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**Validation of the muon momentum resolution
in view of the W boson mass measurement
with the CMS experiment**

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Abstract

In the framework of the Standard Model, the electroweak theory predicts relations among observables which can be measured. After the discovery of the Higgs boson, all the parameters of the Standard Model are known and it is thus possible to predict those observables with increasing precision in order to test the consistency of the model. A small deviation of measured values from those predictions would be an indirect hint of physics beyond the Standard Model.

In particular, the mass of the W boson has a far smaller uncertainty in the theoretical prediction than in the measured value. For this reason, an accurate measurement of the W mass would provide such a test of validity of the Standard Model.

In order to achieve the precision required to have a fair comparison with the theory, it is necessary to control the distributions of the variables entering the measurement at the permil level or even better.

The CMS experiment is planning to deliver a precise measurement of M_W within the next years, analysing events of W decaying to a muon and a neutrino. One of the main systematics on the measurements is the knowledge of the muon momentum scale and resolution, precisely determined from resonances decaying in two muons. The aim of this thesis is to provide a validation of the muon momentum resolution at better than 5% enabling a measurement of the muon momentum scale at 0.1 permil level.

This thesis contains also a preliminary study on one of the CMS subdetectors, allowing to measure its material budget at a few percent level.

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Introduction

After the discovery of the Higgs boson all the parameters of the Standard Model (SM) have been measured and therefore it is possible to exploit the predictive power of the theory to set more stringent limits to known observables. A significant deviation of the measured values from the prediction would be an indirect hint of physics beyond the SM.

The precision with which the W boson mass (M_W) is predicted (8 MeV) almost doubles the precision on the world averaged measured value (15 MeV). For this reason, a measurement of the W mass with an accuracy of 10 MeV provides a crucial test of internal consistency of the SM.

The Compact Muon Solenoid (CMS) experiment at the Large Hadron Collider (LHC) is planning to deliver a precise measurement of M_W within the next years. A measurement claiming such a precision is a very challenging one, implying the control of all the observables sensitive to M_W at levels which are orders of magnitude better than the ones requested for a typical analysis in the same experiment.

The CMS analysis will be performed on the W boson events produced in proton-proton collisions and decaying into a muon and a neutrino. In this sample, the transverse¹ momentum (p_T) of the muon and the hadronic recoil (the transverse component of the vectorial sum of the momenta of all particles produced together with the W) are the key experimental variables to extract M_W . The typical scale of the two objects (40 GeV and 5 GeV) imposes the level of relative precision needed to obtain an absolute uncertainty of ~ 10 MeV on M_W : 10^{-4} on the central value of the p_T of the muon and 10^{-3} on the hadronic recoil.

The work described in this thesis aims at providing a validation of the muon momentum² resolution at better than 5%. This precision is needed to eliminate convolution biases at 10^{-4} level in the closure test of the momentum calibration done with muons from the Z decay.

In CMS, the momentum of the muons is measured by the silicon tracker. The calibration of the muon momentum scale is performed using a physics-motivated model that parametrises the biasing effects in the track recon-

¹A transverse quantity is measured in the transverse plane with respect to the beamline.

²From this point on, the word "momentum" and the symbol p will always refer to the transverse momentum p_T .

struction.

The main contributions arise from the mismodelling of the magnetic field, the mismodelling of the energy loss in the detector and the residual misalignment of the tracker modules (weak modes).

The momentum scale is calibrated using large samples of J/Ψ and Υ decaying into muons. The calibration is validated using Z decays into muons and comparing the reconstructed mass in data and simulation after applying the momentum corrections, as a function of the momentum and angle of one of the muons (closure test). What matters is the correct match of the scale between data and simulation. The comparison of the scale at different momenta using resonances is sensitive to the underlying distribution of the momenta of the muons of the resonance under study. Since the distribution in momentum of the muons from Z decays has a steep slope, the resolution introduces a bias in the true scale which is proportional to the derivative of the momentum spectrum. It is therefore important to precisely measure the resolution and match it correctly between data and simulation in order to perform the closure test with the desired precision.

The physics effects that enter the resolution are the intrinsic hit resolution of the silicon detectors and the multiple scattering that the particles undergo when traversing the tracker material. To correctly derive the parametrisation we have carried out the complete calculation of the momentum resolution keeping also into account the correlation between these two effects, as this result is not mentioned in the existing literature. The result we have obtained is approximated by the following formula:

$$\left(\frac{\sigma_p}{p}\right)_{hit+ms}^2 = a^2 + c^2 p^2 + \frac{b^2}{1 + \frac{d^2}{p^2}} \quad (1)$$

where a^2 is the parameter encoding the multiple scattering and c^2 parametrises the contribution from the hit resolution. b^2 and d^2 describe the correlation between the two. This new model has been tested and found to work very well in the CMS simulation.

The measurement of the momentum resolution is performed using events of J/Ψ and Z decaying into muons collected by CMS at Run2 of LHC. The momentum resolution is of the order of 1 – 2% and it is precisely measured from the width of the resonances. Since the decay angle is measured very precisely, the relative resolution on the mass square depends only on the relative resolution of the momenta of the two muons.

The J/Ψ has a natural width that is much smaller than detector resolution: the width of the measured mass peak is a good estimator of the momentum resolution of the two muons forming the resonance. Moreover, the two muons have typically low momentum and therefore their resolution is driven by the multiple scattering term a^2 . The muons from the Z decays have higher

momenta and probe the muon momentum resolution in the region needed for the closure test. The Z has a natural width comparable to the detector resolution and the resolution is extracted from the increase of the measured width compared to the natural width.

While extracting the momentum resolution from the J/Ψ we have validated the material budget in the tracker at a few percent level since the term a^2 of equation 5.14 is very correlated with the integral of the inverse radiation length of the material crossed by the muon. The precise knowledge of the material in the tracker is needed to compute the energy lost by charged particles during the track reconstruction, which is one of the biasing effects in the muon momentum scale. An interlude during this work is the study of how a measurement of the multiple scattering in the tracker can constrain the momentum bias due to the energy loss in the tracker material.

We present a preliminary study on the measurement of the material budget of a single layer of the detector that gives useful information to constrain the energy loss.

Chapter 1

The Standard Model of particle physics

The Standard Model (SM) of particle physics summarises our understanding of the fundamental components of matter and their interactions. It was formulated in the framework of a Quantum Field Theory, in which the particles composing the matter are fermions with spin $1/2$ and those mediating the interactions are bosons with spin 1 . There is also a scalar particle, the Higgs boson, which interacts with both fermions and bosons, including itself.

This chapter is devoted to describe the SM accurately enough to serve as a theoretical background to the main subject of this thesis. A complete reference can be found in the original papers, such as [1], [2], [3], [4] and in the classic textbooks, such as [5].

1.1 Quarks, leptons and gauge bosons

The fermions consist in six quarks and six leptons, organised into three generations. The particles of the higher generations decay to the lower ones. Each generation consists of two quarks, of charge $+2/3$, (u , s , t) and $-1/3$, (d , c , b), a lepton of charge -1 , (e , μ , τ) and the related neutrino, (ν_e , ν_μ , ν_τ), chargeless. The neutrinos were introduced in the SM as massless particles, but experiments have shown that they have a small but non-zero mass. Moreover, each fermion has an associated *anti*-fermion having opposite charge. Each fundamental force is associated to one or more mediator particle. The strong force is mediated by eight coloured gluons, the electromagnetic force by the photon, while the weak interactions are mediated by the $W^\pm$ and the Z bosons. A summary chart of the SM is shown in Figure 1.1.

The SM was formulated mathematically as a Quantum Field Theory under the symmetry $SU(3) \times SU(2)_L \times U(1)_Y$, $SU(3)$ describing the strong interactions between quarks and gluons (QCD) and $SU(2)_L \times U(1)_Y$ describing the unified electromagnetic and weak, *electroweak* force. As Noether's theorem asserts, each symmetry corresponds to a conserved quantity, which in the SM are colour, weak isospin and hypercharge. While the $SU(2)_L \times U(1)_Y$

symmetry is unbroken, the fermions and the gauge bosons are massless.

	I	II	III	
mass→	2.4 MeV/c ²	1.27 GeV/c ²	171.2 GeV/c ²	0
charge→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin→	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name→	u up	c charm	t top	γ photon
Quarks	4.8 MeV/c ²	104 MeV/c ²	4.2 GeV/c ²	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	d down	s strange	b bottom	g gluon
Leptons	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	91.2 GeV/c ²
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	Z ⁰ Z boson
Gauge Bosons	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	80.4 GeV/c ²
	-1	-1	-1	± 1
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	e electron	μ muon	τ tau	W [±] W boson

Figure 1.1: Summary chart of the SM. [6]

1.1.1 The gauge boson masses at leading order

In order for the gauge bosons to acquire mass it is necessary to introduce in the theory a way to break the electroweak symmetry. This mechanism has been proposed by Higgs [7, 8], Brout and Englert [9].

The subtle point is that adding a direct mass term in the lagrangian for the fermions and gauge bosons would break the gauge symmetry that was assumed as a starting ingredient while building the SM. This would result into a *non-renormalisable* theory without any predictive power.

In this section we briefly illustrate the mechanism through which the gauge bosons acquire mass, which is functional to main purpose of this thesis. A very similar procedure that allows the fermions to get mass is not treated here: the interested reader is referred to [5].

The electroweak lagrangian must be invariant under $SU(2)_L \times U(1)_Y$ transformations. We thus introduce four fields for the gauge boson part: three fields W_μ^a (with $a = 1, 2, 3$) transforming as a triplet under $SU(2)_L$, with coupling constant g , and the field associated to the $U(1)_Y$ symmetry, denoted as B_μ , with coupling constant g' .

The trick is adding to the lagrangian two new complex scalar fields that transform as a weak isospin doublet with hypercharge $Y = 1$:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \varphi_1 + i\varphi_2 \\ \varphi_3 + i\varphi_4 \end{pmatrix} \quad (1.1)$$

through which we define the Higgs potential:

$$V(\phi) = \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad (1.2)$$

with $\mu^2 > 0$ and $\lambda > 0$. This potential has the property of having a set of degenerate minima when $\phi^\dagger \phi = \mu^2/2\lambda = v^2$. A sketch of the Higgs potential is shown in Figure 1.2. We break the symmetry choosing one particular vacuum expectation value in the set and parametrise the oscillations around this minimum using a scalar Higgs field $h(x)$.

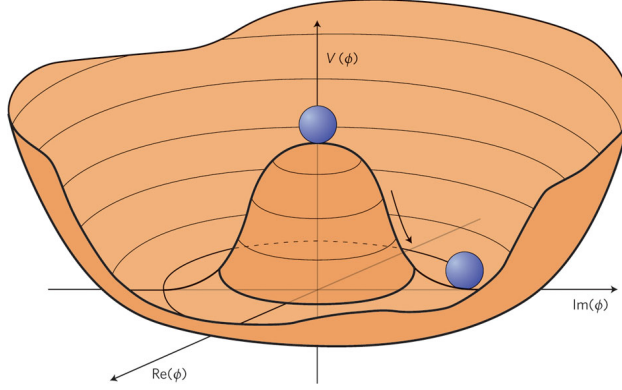


Figure 1.2: Sketch illustrating the Higgs potential. Figure from [10].

Interacting with $h(x)$, the fields W_μ^a and B_μ introduced before get mixed through the Weinberg angle θ_W :

$$A_\mu = \cos \theta_W B_\mu + \sin \theta_W W_\mu^3, \quad (1.3)$$

$$Z_\mu = -\sin \theta_W B_\mu + \cos \theta_W W_\mu^3, \quad (1.4)$$

while the field $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ stays unmixed. The physical fields A_μ (corresponding to the photon), Z_μ , W_μ^+ and W_μ^- , corresponding to the Z and $W^\pm$, are then defined as a linear combination of the W_μ^a and B_μ fields. The coupling constants g and g' are linked to θ_W and to the electromagnetic coupling constant e through the relations:

$$\tan \theta_W = \frac{g'}{g} \quad \text{and} \quad g' \cos \theta_W = e. \quad (1.5)$$

Thanks to this mechanism, the gauge bosons have acquired mass, given by, at the leading order of the perturbation theory (*tree level*):

$$M_W = \frac{gv}{2} \quad (1.6)$$

$$M_Z = \frac{v}{2} \sqrt{g^2 + g'^2} \quad (1.7)$$

Moreover, the particular choice of ϕ as a Higgs doublet constrains the quantity:

$$\rho \equiv \frac{M_W}{M_Z \cos \theta_W} \quad (1.8)$$

to be equal to 1, as follows from eq. 1.5, 1.7 and 1.6.

1.1.2 Electroweak corrections to the W mass

The expression at tree level for the W mass given in eq 1.6 can be rewritten in terms of α , the fine structure constant, $G_F = 1.166 \times 10^{-5} \text{GeV}^{-2}$, the Fermi constant and θ_W :

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W}, \quad (1.9)$$

If we take into account the electroweak corrections, i.e. radiative corrections to the leading order arising from electromagnetic and weak effects, it can be corrected in the following way [11]:

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}G_F \sin^2 \theta_W} (1 + \Delta r). \quad (1.10)$$

Δr receives inputs from various contributions:

$$\Delta r = \Delta\alpha - \frac{\cos^2 \theta_W}{\sin^2 \theta_W} \Delta\rho + \Delta r_{rem}. \quad (1.11)$$

$\Delta\alpha$ is the correction to α due to photon vacuum polarisation and it is sensitive to the fermion masses through $\ln(s/m_f^2)$, where s is the energy of the centre of mass. $\Delta\rho$ is the deviation from 1 of the ρ parameter defined in eq. 1.8.

In the leading terms this deviation comes from corrections to the W propagator with loops containing isodoublets and it is proportional to the isodoublet mass splitting $|m_{f1}^2 - m_{f2}^2|$. The most important contribution comes then from the top-bottom doublet, whose Feynman diagram is shown in Figure 1.3a. Finally, there is a logarithmic correction due to the Higgs boson mass, shown in Figure 1.3b. The contribution in Δr_{rem} are due to the remnant processes (i.e. box diagrams and vertex corrections) and are all negligible except for the ones coming from the top quark and the Higgs boson.

1.2 High precision physics in the electroweak sector

After the discovery of the Higgs boson by the CMS [12] and ATLAS [13] experiments, all the parameters of the SM have been measured and at the

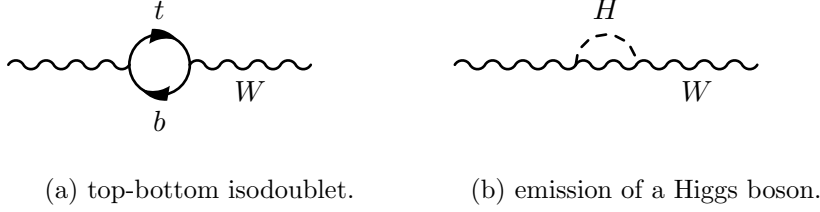


Figure 1.3: Most significant corrections to the W propagator at one-loop level. The top-bottom isodoublet correction grows with the difference of the squares of the masses in the isodoublet, while the correction from the emission of the Higgs boson is logarithmic in M_H .

same time a consistent progress has been made in the theoretical calculation of the same observables. It is then possible to exploit the predictive power of the theory to set more stringent limits to known observables and test the SM for internal consistency. This comparison can be realised through the electroweak fit that, using accurate measurements and calculations as input, returns predictions for key observables in the SM whose precision can even exceed the direct measurement, as in the case of the W mass.

1.2.1 The electroweak fit

The two ingredients needed for a global fit are accurate measurements and precise calculations.

The experimental measurements are used as input. At tree level all the variables in the SM can be computed by three parameters, chosen to be the most precisely measured: α , the fine structure constant, known to a relative precision of $3 \cdot 10^{-10}$, G_F , the Fermi constant, known to a relative precision of $5 \cdot 10^{-7}$ and the mass of the Z , M_Z , measured with a precision of 2 MeV ($2 \cdot 10^{-5}$ relative precision) in the scan of the Z resonance in e^+e^- collisions. In order to compute higher order corrections more input is needed: the strong coupling constant α_s , the running of the fine structure constant to the Z mass $\Delta\alpha$, the mass of the Higgs bosons and the masses of the fermions, but in practice only the mass of the top quark m_t among the fermions matters, since it is orders of magnitude more massive than the others and the corrections grow with the mass value.

As for the theoretical calculations, generally all the observables are calculated to the two-loop order precision. The W mass is calculated to the same order with the addition of four-loop QCD corrections [14]. In section 1.1.2 we have shown the results of the calculation at the one-loop order precision as a proof of principle.

The contribution of the loops is essential since, including those effects in the calculations, physics can be studied at much higher energy scales than available in the center-of-mass energies of the existing colliders. This means

that we could indirectly detect new particles through their contributions to the loops which do not show up in the SM calculations and are responsible for a disagreement between measurements and theoretical predictions.

Global fits of the electroweak sector are possible if the input parameters are over-constrained with a prediction better than the effects from new particles at the one-loop level. Historically, the concept of the global fits has shown to be successful in predicting the top and Higgs masses, but, after the discovery of the Higgs boson, for the first time all the parameters in the SM are known and thus all the electroweak observables can be predicted at the the loop level, allowing for important checks of consistency.

The prediction of one parameter of the SM from the fit is obtained removing the input value of that parameter and fitting all the other parameters to the theory. Table 1.1 shows the result of the global electroweak fit as the latest result currently delivered, as reported in [14].

Table 1.1: Results of the electroweak global fit as reported in [14]. The second column contains the world average experimental value used as input in the fit and the third column contains the result of the fit to all the observables except for the one considered.

Parameter	Experimental value	Fit result
M_H [GeV]	125.14 ± 0.24	93^{+25}_{-21}
M_W [GeV]	80.385 ± 0.015	80.358 ± 0.008
Γ_W [GeV]	2.085 ± 0.042	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	91.200 ± 0.011
Γ_Z [GeV]	2.4952 ± 0.0023	2.4946 ± 0.0016
m_t [GeV]	173.34 ± 0.76	$177.0^{+2.3}_{-2.4}$

1.2.2 The W mass as a hint for new physics

As evident from Table 1.1, the precision with which the W mass is predicted (8 MeV) almost doubles the precision on the world averaged measured value (15 MeV). The difference between measurement and prediction is 27 ± 17 MeV. In this discrepancy lies the motivation of the community to lower the uncertainty on the experimental value. Figure 1.4 shows a scan of the confidence level profile of M_W vs m_t for the scenario where M_H is included in the fit (blue) or not (grey). Both contours agree with the direct measurement (green bands and ellipse) but it is clear that a shrinkage of the error band of the M_W could reveal a slight disagreement.

This could be due to additional contributions to the parameter Δr in eq. 1.11 arising from extensions of the SM. For instance, contributions from supersymmetric particles would be dominated by *squark* loops, identical to the ones shown in Figure 1.3a and 1.3b up to substituting the top-bottom isodoublet and the Higgs boson with *squarks* and *antisquarks*.

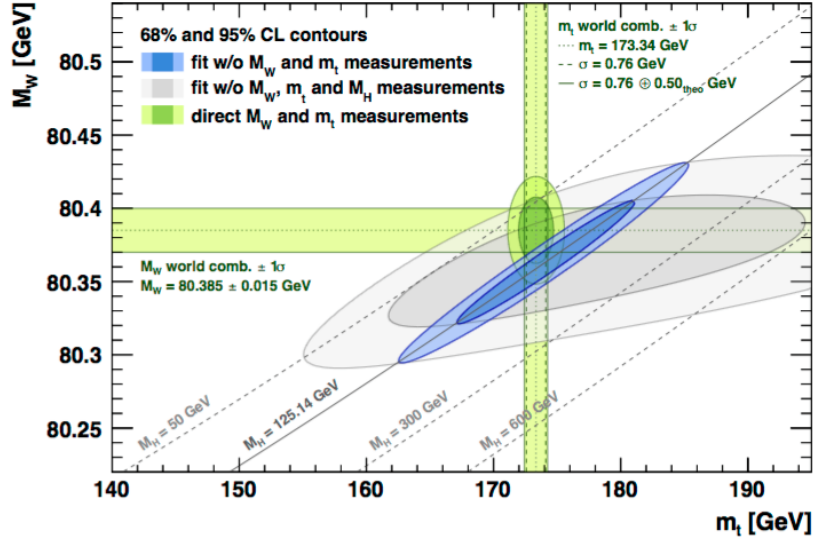


Figure 1.4: Scan of the confidence level profile of M_W vs m_t for the scenario where M_H is included in the fit (blue) or not (grey). The green band and ellipse represent the direct measurements. Figure from [14].

Thanks to their sensitivity to new physics, the precision measurements such as the W mass measurement are a complementary field to the direct searches in physics beyond the SM.

Chapter 2

The CMS experiment at LHC

This chapter contains the essential background to the facilities used in the analysis shown throughout this thesis. The first two sections give an overview on the Large Hadron Collider (LHC) and the characteristics of the physics events produced in its interaction points. The third section describes briefly the Compact Muon Solenoid (CMS) experiment, whose collected data have been employed for this work. The last sections deal with the tracking system of CMS, whose details are essential to understand the core of the following chapters, and the reconstruction of muons, which constitute an important tool in the CMS physics program.

2.1 The Large Hadron Collider

The Large Hadron Collider (LHC) is a circular proton-proton collider located at the European Organisation for Nuclear Research (CERN).

It is contained into a circular tunnel of 27 km of circumference, inside which, in two adjacent parallel beam pipes, the proton beams circulate in opposite directions in ultra-high vacuum. The beams are guided inside the pipes by a strong magnetic field of 8.33 T provided by 1232 superconducting dipole magnets. Moreover, 392 quadrupole magnets are used to focus and steer the beam.

The protons circulating in the pipes are gathered in bunches of $\sim 10^{11}$ protons. Each bunch is accelerated crossing 8 radio frequency cavities per turn, the field of the radio frequency being synchronised with the spacing between the bunches. Since for relativistic particles the orbit grows with their energy, the magnetic field must grow synchronously with the energy gained at each stage.

Once the magnetic field reaches its maximum value, the beams are brought into collision at four points around the ring, which host the four experiments of LHC: ATLAS [15], CMS [16], LHCb [17] and ALICE [18]. The first two are multi-purpose experiments, designed to search for the Higgs boson and new particles with masses at the TeV scale. LHCb studies the properties of

charm and beauty hadrons and ALICE analyses the data from relativistic heavy ion collisions to study the hadronic matter in extreme temperature and density conditions.

Since the LHC can not accelerate the protons from zero energy, various preliminary steps are necessary before the injection in the main accelerator. The full acceleration facility is composed of a linear accelerator (LINAC) and a chain of three synchrotrons: Booster, the Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS) where they are accelerated up to 450 GeV. The scheme representing the full CERN acceleration complex is shown in Figure 2.1

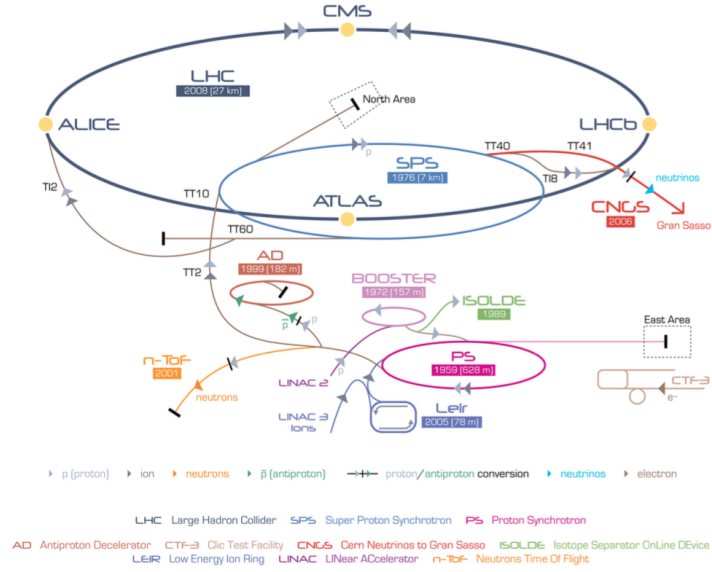


Figure 2.1: Scheme of the facilities of the CERN acceleration complex.

The LHC has been designed to collide protons at a nominal center-of-mass energy $\sqrt{s} = 14$ TeV. Another important aspect is the beam collision rate, which is proportional to the instantaneous luminosity $\mathcal{L}$:

$$\mathcal{L} = \frac{f k n_p^2}{4\pi \sigma_x \sigma_y}, \quad (2.1)$$

where f is the bunch revolution frequency, k the number of bunches, n_p the number of protons per bunch and σ_x, σ_y their transverse dispersion along the x and y axis. At the nominal 14 TeV LHC conditions ($\mathcal{L} = 10^{34} \text{ cm}^{-2}\text{s}^{-1}$) the parameter values are: $k = 2808$, $n_p = 1.5 \times 10^{11}$ and $\sigma_x = \sigma_y = 16.6 \text{ } \mu\text{m}$ (with $\sigma_z = 7.6 \text{ cm}$ along the beam).

The rate of production of a given process is proportional to $\mathcal{L}$ through the cross section σ . Therefore, if the process is very rare, it is necessary to

maximise $\mathcal{L}$ to collect enough statistics. The integrated luminosity is defined as $L = \int \mathcal{L} dt$ and it is usually quoted to show the amount of data available for the analyses.

The work described in this thesis was carried out on data collected by CMS and therefore this chapter will give a more detailed overview of this experiment.

The LHC started its research program in spring 2010 at a center-of-mass energy of 7 TeV, and CMS collected a total integrated luminosity of 5.6 fb^{-1} with a record peak luminosity of $4.0 \cdot 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$. In 2012 the center-of-mass energy has been increased to 8 TeV. CMS collected 22 fb^{-1} with a record peak luminosity of $7.7 \cdot 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$ until the beginning of 2013, when the LHC has been shut down to prepare the Run 2, at increased center-of-mass energy and luminosity.

The LHC has been reactivated in early 2015, and it is still running at 13 TeV energy in the center of mass. The work presented in this thesis has been performed on data collected by CMS in 2016.

2.2 Phenomenology of pp collisions

The proton-proton (pp) interaction at the intersection points of LHC is a complex phenomenon that involves the strong interaction of quarks and gluons composing the protons, also called partons. The interesting part of the collision is called hard scattering and it consists of the interaction of two partons at high transferred momentum q^2 .

Along with this, the event is accompanied by several other processes: the initial and final state radiation of gluons from the quarks and the soft scattering (i.e. at low q^2) of the remnants of the protons. All these interactions together are designated underlying event. Due to colour confinement¹, the quarks created in the scatterings must combine with other quarks and anti-quarks created from the vacuum and they form composite particles, hadrons. This process is known as hadronisation and it is not fully understood since it involves non-perturbative QCD. However, there are some models and parametrisations that describe it quite accurately and are used in the Monte Carlo simulations.

In addition to the underlying events, pp interactions at LHC are characterised by the presence of pileup events. The pileup is constituted of minimum bias events (i.e. events with no hard interaction) among protons in the same bunch crossing that cause the presence of many low energy particles. Figure 2.2 summarises with an illustration the complex of pp interactions.

¹Colour confinement is a feature of the QCD interaction that states that only non-QCD coloured objects can exist individually.

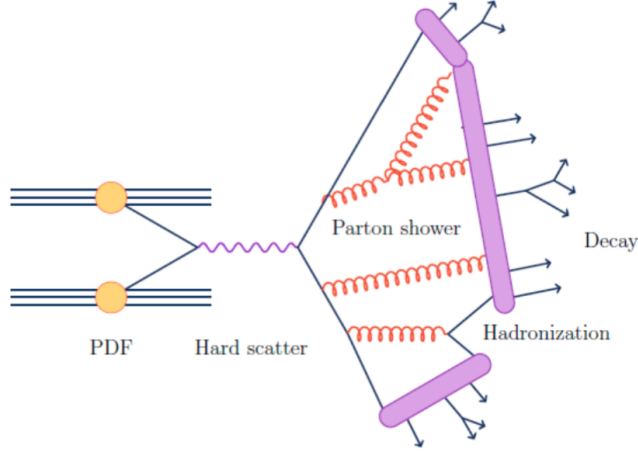


Figure 2.2: Illustration of the hard scattering process, parton shower, hadronization and decay during the generation of an event with two quarks in the final state. Figure from [6].

