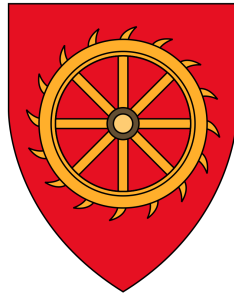




UNIVERSITY OF
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Studies of $ZZ\gamma$ production in pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector



Tommy Martinov

Cavendish Laboratory
University of Cambridge

This dissertation is submitted for the degree of
Master's of Philosophy



Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 15,000 words.

Tommy Martinov
April 2024

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Finally, everything I am and I have is owed to my parents. I am so grateful to them for making me crazy enough to pursue a career in fundamental physics and despite this mad mind, loving me unconditionally and offering me their eternal support in every goal I have ever fixed myself.

Abstract

This thesis presents a study of the feasibility of observing $ZZ\gamma$ triboson production in proton-proton collisions at a centre of mass energy $\sqrt{s} = 13$ TeV using data collected by the ATLAS detector during Run 2 of the LHC. This process has not yet been studied experimentally because of the small cross-section expected. The full Run 2 data set offers an integrated luminosity of 139 fb^{-1} . Using a cut-based method to select $ZZ\gamma$ candidate events, an expected signal significance of 1.8σ is achieved. Machine learning processes such as Boosted Decision Trees are used to raise the expected significance to 2.5σ which gives hope of finding evidence of $ZZ\gamma$ triboson production. Background contributions considered include those due to $t\bar{t}\gamma$ and $WZ\gamma$ production, as well as photons from pile-up interactions and from jets misidentified as photons (fake photons). In order to estimate the latter background, usual methods suffer from the low number of events and a novel technique is thus proposed. The contribution from final state radiation (FSR) is also included as part of the estimated background. The FSR process corresponds to the production of a pair of Z bosons where one of the four decay leptons radiates a photon. By treating the FSR process as signal rather than background, it is also possible to study inclusive $llll\gamma$ production. In this case, the predicted signal significance obtained using the cut-based analysis is 8.0σ . A longer-term goal of the $ZZ\gamma$ study is to search for the presence of anomalous gauge couplings, and this is also discussed here.

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Chapter 1

Introduction

Since the start of full-scale Large Hadron Collider (LHC) operation in 2010, the study of electroweak diboson production (ZZ , WZ , WW , $Z\gamma$, $W\gamma$, $\gamma\gamma$) in proton-proton collisions has been one of the cornerstones of the LHC physics program. Measurements of these processes provide precision tests of the electroweak sector of the Standard Model (SM), and provide significant constraints on the presence of anomalous triple and quartic gauge couplings beyond those expected in the SM. Diboson production also represents an important background to other key physics processes such as Higgs boson production and searches for new particles such as new high mass resonances and supersymmetric particles.

The electroweak structure of the SM can also be tested through measurements of electroweak triboson production (VVV , $VV\gamma$, $V\gamma\gamma$, $\gamma\gamma\gamma$ with $V = W, Z$). Since triboson production cross-sections are much smaller than those for diboson production, triboson production has been studied less extensively. In Run 1, triboson studies were limited to $Z\gamma\gamma$, $WZ\gamma$, $WW\gamma$ and $\gamma\gamma\gamma$, but nevertheless provided significant constraints on anomalous gauge couplings. With the increased energy and luminosity provided by Run 2 of the LHC, the full range of triboson processes is now becoming potentially accessible. Measurements of WWW and WWZ production using the full Run 2 dataset have already been published by the ATLAS and CMS experiments. This thesis will be concerned with $ZZ\gamma$ production in pp collisions, a process which has not been observed yet, and for which there are as yet no published upper limits. The $ZZ\gamma$ studies in this thesis build on the published ATLAS measurement of $Z\gamma$ production using the Run 2 data, and are being carried out in close collaboration with ongoing analyses of $Z\gamma\gamma$ and $WZ\gamma$ production.

The Z boson is a relatively heavy particle ($m_Z = 91.19$ GeV) and interacts only weakly [1]. Thus, the production of two Z bosons requires relatively high energy and the production cross-section is expected to be small compared to similar processes such as $Z\gamma$ or $WZ\gamma$ production (or $t\bar{t}\gamma$ since quarks interact through the strong force). A leading-order Feynman diagram for $ZZ\gamma$ production is shown in Fig. 1.1.

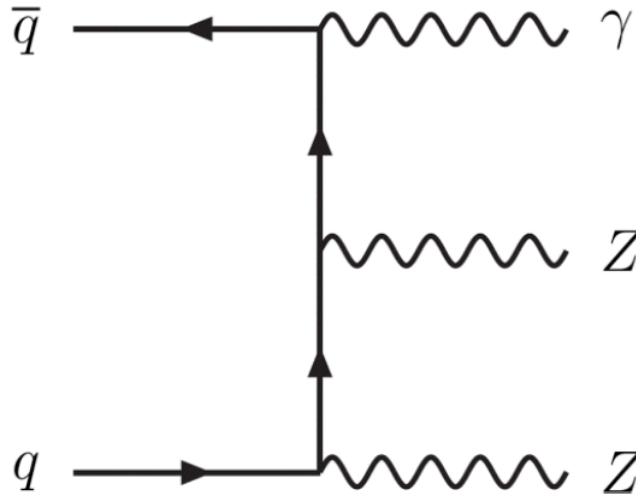


Fig. 1.1 : Feynman diagram of the $ZZ\gamma$ interaction.

The goal of the work described in this thesis is to study simulation samples of $ZZ\gamma$ interactions and potential background contributions in preparation for the analysis of ATLAS Run 2 data samples. From these simulation samples, numbers of signal and background events are predicted. Different selection cuts on leptons and photons are tested including a selection done with machine learning in order to minimise the expected background processes and maximise the signal yield. Estimation of some of these backgrounds requires specialised methods to be developed. These strategies are tested and evaluated before being used on real data. Ultimately, the objective will be to obtain the first evidence for $ZZ\gamma$ production. As the statistics are expected to be limited, a measurement of the $ZZ\gamma$ cross-section might not be possible and thus only an upper limit will be determined. As mentioned above, anomalous couplings will also be studied. Limits on dimension > 4 operators will be found by combining the results with those obtained by the $WZ\gamma$ analysis.

Only the fully-leptonic decay channels of the Z boson are considered. In ZZ events there are three such channels: four electrons (positrons) $4e$, four muons (anti-muons) 4μ and two electrons and two muons $2e2\mu$. The decays into τ leptons are considered as background because of the difficulty to reconstruct them precisely and evaluating their momentum and energy in the detector. Final state radiation (FSR) corresponds to events where the photon is radiated from one of the leptons coming from the decay of the Z bosons. These events are considered as background as they correspond to the production of a Z boson pair rather than a triboson interaction. Another important background is the fake photon background. A fake photon is a jet that is misidentified as a photon. The photon comes from the fragmentation of a quark or a gluon emitted during the interaction. Moreover, pile-up photons are another background contribution. In this case, the photon comes from another pp interaction happening in the same bunch crossing. Finally, different processes that can produce four leptons together with a photon

are considered, such as $t\bar{t}\gamma$ and $WZ\gamma$ production.

The remainder of this thesis is organised as follows: Chapter 2 outlines the theory of $ZZ\gamma$ production and anomalous gauge couplings. Chapter 3 describes the main features of the LHC and ATLAS detector. Chapter 4 summarises the production of data and Monte Carlo samples. Chapter 5 details the differences between initial and final state radiation $ZZ\gamma$ events and defines the different signal regions used for the $ZZ\gamma$ analysis. Chapter 6 lists all the processes considered as background for $ZZ\gamma$ production and explains how these backgrounds were estimated. Chapter 7 provides a summary of the machine learning methods applied on $ZZ\gamma$ signal and background samples. Finally, Chapter 8 summarises the main results obtained in the thesis, and considers the remaining issues which must be tackled in order to carry out a complete analysis of $ZZ\gamma$ production on the full ATLAS Run 2 dataset.

Chapter 2

Theory

2.1 Electroweak interaction

2.1.1 Standard Model

The main goal of this analysis is to study the electroweak sector of the Standard Model (SM). The electroweak interaction unifies two of the four fundamental forces, electromagnetic and weak, in a non-Abelian gauge theory with gauge symmetry $SU(2) \times U(1)$. The four vector bosons carrying this interaction are the neutral Z , the two charged $W^\pm$ and the photon γ [2]. The electroweak Lagrangian can be written as [3]:

$$\mathcal{L}_{EW} = \mathcal{L}_K + \mathcal{L}_N + \mathcal{L}_C + \mathcal{L}_H + \mathcal{L}_{HV} + \mathcal{L}_{WWV} + \mathcal{L}_{WWVV} + \mathcal{L}_Y \quad (2.1)$$

where each term has the following meaning:

- $\mathcal{L}_K$: kinematic component
- $\mathcal{L}_N$: neutral boson couplings to fermions
- $\mathcal{L}_C$: charged bosons couplings to fermions
- $\mathcal{L}_H$: Higgs boson self-interactions
- $\mathcal{L}_{HV}$: Higgs boson couplings to Z and W bosons
- $\mathcal{L}_{WWV}$: triple gauge couplings (TGCs)
- $\mathcal{L}_{WWVV}$: quartic gauge couplings (QGCs)
- $\mathcal{L}_Y$: Yukawa couplings

The sixth and seventh terms are a consequence of the non-Abelian nature of the electroweak interaction, and are particularly important for diboson and triboson production. TGC and QGC vertices involving W bosons together with Z bosons or photons are allowed in the SM but neutral

current TGC and QGC vertices involving Z bosons and photons alone are forbidden. Hence, both TGC and QGC interactions can contribute to $WZ\gamma$ production (cf. Fig. 2.1), but there are no equivalent TGC or QGC diagrams for $ZZ\gamma$ (cf. Fig. 2.2). The possible existence of new physics beyond the SM (BSM) could modify these charged-current gauge interactions and introduce non-zero neutral current couplings. The presence of such anomalous gauge couplings can be searched for by studying diboson and triboson production, and can be introduced theoretically in a non-renormalisable effective field theory (EFT) framework for example. With the $ZZ\gamma$ analysis, limits on EFT operators could be placed. This is detailed in Section 2.1.2.

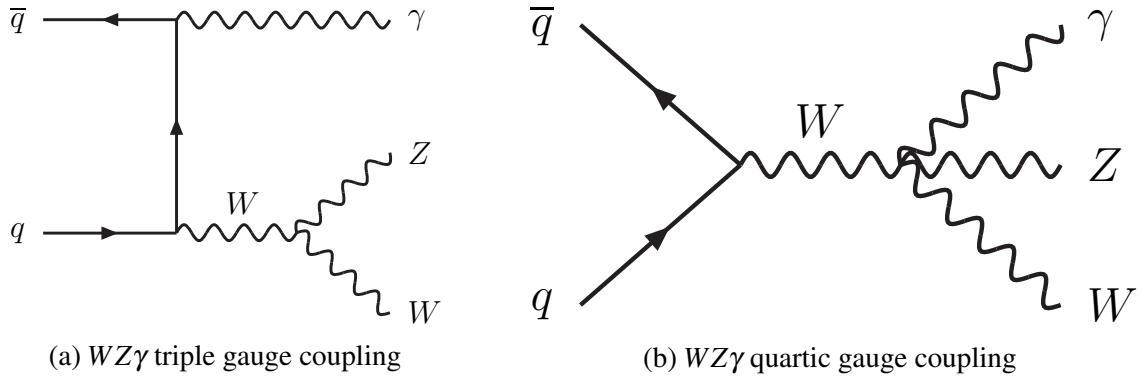


Fig. 2.1 : Triple (left) and quartic (right) gauge couplings for $WZ\gamma$ production.

2.1.2 Anomalous gauge couplings

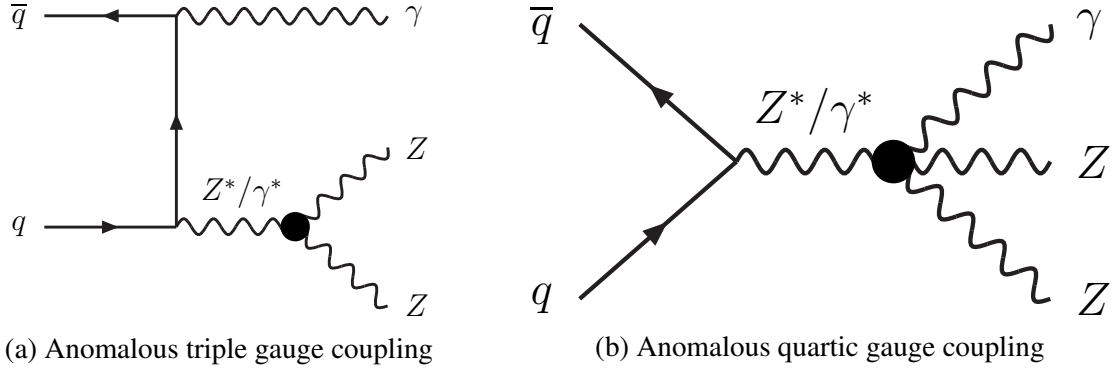
Gauge coupling deviations from the SM are described by additional higher-order operators introducing photon-to- Z couplings. Examples of interactions forbidden in the SM are shown in Fig. 2.2. In this context, the SM Lagrangian is extended to [4]:

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \frac{f_n}{\Lambda^{n-4}} \mathcal{O}_n \quad (2.2)$$

where $\mathcal{O}_n$ are BSM operators, n the dimension of these operators, f_n are the corresponding coefficients and Λ is the energy scale at which BSM physics effects can be observed.

For TGCs, in order to express potential BSM $Z\gamma V$ gauge couplings, with $V = Z, \gamma$ being off-shell and the other two bosons being on-shell, four CP-conserving complex parameters $h_{3,4}^V$ are used [5]. Potential ZZV couplings are described by the parameters $f_{4,5}^V$.

The lowest dimension in the Lagrangian giving rise to QGCs without TGC vertices is dimension $n = 8$. In the Eboli model of anomalous couplings [4], three types of anomalous QGC parameters are defined. The eight parameters $f_{T,i}$ are called transverse, the three parameters $f_{S,i}$ are called longitudinal and the seven parameters $f_{M,i}$ are called mixed transverse and longitudinal. The contribution from the corresponding operators is suppressed by a factor Λ^4 . Thus, in the Eboli

Fig. 2.2 : Examples of anomalous $ZZ\gamma$ gauge couplings.

model, the dimension-8 Lagrangian can be written as [4]:

$$\mathcal{L}_{n=8} = \sum_{i=0,1,2} \frac{f_{S,i}}{\Lambda^4} \mathcal{O}_{S,i} + \sum_{j=0,\dots,7} \frac{f_{T,j}}{\Lambda^4} \mathcal{O}_{T,j} + \sum_{k=0,\dots,6} \frac{f_{M,k}}{\Lambda^4} \mathcal{O}_{M,k}.$$

For example, the eighth transverse operator $\mathcal{O}_{T,9}$, which governs among others the $ZZZ\gamma$ couplings is written as [4]:

$$\mathcal{O}_{T,9} = B_{\alpha\mu} B^{\mu\beta} B_{\beta\nu} B^{\nu\alpha}$$

where B are four field strength tensors. $ZZ\gamma$ production is sensitive to $ZZ\gamma\gamma$ and $ZZZ\gamma$ couplings. In diboson and triboson studies in the electroweak sector, limits on anomalous gauge couplings have been measured. A summary of all these limits is given in the next section.

2.2 Summary of current results

2.2.1 Cross-section measurements

A wide variety of electroweak diboson and triboson production studies have been published by the CMS and ATLAS collaborations.

In Run 1, the three production processes $Z\gamma$, ZZ and $Z\gamma\gamma$ have been studied by both ATLAS and CMS. ATLAS combined the $Z\gamma$ and $Z\gamma\gamma$ analysis at $\sqrt{s} = 8$ TeV with an integrated luminosity of 20.3 fb^{-1} [6] where only the decays of Z bosons to electrons and muons were considered. In the $Z\gamma$ [7] and $Z\gamma\gamma$ [8] (combined with $W\gamma\gamma$) studies, CMS also considered only the electron and muon decay channels of the Z boson. Both analyses were performed with center of mass energy of $\sqrt{s} = 8$ TeV and 19.5 fb^{-1} of data. For the ZZ Run 1 study, the decays to both leptons and neutrinos were considered in ATLAS [9] and the decays to τ leptons were considered in CMS [10]. A measurement of the inclusive $pp \rightarrow ZZ$ cross-section was extrapolated from both ZZ studies. All the measured cross-sections are summarised in Tab. 2.1.

In Run 2, at $\sqrt{s} = 13$ TeV, ATLAS has studied $Z\gamma$ [11] and ZZ [12] production with Z decaying to pairs of electrons or muons. The former study was performed with the full Run 2 dataset of 139 fb^{-1} , while the latter used the data collected over the first two years of Run 2, 2015 and 2016, amounting to an integrated luminosity of 36.1 fb^{-1} . CMS have measured the $pp \rightarrow ZZ(\rightarrow 2\ell 2\ell')$ production cross-section in Run 2 ($\ell, \ell' = e, \mu$) [13]. Their data sample corresponds to an integrated luminosity of 137 fb^{-1} . Recently, CMS published its Run 2 $Z\gamma\gamma$ and $W\gamma\gamma$ combined study [14], with Z decaying to electrons and muons. The measured cross-sections are summarised in Tab. 2.1. No equivalent triboson analyses in the electroweak sector have been published yet by ATLAS. $ZZ\gamma$ production in particular is a rare SM process and no experimental measurements have been published to date due to the small number of events expected in data samples. Triboson studies are currently ongoing in ATLAS as outlined in Section 2.3.

2.2.2 Limits on anomalous couplings

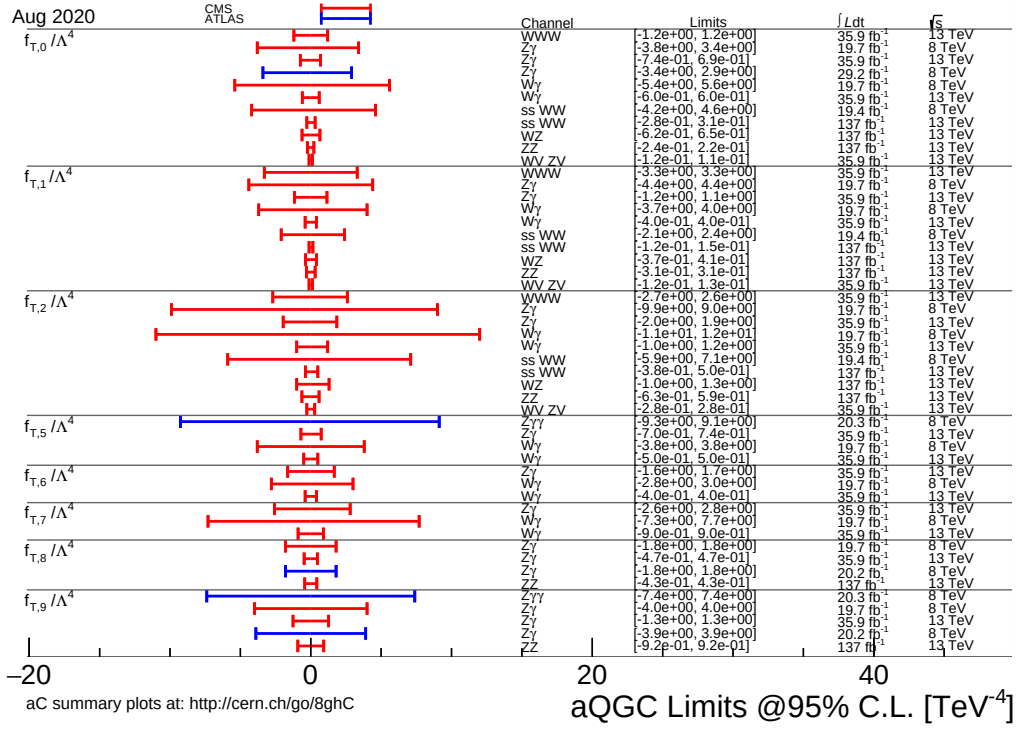
All electroweak diboson and triboson cross-section measurements to date are in agreement with the SM predictions, and can be used to set limits on neutral anomalous gauge couplings. Each diboson or triboson interaction has different levels of sensitivity to the various couplings. For example, $Z\gamma\gamma$ production is sensitive to $ZZ\gamma\gamma$ and $Z\gamma\gamma\gamma$ anomalous quartic gauge couplings. The anomalous TGC couplings $f_{4,5}^V$ and $h_{3,4}^V$ can in principle be studied via the triboson production channels $Z\gamma\gamma$ and $ZZ\gamma$. However these triboson processes are expected to be less sensitive to these couplings than diboson production, and are instead used primarily to constrain anomalous QGC couplings. The QGC coefficients and their current limits are shown in Fig. 2.3a ($f_{T,i}$) and Fig. 2.3b ($f_{M,i}$). In the case of $ZZ\gamma$ production, the quartic gauge couplings $ZZ\gamma\gamma$ and $ZZZ\gamma$ could potentially be studied.

2.3 Run 2 $Z\gamma\gamma$, $WZ\gamma$ and $ZZ\gamma$ analyses in ATLAS

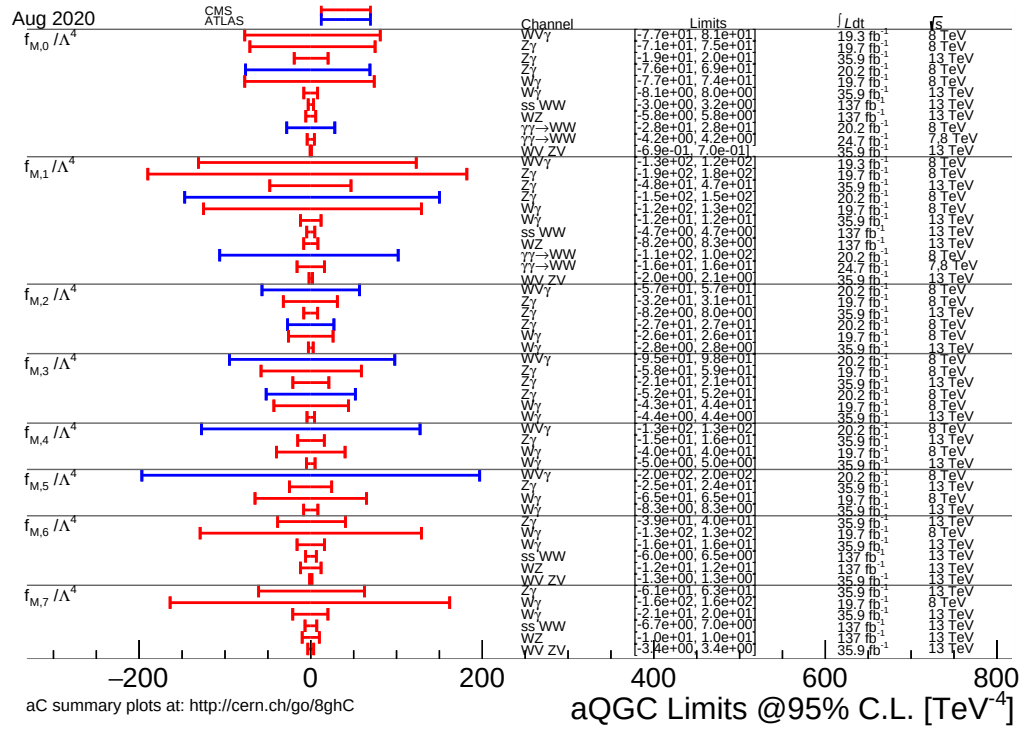
With the ATLAS Run 2 dataset, $Z\gamma\gamma$, $WZ\gamma$ and $ZZ\gamma$ production are currently under investigation. All three of these analyses build on the methods used in the published ATLAS analysis of $Z\gamma$ production at $\sqrt{s} = 13$ TeV [11]. For instance, the estimation of fake photon background in $ZZ\gamma$ and $WZ\gamma$ uses the $Z\gamma$ result for extrapolation. Compared to the other channels, the $Z\gamma\gamma$ channel has the highest predicted cross-section and the cleanest signature, and the analysis is correspondingly the most advanced [16]. 25% of the data has been unblinded. The $Z\gamma\gamma$ preliminary study predicted cross-sections of the order of 2 fb for the $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ channels. Leading-order and next-to-leading order Monte Carlo samples are used for these predictions. The main difficulty with $Z\gamma\gamma$ production comes from the photon backgrounds (fake and pile-up, cf. Chapter 6). $Z\gamma\gamma$ production is sensitive to $ZZ\gamma\gamma$ and $Z\gamma\gamma\gamma$ anomalous vertices. Limits on corresponding dimension-8 operators are currently being computed.

