstructed and generated events.

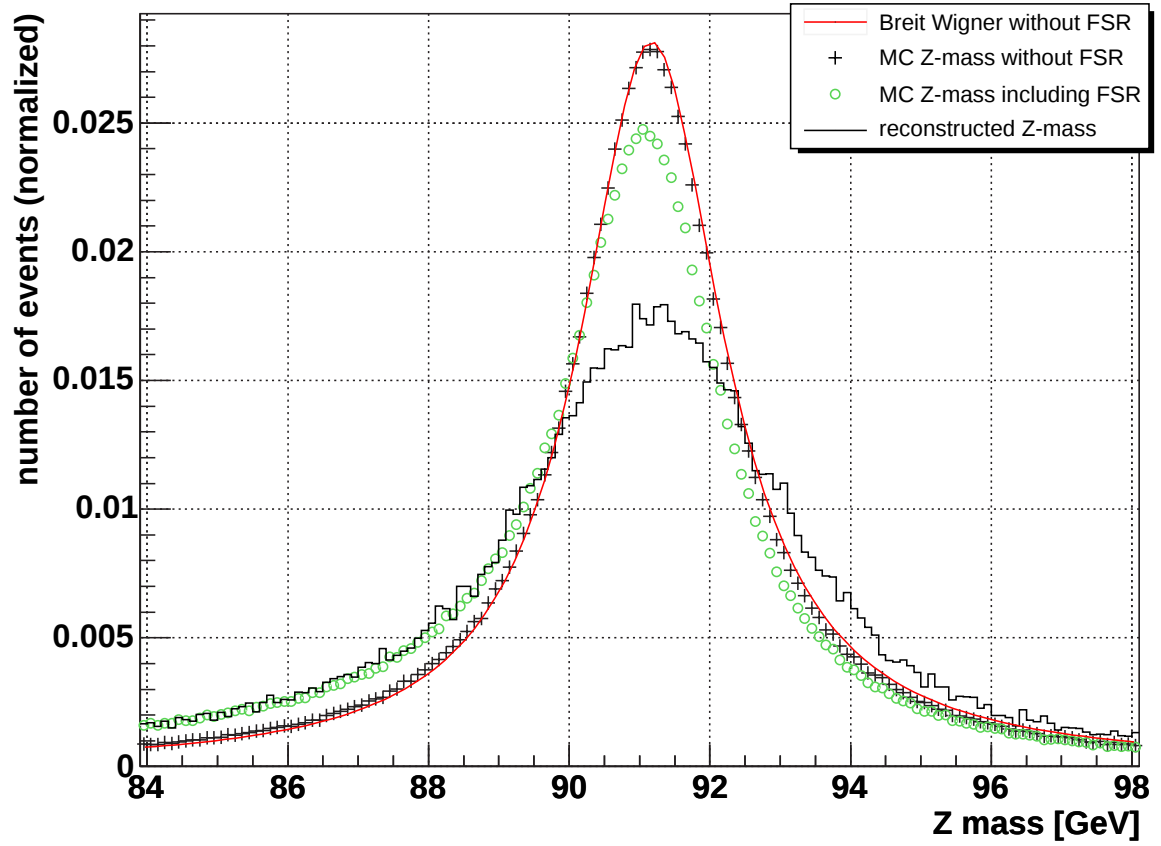


Figure 5.14: Generated and reconstructed  $Z$  mass distributions for the case with and without final state radiation compared to a Breit-Wigner curve with mass 91.154 GeV and width 2.47813 GeV.

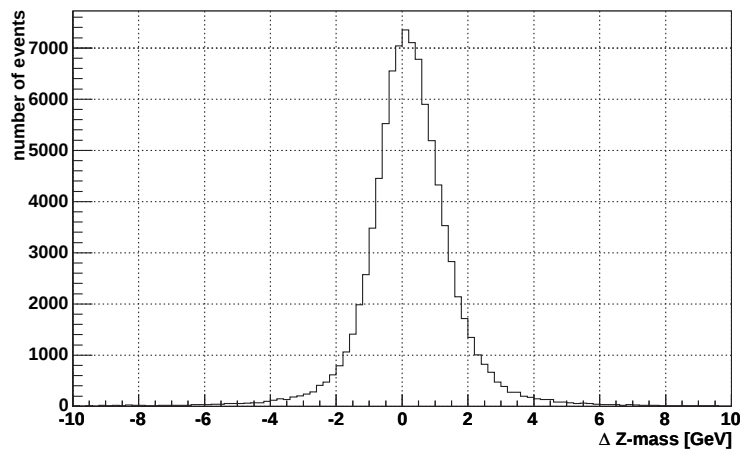


Figure 5.15: Resolution of the  $Z$  mass. It has a mean value of 0.2338 GeV and a RMS of 1.455 GeV.

## Chapter 6

# Determination of the $W$ mass

### 6.1 Strategy of the Analysis

The aim is to extract the  $W$  mass from the distribution of the transverse mass of the  $W$ -boson. One possibility to achieve this is to construct a Monte Carlo template of the  $M_T$  distribution that takes all relevant quantities into account: cross section, initial and final state radiation, angular distributions, recoil model, detector resolution and so on. The remaining parameters, the  $W$  mass and width, are then extracted by fitting this template to the data. This method has been used very successfully, e.g. at CDF [25]. However, this approach has some fundamental uncertainties in the case of LHC and CMS, since the theoretical description and detector simulation are not expected to match the real world to a sufficient degree in the first year of operation.

In this study, an alternative approach, which is as independent of theory and detector description as possible, is investigated. To achieve this, another electroweak process, the decay of the  $Z$ -boson, is utilized. The properties of  $Z$ -bosons are well known from experiments at LEP. Due to the very clean signature and the fully definable final state of a leptonic  $Z$ -boson decay ( $Z \rightarrow l^+l^-$ ), the  $Z$  can be reconstructed very precisely without loss of information and it is not necessary to deal with missing energy. The idea is to determine properties of  $W$ -bosons by a direct comparison of  $W$  and  $Z$  events. In our case, we try to get a scaling factor between  $W$  and  $Z$  mass, but the same method could be applied to measure other quantities. Since these processes are very similar concerning the theoretical description, it is possible to interpret a  $Z$  event as a  $W$  event by substituting a muon by a neutrino. Instead of fitting a complicated Monte Carlo model to the  $W$  data, the  $Z$  data are fitted to the  $W$  data. This way, uncertainties due to the detector simulation and the theoretical description are avoided.

However, it is not possible to eliminate all theoretical input. The knowledge of small differences in the production mechanism (see section 6.3) is still necessary, but these differences have a rather small influence on the distributions of the transverse mass. These differences are compensated by applying weights to the  $Z$  events. These weights have to be extracted from the theory, i.e. Monte Carlo generators (see section 6.3), but the generators are not used to produce the absolute distributions.

In the simulation sections of this thesis, the detector simulation is used to get the resolution of the relevant quantities based on the current state of knowledge. Afterwards, the input variables are smeared with resolution effects and the bias on the result is studied.

To summarize, the approach is to establish a method that extracts a scaling factor between

$W$  and  $Z$  mass by comparing data from  $W$  and  $Z$  events in order to be as independent of the theoretical description as possible. The transformation from a  $Z$  event to a  $W$  event is done in a number of steps that are described in section 6.3.6. The precision of this method based on the current status of the theory and the detector simulation is estimated.

## 6.2 Event Selection

The cross section for the process  $pp \rightarrow W + X$  with  $W \rightarrow l\nu$  and  $l = e, \mu$  is 300 nb at LHC conditions [1]. In one year of low luminosity with an assumed integrated luminosity of  $10 \text{ fb}^{-1}$  about 300 million events will be produced in the experiment. About 150 million events have a muon decay. A clean  $W$  signal will require the following selection cuts:

- An isolated charged lepton with  $P_T > 25 \text{ GeV}$  and  $|\eta| < 2.4$ . The  $P_T$  cut reduces the background from other processes as discussed in section 6.4.
- Missing transverse energy  $E_T^{\text{miss}} > 25 \text{ GeV}$ . The missing energy corresponds to the neutrino energy which should satisfy the same criteria as the muon.
- Recoil  $|\vec{u}| < 20 \text{ GeV}$ . This cut reduces the number of accepted  $W$ 's with high  $P_T$ . For high  $P_T$  the QCD background increases and the transverse mass resolution decreases as shown in section 5.1.

The number of remaining events after the cuts are shown in table 6.1.

| Cut                                  | % Remaining | Events Remaining  |
|--------------------------------------|-------------|-------------------|
| $P_T^\mu > 25 \text{ GeV}$           | 68.9%       | $1.03 \cdot 10^8$ |
| $ \eta^\mu  < 2.4$                   | 55.4%       | $8.3 \cdot 10^7$  |
| $E_T^{\text{miss}} > 25 \text{ GeV}$ | 46.84%      | $7 \cdot 10^7$    |
| $ \vec{u}  < 20 \text{ GeV}$         | 35.2%       | $5.3 \cdot 10^7$  |

Table 6.1: Consecutive selection cuts and remaining events for  $W \rightarrow \mu\nu$  on generator level.

The  $Z$  sample is used to derive the boson's transverse momentum distribution  $P_T$  and other observables. To keep the bias from  $Z$  selection small, it is important to choose the  $Z$  sample as similar as possible to the  $W$  event selection. The bias from slightly different selection criteria and from differences in the production mechanism is studied in section 6.3. The cross section for  $pp \rightarrow Z + X$  with  $Z \rightarrow l^+l^-$  is about 1/10 of the  $W$  cross section and we expect 15 million events after  $10 \text{ fb}^{-1}$  of integrated luminosity.

As described in section 6.1, it is crucial to use exactly the same conditions for the  $W$  and  $Z$  samples. This is necessary because even small differences in the event selection may have large impact on derived quantities like the transverse mass. However, in the real experiment this is not attainable for several reasons.

The neutrino in  $W$  decays, corresponding to one of the muons in  $Z$  decays, is invisible. But on the muon several kinematical constraints are imposed that can not be applied to the neutrino. This leads to fundamental differences in the kinematics of the events considered.

It is common practice to apply preselection cuts to the generated events. By omitting events that would not be triggered in the detector simulation, the efficiency of the simulation

can be increased. This applies to the muon trigger, which has a certain  $\eta$  region of sensitivity ( $\sim |\eta| < 2.5$ ). In addition, the trigger has a lower efficiency at lower transverse momentum.

One possibility to get similar samples is to reconstruct the neutrino by a  $W$  mass constraint and to require that it is in the region where a muon from a  $Z$  would be triggered ( $\sim |\eta| < 2.5$ ). This method uses the missing transverse energy, the  $W$  mass and the lepton four-vector to reconstruct the longitudinal component of the neutrino, but there are always two possible solutions. If we require that both of the solutions have to be in the range of muons from  $Z$  decays, we could get a  $W$  sample with similar properties as the  $Z$  sample. Since the resolution of the missing energy, and therefore the resolution of the neutrino, is worse than the resolution of a muon, we would have to make sure that we don't get neutrinos with  $|\eta| > 2.5$ . This could be achieved by a very strong constraint of the allowed  $|\eta|$  to the barrel region. The same constraint has to be applied to the  $Z$  sample.

The main effect of these selection criteria is a reduction of the number of accepted events. Since the expected number of  $W$  events is large enough this should not be a problem. The constraint to the barrel even improves the sharpness of the Jacobian Edge in the  $M_T$  distribution. The implementation of the analysis with generator data, that is presented in the following, does not apply any of these constraints since the event number that was available does not match realistic numbers anyway.