2.2.1 Parton distribution functions

In pp interactions the energy available in the center of mass is not fixed, but it depends on the fraction of the proton momentum carried by the partons that participate to the hard scattering. Therefore, the computation of the cross section of a given process must be weighted with the functions that encode the probability that the interacting partons carry a fraction x of the momentum of the proton.

These functions are known as Parton Distribution Functions (PDFs), which have to be determined experimentally. Thanks to the DGLAP [19] equations, which describe the q^2 dependence of the PDFs, the functions measured at other experiments can be scaled to fit the scale of energy of LHC. However, the knowledge of the PDFs is today one of the major theoretical systematics affecting the LHC precision measurements. There is currently an on-going effort from multiple groups to incorporate LHC data into the PDFs fits in order to reduce this uncertainty.

2.3 The CMS experiment

The Compact Muon Solenoid (CMS) detector is roughly cylindrical, 21.5 m long and with an outer radius of 7 m. It was designed around a 3.8 T magnetic field provided by a superconducting solenoid. An overview of the detector is given in Figure 2.3.

The reference frame used to describe the CMS detector and the collected events has its origin in the geometrical center of the solenoid. In a right-handed Cartesian coordinate system, the x axis points to the center of the

LHC ring, the y axis points upwards, perpendicular to the LHC plane and the z axis points towards the beam line. A cylindrical coordinate system is more often used. The $r = \sqrt{x^2 + y^2}$ coordinate points from the geometrical center of the cylinder outwards, and the two angles ϕ and θ are such that $\tan \phi = y/x$ and $\tan \theta = r/z$. Instead of the angle θ , the pseudorapidity, defined as:

$$\eta = -\ln \tan \frac{\theta}{2} \quad (2.2)$$

is more often used, since it is additive under boosts along the z axis. In this chapter, it is given a brief overview of the CMS detector, with particular emphasis on the subdetectors which are used in the analysis presented in this thesis. A more complete description of the CMS detector can be found in [16].

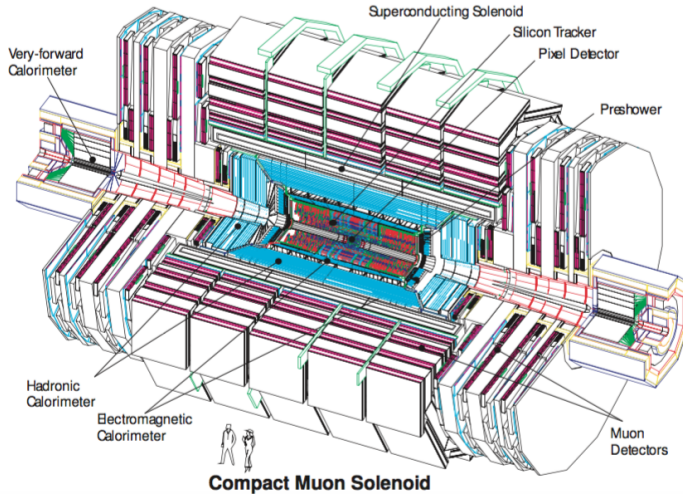


Figure 2.3: Overview of the CMS detector. Figure from [16].

2.3.1 Overview of the detector

The CMS detector consists of a cylindrical barrel up to $|\eta| < 1.2$ and the endcaps from $|\eta| = 1.2$ on. This two parts almost cover the whole solid angle, up to $|\eta| \sim 5$. The main detector is composed of several subdetectors that form layers at increasing values of r in the barrel and increasing values of $|z|$ in the endcaps. Each layer has a different role in the detection and identification of the particles generated in the collisions.

The experiment is built around the solenoid magnet that takes the form of a cylindrical coil of superconducting cables. The whole structure is supported by a steel yoke that forms the bulk of the detector and confines the magnetic

field. The tracker, the electromagnetic calorimeter (ECAL) and the hadronic calorimeter (HCAL) are located inside the solenoid, while in the outside the magnetic field is returned by an iron structure. Here the muon chambers are hosted, which constitute the last layer of the CMS experiment.

Tracker

The tracker is the innermost subdetector and the closest to the interaction point. It measures 5.4 m in length and 1.1 m in radius and its coverage extends up to $|\eta| = 2.5$. It is used to observe the charged particles and measure their momentum from their curvature in the magnetic field. The tracker is equipped with silicon detector: pixel modules in the inner region, to provide enough granularity close to the beam spot, and microstrip modules in the outer region.

A charged particle traversing the modules of the tracker knocks electrons off the valence band, thus creating electron-hole pairs. In the presence of an externally applied electric field the electrons and the holes are separated and collected by the electrodes, producing a signal proportional to the energy lost by the passing particle. In the language of tracking, a signal left in one module is called hit. Through algorithms that will be described in more details in section 2.4.2, the most probable trajectory of a charged particle in the tracker is fitted to a set of hits, allowing for the precise measurement of the momentum.

Calorimeters

The calorimeters are designed to absorb the particles to measure their energy. The ECAL is an homogeneous calorimeter, where the absorber material is the same as the sensitive one, made of lead tungstate (PbWO_4). It contains the electromagnetic showers of charged particles and photons and it produces light proportional to the energy of the initial particle, which is read out by photomultipliers. The ECAL has a barrel section and two endcaps for a total coverage up to $|\eta| = 3$. The thickness is 23 cm in the barrel and 22 cm in the endcaps.

The HCAL is designed to detect and absorbs hadrons. It is composed of layers of brass to stop the hadrons interleaved with tiles of plastic scintillators, whose signal is read out by photodiodes. Also the HCAL has a barrel section and two endcaps for a total coverage up to $|\eta| = 3$ and its thickness in the barrel amounts to 1.2 m.

The calorimeters are complemented in the high pseudorapidity region with two forward hadronic calorimeters (HF) which extend the coverage to $|\eta| = 5$. They consist of layers of steel and quartz read out by photomultiplier tubes.

Muon chambers

The muon system is equipped with various kinds of technologies. In the region $|\eta| < 1.3$ the muon tracking system is made of drift tubes and in the region $0.9 < |\eta| < 2.4$ of cathode strip chambers. Resistive plate chambers are installed in the region $|\eta| < 2.1$ to provide an additional trigger system. A more detailed description of the muon system is given in section 2.5.1.

2.3.2 Trigger

The trigger is an essential tool in the LHC experiments, as the amount of data produced per second is huge. It is then necessary to introduce the trigger, which makes an on-line choice on the data to be kept by the experiment on the basis of their physics content.

In CMS the triggering system is divided in two steps. The first one is the Level-1 Trigger (L1), implemented only in hardware. It exploits the information from the calorimeters and the muon chambers to identify possible interesting processes. This first step is completed in about $1\ \mu\text{s}$, and reduces the rate down to 100 kHz.

If an event passes the L1, it is passed to the High-Level Trigger (HLT), which exploits a simplified form of the software used to off-line analyses for partially reconstructing the event and therefore choosing the most interesting ones. The HLT reduces the event rate further down to a few 100Hz.

2.4 The tracking system of CMS

The tracker is an essential tool in a multi-purpose detector like CMS, as the reconstructed trajectories of the charged particles are a vital ingredient in the measurement of the majority of observables used in physics analyses.

2.4.1 The CMS tracker in a nutshell

The CMS tracker is entirely based on silicon detector technology. While a large number of read-out channels is desirable to increase the granularity of the system, the material used for sensors, electronics, support structures and services must be light enough in order to interact as little as possible with the particles produced in the collisions.

The tracker is composed of 1440 silicon pixel modules and 15148 silicon microstrip modules organised in layers around the interaction point. The regions very close to the interaction point have a higher density of particles and therefore high granularity pixel detectors are needed. The intermediate and outer regions, where the density of particles is reduced, are equipped with microstrip detectors. An overview of the tracker is given in Figure 2.4.

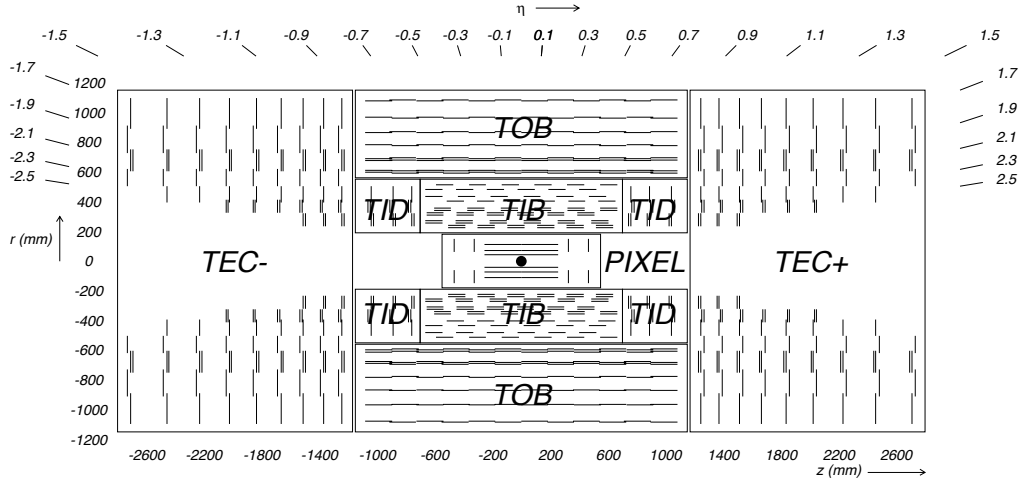


Figure 2.4: Schematic cross section of the CMS tracker in the $r - z$ plane. The strip tracker modules that provided 2D hits are shown by thin, black lines, while those allowing the reconstruction of the third coordinate are shown by closely spaced double lines. The pixel modules also give 3D hits. Within a given layer, each module is shifted slightly in r or z with respect to its neighbours to avoid gaps in the acceptance. Figure from [16].

Pixel detectors

The pixel modules are arranged around three concentric layers installed at radii 4.4 cm, 7.3 cm and 10.2 cm. They are completed to each side of the barrel by two endcaps consisting in 2 disks of pixel detectors, extending from 6 cm to 15 cm in radius at 34.5 cm and 46.5 cm from the nominal interaction point. This ensures the existence of three measurement points for each track over almost the whole acceptance. All the pixel sensors have a cell size of $100 \times 150 \mu\text{m}^2$ with the larger side along the z coordinate, and they can deliver a measurement of the hit position in the three coordinates r , ϕ and z .

The charge carriers that produce the signal in the pixel sensors are the electrons. The effect of the magnetic field in the barrel (Lorentz angle, described in [20]) and the special arrangement with slightly tilted modules in the endcaps ensures the charge signal to be spread over more than one pixel. This permits to reduce the spatial resolution to $\sim 15 \mu\text{m}$ for high momentum tracks that cross the modules perpendicularly.

Microstrip detectors

Further away from the interaction point the silicon strip tracker is installed. The inner silicon strip tracker is composed of the tracker inner barrel (TIB) with four layers and the tracker inner disks (TID) with three endcap layers. The outer part consists of the tracker outer barrel (TOB) composed of six

layers and the tracker endcaps (TEC) composed of nine disks. Within a given layer, each module is shifted slightly in r or z with respect to its neighbour, and the modules overlap, to avoid gaps in the acceptance.

The typical silicon strip module size is $10\text{ cm} \times 5\text{ cm}$ with a strip pitch of $80\text{ }\mu\text{m}$ in the inner regions and $20\text{ cm} \times 10\text{ cm}$ with a strip pitch of $140\text{ }\mu\text{m}$ in the outer regions. In the barrel, the strips are parallel to the z axis and in the endcap they are placed along the radial coordinate. The modules provide a measurement of the $r - \phi$ coordinate with a resolution of $20 - 50\text{ }\mu\text{m}$.

To measure the z coordinate in the barrel and the r coordinate in the endcap with a precision better than the strip length some layers have an additional set of modules, tilted with respect to the original ones by a stereo angle of 100 mrad . The measurement of the third coordinate is obtained through the matching of the hits measured by the tilted modules with the ones obtained with the regular ones.

2.4.2 Algorithms for track reconstruction

The procedure of fitting the trajectory of charged particles to the hit pattern is called track reconstruction.

The charged particles travel into the tracker in the magnetic field on a helical trajectory, described by 5 parameters: the curvature $k = q/p_T$, the azimuthal and polar angles ϕ and η , the transverse impact parameter d_{xy} and the longitudinal impact parameter d_z . The impact parameters d_{xy} and d_z are respectively the r and z coordinate of the track at the point of closest approach to the beamline. The standard algorithm used in CMS for track reconstruction is the Combinatorial Track Finder (CTF) algorithm [21], which is developed in three steps: track seeding, track finding and track fitting.

The track seeding consists in a loop on all pairs of hits compatible with some kinematical cuts. The seeding starts from the innermost pixel detectors since the high resolution on the position of the hit reduces the number of options to consider.

The track finding and fitting steps are based on a standard Kalman filter pattern recognition approach, starting with the seed parameters. This method is used to estimate the state of a dynamic system from a series of measurements with corresponding uncertainties. In the track reconstruction the state is given by the helicoidal parameters that are propagated layer by layer with an inside-out approach and fitted to the hits with a χ^2 fit. At each step the computation of the parameters is updated with local information on the magnetic field, the spatial uncertainty and the quantity of material crossed by the particle. This is shown in Figure 2.5 expressed in terms of fractional radiation lengths, is taken from the simulation and has never been measured precisely on data so far. This topic will be extensively discussed in chapter 5 and 7.

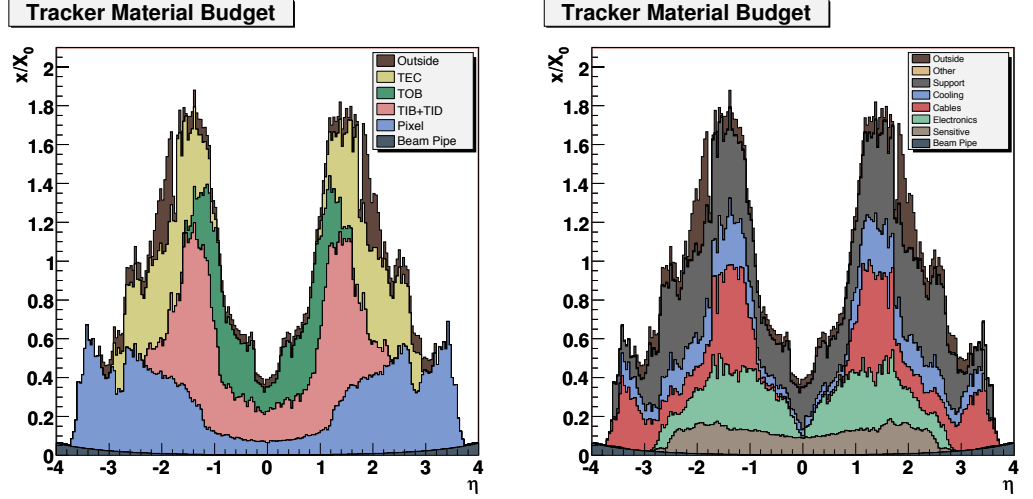


Figure 2.5: Material budget seen by a particle produced in the center of CMS and crossing the whole volume of the Tracker. The material is expressed in terms of fractional radiation lengths as a function of the particle pseudorapidity and shown for the different sub-detectors (left panel) and broken down into the functional contributions (right panel). Figure from [16].

The tracks are assigned a quality based on the χ^2 and the number of missing hits, and only the best quality tracks are kept. Once the whole information is available, the Kalman filter is re-run with an outside-in approach. The final output is the full set of helix parameters and the full covariance matrix describing the correlations of the parameters and their errors. Figure 2.6 shows the expected muon reconstruction efficiencies and muon transverse momentum resolution for the CMS tracker.

Once the full track is reconstructed, the sagitta of the trajectory due to magnetic field is measured and used to derive the transverse momentum² of the particle³. The bending radius of the trajectory due to magnetic field is linked to the transverse momentum by:

$$p_T [\text{GeV}] = 0.3B\rho [\text{Tm}] \quad (2.3)$$

and to the sagitta by:

²The transverse momentum is the component of the momentum projected along the transverse plane with respect to the beamline.

³This is a simplifying statement. As discussed in chapter 4 the map of the magnetic field is used to measure the bending radius and the transverse momentum of the track at the point of closest approach to the beam line.

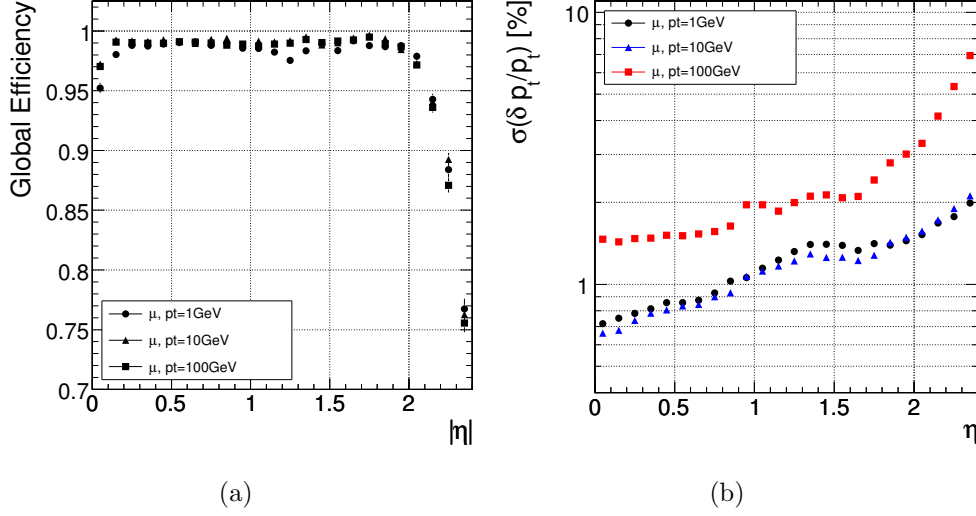


Figure 2.6: Global track reconstruction efficiencies (a) and resolution of transverse momentum (b) for single muons with transverse momenta of 1, 10 and 100 GeV, as computed from simulation before the startup of the experiment in the year 2008. Figures from [16].

$$s = \frac{L^2}{8\rho} = \frac{0.3BL^2}{8p_T}, \quad (2.4)$$

where L is the length of the track measured on the transverse plane.

Tracker alignment

One major challenge in tracking is represented by the alignment of the modules of the tracker. In order to properly reconstruct particle trajectories, the relative positions of the tracker components must be known to a precision better than the intrinsic resolution of the modules.

The alignment procedure is run periodically in the CMS tracker. This optimisation problem can be formulated in the context of linear least squares. The goal is to derive a set of alignment parameters, the modules position corrections, minimising through a χ^2 fit the distance between the track impact point and the related hit. In practice the reconstructed tracks are made better fit the hits by adjusting the assumed positions of the modules.

The procedure consists of various steps. The first step is to adjust the absolute position and orientation of the tracker relative to the magnetic field. This is done using two alignment parameters that describe the tilts around the x and y axes. After that, the individual modules are aligned. The position of each module is parametrised using nine parameters. Three of these describe the translational shift of the module from the nominal position and three describe the rotational shift. The last three parameters describe deviations in the module geometry from a flat plane. As described

in 2.4.1, within a given layer, each module overlaps slightly in r or z with its neighbours. This feature offers a further constraint on the fit of the alignment parameters, as the overlaps between the modules tightly constrain their relative position on the circumference of each barrel layer and each endcap ring. The length scale of the tracker is given by the position of the strips and pixel in a given silicon module, which is controlled to a precision of 10^{-4} by lithography.

The alignment is performed using cosmic ray data as well as data recorded during the LHC operations. The cosmic data are very useful since they impose vertical constraints, while the collision data constrain the center of the tracker. The tracker geometry is found to be very stable with time and the statistical accuracy of the alignment procedure is such that misalignment effects are small compared to the intrinsic hit resolution of the modules.

This calibration provides a correction for the major part of the alignment problem. However, a residual in the correction after the procedure is represented by so-called weak modes. These result from combinations of alignment parameters that do not change the track-hit residuals and therefore do not alter the total χ^2 . A weak mode can result for instance from a twist where modules are moved coherently in ϕ by an amount proportional to the position along the z axis. Another example of weak mode is the alteration of the scale of the tracker due to deformations of the tracker shape from a perfect cylinder.

The weak modes have an impact on the momentum scale of tracks which is far below the nominal resolution. However, as extensively described in chapter 4, if the level precision needed for this observable is very high, it becomes important to correct for this effect.

2.5 Muons in CMS

Muons are very important tools in CMS. They are the only particles that can traverse the whole detectors essentially unharmed and leave clear signatures in the muon chambers. Moreover, they can not be produced directly in soft QCD interactions that dominate the physics production at LHC and constitute a hint for more interesting events. For this reason, they are the main tool for triggering. On average, a muon in the barrel loses 3 GeV of transverse momentum before it reaches the first muon station and another 3 GeV between the first and the last muon station. Muons with lower energy can therefore not be reconstructed.

2.5.1 Muon system

The muon system is shown schematically in Figure 2.7. It is composed of different types of gas detectors organised in layers in the return yoke of the magnet. In the barrel, up to $|\eta| = 1.3$, the muon system consists of four

but adding the constraint that the muons come from the beamspot it is possible to exploit the full lever arm of 12 Tm.

Afterwards, the tracks are combined following two possible approaches. The first one is to start from a standalone muon and search for a matching track, then refit into a global muon combining the hits. This gives major improvements in the measurement of the momentum for very high energy muons.

The second one is to extrapolate the tracker track to the muon system taking into account magnetic field, energy losses and multiple scattering, and search for at least a matching segment in the muon chambers. This improves the measurement of the momentum for low energy muons that stop inside the muon chambers. The majority of muons are reconstructed using both approaches.

In practice, muons with transverse momentum below 200 GeV undergo significant multiple scattering in the muon system which reduces the resolution and therefore the tracker drives the momentum resolution.

Chapter 3

A precise measurement of the W mass

As extensively discussed in section 1.2.2, a high precision measurement of the W mass represents a crucial test of internal consistency for the SM. The electroweak global fit after the Higgs discovery leads to a value of M_W of 80.358 ± 0.008 GeV. This number can be compared to the world average experimental value which is 80.385 ± 0.015 GeV. If a measurement with improved precision is achieved, the level of agreement between the measured and predicted value could shed new light on our understanding of the fundamental interactions.

In this chapter we give an overview of the past and foreseen measurements of the W mass.

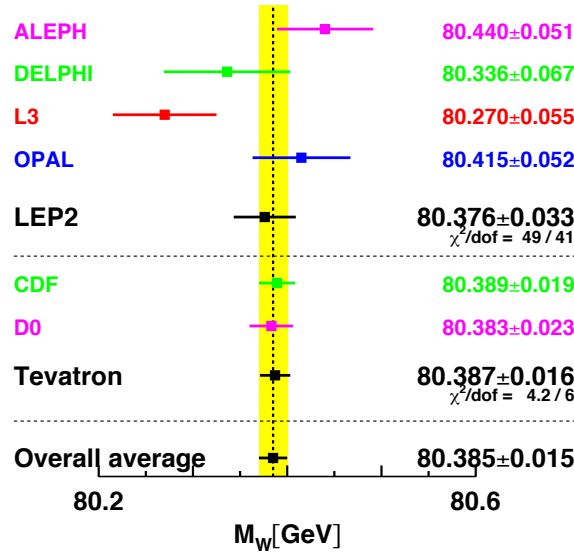


Figure 3.1: Summary of the past measurements of the W mass. The most precise measurement is quoted by CDF (19 MeV) while the current world average is (15 MeV). Figure from [22].

3.1 Historical overview

The W mass has been measured in the past years by experiments at LEP and Tevatron. Figure 3.1 summarises their results. Currently the most precise measurement has been quoted by CDF using the data from the Run 2 of Tevatron (19 MeV) while the current world average is 15 MeV. Below follows a brief description of these analyses. We discuss separately the measurements at lepton and hadron colliders since the physics involved is of different nature.

3.1.1 The W mass measurement at lepton colliders

At lepton colliders the W bosons are mainly produced in pairs through the reaction $e^+e^- \rightarrow W^+W^-$.

Each W subsequently decays either hadronically ($q\bar{q}$) or leptonically ($l\nu$, $l=e,\mu,\tau$). There are three possible four-fermion final states, fully hadronic ($q\bar{q}q\bar{q}$), semileptonic ($q\bar{q}l\nu$), and leptonic ($l\nu l\nu$), with branching ratios of 46%, 44% and 10% respectively.

There are two main methods to measure M_W at lepton colliders.

The first one exploits the fact that the W^+W^- production cross section σ_{WW} is sensitive to M_W in the threshold region, $\sqrt{s} \sim 2M_W$. A measurement of σ_{WW} can then be used to measure M_W . Figure 3.2 shows the W pair production cross section as a function of the center of mass energy for various values of M_W . Only near the energy threshold the three curves are enough separate, allowing for a determination of M_W from σ_{WW} .

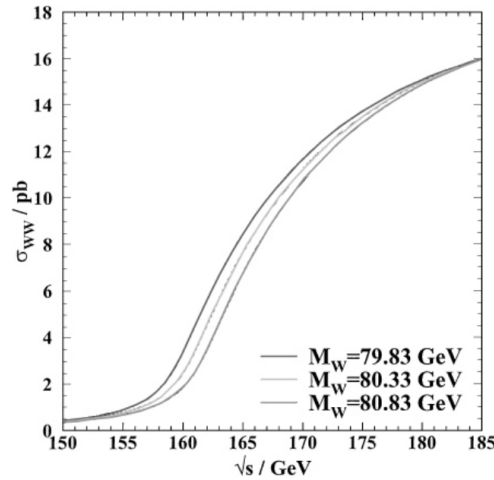


Figure 3.2: $e^+e^- \rightarrow W^+W^-$ cross section as a function of the center of mass energy for various M_W values. Figure from [23].

The second method uses the shape of the reconstructed invariant mass distribution, constraining the initial momentum to be zero.

LEP

From 1996 to 2000 LEP ran at center-of-mass energy above the WW production threshold, $\sqrt{s} \sim 2M_W$. With a subset of the data collected, $\sim 10 \text{ pb}^{-1}$ at $\sqrt{s} \sim 161 \text{ GeV}$, a first measurement of the M_W has been delivered using the production cross section σ_{WW} :

$$M_W = 80400 \pm 200(\text{stat}) \pm 70(\text{syst}) \pm 30(E_{\text{beam}}) \text{ MeV}. \quad (3.1)$$

The statistical uncertainty dominates the systematics. This latter is mainly driven by the modelling of fragmentation and hadronisation of quarks which has a large effect on the fully hadronic decay channel and contributes to the final measurement for $\sim 50 \text{ MeV}$.

With the full dataset of about 700 pb^{-1} LEP published its final measurement fitting the invariant mass shape of the decay products of each W produced in $e^+e^- \rightarrow W^+W^-$ events. This variable is the most sensitive observable to M_W in those kind of events and it can be precisely reconstructed because the energy of the initial state e^+e^- is fixed by the beam energy. Only in fully leptonic decays the final state is not completely known due to the two neutrinos and in that case the leptonic energy spectrum is used, since it peaks at $M_W/2$ and thus it is sensitive to M_W . The final result is:

$$M_W = 80375 \pm 25(\text{stat}) \pm 22(\text{syst}) \text{ MeV}. \quad (3.2)$$

The main sources of systematic uncertainty in this measurement are shown in the summary table in Figure 3.3.

Source	Systematic error on M_W (MeV)		
	$q\bar{q}l\bar{\nu}_l$	$q\bar{q}q\bar{q}$	Combined
ISR/FSR	8	5	7
Hadronization	13	19	14
Detector systematics	10	8	10
LEP beam energy	9	9	9
Color reconnection	–	35	8
Bose–Einstein correlations	–	7	2
Other	3	11	4
Total systematic	21	44	22
Statistical	30	40	25
Total	36	59	33

Figure 3.3: Summary of the uncertainties in the combined LEP measurement of M_W based on direct mass reconstruction in semileptonic and fully hadronic channels. From [24].

The most important contribution to the systematics are given by the modelling of the hadronisation and fragmentation of quarks together with the colour reconnection which has the biggest impact on fully hadronic events. This effect changes the colour flow between the four quarks in the final

state and alters the hadronisation process due to the transfer of momentum between the W 's.

3.1.2 W production and decay at hadron colliders

At hadron colliders the W boson is produced at leading order (LO) directly through the annihilation of a quark and an antiquark, as shown in Figure 3.4.