	Study	ATLAS	CMS
Run 1	$Z\gamma$	$1507 \pm 10^{+78}_{-73} {}^{+29}_{-28}$ fb (1483 $^{+19}_{-37}$ fb)	$2063 \pm 19 \pm 98 \pm 54$ fb (2100 $\pm$ 120 fb)
	$Z\gamma\gamma$	$5.07^{+0.73}_{-0.68} {}^{+0.41}_{-0.38} \pm 10$ fb (3.70 $^{+0.21}_{-0.11}$ fb)	$12.7 \pm 1.4 \pm 1.8 \pm 0.3$ fb (13.0 $\pm$ 1.5 fb)
	ZZ	$7.3 \pm 0.4 \pm 0.3 {}^{+0.2}_{-0.1}$ pb (6.6 $^{+0.7}_{-0.6}$ pb)	$7.7 \pm 0.5 {}^{+0.5}_{-0.4} \pm 0.2 \pm 0.4$ (theo) pb (7.7 $\pm$ 0.6 pb)
Run 2	$Z\gamma$	$533.7 \pm 2.1 \pm 12.4 \pm 9.1$ fb (518.9 $^{+20.4}_{-19.2}$ fb)	$\diagdown$
	$Z\gamma\gamma$	$\diagdown$	$5.41^{+0.58}_{-0.55} {}^{+0.64}_{-0.70} \pm 0.06$ (theo) fb (5.96 $\pm$ 0.06 fb)
	ZZ	$46.2 \pm 1.5 {}^{+1.2}_{-1.1} {}^{+1.6}_{-1.4}$ fb (42.9 $^{+1.9}_{-1.5}$ fb)	$40.5 \pm 0.7 \pm 1.1 \pm 0.7$ fb (39.3 $^{+1.0}_{-0.9}$ fb)

Table 2.1 : Summary of the electroweak diboson and triboson production cross-section measurements in ATLAS and CMS in the the first two runs of the LHC. The first, second and third errors shown are respectively the statistical, systematic and luminosity uncertainties (on the $Z\gamma\gamma$ Run 2 analysis made by CMS, the luminosity uncertainty was included in the systematic uncertainty). When an additional theoretical error is considered, it is indicated in brackets. No Run 2 cross-section measurements have been published for $Z\gamma$ in CMS and $Z\gamma\gamma$ in ATLAS yet. The Run 1 ZZ cross-sections quoted here are inclusive measurements. The other cross-section measurements were performed in different fiducial regions. The numbers in brackets below each measurement are the corresponding Standard Model predictions.



(a) Limits on transverse coefficients



(b) Limits on mixed coefficients

Fig. 2.3 : Summary of the limits measured on dimension-8 operator coefficients in ATLAS (blue) and CMS (red). The top and bottom figures respectively show limits on transverse and mixed coefficients measured from various diboson and triboson channels. The two figures come from Ref. [15].

Finally, the study of $WZ\gamma$ and $ZZ\gamma$ production is currently ongoing. As these are rare processes which have not yet been observed, the data must remain blinded until all event selection cuts have been fixed and the methods to be used to determine the various background contributions have been established. The $WZ\gamma$ production is predicted to have a cross-section about an order of magnitude greater than $ZZ\gamma$. This interaction involves TGCs and QGCs, as shown in Fig. 2.1. The decay of the W boson to a lepton (e, μ) and a neutrino is considered. The $WZ\gamma$ and $ZZ\gamma$ analyses use similar methods to estimate the photon backgrounds. Combination of results from the $WZ\gamma$ and $ZZ\gamma$ analyses could set further limits on different anomalous gauge coupling coefficients.

Chapter 3

The LHC and the ATLAS experiment

3.1 Large Hadron Collider

The Large Hadron Collider (LHC) is a circular collider situated at the border between France and Switzerland at CERN near Geneva. It has a circumference of 27 km [17]. An aerial picture of the LHC is shown in Fig. 3.1. Protons, and less often, heavy ions are accelerated at velocities close to the speed of light. Counter-rotating beams are brought into collision at key points around the ring where experiments are located. The four main experiments are ALICE (quark-gluon plasma studies), LHCb (b -physics), CMS (general-purpose) and ATLAS. Two major data-taking runs have taken place at the LHC. During the first run (Run 1), from 2010-2012, the Higgs boson was discovered. Run 1 started with a center of mass energy of $\sqrt{s} = 7$ TeV which was then increased to $\sqrt{s} = 8$ TeV. During the second run (Run 2), from 2015-2018, the center of mass energy was raised to $\sqrt{s} = 13$ TeV. The prospects for observing $ZZ\gamma$ triboson production in the Run 2 dataset is the subject of this thesis.

Protons circulate in the LHC ring in bunches separated by either 25ns or 50ns [17]. Superconducting dipole magnets located around the ring provide a highly uniform magnetic field of 8.3T which guides the protons around an approximately circular trajectory. Triplets of quadrupole magnets located either side of the experimental interaction points focus the beams to maximise the collision rate. Due to the high beam intensity, most bunch crossings result in multiple proton-proton collisions. This is referred to as pile-up. The average pile-up $\langle\mu\rangle$ corresponds to the average number of inelastic proton-proton interactions occurring at each bunch crossing. During Run 2, for ATLAS, the average value of the pile-up parameter was $\langle\mu\rangle = 33.7$.



Fig. 3.1 : Aerial picture of the LHC (under ground) and the four main experiments [18].

3.2 A Toroidal LHC Apparatus

ATLAS properties and characteristics

The ATLAS detector is, with CMS, one of the two general purpose experiments around the LHC. ATLAS was involved in the discovery of the Higgs boson for instance. Since the first LHC run, ATLAS has also measured cross-sections for many different processes, in the electroweak sector for example. The main features of the ATLAS detector are outlined in this section. Not all events occurring at the ATLAS detection point are recorded for further analysis. A complex system of hardware and software triggers reduces the frequency of stored collisions from 40MHz (25ns) to 1kHz on average [19]. More details about the ATLAS trigger system are given below. By increasing the energy of the beam and reducing its size in the transverse plane at the interaction point (10 μm in the x and y directions), a full integrated luminosity $\int L dt = 139 \text{ fb}^{-1}$ was achieved in ATLAS during Run 2.

A standard right-handed coordinate system is used to describe the geometry of the ATLAS detector and for the reconstruction of the objects used in physics analysis. The origin of this coordinate system is taken to be at the geometric centre of the ATLAS detector. The beam line is taken to be the z -axis, while the x -axis points toward the centre of the LHC ring and the y -axis points upwards, perpendicularly to the plane of the LHC ring. A cylindrical coordinate system is mainly used. The angles ϕ and θ are respectively the azimuth angle and the polar angle with respect to the beam axis. The latter defines the pseudorapidity as $\eta \equiv -\ln[\tan(\frac{\theta}{2})]$. In the $(\phi; \eta)$ space, the separation between two objects is calculated as $\Delta R = \sqrt{(\Delta\phi)^2 + (\Delta\eta)^2}$. Rather than

the momentum, the transverse momentum p_T is often a more useful parameter. It corresponds to the component of the momentum in the plane transverse to the beam axis and can be computed as $p_T = |\vec{p}| / \cosh \eta$.

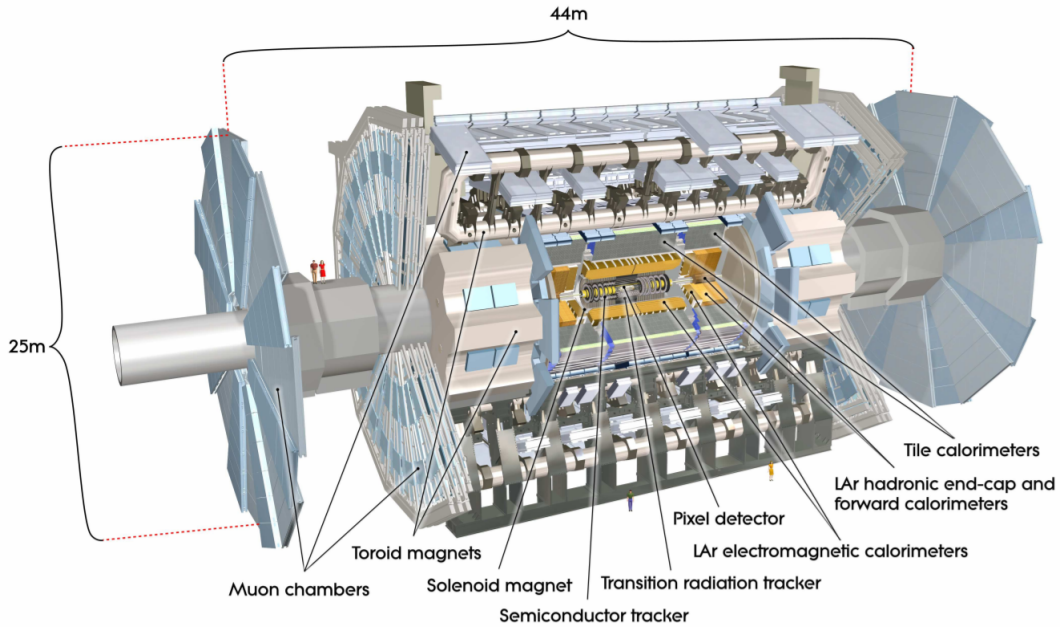


Fig. 3.2 : The ATLAS detector [20].

ATLAS components

The ATLAS detector is described in detail in Ref. [19]. The main layers of ATLAS from innermost to outermost are, as illustrated in Fig. 3.2:

- Inner detector (ID) [21]: the ID has three main components, the silicon pixel and strip detectors and the transition radiation tracker. The ID detects charged particles with $p_T > 0.5$ GeV. A uniform 2T magnetic field is applied in the direction of the z -axis. In the magnetic field, charged particles follow a curved trajectory, which can be determined from space-points reconstructed using the ID hit information. By determining the radius of curvature and polar angle of this trajectory, the three-momentum of charged particles can be reconstructed. Primary vertices can be reconstructed by extrapolating reconstructed ID tracks inwards towards the beam line. The transverse (d_0) and longitudinal (z_0) impact parameters of charged tracks with respect to the primary vertices can also be reconstructed. The inner detector is illustrated in Fig. 3.3.
- Electromagnetic calorimeter (ECAL) [23]: when travelling through material, photons interact with nuclei through e^+e^- pair production and electrons interact with nuclei through the bremsstrahlung process [2]. When an energetic electron or photon enters the ECAL, an electromagnetic shower is therefore produced. An electron is distinguished

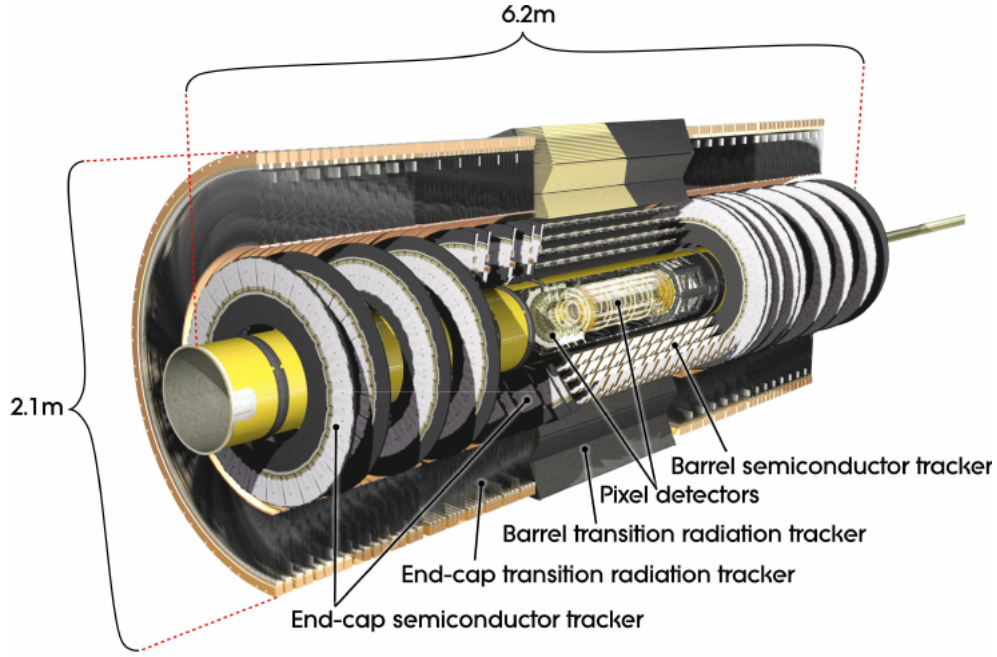


Fig. 3.3 : The ATLAS inner detector [22].

from a photon by the presence of an associated charged particle trajectory reconstructed in the ID. The energy resolution provided by the ECAL is typically of the order of a few percent. In ATLAS, the ECAL is a liquid argon calorimeter interspersed with lead absorbers. In addition to the ECAL, the electromagnetic end-cap calorimeter and the forward calorimeter extend the coverage of electrons and photons to the $|\eta| > 2.5$ region. In the region $|\eta| < 2.5$, the ECAL is segmented longitudinally (in shower depth) into three layers. In the barrel region, the first layer of the ECAL contains cells of dimension $\Delta\eta \times \Delta\phi = 0.0031 \times 0.025$. With these fine granularity cells, high electron reconstruction efficiency is possible and photons produced with a small separation (such as photons coming from π^0) can be distinguished. The second layer collects most of the shower energy and has a granularity of $\Delta\eta \times \Delta\phi = 0.025 \times 0.025$. The ECAL is illustrated in Fig. 3.4.

- Hadron calorimeter (HCAL) [23]: the HCAL consists of alternating layers of steel absorber and plastic scintillator tiles. When energetic hadrons enter the HCAL, they produce hadronic showers which occupy a larger volume than EM showers. Because of variations in the electromagnetic fraction of hadron showers and energy lost in collisions with nuclei, the precision of the energy measurement is not as good as in the ECAL - approximately one order of magnitude worse [2]. The HCAL is illustrated in Fig. 3.4.
- Muon spectrometer (MS) [25]: muons mainly interact through ionisation in a detector. They thus lose energy more slowly than electrons and hadrons and they travel greater distances through material [2]. They leave a trail of ionization and they are detected at large radius in the MS. The MS is immersed in a 1T toroidal magnetic field, allowing a

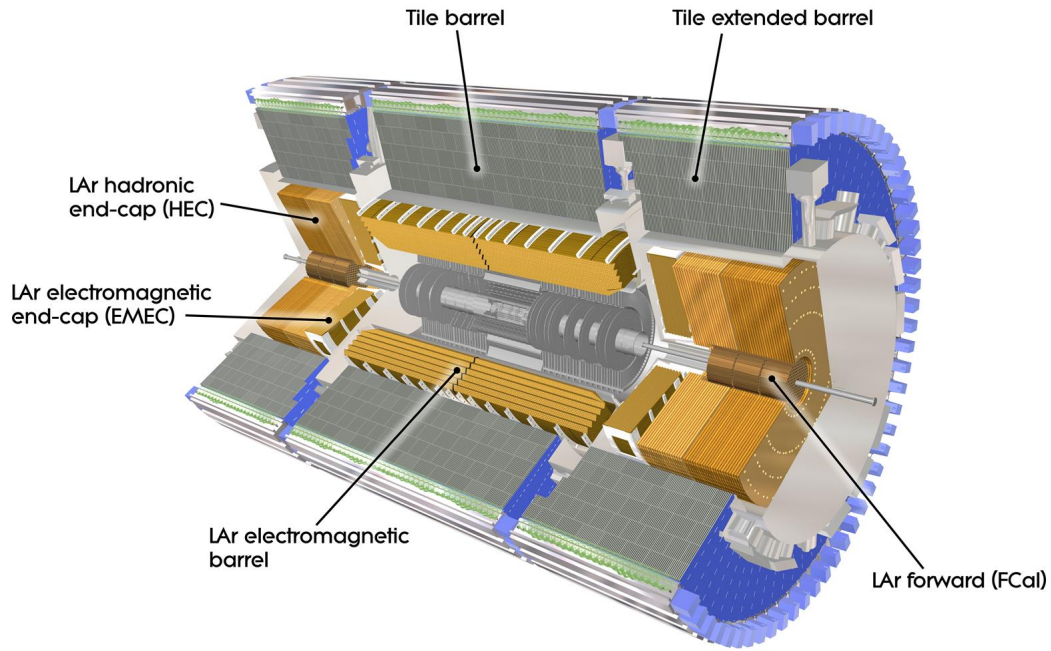


Fig. 3.4 : The ATLAS electromagnetic and hadronic calorimeters [24].

measurement of the muon momentum which is independent to that made in the ID. In ATLAS, the muon spectrometer is made up of 4,000 individual muon chambers. Four different components constitute the MS: thin gap chambers and resistive plate chambers used for triggering and coordinate measurement, monitored drift tubes to measure track curvature and thus muon momentum and finally cathode strip chambers to measure precision coordinates. The MS is illustrated in Fig. 3.5.

Neutrinos interact very weakly and they require specific technology to be detected. Hence, in ATLAS they are characterised as missing energy in analysis where neutrino production is expected. Jets are recurrent in ATLAS. They are formed by the hadronization of quarks and gluons produced during collisions. Even though they are a collection of different particles, they can be treated as individual objects with their own mass, energy and position.

Trigger system

The ATLAS detector contains approximately 100 million readout channels carrying signals produced by particles from proton-proton collisions occurring at a rate of 40 MHz. This amount of data is too large for it all to be stored. Moreover, most events are of no interest for physics analysis. For example, electroweak and top physics events appear in one out of 1-100 million collisions. Hence, a trigger system is implemented in ATLAS to keep only a fraction of all the events [27]. First, the hardware based Level 1 trigger system makes a coarse selection based on information from the calorimeter and the muon detectors. The measurement rate is reduced to a

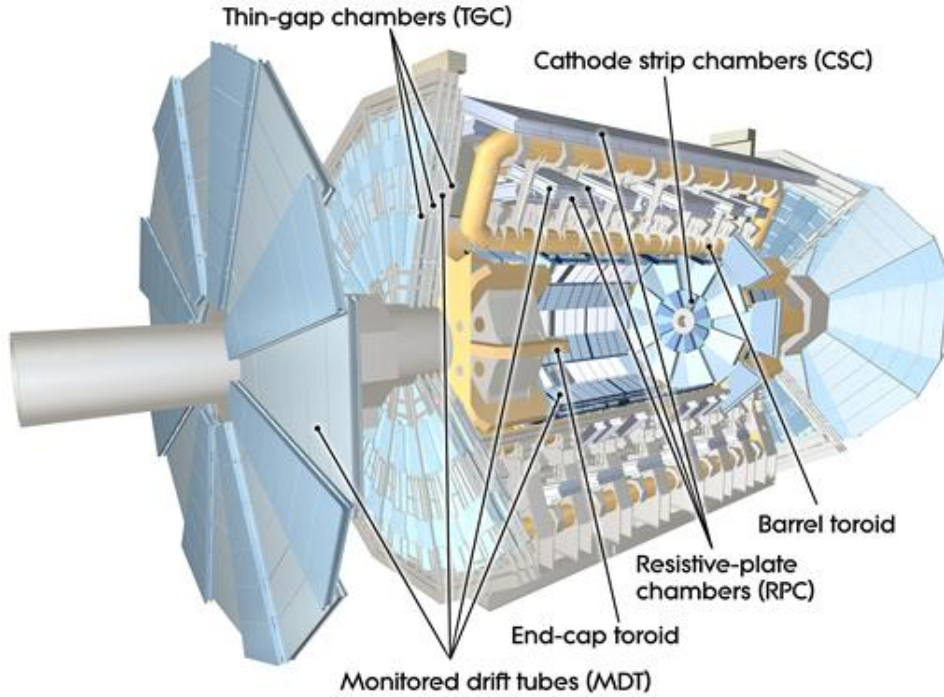


Fig. 3.5 : The ATLAS muon spectrometer [26].

100 kHz. Then, the software based High Level Trigger (HLT) system reduces the measurement rate to 1 kHz on average. Based on offline algorithms, the HLT system processes information about clusters, tracks and their matching in order to reject events. A wide variety of trigger channels operate in parallel, including triggers requiring the presence in an event of at least one high transverse momentum electron or muon. During the first year of Run 2 (2015), the lowest threshold unprescaled single-electron (single-muon) trigger had a nominal threshold of $p_T \geq 24$ GeV ($p_T \geq 20$ GeV). Over the following three years, these thresholds were raised to 26 GeV for both electrons and muons, and more restrictive isolation requirements were imposed. Single-lepton triggers with higher p_T thresholds but no isolation requirements are also available, as are dilepton and trilepton triggers requiring two or three high p_T electrons or muons. For the $ZZ\gamma$ analysis considered in this thesis, an event is accepted if any single-lepton trigger has fired. A study carried out for the $WZ\gamma$ channel showed that no useful gain in signal efficiency was observed by adding multi-lepton triggers. It is likely that multi-lepton triggers will not benefit the $ZZ\gamma$ analysis either, though this remains to be studied.

Chapter 4

Data samples

This chapter describes how data is processed from the detection of particles to the production of the reconstructed objects used for physics analysis. As the ATLAS data for $ZZ\gamma$ are still blinded, mainly Monte Carlo (MC) samples are used during the preliminary analysis. Their production is also described. All the MC samples used are listed below.

4.1 Data processing

The raw data are reconstructed and placed into Analysis Object Data (AOD or xAOD) data sets. The raw data is also used for calibration of the detector. Even though AODs contain reconstructed objects (electrons, muons, jets...) with their different properties (p_T , η ...) they are not used directly for analysis. Objects can be updated after reconstruction (b -tagging for example) and AODs are extremely large data sets containing a lot of unneeded information for a given analysis. To reduce the dataset size, Derived AODs (DAODs) are created. DAODs are specific for each analysis. In the $ZZ\gamma$ analysis the main derivation used is DAOD_HIGG1D2 corresponding to the second derivation produced by/for the Higgs group number 1 (which studies the decay $H \rightarrow \gamma\gamma$ of the Higgs boson to two photons). DAOD_TRUTH1 samples are also used. They contain detailed information about simulated particles, known as truth information. For MC samples, a subset of the truth information is also saved on the DAODs and on files derived from them.