## 6.3 Generator Study

It is necessary to perform a detailed study of the kinematical differences of  $W$  and  $Z$  distributions, followed by an investigation of a reweighting procedure to compensate the differences.

For these studies, one million  $W \rightarrow \mu\nu$  events and the same number of  $Z \rightarrow \mu^+\mu^-$  events have been produced with PYTHIA [13] and analyzed with the PAX interface to HEPEVT ntuples (section 4.3).

### 6.3.1 Transverse Momentum of the Boson

The bosons are produced with transverse momentum  $P_T$  resulting from the hadronic recoil. The distributions in figure 6.1 show that there are more  $W$  events with low  $P_T$  than  $Z$  events, the ratio shows an approximately linear decrease. No significant differences between  $W^+$  and  $W^-$  events have been found.

### 6.3.2 Pseudo-Rapidity of Bosons and $W^+ - W^-$ -Asymmetry

The Pseudo-Rapidity  $\eta$  of the bosons depends on the initial state of the Drell-Yan process. Including the transverse momentum  $P_T$ , the system is fully defined, except for the azimuthal angle. Therefore,  $P_T$  and  $\eta$  should be sufficient to determine the kinematical differences in the production mechanism of the bosons.

At this point, a very important difference between  $W^+$  and  $W^-$  appears. As shown in figure 6.2 the Pseudo-Rapidity of  $W^+$  bosons tends to be larger than that of  $W^-$  bosons. Furthermore, the number of  $W^+$  events is larger than the number of  $W^-$  events.

|       | ratio of events |
|-------|-----------------|
| $W^+$ | 58.3%           |
| $W^-$ | 41.7%           |

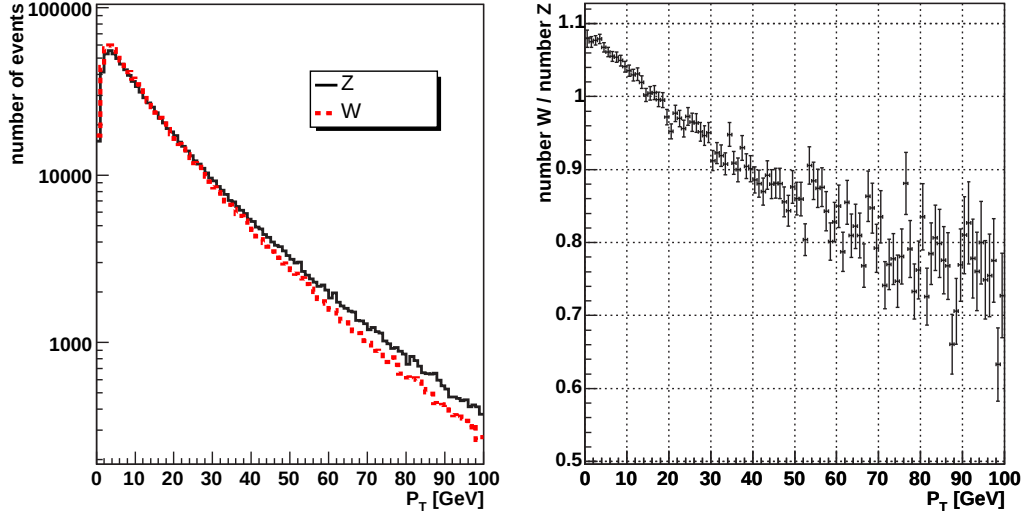


Figure 6.1: The  $P_T$  distributions of  $W$  and  $Z$ -bosons without any cuts. The right plot shows the ratio of the event numbers. The total number of events is 1,000,000 each.

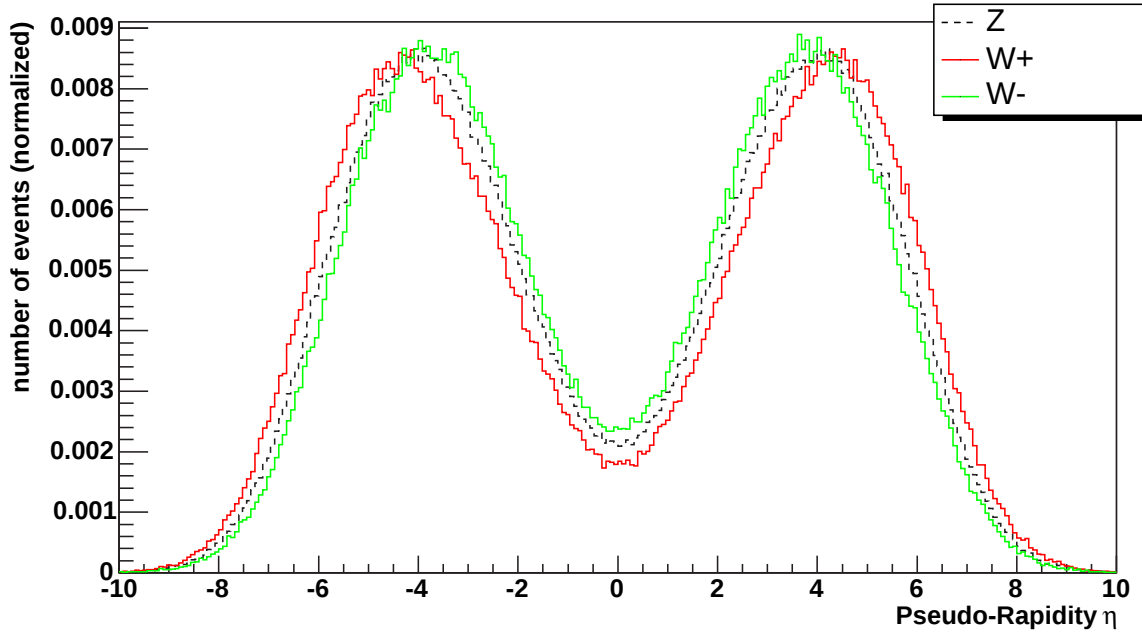


Figure 6.2: Pseudo-Rapidity of  $W^+$ ,  $W^-$  and  $Z$ -bosons compared. The number of events is normalized to 1, but the absolute number of  $W^+$  and  $W^-$  events is different.

This effect has been observed at proton-antiproton colliders as a forward-backward asymmetry. With proton-proton colliders, there is no possibility to distinguish between forward and backward. Here, this effect emerges in a difference between  $W^+$  and  $W^-$  distributions.

The source of this effect is to be found in the parton distribution functions of the proton. These distribution functions are shown in figure 6.3 for the MRST2002NLO case. The  $u\bar{d}$

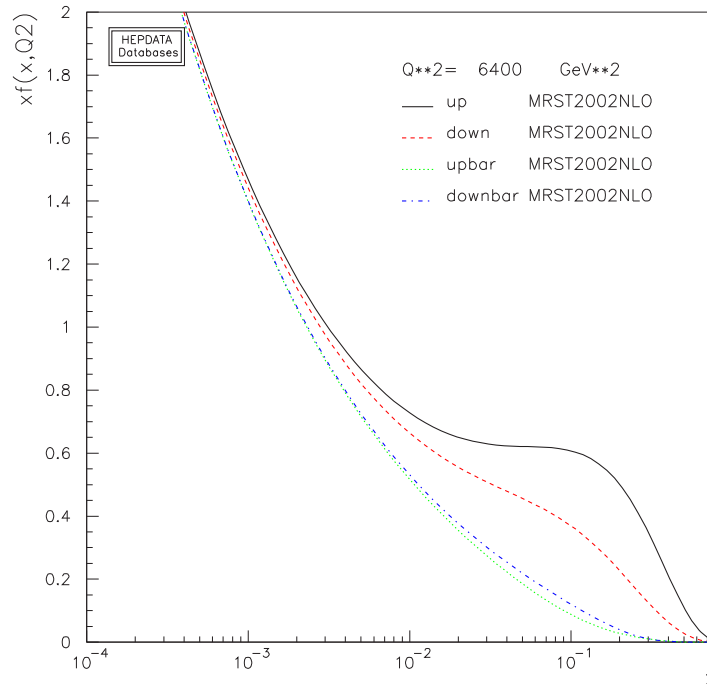


Figure 6.3: MRST parton distribution functions of the proton for the relevant partons that participate in a Drell-Yan process [26].

process occurs more frequently than the  $\bar{u}d$  process. This is quite intuitive, because there are twice as much up valence-quarks than down valence-quarks. The differences in the pdf's result in an asymmetry in the initial state of the Drell-Yan process. On average, the  $u\bar{d}$  system has a larger longitudinal momentum than the  $\bar{u}d$  system, hence a larger Pseudo-Rapidity.

### 6.3.3 Angular Distribution of Leptons from Boson Decay

The Pseudo-Rapidity of muons from decaying bosons depends on the decay-angle in the boson rest frame and on the boosts along z-axis and transverse axis. The angular distribution in the rest frame is explained in section 3.3. The according generator output is shown in figure 6.4. The distributions show no significant differences between the charged and neutral bosons. We would expect a different angular distribution for the  $Z$ -boson since it couples right handed states too and interferes with the photon as explained in section 3.4. Obviously this is not implemented in PYTHIA. However, the differences in the distributions would have been balanced by weights since we want to reweight  $Z$  distributions so they match  $W$  distributions. We just accept the PYTHIA output and do not reweight for the decay angles.

The  $\eta$  distributions of muons and neutrinos can then be understood by boosting the leptons into the laboratory frame. Figure 6.5 shows these distributions for the  $W^+$  case.

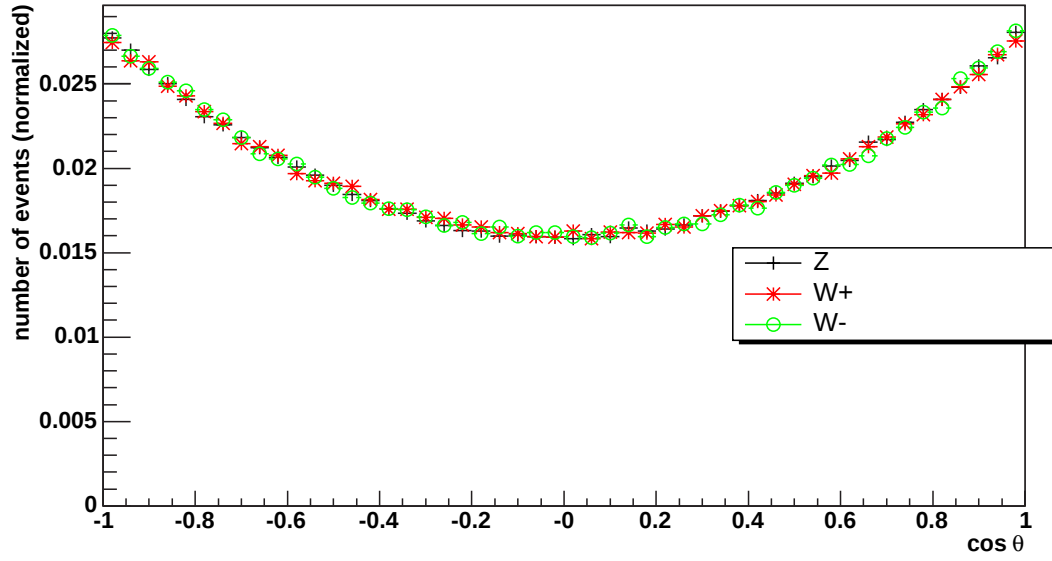


Figure 6.4: Decay angle of the muon in the rest frame of the boson.