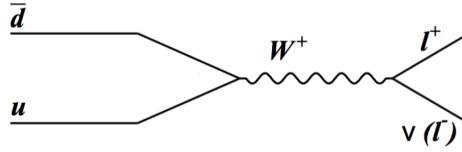


Figure 3.4: Leading-order process from the annihilation of a quark and antiquark inside the colliding (anti)protons, producing a W boson. The $d\bar{u} \rightarrow W^-$ process is similar.

At $p\bar{p}$ colliders, such as Tevatron, the hard scattering is mainly among valence quarks, while at LHC the hard scattering quarks are mainly u and d valence quarks with sea anti-quarks, with a smaller contribution from other flavour sea quarks. The ingredients needed for determining the production cross section are the amplitude, which is a purely electroweak process and can be calculated perturbatively, and the PDFs of the proton, which are determined from data.

Figure 3.5 shows the predicted and measured cross sections in CMS for different processes. Predictions and measurements for the W boson match very well, and moreover its production rate is quite abundant, allowing for a large dataset.

Contrary to the lepton colliders, at hadron colliders the energy of the initial state is not fixed. Therefore, it is very difficult to reconstruct precisely the final state of a W decaying hadronically. Moreover, this channel has an overwhelming background due to QCD. The semileptonic channel is then preferred, and particularly the muon channel, given the purity with which the muons are reconstructed. However, the presence of the undetected neutrino in the final state, whose properties must be inferred from conservation principles, precludes the possibility to fully reconstruct the invariant mass of the event. Since the initial partonic p_z are not known, it is only possible to apply the conservation of momentum in the transverse plane, where the sum of the momenta of the colliding partons is assumed with very good approximation to be zero.

A W event is then characterised by the transverse momentum of the visible lepton, p_T^l and the $E_T^{\text{miss}} = -\sum^{\text{all particles}} \vec{p}_T$ ¹, the balance of the missing

¹For historical reason it is common practise to call the missing transverse momentum as missing transverse energy.

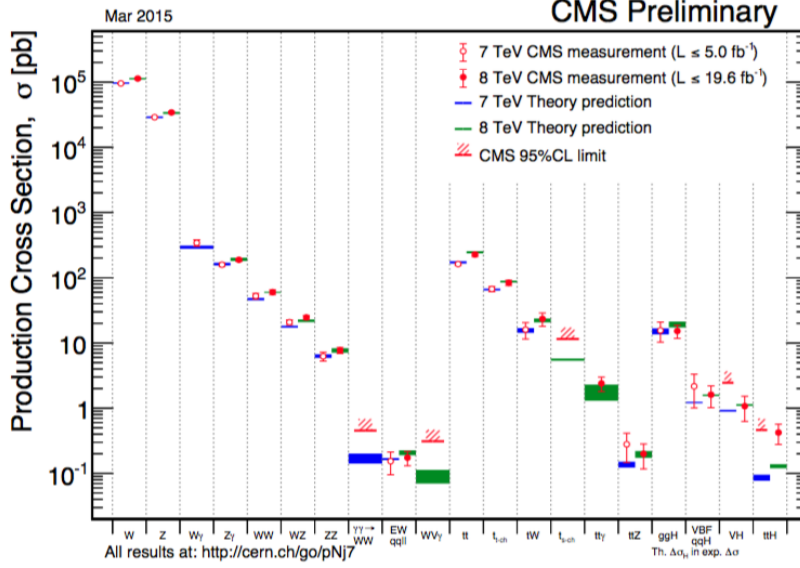


Figure 3.5: Predicted and measured cross sections in CMS for different processes at the LHC at 7 and 8 TeV.

transverse energy in the detector. Using these observables we introduce the *transverse mass*, defined as:

$$M_T = \sqrt{2p_T^l E_T^{miss}(1 - \cos \phi)}, \quad (3.3)$$

where ϕ is the angle between the lepton transverse momentum and the E_T^{miss} in the transverse plane. This observable coincides with the invariant mass of the W if $\Delta\eta$ of muon and neutrino is zero. The peak at the value of M_W is also known as Jacobian peak. The lepton transverse momentum and the E_T^{miss} peak at $M_W/2$. Therefore, all the three variables are in principle sensitive to M_W . When measured, the M_T , E_T^{miss} and p_T^l distributions are to be convolved with the detector and production effects which smear their peaks, as shown in Figure 3.6. The lepton p_T distribution is significantly affected by the boson p_T , while the M_T distribution is more sensitive to detector effects.

3.1.3 Measurement strategy at hadron colliders

To extract the W mass the template method is used. The distributions of the M_T , E_T^{miss} and p_T^l in data are compared to the templates generated by a calibrated Monte Carlo simulation varying the mass of the W . For each value of M_W and for each distribution a maximum likelihood binned fit picks the value of M_W that minimises the difference between data and calibrated Monte Carlo. This value is taken as the best estimation of M_W .

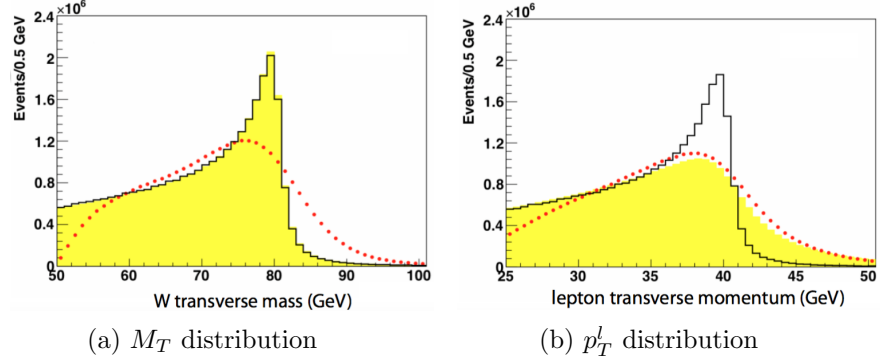


Figure 3.6: Effect of the W boson p_T and detector smearing on the W m_T (a) and lepton p_T (b) distributions. The generator level distributions with W boson at rest are represented by the solid black histogram; they become the filled yellow histograms once the boson p_T effects are considered and the red dashed curves once the smearing detector effects are taken into account. The lepton p_T distribution is significantly affected by the boson p_T , while the M_T distribution is more sensitive to detector effects.

Together with the experimental uncertainties which affect the measurement of the aforementioned observables, the measurement of the W mass at hadron colliders is affected by theoretical systematics. These uncertainties are an estimate of our level of knowledge of physical models used to simulate the events in the Monte Carlo. The most relevant are the degree of knowledge of the PDFs, the electroweak corrections due to final state radiation and of distribution of the transverse momentum of the W , p_W . This latter appears when QCD corrections to gauge bosons production are considered and therefore predicting precisely p_W means dealing with calculations that, at low energy scale, are non-perturbative.

Tevatron

Both CDF and D0 experiments at Tevatron delivered M_W measurements during the Run 1 and Run 2. To calibrate the E_T^{miss} and p_T^l they have used resonances such as Z , J/ψ and Υ decaying to leptons, reducing the experimental systematics to very competitive ones. Also the quoted theoretical systematics are quite small numbers. The details of the calibrations can be found in [24]. In the table in Figure 3.7 we show the uncertainties quoted by CDF which to date has delivered the most precise measurement of M_W :

$$M_W = 80387 \pm 12(\text{stat}) \pm 15(\text{syst}) \text{ MeV.} \quad (3.4)$$

Source	Uncertainty (MeV)
Lepton energy scale and resolution	7
Recoil energy scale and resolution	6
Lepton removal	2
Backgrounds	3
$p_T(W)$ model	5
Parton distributions	10
QED radiation	4
W -boson statistics	12
Total	19

Figure 3.7: Summary of the uncertainties in the CDF W mass measurement at Run 2 (2.2 fb⁻¹). From [25].

3.2 Prospects for the first measurement of the W mass at CMS

A W mass measurement at LHC is highly wished to eventually reach the requested experimental precision (~ 10 MeV) to allow a fair comparison with the predictions. It must be highlighted that a measurement claiming such high levels of precision is a very challenging one, implying the calibration of all the observables sensitive to M_W at levels which are orders of magnitude better than the ones requested for a typical analysis in the LHC experiments. Also, the reduction of the theoretical systematics to the level of the ones in Tevatron needs a huge effort from both the experimental and theoretical community.

However, there are many advantages that LHC has with respect to the past facilities. First of all, the amount of data collected during the Run 1 allows a significative reduction of the statistical uncertainty on the measurement. The dataset at 7 TeV amounts to 4.5 fb⁻¹, propagating to the final measurement as 10 MeV. If we add the dataset at 8 TeV (20 fb⁻¹), the statistical uncertainty is reduced to 5 MeV, meaning that for the first time the W mass measurement would be dominated by the systematics. What is more, bigger samples allow for better calibrations, essential tools to validate the level of control of the observables sensitive to M_W .

On the other hand, there are some challenging aspects. The first one is the pileup conditions, affecting mostly the E_T^{miss} distribution. This calls for a robust definition of E_T^{miss} in order to overcome this issue. What is more, the new center-of-mass energy regime with respect to Tevatron imposes in principle a higher systematics on the PDFs.

Another feature of the W production at the LHC is the asymmetric W^+ and W^- production, forcing the analysis to be charge-dependent.

Today CMS has reached the important milestone of measuring the Z mass

using experimental techniques which are very close to the ones which will be used for the W mass measurement, showing the community that it is possible to reach the desired level of precision in the calibration of the experimental observables. Nevertheless, getting a competitive measurement of the W mass still needs various refinements both in the calibrations and in the precise understanding of the theoretical systematics.

3.2.1 The $Z W_{like}$ measurement

The $Z W_{like}$ measurement in CMS has been carried on a sample of Z dimuon events corresponding to an integrated luminosity of 4.5 fb^{-1} . The purpose of this analysis has been to provide a quantitative validation of the techniques that are being developed for the W mass measurement. The most important aspect of this analysis are the calibration of the observables sensitive to M_W . The details for this measurement can be found in [26].

To obtain a W_{like} event we remove one of the two muons from the Z events and we compute the E_T^{miss} . Events of this kind are characterised by the leading muon and the hadronic recoil: the sum of the transverse momenta of all other particles produced at the same vertex of the W . The scale of the first object is $\sim 45 \text{ GeV}$, while the hadronic recoil peaks at $\sim 5 \text{ GeV}$. If we recall the definition of the transverse mass, eq. 3.3 and we substitute $E_T^{miss} = -\vec{p}_T^l - \vec{h}$, where $\vec{h}$ is the hadronic recoil and make a Taylor expansion around the small quantity $|\vec{h}|/p_T^l$ we obtain:

$$M_T \sim 2p_T^l + h_{\parallel} \quad (3.5)$$

where $h_{\parallel}$ is the projection of the hadronic recoil along the direction of the transverse momentum of the muon. From eq. 3.5 we deduce that if we want to measure M_W with an uncertainty of 10 MeV, we must control the scales of the transverse momentum of the muon and of the hadronic recoil at a level of 10^{-4} and 10^{-3} respectively. In order to get this degree of precision it is necessary to devise dedicated calibration procedures for these two observables.

Calibration of the experimental observables

The calibration of the muon momentum scale is discussed extensively in section 4.

The calibration of the hadronic recoil is made on $Z \rightarrow \mu\mu$ events since this sample has low background and the topology of the event is very similar to the W one.

To effectively study the properties of the hadronic recoil, we project the recoil vector along the directions parallel and perpendicular to the Z transverse momentum direction: the parallel component should be proportional

to the boson transverse momentum, while the perpendicular component gets mainly contributions from the underlying event and it is expected to be distributed around zero. A sketch of the kinematics of Z events is shown in Figure 3.8.

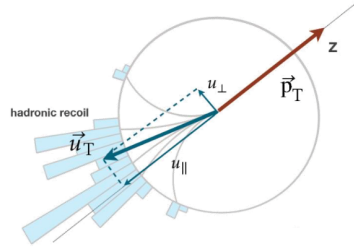


Figure 3.8: Sketch illustrating the kinematics of Z events.

Since the hadronic recoil is affected by the pileup conditions, the task of calibrating translates into finding an appropriate definition of this observable that minimises the systematics on its scale. The choice was obtained by using all the reconstructed charged tracks compatible with the muon vertex. While this definition, called $tkMET$, has the drawback of only retaining 40% of the hadronic recoil probed using all the tracks, it has the advantages of exhibiting a better data-MC agreement and of being essentially insensitive to pileup. More importantly, $tkMET$ provides, in the presence of pileup, the best discriminating power for the transverse mass Jacobian peak, as it is shown in Figure 3.9.

The result obtained on the W_{like} measurement is compatible with the value on the PDG and, more importantly, the experimental systematics are of the order of 20 MeV, meaning that we are on the right path to get a competitive measurement of the W mass.

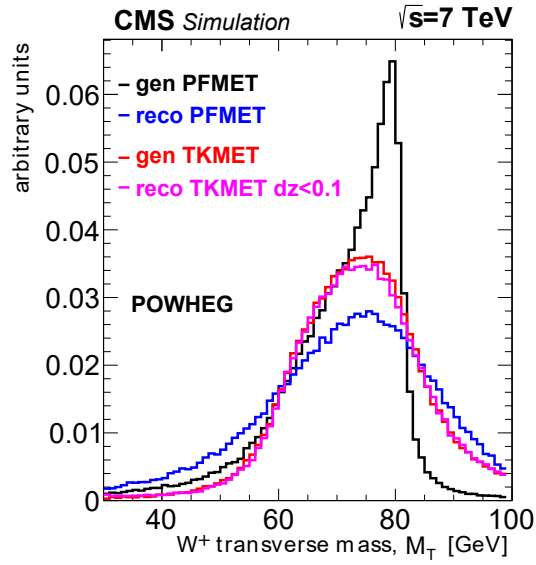


Figure 3.9: This plot shows the M_T built using different definitions of the E_T^{miss} : all the particles in the event except the muon from W decay (blue), all the generated particles in the event except the muon from W decay (black), only the reconstructed tracks - $tkMET$ - (pink), only the reconstructed generated tracks (red). The $tkMET$ provides the best discriminating power for the Jacobian peak, as it is the closest curve to the black one. Figure from [26].

Chapter 4

Calibration of the muon momentum scale

The precise knowledge of the muon momentum scale is one of the most important systematics in the W mass measurement, as extensively discussed in chapter 3. In order to achieve the precision required on the final measurement, the muon momentum scale must be calibrated at the level of 10^{-4} . This implies a deep understanding and control of all the detector effects that could bias this observable. This chapter is devoted to the detailed description of the calibration procedure and results.

4.1 Detector effects that bias the muon momentum scale

The transverse momentum p_T is measured from the sagitta of its track in magnetic field, as discussed in section 2.4.2. The track is reconstructed using information on the location of each module, on the magnetic field and on the material in the tracker. Therefore, any flaws in the knowledge of these inputs will result in a bias in the muon momentum scale that has to be parametrised and corrected. In this section we discuss the causes of the mismodelling of the magnetic field, of the alignment of the modules and of the energy loss in the tracker and how they propagate to the muon momentum scale.

4.1.1 Magnetic field

The map of the magnetic field in CMS had been precisely measured before the experiment was installed in the cavern. However, in order to save computing time, during the reconstruction of the tracks an approximate map is used instead of the real one. Figure 4.1 shows the ratio of the field integral, along straight lines from the origin to the last point in the tracker, computed with the real and approximate map as a function of η and ϕ . It shows varia-

tions of the order of 10^{-3} that propagate to the muon momentum scale since the transverse momentum is directly proportional to the magnetic field, as eq. 2.3 shows. Moreover, after the installation underground, the variation of the magnetic field has been measured using NMR probes. It was measured to be 0.9992 and it is not clear how these effects propagate to the tracker volume, but it is plausible that the magnetic field is underestimated of the same amount. Therefore, the bias on the momentum scale would be of the order of $8 \cdot 10^{-4}$ which exceeds the desired precision.

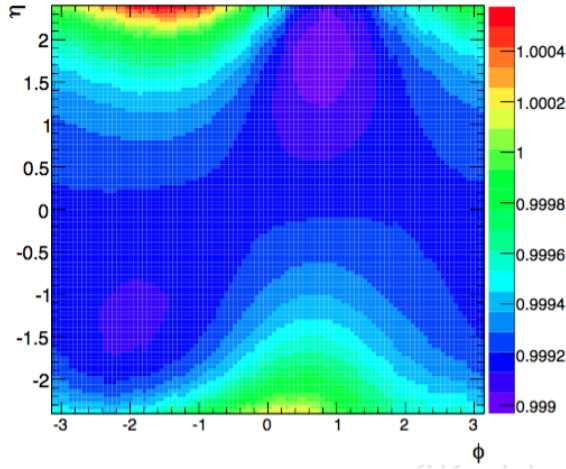


Figure 4.1: Ratio between two CMS magnetic field maps, in the η vs. ϕ plane: approximate map over measured map

4.1.2 Misalignment

The alignment procedure of the tracker is described in section 2.4.2. After this procedure the positions of the modules are known with a statistical accuracy below the intrinsic measurement precision of the sensors. However, there is a residual bias introduced by the weak modes that propagates to the muon momentum scale which is comparable to the level of precision we wish to achieve.

4.1.3 Energy loss

A muon traversing the tracker loses energy due to ionisation. This effect is described by the Bethe-Bloch formula, which models the energy loss of muons given the properties of the traversed material. For a tracker of uniform material with density 0.2 g cm^{-3} we would have a loss of $\sim 60 \text{ MeV}$ for a muon of 60 GeV . As the material with which the tracker of CMS was built is far from being constant, as extensively discussed in Chapter 7, the energy loss is computed from the local information of the material in the

tracker during the reconstruction of the tracks using the information from the Bethe-Bloch formula and a model of the material distribution. Therefore, an imperfect modelling of the local material budget would result in a bias in the energy loss and would in turn affect the momentum measurement at a level above the precision required. This problem will be further discussed in Chapter 7.

4.2 Samples used in the calibration

The calibration of the muon momentum scale and resolution is performed on measured and simulated samples of J/ψ and the closure is checked with samples of Z dimuon events at 13 TeV c.o.m. energy collected by CMS in fall 2015. The simulated samples used in this analysis are generated, respectively, with PYTHIA8 [27] and with POWHEG [28] interfaced with PYTHIA8.

The events of J/ψ dimuon are selected in subsequent trigger levels, where the muons are paired and required to have an invariant mass in the regions 2.9-3.3 GeV. The dimuon transverse momentum must be above 10 GeV. In this analysis we further require the muons to have $p_T > 4$ GeV. The events are collected up to $|\eta| < 2.5$. Figure 4.2 shows the kinematic distributions for the measured and simulated samples. Data and simulation show an overall agreement. The difference in the distribution of η in the J/ψ sample is due to the trigger which is not simulated in Monte Carlo.

The events in the Z sample are selected at trigger level by requiring the most energetic muon to fulfill $p_T > 17$ GeV and the other to fulfill $p_T > 8$ GeV. In this analysis we further request the invariant dimuon mass in a window 70-120 GeV. Figure 4.3 shows the kinematic distributions for the Z samples.

4.3 Calibration procedure

The calibration of the muon momentum scale is performed using a physics-motivated model that parametrises the three biasing effects described in section 4.1 and propagates their effect to the dimuon mass. Since these effects are assumed to vary with the geometry of the tracker, the solid angle is divided in bins of η and ϕ . In each bin the parameters are disentangled and extracted from a fit to the invariant dimuon mass peak of the control samples. In this way we derive a set of correction factors which can be applied on each event in the sample used to measure the W mass to correct the muon momentum scale.

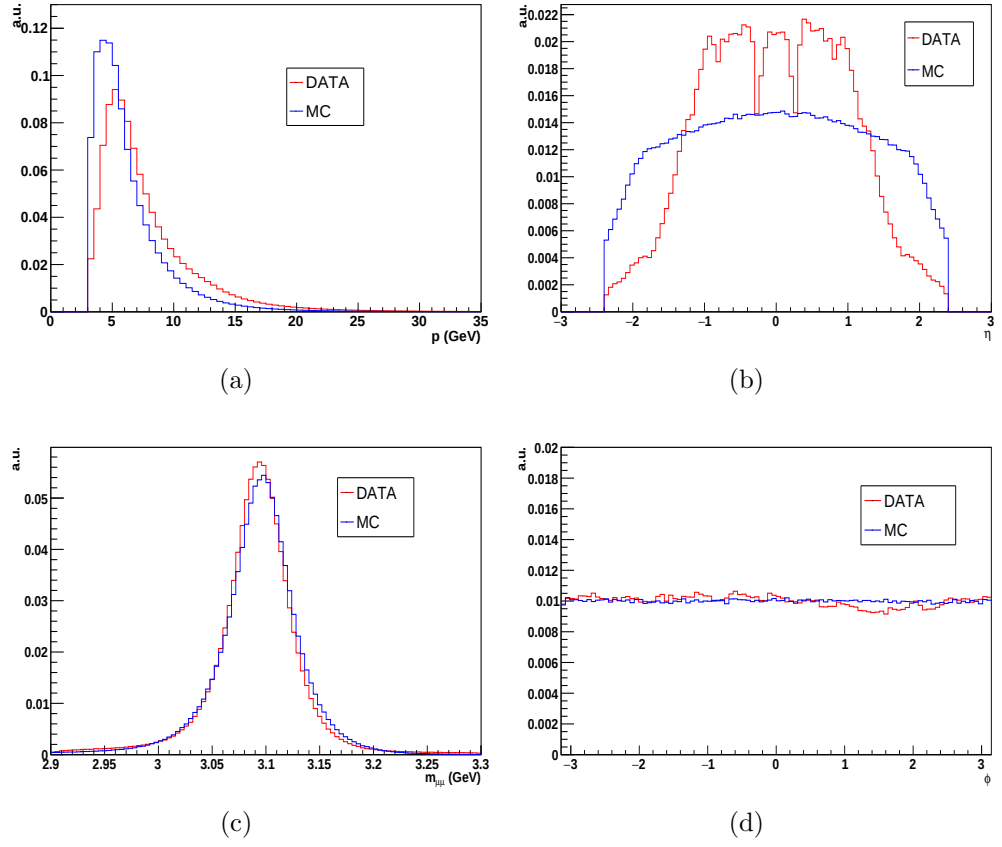


Figure 4.2: Measured (red) and simulated (blue) kinematic distributions for dimuon J/ψ decays. (a): p_T of the positive muon. (b): η of the positive muon. (c): dimuon mass distribution. (d): ϕ of the positive muon.

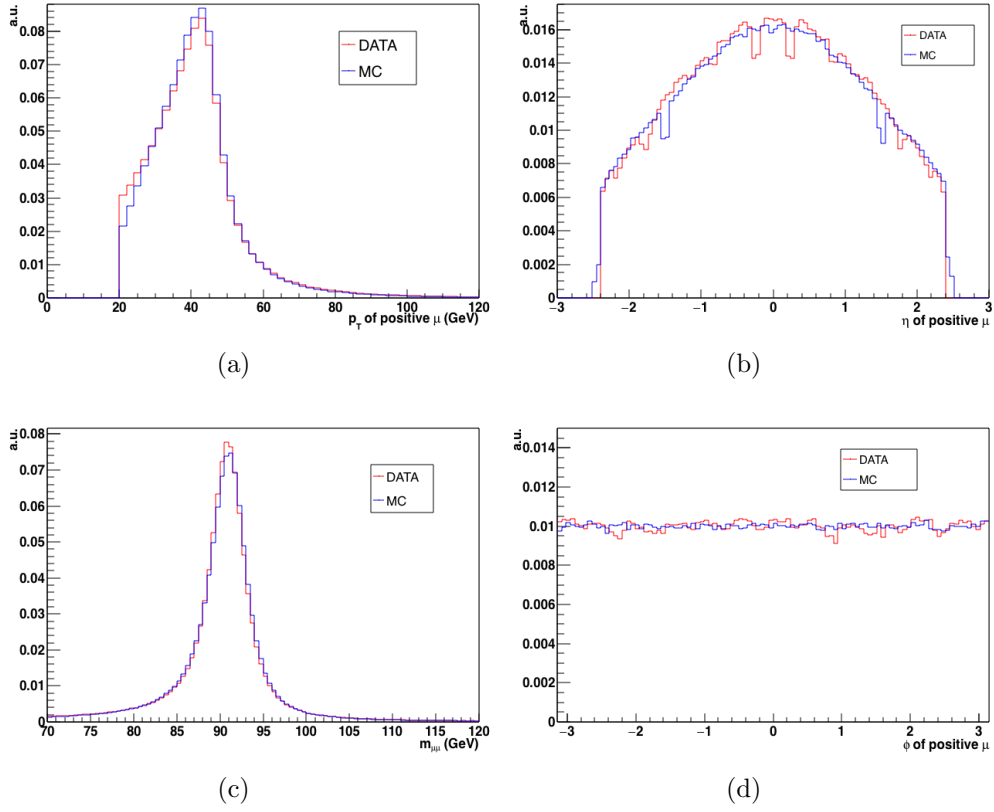


Figure 4.3: Measured (red) and simulated (blue) kinematic distributions for dimuon Z decays. (a): p_T of the positive muon. (b): η of the positive muons. (c): dimuon mass distribution. (d): ϕ of the positive muon.

4.3.1 Parametrisation

The calibration of the muon momentum scale is performed on the unsigned curvature $k = 1/p_T$. As from eq. 2.3, the inverse of the momentum is proportional to the sagitta. The latter is the quantity directly measured in the track fit and is distributed as a Gaussian. In each bin of η and ϕ the measured curvature k is corrected into k^c accounting for the three biasing effects described in section 4.1: the mismodelling of the magnetic field, the misalignment of the tracker modules and the mismodelling of the energy loss in the tracker. To parametrise those effects we start from the equation 2.3 that links the curvature k to the magnetic field B and the radius of curvature ρ :

$$k = \frac{1}{0.3B\rho}, \quad (4.1)$$

from which it is clear that any flaws in the magnetic field map propagates to k with a multiplicative factor:

$$k^c = A \cdot k. \quad (4.2)$$

The misalignment of the modules in the tracker causes an additive bias in the muon momentum scale which is opposite in sign for positive and negative muons, as it can be understood from Figure 4.4. The correction of the curvature can thus be expressed as:

$$k^c = k + qM, \quad (4.3)$$

where q is the muon charge and M is the magnitude of the correction.

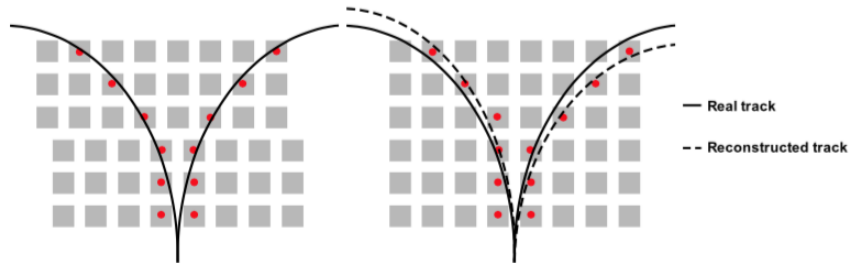


Figure 4.4: Misalignment of the tracker modules. The trajectories of the two muons in the tracker bend in opposite directions due to the magnetic field in the tracker. The actual trajectories in the misaligned tracker (left) will be reconstructed as if the detector was perfectly aligned (right). This leads to an additive term to the curvature, opposite in sign for the two muons.

Finally, any flaws in the modelling of the energy loss will result in a charge-independent additive bias to the measured energy. Since in our samples the muons are ultrarelativistic, the energy is equal to the modulus of the momentum (3D). Therefore:

$$E^c = E + \epsilon, \quad (4.4)$$

which propagates to the curvature as:

$$k = \frac{1}{E \sin \theta} \rightarrow k^c = \frac{1}{E \sin \theta + \epsilon \sin \theta} = \frac{k}{1 + k\epsilon \sin \theta}. \quad (4.5)$$

Accounting for all the effects, the corrected curvature can be expressed as:

$$k^c = (A - 1)k + \frac{k}{1 + k\epsilon \sin \theta} + qM, \quad (4.6)$$

where $(A - 1)$, ϵ and M are the correction parameters for each η and ϕ bin. The values of these parameters are expected to be small according to the discussion of section 4.1. A should differ from 1 for less than 10^{-3} , M is expected to be less than 10^{-4} GeV and ϵ should be of the order of a few MeV.