DAODs are still large data sets (can be as high as a few Terabytes). To be able to carry out a particular physics analysis, DAODs are further derived into samples called Mini xAODs (MxAODs). For the $ZZ\gamma$ analysis, a framework developed by the $H\gamma\gamma^*$ analysis group is used to produce $ZZ\gamma$ -specific MxAOD samples. During this process, selection cuts on different particle properties (such as lepton p_T) are applied as explained in detail in Chapter 5.

Electrons and photons are reconstructed from ID tracks and ECAL clusters [28]. Electron clusters are matched to tracks measured in the inner detector. Photon clusters are matched to conversion vertices which are reconstructed from the available information on the e^+e^- pair

produced from each photon if the photon converts in the ID. Most photons however are unconverted and interact only in the ECAL. After calibrations and corrections are applied, the analysis objects containing detailed information about the reconstructed electrons and photons are created. Similarly, muons are reconstructed using information from the muons spectrometer and the inner detector [29]. Tracks from the MS and the ID are matched and combined. Information from calorimeter hits can also be used.

Important characteristics of electrons, muons and photons in ATLAS are their identification and isolation. The identification of a particle has several different levels quantifying the confidence with which an object can be identified as a particular type of SM particle. The identification can be 'Loose', 'Medium' or 'Tight'. The identification evaluation is based on cluster and track properties. These are compared to shower shapes expected for a given particle. For electrons, a likelihood discriminant is constructed from the variables characterising each object [28]. A selection is applied on the discriminant to determine the electron identification level. Additional requirements may be applied (for an electron to be 'Tight' for instance, it must satisfy $E/p < 10$ and $p_T > 2$ GeV). The photon identification is constructed from a cut-based selection on the shower shape variables. The 'Tight' criterion is optimised using machine learning algorithms [28]. Depending on the available information for a muon, five main reconstruction strategies are used [29]. These assign a type to each muon. The muon identification is constructed from the muon type, the number of muon spectrometer hits and other requirements on the properties of the muon.

Several different isolation variables quantifying the level of activity close to a given particle are used within ATLAS [30, 31]. Well isolated particles come from prompt production and not from jets. Thus, leptons and photons coming from the weak decays of hadrons produced during hadronization can be discarded by requiring a high degree of isolation. For the $ZZ\gamma$ signal, only prompt leptons are required. In order to evaluate the isolation, the environment in a cone of size $\Delta R = 0.2, 0.3$ or 0.4 around a particle in the ID and/or the calorimeters is scanned. The energy and momentum of clusters and tracks inside the cone are summed, excluding the cluster or track of interest. Different variables are thus calculated (depending on the measured parameter and the size of the cone). A working point is finally built by applying requirements on these variables. The working points used in the $ZZ\gamma$ analysis are given in section 5.1.

Primary vertex candidates are reconstructed from a minimum of two tracks with $p_T > 0.4$ GeV [11]. After reconstruction of all the candidates, the primary vertex with the highest sum of p_T^2 of the associated tracks is selected. Electron and muon candidates are required to originate from the primary vertex by demanding that $|d_0|/\sigma_{d_0} < 3$ for muons and $|d_0|/\sigma_{d_0} < 5$ for electrons, d_0 being the track transverse impact parameter measured relative to the beam trajectory and σ_{d_0} being the associated uncertainty. Furthermore, it is required that $\Delta z_0 = z_0 - z_{PV}$, z_0 being the z coordinate of the track point defining d_0 and z_{PV} being the z coordinate of the primary

vertex, satisfies $|\Delta z_0 \cdot \sin \theta| < 0.5$ mm for electrons and muons.

Jets, as a collection of objects rather than a unique particle, are reconstructed using specific clustering algorithms. Various such algorithms exist and the one used in ATLAS is called "anti- k_t " (k_t being the transverse momentum of an object) [32]. In this algorithm, two distances are defined: the distance between two objects (particles, jets) $d_{i,j}$ and the distance between an object and the beam $d_{i,B}$. The first distance is calculated using the inverse of the object's transverse momentum squared and the distance between the two objects in the $(y; \phi)$ plane (y being the rapidity). This distance is inversely proportional to the square of the radius parameter R . The second distance is equal to $1/k_t^2$. If $d_{i,j}$ is smaller than $d_{i,B}$, the two objects are combined and if $d_{i,B}$ is smaller, the object is called a jet and removed from the list of objects. This process is repeated until no objects are left. Once jets are defined, their properties can be extracted (η , p_T , ϕ , mass...). For the MC samples used in this thesis, the parameter R is set to 0.4. For Monte Carlo samples, the inputs for the anti- k_t algorithm are particle flow objects *i.e.* tracks and modified clusters [33, 34]. The jets are b -tagged. For the data samples used, the inputs are clusters of calorimeter cells. Jets with $p_T > 10$ GeV and $|\eta| < 4.4$ are selected. It is also required that the ΔR separation between jets and photons is greater than 0.4 and that the separation between jets and leptons is greater than 0.2. More details about ΔR requirements are given in Section 5.1.

As the $ZZ\gamma$ data is blinded, mainly Monte Carlo simulation samples are used throughout the thesis. These samples are described in the next section. Samples of $Z\gamma$ events selected from real data were used for estimating the fake photon background as detailed in Section 6.3. Four samples are used in total, one for each year of Run 2 data-taking.

4.2 Monte Carlo samples

Before unblinding the data, only Monte Carlo (MC) simulation samples are used. To produce each MC sample, a set of events corresponding to a given hard-scattering process of interest is generated first. Several different event generators exist for simulating the physics processes including SHERPA, POWHEG and MADGRAPH. For simulations of pp collisions in the LHC, the event generators model the hard scatter interaction between partons (quarks, gluons) inside protons. Soft gluons and quarks coming from partons not involved in the main interaction are produced. The parton-parton interaction modeling depends on the chosen Parton Distribution Function (PDF) set. PDFs are defined as the probability density functions for finding partons in a proton, each parton carrying a fraction x of the proton's momentum [35].

The interaction of the particles produced by the event generator with the detector are simulated with GEANT4 [36]. The next step, called digitisation, consists in translating the interactions with the detector into realistic signals. Finally, particles are reconstructed from the signals,

exactly like real data. Below, are detailed all the signal and background MC samples used during the $ZZ\gamma$ study.

In ATLAS, to model the additional pile-up interactions occurring in each bunch crossing, fully simulated inelastic proton-proton collisions (33.7 on average) are added the simulated hard-scatter event. As discussed in Section 6.2, the simulation of high p_T isolated photons from pile-up interactions is not sufficiently reliable to be used directly, and data-driven methods are preferred to estimate pile-up backgrounds. The distribution of the mean number of inelastic pp interactions occurring in each bunch crossing, $\langle\mu\rangle$, for each year of the Run 2 data-taking is shown in Fig. 4.1a. The number of reconstructed primary vertices in each event is found to correlate well with $\langle\mu\rangle$, as shown in Fig. 4.1b for a MC sample of simulated $ZZ\gamma$ signal events.

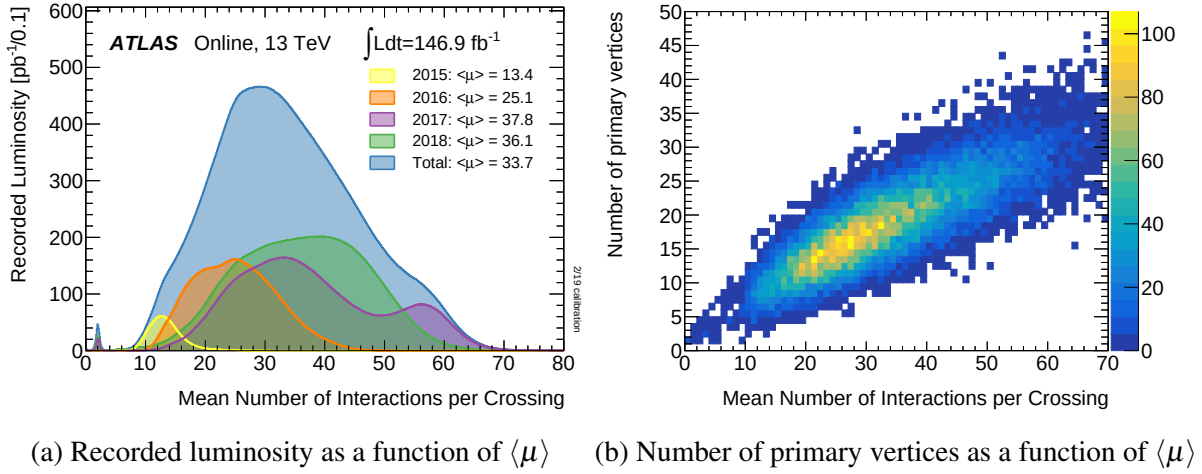


Fig. 4.1 : On the left-hand side is plotted the mean number of interactions per bunch crossing against the ATLAS recorded luminosity by year during Run 2 [37]. On the right-hand side is plotted the mean number of interactions per bunch crossing against the number of reconstructed primary vertices in a $ZZ\gamma$ SHERPA Monte Carlo simulation.

The signal sample used throughout this analysis is a $ZZ\gamma$ on-shell sample without final state radiation contribution (cf. section 5.2). The sample is generated using event generator SHERPA 2.2.5 [38] with PDF set NNPDF3.0 [39] NNLO. The Z bosons decay to leptons (e , μ , τ). Even though decay to pairs of τ leptons are included, these events are usually considered as background in ATLAS.

To model events containing final state radiation, two $llll$ inclusive samples are used. They include final state radiation and jet production (needed for estimating the fake photon background as explained in section 6.3). They also include fake and pile-up photons (cf. chapter 6). In both of these samples, inclusive $llll$ events are generated for which all same flavour opposite sign (SFOS) lepton pairs have a mass greater than 4 GeV. The first sample is generated with SHERPA 2.2.2 using the NNPDF3.0 NNLO PDF set and the second one is generated with POWHEG-BOX [40] interfaced to PYTHIA8 [41] for hadron showering with CT10nloME [42]

PDF set. For soft particles processes it uses the AZNLO [43] parameter tune. The $t\bar{t}\gamma$ background is generated with MADGRAPH5_aMC@NLO [44] at LO with NNPDF 2.3 LO PDF set. The $W(\rightarrow \ell\nu)Z(\rightarrow \ell\ell)\gamma$ background is generated with SHERPA 2.2.5 and uses NNPDF3.0 NNLO PDF set. Finally, for the fake photon estimation, Z+jets samples are needed. Separate samples are generated for the $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ decay channels. The two samples are generated with POWHEG-BOX interfaced with PYTHIA 8.210 with CTEQ6L1 PDF set and AZNLO tune. The list of samples described above is summarised in Tab. 4.1.

Process	Generator	PDF set
Signal $Z(\rightarrow \ell\ell)Z(\rightarrow \ell\ell)\gamma$	SHERPA	NNPDF3.0 NNLO
$llll$	SHERPA	NNPDF3.0 NNLO
$llll$	POWHEG	CT10nloME
$t\bar{t}\gamma$	MADGRAPH	NNPDF 2.3 LO
$WZ\gamma$	SHERPA	NNPDF3.0 NNLO
$Z(\rightarrow ee)+\text{jets}$	POWHEG	CTEQ6L1
$Z(\rightarrow \mu\mu)+\text{jets}$	POWHEG	CTEQ6L1

Table 4.1 : Summary of all MC samples used for this analysis

With the currently available MC samples described above, the inclusive $llll\gamma$ final state has to be modelled by combining two separate samples: an on-shell $ZZ\gamma$ sample for ISR events and an inclusive $llll$ sample for FSR events (the ISR and FSR processes are discussed in more detail in the next chapter). The fraction of $llll\gamma_{\text{FSR}}$ events in the inclusive $llll$ sample is small, and the number of simulated FSR events available is currently somewhat limited. An inclusive $llll\gamma$ sample with SFOS lepton pair mass greater than 20 GeV is under preparation. This will substantially improve the available statistics for FSR events. It will also take into account interference between the ISR and FSR processes, though this is likely a small effect.

Chapter 5

Signal

In this chapter, the cuts used to select candidate $ZZ\gamma$ signal events are defined. The aim is to select events containing four isolated leptons and one isolated photon consistent with $ZZ\gamma$ production, while at the same time rejecting events due to other processes or containing misidentified leptons or photons. The separation between initial state radiation (ISR) and final state radiation (FSR) is studied. Two signal regions are defined, an inclusive ISR and FSR region and an ISR-enhanced region. A looser signal region which will be used later for machine learning studies (cf. chapter 7) is also defined.

5.1 Signal selection

The signal region is characterised by cuts placed on the properties of reconstructed electrons, muons and photons. Cuts which are applied to ensure that the selected particles are in regions of high detector acceptance are described first. This is followed by a description of the selection cuts applied which are specific to the $ZZ\gamma$ final state.

Cuts on pseudorapidity $|\eta|$ are applied during MxAOD production. In order to distinguish photons from jets, the fine segmentation of the electromagnetic calorimeter's first layer is exploited [23]. Photons in the region outside the acceptance of the first layer *i.e.* where the photons pseudorapidity is $|\eta^\gamma| > 2.37$ are excluded. The kinematic acceptance for electrons corresponds to the region where $|\eta^e| < 2.47$. Furthermore, the ECAL has a crack region between the barrel and the end cap where photons and electrons are relatively poorly measured. Thus, the region $1.37 < |\eta| < 1.52$ is excluded for both types of particle. The muon detector acceptance covers the region $|\eta^\mu| < 2.7$ [25]. However, the inner detector does not detect charged particles with $|\eta| > 2.5$ and therefore muons with $|\eta^\mu| > 2.5$ are discarded. Muons with $|\eta^\mu| < 0.1$ have reduced acceptance because the muon spectrometer has a small gap in this region to provide space for services to other parts of the detector and support structures. The acceptances of all three types of particles described here can be seen on the pseudorapidity distributions of reconstructed electrons, photons and muons in Fig. 5.1. The upper limits on $|\eta|$ for all three types

of particles, the lower limit for muons and the excluded calorimeter crack region for electrons and photons are clearly visible. The edges of each region are smeared because the cuts are placed on the pseudorapidity of the electromagnetic cluster reconstructed in the ECAL, not on the overall value of η computed using information from all detector components.

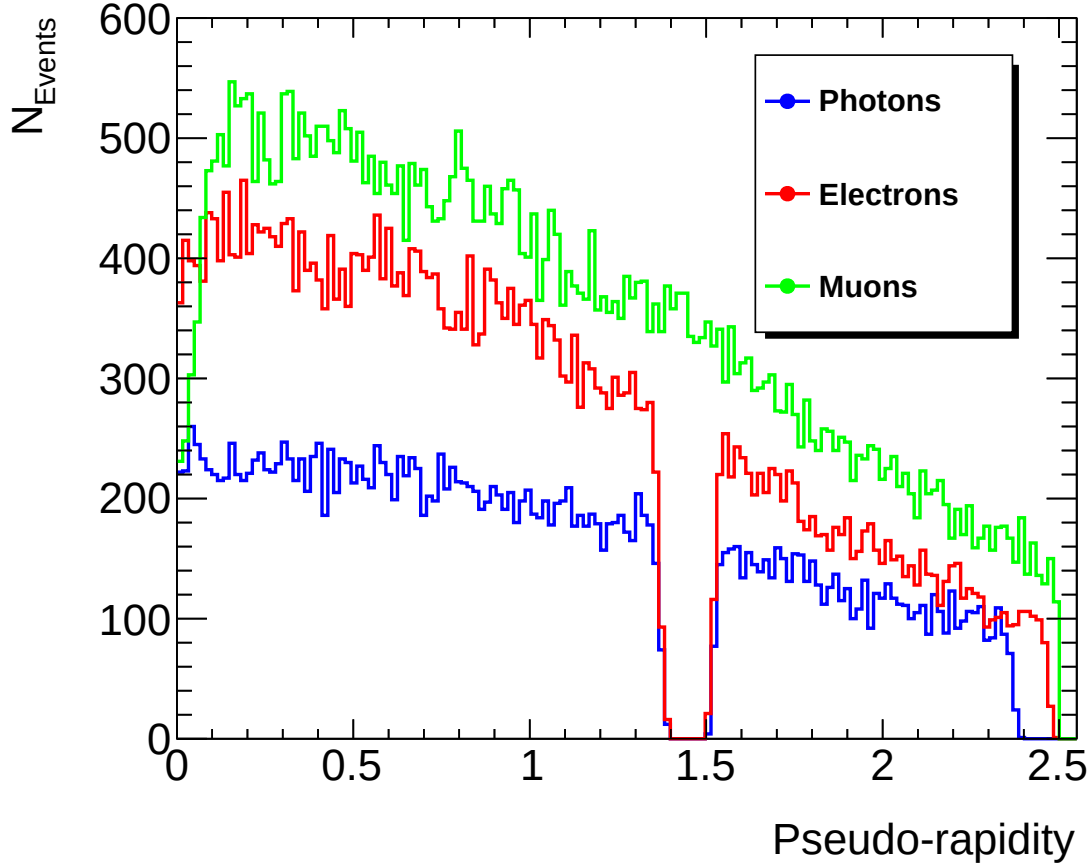


Fig. 5.1 : Pseudorapidity (absolute value) distribution of selected photons (blue), electrons (red) and muons (green) in the $ZZ\gamma$ signal MC sample in region SR2.

To ensure they are reconstructed reliably, leptons and photons are required to be well separated from each other. The separation ΔR defined in section 3.2 is used for that purpose. If a photon is found within a cone with $\Delta R < 0.4$ around a lepton, the event is removed. Similarly, if an electron is separated by $\Delta R < 0.2$ of a muon, the event is discarded. Distributions of $\Delta R(\ell, \gamma)$ and $\Delta R(\ell, \ell)$ are shown in Fig. 5.2. The ΔR cuts at 0.4 and 0.2 are visible on the left (Fig. 5.2a) and right (Fig. 5.2b) hand side plot respectively.

Additional cuts on leptons and photons are required during sample production. There must be at least one 'Loose' photon with $p_T > 15$ GeV (the photon p_T distributions in the signal and background samples are shown in Fig. 5.3), one quadruplet of leptons and a same flavour opposite sign (SFOS) lepton pair. As only decays of Z bosons to photons and muons are considered, the three possible channels are $e^+e^-e^+e^-$, $\mu^+\mu^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$. For the $4e$ and

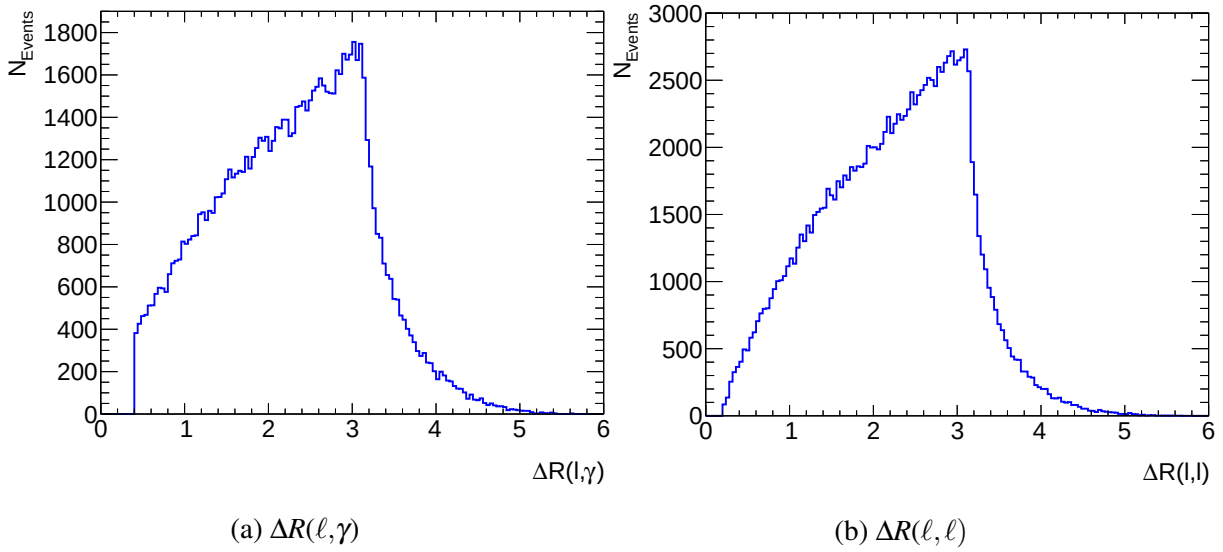


Fig. 5.2 : Distribution of separations ΔR between leptons and photons (left) and between pairs of leptons (right) in the $ZZ\gamma$ signal MC sample in region SR2.

4μ channels there is a two-fold ambiguity in the way that the four leptons can be assigned to two separate SFOS lepton pairs. The two positive leptons are indistinguishable, as are the two negative leptons. Thus, when looking at four identical leptons (two positive and two negative) there are two possible sets of pairs. In order to distinguish between them, the sum of mass differences $|m_{\ell\ell 1} - m_Z| + |m_{\ell\ell 2} - m_Z|$ is considered, where the $m_{\ell\ell}$ are the masses of the lepton pairs and m_Z is the mass of the Z boson, $m_Z = 91.19$ GeV. The mass-difference sum is computed for both possible sets of pairs and the pairing assignment chosen is the one which minimizes its value. In addition, each one of the lepton pairs must have a mass $m_{\ell\ell}$ greater than 40 GeV. The highest p_T lepton is required to have $p_T > 25$ GeV. All other leptons must have a p_T greater than 10 GeV to eliminate background processes which are dominant at low energies. All the cuts given above are applied during sample production. They define a loose signal region that will be called SR2.

The loose signal region SR2 forms the basis for the definition of two further, tighter signal regions SR1 and SR0. The former is an inclusive $\ell\ell\ell\ell\gamma$ region that includes both FSR and ISR events. The latter selects on-shell $ZZ\gamma$ events dominated by ISR processes. The region SR1 is a subset of SR2 and the region SR0 is a subset of SR1. The cuts used to define the regions SR1 and SR0 are given below.

Only well identified leptons are required and thus, a 'Tight' identification is chosen for electrons, muons and photons. The isolation working points chosen for electrons, muons and photons are respectively LooseVarRad, PflowLoose_FixedRad and FixedCutLoose. Chapter 4 and Appendix A give more details about the isolation working points.

Only events containing exactly four leptons are kept. The leptons are labelled according to their p_T : the highest p_T lepton, also called the leading lepton, is labelled ℓ_1 and so on. Four different lepton p_T cuts are applied, one for each lepton. The leading lepton cut is tightened to 30 GeV, as the standard lepton trigger is already at 27 GeV. The second and third lepton cuts have been defined by looking at their p_T distributions in signal and background samples as shown in Fig. 5.4. They are respectively set at 25 GeV and 13 GeV. Since only a small number of $ZZ\gamma$ signal events is expected, the cuts have been placed to maintain a high acceptance for $ZZ\gamma$ events. The fourth lepton cut is set at 10 GeV. Below this limit, the different background processes are dominant compared to the signal. The background processes plotted in Fig. 5.4 include final state radiation (cf. section 5.2), pile-up photons and fake photons as modelled in the $\ell\ell\ell\ell$ SHERPA sample (cf. chapter 6).