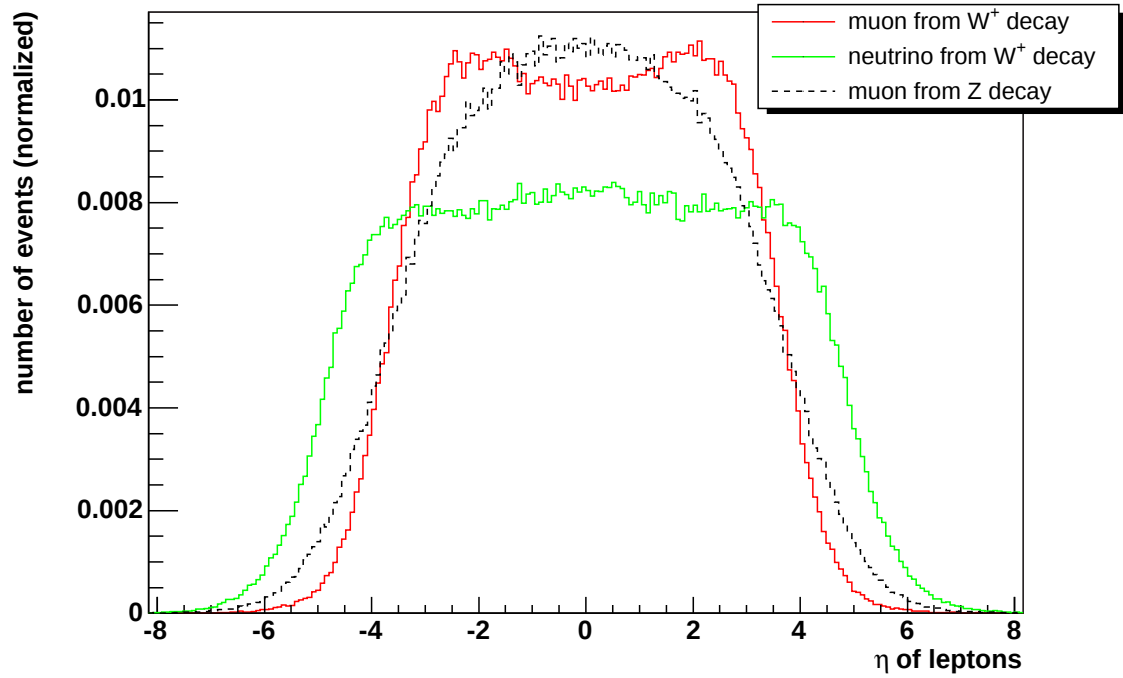


Figure 6.5: Pseudo-Rapidity of muons and neutrinos from boson decay. Final state radiation is not included in this plot.

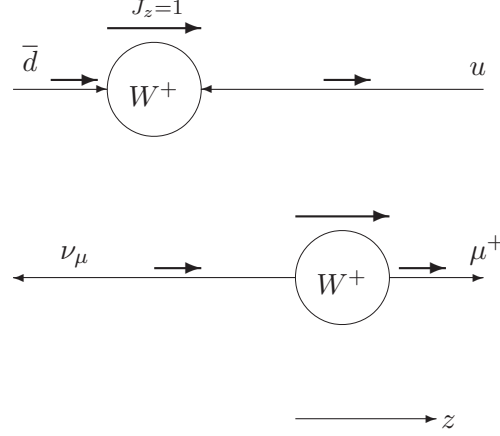


Figure 6.6: The incoming  $u$ -quark has a large momentum. The small arrows show the spin of the particles. Particles participating in weak interactions are left-handed and anti-particles are right handed. The neutrino is then boosted into the direction of the  $W$  momentum.

Obviously, the neutrino is preferably emitted in the forward direction. This effect results from parity violation and the parton distribution functions. The kinematical situation is depicted in figure 6.6.

The incoming  $u$ -quark has a larger longitudinal momentum than the  $\bar{d}$ -quark. Since the neutrino is left-handed, it is emitted into the direction of the boson's momentum. Therefore the  $|\eta|$  of the neutrino tends to be larger than that of the muon. This is a fundamental difference to the  $Z$  decay, where the muon distributions are identical. Fortunately, this is a “longitudinal effect” and the transverse mass measurement will not be affected by it.

However, if transverse quantities are considered, there is still a difference in the distributions of the leptons. In the boson rest frame, the  $P_T$  distributions are identical. But after the boost into the laboratory frame, the neutrino has a lower  $P_T$  than the muon (figure 6.7).

It is important to point out that these effects are not expected to affect the transverse mass measurement. The differences in lepton  $P_T$  cancel since the transverse mass is given by

$$M_T = \sqrt{(E_T^l + E_T^\nu)^2 - (\vec{E}_T^l + \vec{E}_T^\nu)^2}.$$

If detector resolution effects were included, the differences in the resolution for muons and neutrinos (missing  $E_T$ ) would have to be taken into account. Since the difference in  $P_T$  is rather small, this is not considered to be a serious problem.

#### 6.3.4 Final State Radiation

In this thesis, the topic of final state radiation is not discussed in detail. The differences between  $Z$  and  $W$  samples have to be taken into account correctly. The  $Z$  sample has two muons that both may radiate photons. The  $W$  sample has only one muon with lower energy. This effect causes a tail in the distributions of the invariant mass of the bosons. The shape of the tail depends on the event topology and the energies and is therefore slightly different for the  $Z$  and the  $W$  case respectively. This is an effect resulting from quantum electrodynamics,

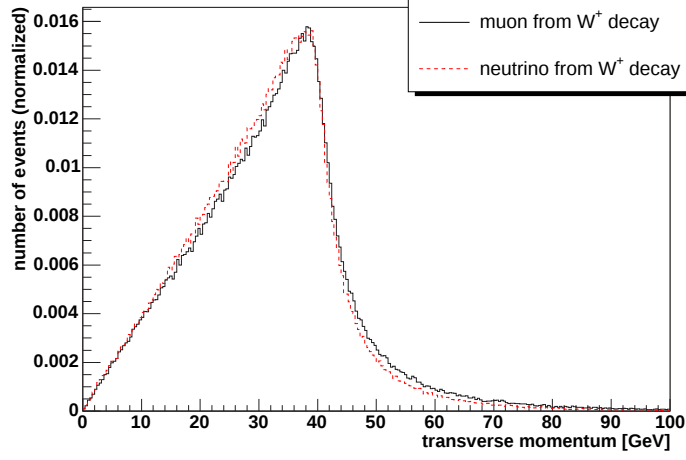


Figure 6.7: Transverse momenta of muon and neutrino in a  $W^+$  decay. Final state radiation is not included.

which is well understood. A correct description of FSR by the Monte Carlo generator used is assumed.

The amount of radiated photons for the  $Z$  and  $W$  case and the impact on the Breit-Wigner distribution is shown in figure 6.8.

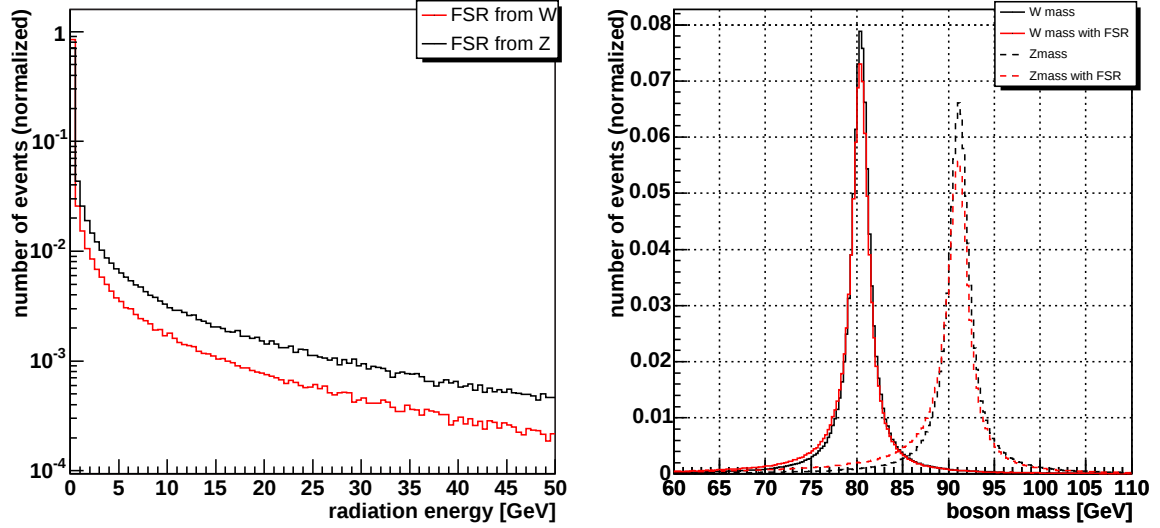


Figure 6.8: Left plot: Energy sum of the photons from final state radiation. Right plot: Impact of electromagnetic radiation on the boson's mass distributions.

### 6.3.5 Extraction of Weights

The aim is to assign a weight to each  $Z$  event in a way that weighted  $Z$  distributions match the according  $W$  distributions.

The differences in  $P_T$  and  $\eta$  were already discussed in sections 6.3.1 and 6.3.2. The right plot of figure 6.1 could be used to determine weights in  $P_T$ . Since  $P_T$  and  $\eta$  are not supposed to be independent of each other, a 2-dimensional weighting histogram could be used. If the weight from width and final state radiation is included, one would have to deal with a 3-dimensional weighting histogram, which in our case is limited by statistics.

The procedure is simple: The variables  $P_T$  and  $\eta$  for the  $W$  and  $Z$  case respectively are histogrammed and the resulting 2-dimensional histograms are divided. An alternative approach manages to get 1-dimensional histograms: First, the weights in  $P_T$  are extracted and then the  $\eta$  distribution is produced with the  $P_T$  weights applied. Then the weighted  $\eta$  distribution is used to extract the weights in  $\eta$ . This way, the differences are not counted twice and one has 1-dimensional histograms, that are easier to deal with.

To reduce the effect of fluctuations for bins with only a few entries, polynomials are fitted to the weights. For the  $P_T$  weights, a third order polynomial  $a + bx + cx^2 + dx^3$  is used for the range 0-70 GeV and a polynomial of first order in 70-300 GeV, as shown in figure 6.9.

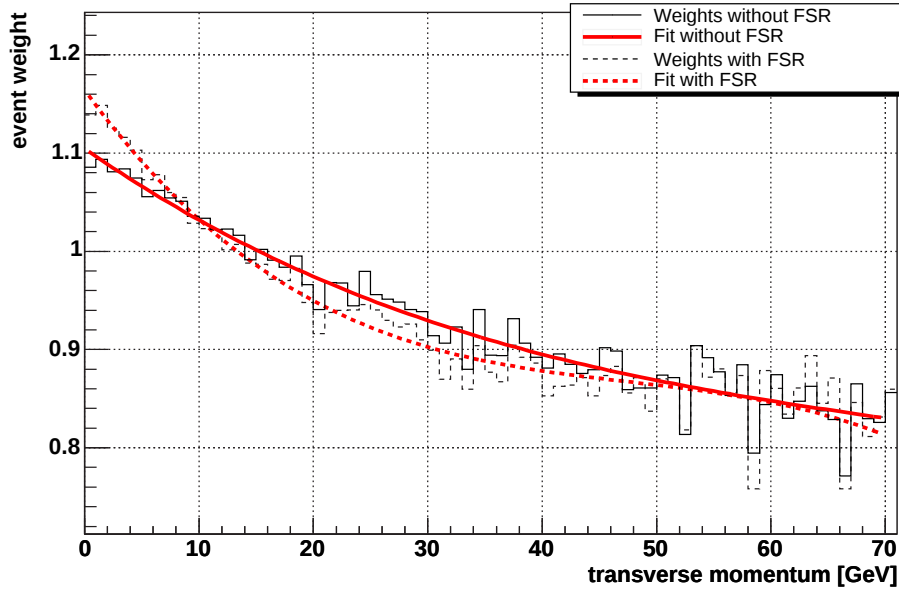


Figure 6.9:  $P_T$  weights and the polynomial fits in the range 0 – 70 GeV.