The invariant dimuon mass can be expressed as a function of the curvature of the positive and negative muons as:

$$m^2 = \frac{1}{k_+} \frac{1}{k_-} [e^{\Delta\eta} + e^{-\Delta\eta} - 2 \cos \Delta\phi], \quad (4.7)$$

where $\Delta\eta$ and $\Delta\phi$ are the difference between the pseudorapidity and the azimuthal angles of the two muons. Here we assume that the bias on the dimuon mass only originates from the bias on the measured curvature, since the angles η and ϕ are measured with a much better precision. Then we can express the ratio between the reconstructed and true mass as:

$$\frac{m_{reco}^2}{m_{true}^2} = \frac{k_+^c k_-^c}{k_+ k_-}. \quad (4.8)$$

Since the two muons can be in a different η and ϕ bin, the parameters A , M and ϵ can be different, resulting into:

$$\frac{m_{reco}^2}{m_{true}^2} = \left(A_+ - 1 + \frac{1}{1 + k_+ \epsilon_+ \sin \theta_+} + \frac{M_+}{k_+} \right) \left(A_- - 1 + \frac{1}{1 + k_- \epsilon_- \sin \theta_-} + \frac{M_-}{k_-} \right) \quad (4.9)$$

4.3.2 Fit of the correction parameters

Equation 4.9 can be used to extract the correction factors from the samples we have described in section 4.2. During the fit the mass of the observed resonance is compared to the expected mass m_{true}^2 after final state radiation from muons, FSR, and the mass shift due to parton distribution functions, PDFs. m_{true}^2 is taken as the average on a sample of simulated events at generator level.

The challenge of this fit resides into the correct assignment of a mass shift in the measured mass to any of the three main physics effects in the positive or negative muon. This is done using a Kalman filter approach, already described in section 2.4.2. The state of the Kalman filter is updated with a new estimate of the correction factors every time a new event is processed. Each event is weighted with the event by event mass resolution obtained propagating the covariance matrices of the track fit to the dimuon mass.

4.3.3 Closure of the scale calibration

The closure of the fit is established by fitting the mass peak of the resonances in the control samples described in section 4.2, after correcting the momentum scale of the muons with the parameters derived in the Kalman fit. The mass fit is performed in different bins of η and p and its purpose is to estimate the deviation of the mass peak from the expected scale. The parameter of interest in the closure is

$$r = \frac{m_{corr}}{m_{true}} \quad (4.10)$$

where m_{corr} is the average reconstructed mass after calibration and m_{corr} has been defined in section 4.3.2. A perfect calibration takes the reconstructed mass to the real mass and returns $r = 1$. Any deviation from this value determines the systematic error of the method. Since in the W mass analysis what matters is the comparison between data and simulation, the following ratios are defined:

$$r_{data} = \frac{m_{data}}{m_{true}} \quad (4.11)$$

$$r_{sim} = \frac{m_{sim}}{m_{true}} \quad (4.12)$$

and

$$r_{abs} = r_{data} - r_{sim} \quad (4.13)$$

Figure 4.5 shows a plot of the closure test, where r_{abs} is plotted as a function of p_T and η of the positive muon. The closure is run on J/ψ , Υ and Z dimuon events in a sample at 7 TeV. The calibration is performed using J/ψ and Υ events. Z events are used only in the closure test. In this sample, the relative systematics on the closure is of the order of $2 \cdot 10^{-4}$, propagating on the W mass with an uncertainty of ~ 8 MeV.

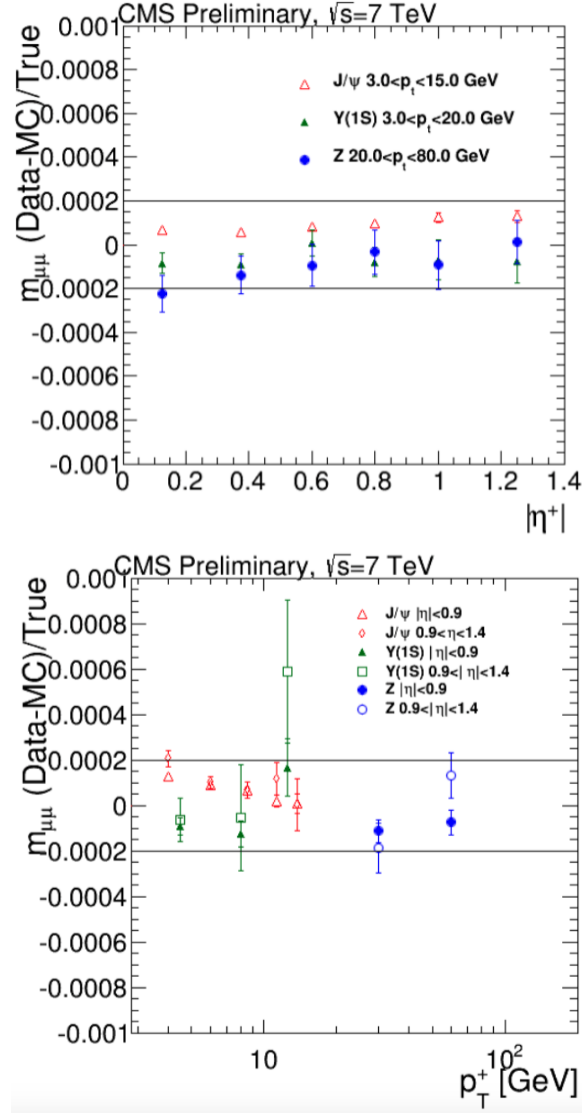


Figure 4.5: Closure test of the scale calibration performed on J/ψ , Υ and Z dimuon events in a sample at 7 TeV, as a function of p_T (top panel) and η (bottom panel) of the positive muon.

Chapter 5

Validation of the material budget in the CMS tracker

The material budget of the CMS tracker is an important input in the reconstruction of tracks, as extensively explained in section 2.4.2. The track reconstruction algorithm relies on the information from the simulation of the detector, since the material budget has never been precisely measured in data so far.

The muon transverse momentum resolution is an observable which is sensitive to this quantity expressed in fractional radiation lengths through its dependence on the multiple scattering of these particles in the tracker. In this chapter we explain the procedure we have followed to validate the material budget in the tracker of CMS exploiting the muon transverse momentum resolution, introducing firstly the physics-motivated model and the parametrisation that we have derived. Then, we describe the method we have devised to extract the parameters of interest and finally we summarise the results that we have obtained.

5.1 Multiple scattering of muons in the tracker

The Coulomb scatterings with the nuclei of the material cause the muon trajectory in the tracker to be deflected by a small angle which is approximately Gaussian distributed with mean zero and r.m.s. given by:

$$\theta_0 = \frac{13.6 \text{ MeV}}{\beta c p} \sqrt{\frac{x}{X_0}} \left[1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right], \quad (5.1)$$

where βc and p are respectively velocity and momentum and of the incident muon and x/X_0 is the thickness of the scattering medium in radiation lengths. An exhaustive description of these phenomena can be found in [29], where a complete derivation of the physical model is given.

As extensively discussed later, the transverse momentum resolution σ_{p_T}/p_T in the CMS tracker is limited by multiple scattering and $(\sigma_{p_T}/p_T)^2$ at low

momenta is correlated to the integral of the inverse radiation length along the trajectory. This quantity obtained from the simulation of the CMS detector is plotted in Figure 2.5 as a function of η and we show it again here, magnified, in Figure 5.1.

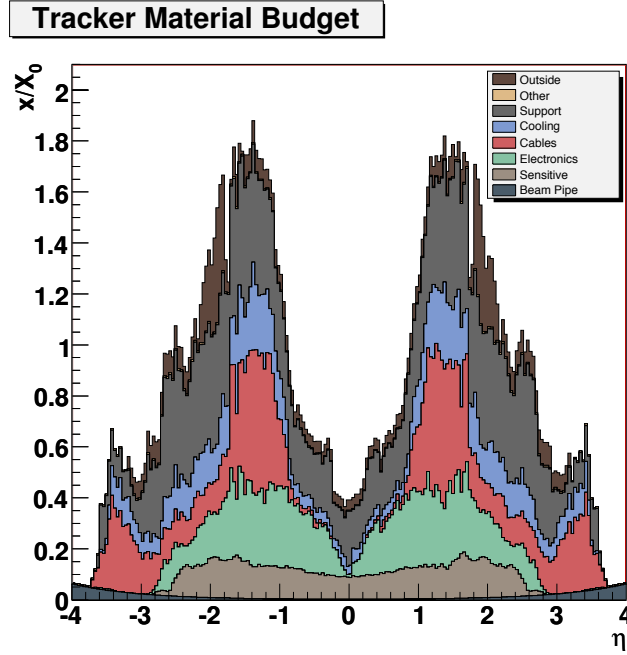


Figure 5.1: Material budget seen by a particle produced in the center of CMS and crossing the whole volume of the tracker. The material is expressed in terms of fractional radiation lengths as a function of the particle pseudorapidity. The different colours highlight the contributions from the different components of the tracker. From [16].

From Figure 5.1 we notice that the silicon sensors contribute to the material budget uniformly in the range $|\eta| < 2.5$ while the material associated to the support structures varies by a factor of two between barrel and endcap. Moreover, the material corresponding to cables and cooling is almost negligible in the barrel while it constitutes about the 50% of the total budget in the transition region between barrel and endcap.

This inhomogeneity can be clarified observing Figure 5.2 which shows the distribution of the simulated material of the tracker (expressed in inverse radiation lengths) in the $r - z$ plane. In order to avoid dead zones in the cylindric layers of the tracker, all the services in the barrel are routed through the two gaps between the barrel and the endcaps. Therefore, a particle with $|\eta|$ between 1.1 and 1.4 crosses the same cables and cooling pipes up to three times.

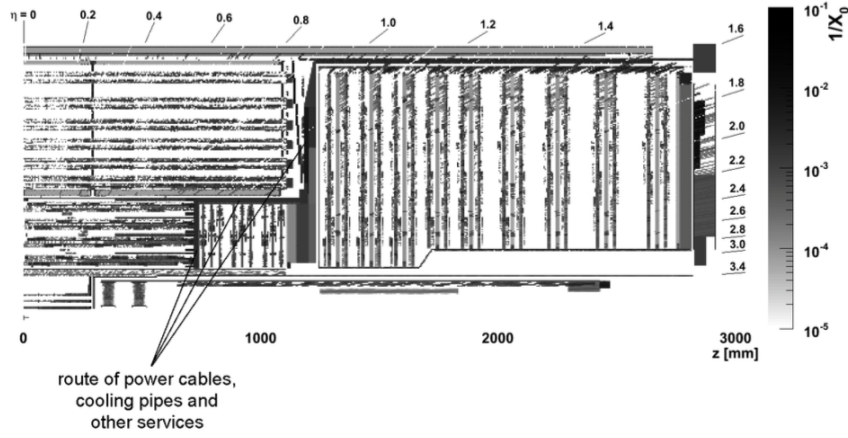


Figure 5.2: Longitudinal $r - z$ view of the silicon tracker and material budget expressed in inverse radiation lengths. The darker area within the gap between the barrel and the endcap is mostly due to cables, cooling and other services. Figure from [20].

5.2 Measurement of the material budget

In the reconstruction of tracks the uncertainty on the sagitta is affected by the position resolution of the hits in the silicon detectors and by the multiple scattering. If we are able to correctly model the transverse momentum resolution and decouple these effects we can directly extract the material budget from multiple scattering using eq. 5.1.

5.2.1 Parametrisation of the momentum resolution

Here we present a simplified description of the physical model that has been used to parametrise the momentum resolution. The complete calculation can be found in Appendix A.

As explained in section 2.4.2, in the CMS tracker the measurement of the momentum of the tracks is made through the measurement of the sagitta. For a track of transverse length L in a uniform and constant magnetic field B the sagitta s is linked to the transverse momentum p^1 by:

$$p \text{ [GeV]} = \frac{0.3BL^2}{8s} \text{ [T m]}. \quad (5.2)$$

The uncertainty on the sagitta is propagated to the momentum by:

$$\sigma_p = \frac{0.3BL^2}{8s^2} \sigma_s = p^2 \frac{8}{0.3BL^2} \sigma_s. \quad (5.3)$$

¹From this point on, in this thesis the word "momentum" and the symbol p will always refer to the transverse momentum p_T .

The intrinsic spatial hit resolution

The resolution of the sagitta is affected by the spatial resolution of the silicon sensors with which the hits are measured. The contribution arising from this effect has been calculated in [30] as function of the number of hits N , assuming equal position uncertainties σ and equally spaced detectors:

$$\sigma_s^{hit} = \frac{\sigma}{8} \sqrt{\frac{720}{N+5}}. \quad (5.4)$$

The related uncertainty on the momentum is given by eq. 5.3:

$$\sigma_p^{hit} = p^2 \frac{\sigma}{0.3BL^2} \sqrt{\frac{720}{N+5}}. \quad (5.5)$$

In the general case formula 5.4 is still a reasonable approximation and the relative momentum resolution will be proportional to the momentum itself through a constant c :

$$\left(\frac{\sigma_p}{p} \right)_{hit} = c \cdot p. \quad (5.6)$$

In the CMS tracker, $L \sim 1$ m, $N \sim 14$ and $B \sim 3.8$ T, while σ varies between $15 - 100 \mu\text{m}$ as discussed in section 2.4.1. Choosing $\sigma = 30 \mu\text{m}$ and using formula 5.5, we obtain a value of $c^2 = 2.6 \cdot 10^{-8} \text{GeV}^{-2}$.

The multiple scattering

For a uniformly distributed material the error on the sagitta due to multiple scattering is given by [22]:

$$\sigma_s^{ms} = \frac{L}{4\sqrt{3}} \frac{13.6 \text{ MeV}}{p} \sqrt{\frac{L}{X_0}}. \quad (5.7)$$

The related error on the momentum is given by eq. 5.3:

$$\sigma_p^{ms} = p^2 \frac{8}{0.3BL^2} \frac{L}{4\sqrt{3}} \frac{13.6 \text{ MeV}}{p} \sqrt{\frac{L}{X_0}}, \quad (5.8)$$

thus the relative resolution is independent of momentum:

$$\left(\frac{\sigma_p}{p} \right)_{ms} = a, \quad (5.9)$$

where a is a constant defined by:

$$a = \frac{2 \cdot 13.6 \text{ MeV}}{\sqrt{3} \cdot 0.3BL} \sqrt{\frac{L}{X_0}}. \quad (5.10)$$

We see here that a^2 for uniform material is proportional to L/X_0 , the material budget measured in number of radiation lengths. Moreover, the constant of proportionality contains the transverse track length L .

The correlation terms

In addition to the previous contributions, to correctly describe the physics of the momentum resolution it is necessary to introduce two new terms that parametrise the correlation between the spatial hit resolution and the multiple scattering. This is extensively discussed in Appendix A.

A simple example gives an idea why this happens. Consider the tracker of length $2L$ shown in Figure 5.3. There are $2N$ measuring layers equally spaced and there is no material, with the exception of a scattering layer positioned in the middle.

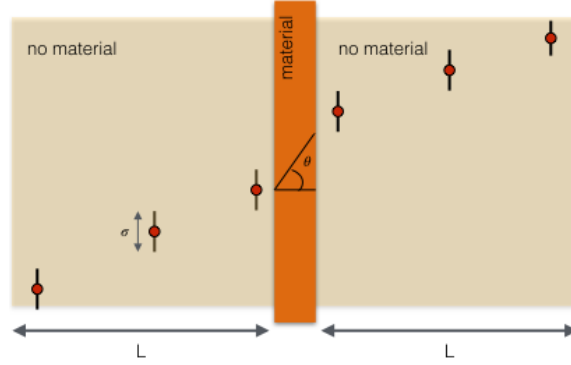


Figure 5.3: Cartoon illustrating the correlation between multiple scattering and spatial hit resolution on the momentum resolution.

If the material in the scattering layer is negligible, the scattering can be ignored and the momentum resolution σ_p of this tracker can be computed using equation 5.5:

$$\sigma_p^{no \text{ mat}} = p^2 \frac{\sigma}{0.3B} \frac{1}{4L^2} \sqrt{\frac{720}{2N+5}}. \quad (5.11)$$

The other limiting case is when the angle θ in the scattering layer is large compared to σ/L . In this case the measurement of the momentum in the first and second half of the tracker are not correlated and the resulting momentum

resolution is simply $1/\sqrt{2}$ of the momentum resolution computed with one half of the tracker:

$$\sigma_p^{mat} = \frac{1}{\sqrt{2}} p^2 \frac{\sigma}{0.3B} \frac{1}{L^2} \sqrt{\frac{720}{N+5}} \quad (5.12)$$

The ratio between the two limiting cases is:

$$\frac{\sigma_p^{mat}}{\sigma_p^{no\ mat}} = \frac{4}{\sqrt{2}} \sqrt{\frac{2N+5}{N+5}} \sim 4 \quad (5.13)$$

In the general case, when the scattering angle θ is comparable to σ/L the momentum resolution is given by an expression that contains the correlation between the scattering angle and the position resolution.

In Appendix A we carry out the complete calculation of the momentum resolution keeping also into account this correlation, as this result is not mentioned in the related literature. The result we have obtained is approximated by the following formula:

$$\left(\frac{\sigma_p}{p} \right)_{hit+ms}^2 = a^2 + c^2 p^2 + \frac{b^2}{1 + \frac{d^2}{p^2}} \quad (5.14)$$

where a and c are the uncorrelated parameters that we introduced before and b and d are the additional ones that arise from the correlation effect.

We can take two opposite limits of eq. 5.14 in momentum:

- limit for low momentum: $\left(\frac{\sigma_p}{p} \right)^2 \rightarrow a^2$, the resolution is driven by multiple scattering
- limit for high momentum: $\left(\frac{\sigma_p}{p} \right)^2 \rightarrow c^2 p^2$, the resolution is driven by the hit position resolution

This observation plays a fundamental role in the choice of the samples to use in the fit of the resolution: we have to use low momentum muons to probe the material budget and high momentum ones to measure the hit position resolution.

5.2.2 Dependence of the resolution on η

As discussed in section 5.2.1 the momentum resolution depends on the momentum p , on the position resolution σ , on the the material budget L/X_0 and on the transverse track length L .

The dependence on the track length is strong. The term c of equation 5.9 is proportional to $1/L^2$ and the term a of equation 5.3 is proportional to

$1/L$. Looking at Figure 2.4 we notice that L is, in first approximation, constant up to $\eta = 1.6$ and then decreases when the track crosses the end of the tracker endcap. It is useful to redefine the terms a , b , c and d used in the fitting function as

$$\begin{aligned} a &\rightarrow a \cdot r_L(\eta) \\ c &\rightarrow c \cdot r_L^2(\eta) \\ b &\rightarrow b \cdot r_L(\eta) \\ d &\rightarrow d / r_L(\eta) \end{aligned} \tag{5.15}$$

and $r_L(\eta)$ is defined as follows: $r_L(\eta) = L_0/L$, where $L_0 = 108.0 - 4.4$ cm and

$$L = \begin{cases} L_0, & \text{if } |\eta| < 1.6 \\ \min(r_{max}, 108.0) - 4.4 \text{ cm}, & \text{if } |\eta| > 1.6 \end{cases} \tag{5.16}$$

defining

$$r_{max} = \begin{cases} (267.0 \text{ cm}) \cdot \tan \theta, & \text{if } \eta > 1.6 \\ (-267.0 \text{ cm}) \cdot \tan \theta, & \text{if } \eta < -1.6 \end{cases} \tag{5.17}$$

where $\tan \theta$ is easily derived from η through:

$$\tan \theta = \frac{2}{e^\eta - e^{-\eta}}.$$

The numerical values in 5.16 and 5.17 can be easily understood looking at Figure 2.4. The average radius of the first hit of the trajectory in the pixel is 4.4 cm. The average radius of the last point to the trajectory for a track leaving the tracker through the barrel is 108 cm and the z position of the outermost disk of the TEC is 267 cm.

In this way the term c^2 is only mildly dependent on η since the spatial resolution in the tracker is almost constant and the term a^2 is roughly proportional to the material budget also for large η when the transverse track length L decreases.

It is worth recalling that this is an approximation: the function $r_L(\eta)$ takes out from a and c the bulk of the dependence on the track length, however it is not an exact measurement of the track length that depends in a complex way on the geometry. This complex behaviour is fully described by the Monte Carlo simulation.

5.2.3 About the material non-uniformity

The modelling of the term a^2 in eq. 5.10 is valid only in the approximation of uniform material. As discussed in section 7.1, the material budget of the tracker is quite far from being uniform. To take care of this non uniformity, the previous definition is be modified as

$$a = \frac{2 \cdot 13.6 \text{ MeV}}{\sqrt{3} \cdot 0.3BL} \sqrt{\frac{\int w(x)dx/X_0}{\int w(x)dx}}. \quad (5.18)$$

where $w(x)$ is the square of the distance from the closest between the first and the last point of the measured trajectory. The complete calculation to derive the latter formula is shown in Appendix B.

The importance of this weighting is shown in Figure 5.4 where the two plots show a^2 modelled by equation 5.10 (uniform material) and 5.18 (weighted material), using the simulated material in the tracker². We see that the effect of the non-uniformity is especially important near $|\eta| \sim 1.4$. This can be understood from figure 5.2, which shows that a track near these values of pseudorapidity crosses the cables up to three times near the middle of the trajectory, where the weight w in formula 5.18 is largest.

5.2.4 Measurement of the material budget from simulation

Using samples of simulated J/ψ and Z decaying in two muons we measure the resolution comparing the generated and reconstructed momentum of each track. The relative momentum resolution σ_p/p is given by the r.m.s. of the distribution of

$$\frac{(1/p_{rec} - 1/p_{gen})}{1/p_{gen}}, \quad (5.19)$$

where p_{rec} is the momentum of the reconstructed muon and p_{gen} is the momentum of the matched generated muon.

The resolution σ is extracted with a Gaussian³ fit to the distribution of eq. 5.19 performed in bins of η and p of the lepton. There is no need to bin also in ϕ since the material distribution and the hit resolution do not

² The description of the material of the tracker used in this Figure and in Figure 5.9 is obtained from the GEANT files using some approximations that describe the material in small volumes as a function of η and r . It is the same file used for drawing Figure 5.2. However this description of the material is not so precise and there are some places with large fluctuations among nearby bins. The agreement between this description of the material and the full simulation is only qualitative.

³ As shown by equation 5.2, the inverse of the momentum is proportional to the sagitta. The latter is the quantity directly measured in the track fit and is distributed as a Gaussian.

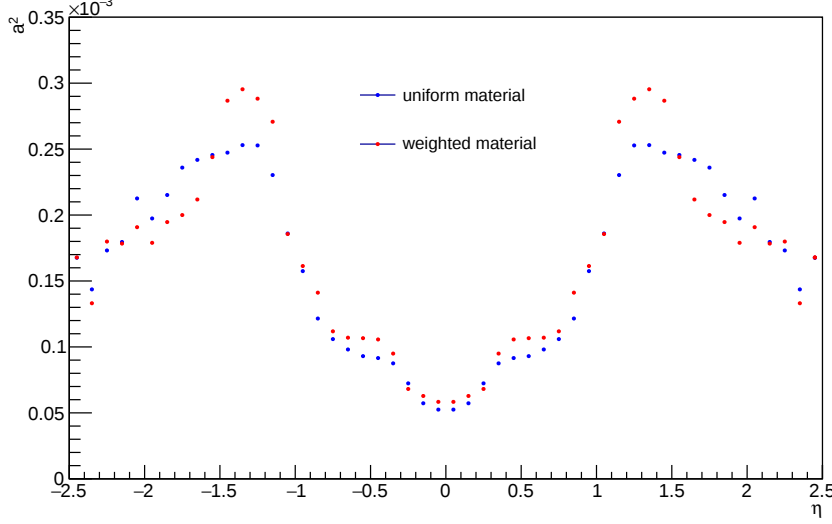


Figure 5.4: a^2 modelled by equation 5.10 (uniform material, blue) and 5.18 (weighted material, red) calculated with the simulation of the material in the tracker.

depend on ϕ to very good approximation. Then, we fit σ^2 as a function of p^2 in bins of η using eq. 5.14:

$$\left(\frac{\sigma_p}{p}\right)^2 = a^2(\eta) \cdot r_L^2(\eta) + c^2(\eta)p^2 \cdot r_L^4(\eta) + \frac{b^2(\eta) \cdot r_L^2(\eta)}{1 + \frac{d(\eta)^2}{p^2} \cdot \frac{1}{r_L^2(\eta)}}, \quad (5.20)$$

Here the terms a , c , b and d have been scaled using equation 5.15.

This sample is used to check the validity of our model of the resolution. By definition, eq. 5.19 gives the exact resolution, while our model has been computed based on some assumptions on the geometry and on the material distribution. In addition we have kept in eq. 5.20 only one term in the sum of eq. A.6.

The fit works well, as we see in Figure 5.5, where we show σ_p/p in the bins of η : -1.85, -0.15, 0.75 and 1.45 with the related fitted parameters.

Results

Figure 5.6 shows the four terms of equation 5.20 as obtained from the fit of the momentum dependence of the resolution in the various η bins. The plots show the expected symmetry between $\pm\eta$ that is not imposed by construction.

We notice that the material budget term a^2 is measured with good precision. The three other terms show large errors in some bins. This is due to the correlation among the three parameters b , c and d that is expected from

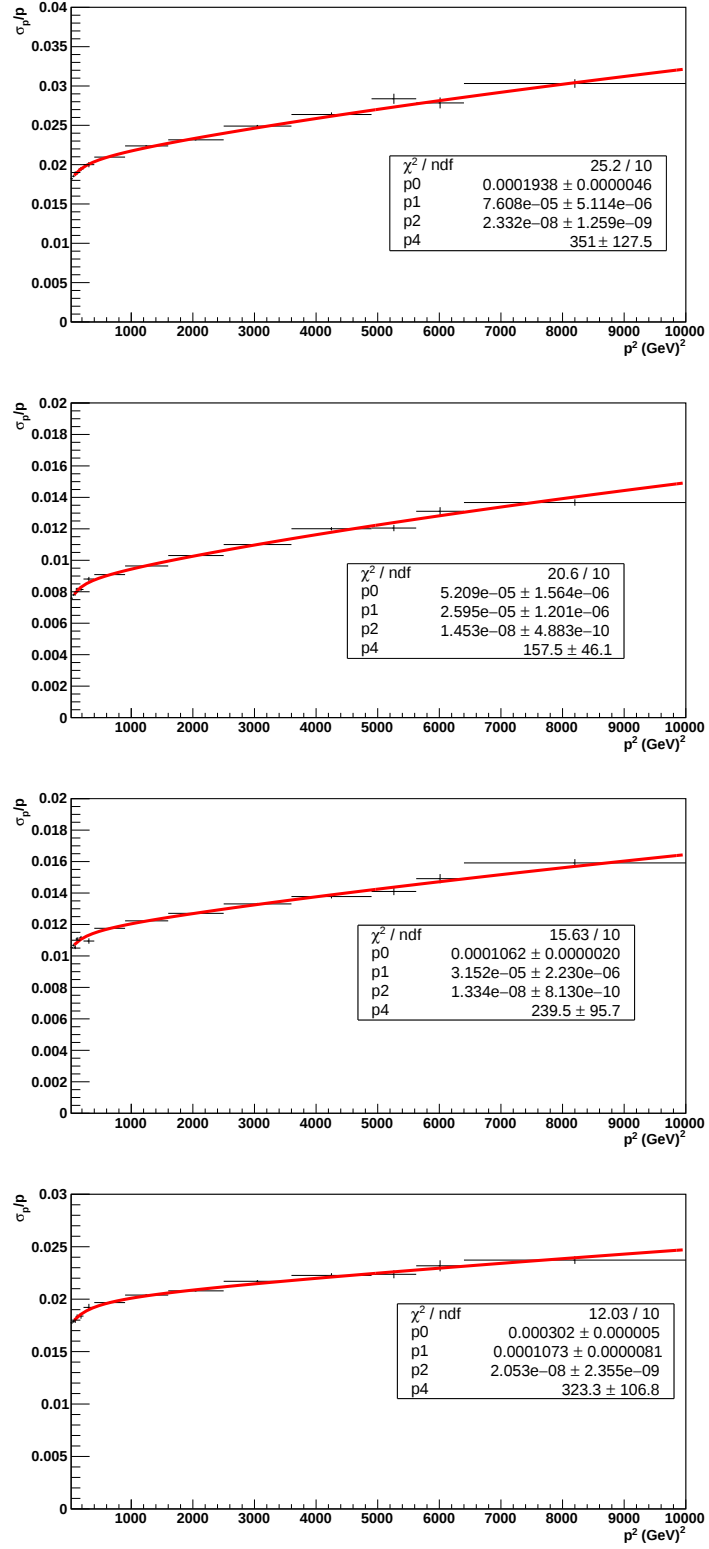


Figure 5.5: Fitted distributions of the true resolution in various bins of η . From top to bottom: $\eta = -1.85, -0.15, 0.75$ and 1.45

equation 5.20. Moreover, the correlation is larger near $\eta \sim 1.5$ where the material budget is large.