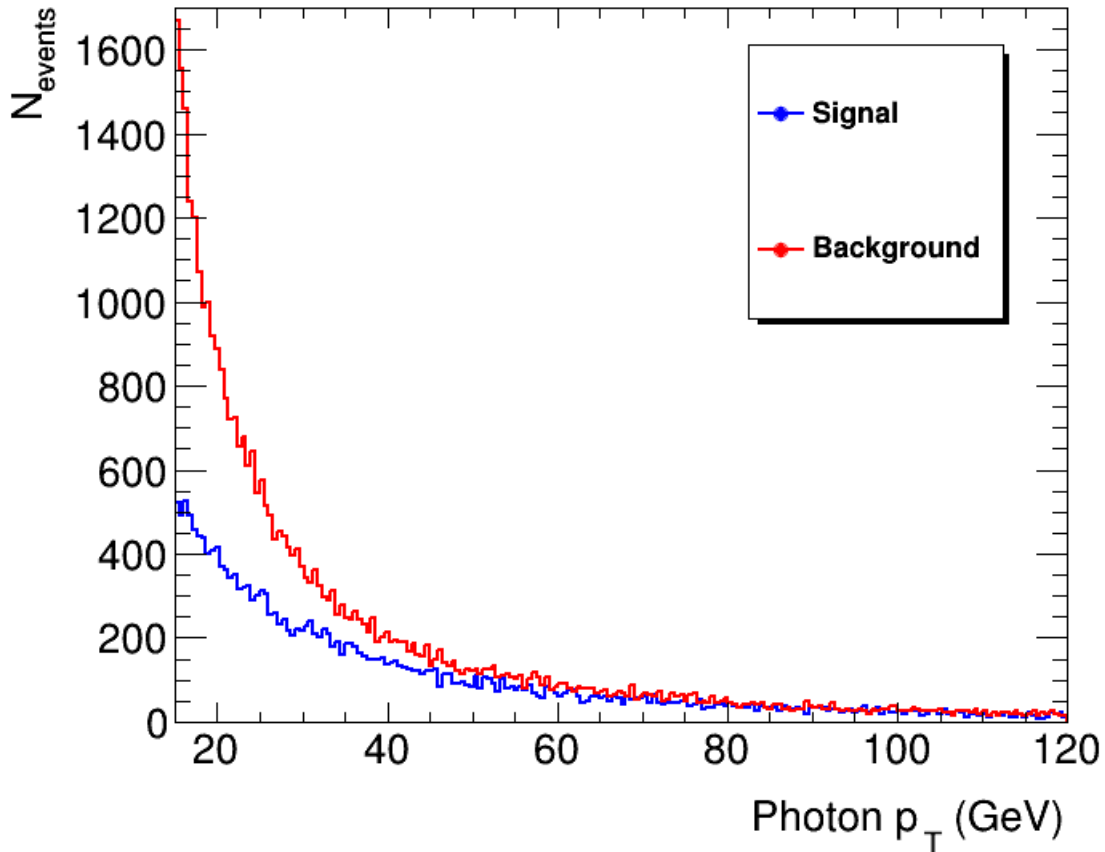


Fig. 5.3 : Photon p_T distribution in SR2 in the $ZZ\gamma$ signal sample (blue) and the $\ell\ell\ell\ell$ SHERPA sample (red).

In order to ensure that two pairs of particles are observed, in addition to the SFOS requirement, it is checked that an event is comprised of exactly four leptons, that the sum of the lepton charges equals zero and finally that there is always an even number of electrons and muons (either 0, 2 or 4). This ensures that only events with one of the three decay channels $e^+e^-e^+e^-$, $\mu^+\mu^-\mu^+\mu^-$ and $e^+e^-\mu^+\mu^-$ are kept.

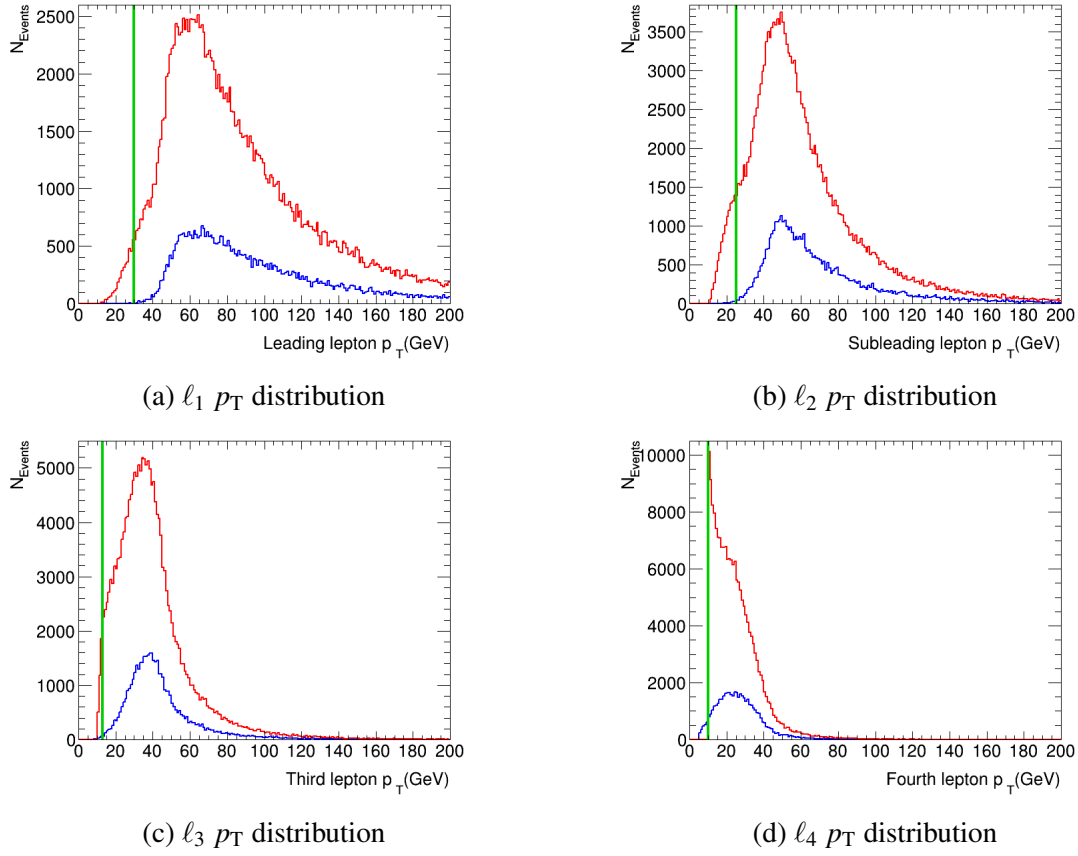


Fig. 5.4 : p_T distributions for the four leading leptons in events passing the SR2 selection requirements. The $ZZ\gamma$ signal MC sample distribution is shown in blue and the $llll$ background SHERPA sample distribution in red. The green vertical line indicates where the cut defining SR1 and SR0 is placed.

	Photons	Electrons	Muons
Ident.	Tight	Tight	Tight
Isolation	FixedCutLoose	LooseVarRad	PflowLooseFixedRad
p_{T}	$> 15 \text{ GeV}$	$> 30, 25, 13, 10 \text{ GeV}$	
$ \eta $	< 2.37	< 2.47	< 2.5
ΔR	$\Delta R(\ell, \gamma) > 0.4$	$\Delta R(\ell, \ell) > 0.2$	
$m_{\ell\ell}$	$\searrow$	$m_{\ell\ell} > 40 \text{ GeV}$	
Anti-FSR	Cut A: $ m_{\ell\ell 1} - m_Z < 10\text{GeV}$ and $ m_{\ell\ell 2} - m_Z < 10\text{GeV}$ Cut B: $m_{\ell\ell 1} + m_{\ell\ell\gamma 1} > 2m_Z$ and $m_{\ell\ell 2} + m_{\ell\ell\gamma 2} > 2m_Z$		

Table 5.1 : Signal selection cuts. In row 1 is given the identification and in row 2, the isolation working points. For photons and electrons, the pseudorapidity region $1.37 < |\eta| < 1.52$ is excluded. The first six rows define the signal region SR1. Including also the anti-FSR cuts in the bottom row defines the signal region SR0.

All the cuts discussed above define the $llll\gamma$ signal region known as SR1. The cuts are summarised in Tab. 5.1.

In Tab. 5.2 are shown the predicted numbers of selected $ZZ\gamma$ signal events and $ZZ\gamma$ FSR events in Run 2. The $ZZ\gamma$ SHERPA sample and the $llll$ SHERPA sample are used respectively. The numbers are given for signal regions SR1 and SR0 (to be defined in the next section) in each of the three lepton decay channels as well as the total of all channels. The smaller number of $4e$ events compared to the number of 4μ events can be explained by the lower reconstruction efficiency of an electron relative to a muon. The difference is enhanced as four leptons have to be reconstructed for each event.

region	$eeee$	$\mu\mu\mu\mu$	$ee\mu\mu$	total
$ZZ\gamma$				
SR1	0.74 ± 0.02	1.56 ± 0.03	2.27 ± 0.03	4.57 ± 0.05
SR0	0.61 ± 0.02	1.23 ± 0.02	1.82 ± 0.03	3.66 ± 0.04
$llll\gamma_{\text{FSR}}$				
SR1	3.02 ± 0.16	8.14 ± 0.27	9.32 ± 0.38	20.48 ± 0.48
SR0	0.30 ± 0.04	0.68 ± 0.08	0.70 ± 0.07	1.68 ± 0.11

Table 5.2 : Number of $ZZ\gamma$ and $llll\gamma_{\text{FSR}}$ events expected in SR0 and SR1 for each decay channel in Run 2 data. The numbers come from the $ZZ\gamma$ signal MC sample and the $llll$ SHERPA background sample. The statistical uncertainty is shown.

In Fig. 5.5 are shown the distributions of electron and muon pair masses in SR1 in the $ZZ\gamma$ signal sample and the SHERPA $llll$ background sample. The signal distributions present a peak around the Z boson mass as expected. However, in the background distributions the mass range below 80 GeV is significantly populated. These events come mostly from FSR processes as explained in the next section.

In Fig. 5.6, the mass of electron pairs are plotted versus the mass of muon pairs for events satisfying the SR1 selection requirements. In the signal plot, Fig 5.6a, most events lie close to the $(m_Z; m_Z)$ point as expected. This confirms what is shown in Fig. 5.5a. In the background two-dimensional plot of lepton pairs, a significant population of events below the Z mass can be seen. Again, this is confirmed by Fig. 5.5b. The two visible lines ($x = m_Z, y = m_Z$) of events with masses below the Z mass are due to FSR. Separation between ISR and FSR events will be considered in the next section. In Fig 5.6b, a small but significant population of events along the diagonal region below the Z mass can be seen. The origin of these diagonal events is under investigation, but seems to be due to an anomaly in the event generator SHERPA.

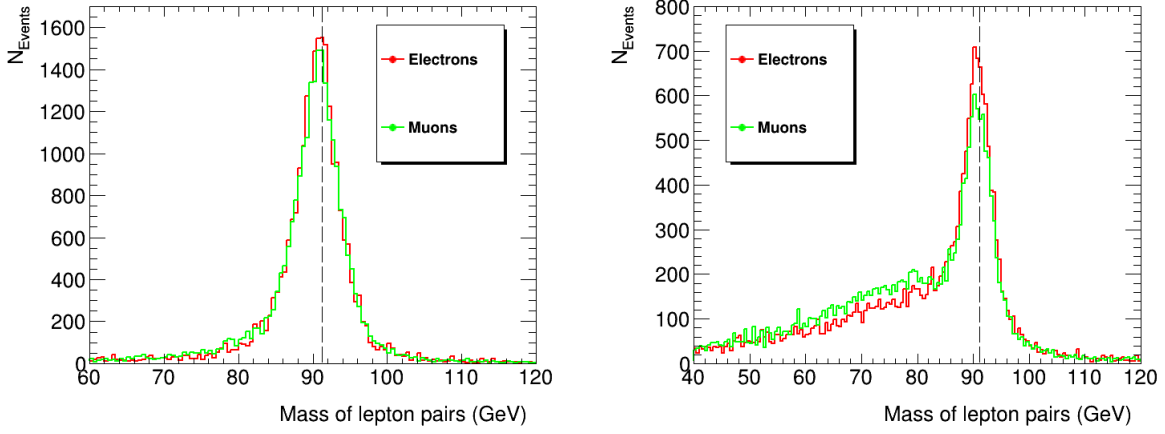
(a) $m_{\ell\ell}$ distributions in $ZZ\gamma$ signal sample(b) $m_{\ell\ell}$ distributions in $llll$ background sample

Fig. 5.5 : Lepton pair mass distributions (in GeV) in SR1 from the $ZZ\gamma$ signal sample (left) and the $llll$ SHERPA background sample (right). Electron pairs are in red and muon pairs in green. The black dotted line indicates $m_Z = 91.19$ GeV.

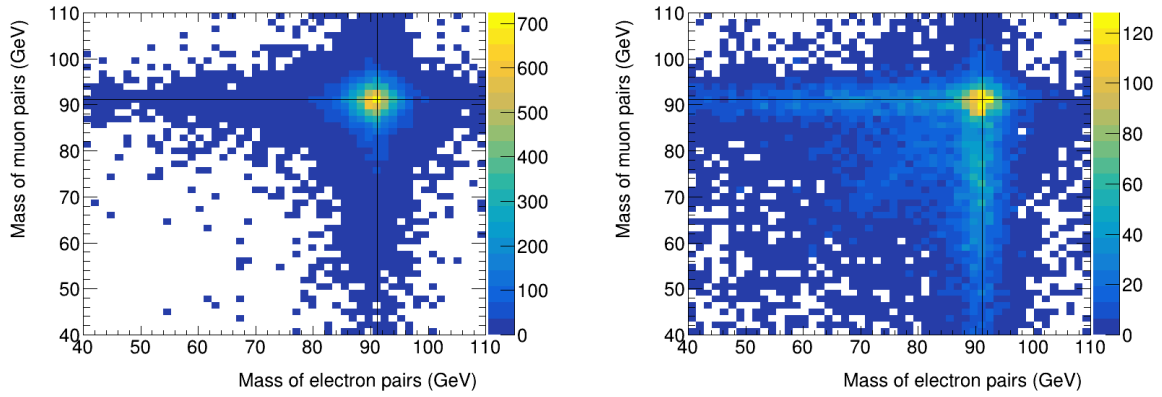
(a) m_{ee} vs $m_{\mu\mu}$ in GeV in $ZZ\gamma$ signal sample(b) m_{ee} vs $m_{\mu\mu}$ in GeV in $llll$ background sample

Fig. 5.6 : Two dimensional plots of electron pairs mass versus muon pairs mass (in GeV) in SR1 from the $ZZ\gamma$ signal sample (left) and the $llll$ SHERPA background sample (right). The horizontal and vertical black lines indicate the mass of the Z boson $m_Z = 91.19$ GeV.

5.2 Final State Radiation

Final state radiation events correspond to diboson ZZ production where one of the four decay leptons radiates a photon. Thus, one of the two Z bosons decays as $Z \rightarrow \ell^+ \ell^- \gamma$. FSR events can be distinguished from $ZZ\gamma$ triboson events where the photon comes from initial state radiation. Representative leading-order Feynman diagrams for the two types of process are shown in Fig. 5.7. In this section, the FSR process is considered as background (being due to diboson, not triboson production) and the feasibility of measuring $ZZ\gamma$ production via the ISR (triboson) process alone is considered. This is not always the approach taken in similar electroweak sector ATLAS and CMS analyses involving photon production. For example, in the CMS Run 1 $Z\gamma$ analysis [7], the goal was to study every interaction resulting in a $\ell\ell\gamma$ final state. On the other hand, the ATLAS Run 2 $Z\gamma$ analysis chose to study $\ell\ell\gamma$ events in a phase space region dominated by ISR events.

To reduce the level of FSR background, two cuts are considered:

- Cut A: each of the lepton pairs is required to have a mass within a range of 10 GeV around the Z boson mass: $|m_{\ell\ell 1} - m_Z| < 10 \text{ GeV}$ and $|m_{\ell\ell 2} - m_Z| < 10 \text{ GeV}$.
- Cut B: $m_{\ell\ell 1} + m_{\ell\ell\gamma 1} > 2m_Z$ and $m_{\ell\ell 2} + m_{\ell\ell\gamma 2} > 2m_Z$.

In Fig. 5.7b it can be seen that for FSR events, the mass of the lower lepton pair in the Feynman diagram must be smaller than m_Z because a photon has been radiated, while $m_{\ell\ell\gamma}$ is in principle equal to the mass of the Z . This is not the case for ISR events as $m_{\ell\ell} = m_Z$ and $m_{\ell\ell\gamma} > m_Z$ as illustrated in Fig. 5.7a¹. The plots shown in Fig. 5.8 confirm the intuition coming from Feynman diagrams and demonstrate the effectiveness of cut B. When taken together, these cuts reject approximately 90% of the FSR background while leaving the signal almost untouched as is summarised in Tab. 5.3. In this table, the final column gives the significance of observing the predicted number of $ZZ\gamma$ signal events (ISR) given the predicted number of background events (FSR) and the uncertainty on the background prediction. The significance values shown throughout this thesis are computed using the recommended ATLAS formula (cf. Appendix B) unless otherwise stated. The error on this significance is computed from the signal total uncertainty, the background uncertainty being already included in the significance formula. From these numbers it can be deduced that the best ISR/FSR separation is achieved by applying both Cut A and Cut B.

¹For the particular Feynman diagram shown in Fig. 5.7a, the photon is in fact not strictly due to initial state radiation as it is not emitted directly from one of the incoming partons. In this sense the term ISR is being used as a convenient label to refer to all non-FSR diagrams.

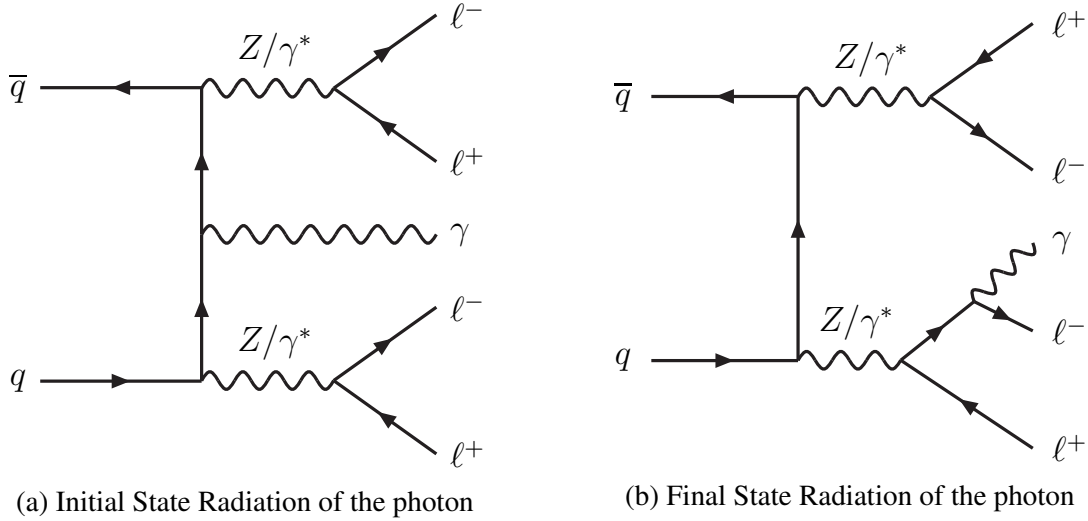


Fig. 5.7 : Feynman diagrams of $ZZ\gamma$ ISR (left) and FSR (right) production.

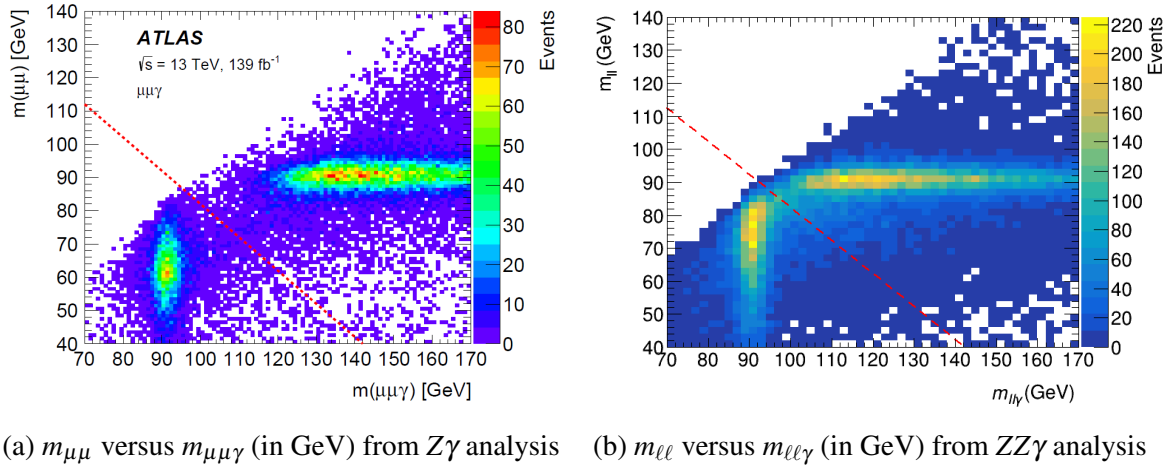


Fig. 5.8 : Two-dimensional distribution of lepton-pair mass and three-body $\ell\ell\gamma$ invariant mass. The dotted and dashed red lines show the equation $m_{\ell\ell} + m_{\ell\ell\gamma} = 2m_Z$ used in the definition of Cut B. The regions below these lines are dominated by FSR events and the regions above by ISR events. The plot on the left is taken from [11] and was made for selected $\mu^+\mu^-\gamma$ events in data. The plot on the right is obtained for the sum of the $ZZ\gamma$ signal MC sample and the $\ell\ell\ell\ell$ SHERPA sample for events in the loose signal region SR2. The right-hand plot contains two entries for each selected event.

The signal region SR0 is defined by adding the two anti-FSR cuts (Cut A and Cut B) to the cuts defining SR1 (cf. Tab. 5.1). The region SR0 contains an enhanced proportion of events due to on-shell triboson $ZZ\gamma$ production (ISR).

5.3 Summary

In this chapter, the various selection cuts used to define the three signal regions SR2, SR1 and SR0 were defined and justified. The region SR2 is used as the starting point to define the other

	Signal	$llll\gamma_{\text{FSR}}$	Significance
No cuts (SR1)	4.57 ± 0.05	20.48 ± 0.48	0.97 ± 0.01
Cut A/No cut B	3.67 ± 0.04	2.26 ± 0.13	2.01 ± 0.05
No cut A/cut B	4.39 ± 0.05	7.80 ± 0.24	1.44 ± 0.02
Cut A/Cut B (SR0)	3.66 ± 0.04	1.68 ± 0.11	2.23 ± 0.05

Table 5.3 : Predicted number of signal (first column) and FSR (second column) events in 139fb^{-1} when FSR cuts are applied or not. The last column shows the significance of observing the number of signal events given the number of FSR background events. The displayed errors are statistical.

two regions. This loose signal region will be used to evaluate the effectiveness of multivariate algorithms at discriminating between signal and background (cf. chapter 7). The regions SR1 and SR0 distinguish triboson ISR $ZZ\gamma$ production events from inclusive $llll\gamma$ production (including FSR). FSR events, which are essentially due to diboson rather than triboson production, can be largely removed using a pair of cuts on the $\ell\ell$ and $\ell\ell\gamma$ invariant masses. The studies presented above indicate that it is unlikely that evidence for $ZZ\gamma$ production via the ISR process will be found in SR0 in data as the predicted significance is below 3σ . In the next chapter, these results are extended by considering also the dominant expected background contributions and the associated systematic uncertainties. In addition, the possibility of observing an inclusive $llll\gamma$ signal due to ISR and FSR processes combined is investigated.