The weighted  $\eta$  histograms and the according fits are shown in figure 6.10. Polynomials of higher order (up to 8) for the fit between  $-8$  and  $8$  are used. The ranges  $> 8$  and  $< -8$  are treated separately.

The differences in the shape of the mass distributions result from final state radiation and width only. A proof for this is given in figure 6.11. The weighting factors shown in figure 6.12 are extracted by dividing the event numbers in each bin of the left plot in figure 6.11. A correction for both width and final state radiation is achieved by applying these factors to each  $Z$  event. Since the differences between the Breit-Wigner distributions have been proved

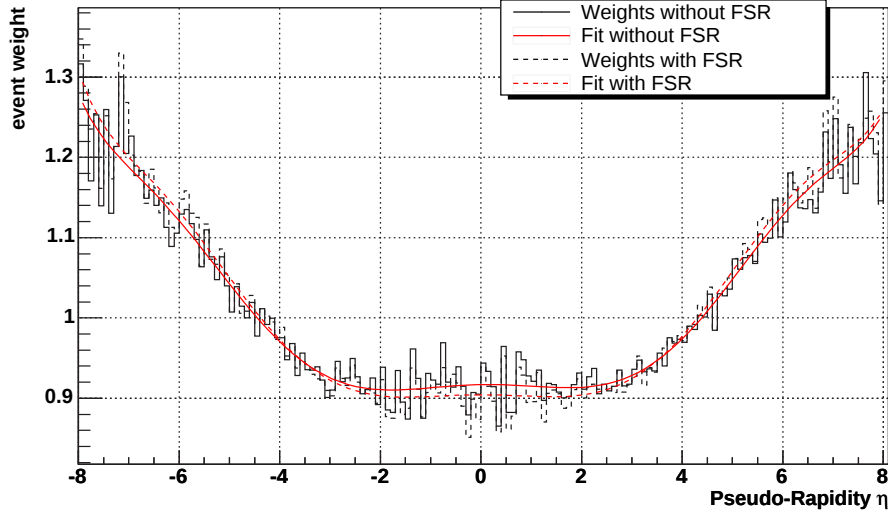


Figure 6.10:  $\eta$  weights and the polynomial fits in the range -8 to 8.

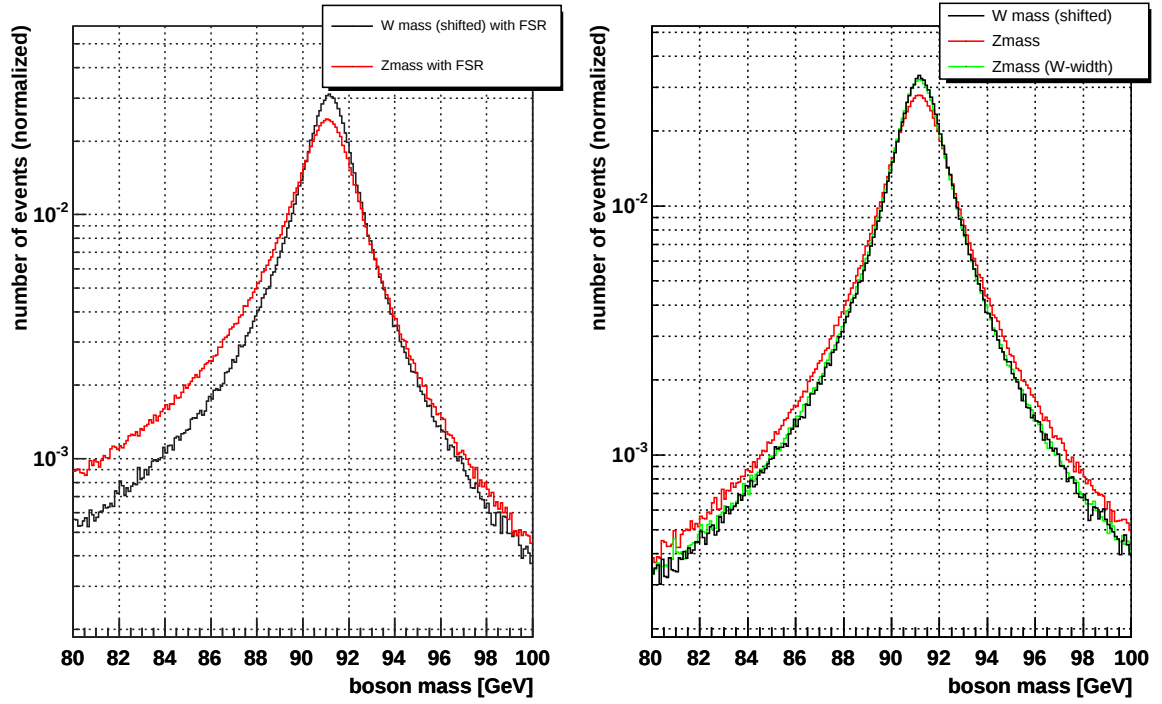


Figure 6.11: The left plot shows the mass distributions of  $W$ - and  $Z$ -bosons including the effect of final state radiation. The  $W$  distribution is already shifted to the  $Z$  mass. In the right plot the final state radiation is omitted. The only difference is the width. After scaling each  $Z$  event with a width factor  $\Gamma_W/\Gamma_Z$  the  $Z$  distribution matches the shifted  $W$  distribution almost exactly.

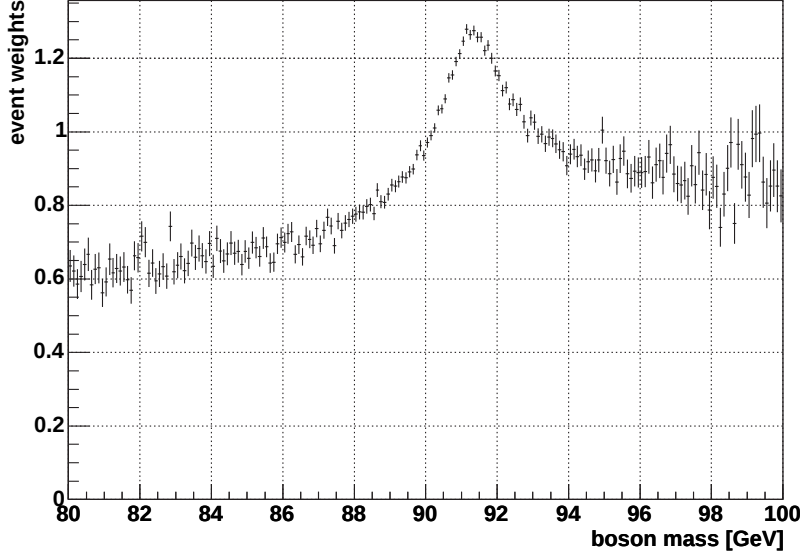


Figure 6.12: The event weights resulting from dividing the Breit-Wigner distributions including FSR.

to be well known, this procedure is justified. To get the weighting factors for another  $W$  width, the  $W$  distribution has to be corrected by the corresponding  $\Gamma_W/\Gamma_W^{generator}$  factor.

Again, one could fit polynomials to the width and FSR weights in figure 6.12, but since the analytical description of the Breit-Wigner distribution of the  $Z$  and  $W$  mass are well known, the two Breit-Wigner functions can just be fitted and the resulting fit-functions can be divided. This way we get an analytical description of the weights without statistical fluctuations. A complication arises if final state radiation is included. It is not possible to fit a plain Breit-Wigner function in this case. The best results have been achieved with a polynomial of second order added to the Breit-Wigner function for three separate energy ranges (0 – 90 GeV; 90 – 92 GeV and >92 GeV).

To get the total weight of the event, the weights from the  $P_T$  and  $\eta$  corrections have to be multiplied by the weight from the width and FSR corrections.

### 6.3.6 Performing the Analysis

In this section the “proof of principle” is presented. The analysis as outlined in section 6.1 is performed with generator data first. Afterwards the detector resolution from section 5.1 is used and the input variables are redistributed accordingly. This way one gets an estimation of the precision of this method using high statistics from the Monte Carlo generator.

The transformation of the  $M_T$  distribution using  $Z$  input variables is performed in the following steps:

- First, the muons from a  $Z$  event are used to reconstruct the  $Z$ -boson four-vector.
- Then, the weight from width and final state radiation is assigned to the event. These weights are multiplied by the weight from  $P_T$  and  $\eta$ .

- The muon four-vectors are transformed to match  $W$ -boson energies:
  - The muons are boosted into the rest frame of the  $Z$  boson. Assuming a two-body decay, the muons have exactly opposite momenta in this frame.
  - Now the energy of the  $Z$ -boson  $M_Z$  is set to the new value, i.e. the mass difference  $M_Z - M_W$  is subtracted from the  $Z$  mass. This value will be the fit parameter at the end.
  - The muon four-vectors are calculated assuming a two-body decay. According to relativistic kinematics, the energy of the muon is

$$E_\mu = \frac{1}{2}(E_Z + \frac{m_\mu^2}{E_Z}) \approx \frac{1}{2}E_Z ,$$

where  $E_Z$  is the new energy of the  $Z$ -boson and  $m_\mu$  is the muon mass, the neutrino is assumed to be massless.

- $Z$ -boson and muons are boosted back into the lab frame. The momentum of the  $Z$ -boson must be the same as before, the boost is calculated accordingly.
- One of the muons is selected randomly to match the neutrino from  $W$  decay.
- The transverse mass is calculated using hadronic recoil and one muon as explained in section 3.5.

Finally the resulting distribution of the transverse mass is compared to the distribution from the  $W$  event. Figure 6.13 shows several transverse mass distributions compared to each other. The distribution that is transformed to the simulated  $W$  mass is in good agreement with the original  $W$  distribution.

With the objective of a measurement of the  $W$  mass, a large number of  $M_T$  distributions from  $Z$  events belonging to different transformed masses are produced. A quantity  $\chi^2$  that characterizes the compatibility of the histograms is defined:

$$\chi^2 = \sum_i \frac{(N_i^W - N_i^Z)^2}{\sigma_{i,W}^2 + \sigma_{i,Z}^2}$$

The sum includes a certain range of bins in the histogram of the transverse mass which is detailed below.  $N_i$  denominates the number of entries in bin  $i$ . For the  $Z$  distribution this is equal to the sum of weights  $\sum_j w_{ij}$ . The error  $\sigma$  is  $1/\sqrt{N_i}$  or  $1/\sqrt{\sum_i w_{ij}^2}$  for weighted histograms. Around the minimum, an approximately parabolic curve for  $\chi^2(M)$  is expected. The statistical  $1\sigma$  error corresponds to an increase of  $\pm 1$  in  $\chi^2(M)$ . The quantity  $\chi^2$  is calculated for a number of cases in the following subsections.

### The Analysis Without Final State Radiation

First, the analysis is performed with a cut on final state radiation. On generator level, the energy of the vector sum of the photons is required to be less than 1 MeV. 49.3% of the  $Z$  events and 72.6% of the  $W^+$  events pass this cut.