In conclusion, we see that the model derived in the Appendix A for describing the resolution of the tracker works well and that measuring the resolution one can extract the material budget of the tracker.

Figure 5.6, last panel from top to bottom, shows that the parameter d^2 describing the correlation between multiple scattering and hit resolution is compatible to be constant at a value of $\sim (19 \text{ GeV})^2$. Moreover this parameter is measured with large errors in the high η region. The fits have been repeated fixing the term d^2 at 370 GeV^2 and the quality of the fit evaluated from the χ^2 per degree of freedom is unchanged. Figure 5.7 shows a comparison of the term a^2 obtained with and without fixing d^2 in the fit, while Figure 5.8 shows the term c^2 after fixing d^2 .

Figure 5.9 shows the comparison of the radiation lengths in the tracker computed using the term a^2 obtained from the fit to the momentum resolution with equation 5.20 and formula 5.18 with the value computed from the simulation of the material.

We see a good qualitative agreement. In particular in the pseudorapidity region $|\eta| \sim [1.1, 1.5]$ it is shown that the weighted material describes the fitted resolution.

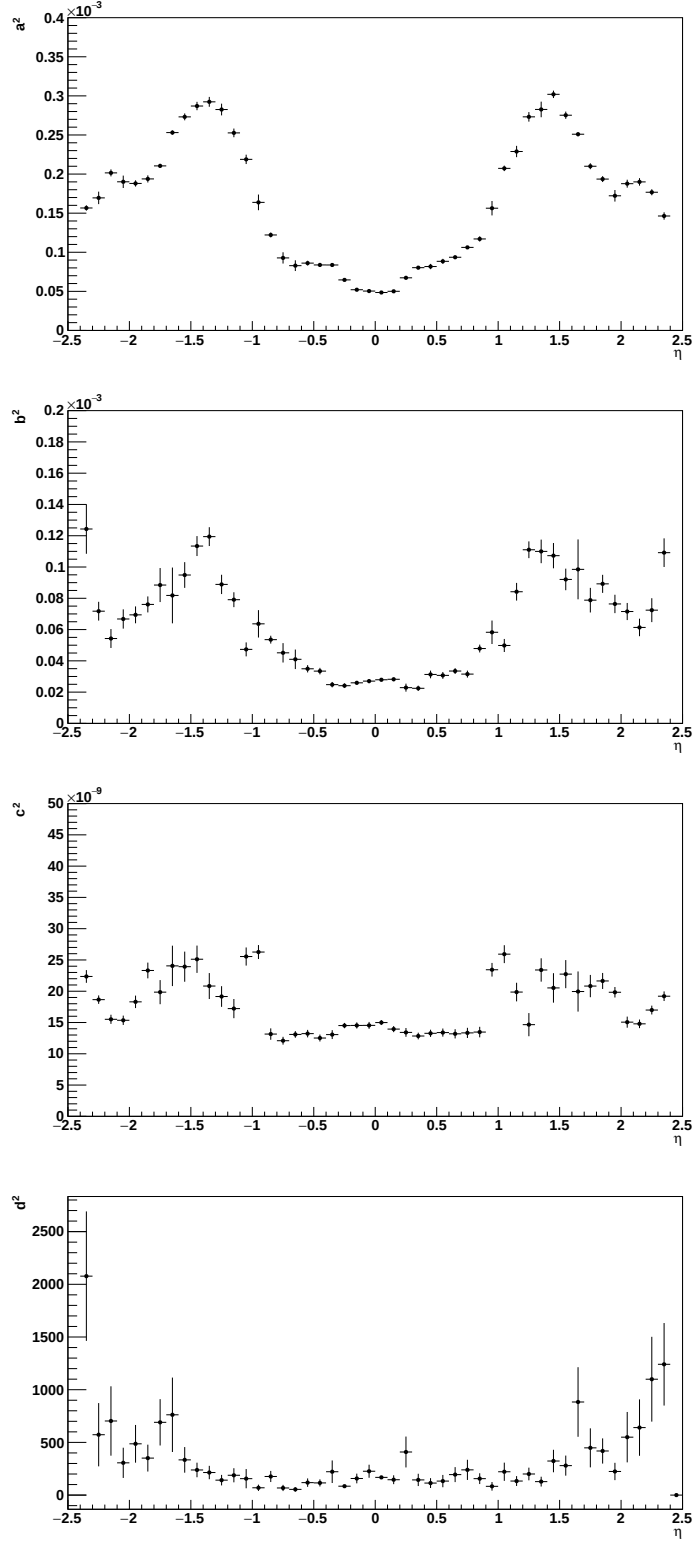


Figure 5.6: This figure shows as a function of η the four parameters of formula 5.20 obtained via a fit of to true resolution of the tracker defined by 5.19.

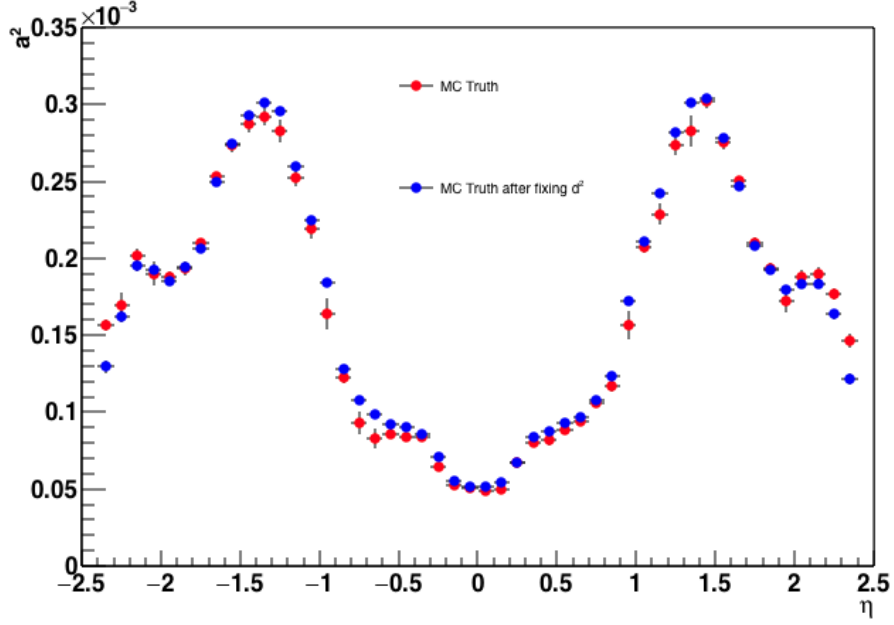


Figure 5.7: Comparison of the a^2 term obtained via a fit of to true resolution of the tracker defined by 5.19, with free parameters (red) and fixing d^2 to 370 GeV^2 (blue).

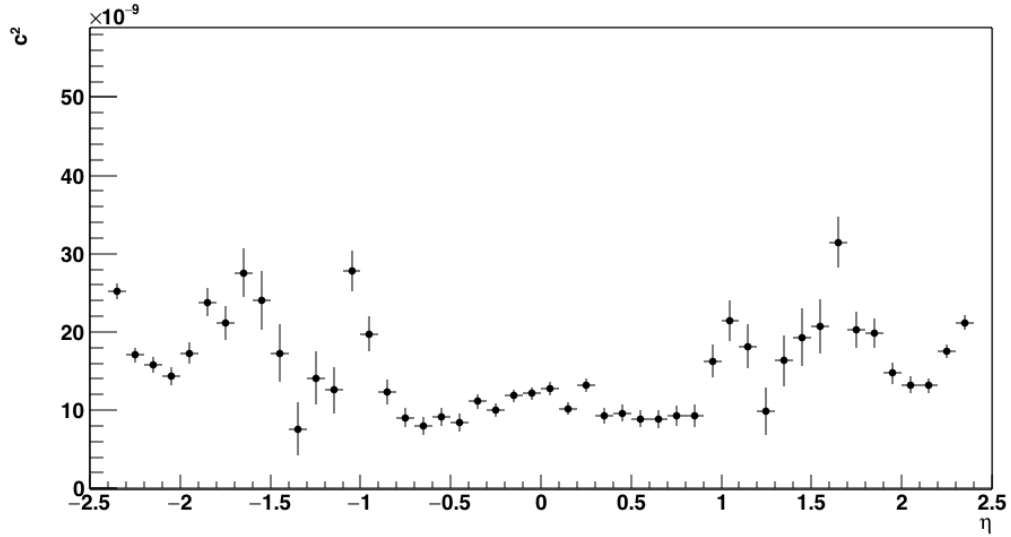


Figure 5.8: Plot of the term c^2 after fixing d^2 at 370 GeV^2 .

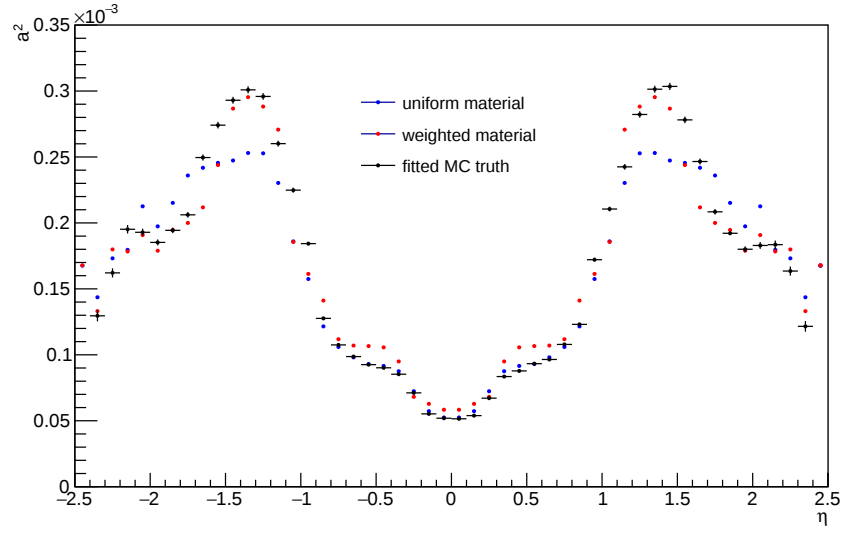


Figure 5.9: a^2 modelled by equation 5.10 (uniform material, blue) and 5.18 (weighted material, red) calculated with the simulation of the material in the tracker, compared compared (black) to the value of a^2 obtained to the fit of equation 5.20 to the momentum of the simulated muons.

5.2.5 Measurement of the material budget from CMS data

The procedure we follow to measure the momentum resolution in data is different. We use a sample of J/ψ dimuon events described in section 4.2 and measure the mass resolution of the dimuon system. The invariant mass distribution is given by:

$$m^2 = p_1 p_2 [e^{\Delta\eta} + e^{-\Delta\eta} - 2 \cos \Delta\phi], \quad (5.21)$$

where p_1 and p_2 denote the transverse momentum of the two muons in the decay and $\Delta\eta$ and $\Delta\phi$ are the difference between the pseudorapidity and the azimuthal angles of the two muons. The angles ϕ and η are typically measured in CMS with a precision of 10^{-3} , while the momentum resolution effect is of the order of $1-2\%$. Therefore, the uncertainties in measuring the angles can be neglected in the calculation of the uncertainty of the invariant mass. Then we have:

$$\left(\frac{\sigma_{m^2}}{m^2}\right)^2 = \left(\frac{2\sigma_m}{m}\right)^2 = \left(\frac{\sigma_{p_1}}{p_1}\right)^2 + \left(\frac{\sigma_{p_2}}{p_2}\right)^2, \quad (5.22)$$

and for each muon (σ_p/p) is given by eq. 5.20.

To measure the momentum resolution we make the hypothesis that the width we observe in the invariant mass distribution is all due to detector effects. This is true for the J/ψ , being a very narrow resonance in the dimuon spectrum. Moreover, the two muons emitted have low momentum, as shown in Figure 4.2a and their momentum resolution is mainly driven by multiple scattering, as we have pointed out before. We fit the measured mass resolution using equation 5.20 and 5.22.

Binning the dataset

The first step we make is splitting the dataset. In principle it is possible to perform an unbinned fit of the whole sample. The splitting gives us the possibility to do multiple fits separately at the same time. Since the resolution depends on η , we divide the dataset in 50 bins of η in the range $(-2.5, 2.5)$, choosing only events in which the two muons are in the same η bin.

This choice does not reduce heavily the size of the sample since the majority of the J/ψ collected by CMS are boosted because of the trigger threshold on the momentum of the muons and the polar angle between the decay products in this configuration is quite small. In addition, in a sizeable fraction of the decays the opening angle between the muons has a large component in the ϕ direction: the two muons have the same η in spite of an opening angle larger than the η bin width. In each η bin we fit the momentum dependence of the mass resolution.

Fit of the momentum resolution

To extract the mass resolution we model the signal distribution using a Crystal Ball function⁴ which accounts for radiative losses and we add an underlying exponential distribution for the (small) background. In each η bin we fit the invariant mass constraining the σ parameter of the Crystal Ball to satisfy eq. 5.22 and be a function of p through the parameters given in eq. 5.20.

To help the fit converge we give as starting point to the fit the information from the real resolution fitted in Monte Carlo described in section 5.2.4 : the term a^2 is free to vary in a range $(0.1 \cdot \text{MC}, 10 \cdot \text{MC})$, where MC is the related parameter in the fit of Monte Carlo, while the terms b^2 , c^2 and d^2 are fixed respectively at the value of the fit of Monte Carlo and to 370 GeV^2 .

The result of this fits is simultaneously the lineshape of the J/ψ described by the parameters fitted by the Crystal Ball and exponential distributions and the parameter a^2 of eq. 5.20. We estimate the systematics on this procedure repeating the fit fixing the parameters b^2 and c^2 to a fraction 0.9 and 1.1 of their values. In this way we obtain four other results for the a^2 parameter (one for every combination of the variations) in addition to the central value previously fitted. We find that the difference between the central value and the value obtained with the varied fit that has the biggest deviation from the central value is negligible in the sum in quadrature with the statistical error from the fit.

Validation of the procedure with simulated J/ψ

In order to validate the procedure we run the same fit on a sample of simulated J/ψ with spectrum similar to those collected by CMS, neglecting the background which is not present in the simulation. If the results are compatible with those we obtained fitting the true resolution (Figure 5.6) in section 5.2.4, then we can be confident of the results we obtain when fitting the data.

Figure 5.10 top panel shows the comparison between the parameter a^2 of the fit to the simulated J/ψ samples (called MC Datalike) and the parameter a^2 obtained from the fit of single muons (called MC Truth) in section 5.2.4. The two distributions match very well, at less than 10% level, as shown by the plot of their ratio in Figure 5.10, bottom panel. This plot shows a small overall bias, less than 5% and no η dependence. Therefore, we consider the fit procedure to be validated.

⁴The Crystal Ball function consists of a Gaussian core portion, described by μ and σ parameters and a power-law low-end tail, below a certain threshold described by a parameter α .

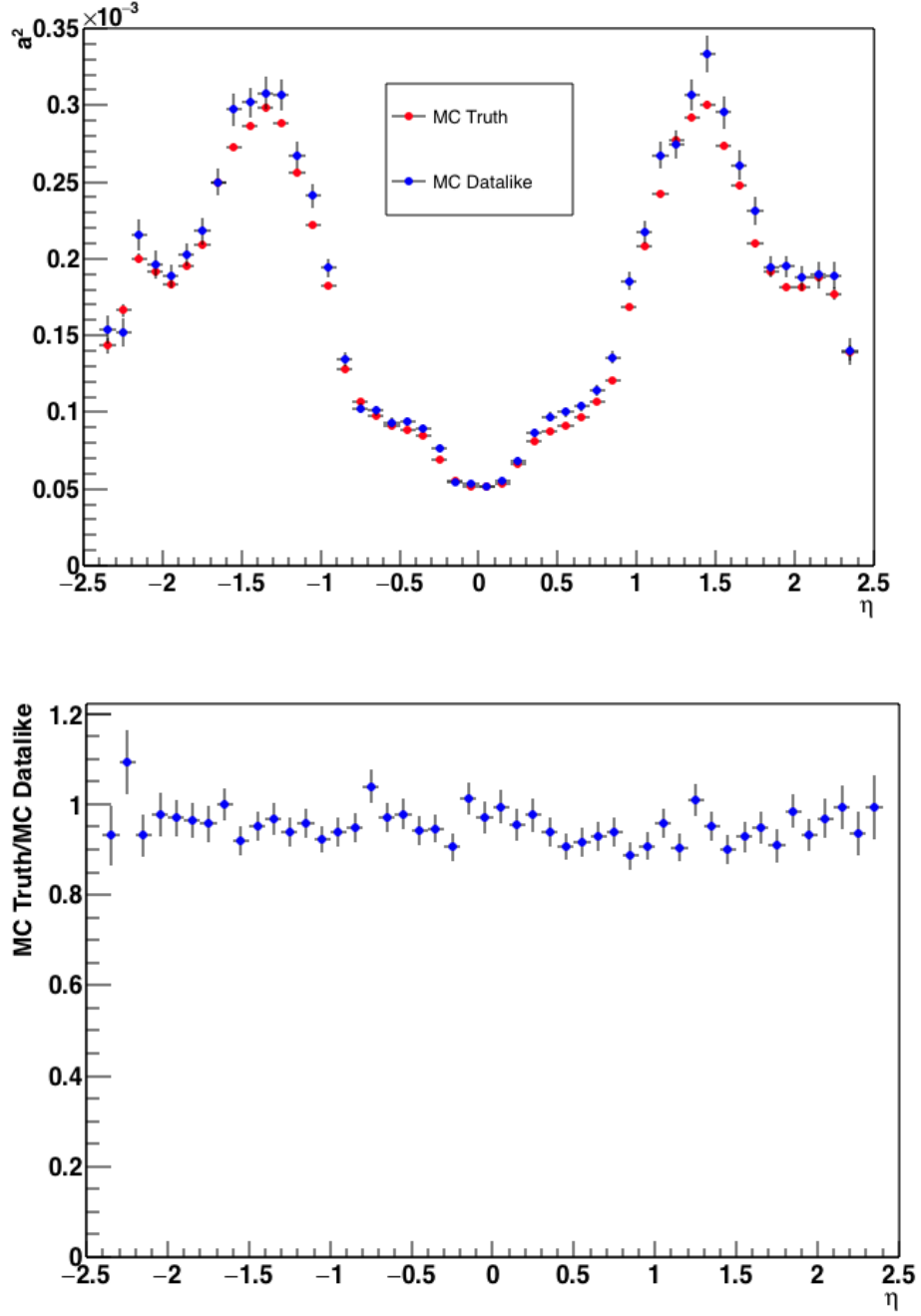


Figure 5.10: Top panel: a^2 parameter measured in Monte Carlo as described in section 5.2.4 (red) and in the same sample using the same procedure as in data (blue). Bottom panel: ratio of the two curves.

Results

In the set of plots in Figure 5.11 we show the measured mass distribution and the fitted lineshape of the J/ψ in the bins of η -1.85, -0.15, 0.75 and 1.45 with the related reduced χ^2 . In Figure 5.12 we show the results for the parameter a^2 and we compare the results obtained in data and Monte Carlo Datalike in Figure 5.13 where the ratio of the two curves is shown. The ratio data/simulation is measured in 50 bins of steps $\Delta\eta = 0.1$ with a precision of $\sim 2\%$ in the central bins to $\sim 10\%$ in the large η region. It shows that the material in the simulation is underestimated by about 10%.

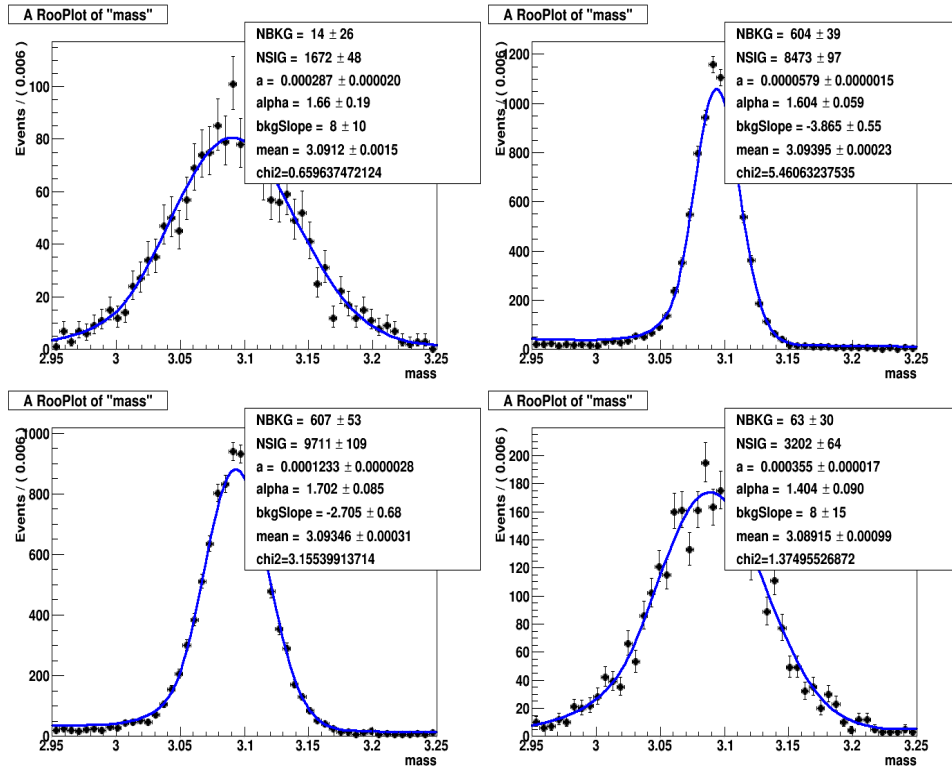
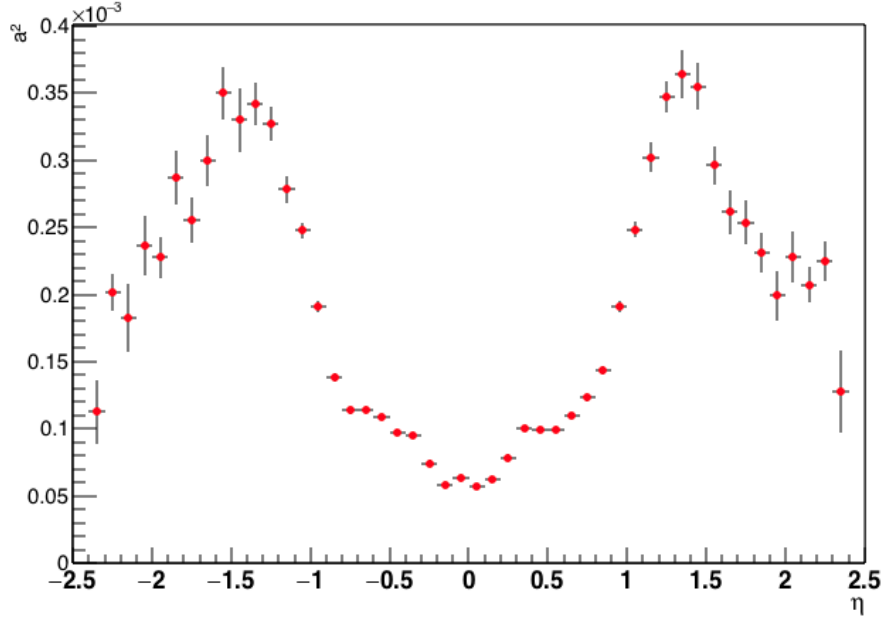
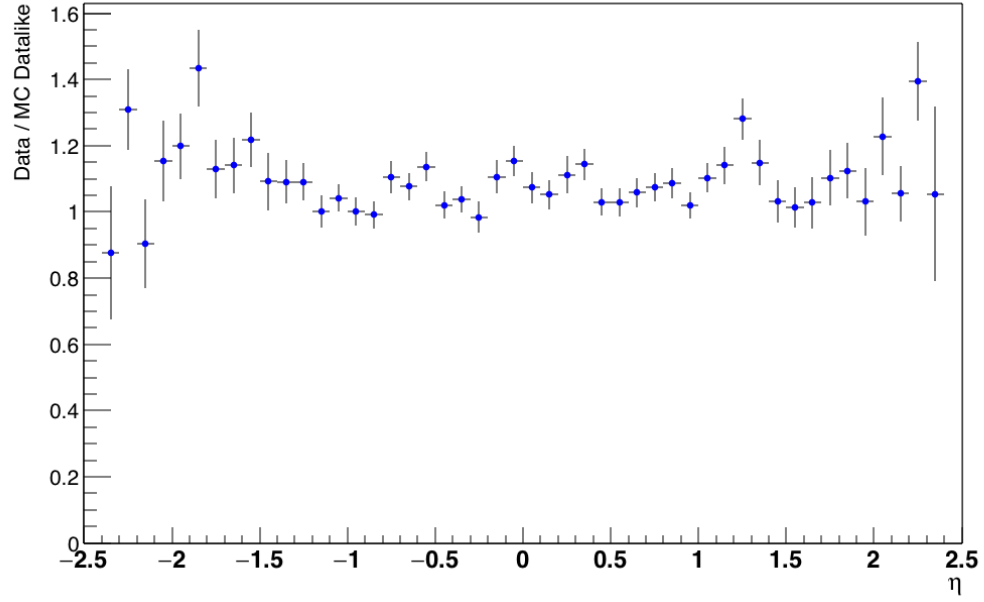


Figure 5.11: Fitted distributions of the J/ψ mass in various bins of η .

In order to test the difference in the material between data and simulation, with better precision, we repeated the fits dividing the dataset in bins of η of only one muon, while the other could be anywhere in the range $(-2.5, 2.5)$. We show in Figure 5.14 the ratio data/simulation of the a^2 parameter as obtained with this procedure. The enlarged statistics allows us to appreciate the variation of the material between data and simulation. In particular, we observe that there is a great discrepancy in the negative endcap, in the range $\eta < -2$. We know from independent analyses that this part of the detector suffers from alignment problems.

Figure 5.12: a^2 parameter measured in data.Figure 5.13: Ratio of the a^2 parameter measured in data and in the simulation using the same procedure as in data (MC Datalike).

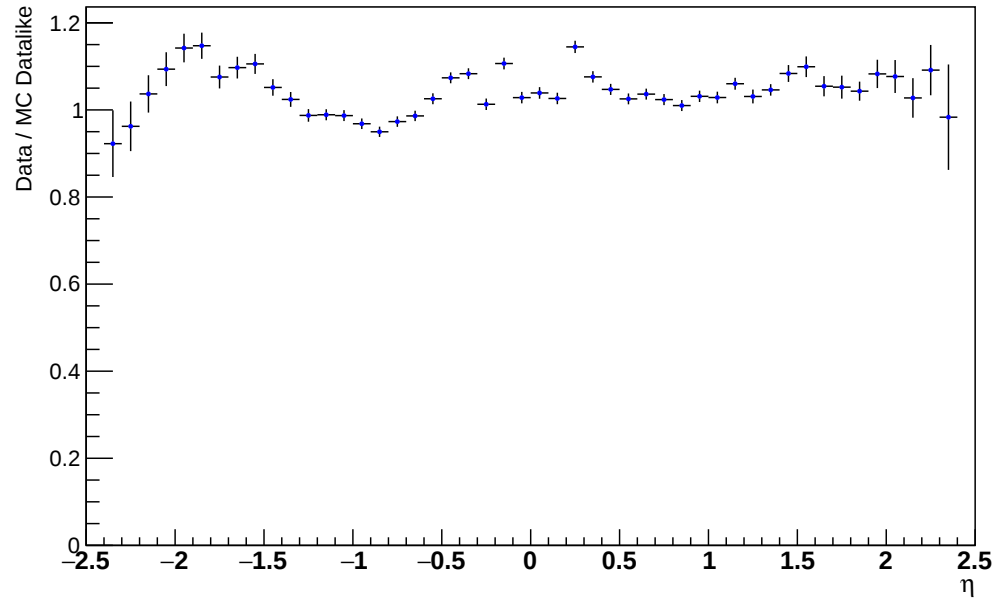


Figure 5.14: Ratio of the a^2 parameter measured in data and in the simulation using the same procedure as in data (MC Datalike), measured with the new selection procedure.