Chapter 6

Background

In this chapter, the main sources of background to the $ZZ\gamma$ signal and the methods used to estimate their contributions are described. As explained in Chapter 5, final state radiation is considered as background in the signal region SR0, but as part of the signal in SR1. As mentioned previously, events where one of the Z bosons in a $ZZ\gamma$ event decays to a pair of τ leptons are considered as background because of the difficulty to identify and reconstruct taus. The amount of tau background is expected to be negligible in both signal regions SR0 and SR1.

In the next section, prompt-photon background processes are described. These processes produce an isolated photon directly in the hard-scatter interaction and can produce in addition four well-identified isolated leptons. The two prompt-photon backgrounds discussed here are $t\bar{t}\gamma$ and $WZ\gamma$ production. Then, non-prompt photon background processes are described. In this type of background, the photon does not originate from the hard scattering process. The first possibility considered is that the photon and the four isolated leptons are produced in two different pp interactions occurring in the same bunch crossing. This is called pile-up photon background. In a similar process, known as double-parton scattering, the photon and the four leptons are produced in separate parton-parton interactions occurring within the same pp interaction. Finally, the fake photon background is discussed, where a jet is misreconstructed as a photon. The estimation of this background requires data-driven techniques and is quite complex in the $ZZ\gamma$ case because of the small number of events expected.

6.1 Prompt photon backgrounds: $t\bar{t}\gamma$, $WZ\gamma$

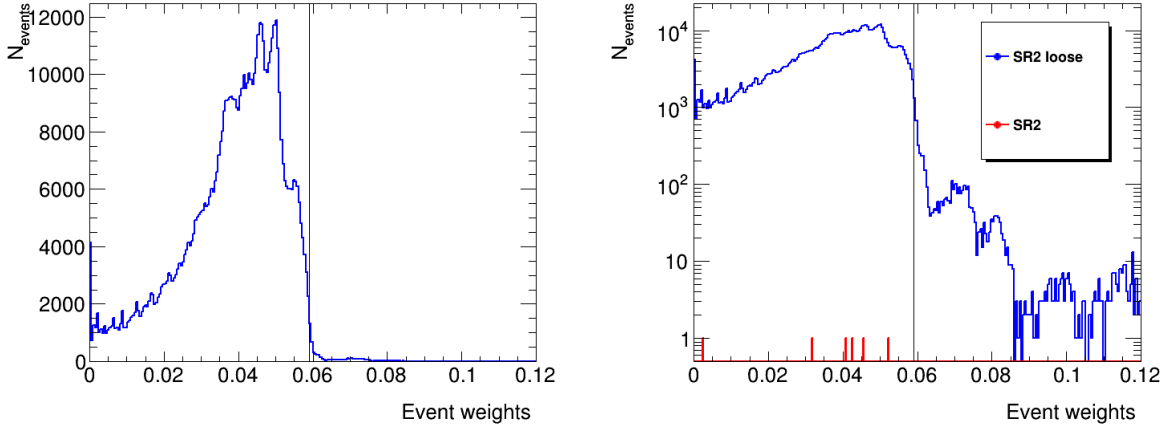
In this section, background processes which produce an isolated photon and could produce four isolated leptons are discussed. An example of such a process is $t\bar{t}\gamma$ production. The two top quarks will mainly decay to a pair of W bosons and a pair of beauty quarks (the branching fraction of the top quark for this decay is known to be close to 100% [1]). Each of the W bosons decays to a lepton and a neutrino. When the two b quarks hadronise, two more leptons could be produced from the decays of the resulting b -hadrons. Because of the relatively high mass of the b

quarks, these leptons can have a large transverse momentum relative to the b -quark direction, and hence be relatively isolated from the other particles produced in the b -quark hadronisation [1, 2]. The other prompt-photon background contribution considered here is $WZ\gamma$ production. In a $WZ\gamma$ process, if the W decays to a lepton and a neutrino and an additional fake lepton is produced (a jet misreconstructed as a lepton for example), the interaction could be misidentified as $ZZ\gamma$ production [1, 2]. The additional lepton could also come from an FSR photon reconstructed as an electron. Events due to $t\bar{t}\gamma$ or $WZ\gamma$ production are expected to have only a small probability to pass all the $ZZ\gamma$ signal selection cuts. However, because these interactions have larger cross-sections than $ZZ\gamma$ (especially $t\bar{t}\gamma$ production), their contributions cannot be assumed to be negligible. Other processes such as $t\bar{t}Z$ or $H \rightarrow ZZ^*$ production could also produce background, but their contributions are expected to be smaller than those from non-prompt photon processes and have not been investigated for now.

When applying the SR0 selection cuts on $t\bar{t}\gamma$ and $WZ\gamma$ samples, no MC events were selected. In the case of SR1, no $t\bar{t}\gamma$ or $WZ\gamma$ MC events are selected, but one $t\bar{t}\gamma$ event and two $WZ\gamma$ events failed only one of the selection cuts - respectively the fourth lepton p_T selection (with $p_T = 8.6$ GeV) and the photon p_T selection (with $p_T = 11$ GeV for both photons). The weight, *i.e.* the contribution of each event to the total number of events predicted¹, of the $t\bar{t}\gamma$ event is equal to 0.003 and the two $WZ\gamma$ events have a total weight of 0.008. For the signal region SR2, six $t\bar{t}\gamma$ and eleven $WZ\gamma$ MC events were selected, amounting to 0.22 ± 0.10 and 0.04 ± 0.01 expected events respectively after reweighting. The event weight distributions in SR2 are shown in Figs. 6.1b and 6.2b. The distributions for $t\bar{t}\gamma$ and $WZ\gamma$ in a looser SR2 region, where the $m_{\ell\ell} > 40$ GeV cut has been relaxed in order to obtain a significant amount of events, are also shown in Figs. 6.1 and 6.2 below. For both processes, the weights distributions in the looser SR2 region are also plotted with logarithmic y-scale. For the $t\bar{t}\gamma$ process, there is a tail of events with relatively high event weight, above about 0.06, which could potentially result in a significant background contribution were they to pass the SR0 or SR1 selection requirements. However, based on the information above, there is no indication of a population of $t\bar{t}\gamma$ or $WZ\gamma$ MC events lying close to the $ZZ\gamma$ signal regions, or that MC events with high weight are more likely to pass the signal selection cuts. Therefore, it is unlikely that the $t\bar{t}\gamma$ and the $WZ\gamma$ processes contribute significantly to the $ZZ\gamma$ background.

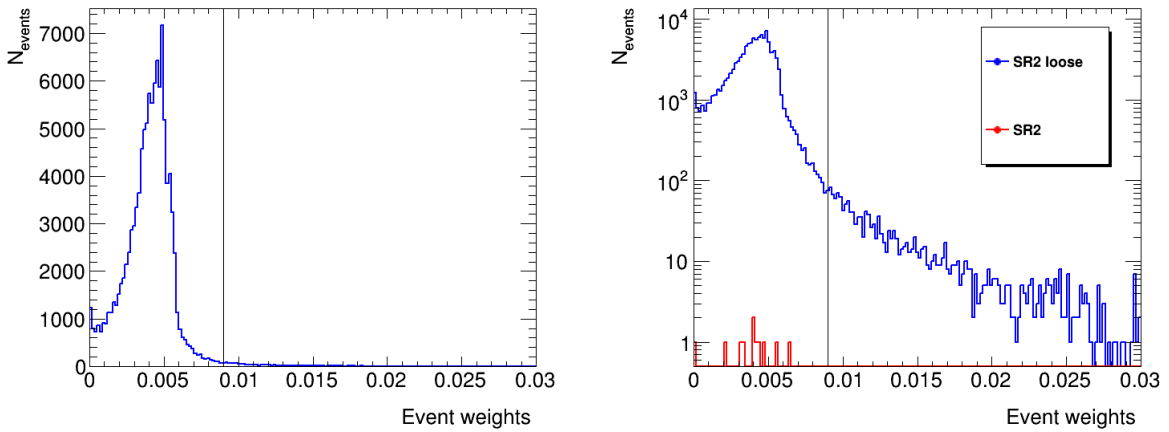
Since no MC events pass all the SR0 or SR1 selection requirements, an upper limit on these background contributions is derived. Confidence intervals are built using the Neyman construction [1]. A probability density function (p.d.f.) $f(x; \theta)$ is considered, where x represents the outcome of the experiment and θ is the parameter that the confidence interval is built for. The variable x represents an estimator of θ . For given boundaries x_1 and x_2 , the probability for x to

¹The weight of each event is directly calculated by the MC simulation. The weight depends on the integrated luminosity generated and the theoretical cross-section of a given process.



(a) $t\bar{t}\gamma$ weights distribution with linear y-axis (b) $t\bar{t}\gamma$ weights distribution with logarithmic y-axis

Fig. 6.1 : Distribution of event weights for $t\bar{t}\gamma$ events passing the relaxed SR2 selection in linear (left) and logarithmic (right) scale. The vertical black line indicates where the 99% confidence level is placed (0.059). On the right-hand side plot, the weight distribution for events passing the SR2 selection is also shown in red.



(a) $WZ\gamma$ weights distribution with linear y-axis (b) $WZ\gamma$ weights distribution with logarithmic y-axis

Fig. 6.2 : Distribution of event weights for $WZ\gamma$ events passing the relaxed SR2 selection in linear (left) and logarithmic (right) scale. The vertical black line indicates where the 99% confidence level is placed (0.009). On the right-hand side plot, the weight distribution for events passing the SR2 selection is also shown in red.

sit in the interval is then:

$$P(x_1 < x < x_2; \theta) = \int_{x_1}^{x_2} f(x; \theta) dx. \quad (6.1)$$

In the $ZZ\gamma$ case, the p.d.f. f is taken to be the weight distribution for MC events passing the relaxed SR2 region (cf. Figs. 6.1 and 6.2). In Equation 6.1, x_1 is set to zero. Then, for the desired confidence level, the value of x_2 is determined such that the equality above stands². The confidence intervals for the $t\bar{t}\gamma$ and $WZ\gamma$ processes for different confidence levels are given in Tab. 6.1.

Confidence level	$t\bar{t}\gamma$	$WZ\gamma$
90%	<0.054	<0.006
95%	<0.056	<0.007
99%	<0.059	<0.009

Table 6.1 : Upper limits on expected number of $t\bar{t}\gamma$ and $WZ\gamma$ background events at 90%, 95% and 99% confidence level.

6.2 Pile-up background

A small fraction of photons undergo conversion to an electron-positron pair in the material of the Inner Detector (ID) [28]. If the electron and positron are both reconstructed as ID tracks, then the vertex associated with the conversion can be reconstructed. If only one of the conversion particles is reconstructed, the conversion vertex is taken to be the first (smallest radius) measurement of the ID track. Combining the conversion vertex position with the reconstructed position of the ECAL cluster produced by the electron and/or positron, and after extrapolation inwards towards the position of the beam axis, the longitudinal position z_γ of the photon production point can be determined. However, most photons remain unconverted before they reach the ECAL. In this case, the photon is reconstructed from the ECAL cluster alone. The longitudinal position z_γ is then determined by extrapolating the ECAL shower axis (formed using the segmented layer information from the ECAL) inwards from the ECAL to the beam axis. For converted photons, the precision on z_γ is typically sufficient to determine whether or not the photon originated from the same primary vertex as the four-lepton system. For unconverted photons however, due to the longer lever arm associated with the inwards extrapolation, the precision on z_γ is much poorer and this is not possible. Thus a four-lepton event together with an additional photon coming from another pp interaction occurring in the same bunch crossing could be misidentified as a $ZZ\gamma$ event. This is called pile-up background and is illustrated in Fig. 6.3.

²The distributions f are histograms and thus not continuous functions. Hence, the $=$ sign in Equation 6.1 should be replaced by a $\geq$ sign.

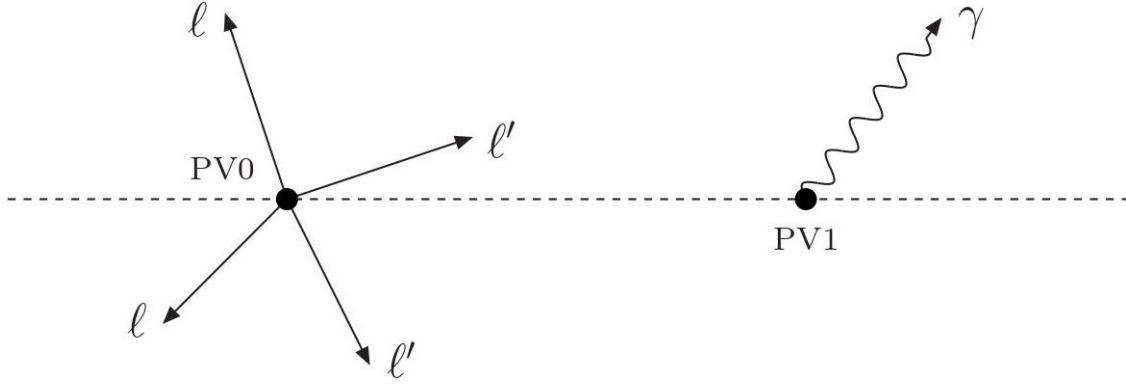


Fig. 6.3 : Illustration of pile-up background. The four-lepton system and the photon are produced in separate pp interactions, with primary vertices PV0 and PV1, occurring in the same bunch crossing.

An order-of-magnitude estimate of the level of pile-up background, $N_{ZZ\gamma}^{\text{PU}}$, can be obtained using the following expression:

$$N_{ZZ\gamma}^{\text{PU}} = \langle \mu \rangle \mathcal{L} \frac{\sigma_{ZZ}\sigma_{\gamma}}{\sigma_{\text{inel}}} , \quad (6.2)$$

where the pile-up parameter $\langle \mu \rangle$ was introduced in Section 3.1 ($\langle \mu \rangle = 33.7$ for ATLAS Run 2 data), $\mathcal{L}$ is the integrated luminosity ($\mathcal{L} = 139 \text{ fb}^{-1}$), σ_{ZZ} is the ZZ production cross-section for the four-lepton channel obtained in ATLAS with similar cuts to the $ZZ\gamma$ selection in Ref. [12], σ_{γ} is the inclusive γ production cross-section for p_T^{γ} greater than 15 or 20 GeV extrapolated from the ATLAS inclusive isolated-photon production analysis [45] and $\sigma_{\text{inel}} = 80 \text{ mb}$ is the inelastic pp interaction cross-section [46]. The formula 6.2 is derived in Appendix C. An approximate estimation of the number of $ZZ\gamma$ pile-up events is $N_{ZZ\gamma}^{\text{PU}} \sim 0.2$ with a photon p_T cut at 15 GeV and $N_{ZZ\gamma}^{\text{PU}} \sim 0.1$ with a photon p_T cut at 20 GeV. This indicates that pile-up background is at the level of a few percent of the expected $ZZ\gamma$ signal.

The modelling of additional pile-up interactions in the MC simulation cannot be relied upon to accurately determine the pile-up background contribution in the $ZZ\gamma$ signal regions. Alternative dedicated MC simulations, or, preferably, data-driven methods must be used. In the ATLAS $Z\gamma$ analysis [11] for example, the longitudinal separation $\Delta z = z_{\gamma} - z_{\text{PV}}$ for converted photons in data was used to determine the pile-up background contribution³. A relative uncertainty of 20% was achieved. Ultimately, the goal in $ZZ\gamma$ would be to use a purely data-driven method. For instance, a ratio to the number of pile-up photons in $Z\gamma$ could be used. In the $Z\gamma$ analysis, the pile-up background was obtained with an uncertainty of around 20%. For now however, a MC simulation approach is used, based on overlaying particles from separate simulated four-lepton

³This was done using photons which converted within the volume occupied by the pixel detector. For such conversions, the separation Δz is particularly well determined.

and photon interactions [11, 47].

Pile-up interactions are simulated by overlaying, at truth level, events from the $ZZ \rightarrow 4\ell$ and inclusive photon MC samples. Systematic uncertainties of 30% and 10% are associated to the modeling of photons and $ZZ \rightarrow \ell\ell\ell\ell$ production respectively. As these samples contain particle-level information and not reconstruction-level information, a selection efficiency coefficient is applied on the samples. The selection efficiency is taken from the $ZZ\gamma$ signal MC. The E_T^γ spectrum from the $ZZ\gamma$ MC sample should be reweighted to match the E_T^γ spectrum in the single-photon sample, but this could not be done in the time available. When using this overlay method on $WZ\gamma$ events, it was found that the reweighting changed the pile-up photon estimate by approximately 10%. Hence, a 10% systematic uncertainty is added on the pile-up photon estimate. After applying the SR0 signal cuts, the number of pile-up events, with photon selection cut $p_T^\gamma > 15$ GeV and $p_T^\gamma > 20$ GeV respectively, is estimated to be [47]:

$$N_{ZZ\gamma}^{\text{PU}}(p_T^\gamma > 15 \text{ GeV}) = 0.20 \pm 0.02(\text{stat}) \pm 0.07(\text{syst}), \quad (6.3)$$

$$N_{ZZ\gamma}^{\text{PU}}(p_T^\gamma > 20 \text{ GeV}) = 0.12 \pm 0.01(\text{stat}) \pm 0.04(\text{syst}). \quad (6.4)$$

These estimates are consistent with the order-of-magnitude estimates obtained earlier.

Double-parton scattering

Double-parton scattering (DPS) background [48], where the two Z bosons and the photon are produced in the same pp collision but from two different parton-parton interactions (for example $q\bar{q} \rightarrow 4\ell$ together with $q\bar{q} \rightarrow \gamma + X$ within the same pp interaction), can be estimated with a similar formula to Equation 6.2. The pile-up $\langle\mu\rangle$ is removed from the formula as in DPS only one pp collision is considered and the cross-section σ_{inel} is replaced by an effective cross-section, σ_{DPS} , which must be determined empirically. This cross-section is estimated from a combination of DPS measurements. It is estimated to be approximately 15 mb but the associated uncertainty is large. A summary of measurements and limits on σ_{DPS} is shown in Fig. 4 in Ref. [48]. On this plot the wide spread of σ_{DPS} measurements and the lack of precision of these measurements can be seen. The latest ATLAS and CMS measurements only produced a lower limit for σ_{DPS} , for instance. An estimate of the level of DPS background can therefore be obtained using:

$$N_{\text{DPS}} = \mathcal{L} \frac{\sigma_{ZZ}\sigma_\gamma}{\sigma_{\text{DPS}}}. \quad (6.5)$$

For the $ZZ\gamma$ signal region SR0, two channels contribute to the DPS background. First, when a pair of Z bosons is produced in one parton-parton interaction and a photon is produced in another and second, when a Z boson and a photon are produced in one parton scattering and the second Z in another. Using Equation 6.5 and summing the $(ZZ)+(\gamma)$ and $(Z\gamma)+(Z)$ DPS channels, the total contribution is estimated to be approximately 0.04 events in SR0 for $p_T^\gamma > 15$ GeV and 0.02

for $p_T^\gamma > 20$ GeV. This estimation is derived in Appendix C. As the DPS estimations are very approximate (because of the large uncertainties on σ_{DPS} and σ_γ mainly), an error of 100% is associated to them.

For estimating the DPS background in SR1, an additional possibility needs to be considered because FSR events may now also contribute. Two Z bosons could be produced in two different parton-parton interactions within the same pp collision, with one of the Z bosons decaying to a pair of leptons and the other decaying to a pair of leptons and a photon from FSR decay. However, no suitable $Z \rightarrow \ell\ell\gamma_{\text{FSR}}$ cross-section measurement is available. It is assumed instead that this cross-section is approximately equal to the $Z\gamma$ production cross-section. This assumption is motivated by Fig. 5.8a where the number of ISR and FSR $Z\gamma$ events are seen to be of similar magnitude. The assumption is confirmed by counting the number of $\ell\ell\gamma_{\text{FSR}}$ events and the number of total events in the $Z\gamma$ data samples. The number of DPS background events in the channel $(Z \rightarrow \ell\ell) + (Z \rightarrow \ell\ell\gamma)$ contributing to $\ell\ell\ell\ell\gamma_{\text{FSR}}$ events is estimated to be 0.01 and therefore, by adding this estimate to the SR0 estimates, $N_{\text{DPS}} = 0.05$ events are predicted in SR1 when $p_T^\gamma > 15$ GeV and 0.03 events are predicted when $p_T^\gamma > 20$ GeV. A relative systematic uncertainty of 100% is again associated to these estimates.

6.3 Fake photon background

A fake photon comes from the fragmentation of a gluon or a quark so it is in fact a jet misidentified as a photon. Fake photon production is illustrated in Fig. 6.4. For example, hadronisation could produce a jet with an energetic neutral pion taking most of the jet energy which will then decay into a pair of photons, $\pi^0 \rightarrow \gamma\gamma$. The photon pair can produce overlapping electromagnetic showers in the ECAL which are reconstructed as a single ECAL cluster and identified as a single isolated photon.

Fake photons are not well modelled by the MC simulation. For instance, the fake photon background in the ATLAS $Z\gamma$ analysis is overestimated by a factor of two by the MC simulation: the Z+jets SHERPA sample predicted ~17,000 events but ~9,000 events were estimated by a data-driven method [11]. Furthermore, the number of fake photons in $WZ\gamma$ varies by one order of magnitude between SHERPA (~40 events) and POWHEG samples (~4 events) whereas for $ZZ\gamma$ this number is multiplied by two from SHERPA (~0.1) to POWHEG (~0.2). Thus, data-driven methods are required.

A widely used method within ATLAS is the ABCD method [49]. In this method, the data sample is separated into four regions based on photon identification and isolation requirements. One region is the signal region and the other three are control regions. These three control regions are defined by inverting the photon 'Tight' identification requirement and/or the photon

isolation requirement. Based on the number of events in each region, the number of ZZ +fake photon background events in the signal region can be estimated. However, in the $ZZ\gamma$ case, from MC, the number of events in the control regions is predicted to be only $O(10^{-1})$, which is too small for the ABCD method to be applied, as either an unphysical result will be obtained or the statistical precision will be greater than 100%. A novel approach is therefore suggested here to estimate the fake photon background for $ZZ\gamma$. It exploits the relatively high number of events of the ATLAS $Z\gamma$ analysis, where a fake photon background estimate with a precision of the order of 10% was obtained.