It is found that the  $\chi^2$  distribution depends on the range in the input histograms that is taken into account. It also depends on the binning of the input histograms. If Poisson errors

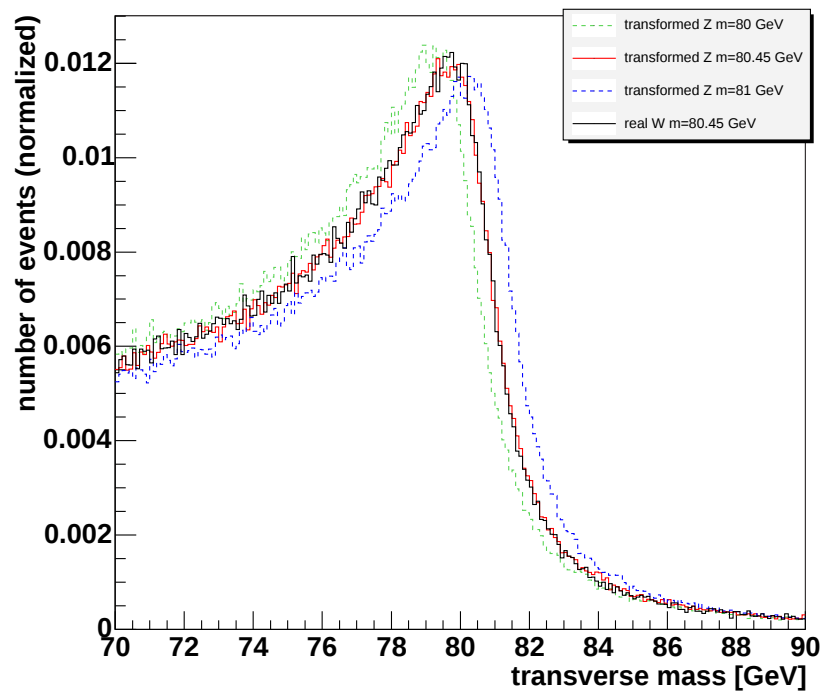


Figure 6.13: Transformed and weighted transverse mass distributions for 3 different masses. The black histogram shows the transverse mass distribution of the  $W$ -boson from the Monte Carlo generator.

are assumed in the bins, the statistical error of the result should not depend on the binning of the input histograms. However, if a small range in the  $M_T$  histograms is considered a finer binning may lead to a jumping  $\chi^2$  if statistical fluctuations are shifted into the range and if events with large weights change their bin locations. The fluctuations in  $\chi^2$  are due to the fact that there is no smooth theory curve that is used for the calculation. This problem could be solved by the usage of higher event numbers.

Therefore, a binning that yields a sufficiently smooth  $\chi^2$  curve is chosen. A nice parabolic distribution of  $\chi^2$  has been achieved with a binning of 3.3 bins/GeV in the  $M_T$  distribution. The  $\chi^2$  has been calculated in the range 80 – 82 GeV, because the Jacobian edge in that range has the greatest sensitivity to the mass. A larger range results in a higher  $\chi^2$  because the tails in  $M_T$  are not very sensitive to the mass and contain large statistical fluctuations. The left plot of figure 6.14 shows the  $\chi^2$  values in the  $3\sigma$  range ( $\Delta\chi^2 = 9$ ), where a minimum

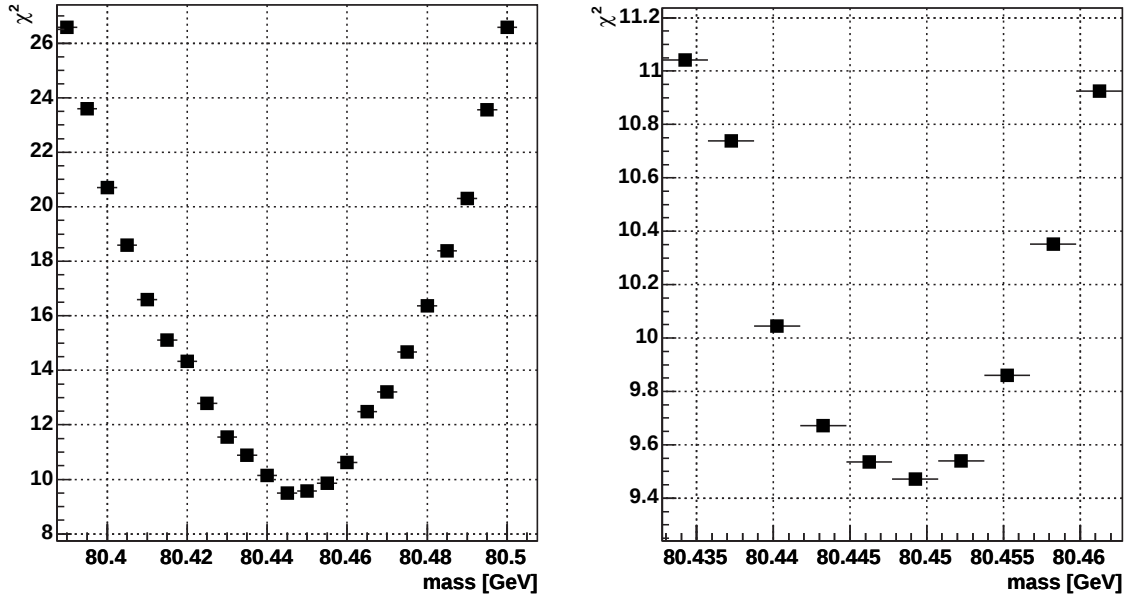


Figure 6.14:  $\chi^2$  distributions without final state radiation. The true value of the  $W$  mass is 80.45 GeV. The left plot has 2 points per 10 MeV, while the right plot uses 20 points per 10 MeV but 6 points are combined to 1 by calculating the mean value.

at 80.445 GeV is recognized. The right plot of figure 6.14 is zoomed in to the  $1\sigma$  range of  $\Delta\chi^2 = 1$ . In contrast to the left plot, where the  $\chi^2$  values as they are calculated for each point are shown, the right plot is generated by using a larger number of points and showing the mean value of six  $\chi^2$  points. This way some of the statistical fluctuations are compensated. The minimum value of  $\chi^2$  in the right plot is 80.449 GeV. This is in extremely good agreement with the generated  $W$  mass of 80.450 GeV. The statistical error with  $\Delta\chi^2 = 1$  can be deduced to be  $\sigma \approx 10$  MeV.

### The Analysis Including Final State Radiation

If final state radiation is included, then also the weights have to be calculated including FSR. The same binning and conditions as in the case without FSR are used. The result is shown in figure 6.15. The  $\chi^2$  distribution shows larger fluctuations and it does not form a symmetric

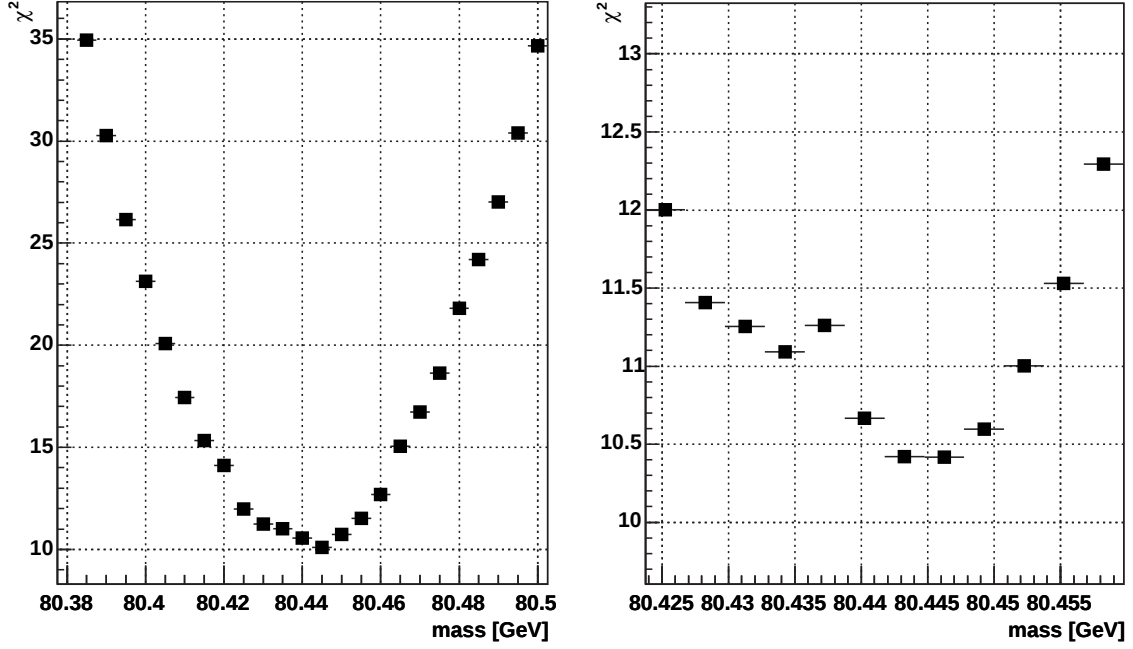


Figure 6.15: The  $\chi^2$  plot including FSR. As in the previous plot, the left histogram has 2 points per 10 MeV, while the right one has 20 points per 10 MeV from which 6 points are combined to one by calculating the mean value.

parabola. However, the minimum at 80.445 GeV is still in good agreement with the generated value of 80.450 GeV. The statistical error to the left of minimum is larger and amounts to  $\sigma \approx 15$  MeV.

### The Analysis with Smeared Input Parameters

The results of chapter 5, where the detector resolution was estimated with a full detector simulation, are used to smear the muon's momenta according to the histogram in figure 5.4. The resulting mass distributions are compared in figure 6.16.

The smearing yields a mass distribution that is in good agreement with the reconstructed values.

The event weights are recalculated according to the description in the previous sections, but now the smeared input parameters for the calculation of the weights are used. Figure 6.17 shows the result for the case of smeared muon momenta. The minimum value is found at 80.446 GeV and the error is  $\sigma \approx 16$  GeV. The smearing of the input muons does not deteriorate the measurement too much.

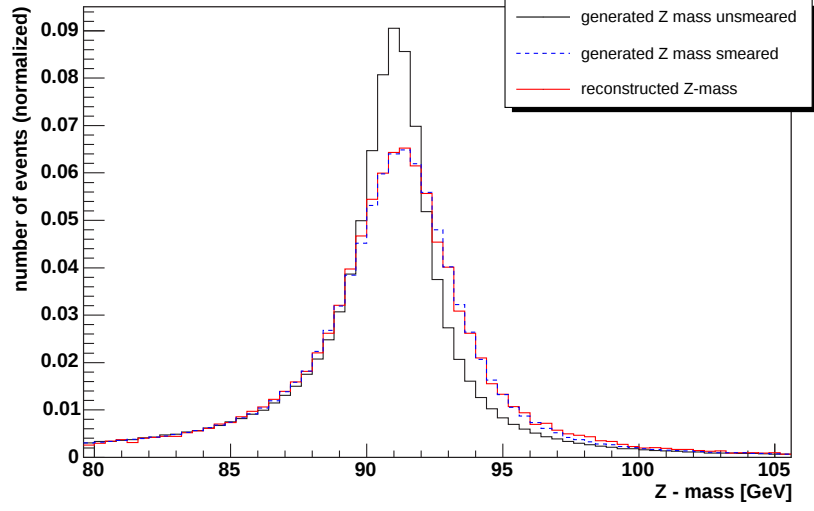


Figure 6.16:  $Z$  mass with smeared and unsmeared input variables compared to the reconstructed  $Z$  mass.