Chapter 6

Validation of the muon momentum resolution

The muon momentum resolution plays an important role in the calibration of the muon momentum scale. A mismatch in the resolution between data and Monte Carlo bias the closure of the calibration of the momentum scale. If we measure the momentum resolution accurately enough, this bias can be removed.

In this chapter we validate the calibration of the momentum resolution in data and Monte Carlo using the fit procedure we extensively discussed in chapter 5.

6.1 Resolution effects in the calibration of the muon momentum scale

The calibration of the muon momentum scale discussed in chapter 4 is validated comparing the average Z mass as a function of the positive (or negative) muon momentum and angle in data and Monte Carlo simulation. This plot is shown in Figure 4.5.

The distribution of the momentum of the positive muon in the Z decays is shown in Figure 4.3a. The measured momentum distribution is the convolution of the true momentum distribution with the momentum resolution. When events are selected in a bin of *measured* momentum, the average *true* momentum of the selected event is not the center of the bin and there is a small bias ϵ_p :

$$\epsilon_p = \sigma_p^2 \frac{1}{f(p)} \frac{\partial f}{\partial p}. \quad (6.1)$$

Here σ_p^2 is the square of the momentum resolution, $f(p)$ the momentum distribution of the muons from Z decays and $\partial f / \partial p$ is the derivative of the spectrum. This effect can be understood looking at Figure 4.3a. Selecting

measured muons in a small bin around 35 GeV, because of the resolution we will select events with true momentum above 35 GeV and below 35 GeV. Since there are more events above 35 GeV than below 35 GeV the average *true* momentum of the selected events will be higher than 35 GeV by the small amount shown in eq. 6.1.

Because of this effect when we plot a variable as a function of the measured momentum we introduce a bias that depends on the resolution. One simple example is the plot of the average of p^{meas}/p^{gen} as a function of the momentum in muons from the Z decay using simulated events. If we plot this average as a function of the *true* momentum the distribution is centred at 1 for all momenta because $1/p^{meas}$ is indeed distributed as a Gaussian around $1/p^{gen}$. If we plot the same quantity as a function of the *measured* momentum the average value of $1/p^{gen}$ is biased because of the resolution:

$$\langle \frac{1}{p^{gen}} \rangle = \frac{1}{p^{meas}} - \frac{\epsilon_p}{p^2} \quad (6.2)$$

and the distribution centered at $1 - \epsilon_p/p$.

When we plot the average Z mass as function of the measured momentum of the muon we also introduce a bias. This effect can be tested computing in simulated events the Z mass m_{gen} using equation 5.21 with the generated momenta and comparing it to the mass m obtained with the *measured* momenta. The ratio m/m_{gen} is equal to the square root of the ratio $(p_1^{meas} p_2^{meas})/(p_1^{gen} p_2^{gen})$ where p_1 and p_2 are the momenta of the two muons. The ratio m/m_{gen} is plotted in Figure 6.1 as a function of the *measured* momentum of the positive muon. In producing this plot we have smeared the generated momenta of the two muons with the expected resolution as a proxy of the reconstructed momenta. Because of the resolution there is a bias at the level of 0.1%. The bias is negative for momenta below 45 GeV and positive for momenta above 45 GeV. The same plot shows also the effect of smearing the events increasing the resolution by 10% and 20%.

Figure 6.1 shows that the resolution produces a bias of $2 \cdot 10^{-3}$ on the ratio m/m_{gen} . This bias is removed when plotting the variable r_{abs} introduced in section 4.3.3 that compares the reconstructed mass in data and simulation.

The cancellation is complete if the resolution in data and simulation are identical, otherwise there is a residual bias that can be as large as $3 \cdot 10^{-4}$. We conclude that we need to control the resolution in data and in simulation to better than 3% in order to not bias the closure test of the calibration of the momentum scale.

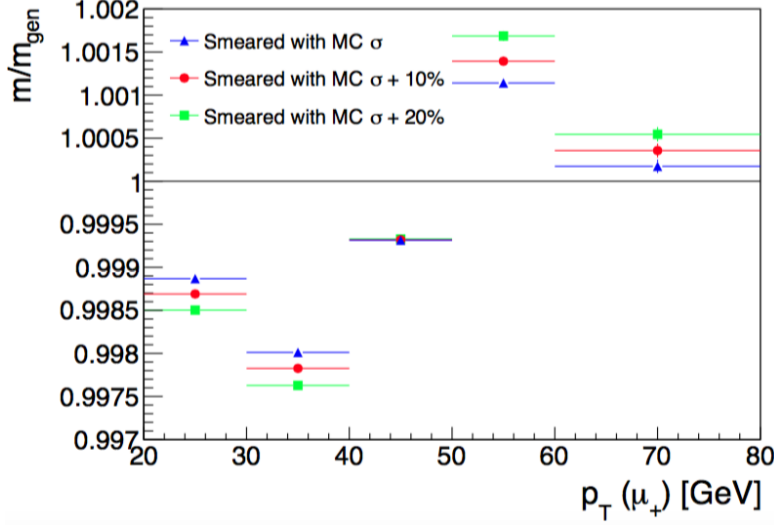


Figure 6.1: Mass shift in smeared Z generator sample as a function of the transverse momentum of the positive muon. The sample is smeared using different fractions of the resolution σ in the Montecarlo, $1 \cdot \sigma$ (blue triangle), $1.1 \cdot \sigma$ (red circle) and $1.2 \cdot \sigma$ (green square).

6.2 Extracting the momentum resolution at high muon momenta

In this section we discuss again the fit of the momentum resolution using dimuon resonances. The fit procedure has been described in section 5.2.5 where we have measured the material budget of the tracker using the parameter a^2 of eq. 5.20.

Here we discuss the procedure for extracting the other parameters of the fit in order to describe correctly the resolution at all momenta. As we already pointed out in section 5.2.1, to probe correctly the slope of the momentum at high values of p_T we need sufficiently energetic muons. The muons from the J/ψ sample we use for measuring the material budget do not guarantee enough lever arm. Therefore, we have repeated the fits including also a sample of Z decays whose dimuons have a much higher momentum spectrum, shown in Figure 4.3a.

However, when extending the fit procedure including the Z sample, we encounter some further difficulties. While the J/ψ is a very narrow resonance, the Z has a natural width of 2.5 GeV as from [22]. A change in resolution corresponds to a relative small change in the measured width compared to the natural width. This is shown in Figure 6.2 where the mass of the Z obtained with generated and reconstructed momenta is shown for simulated events.

In addition the Z lineshape simulated by the Monte Carlo depends on the

knowledge of the PDFs introducing some systematic effects when comparing the resolution in data and in simulation.

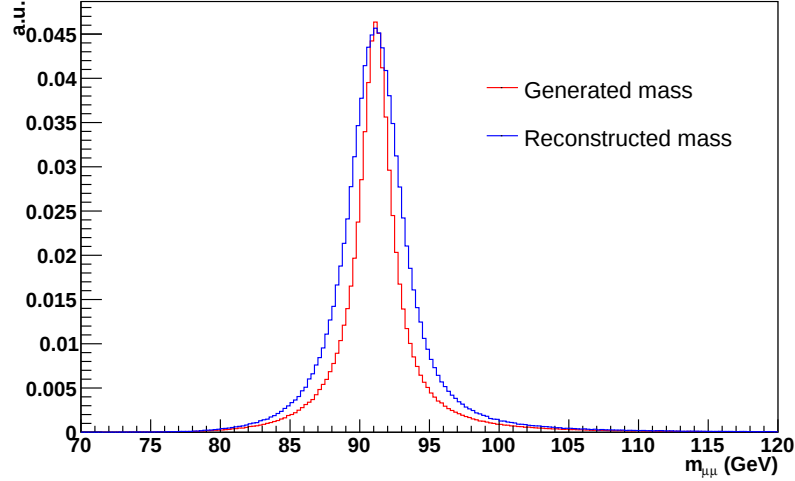


Figure 6.2: Invariant mass of the Z obtained with generated (red) and reconstructed (blue) muon momenta.

6.2.1 Procedure for fitting the resolution in Z dimuon events

Due to the previous considerations, we have introduced a new model to fit the invariant mass of the Z .

Extracting the Z width

The first step we take is to measure the shape of the Z resonance using a sample of generated events. This preliminary step is necessary because we measure the dimuon invariant mass after final state radiation. In addition the lineshape is also modified by the PDFs. This effect may depend on the rapidity of the Z , that in our selection is equal to the pseudorapidity of the muons.

As described in section 5.2.5, we bin the sample in η , selecting only the events having both muons in the same η bin. This is done to keep this procedure as close as possible to the fit in data. In each η bin we compute the invariant mass of the Z using the momenta of the generated muons after FSR and we fit its width using a Breit-Wigner distribution.

We observe that this value is found to be approximately constant as function of η and higher than the value quoted in PDG by about 400 MeV. Therefore, we take as our best estimate of the width of the Z a value of 2.91 and we assign on the final fit a systematics based on the validity of this assumption.

Fit of the resolution in data and simulation

We perform the fits of the resolution in Z binning the dataset in 7 bins of η in the range $(-2.5, 2.5)$ and we select only events having both muons in the same η bin, exactly as described in section 5.2.5. However, fitting on the Z sample we had to choose a less fine binning since, differently from J/ψ events, it is rare for muons emitted from the Z to be very close to each other. To fit the resolution at high momenta we modify eq. 5.20 as follows:

$$\left(\frac{\sigma_p}{p}\right)^2 = a^2(\eta) \cdot r_L^2(\eta) + c^2(\eta)p^2 \cdot r_L^4(\eta) + \frac{b^2(\eta) \cdot r_L^2(\eta)}{1 + \frac{d(\eta)^2}{p^2} \cdot \frac{1}{r_L^2(\eta)}} + e^2(\eta)p^2 \cdot r_L^4(\eta), \quad (6.3)$$

introducing a new parameter e^2 encoding the slope of the resolution at high momenta. This parameter has the same p dependence as c^2 , but the two can not be put together because they are measured with a different pseudorapidity binning.

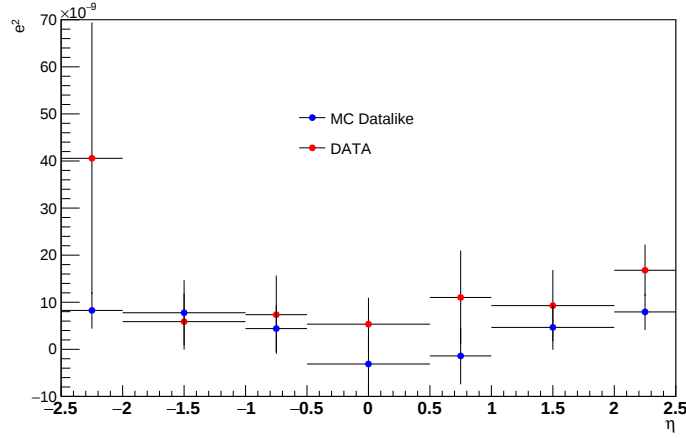
We model the invariant mass of the Z with a Breit-Wigner convoluted with a Gaussian. In the fit we fix the width of the Breit-Wigner to be equal to the value previously obtained from the generated sample. Furthermore, we constrain the σ parameter of the Gaussian to satisfy eq. 5.22 and depend on p through the parameters given in eq. 6.3. The parameter a^2 is fixed to the values obtained through the fit on the J/ψ , shown in Figure 5.10 and 5.12, while the parameters b^2 and c^2 are fixed to the values obtained in the MC Truth, as in J/ψ fit. The parameter e^2 is left free to converge to the appropriate values, positive or negative.

Results

The result of the fit for the parameter e^2 is shown in Figure 6.3 for data and Monte Carlo as a function of η . We observe that the value of e^2 is almost flat in both samples. The error bars on e^2 contain the statistical error on the fit and a systematic error due to our choice of the width of the Z . We compute this systematics as the difference in quadrature of the central value obtained with this fit and the one obtained with another fit with the Z width equal to 2.95 i.e. a variation of 10% of the difference between the natural width of the Z and the one we fit after final state radiation.

6.3 Comparing the measured resolution in data and simulation

Having obtained from J/ψ and Z events all the parameters describing the resolution in our model, we can put all the results together to estimate the


 Figure 6.3: Results of the fitted e^2 parameter in data and simulation.

muon momentum resolution as function of η in the range of momentum that we have considered. In Figure 6.4 and 6.5 we show the measured resolution in data, MC Datalike and MC Truth for $p = 10$ GeV, $p = 25$ GeV, $p = 50$ GeV and $p = 70$ GeV, while in Figure 6.6 and 6.7 we show the ratio of the measured resolution in data and MC Datalike, for the same values of p .

The four plots showing the ratio between the resolution measured in data and in the simulation (MC Datalike) are the conclusive plots of this analysis. The region at $\eta < -2$ is somewhat problematic, as this is the same region already discussed in section 5.2.5. We know that in this region the data suffer an alignment problem.

Comparing the four plots one notices that in the region of the (η, p) phase space where the contribution of the material is dominant (small momenta, or $|\eta|$ near 1.5, whenever the term a^2 in formula 6.3 is large compared to $e^2 p^2$) the ratio of the resolutions is measured with an uncertainty of about 3% in each bin. This because the material budget is constrained very precisely with the J/ψ data.

When the contribution of the $e^2 p^2$ term of equation 6.3 becomes dominant with respect to the a^2 term (as at $\eta \sim 0$ and $p=70$ GeV) we depend on the fit of the Z data, that is more challenging. The largest uncertainty in the plot at $p=50$ GeV in a single bin is 10%. It is important to recall that the e^2 fit is done in 7 bins and not in 50, so the errors in nearby bins of the plots in Figure 6.6 and 6.7 are largely correlated whenever the $e^2 p^2 \gg a^2$.

The closure plot 6.1 is done averaging the eta dependence at fixed momentum. This is equivalent to make the η average of one of the plots in Figure 6.6 and 6.7. When averaging over these uncertainties, taking into account the correlations among bins, the ratio of the resolution data and simulation is measured with an uncertainty that is smaller than 3%.

In the future the fit of the Z can be improved with a better modelling of the Z effective width that may reduce the systematic error. In addition a different fit strategy may be adopted in order to fully profit from the Z statistics.

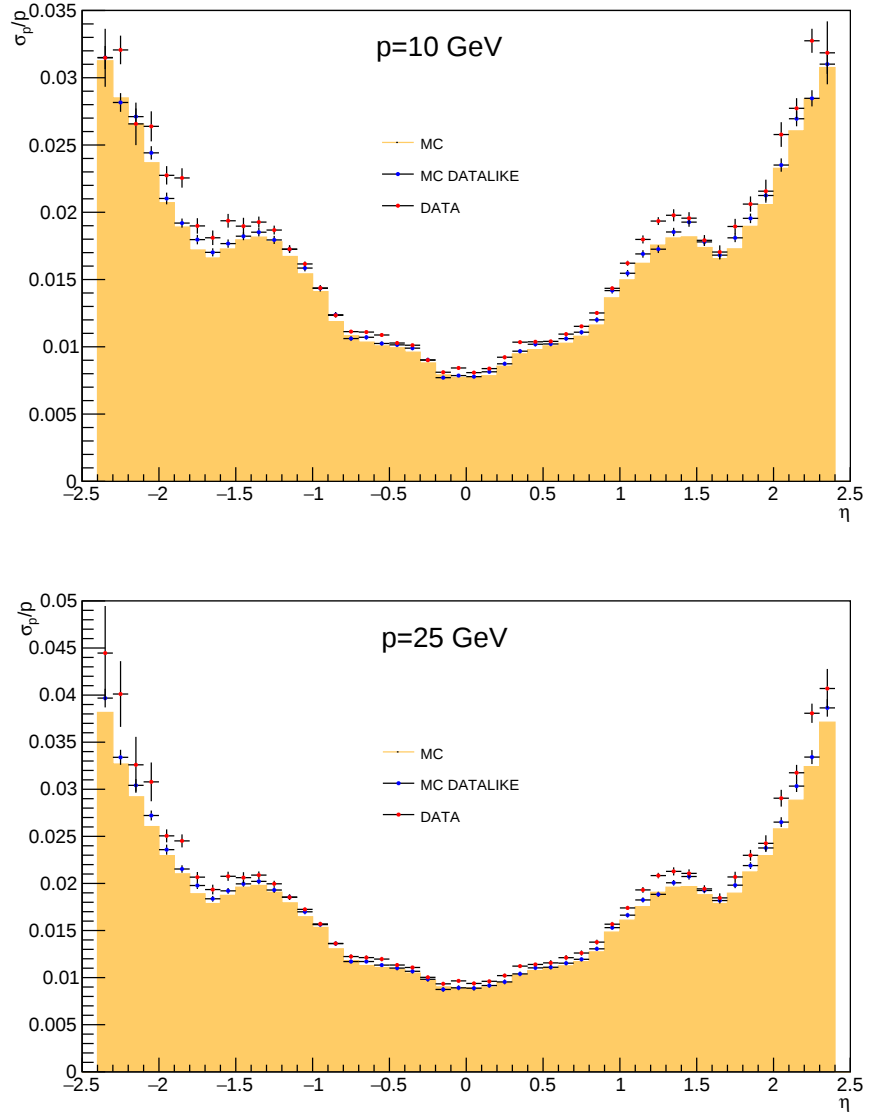


Figure 6.4: Measured resolution in data (red), MC Datalike (blue) and MC Truth (full orange), for $p = 10$ GeV (top) and $p = 25$ GeV (bottom).

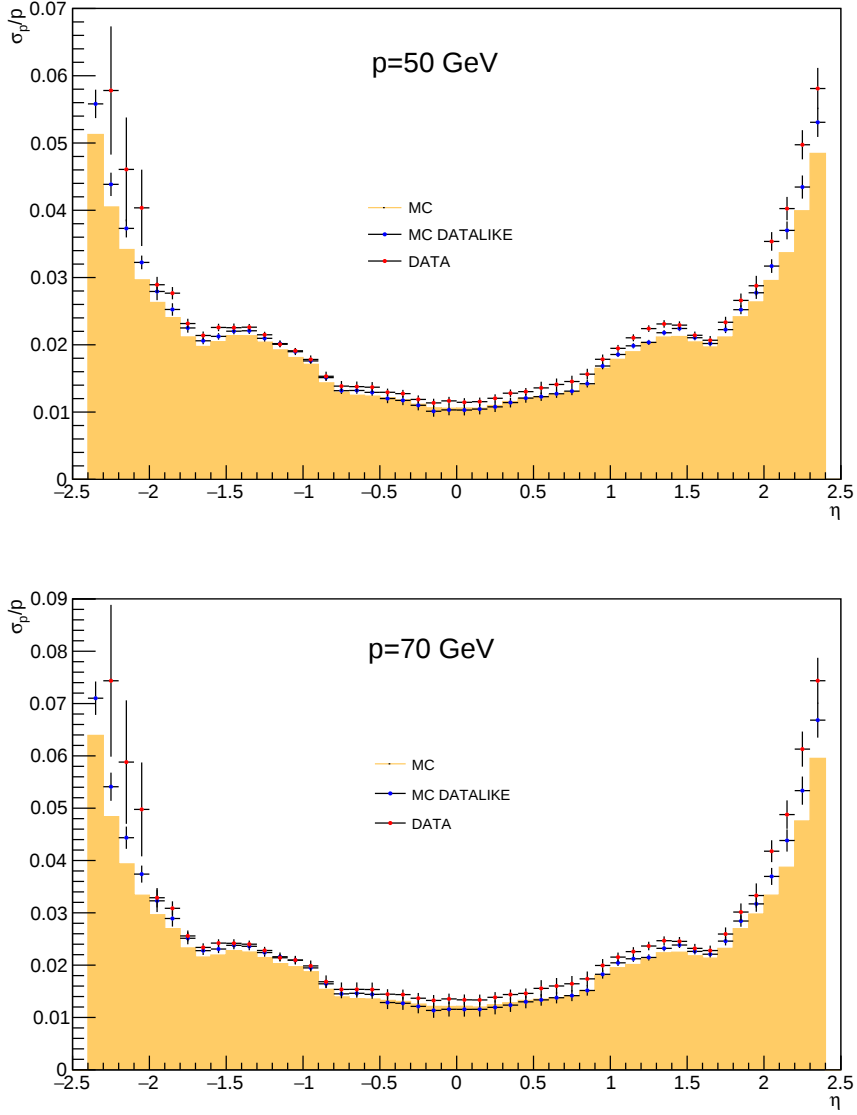


Figure 6.5: Measured resolution in data (red), MC Datalike (blue) and MC Truth (full orange), for $p = 50$ GeV (top) and $p = 70$ GeV (bottom).

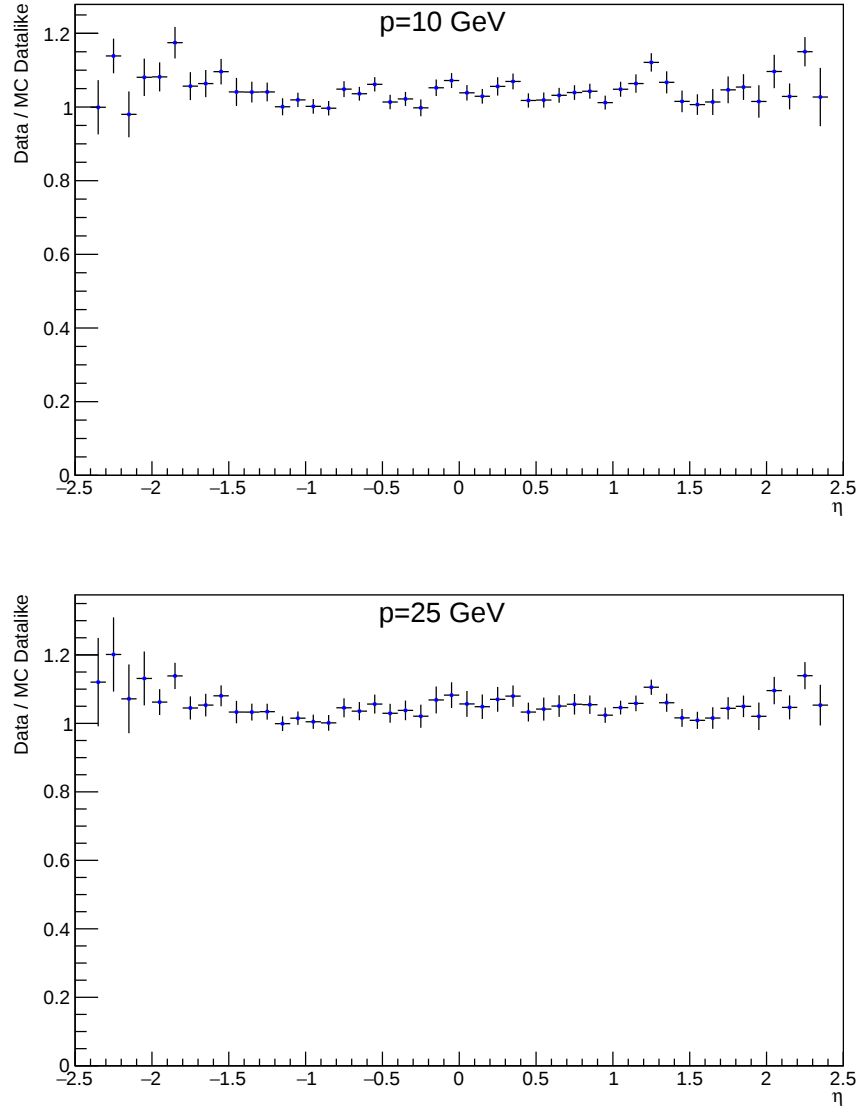


Figure 6.6: Ratio of the resolutions measured in data and MC Datalike, for $p = 10$ GeV (top) and $p = 25$ GeV (bottom).

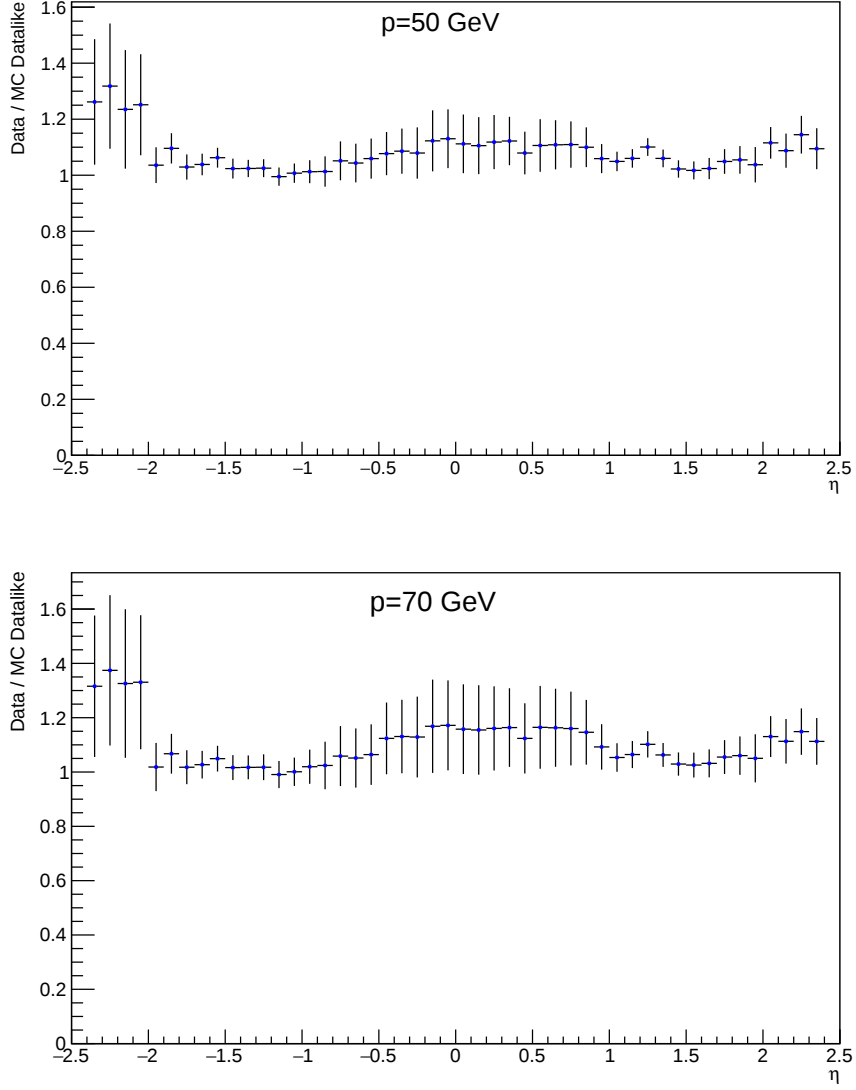


Figure 6.7: Ratio of the resolutions measured in data and MC Datalike, for $p = 50$ GeV (top) and $p = 70$ GeV (bottom).

Chapter 7

A study of the energy loss in the CMS tracker

In chapter 4 we have shown that a muon crossing the tracker loses about 60 MeV and the measurement of its momentum will be biased because of the energy loss. Since our goal is to calibrate the momentum scale of the tracker at $p = 50$ GeV with a precision of 10^{-4} this loss is not negligible. In chapter 5 we have validated the material budget of the tracker defined as the integral of the radiation length of the material encountered by a high energy muon as function of η . Here we want to extend this validation to the energy loss of the muon.

7.1 Energy loss of muons crossing the tracker

The energy loss of muons is precisely predicted by the Bethe-Bloch equation to depend on the characteristics of the traversed material:

$$\frac{dE}{dx} \propto \frac{Z}{A} \rho, \quad (7.1)$$

where Z and A are respectively the atomic and mass number of the material and ρ its density.

The complete formula, available in [22], is plotted in Figure 7.1, where dE/dx for muons is shown on the y axis in units of $\text{MeV g}^{-1} \text{ cm}^2$ as function of their momentum p for various traversed material. The curves show a minimum at $p \sim 0.5$ GeV and a logarithmic increase with p . At the minimum of the curves, also called Minimum Ionising Point (MIP), the energy loss weighted with the inverse of the material density is roughly independent from the material and its value is approximately $2 \text{ MeV g}^{-1} \text{ cm}^2$. Therefore, the average tracker density being 0.2 gr cm^{-2} , the typical energy loss of a muon crossing 1.5 m in the tracker is 60 MeV.