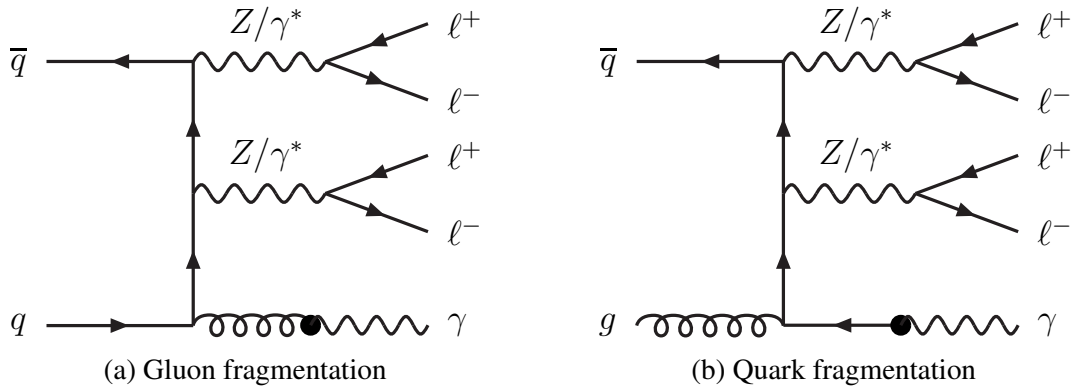


Fig. 6.4 : Feynman diagram of ZZ interactions producing a gluon or a quark which then will hadronize and can produce a fake photon.

Jet ratio method

The number of fake photons depends on the number of jets that are produced. However, after a jet has formed, for a given jet energy and direction, the probability that it is misidentified as a photon should be approximately independent of the jet production process. For the case of $ZZ\gamma$ and $Z\gamma$ interactions, this assumption translates as the following equation:

$$\frac{N_{ZZ\gamma}^{j \rightarrow \gamma}}{N_{ZZ}^{\text{jet}}} = \frac{N_{Z\gamma}^{j \rightarrow \gamma}}{N_Z^{\text{jet}}}, \quad (6.6)$$

where N_{ZZ}^{jet} and N_Z^{jet} are the number of jets formed in ZZ and Z production events respectively and $N_{ZZ\gamma}^{j \rightarrow \gamma}$ and $N_{Z\gamma}^{j \rightarrow \gamma}$ are the number of isolated fake photons generated in ZZ and Z production respectively. For Equation 6.6 to be valid, the cuts applied to the four-lepton (two-lepton) system to count N_{ZZ}^{jet} (N_Z^{jet}) must be the same as those used to count $N_{ZZ\gamma}^{j \rightarrow \gamma}$ ($N_{Z\gamma}^{j \rightarrow \gamma}$). From Equation 6.6, the fake photon background $N_{ZZ\gamma}^{j \rightarrow \gamma}$ can be obtained using the following equation:

$$N_{ZZ\gamma}^{j \rightarrow \gamma} = N_{Z\gamma}^{j \rightarrow \gamma} \cdot \frac{N_{ZZ}^{\text{jet}}}{N_Z^{\text{jet}}}. \quad (6.7)$$

All three quantities on the right-hand side of Equation 6.7 are estimated independently. The number of fake photons in $Z\gamma$ is estimated using the ABCD method on real data. Ideally, in order to have a fully data-driven method, real data samples should also be used to estimate N_Z^{jet} and N_{ZZ}^{jet} . As suitable data samples could not be produced in the time available⁴, the ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$ was instead obtained from the MC prediction. The POWHEG $\ell\ell\ell\ell$ sample is used for computing N_{ZZ}^{jet} and the POWHEG $Z \rightarrow \ell\ell$ samples are used for N_Z^{jet} (the details about these samples are given in Section 4.2). It is well-known that jets are not well modelled beyond the second jet in an event. However, because a ratio is considered in Equation 6.7, most systematics should cancel and the accuracy and precision of the ratio should be better than those of the individual numbers. This is discussed later in this section.

All three quantities on the right-hand side of Equation 6.7 are estimated in a phase space region as similar as possible to the $ZZ\gamma$ signal region SR0 regarding leptons, the only difference being the identification and isolation cuts applied on $Z\gamma$ data. The samples used were produced with earlier versions of the $H\gamma\gamma^*$ framework with different working points. No equivalent working points can be found. For the first two leptons, the p_T cuts are respectively set at 30 and 25 GeV and for the two additional leptons in $ZZ(\gamma)$ interactions, the cuts are set at 13 and 10 GeV (cf. Section 5.1). The same lepton identification and isolation cuts are applied on N_Z^{jet} and N_{ZZ}^{jet} as in SR0 in order to cancel lepton systematics as much as possible. However, because the numerator corresponds to four-lepton events whereas the denominator to two-lepton events and the lepton p_T spectra are different, lepton systematics cannot cancel completely. When counting jets, in order to consider a region as inclusive as possible no photon is required, *i.e.* the one-photon requirement is switched off during MxAOD production and thus no cuts are applied on photons if present. Nevertheless, as the counted jets are potential fake photons, the same p_T (> 15 GeV) and η ($|\eta| < 2.37$ excluding $1.37 < |\eta| < 1.52$) cuts are applied on these jets as on photons in the signal region. Finally, two overlap removal requirements are applied. Jets must be separated from photons by $\Delta R(\text{jet}, \gamma) > 0.4$ and from leptons by $\Delta R(\text{jet}, \ell) > 0.2$.

The validity of Equation 6.6 was tested using MC samples. Even though the modelling of fake photons in MC is unreliable, Equation 6.6 should still hold when all quantities are evaluated using a common MC simulation. Because of the very low number of fake photons in $ZZ\gamma$ MC, it is not possible to apply this equation in bins of p_T as it is done below. When computed, the number $N_{ZZ\gamma}^{j \rightarrow \gamma}$ has a statistical error of the order of 30%. Taking into account the modeling of fake photons in the MC, the overall error is of the order of 45%. When computed in MC, the numbers on the left-hand side and right-hand side of Equation 6.6 are respectively equal to 0.0021 and 0.0034, which gives:

$$0.0034/0.0021 = 1.62 \pm 0.73.$$

⁴The samples used for the ZZ +dijets analysis in ATLAS [50] could potentially be used for instance, but they are probably not suitable in the specific case described here as the jet production is enhanced in these samples and the analysis is focused on the first two jets, not all jets produced.

This number is compatible with unity, but with a large relative uncertainty.

The fake photon yield estimated with the method described above, includes the background contribution from fake photons produced in pile-up interactions. The pile-up photon background estimate from the overlay MC obtained in Section 6.2 only takes prompt pile-up photons into account.

Ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$

The ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$ depends significantly on the transverse momentum of the jet, as shown in Fig. 6.5a. No significant dependence on other kinematic quantities was found. For example, Fig. 6.5b shows a plot of the ratio as a function of the jet pseudorapidity. All quantities in Equation 6.6 are therefore computed in bins of p_T . The following choice of bins has been applied: [15, 20, 30, 40, 50, 60, 80, 100, 120, 150, 1200] in GeV. This choice offers flexibility on the photon p_T cut as discussed later.

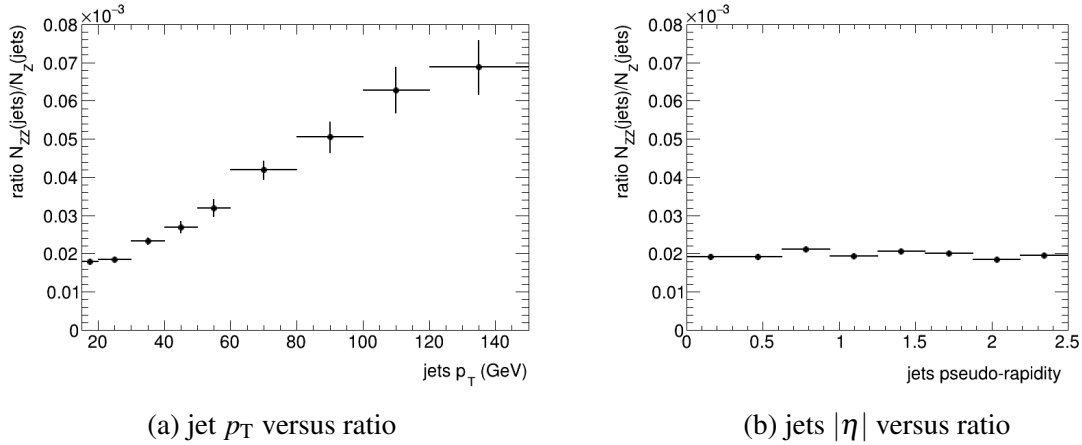


Fig. 6.5 : Ratio of number of jets in ZZ over number of jets in Z from POWHEG samples (same ratio as in Equation 6.7) as a function of jets p_T in GeV (left) and $|\eta|$ (right). The last bin from 150 to 1200 GeV is not present on the left graphic for better visualisation, but is used in the calculation. The error bars are only statistical uncertainties.

In order to compute the number of jets in ZZ and Z processes, the true number of jets from MC samples is taken after all the necessary cuts have been applied. To estimate the systematic uncertainty on the ratio due to possible MC mismodelling of jet production, the ratio is computed using two different generators, SHERPA and POWHEG, and the results are compared. Due to technical difficulties, Z+jets SHERPA samples could not be used so they were replaced by WZ+jets samples instead where jets are counted in the same region as jets in Z and ZZ events. The W boson decays to a lepton and a neutrino so the same p_T , η , identification and isolation cuts as in ZZ γ SR0 were applied to the three leading leptons in a WZ+jets event and a requirement that missing transverse energy E_T^{miss} is greater than 30 GeV is added to take the neutrino into account. The cuts applied on WZ+jets events are summarised in Tab. 6.2.

	Jets	Electrons	Muons
Ident.	↘	Tight	Tight
Isolation	↘	LooseVarRad	PflowLooseFixedRad
p_{T}	$> 15 \text{ GeV}$	$> 30, 25, 13 \text{ GeV}$	
$ \eta $	< 2.37	< 2.47	< 2.5
ΔR	$\Delta R(\text{jet}, \ell) > 0.2$	$\Delta R(\ell, \ell) > 0.2$	
$m_{\ell\ell}$	↘	$m_{\ell\ell} > 40 \text{ GeV}$	
$E_{\text{T}}^{\text{miss}}$	$> 30 \text{ GeV}$		

Table 6.2 : Summary of the selection cuts applied on WZ +jets events.

In Fig. 6.6, jets distributions as a function of transverse momentum in POWHEG and SHERPA for ZZ and WZ samples are compared. SHERPA overestimates the number of jets in all bins compared to POWHEG. However, when considering the ratio $N_{ZZ}^{\text{jet}}/N_{WZ}^{\text{jet}}$ from the same samples, a good agreement between POWHEG and SHERPA is seen as in Fig. 6.7. The total numbers (*i.e.* the numbers summed over all bins) are shown in Tab. 6.3. Even though the individual jet numbers vary from one generator to the other, the difference is largely suppressed when considering the ratios of these numbers. The overall difference is equal to $6 \pm 2 \%$ ⁵. However, because $WZ\gamma$ samples are used rather than $Z\gamma$ samples to estimate this uncertainty, an additional error must be added. It is difficult to estimate exactly the difference between $WZ\gamma$ samples and $Z\gamma$ samples. A 10% systematic error is applied on the ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$. A similar method to estimate the fake photon background is used for the $WZ\gamma$ analysis [51] where a systematic error of 10.8% is applied on the jet ratio. This justifies the systematic uncertainty applied on the ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$ in the method described here.

	N_{ZZ}^{jet}	N_{WZ}^{jet}	$N_{ZZ}^{\text{jet}}/N_{WZ}^{\text{jet}}$
POWHEG	101.7 ± 1.2	1471.0 ± 5.3	0.069 ± 0.001
SHERPA	136.1 ± 2.8	2089.2 ± 7.4	0.065 ± 0.001
Percentage difference	$29 \pm 2 \%$	$35 \pm 1 \%$	$6 \pm 2 \%$

Table 6.3 : Number of jets in ZZ and WZ samples and their ratio for two different event generators, POWHEG and SHERPA. The cuts applied to count the jets are the same as the ones defined above and used for the numbers in Equation 6.7.

Fake photon background in $Z\gamma$

To obtain the number of fake photon events in $Z\gamma$, $N_{Z\gamma}^{\text{j} \rightarrow \gamma}$, the method applied in the ATLAS $Z\gamma$ analysis at 13 TeV [49] is used. The $Z\gamma$ analysis applied a 30 GeV p_T cut on photons and a 40 GeV cut on lepton pair mass whereas for $ZZ\gamma$, a 15 GeV cut is applied on photons and it is

⁵For two positive values x and y , the percentage difference is calculated as the fraction of the difference between x and y relative to their average: $|x - y| / [(x + y)/2]$.

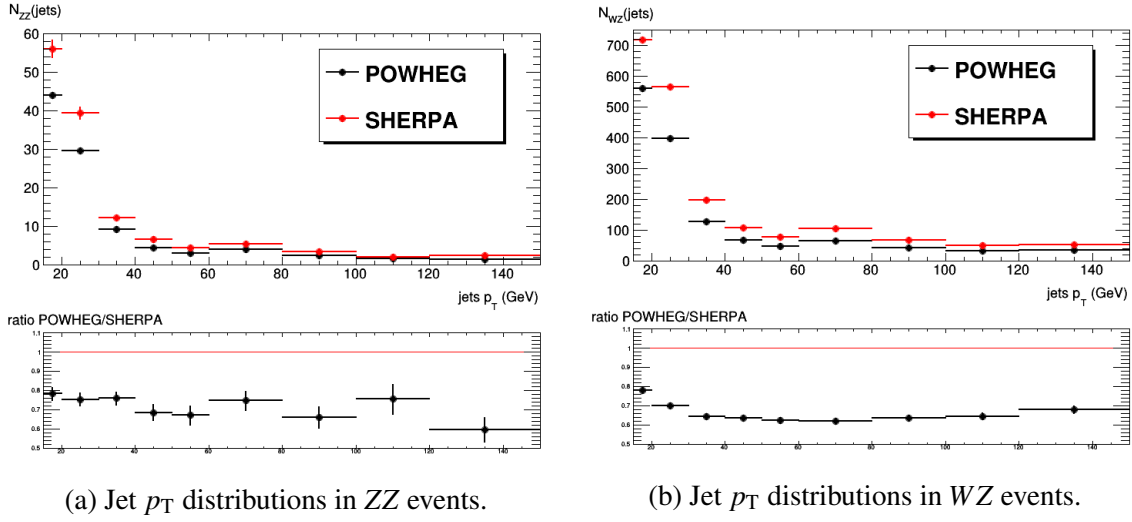


Fig. 6.6 : Comparison of jet p_T distributions from POWHEG (black) and SHERPA (red) ZZ (left) and WZ (right) samples. On the top plots are plotted the numbers of jets as a function of jet p_T and on the bottom plots, the ratios of numbers obtained in POWHEG relative to numbers from SHERPA as a function of jet p_T . On the ratio plots, the red line marks unity (*i.e.* where all points would be if POWHEG and SHERPA modelled jets in the exact same way).

required that $|m_{\ell\ell} - m_Z| < 10$ GeV (cf. Section 5.1). Thus, the estimate of $N_{Z\gamma}^{j\rightarrow\gamma}$ needs to be reevaluated using the cuts appropriate to the $ZZ\gamma$ analysis. To this end, the number of observed events in data passing the $Z\gamma$ N_{obs} , selection is written as:

$$N_{\text{obs}} = N_{\text{sig}} + N_{\text{bkg}} + N_{Z\gamma}^{j\rightarrow\gamma}, \quad (6.8)$$

where N_{sig} is the number of $Z\gamma$ signal events, N_{bkg} is the number of events that correspond to backgrounds other than fake photon background and $N_{Z\gamma}^{j\rightarrow\gamma}$ is the number of fake photon events. The $Z\gamma$ background processes considered are: $t\bar{t}\gamma$, WZ , $WW\gamma$ and ZZ ⁶. Equation 6.8 can be rearranged as [49]:

$$N_{Z\gamma}^{j\rightarrow\gamma} = (N_{\text{obs}} - N_{\text{bkg}}) \cdot (1 - P), \quad (6.9)$$

where P is the purity in the signal region, defined as the ratio of signal events over the sum of signal events and fake photon:

$$P = \frac{N_{\text{sig}}}{N_{\text{sig}} + N_{Z\gamma}^{j\rightarrow\gamma}}.$$

The right-hand side of Equation 6.9 will be evaluated with N_{obs} obtained from the full Run 2 dataset, N_{bkg} obtained from MC samples and P taken from the published $Z\gamma$ analysis. The purity was obtained by the $Z\gamma$ analysis using the ABCD method. The purity as a function of photon transverse energy is shown in Fig. 6.8. The purity P depends on the photon isolation

⁶ Z decay to a tau pair was also considered as a background. The same $\tau\tau\gamma$ samples could not be used. The number of predicted events was however extremely small compared the number of signal events so this background is neglected for the $ZZ\gamma$ fake photon estimate for now.

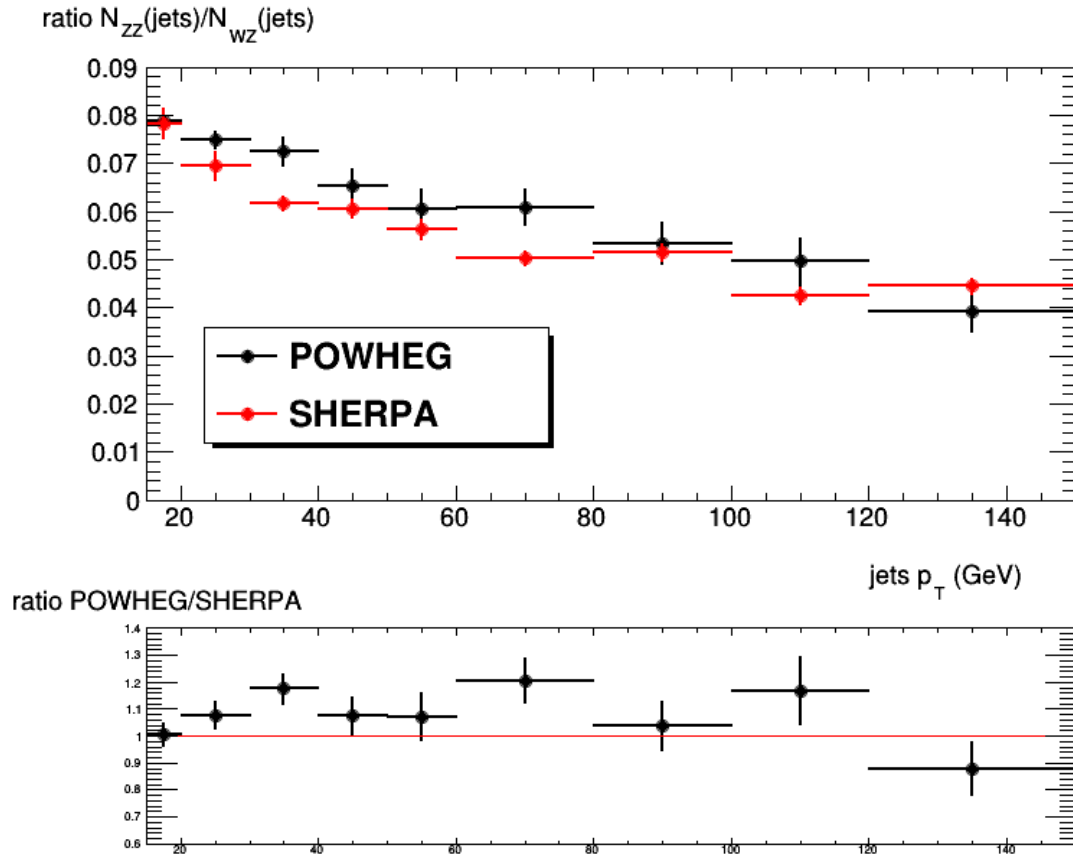


Fig. 6.7 : The top plot shows the ratio of number of jets in ZZ processes over the number of jets in WZ processes for two different event generators, POWHEG and SHERPA, as a function of jets' p_T . The bottom plot shows the ratio of POWHEG over SHERPA numbers from the top plot as a function of jets' p_T . The error bars are only statistical uncertainties.

and identification requirements applied, so the values from Fig. 6.8 can be used as the same requirements are applied on the $Z\gamma$ data samples as those used during the $Z\gamma$ analysis. The identification selection cut used is called LoosePrime4. The LoosePrimeN requirements are various relaxed versions of the photon 'Tight' requirement (cf. Fig. 6.8). The purity also varies with the energy gap between the isolated and non-isolated regions, which is by default set at 2 GeV. Thus, the blue points are used in Fig. 6.8. The numbers of observed and background events are computed in the signal region in the same p_T bins as P which are the same bins used for the ratio in Equation 6.7. The identification and isolation selection cuts used to count the background contributions are different than those used for N_{obs} and P as the MxAODs contain only the latest working points and not those used during the $Z\gamma$ analysis. In order to cross-check the estimations of N_{obs} and the different N_{bkg} contributions, the results from the $Z\gamma$ analysis are retrieved by applying the same cuts. This is detailed in Appendix D. The numbers found are slightly different, which is due to the update of isolation and identification working points. The numbers are however compatible within uncertainties. The numbers of observed events and estimated backgrounds used for estimating $N_{Z\gamma}^{j\rightarrow\gamma}$ are summarised in Tab. 6.4. A 15% systematic error is applied on $t\bar{t}\gamma$ and 30% on multiboson backgrounds [11]. These uncertainties were estimated directly from MC. They account for the possible misreconstruction of an electron as a photon and for higher order contributions. The $N_{Z\gamma}^{j\rightarrow\gamma}$ distribution as a function of photon p_T obtained from Equation 6.9 is shown in Fig. 6.9a.

Photon p_T cut	> 15 GeV	> 20 GeV
N_{obs}	169,740	115,733
$N_{t\bar{t}\gamma}$	$925.3 \pm 7.9 \pm 121.0$	$799.9 \pm 7.3 \pm 120.0$
N_{WZ}	$431.0 \pm 5.1 \pm 129.0$	$385.9 \pm 4.8 \pm 115.8$
$N_{WW\gamma}$	$42.5 \pm 1.3 \pm 12.8$	$36.8 \pm 1.2 \pm 11.0$
N_{ZZ}	$173.1 \pm 2.6 \pm 51.9$	$155.5 \pm 2.4 \pm 46.7$
N_{bkg}	$1571.9 \pm 9.8 \pm 184.8$	$1378.1 \pm 9.1 \pm 173.5$

Table 6.4 : The first row gives the number of events from the Run 2 data passing the $Z(\rightarrow \ell\ell)\gamma$ selection requirements. The next four rows give the estimated background contributions to the $Z\gamma$ sample obtained from MC event samples. The bottom row gives the total estimated background. The first error error is the statistical uncertainty and the second error is the systematic uncertainty.

Fake photon background in $ZZ\gamma$

Using the ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$ p_T distribution shown in Fig. 6.5a and $Z\gamma$ fake photon p_T distribution shown in Fig. 6.9a, the number of fake photon background events in the $ZZ\gamma$ signal region can now be obtained using Equation 6.7. The integrated number of fake photon events for $p_T^\gamma > 15$ GeV and $p_T^\gamma > 20$ GeV is given in Equations 6.10 and 6.11 respectively. The estimated fake photon p_T distribution is shown in Fig. 6.9b.