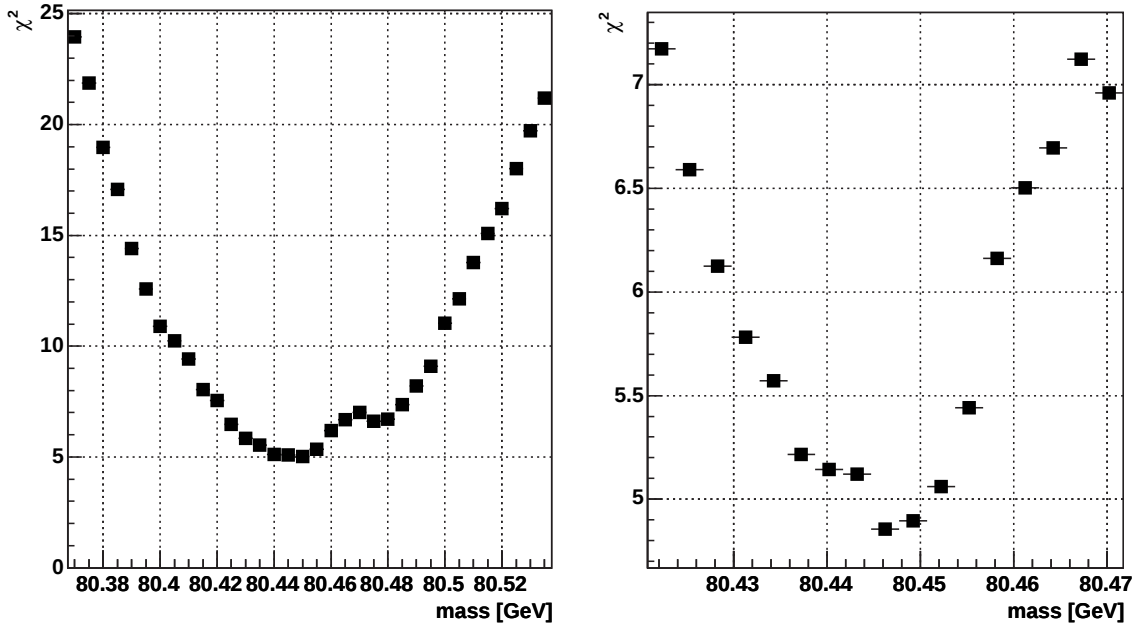


Figure 6.17:  $\chi^2$  distributions with smeared muon momentum. As before, the true value of the  $W$  mass is 80.450 GeV. The left histogram has 2 points per 10 MeV, while the right one has 20 points per 10 MeV from which 6 points are combined to one by calculating the mean value.

The last step is the smearing of the recoil. Since the recoil is not used in the reweighting procedure, it is not necessary to recalculate any weights. In contrast to the muons, the recoil resolution is almost independent of the absolute value of its energy (see figure 5.9). For large values, the amount of reconstructed energy is less than the amount of generated energy. This displacement is ignored because the final analysis has a cut on the recoil of 20 GeV (see section 6.2). The recoil is smeared according to a Gaussian distribution with  $\sigma = 10$  GeV and mean 0. The distribution of the transverse mass is smeared as shown in figure 6.18. It turns

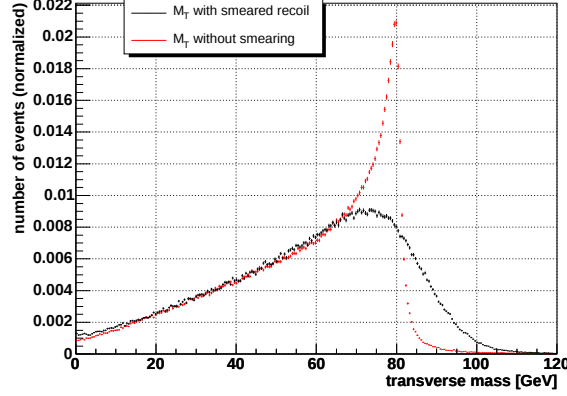


Figure 6.18: Transverse mass of the  $W$ -boson including smearing of the muon and the recoil compared to the case without smearing.

out to be necessary to extend the range of the  $\chi^2$  calculation because the Jacobian Edge is stretched to a range of 76 - 100 GeV. Figure 6.19 shows the result with the same binning in  $M_T$  as before. The minimum is found at 80.49 GeV, about 40 MeV too high. The  $3\sigma$  error is 100 MeV so the statistical error in this case is 33 MeV.

To summarize, one can say that in principle, the method is working. A very good measurement of the  $W$  mass is achieved in the case of unsmeared recoil. However, if the recoil is smeared by  $\sigma = 10$  GeV, a deviation in the measurement of about 40 MeV arises. This is quite small compared to the uncertainty in the recoil. It turns out that the  $\chi^2$  distributions have large statistical fluctuations. This is expected because there is no smooth theory prediction to calculate the  $\chi^2$ . The theory prediction is substituted by a measurement which implies statistical effects. This also shows the limitations of this method since it is only feasible with a certain amount of statistics. The remaining problems and possible solutions are discussed in chapter 7. The main requirement is a study with full simulation.

### 6.3.7 Error Estimation for One Year of LHC Data Taking

In this study, 1 million generated  $Z$  events and 1 million generated  $W$  events have been analyzed. For the proof of principle only  $W^+$  events which contribute about 58.3% have been used. In section 6.2 the expected number of events has been calculated to be about 50 million for  $W$  and 5 million for  $Z$ .

It is possible to extrapolate the error to the expected event numbers by a simple calculation. The error of the mean value of a distribution is  $\sigma/\sqrt{N}$ , where  $\sigma$  is the RMS of the distribution and  $N$  the total number of events. In the case of perturbed input values, the

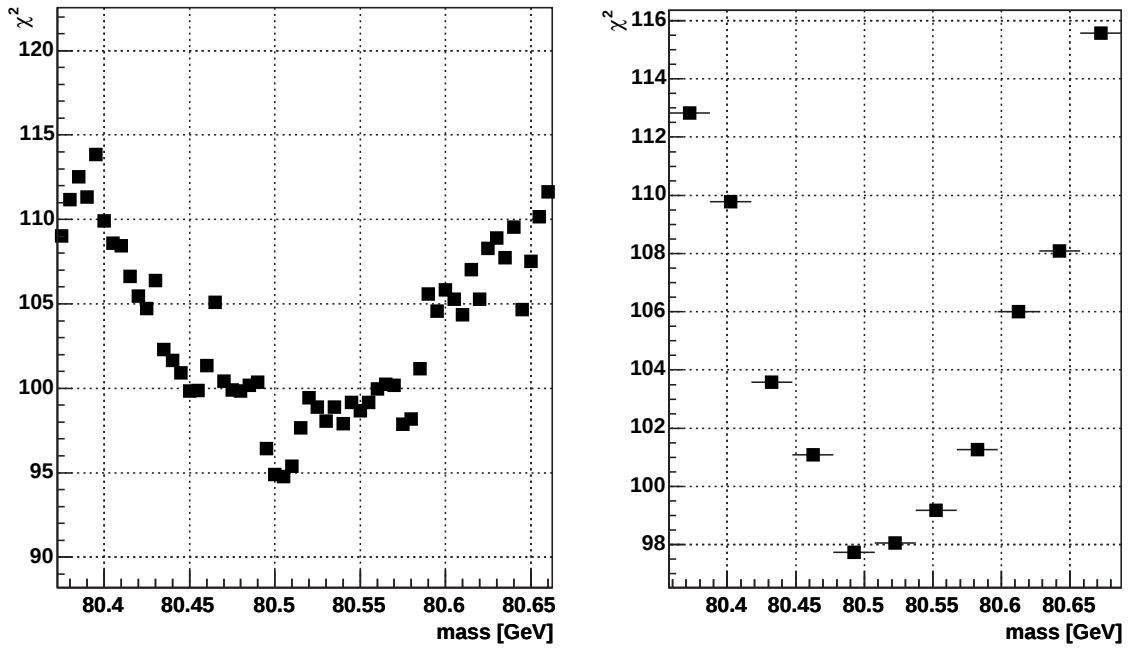


Figure 6.19:  $\chi^2$  distributions with smeared muon momentum and smeared recoil. The range in  $M_T$  is increased to 76 - 100 GeV. In this case the right plot results from the left plot, which has 2 points per 10 MeV, by combining 6 bins into one by calculating the mean value.

$M_T^{W^+}$  histogram has 583 073 events and an RMS of 22.84 GeV, while the  $M_T^Z$  histogram has 1 000 000 events and the same RMS. These numbers yield

$$\begin{aligned}\sigma_W &= 30 \text{ MeV} \\ \sigma_Z &= 22.8 \text{ MeV} \\ \sigma = \sqrt{\sigma_W^2 + \sigma_Z^2} &\approx 38 \text{ MeV} .\end{aligned}$$

This is about the same error as the result of the  $\chi^2$  fit. Hence, this estimation yields approximately the right value. To get an estimation of the error with realistic event numbers, the event numbers of section 6.2, that is 50 million  $W$  events (30 million  $W^+$ ) and 5 million  $Z$  events, are used:

$$\begin{aligned}\sigma_W &= 4.1 \text{ MeV} \\ \sigma_Z &= 10 \text{ MeV} \\ \sigma = \sqrt{\sigma_W^2 + \sigma_Z^2} &\approx 10.8 \text{ MeV} .\end{aligned}$$

This error is in agreement with previous estimations of about 15 MeV [1]. A systematic error is not yet included in this study.

It turns out that a realistic study requires an order of magnitude higher statistics. The CPU time needed for this task was not available on the time scale of this thesis.

## 6.4 Background

In this study background is not included. A realistic analysis would require a detailed simulation of background events and a study of the impact on the precision of the measurements. Backgrounds distort the transverse mass distributions; uncertainties in normalization and shape translate into an error on the  $W$  mass. The main contributors to the background are listed in the following:

- A large background comes from  $Z \rightarrow \mu^+ \mu^-$  where one muon is lost. If one muon has a large  $\eta$  it is not detected and gives rise to a large missing  $P_T$ . The remaining muon is misidentified as a  $W$  decay muon. The contribution of this background is estimated to be 4% [1].
- The contribution of  $W \rightarrow \tau \nu$  with  $\tau \rightarrow \mu \nu \nu \nu$  is estimated to amount to 1.3% [1]. The additional neutrinos appear as missing energy. This background can be suppressed by a cut on the muon momentum because the neutrinos carry away a large fraction of the momentum.
- QCD background: Di-jet events may pass the selection cuts, if one jet is mismeasured and one jet contains a muon. The mismeasured jet creates missing  $E_T$ . QCD backgrounds are estimated to be negligible [1].



## Chapter 7

# Conclusion and Outlook

In this study we implemented and tested a method for the measurement of the  $W$  mass. It employs the leptonic decay channel  $W \rightarrow \mu\nu$  only since this channel promises the highest precision. The difficulty of the proposed procedure is the measurement of the neutrino. It does not interact with the detector so it is necessary to use missing energy to reconstruct it. Since the longitudinal component of the missing energy is unknown, one has to rely on transverse quantities only.

The aim is to extract the value of the  $W$  mass from the measured distribution of the transverse mass  $M_T$ . In contrast to previous studies, where a Monte Carlo model was used to predict the  $M_T$  distribution, we use the  $M_T$  distribution of  $Z$ -bosons instead. The current error on the  $Z$  mass is only about 2 MeV while the error on the  $W$  mass is 31 MeV [2, 4]. The idea is to extract a scaling factor between  $Z$  mass and  $W$  mass by a comparison of data from  $Z$  events to data from  $W$  events. These processes are very similar if one muon from the  $Z$  event is exchanged by a neutrino. To do so, it is necessary to perform a transformation of  $Z$  events in a way that the  $W$  mass is accessible as a parameter of the measurement.

With the objective of a measurement of the  $W$  mass, a large number of  $M_T$  distributions from  $Z$  data, belonging to different transformed masses is produced. The compatibility of the transformed  $M_T$  histograms with the measurement of  $W$  data is quantified by a  $\chi^2$  criterion.