The density of the CMS tracker is not uniform since there is a great variety of materials in the different components and a large part of the trajectory

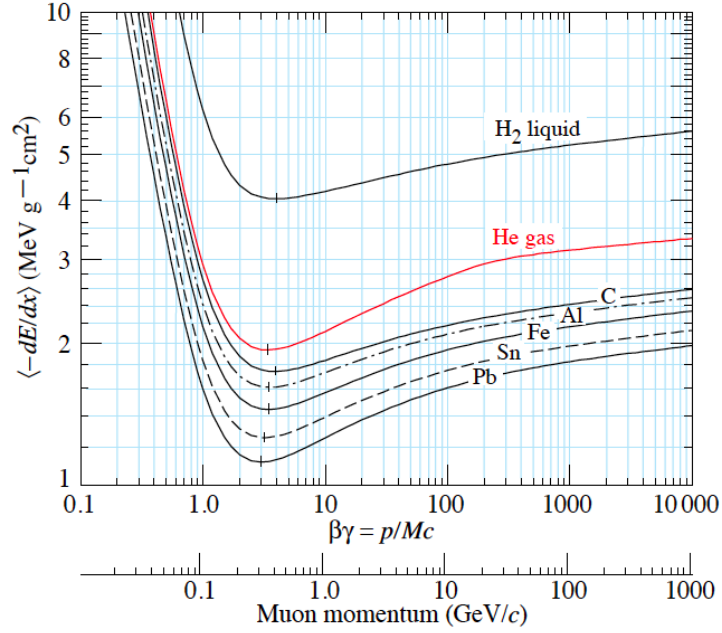


Figure 7.1: Energy loss for muons

is in the inert gas. The material of the measurement layers is concentrated in shells and disks, and the aluminum cables and pipes are routed outside the sensitive volume. In addition there are support structures.

The silicon sensors are typically less than 20% of the budget. The cooling pipes are made in Aluminum and the cooling fluid is C₆F₁₄. The supporting structure of the modules are in Carbon fibre and the substrate of the hybrids is in Alumina (Al₂O₃). Cables are also made of Aluminum. The large supporting structures are in Carbon and electronic boards contain traces of copper. Large part of the trajectory extends in the inert (Nitrogen) gas of the tracker volume. Table 7.1 shows relevant properties of the different materials listed above and Figure 7.2 shows a map of the density of the tracker derived from the GEANT simulation of the detector.

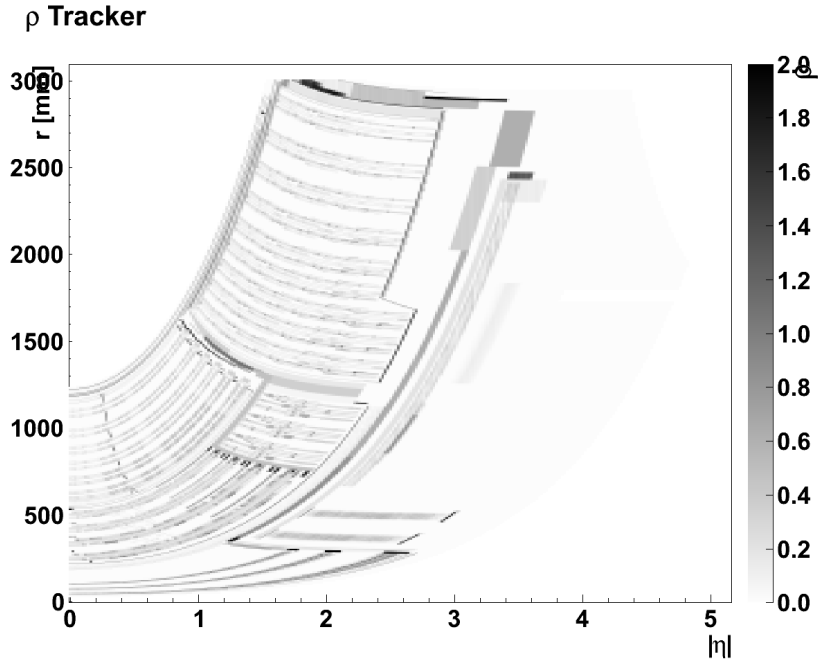
In addition, Table 7.1 shows that for most of the materials present in the tracker the values of the energy loss at minimum are very similar, equal to better than 1-2%. The largest difference is between Carbon and Aluminum that is 5%. The density of Nitrogen is so small that the energy loss in it can be safely neglected.

7.1.1 Momentum bias due to a localised energy loss

Depending on the point in space where the energy loss occurs, there is a bias in the measured transverse momentum.

Table 7.1: Properties of different materials in the tracker.

Material	ρ gr cm ⁻³	$1/X_0$ cm ⁻¹	$dE/dx(\text{min})$ MeV g ⁻¹ cm ²
Carbon	2.2	0.051	1.74
Silicon	2.3	0.11	1.66
Aluminum	2.7	0.11	1.62
Coolant C ₆ F ₁₄	1.7	0.050	1.69
Alumina (AL ₂ O ₃)	3.95	0.14	1.65
Copper	9.0	0.70	1.40
Nitrogen gas	1.2 10 ⁻³	3.2 10 ⁻⁵	1.82


 Figure 7.2: Map of the density in the tracker as a function of η and the distance r_s from the nominal origin.

Call $\Delta E = \Delta E_0 \sin \theta$ the loss in momentum¹. If it occurs before the first measurement, the bias will be ΔE . In the general case consider a the trajectory of a track with curvature $1/\rho = 0.3B/p$ on a transverse length L when there is a (small) momentum loss at $x = x_0$.

The trajectory is:

¹Since we treat only relativistic muons we will use always the approximation $E \sin \theta = p$ where p is the transverse momentum and θ the polar angle.

$$y = \frac{1}{2\rho} \left(x(x-L) + \frac{\Delta E}{p} (x-x_0)^2 \right) \quad (7.2)$$

where the second term should be considered only for $x > x_0$.

Assuming uniformly spaced measurements one can minimise the χ^2 of a parabolic fit that ignores the loss and compute the bias Δp on the momentum:

$$\frac{\Delta p}{p} = \frac{\Delta E_0 \sin \theta}{p} \left(1 - \frac{5}{2} \frac{x_0}{L} + \frac{5}{3} \left(\frac{x_0}{L} \right)^2 - \frac{1}{6} \left(\frac{x_0}{L} \right)^5 \right) \quad (7.3)$$

The function that multiplies $\Delta E_0 \sin \theta / p$ is plotted in Figure 7.3. As expected, the weight of the first part of the trajectory is much larger than the second part.

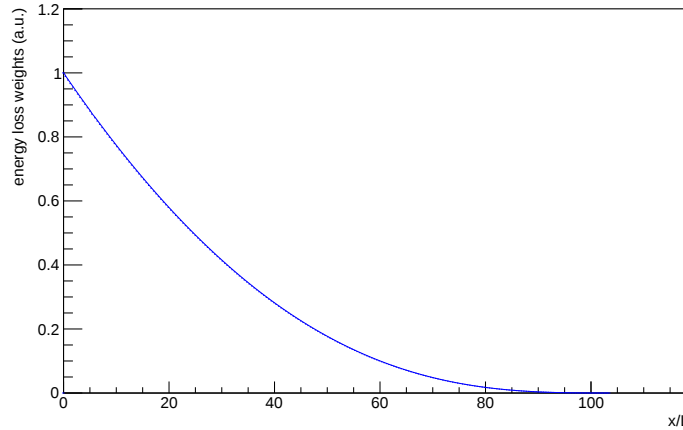


Figure 7.3: Fraction of the momentum bias due to an energy loss occurring at a position x/L along the trajectory plotted vs x/L .

During the track fit in CMS the energy loss is included in the track propagation with an approximation that assumes the material to be concentrated in the measurement layers.

Comparing Figure 7.3 and Figure 7.2 we notice that what matters for the momentum bias is the material in the first half of the trajectory, where the tracker is pretty light until $\eta = 1.2$. The service tubes of the pixels and the cables between TIB and TID appear as the most critical places.

The energy loss ΔE_0 at $p = 50$ GeV can be computed using the simulation of the tracker material. This quantity is shown as function of η in Figure 7.4 top panel. From this plot we see that at $\eta = 0.5$ the energy loss is ~ 30 MeV and at $\eta = 1.4$ is ~ 89 MeV.

The bias in the transverse momentum $p = 50$ GeV can be computed weighting the energy loss along the trajectory using equation 7.3 and projecting it on the transverse plane. This is shown as function of η in Figure 7.4 bottom panel. We see that $\Delta p \sim 8$ MeV at $\eta = 0.5$ and $\Delta p \sim 14$ MeV at $\eta = 1.4$.

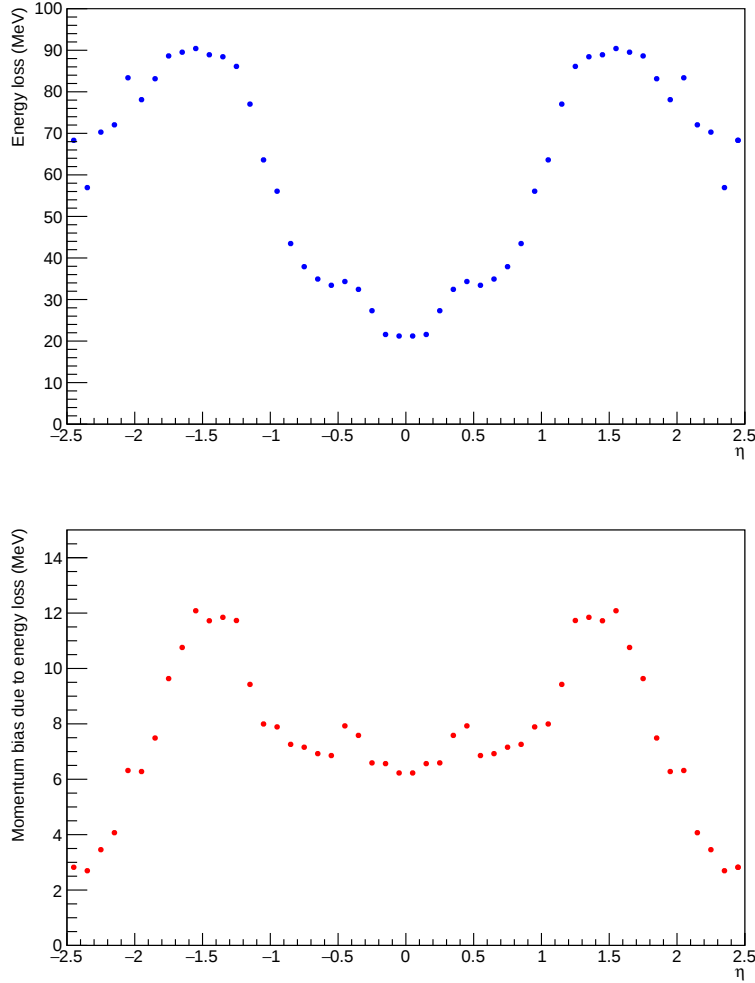


Figure 7.4: Top panel: energy loss at $p = 50$ GeV as function of η . Bottom panel: momentum bias at $p = 50$ GeV due to energy loss as function of η .

7.1.2 Precision needed in the control of the energy loss

Our goal is to calibrate the momentum scale of the tracker at $p = 50$ GeV with a precision of 10^{-4} . We see that the momentum bias due to the energy loss at 50 GeV is $1.5 \cdot 10^{-4}$ at $\eta = 0.5$ and $3 \cdot 10^{-4}$ at $\eta = 1.4$. A description of the tracker material accurate to few 10% would make this effect negligible.

However, there is an important correlation effect between energy loss and magnetic field in the calibration procedure that calls for a better knowledge of the energy loss. The energy loss is fitted in each η bin with an error smaller than 1 MeV in the Kalman fit for scale determination (see chapter 4), together with other parameters affecting the momentum scale: magnetic field and sagitta distortions due to alignment errors.

The calibration is done with J/ψ decays in two muons whose transverse momentum distribution is highly peaked at low momenta. For this reason the fit has reduced power in disentangling the effect of a change of magnetic field from an energy loss in the material: the two effects are indistinguishable at fixed muon momentum. This results in a large correlation ($\sim 90\%$) between the magnetic field scale parameters and the energy loss parameters in the calibration.

The calibration performed on ~ 5 GeV muons is later used on muons of ~ 50 GeV. The relative weight of the corrections for magnetic field calibration (proportional to the momentum) and energy loss calibration (constant at the 10% level between $p=5$ GeV and $p=50$ GeV) changes by a factor of about 10, reducing the precision on the relative scale determination at 50 GeV compared to 5 GeV. Since the muon calibration is done with ~ 5 GeV muons we can reduce significantly the correlation in the calibration procedure with a knowledge of the material density with a precision of 10%.

7.1.3 Material budget: energy loss and momentum resolution

In chapter 5 we have shown that measuring the resolution on the J/ψ mass as a function of the momenta (and directions) of the two decay muons it is possible to compute a quantity that is correlated with the integral of the radiation length:

$$\frac{\int w(x)dx/X_0}{\int w(x)dx}, \quad (7.4)$$

where $w(x)$ is the square of the distance from the closest between the first and the last point of the measured trajectory.

Figure 7.5 top panel shows how the distribution of the material along the track affects in a different way the momentum bias due to energy loss and the momentum resolution. The momentum bias due to energy loss depends on the material distribution in the first part of the track, while the momentum resolution due to multiple scattering samples the center of the trajectory.

On the other hand, Figure 7.5 bottom panel shows the density along a track at $\eta = 1.4$ multiplied by the weight relevant for the momentum bias due to energy loss and the density multiplied by the weight relevant for the momentum resolution. We see that the two measurements sample different

parts of the tracker and that the two integrals along the trajectory are quite uncorrelated.

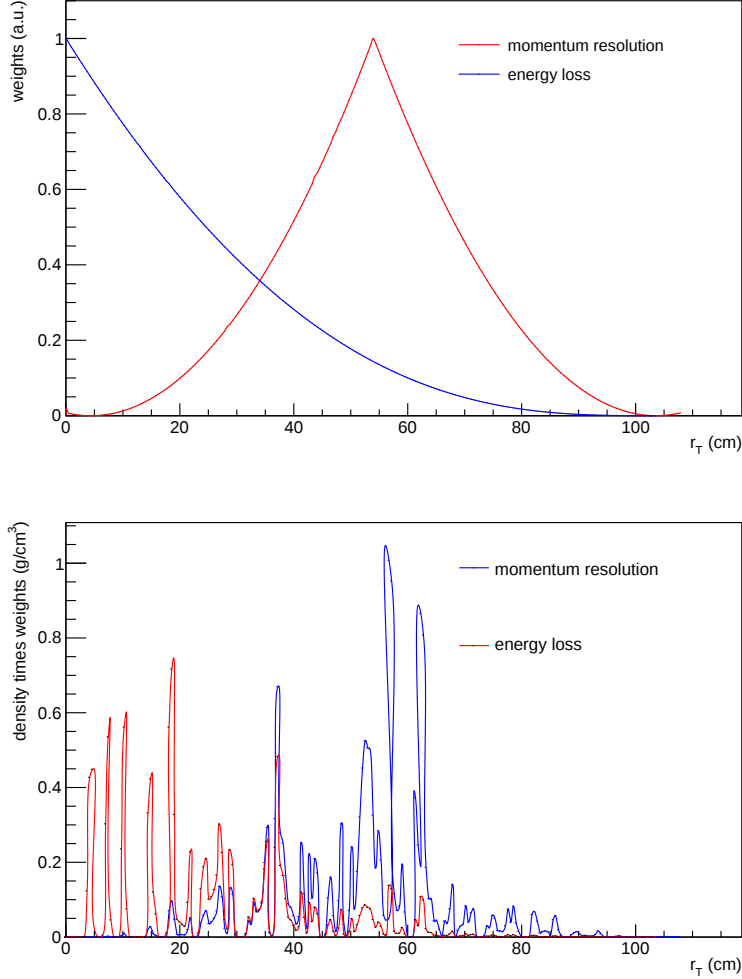


Figure 7.5: Top panel: weights of the material located at the transverse radius r_T in the momentum bias due to energy loss as function of r_T (blue curve). This is the same plot as fig. 7.3. The red curve contains the weights of the material located at r_T in the momentum resolution. Bottom panel: weights of the top panel multiplied by the density along the track.

The momentum resolution tests the material budget mainly in the central part of the detector while the momentum bias due to the energy loss depends on the material at the beginning of the trajectory: the material budget measured with momentum resolution cannot be used to validate the momentum bias due to the energy loss. This limitation can be solved if we measure the material budget locally, as a function of the distance of the track from

the origin. A preliminary study of this measurement is presented in the following section.

7.2 Local measurement of the material budget

In the tracker the transverse momentum is computed using several, typically 13, silicon layers. The track fit gives an estimate of the transverse momentum on each layer (called here p) taking into account the energy loss estimated from the material description. In the following we will assume that the sagitta measured using three consecutive layers is largely uncorrelated from p .

7.2.1 A model in the framework of tracking

Consider the measurements of the track position done by three consecutive layers. These measurements are projected on the transverse plane (r - ϕ plane) of the global CMS system and each measurement is identified by a 2D vector in this plane. The measurement in the local system of the sensor are roto-translated in the the global system using the alignment information. In the case of strip sensors the coordinate along the strip in the local system is set to the intercept of the fitted track with the module before applying the roto-traslation.

Using the measured coordinates we define two 2D vectors: $\vec{a} = \vec{x}_2 - \vec{x}_1$ and $\vec{b} = \vec{x}_3 - \vec{x}_1$ where $\vec{x}_1, \vec{x}_2, \vec{x}_3$ are the measurements of the three points in the global system (ordered in increasing r).

The sine of the angle between the two vectors $\vec{a}$ and $\vec{b}$ is given by

$$\sin \alpha = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}, \quad (7.5)$$

and the distance between the point $\vec{x}_2$ from the line joining $\vec{x}_1$ and $\vec{x}_3$ is

$$s = \frac{|\vec{a} \times \vec{b}|}{|\vec{b}|} \quad (7.6)$$

If the projection of $\vec{a}$ on $\vec{b}$ is half of the length of $\vec{b}$ then s is the local sagitta of the track.

The geometry of the system is shown in Figure 7.6.

Consecutive measurements in the CMS tracker

We consider now a local reference system with origin on $\vec{x}_1$, the x axis along the line joining $\vec{x}_1$ and $\vec{x}_3$ and the y axis in the transverse plane of CMS

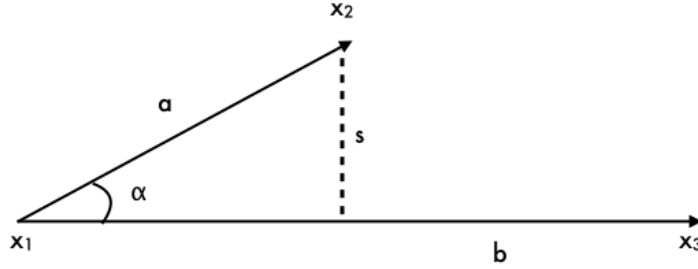


Figure 7.6: Geometry of the system of three measurements from three consecutive layers in the CMS tracker.

where the vectors $\vec{a}$ and $\vec{b}$ lay. In this system the local trajectory, in absence of multiple scattering, is a circle² of radius ρ that can be approximated with a parabola assuming $\rho \gg |\vec{b}|$. We will test later in this section the validity of this approximation. The equation of the trajectory is:

$$y = -\frac{x^2 - bx}{2\rho}. \quad (7.7)$$

Assuming a multiple scattering angle θ at $x_0 = |\vec{a}| \cos \alpha$ the trajectory gets modified as:

$$\begin{aligned} y &= -\frac{x^2 - bx}{2\rho} - \theta \frac{b - x_0}{b} x & x < x_0 \\ y &= -\frac{x^2 - bx}{2\rho} - \theta \frac{b - x_0}{b} x + \theta(x - x_0) & x > x_0 \end{aligned} \quad (7.8)$$

and the distance s of the point $\vec{x}_2$ from the line joining $\vec{x}_1$ and $\vec{x}_3$ is:

$$s = -\frac{x_0^2 - bx_0}{2\rho} - \theta \frac{b - x_0}{b} x_0. \quad (7.9)$$

This equation can be rewritten as:

$$\frac{s}{x_0(b - x_0)} = \frac{1}{2\rho} - \frac{\theta}{b}, \quad (7.10)$$

If the material is not concentrated on the central layer this equation is still a good approximation and the angle θ is interpretable as the multiple scattering weighted with some geometrical function. The more the material is close to the first or the last layer, the smaller is the angle.

²The radius of a 1 GeV particle in CMS is 88 cm.

We can now multiply by p and we obtain

$$t \equiv \frac{ps}{x_0(b-x_0)} = \frac{p}{2\rho} - \frac{p\theta}{b}. \quad (7.11)$$

The first term on the right side of the equation is about constant in modulus and changes sign for positive and negative tracks.

The momentum p and the global radius ρ_G of curvature of the track are linked to the magnetic field B by:

$$p = 0.3B\rho_G/100, \quad (7.12)$$

where p is measured in GeV, B in Tesla and ρ_G in centimetres.

Assuming the mean value of the magnetic field to be $\bar{B} = 3.8$ T in CMS, the first term of eq. 7.11 can be written as $\rho_G/\rho \cdot 5.7 \cdot 10^{-3}$ GeV/cm.

Since $\rho_G/\rho \sim 1$, we expect that if we measure perfectly all the variables contributing to the left side of equation 7.11 we obtain two peaks centred at $5.7 \cdot 10^{-3}$ GeV/cm since the scattering angle θ averages at zero.

The second term on the right side of eq. 7.11 is due to multiple scattering. The angle θ is distributed with average equal to zero and r.m.s equal to θ_0 given by eq. 5.1, that is inversely proportional to p .

The term of eq. 7.11 $p\theta/b$ has r.m.s. equal to $(13.6 \cdot 10^{-3} \sqrt{L/X_0})/b$. For $b = 10$ cm and $L/X_0 = 0.05$ it gives $\text{r.m.s}_{MS} = 3 \cdot 10^{-4}$ GeV/cm, i.e. a contribution of $\sim 5\%$ to the width of the peaks.

Validity of the approximation of a circle with a parabola

The distance between the modules is ~ 10 cm and the radius of curvature of a particle of 1 GeV momentum is 88 cm. The parabola approximation of the circumference may introduce a bias.

Introducing a second term in the approximation of the circle with a parabola modifies the equation as:

$$y = -\frac{x^2 - bx}{2\rho} \rightarrow y = -\frac{x^2 - bx}{2\rho} - \frac{x^4 - 2x^3b + 3/2 x^2b^2 - 1/2 xb^3}{8\rho^3} \quad (7.13)$$

The ratio between the two terms at $x = b/2$ is $b^2/(32\rho^2)$ that is 0.04% for $b=10$ cm and $p=1$ GeV.

7.2.2 Testing the model on CMS data

The validity of this model has been tested on a sample of data collected by CMS in Spring 2016. These events have been selected using a Zero Bias trigger, i.e. randomly reading out the detector whenever a collision is possible. Since the LHC production is dominated by soft QCD interactions, the

majority of these events do not contain any hard scattering and are composed mainly of low energy hadrons, especially pions and kaons, uniformly distributed in η and characterised by an exponentially-decreasing spectrum in transverse momentum p . Furthermore, a sample of Monte Carlo simulation with the same characteristics has been compared to the data. The distributions of these kinematical variables are plotted in Figure 7.7. An overall agreement is observed between data and simulation. The eta plot shows some left right asymmetries due to some non working modules, amplified by the fact that we require a hit on each silicon layer crossed by the track.

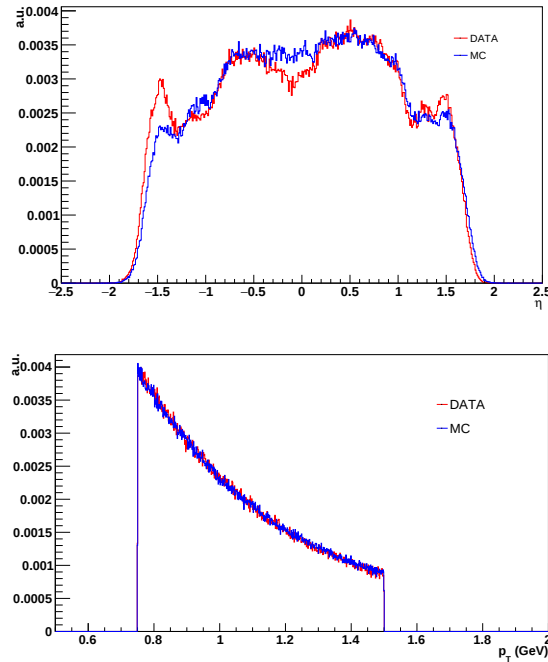


Figure 7.7: Distributions of η (top) and transverse momentum p (bottom) in the samples of data (red) and simulation (blue).

Event selection

For each event in data and simulation we have the parameters of each fitted track and in addition we have access to the estimated values of these parameters in each layer of the tracker. Before running the analysis we select only the tracks fulfilling some quality requirements.

We keep only the tracks in the event having p between 750 MeV and 1.5 GeV. The lower cut is useful to reject the loopers, i.e. low energy particles that do not have enough energy for traversing the whole tracker and make multiple turns in the first layers of the detector. 750 MeV is the minimum

p needed for traversing the whole tracker barrel, from the first pixel layer to the last TOB layer. The upper cut is quite arbitrary and has been chosen large enough to obtain a significant range in p^2 , which is useful to extract the material budget as we will explain precisely later in this section.

Moreover, we require that the tracks have at least 14 hits. This is the number of layers in the tracker barrel plus one: 3 pixel layers, 4 TIB layers and 6 TOB layers. Since there are in addition 4 stereo layers and some overlaps among the modules the number of hits is very often above 14. In particular, at least 2 of these hits must be in the pixel layers.

Finally, we accept only the tracks having a relative uncertainty on the p equal or less than 1%.

Extraction of the local radiation length

In this study we have extracted the local radiation length of the layers of the pixel detector of the CMS tracker. The first step has been to select a subset of events containing one hit in each of the three consecutive following layers:

- pixel layer 1, pixel layer 2, pixel layer 3;
- pixel layer 2, pixel layer 3, TIB layer 1.

Notice that, since with this model we measure the scattering angle in the central point of the triplets, we can not measure the material in the first layer of the pixel detector³.

For each triplet, we plot the variable t of eq. 7.11 in Figure 7.8 in data and simulation, which superimpose almost perfectly. Observing the position of the peaks in these plots we can verify that eq. 7.11 behaves as expected. In Figure 7.9 we show again the distribution of the variable t for the triplet pixel layer 1, pixel layer 2, pixel layer 3 in data, in logarithmic scale in the y axis to highlight the gaussianity of the distribution.

A meaningful operation is taking the sum and the difference of the values of the mean of the two peaks. Their sum must be compatible with 0, and any deviation from this value is due to the misalignment of the modules along the $r - \phi$ direction. On the other hand, their difference must be compatible with the term p/ρ of eq. 7.11 which in turn must be proportional to the magnetic field. Therefore, a difference compatible with the prediction of $11.4 \cdot 10^{-3}$ in both the triplets confirms that the magnetic field is constant in the tracker barrel. Table 7.2 summarises the values of the sum and difference of the means of the positive and negative peak, which confirm the previous statements.

³This analysis, however, can be extended also in that region using as first measurement the position of the primary vertex of the fitted track using the beam envelope information and several tracks.