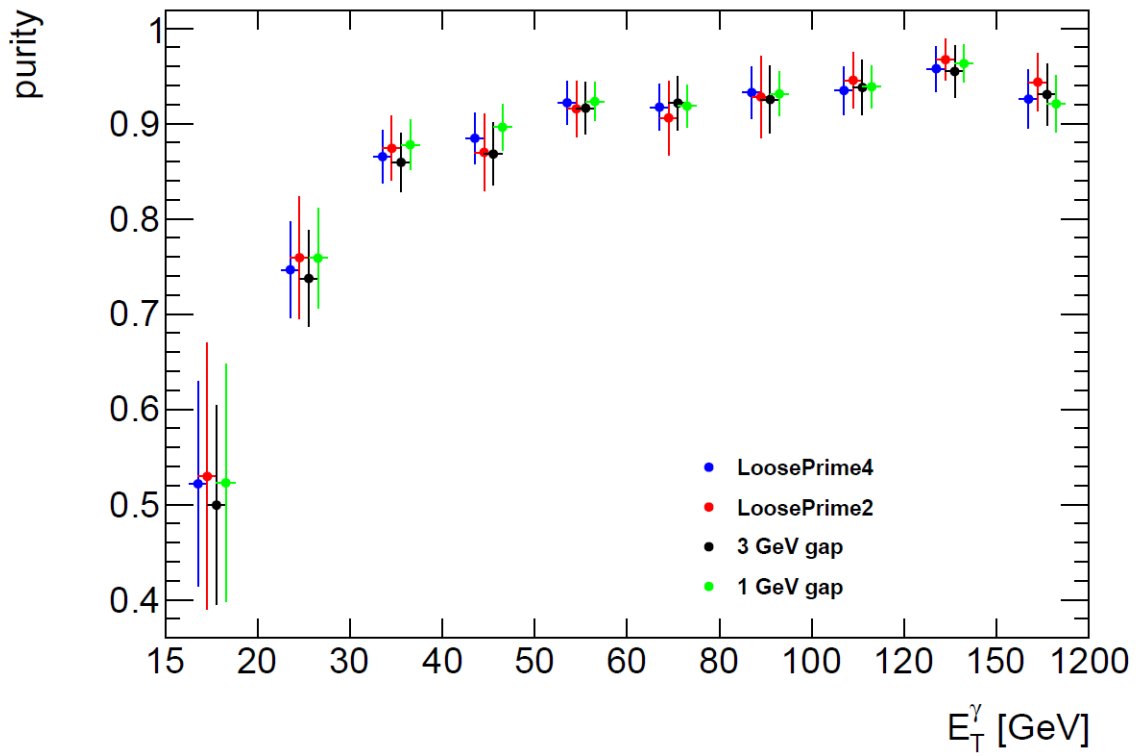


Fig. 6.8 : Values of purity as a function of the transverse energy of photons (in GeV) as estimated by Ref. [49]. The blue points (LoosePrime4 identification, energy gap of 2 GeV) are used for estimating $N_{Z\gamma}^{j\rightarrow\gamma}$ here. The total uncertainties are shown.

$$N_{ZZ\gamma}^{j\rightarrow\gamma}(p_T^\gamma > 15 \text{ GeV}) = 0.922 \pm 0.011(\text{stat}) \pm 0.130(\text{syst}), \quad (6.10)$$

$$N_{ZZ\gamma}^{j\rightarrow\gamma}(p_T^\gamma > 20 \text{ GeV}) = 0.458 \pm 0.007(\text{stat}) \pm 0.058(\text{syst}). \quad (6.11)$$

The systematic uncertainties are computed as follows:

- The error on the purity is taken from the $Z\gamma$ analysis
- On $Z\gamma$ backgrounds, a 30% error is assigned on electroweak multi-boson processes and a 15% error on the $t\bar{t}\gamma$ background
- From comparison between SHERPA and POWHEG samples, a 10% uncertainty is assigned on the ratio $N_{ZZ}^{\text{jet}}/N_Z^{\text{jet}}$

The error on the purity contributes most on the total systematic error. The $Z\gamma$ background errors are negligible compared to the total uncertainty. The contribution of each source of systematic error to the total fake photon systematic uncertainty is given in Tab. 6.5. As the statistical uncertainty is small compared to the systematic uncertainty, the total uncertainty corresponds approximately to the latter one.

Source	$p_T^\gamma > 15 \text{ GeV}$	$p_T^\gamma > 20 \text{ GeV}$
Purity	0.118	0.052
$Z\gamma$ backgrounds	0.0003	0.0003
Ratio	0.053	0.026
Total	0.130	0.058

Table 6.5 : Impact of each source of systematic error on the total fake photon systematic uncertainty (given in Equations 6.10 and 6.11) for $p_T^\gamma > 15 \text{ GeV}$ and $p_T^\gamma > 20 \text{ GeV}$.

Raising the photon p_T cut from 15 to 20 GeV reduces the fake background by a factor of two but the signal only decreases by approximately 14%. This results in a slight increase in the significance of the measurement. Higher cuts were tested but too many signal events are lost. Whether, the final cut will be placed at 15 or 20 GeV is still under discussion.

Validation using $WZ\gamma$ events

In order to validate the method, it is applied on $WZ\gamma$ production and the results are compared with those obtained by the $WZ\gamma$ group with a similar but different approach [51]. Using the method described here, Equation 6.7 can be applied to $WZ\gamma$ and thus becomes:

$$N_{WZ\gamma}^{j\rightarrow\gamma} = N_{Z\gamma}^{j\rightarrow\gamma} \cdot \frac{N_{WZ}^{\text{jet}}}{N_Z^{\text{jet}}}. \quad (6.12)$$

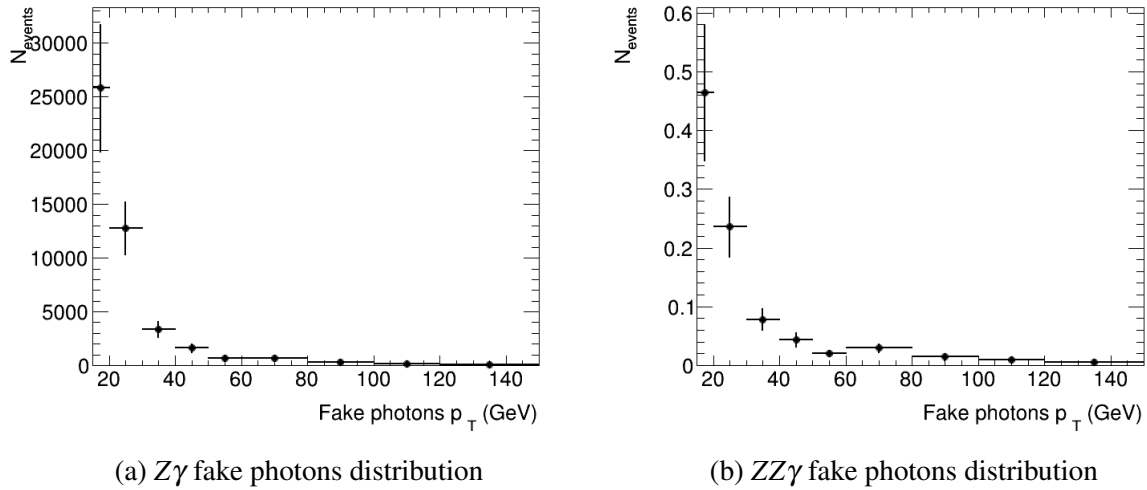


Fig. 6.9 : Fake photon distributions in $Z\gamma$ (left) and $ZZ\gamma$ (right) as a function of transverse momentum in GeV. The displayed error bars correspond to the systematic uncertainty.

The cuts applied on WZ +jets events are shown in Tab. 6.2. The selection cuts applied on photons in the $WZ\gamma$ signal region are the same as those applied on photons in the $ZZ\gamma$ signal region SR0 (cf. Tab. 5.1). The results from the two fake photon methods are summarised in Tab. 6.6. Because MC samples are used for the method described in this section, the statistical error is very small compared to the purely data-driven method used for the $WZ\gamma$ analysis. The two estimates agree within uncertainties. This result is encouraging, and provides a useful consistency check of both methods.

	$N_{WZ\gamma}^{j\rightarrow\gamma}(p_T^\gamma > 15 \text{ GeV})$	$N_{WZ\gamma}^{j\rightarrow\gamma}(p_T^\gamma > 20 \text{ GeV})$
Jets ratio method	$13.04 \pm 0.06 \pm 1.70$	$7.11 \pm 0.04 \pm 0.84$
$WZ\gamma$ method	$17.1 \pm 4.8 \pm 3.3$	$9.7 \pm 3.5 \pm 2.2$

Table 6.6 : Comparison of the number of WZ +fake events found with two different methods: the jet ratio method used here for the $ZZ\gamma$ analysis and the data-driven method used for the $WZ\gamma$ analysis. The second and third columns give the estimates with a photon p_T cut of 15 GeV and 20 GeV respectively. The first uncertainty is statistical and the second one, systematic.

6.4 Summary

The total predicted numbers of signal and background events for photon p_T greater than 15 and 20 GeV are given below in Tab. 6.7 for SR0 and Tab. 6.8 for SR1. The statistical and systematic uncertainty are shown in the tables. The systematic errors for pile-up and fake photons have been discussed in their respective Sections (6.2 and 6.3). The uncertainty on the misidentification of jets photons as prompt photons is expected to be the leading systematic uncertainty. An error of 100% is assigned to the DPS background. For signal and FSR estimates, two types of systematic

uncertainty are considered: theoretical and experimental. The theoretical error comes from uncertainties on the renormalisation (μ_R) and factorisation (μ_F) scales, the PDF and the strong coupling constant [16]. These uncertainties are estimated by varying the values of the different parameters. The experimental errors come from the trigger, reconstruction, identification and isolation of particles as well as corrections to the energy scale and resolution. These two types of uncertainties could not be evaluated for this thesis in the time available, and were instead taken from the $Z\gamma\gamma$ analysis [16]. In the $Z\gamma\gamma$ analysis, the total theoretical uncertainty and experimental uncertainty were estimated to be approximately 15% and 5.6% respectively. In total, the systematic uncertainty is 16%.

In Tables 6.7 and 6.8, the $t\bar{t}\gamma$ and $WZ\gamma$ backgrounds were neglected in the total background sum. The significance is calculated using the recommended ATLAS formula [52] (cf. Appendix B). The significance computed in SR1, *i.e.* in the region where FSR is considered to be part of the $ZZ\gamma$ signal rather than background, is higher than 5σ and thus, inclusive $llll\gamma$ production in ATLAS Run 2 is expected to be observed. However, the significance in SR0 is lower than 3σ so it is not expected to find evidence of $ZZ\gamma$ triboson production. It can be seen that the significance increases slightly when the photon p_T cut is raised from 15 to 20 GeV.

	$p_T^\gamma > 15 \text{ GeV}$	$p_T^\gamma > 20 \text{ GeV}$
Signal	$3.66 \pm 0.04 \pm 0.59$	$3.15 \pm 0.04 \pm 0.50$
FSR	$1.68 \pm 0.11 \pm 0.27$	$1.10 \pm 0.09 \pm 0.18$
Fake γ	$0.92 \pm 0.01 \pm 0.13$	$0.46 \pm 0.01 \pm 0.06$
Pile-up	$0.20 \pm 0.02 \pm 0.07$	$0.12 \pm 0.01 \pm 0.04$
DPS	0.04 ± 0.04	0.02 ± 0.02
$t\bar{t}\gamma$	$<0.059 \text{ (99\% C.L.)}$	
$WZ\gamma$	$<0.009 \text{ (99\% C.L.)}$	
Total background	2.84 ± 0.33	1.70 ± 0.21
Significance	1.81 ± 0.25	1.93 ± 0.26

Table 6.7 : Expected numbers of signal and background events in SR0. In the first four rows the first uncertainty is statistical and the second one is systematic. In the last two rows and the DPS row, the total error is given. The $t\bar{t}\gamma$ and $WZ\gamma$ backgrounds are neglected in the total background sum.

Machine learning algorithms can outperform cut-based analyses regarding signal-to-background differentiation. Thus, if such processes are applied instead of, or in addition to, the selection cuts used so far, the $ZZ\gamma$ significance could possibly be increased by a significant margin. This is considered in the next Chapter.

	$p_T^\gamma > 15 \text{ GeV}$	$p_T^\gamma > 20 \text{ GeV}$
ISR	$4.57 \pm 0.05 \pm 0.73$	$3.94 \pm 0.04 \pm 0.63$
FSR	$20.48 \pm 0.48 \pm 3.28$	$16.58 \pm 0.46 \pm 2.65$
Total signal	$25.05 \pm 0.48 \pm 3.36$	$20.52 \pm 0.46 \pm 2.72$
Fake γ	$2.79 \pm 0.03 \pm 0.44$	$1.19 \pm 0.02 \pm 0.17$
Pile-up	$0.25 \pm 0.02 \pm 0.08$	$0.15 \pm 0.01 \pm 0.05$
DPS	0.05 ± 0.05	0.03 ± 0.03
Total background	3.09 ± 0.45	1.37 ± 0.18
Significance	7.96 ± 0.77	8.60 ± 0.79

Table 6.8 : Expected number of signal and background events in SR1. In the first four rows the first uncertainty is statistical and the second one is systematic. In the last two rows and the DPS row, the total error is given. The $t\bar{t}\gamma$ and $WZ\gamma$ backgrounds are neglected in the total background sum.

Chapter 7

Applying machine learning to $ZZ\gamma$ production

Nowadays, the use of machine learning techniques in particle physics has become widespread. Powerful toolkits such as 'TMVA' in ROOT software (C++) or 'scikit-learn' and 'PyTorch' in Python, can be extremely efficient in optimizing the signal-to-background ratio in a given sample, *i.e.* discriminate between signal and background and reject the latter while keeping the former as high as possible. Many different multivariate processes exist. Here the Toolkit for Multivariate Analysis (TMVA) from ROOT [53] is used and the performance of different processes is compared.

7.1 Methodology

The full machine-learning analysis methodology consists of the following steps:

- The samples on which the process is applied are selected. One signal and one background sample is required. Two MC simulation samples are used here. The signal sample is the $ZZ\gamma$ SHERPA MC sample described in Chapter 4. For the background, an exhaustive sample is needed to tackle as many background processes as possible. The $llll$ SHERPA sample (cf. Chapter 4) is used. It includes FSR, pile-up and fake photon backgrounds. Even though the latter two are not well modelled, they contribute in the multivariate process training.
- Selection cuts can be applied or not. In general, applying cuts can help the multivariate process and improve its efficiency. However, if the selection is too tight and removes too much background, the process will struggle to perform. Statistics must be monitored carefully.

- The input variables must be carefully chosen. A 'good' variable is one that helps discriminate between signal and background, as in the examples shown in Fig. 7.1. A total of 18 variables are used here:
 - pt_{y1} : transverse momentum of the photon
 - $m_{lly1/2}$: invariant mass of the heaviest/lightest $\ell\ell\gamma$ system
 - $m_{ll1/2}$: invariant mass of the heaviest/lightest lepton pair
 - $l1/2/3/4pt$: transverse momentum of the four leptons (based on their p_T , ℓ_1 being the highest p_T lepton)
 - $dR_{l1/2/3/4_y}$: separation between each lepton and the photon
 - $d\Phi_{z2_y}$: difference between lightest lepton pair and photon ϕ angles (absolute value)
 - $m_{l1/2/3/4y}$: mass of the four lepton-photon pairs

In Appendix E, the importance of each variable is given, and distributions of each variable are shown for signal and background. In general, correlations between variables should be avoided. It can bring in overtraining as discussed below. The correlation matrix between all input variables is shown in Fig. 7.2. Correlations between the masses of the $\ell\gamma$ and $\ell\ell\gamma$ systems are seen which is expected as these systems are similar. The four $m_{\ell\gamma}$ variables are also correlated with each other as are the two $m_{\ell\ell\gamma}$ variables. The photon p_T is correlated with these two sets of variables. The photon being massless, the contribution of the photon to $m_{\ell\gamma}$ and $m_{\ell\ell\gamma}$ comes from its energy/momentum. The background correlation matrix is very similar to the signal matrix. Several variables not listed above were also tested: $\Delta\eta(\ell, \gamma)$, $\Delta\phi(\ell, \gamma)$, $\Delta\phi(\ell\ell_1, \gamma)$, $\ell\ell$ and $\ell\ell\gamma p_T$. They were removed because of their poor contribution. The optimal set of variables found is the one given above.

- The two samples are split into a training set and a testing set. The second set is used to evaluate the performance of the trained process. Usual splittings sit between 50%-50% and 75%-25% for training and testing respectively. Here, a splitting of 67%-33% is chosen.
- The multivariate processes are applied. Many different possible algorithms are available in the TMVA package, including linear Fisher discriminants analysis, Artificial Neural Networks (ANNs) and Boosted Decision Trees (BDTs). After applying the process, the output results are evaluated. They are usually summarized in the Receiver Operating Characteristic (ROC) curve, a background rejection versus signal efficiency plot. A variety of methods are compared in Fig. 7.3. The BDT method is chosen for two main reasons: first, it is more efficient than most methods and second, modifying its hyper-parameters is a very intuitive process, especially compared to neural networks. In the BDT method, at each node, the efficiency of a cut applied on one of the input variables is evaluated. This is

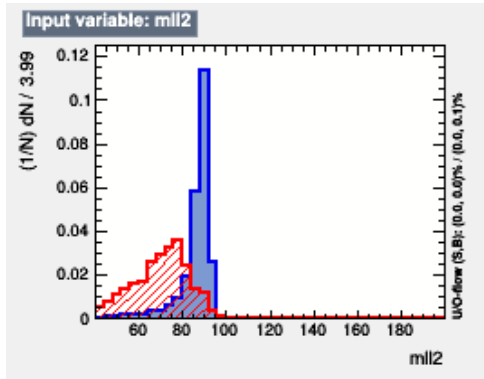
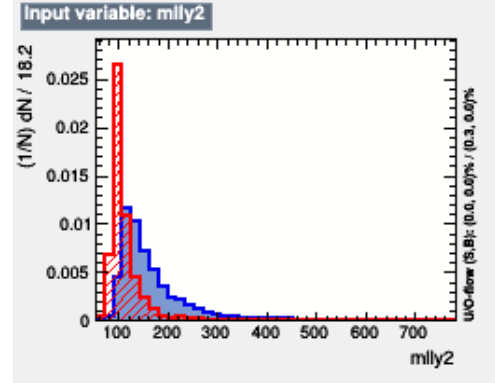
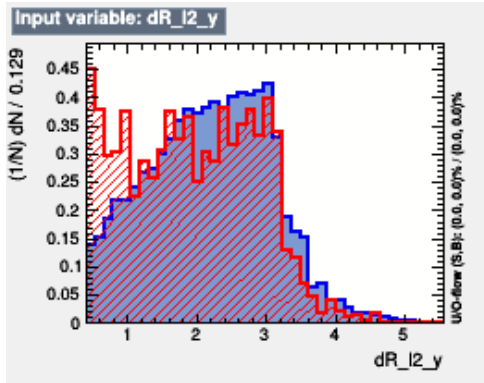
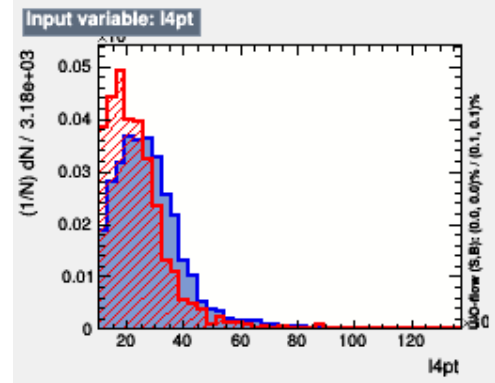
(a) $m_{\ell\ell 2}$ distribution (GeV)(b) $m_{\ell\ell\gamma 2}$ distribution (GeV)(c) $\Delta R(\ell_2; \gamma)$ distribution(d) $\ell_4 p_T$ distribution (GeV)

Fig. 7.1 : Four diagrams showing signal (blue) and background (red) distributions in SR1 (cf. Section 7.2.2 below). (Top left) Mass of the lightest lepton pair; (Top right) Mass of the lightest lepton pair-photon system; (Bottom left) Separation between sub-leading lepton and photon; (Bottom right) Fourth lepton p_T .

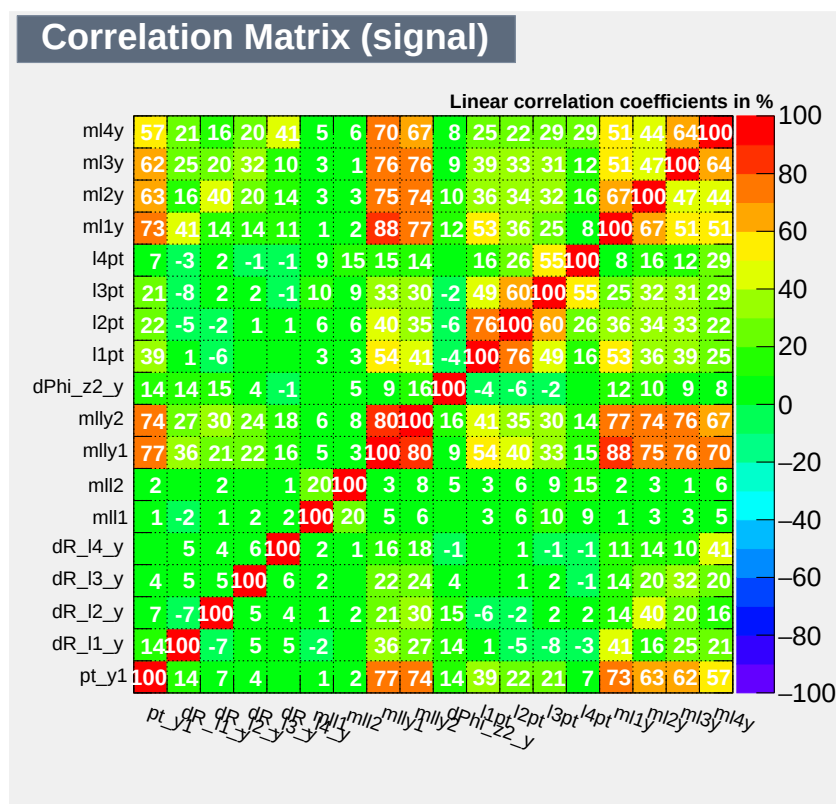


Fig. 7.2 : Correlations between all used variables in the signal sample in SR1 (cf. Section 7.2.2)

repeated over a certain number of layers and trees. An illustration of a BDT is shown in Fig. 7.4. BDTs have the advantage that their performance is not impacted by correlations between variables. This is why, even though strong correlations exist between some of the input variables, as shown in Fig. 7.2, these variables are kept.