After an assumed integrated luminosity of  $10 \text{ fb}^{-1}$ , the estimation of the statistical error of the  $W$  mass measurement is about 10 MeV. This corresponds to earlier estimations of about 15 MeV [1]. A systematic error is not yet included in this analysis. We conclude that this method already shows some promising results at this stage.

There are still some problems that have to be resolved in a next iteration of this analysis. Especially the lack of realistic event numbers causes statistical fluctuations in the  $\chi^2$  distributions if events with large weights change their bin locations in the  $M_T$  histograms. Another problem is that simplified 1-dimensional weighting histograms had to be used. One could use 2-dimensional weights if larger event numbers become available. A shift in the  $W$  mass, if the range taken for the  $\chi^2$  calculation is increased, has been found. This is connected to the angular distribution of the leptons that are not correctly reweighted as mentioned above.

A full detector simulation with realistic event numbers would be desirable but was not yet feasible. Furthermore, a realistic study would require an investigation of the effects of background, simultaneous collisions (minimum bias) and pileup.



## Appendix A

# Approximation of the Transverse Mass

We want to show that  $M_T \approx 2E_T^{rest}$  where  $E_T^{rest}$  is the energy of the outgoing lepton in the boson's rest frame.  $E_T^{rest}$  has a Jacobian Edge that is smeared in the laboratory system by the transverse momentum of the boson. We are interested in  $M_T$  because it keeps the edge in the laboratory system. We start with

$$M_T^2 = (E_T^l + E_T^\nu)^2 - (\vec{E}_T^l + \vec{E}_T^\nu)^2 ,$$

and substitute  $\vec{E}_T^\nu = \vec{P}_T^W - \vec{E}_T^l$ :

$$\begin{aligned} M_T^2 &= (E_T^l + |\vec{P}_T^W - \vec{E}_T^l|)^2 - (P_T^W)^2 \\ &= 2(E_T^l)^2 + 2E_T^l |\vec{P}_T^W - \vec{E}_T^l| - 2\vec{P}_T^W \cdot \vec{E}_T^l \end{aligned}$$

With

$$\begin{aligned} |\vec{P}_T^W - \vec{E}_T^l| &= \sqrt{(\vec{P}_T^W - \vec{E}_T^l)^2} = \sqrt{(P_T^W)^2 - 2\vec{P}_T^W \cdot \vec{E}_T^l + (E_T^l)^2} \\ &= E_T^l \sqrt{\left(\frac{P_T^W}{E_T^l}\right)^2 - 2\frac{\vec{P}_T^W \cdot \vec{E}_T^l}{(E_T^l)^2} + 1} \end{aligned}$$

and after expanding the square root in first order of  $P_T^W/E_T^l$ :

$$|\vec{P}_T^W - \vec{E}_T^l| \approx E_T^l \left( 1 + \frac{1}{2} \left( \frac{P_T^W}{E_T^l} \right)^2 - \frac{\vec{P}_T^W \cdot \vec{E}_T^l}{(E_T^l)^2} \right)$$

We substitute this into  $M_T^2$  and we get

$$\begin{aligned} M_T^2 &= 2(E_T^l)^2 + 2(E_T^l)^2 \left( 1 + \frac{1}{2} \left( \frac{P_T^W}{E_T^l} \right)^2 - \frac{\vec{P}_T^W \cdot \vec{E}_T^l}{(E_T^l)^2} \right) - 2\vec{P}_T^W \cdot \vec{E}_T^l \\ &= 4(E_T^l)^2 - 4\vec{P}_T^W \cdot \vec{E}_T^l + (P_T^W)^2 = (2\vec{E}_T^l - \vec{P}_T^W)^2 . \end{aligned}$$

In the classical approximation of a two-particle decay with identical mass it is

$$\vec{E}_T^{rest} = \vec{E}_T^l - \frac{1}{2}\vec{P}_T^W .$$

This approximation holds in first order of  $P_T^W/E_T^l$  and we get the desired result.



## Appendix B

# PaxExperimentClass

The following code fragments show how the interface to original detector objects could be used for a TTrack object that is associated with a *PaxFourVector*:

```
PaxFourVector *fv = ...;

PaxExperimentClassRelations *ecr_relation =
    new PaxExperimentClassRelations;
string s = "theTracks";
fv->map_experimentclass_relations->add(s,ecr_relation);
```

First a *PaxExperimentClassRelations* has to be registered in the considered *PaxFourVector*.

```
TTrack *Track1 = ...;
PaxExperimentClass *ec = new PaxExperiment<TTrack>(Track1);
ecr_relation->add(1,ec);
```

Here the original experiment information (TTrack) is interfaced to PAX. The integer argument “1” is used as index for the different tracks in the *PaxExperimentClassRelations*.

The original experiment information can be restored in a later stage of the analysis:

```
if(fv->map_experimentclass_relations->findItemByKey(s))
{
    PaxExperiment<TTrack> *TracksRestored;

    PaxIterator<PaxExperimentClass*> *iter =
        fv->map_experimentclass_relations->findItemByKey(s)->getIterator();

    for(iter->First(); !iter->IsDone(); iter->Next()){
        TracksRestored =
            dynamic_cast<PaxExperiment<TTrack>*>(iter->CurrentItem());

        //access member functions of TTrack:
        TracksRestored->data->recHits();
    }
}
```

Using the `dynamic_cast` mechanism makes sure that a 0-pointer is returned if the cast isn't successful, i.e. if the type requested is not available.

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# German Summary - Deutsche Zusammenfassung

Am Europäischen Zentrum für Nuklearforschung (CERN) in Genf wird zurzeit ein neuer Teilchenbeschleuniger gebaut. Es handelt sich dabei um den “Large Hadron Collider” (LHC), der Protonen auf die bisher unerreichte Schwerpunktsenergie von 14 TeV beschleunigen soll. Die Protonenstrahlen werden in dem insgesamt 27 km langen Ring an vier Punkten gekreuzt und zur Kollision gebracht. An diesen Punkten werden die bisher größten und leistungsfähigsten Teilchendetektoren, unter anderem das CMS<sup>1</sup> Experiment, aufgebaut, um die bei der Kollision entstehenden Elementarteilchen nachzuweisen.

Eine der wichtigsten Aufgaben dieses Projektes wird die Klärung einiger der drängendsten Fragen der modernen Physik sein. Beispielsweise ist der Mechanismus, der für die Entstehung der Masse der Elementarteilchen verantwortlich ist, noch ungeklärt. Eine der erfolgversprechendsten Theorien, um diese Frage zu beantworten, ist der “Higgs-Mechanismus”. In dieser Theorie wird ein skalares Hintergrundfeld, das sogenannte “Higgs-Feld”, postuliert, an das die Elementarteilchen koppeln. Durch eine “spontane Symmetriebrechung” dieses Feldes ist es möglich, in der Lagrange-Dichte der schwachen Wechselwirkung einen Massenterm zu erzeugen. Damit verbunden ist ausserdem die Existenz eines weiteren Elementarteilchens, das “Higgs-Boson”. Durch den Nachweis dieses Bosons könnte die Frage der Massenentstehung geklärt werden.

Die Experimente am LHC werden nicht nur nach neuer Physik suchen, sondern auch bisherige Messungen deutlich verbessern. Dies ist notwendig, um die Konsistenz der bisherigen Theorien zu sichern und sie mit den neuen Entdeckungen abzugleichen. So wird zum Beispiel durch die Masse des  $W$ -Bosons und des top-quarks ein theoretisch erlaubtes Gebiet für die Higgs-Masse vorhergesagt. Eine Präzisionsmessung der top- und  $W$ -Masse dient also als wichtiger Abgleich mit der eventuell tatsächlich gefundenen Higgs-Masse.

In der vorliegenden Studie wird eine Methode zur Messung der  $W$ -Masse ausgearbeitet und Anhand einer Detektorsimulation getestet. Ein wesentlicher Teil der Arbeit besteht dabei aus der Computersimulation der auftretenden Prozesse. Zunächst müssen die physikalischen Prozesse auf theoretischer Ebene simuliert werden. Dazu wird der Monte Carlo Generator PYTHIA [13] verwendet, eingebettet in die CMS spezifische Umgebung CMKIN [14]. Anschliessend werden diese Daten in einer Detektorsimulation (CMSIM und ORCA) verarbeitet und als Datenbank, die dann die simulierten Detektor-Daten enthält, auf Massenspeicher gesichert. Insgesamt wurden im Rahmen dieser Studie mehrere tausend CPU-Stunden verwendet wobei mehrere hundert Gigabyte an Daten entstanden sind. Diese Daten werden dann in der Detektor-Rekonstruktion (ORCA) ausgewertet, wobei die Leistungsfähigkeit des De-

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<sup>1</sup>Compact Muon Solenoid

tektors getestet werden kann und Analysen vorbereitet werden können, die dann letztendlich auf realen Daten laufen sollen. Zur Interpretation der Daten wurde das neue Analysewerkzeug PAX (Physics Analysis eXpert) verwendet. Im Verlauf dieser Arbeit wurde ein Beitrag zur Entwicklung und Verbesserung von PAX geleistet.

Die vorliegende Studie beschränkt sich auf den leptonischen Zerfallskanal  $W \rightarrow \mu\nu$ , da hier die größte Präzision zu erwarten ist. Die Schwierigkeit bei der Messung der  $W$ -Masse besteht in der Messung des Neutrinos, da es nicht mit dem Detektor wechselwirkt. Man muss sich daher mit der sogenannten “fehlenden Energie” behelfen. Unter der Annahme, dass der Anfangszustand der Reaktion bekannt ist, kann man nach der vollständigen Messung aller auftretenden Energien einen Fehlbetrag rekonstruieren. Der Fehlbetrag wird dann dem Neutrino zugerechnet. Die Anwendung dieser Methode wird durch den unbekannten longitudinalen Anteil des Anfangszustandes erheblich erschwert. Da es sich bei den kollidierenden Teilchen um Protonen handelt, die eine Substruktur haben, sind die Energien der Quarks gemäß den Parton-Verteilungsfunktionen des Protons verteilt, also im konkreten Ereignis unbekannt. Außerdem verschwindet der größte Teil des Protonrestes im Strahlrohr und wird nicht gemessen. Es bleibt keine andere Möglichkeit als sich auf transversale Größen zu beschränken. Analog zur invarianten Masse

$$M^2 = E^2 - P^2 = (E^\nu + E^\mu)^2 - (\vec{P}^\nu + \vec{P}^\mu)^2 ,$$

wobei  $\nu$  die Größen des Neutrinos und  $\mu$  die des Muons bezeichnen, wird daher die transversale Masse  $M_T$  definiert:

$$M_T^2 = (E_T^\nu + E_T^\mu)^2 - (\vec{P}_T^\nu + \vec{P}_T^\mu)^2$$

Diese Größe kann ausschließlich aus transversalen Messwerten gebildet werden, was durch den Index  $T$  angedeutet wird. Die Energie des Neutrinos muss dabei wie schon gesagt aus der fehlenden transversalen Energie  $E_T^{ehl}$  gewonnen werden. Im Idealfall und in erster Ordnung des Wirkungsquerschnittes des Drell-Yan Prozesses  $W \rightarrow \mu\nu$  wird das  $W$ -Boson ohne transversalen Impuls produziert. Wenn ein zwei-Körper Zerfall angenommen wird, ist in diesem Fall die fehlende transversale Energie  $E_T^{ehl} = -E_T^\mu$ . In höheren Ordnungen gibt es allerdings Gluon-Abstahlungen im Anfangszustand, die vor allem bei den hohen LHC-Energien sehr grosse Werte annehmen können. Es entsteht also ein hadronischer Rückstoß  $\vec{u}$  gegen das Boson. Wenn dieser Rückstoß berücksichtigt wird, ist die fehlende Energie

$$E_T^{ehl} = -|\vec{E}_T^\mu + \vec{u}| ,$$

wobei  $\vec{E}_T^\mu$  die transversale Energie des Muons bezeichnet. Die Verteilung von  $M_T$  hängt natürlich von der Winkelverteilung der Leptonen ab. In jedem Fall hat  $M_T$  aber eine charakteristische Kante, die “Jakobi-Kante”, die exakt bei der Masse des Bosons auftritt, da hier das kinematische Maximum erreicht ist.