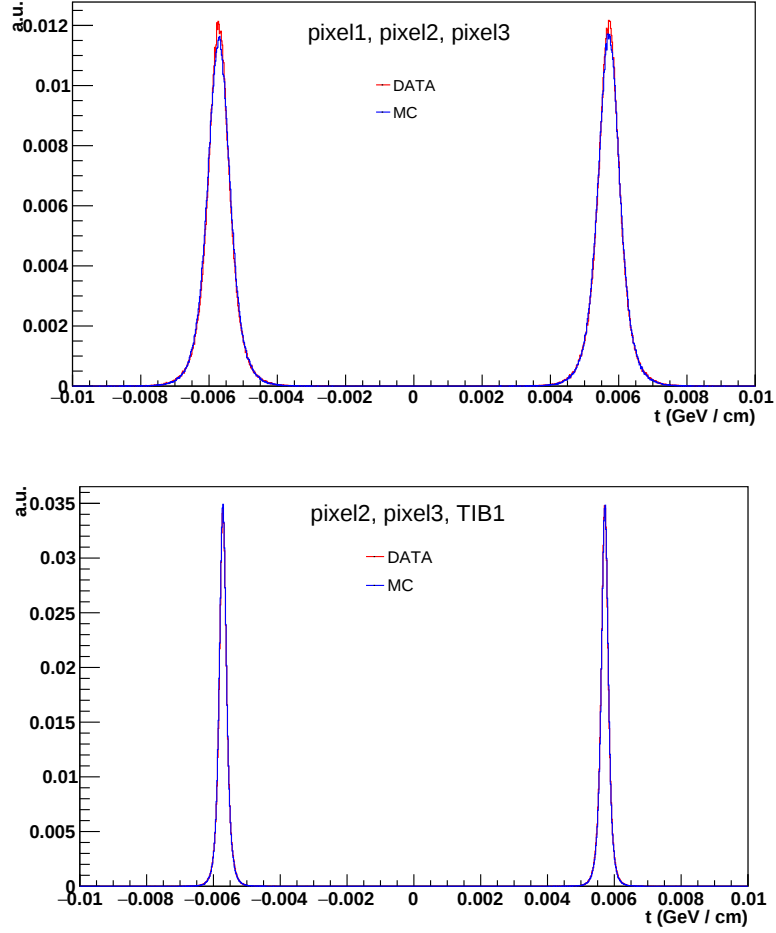


Figure 7.8: Distributions of the variable t in events containing one hit in each of the three consecutive layers: pixel 1, pixel 2, pixel 3 (top) and pixel 2, pixel 3, TIB 1 (bottom). Data (red) and simulation (blue) superimpose almost perfectly and the distributions peak around $\pm 5.7 \cdot 10^{-3}$ GeV/cm as expected. The r.m.s. of the distributions in the two panels is different because the distance between the first and the third measurement layer (b in equation 7.11) is about 3 times larger for the bottom panel.

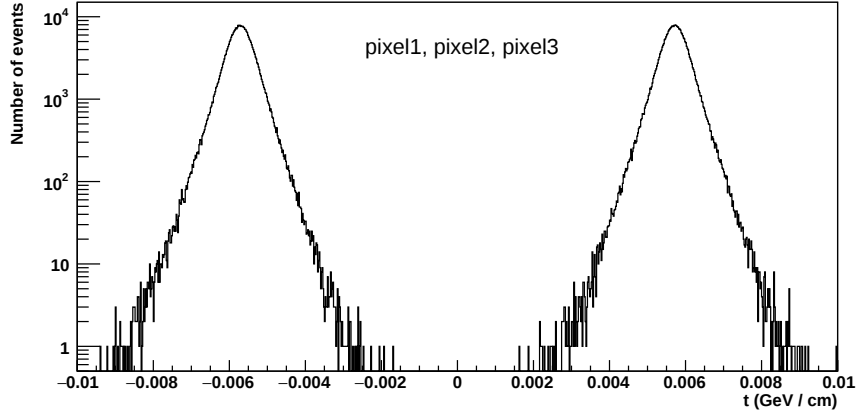


Figure 7.9: Distributions of the variable t in events containing one hit in each of the three consecutive layers: pixel 1, pixel 2, pixel 3 in data. The plot is shown in logarithmic scale in the y axis to highlight the gaussianity of the distribution.

Table 7.2: Summary of the values of the sum and difference of the means of the positive and negative peaks obtained plotting eq. 7.11 .

layer	sum (GeV/cm)	difference (GeV/cm)
pixel 2	$(1 \pm 1) \cdot 10^{-6}$	$(11.427 \pm 0.001) \cdot 10^{-3}$
pixel 3	$(1.1 \pm 0.4) \cdot 10^{-6}$	$(11.4231 \pm 0.0004) \cdot 10^{-3}$

To obtain an histogram distributed as the multiple scattering angle θ times p , we subtract the measured mean from t and multiply for b on an event by event basis, separately for positive and negative tracks. Since we checked that the two r.m.s. are equal in the statistical precision, for every triplet we take as measured width the mean of the r.m.s. of the distributions of $(t - peak) \cdot b$ for positive and negative tracks.

Nevertheless, before extracting the material budget from the width of the peaks, it is necessary to introduce a few caveats.

If we measure perfectly all terms on the left side of eq. 7.11 we obtain two peaks whose variance is proportional to the amount of radiation lengths in the central layer. However the quantities p, s, x_0 and $(b - x_0)$ are measured with finite precision contributing to the variance of the two peaks, adding to the variance given by multiple scattering.

Propagating the uncertainties on 7.11 we obtain:

$$\Delta t = t_0 \frac{\Delta p}{p} \oplus t_0 \frac{\Delta x_0 (b - x_0)}{x_0 (b - x_0)} \oplus t_0 \frac{\Delta s}{s} \quad (7.14)$$

At low momentum (below 10 GeV) the relative error on the momentum of a well measured track is smaller than 1%. The first term of eq. 7.14 is then in

general smaller than the contribution of the multiple scattering. To control this effect we have added to the selection a criterion which accepts only tracks with a measured (using the fit covariance matrix) value of $dp/p < 1\%$. The second term of eq. 7.14 is related to how well we know the alignment of the tracker (the radial position of the modules), that is probably better than $10\ \mu\text{m}$. In addition the distance is fairly constant for a given triplet of modules, i.e. does not change on event by event basis. The size of this effect is about $10\ \mu\text{m}/5\ \text{cm} : 0.02\%$ and can be neglected compared to the other terms.

The last term of eq. 7.14 is more problematic. The sagitta on 10 cm of a track with momentum 1 GeV is 1.4 mm and the combination of the measurement uncertainties on the modules gives few $10\ \mu\text{m}$. The ratio of the uncertainty and the sagitta can be several percent.

For a given triplet of modules what matters is the measurement error: an uncertainty on the relative alignment will contribute to a shift of both peaks by a fixed amount. When integrating on several triplets of modules the uncorrelated shifts will contribute to the width of the peak, however this contribution is smaller than the one caused by the measurement uncertainties since the alignment uncertainty is smaller than the measurement uncertainty.

The most relevant contribution to the variance of the peaks among the terms in eq. 7.14 comes from the uncertainty on the sagitta and can be rewritten as:

$$\Delta t_s = p \frac{\Delta s}{x_0(b - x_0)} \quad (7.15)$$

It is linear in p if the fraction in eq. 7.15 is independent on momentum. Therefore, to remove this uncertainty it is sufficient to study the dependence of the variance of the peaks as function of the momentum and extrapolate it at $p=0$. A linear dependence indicates that the error on the sagitta is independent on momentum.

Since the material budget varies strongly with the pseudorapidity, we have divided the dataset in 25 bins of local pseudorapidity⁴ of the hit in the range local $|\eta| < 2.5$. The reason for using the local pseudorapidity of the hit and not the pseudorapidity of the fitted tracks lies in the fact that the former is calculated with respect to the origin of the reference system, while the latter to the real point of interaction that varies on event by event basis due to the beam envelope.

In each local pseudorapidity bin, we fit the square of the mean of the r.m.s. of the positive and negative distribution as function of local p^2 with a straight line (since the contributions to the variance from eq. 7.15 and the multiple scattering sum in quadrature) and we extract its intercept with the y axis. In Figure 7.10 we show these fits for each triplet in the bin of local pseudora-

⁴The local pseudorapidity is computed with a polar angle such that $\tan \theta = z/r$.

pidity $(-0.2,0)$ for the data sample. The plots on the simulation show slopes very similar to the data. The difference in the value of the fitted slopes in the two triplets are due to the different geometry of the two cases.

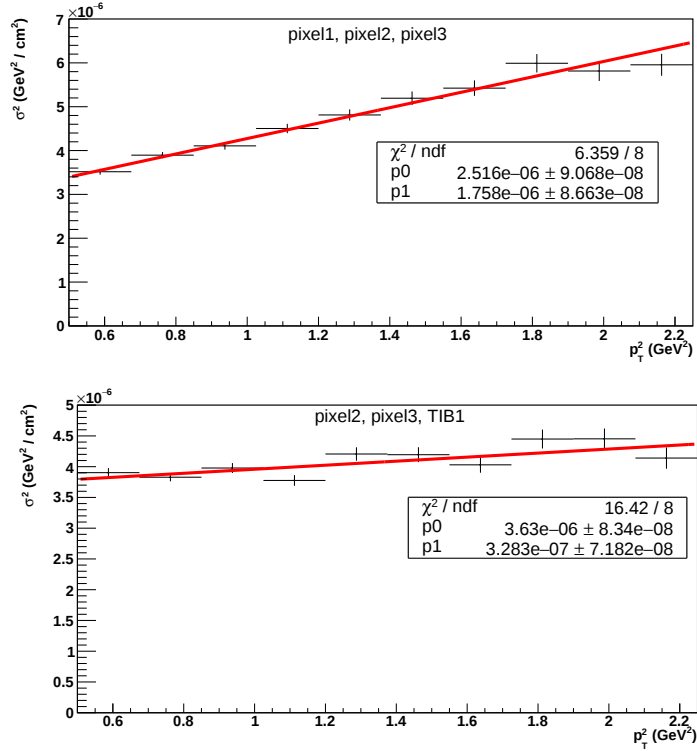


Figure 7.10: Fit of the square of the mean of the r.m.s. of the positive and negative distribution as function of local p_T^2 in the bin of local pseudorapidity $(-0.2,0)$ for the data sample, for the events containing one hit in each of the three consecutive layers: pixel 1, pixel 2, pixel 3 (top) and pixel 2, pixel 3, TIB 1 (bottom).

Having removed the contribution from eq. 7.15, according to the previous discussion, the intercept must be distributed as the multiple scattering angle times p and be equal to the fractional radiation length x/X_0 up to a constant. In Figure 7.11 we plot the distribution of the intercept as function of local η . We observe that the plots show an overall symmetry and that data and simulation have a very good agreement, meaning that the material budget of the tracker is very well simulated.

A way to check that we are correctly probing the material budget is to observe the distribution of the intercept as function of local η of Figure 7.11. For pure geometrical reasons, the quantity of traversed material is proportional to $1/\sin \theta$, where θ is the angle of incidence of the track in the module. In the assumption of constant material in the central layer of the triplet, a rescaling of the plot of the intercept as function of local η by $\sin \theta$ should give a flat distribution.

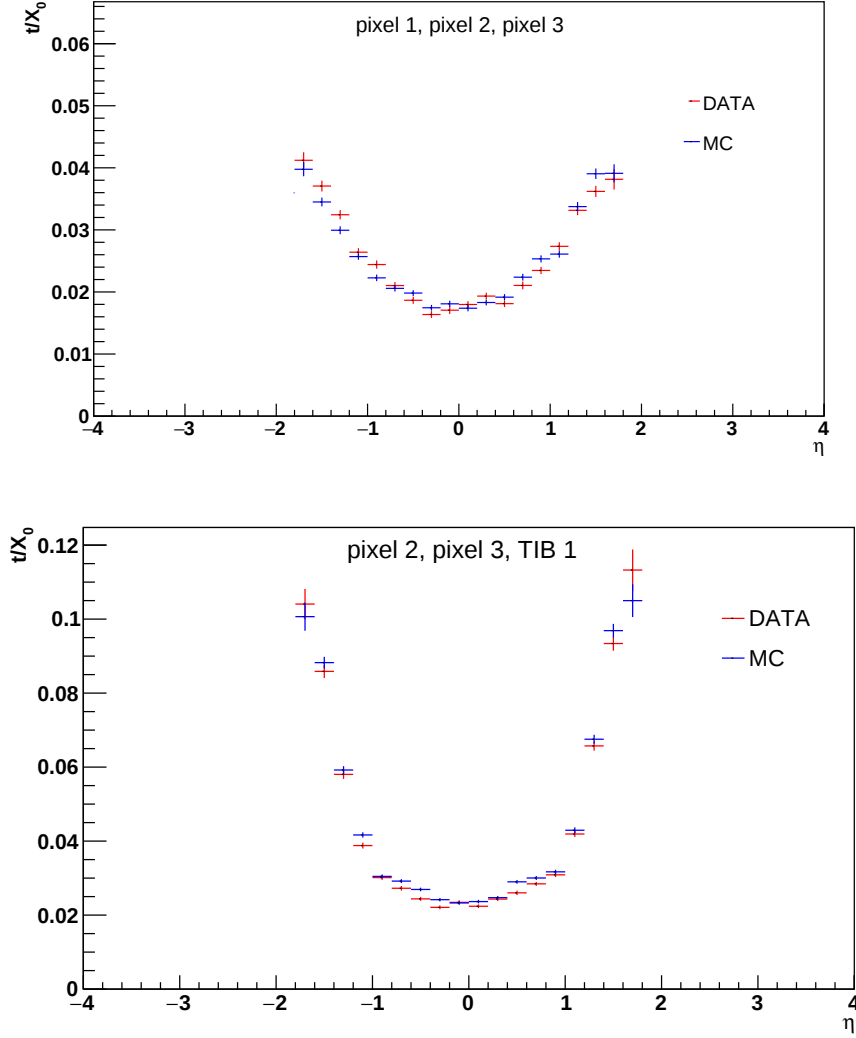


Figure 7.11: Distribution of the fractional radiation length t/X_0 , where t is the thickness of traversed material, as function of the local pseudorapidity, for the events containing one hit in each of the three consecutive layers: pixel 1, pixel 2, pixel 3 (top) and pixel 2, pixel 3, TIB 1 (bottom), for data (red) and simulation (blue).

In Figure 7.12 we show such distributions compared for data and simulation for both triplets. We notice that, while the distribution of the material in pixel layer 2 is almost flat, the one showing the material in pixel layer 3 starts increasing from a value of $|\eta| > 1$. This effect is due to the fact that in the region of pseudorapidity concerned, between the last layer of the pixel detector and the first layer of the TIB, the approximation of constant material fails since the material of the cables and cooling adds to the

measurement weighted with some geometrical function. This can be seen in Figure 7.2: here the trajectory of a track at constant η is a vertical line and we see that at $|\eta| = 1.2$ we start crossing the pixel support tube. This is also the reason why in the central part of the distribution we measure more material in the pixel 3 than in pixel 2: in the first case we are also sensitive to the material of the supports of the TIB detector, while in the second we only measure the effective material of the pixel 2.

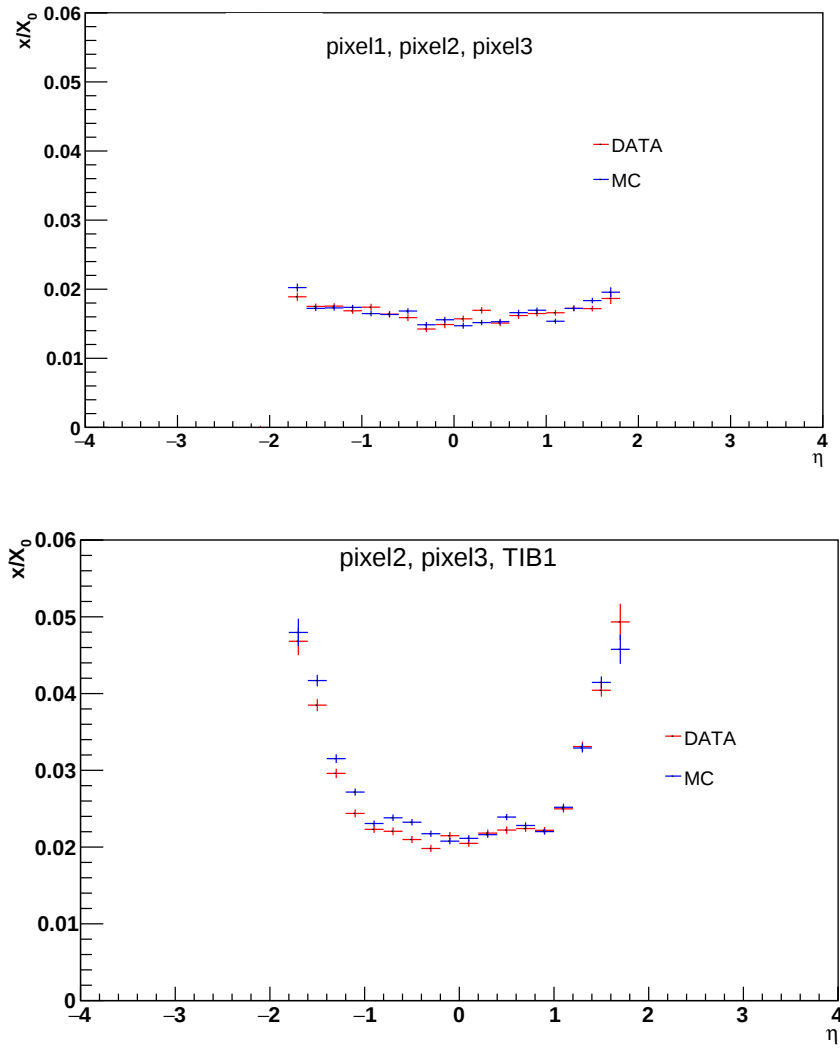


Figure 7.12: Distribution of the fractional radiation length x/X_0 as function of the local pseudorapidity, for the events containing one hit in each of the three consecutive layers: pixel 1, pixel 2, pixel 3 (top) and pixel 2, pixel 3, TIB 1 (bottom), for data (red) and simulation (blue). Here the plot has been rescaled by $\sin \theta$.

To sum up, in this study we have measured the local material budget of the tracker as function of the local pseudorapidity of the central hit of three consecutive measurements. The method we devised allowed us to reach a precision on x/X_0 of less than 10% on each single bin and $\sim 2\%$ integrating in each layer.

Appendix A

Derivation of the parametrisation of momentum resolution

In the CMS tracker the measurement of the momentum of the tracks is made through the measurement of the sagitta, as described in Chap 5.

For a particle of charge e and track length L in a uniform and constant magnetic field B the sagitta s is linked to the transverse momentum p by:

$$p = \frac{eBL^2}{8s} \quad (\text{A.1})$$

The uncertainty on the sagitta is thus propagated to the momentum by:

$$\sigma_p = \frac{eBL^2}{8s^2} \sigma_s = p^2 \frac{8}{eBL^2} \sigma_s \quad (\text{A.2})$$

In the reconstruction of tracks the momentum resolution is affected by the intrinsic hit uncertainty and the multiple scattering that the particles undergo when they cross the tracker.

The trajectory of the track can be parametrised as:

$$y = \alpha + \theta x + \frac{1}{2\rho} x^2 \quad (\text{A.3})$$

and the parameters α , θ and $1/\rho$ are found by minimizing the relative χ^2 . Our interest here is limited in retrieving the uncertainty on the parameter $1/\rho$, the curvature of the track that is proportional to the sagitta $1/\rho = 8s/L^2$.

Following the procedure of the minimization of the χ^2 , the covariance matrix of the fitted parameters is:

$$V = (A^T (V^{yy})^{-1} A)^{-1} \quad (\text{A.4})$$

where A is defined by:

$$A = \begin{pmatrix} 1 & x_0 & x_0^2 \\ 1 & x_1 & x_1^2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ 1 & x_N & x_N^2 \end{pmatrix}$$

and V^{yy} is the covariance matrix of the measurements y_i : $V_{ij}^{yy} = \langle y_i y_j \rangle$. The matrix element $V_{3,3}$ is the square of the uncertainty on the curvature of the track.

We compute now explicitly the covariance matrix V^{yy} in a simple geometry. Let us assume we have $N+1$ layers of detectors, at positions x_i , $i = 0, \dots, N$ along the axis of the original direction of the particle, uniformly spaced and in a constant and uniform magnetic field. Assume that each layer has identical radiation length $\Delta x/X_0$. The variance of the multiple scattering angle on each layer is $\delta\theta_0^2$. The total amount of material is $(N+1)\Delta x/X_0$ and the variance of the scattering angle of a track crossing the whole tracker is $\theta_0^2 = (N+1)\delta\theta_0^2$.

The elements of the covariance matrix V^{yy} are

$$V_y^{i,j} = \sigma^2 \delta_{i,j} + \delta\theta_0^2 \left(\frac{L}{N} \right)^2 \left(\frac{i(i+1)(2i+1)}{6} + (j-i) \frac{i(i+1)}{2} \right) \quad (\text{A.5})$$

where i runs from 0 to N and j runs from $i+1$ to N .

The matrix V has been computed explicitly for a configuration with 5, 7, 9, 11 layers and the momentum resolution has been computed from the V_{33} term and the formulae given above.

$$\left(\frac{\sigma_p}{p} \right)_{hit+ms}^2 = a^2 + c^2 p^2 \frac{1 + \sum_{\ell=1}^m g_\ell^2 \left(\frac{kL}{p\sigma} \right)^{2\ell}}{1 + \sum_{\ell=1}^m d_\ell^2 \left(\frac{kL}{p\sigma} \right)^{2\ell}} \quad (\text{A.6})$$

Here the terms c^2 and a^2 reproduce the terms introduced in equations ?? and 5.10 and there is an additional term that describes the correlation between the multiple scattering and the hit resolution. The sum runs up to a value m that depends on the number of layers and in our calculations it is 1,2,3,4 respectively for 5, 7, 9, 11 layers. In the case of three layers this term is absent because the multiple scattering on the first and on the last layer does not influence the momentum resolution. The term k is proportional to the material in the tracker and has dimension of momentum. The terms g_ℓ and d_ℓ are pure numbers and are reported in table A.1 for the cases we have considered.

Table A.1: Numbers we have derived for the terms g_ℓ and d_ℓ .

	$\frac{g_1^2}{d_1^2}$	$\frac{g_2^2}{d_2^2}$	$\frac{g_3^2}{d_3^2}$	$\frac{g_4^2}{d_4^2}$
5 layers	$2.08 \cdot 10^{-3}$	-	-	-
	$1.34 \cdot 10^{-3}$	-	-	-
7 layers	$3.70 \cdot 10^{-3}$	$1.94 \cdot 10^{-5}$	-	-
	$1.43 \cdot 10^{-3}$	$3.76 \cdot 10^{-6}$	-	-
9 layers	$5.21 \cdot 10^{-3}$	$7.35 \cdot 10^{-6}$	$1.51 \cdot 10^{-8}$	-
	$1.53 \cdot 10^{-3}$	$3.77 \cdot 10^{-6}$	$2.41 \cdot 10^{-9}$	-
11 layers	$6.67 \cdot 10^{-3}$	$3.78 \cdot 10^{-5}$	$5.2 \cdot 10^{-8}$	$2.02 \cdot 10^{-11}$
	$1.67 \cdot 10^{-3}$	$5.50 \cdot 10^{-6}$	$5.83 \cdot 10^{-9}$	$6.36 \cdot 10^{-13}$

When fitting the momentum resolution of the CMS tracker we have truncated the summation to the first term

$$\left(\frac{\sigma_p}{p}\right)_{hit+ms}^2 = a^2 + c^2 p^2 \frac{1 + \frac{g^2}{p^2}}{1 + \frac{d^2}{p^2}} = a^2 + c^2 p^2 + \frac{b^2}{1 + \frac{d^2}{p^2}} \quad (\text{A.7})$$

with $b^2 = c^2(g^2 - d^2)$.

Appendix B

Correction to the sagitta for non uniform material distribution

Consider a particle entering along the x axis in medium of length L . It will be subjected to multiple scattering (MS) resulting in a gaussian distribution of the angle (in any plane perpendicular to its original direction) of average zero and rms:

$$\theta_0 = \frac{13.6\text{MeV}}{p} \sqrt{L/X_0} \quad (\text{B.1})$$

Where p is the momentum of the particle and L/X_0 is the length of the material measured in radiation lengths. The MS will also cause a transverse (call it y direction) displacement of the track that we can compute in the following way.

Consider N small steps Δx along the path of the particle $N\Delta X = L$ and ignore to first order that the steps are not exactly along x . At each step the particle will scatter by a small angle $\delta\theta$ and the transverse displacement at $x = L$ is given by

$$y = \sum_{i=1}^N \delta\theta_i (L - i\Delta X) = \sum_{i=-N/2}^{N/2} \delta\theta_i (L/2 - i\Delta X)$$

Here, for reasons of symmetry that will help later in the calculation, we have assumed that the particle moves along x from $-\frac{L}{2}$ to $\frac{L}{2}$.

The average y , $\langle y \rangle$, is $\langle y \rangle = 0$ and the variance of y is given by

$$\langle yy \rangle = \left\langle \left(\sum_{i=-N/2}^{N/2} \delta\theta_i (L/2 - i\Delta X) \right) \left(\sum_{j=-N/2}^{N/2} \delta\theta_j (L/2 - j\Delta X) \right) \right\rangle$$

since the $\delta\theta$ are all independent the variance of y is given by

$$\langle yy \rangle = \left\langle \sum_{i=-N/2}^{N/2} \delta\theta_i^2 (L/2 - i\Delta X)^2 \right\rangle = \sum_{i=-N/2}^{N/2} \langle \delta\theta_i^2 \rangle (L^2/4 - Li\Delta X + i^2(\Delta X)^2) \quad (\text{B.2})$$

Assuming all $\delta\theta_i^2$ are equal (uniform medium)

$$\langle yy \rangle = N \langle \delta\theta^2 \rangle \left(\frac{L^2}{4} + \frac{(N\Delta X)^2}{4 \cdot 3} \right) = \theta_0^2 \frac{L^2}{3} \quad (\text{B.3})$$

This MS will simulate a sagitta $s = y(0) - (y(L/2) + y(-L/2))/2$

$$\begin{aligned} s &= \sum_{i=-N/2}^0 \delta\theta_i(-i\Delta X) - \frac{1}{2} \sum_{i=-N/2}^{N/2} \delta\theta_i(L/2 - i\Delta X) \\ s &= -\frac{1}{2} \sum_{i=-N/2}^0 \delta\theta_i(L/2 + i\Delta X) - \frac{1}{2} \sum_{i=0}^{N/2} \delta\theta_i(L/2 - i\Delta X) \end{aligned}$$

The average of s is 0 and its variance is

$$\langle ss \rangle = \frac{1}{4} \left\langle \sum_{i=-N/2}^0 \delta\theta_i^2 (L/2 + i\Delta X)^2 + \sum_{i=0}^{N/2} \delta\theta_i^2 (L/2 - i\Delta X)^2 \right\rangle \quad (\text{B.4})$$

$$\langle ss \rangle = \frac{1}{4} \left(\sum_{i=-N/2}^0 \langle \delta\theta_i^2 \rangle (L/2 + i\Delta X)^2 + \sum_{i=0}^{N/2} \langle \delta\theta_i^2 \rangle (L/2 - i\Delta X)^2 \right)$$

Inspecting the previous formula we notice that the scattering angles are weighted with the square of the distance from the closer endpoint of the segment of length L .

Using formula B.1 we define

$$\delta\theta^2 = \left(\frac{13.6 \text{MeV}}{p} \right)^2 \frac{dx}{X_0} = \alpha \frac{dx}{X_0}$$

and equation B.4 can be rewritten in integral form as

$$\langle ss \rangle = \frac{\alpha}{4} \left(\int_{-L/2}^0 (L/2 + x)^2 \frac{dx}{X_0} + \int_0^{L/2} (L/2 - x)^2 \frac{dx}{X_0} \right)$$

Assuming uniform medium (uniform probability of scattering angle) we obtain

$$\langle ss \rangle = \frac{1}{16 \cdot 3} L^2 \frac{\alpha L}{X_0} = \frac{1}{16 \cdot 3} L^2 \theta_0^2 \quad (\text{B.5})$$

Using equation B.1, we see that the correction factor on the variance of the sagitta due to non uniform distribution is

$$C_{\langle ss \rangle} = \frac{12 \int_{-L/2}^0 (L/2 + x)^2 \frac{dx}{X_0} + \int_0^{L/2} (L/2 - x)^2 \frac{dx}{X_0}}{L^2 \int_{-L/2}^{L/2} \frac{dx}{X_0}}$$

$C_{\langle ss \rangle}$ is the ratio between the variance of the sagitta computed taking into account the material distribution along the track and the variance assuming uniform material distribution.

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