Das Ziel ist es nun, aus der gemessenen  $M_T$  Verteilung den Wert der Masse des  $W$ -Bosons zu gewinnen. Eine Möglichkeit, um dies zu erreichen, ist das Erstellen eines Monte Carlo Modells, das alle eingehenden Größen berücksichtigt: Wirkungsquerschnitt, “Initial-State-Radiation”, Winkelverteilung, Modellierung des Rückstoßes, Detektorauflösung von Muon und Rückstoß, “Final-State-Radiation” und eventuelle Korrekturen aufgrund unbekannter Vorgänge bei LHC-Bedingungen. Dieses Modell kann in einem “Fit” an die Messung angepasst werden, wobei die freien Parameter wie  $W$ -Masse festgelegt werden. Diese Vorgehensweise ist allerdings im Falle von LHC und CMS mit erheblichen Unsicherheiten verbunden, da das

theoretische Modell und die Detektorsimulation im ersten Jahr des Betriebes mit Sicherheit nicht die Realität widerspiegeln werden.

In der vorliegenden Studie wird nun eine Methode untersucht, die so weit wie möglich unabhängig von Theorie und Simulation sein soll. Dazu macht man sich einen weiteren elektroschwachen Prozess zunutze: Den Zerfall des  $Z$ -Bosons. Die Eigenschaften des  $Z$ -Bosons wurden in früheren Experimenten bei LEP sehr viel genauer gemessen als die des  $W$ -Bosons. Dies ist möglich, da der Zustand des  $Z$ -Bosons im Zerfallskanal  $Z \rightarrow \mu^+ \mu^-$  vollständig bestimmt werden kann. Muonen können mit hoher Präzision rekonstruiert werden und man muss sich nicht mit fehlender Energie abmühen. Die Idee besteht nun darin, durch einen direkten Vergleich von  $Z$  und  $W$  Ereignissen einen Skalierungsfaktor zwischen beiden Massen zu erhalten. Aufgrund der Ähnlichkeit der beiden Prozesse und der gleichwertigen theoretischen Beschreibung von  $Z$ - und  $W$ -Boson kann man ein  $Z$ -Ereignis als  $W$ -Ereignis interpretieren, sofern man ein Muon durch ein Neutrino ersetzt. Anstatt ein kompliziertes Monte Carlo Modell an die  $W$ -Daten zu fitten, wird also der Versuch unternommen, die  $Z$ -Daten an die  $W$ -Daten zu fitten. Damit umgeht man sowohl die theoretischen Schwierigkeiten bei der Extrapolation zu LHC-Energien als auch die Detektorsimulation.

Bedauerlicherweise ist die Methode nicht blind anzuwenden, da es trotz der Ähnlichkeiten noch einige kleine Unterschiede zwischen  $W$  und  $Z$  gibt. Hauptsächlich der Produktionsmechanismus unterscheidet sich, da das  $W^+$  aus  $u\bar{d}$  Fusion entsteht, während das  $Z$  aus  $u\bar{u}$  produziert wird. Dies äußert sich in leicht unterschiedlichen transversalen Impulsverteilungen  $P_T$  sowie in unterschiedlichen Verteilungen der Pseudo-Rapidity  $\eta$ . Neben der unterschiedlichen Breite besteht ein weiterer wichtiger Unterschied im Zerfall. Die "Final-State-Radiation" tritt im Falle des  $W$ -Zerfalls bei nur einem Muon auf, während beim  $Z$ -Boson beide Muonen strahlen. Die gefundenen Unterschiede können mit einem "Umwichtungstrick" ausgeglichen werden. Dazu ist es allerdings notwendig, die Unterschiede als theoretische Annahme in die Analyse einzuführen. Das gesetzte Ziel, vollständig unabhängig von Eingaben aus der Theorie zu werden kann damit nicht zu 100% erreicht werden. Glücklicherweise sind die Unterschiede sehr gering. Aufgrund der Tatsache, dass keine Absolutverteilungen in die Analyse eingehen, sondern ausschließlich die geringen prozentualen Unterschiede, kann das Ziel, möglichst wenig theoretische Annahmen zu verwenden, in gewissem Maße aufrechterhalten werden.

Um die Daten von  $Z$ - und  $W$ -Ereignissen zu vergleichen, muss eine Transformation durchgeführt werden, um die Unterschiede auszugleichen und schliesslich die  $W$ -Masse als Parameter der Messung zugänglich zu machen. Dies erfolgt in folgenden Schritten:

- Zuerst wird das  $Z$ -Boson anhand der zwei Muonen vollständig rekonstruiert.
- Die Gewichte aus  $\eta$ ,  $P_T$  und der Breite werden bestimmt und dem jeweiligen Ereignis zugewiesen.
- Der wichtigste Schritt besteht in der Transformation der Muon-Vierervektoren, so dass sie der Energie eines  $W$ -Ereignisses entsprechen. Dazu werden die Muonen-Vierervektoren in das Ruhesystem des  $Z$ -Bosons mittels eines "Lorentz-boost" transformiert. Sofern ein zwei-Körper Zerfall angenommen wird, sollten die Impulse der Muonen in diesem System exakt entgegengerichtet sein. Jetzt kann die Energie des  $Z$ -Bosons auf den neuen Wert gesetzt werden, das heißt auf den Wert des  $W$ -Bosons, auf den transformiert werden soll. Anschließend werden die Vierervektoren der Muonen neu berechnet, so dass sie zum neuen  $Z$ -Boson passen. Die Vektoren des  $Z$ -Bosons und die der Muonen werden jetzt zurück in das Laborsystem "geboostet", wobei beachtet werden muss, dass der

Impuls des  $Z$ -Bosons genau so gross ist wie zu Beginn. Der “Boost” muss entsprechend berechnet werden.

- Eines der Muonen wird nun ausgewählt und im folgenden als Neutrino betrachtet.
- Die transversale Masse des  $Z$ -Ereignisses wird mittels des hadronischen Rückstoßes so berechnet wie bei einem  $W$ -Ereignis.

Nach dieser Transformation sollte die Verteilung der transversalen Masse der  $Z$ -Ereignisse der Verteilung der  $W$ -Ereignisse entsprechen. Ein Beispiel für verschiedene Massen ist in Bild 6.13 gezeigt. Man produziert nun zahlreiche  $M_T$  Histogramme aus  $Z$ -Ereignissen mit jeweils unterschiedlichen Massen und überprüft, welches der Histogramme am besten zur gemessenen  $M_T$  Verteilung aus  $W$ -Ereignissen passt. Die Kompatibilität der Histogramme wird durch die Größe  $\chi^2$  quantifiziert, die folgendermaßen definiert ist:

$$\chi^2 = \sum_i \frac{(N_i^W - N_i^Z)^2}{\sigma_{i,W}^2 + \sigma_{i,Z}^2}$$

Die Summation geht hierbei über die Bins im  $M_T$  Histogramm, die zur Analyse herangezogen werden sollen.  $N_i$  bezeichnet die Zahl der Einträge im jeweiligen Histogramm bzw. die Summe der Gewichte  $\sum_j w_{ij}$ . Der statistische Fehler im jeweiligen Bin ist mit  $\sigma$  bezeichnet, das heißt  $\sigma_i = \sqrt{N_i}$  unter Annahme poissonverteilter Einträge bzw.  $\sigma_i = \sqrt{\sum_j w_{ij}^2}$  im Falle gewichteter Einträge. Das Resultat ist eine Verteilung von  $\chi^2$ , die im Idealfall parabelförmig verläuft. Das Minimum dieser Verteilung liegt dann beim gemessenen Wert von  $M_W$ . Aus dem Anstieg um  $\pm 1$  kann der  $1\sigma$  Fehler abgelesen werden. Ein Beispiel für eine solche  $\chi^2$  Verteilung ist in Bild 6.14 angegeben.

Diese Verfahrensweise wird für jeweils 1 Million  $W$ - und  $Z$ -Ereignissen mit Monte Carlo Daten getestet. Die hierbei erzielten statistischen Genauigkeiten in der Bestimmung der  $W$ -Masse liegen im Bereich 15 MeV. Um eine realistische Abschätzung der Präzision dieser Methode zu erhalten, wird die Auflösung der relevanten Größen mittels einer Detektorsimulation untersucht. Anschließend werden die Monte Carlo Größen vor der Durchführung der Analyse entsprechend der jeweiligen Auflösungen “verschmiert”. Dadurch verschlechtert sich die Präzision der  $W$ -Massen-Bestimmung auf etwa 33 MeV. Die hier verwendeten Ereigniszahlen entsprechen nicht den tatsächlich zu erwartenden Zahlen, da die entsprechende CPU-Zeit nicht in der Zeitskala dieser Arbeit zur Verfügung stand. Um trotzdem eine grobe Abschätzung des Fehlers zu erhalten, kann der statistische Fehler unter der Annahme einer Abhängigkeit von  $1/\sqrt{N}$  auf die zu erwartenden Zahlen hochskaliert werden. Unter der Annahme einer integrierten Luminosität von  $10 \text{ fb}^{-1}$  werden 150 Millionen  $W \rightarrow \mu\nu$  Ereignisse erwartet. Um ein klares  $W$ -Signal zu erhalten, wird diese Zahl durch Auswahlkriterien auf etwa 50 Millionen verringert. Die Anzahl der  $Z$ -Ereignisse beträgt etwa 1/10 der Zahl der  $W$ -Ereignisse. Mit diesen Zahlen ergibt sich ein Fehler von etwa 10 MeV. Diese Zahl entspricht näherungsweise früheren Abschätzungen [1], wobei in der vorliegenden Studie noch kein systematischer Fehler abgeschätzt wurde.

Anhand der Ergebnisse kann insgesamt gefolgert werden, dass diese Methode durchaus erfolgversprechende Resultate liefert. Es bleiben allerdings noch eine Reihe von Problemen und Verbesserungsmöglichkeiten, die in künftigen Studien umgesetzt werden können. Insbesondere die geringe Statistik verursacht mitunter starke Schwankungen in der  $\chi^2$  Verteilung. Aus

dem gleichen Grund muss man sich mit vereinfachten Gewichten behelfen, denn die Unterschiede in  $P_T$  und  $\eta$  sind voneinander abhängig, was bedeutet dass 2-dimensionale Gewichte bestimmt werden müssen. Wenn noch “Final-State-Radiation” und die Breite hinzugenommen werden, hat man schnell multidimensionale Gewichte mit einer viel zu hohen Zahl an Bins. In der vorliegenden Studie wurde daher eine Vereinfachung durchgeführt, so dass 1-dimensionale Gewichts-Histogramme verwendet werden können, was aber mit einem Verlust an Information verbunden ist. Für Abhilfe könnte hier eine größere Zahl an Ereignissen, sowie eine geschicktere Auswahl der Größen, auf die gewichtet wird, sorgen. Für eine realistische Studie müssen außerdem die Auswirkungen von “Hintergrund-Ereignissen” untersucht werden.



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