- Finally, overtraining must be evaluated. It occurs when the number of data points is too small compared to the number of model parameters of an algorithm. For instance, BDTs usually suffer from at least partial overtraining due to their large number of nodes. Correlated variables, wrong choices of hyper-parameter values or small data sets can cause overtraining. This leads to a decrease in the overall performance of the method. Overtraining can be evaluated by comparing the training and test output distributions. In Fig. 7.6, three overtraining checks corresponding to the three cases described in the next section are shown. In the first two examples (Fig. 7.6a and 7.6b), the number of available events is sufficient to eliminate most overtraining, as can be seen qualitatively from the good agreement between training and test sample distributions. However, in the third example (Fig. 7.6c), there is clear evidence of overtraining as the distributions are in poor agreement. There are various metrics to obtain a quantitative evaluation of overtraining such as the Kolmogorov-Smirnov test. The KS test is used to compare two samples. If the two distributions come from one parent sample, the test outputs a uniform distribution from 0 to 1 and consequently an average value of 0.5. Thus, the closer the KS test output

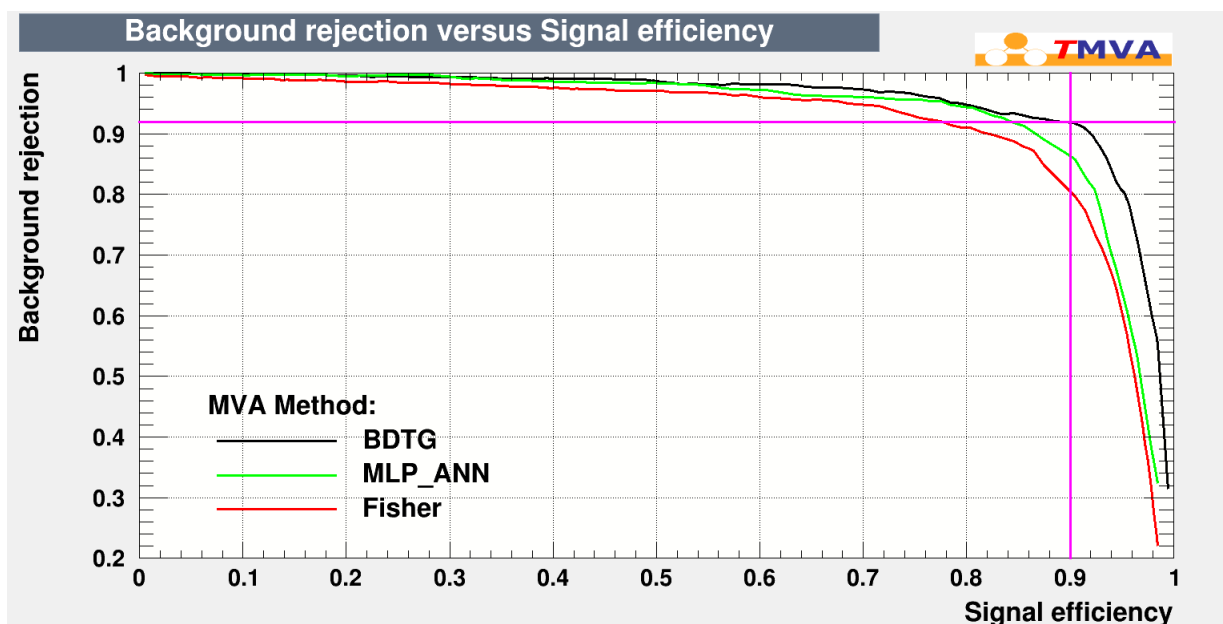


Fig. 7.3 : Receiver Operating Curve: background rejection versus signal efficiency obtained by different machine learning algorithms in SR1 (Case 2, cf. Section 7.2.2). The black curve (BDTG), the green curve (MLP_ANN) and the red curve (Fisher) are respectively the ROC curves for the BDT, ANN and Fisher algorithms. The vertical and horizontal pink lines intersect at the highest significance point (90%; 92%) as calculated by TMVA.

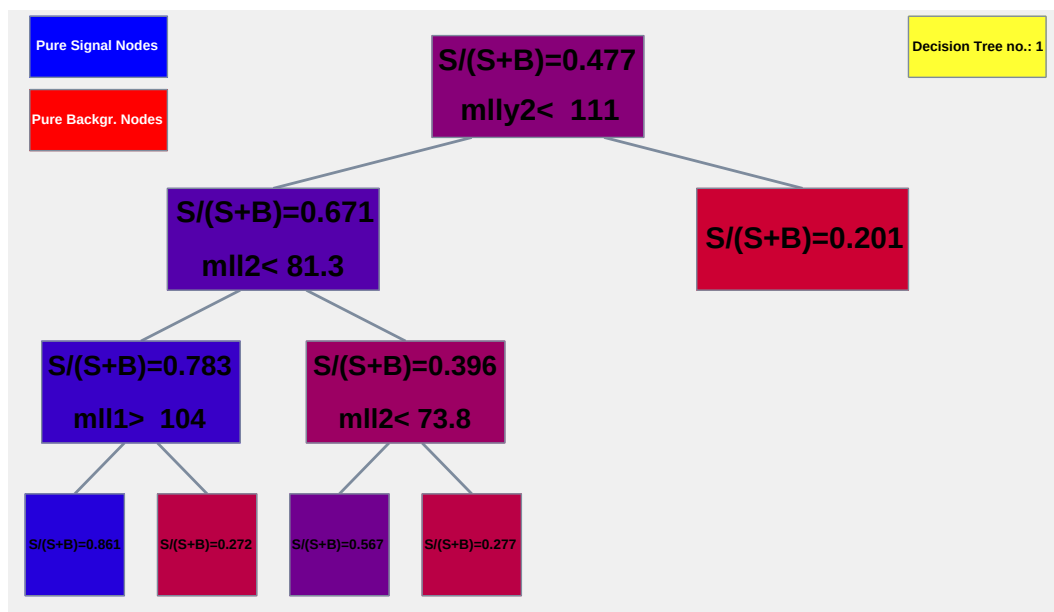


Fig. 7.4 : Illustration of a BDT. This tree is taken from Case 2 (cf. Section 7.2.2 below) where each tree has a maximum of five layers and for a node to be constructed it is required that at least 4% of the available events are present (approximately 900 events here).

number is to 0.5, the most similar the two sample distributions are. In ROOT TMVA though, the KS test is difficult to interpret as small deviations impact the result which also heavily depends on the number of bins used for distributions. In Fig. 7.6, the KS test values are equal or very close to 0. Most often, overtraining is estimated qualitatively by simply comparing by eye the training and test distributions.

7.2 Boosted Decisions Trees and $ZZ\gamma$ production

Three separate BDT processes are considered, corresponding to each of the three signal regions SR2, SR1 and SR0 described in Chapter 5. All three methods are compared to the results obtained with cuts alone for SR0 and 139fb^{-1} , *i.e.* without using any machine learning process. For the cuts-based analysis, the number of signal and background events predicted by MC in Run 2 are $S = 3.66$ and $B = 2.23$ ¹. In the TMVA package in ROOT, the significance is calculated as $S/\sqrt{S+B}$ and this formula will be used in this section. It is thus equal to 1.51 when applying cuts without using BDTs. The signal, background and significance predicted for each case are summarised in Tab. 7.1.

7.2.1 Case 1: SR2

When only the SR2 cuts are applied, the signal and background MC samples contain respectively 23,016 and 39,042 events. After reweighting, this corresponds to $S = 8.42$ and $B = 58.2$ events. As mentioned previously, two-thirds of each sample are used for training and one third for testing. After optimisation of the BDTs hyper-parameters, 95 trees with a maximum depth of 5 layers are used. At each node, at least 3% of the available training events are required. The parameter $nCut$ is set at 50. This parameter specifies the number of points in variable range used to find the optimal cut at each node and quantifies the precision of each cut.

The maximum significance achieved, as shown in Fig. 7.5, is equal to 1.41. It is obtained for a signal efficiency of 89.2% and a background efficiency of 35.7%. When a cut on the BDT output response at the point corresponding to maximum significance is applied and only events above this cut value (-0.0778 in this case, as given in Fig. 7.5) are kept, 7.51 signal events and 20.8 background events are obtained.

Overtraining is evaluated from Fig. 7.6a. As the training and test samples seem to correspond within uncertainties for most points for both signal and background, the samples are not overtrained.

¹This number does not correspond to the sum of the background contributions detailed in Chapters 5 and 6. It is taken directly from the background MC sample used for the BDT studies.

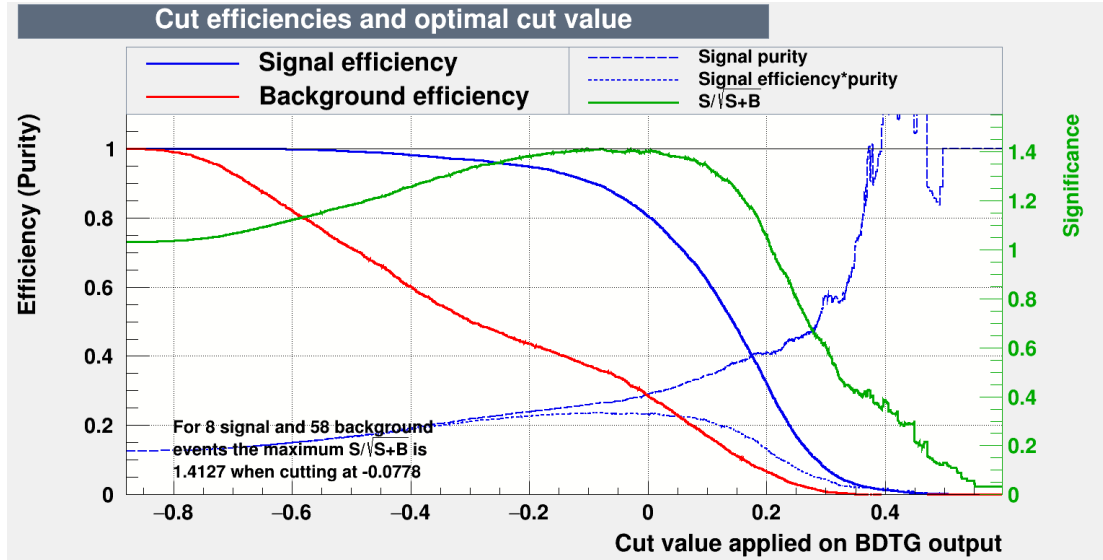


Fig. 7.5 : Plot of the significance (green) versus the BDTs output cut for Case 1. The plots for Cases 2 and 3 look similar. The corresponding signal (blue) and background (red) efficiencies are also plotted. The dashed blue curve corresponds to the signal purity S/B and the dotted blue line to the signal efficiency multiplied by the signal purity.

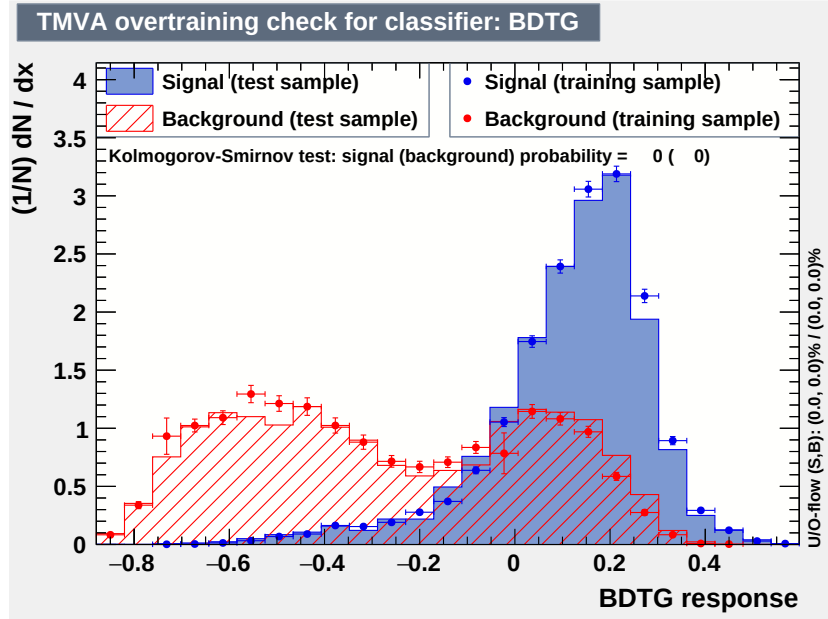
7.2.2 Case 2: SR1

In this case, all signal selection cuts are applied except the two anti-FSR cuts. Compared to the previous case, the lepton and photon identification, isolation and p_T cuts are added. This corresponds to region SR1. The signal sample contains 12,394 events and the background sample contains 10,399 events which correspond to $S = 4.58$ and $B = 21.2$. Because of the lower statistics, 90 trees with a maximum depth of 5 layers are used. The percentage of training events required at each node is increased to 4%. The parameter $nCut$ remains equal to 50. The maximum significance achieved is equal to 1.71, at 90.0% signal efficiency and 8.0% background efficiency. This corresponds to 4.12 signal events and 1.70 background events.

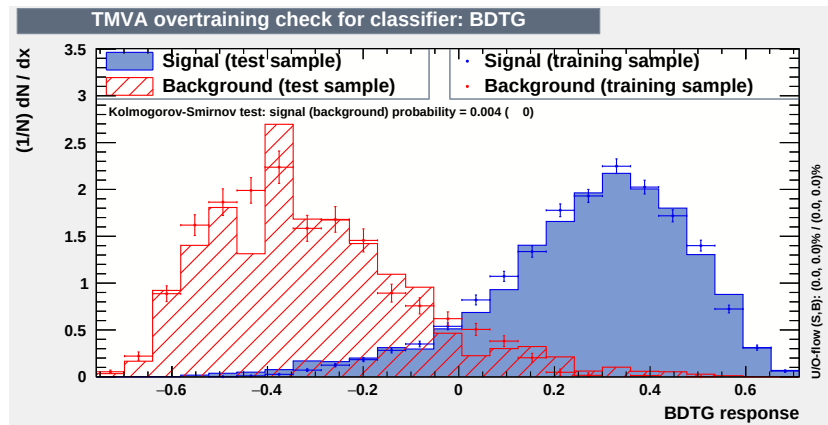
Overtraining is evaluated from Fig. 7.6b. Even though deviations can be seen for some of the points, the training and test distributions are in generally good agreement.

7.2.3 Case 3: SR0

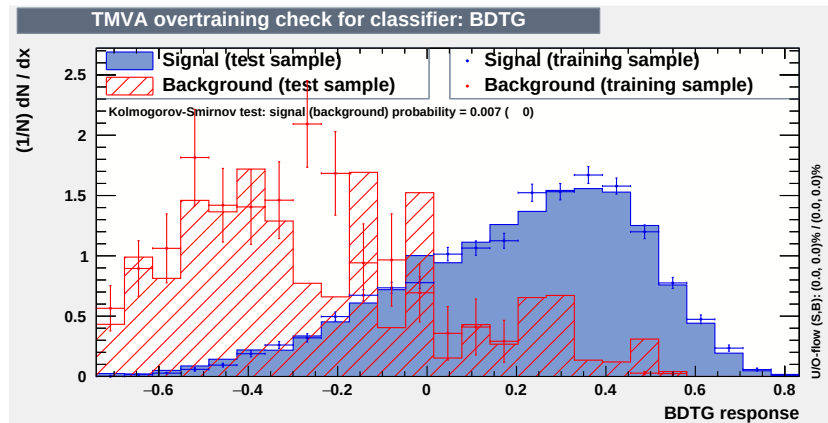
In the final case, all cuts are applied, *i.e.* the two anti-FSR cuts are also added. The considered region in this case is thus SR0. The statistics are here very low compared to the first two cases. There are 9,885 signal events and 1,030 background events. After reweighting, these numbers correspond to those mentioned in the introduction of this section: $S = 3.66$ and $B = 2.23$. The lack of statistics does not allow for the parameters to keep similar values as in the first two cases. 40 trees with a maximum depth of 3 layers are used. At each node 10% of events are required. The overall performance decreases and the overtraining is high as can be seen in Fig. 7.6c. The



(a) Case 1 (SR2) overtraining check



(b) Case 2 (SR1) overtraining check



(c) Case 3 (SR0) overtraining check

Fig. 7.6 : The TMVA BDT overtraining check distributions are plotted for signal (blue) and background (red) samples for the three cases described here. The points correspond to the training samples and the dashed regions to the test samples. The errors on the points are statistical.

deviations between background training and test distributions are larger than in the first two cases. The statistical errors are also larger. At 94.0% signal efficiency, a background efficiency of 45.0% is obtained. This results in $S = 3.44$ signal events and $B = 1.00$ background events. Therefore, the significance is equal to 1.63. However, because of the large overtraining these results cannot be considered reliable. This study should eventually be repeated once larger MC samples become available.

7.3 Summary

	Cut-based	Case 1 (SR2)	Case 2 (SR1)	Case 3 (SR0)
S	3.66 ± 0.04	7.51 ± 0.05	4.12 ± 0.04	3.44 ± 0.04
B	2.23 ± 0.13	20.8 ± 0.29	1.70 ± 0.04	1.00 ± 0.06
S/B	1.64 ± 0.10	0.36 ± 0.01	2.42 ± 0.06	3.44 ± 0.21
$S/(S+B)$	0.62 ± 0.01	0.27 ± 0.01	0.71 ± 0.01	0.77 ± 0.01
$S/\sqrt{S+B}$	1.51 ± 0.02	1.41 ± 0.01	1.71 ± 0.01	1.63 ± 0.02
Significance ATLAS	2.02 ± 0.05	1.56 ± 0.01	2.46 ± 0.03	2.51 ± 0.05

Table 7.1 : Comparison of the results obtained for each BDT case with the results from the cut-based signal selection. S and B are the number of signal and background events predicted in each case. In the last row the significance using the recommended ATLAS formula is shown. The largest (or smallest for B) number in each row is in bold, excluding Case 3 as it is not considered reliable due to overtraining. The errors are statistical uncertainties.

The multivariate analysis described in this chapter shows that an improvement in the signal-background separation can be gained with machine learning algorithms. The performance of each BDT process is evaluated by considering the maximum value obtained for the significance. The results obtained for Case 3 are unreliable due to the low MC statistics available and consequent overtraining. The results in Case 3 nevertheless hint that the $ZZ\gamma$ signal significance might be increased even further by applying a BDT selection after all the SR0 selection cuts have been applied. In Case 1, BDTs struggle to separate signal from background because identification and isolation cuts have not been applied. Particles which are not well identified or isolated (and which are eventually classified as background) cannot be distinguished from signal event particles using the p_T and mass distributions alone. Moreover, in SR2, the number of background events is much higher than in the other two signal regions and therefore even after applying BDTs, this number remains high compared to the other two cases. The best performance is seen in Case 2. This demonstrates that it is preferable to replace the two anti-FSR cuts by a BDT selection.

The results obtained are summarised in Tab. 7.1. In the last row of Tab. 7.1, the significance is calculated using the ATLAS recommended formula where the error comes from the signal uncertainty (cf. Appendix B). Using the ATLAS formula, the highest achieved significance

is equal to $2.46(\pm 0.03)\sigma$ in Case 2. This demonstrates that, with further optimisation of the analysis, it might eventually be possible to obtain evidence at 3σ for $ZZ\gamma$ triboson production using the Run 2 data. Qualitatively the same conclusions are reached using the simpler expression $S/\sqrt{S+B}$ for the significance in place of the recommended ATLAS expression.

There are various motivations for repeating this study, the first one being the lack of statistics. The more training and testing events are inputted in TMVA, the more efficient BDTs will be. Here, only a few thousands of event were used at best. Ideally, the study should be repeated with bigger but similar HIGG1D2 samples. However, if not possible, different types of samples could be used, such as TRUTH1 samples where the number of events is large in general.

The analysis should be repeated with other machine learning algorithms. BDTs were mainly used here. The Fisher method and ANNs did not show promising results. However, it was noticed that in some cases the likelihood estimator could outperform BDTs. Furthermore, Deep Neural Networks (DNNs, different from ANNs) were not tested. An even higher significance than in Case 2 could be obtained by optimising one of these methods.

Finally, a MC sample which combines ISR and FSR $ZZ\gamma$ events is under preparation. This inclusive $llll\gamma$ sample will eventually increase the MC statistics available, especially for the FSR contribution, and give an improved evaluation of the power of multivariate analyses to discriminate between the $ZZ\gamma$ signal and the various backgrounds.

Chapter 8

Summary and outlook

An exploratory study of $ZZ(\rightarrow \ell\ell\ell)\gamma$ production in the ATLAS Run 2 dataset is described in this thesis. Events containing a pair of Z bosons together with a photon are studied for the first time. This is a challenging final state to observe because of the small expected cross-section compared to other, similar electroweak processes. At this stage, the data must remain blinded until the analysis methods to be used are finalised and agreed within the ATLAS collaboration. Accordingly, the studies in this thesis were largely carried out using MC simulation samples.

The contributions to $ZZ\gamma$ production from initial state radiation (ISR) and final state radiation (FSR) must be distinguished when studying $ZZ\gamma$ production. The FSR process corresponds essentially to diboson (ZZ) production rather than triboson ($ZZ\gamma$) production. In this sense, the FSR contribution to the $\ell\ell\ell\gamma$ final state can be considered as a background to the ISR (triboson) contribution which is of primary interest. However, in previous analyses of electroweak final states containing a photon, FSR has not always been considered as background as doing so reduces the overall number of events available for a study. In this thesis, selection cuts were introduced to separate the ISR and FSR contributions. In the $ZZ\gamma$ case, when FSR events are considered as signal, the predicted significance increases from 1.81σ to 7.96σ . Thus, obtaining evidence for triboson $ZZ\gamma$ production via the ISR process will be challenging, but an unambiguous observation of inclusive $\ell\ell\ell\gamma$ should be possible.

The use of machine learning algorithms enhances the signal-background differentiation compared to a cut-based definition of the signal region. Boosted Decision Trees (BDTs) were used for this study as they combine good performance and intuitive implementation. Different BDT processes were studied and it was shown that when the FSR cuts are replaced by a BDT multivariate process, the expected significance can be as high as 2.46σ . This result is encouraging as it shows that a clever combination of selection cuts and machine learning could reject enough background (and at the same time keep enough signal) to obtain evidence for triboson $ZZ\gamma$ production. The machine learning part of the study should be repeated with various optimised algorithms and higher statistics in order to obtain less overtrained - and thus more accurate - output results.

Some of the potential backgrounds must be estimated with data-driven methods, such as the pile-up photon and fake photon backgrounds. In this thesis, Monte Carlo samples were used to estimate both of these backgrounds, and thus their estimation should be repeated using suitable data samples, thereby hopefully reducing the systematic uncertainties. The low number of events is problematic when estimating the fake photon background. Usual methods suffer from the lack of statistics and in consequence a novel technique was proposed. By considering the probability for a gluon/quark to fragment and eventually produce a fake photon and by using the relatively precise fake photon background estimation which is possible for the $Z\gamma$ final state, it is possible to compute an expected fake photon contribution in the $ZZ\gamma$ case. The systematic uncertainty on the jet ratio in this method must be reevaluated as a WZ +jets sample had to be used for that purpose instead of a Z +jets sample. To reduce systematic uncertainties further, the estimation of the jet ratio in the $ZZ\gamma$ fake photon method should be taken directly from data rather than from MC. The $WZ\gamma$ production analysis group has decided to use a similar strategy to that described in this thesis for estimating the fake photon background as the expected number of $WZ\gamma$ events is also too low to apply the usual ABCD method.

Once the analysis methods are finalised, the Run 2 dataset will be unblinded. This will result either in the first evidence for $ZZ\gamma$ triboson production or the first experimental upper limit on the $ZZ\gamma$ production cross-section. Even in the latter case, it is still possible that the $ZZ\gamma$ channel could help to improve existing constraints on the possible presence of anomalous gauge couplings beyond the SM, though this remains to be studied.

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Appendix A

Isolation working points

The following measured variables are used for the isolation working points [30, 31]:

- E_T^{cone20} : sum of transverse energy of clusters in calorimeter in a cone with size $\Delta R = 0.2$
- $E_{\text{neflow}}^{\text{cone20}}$: the neutral particle flow energy in the given cone. Neutral means clusters that were not associated to tracks.
- p_T^{cone20} : the p_T sum of tracks in a cone with $\Delta R = 0.2$ around the object of interest.
- $p_T^{\text{varcone30}}$: the p_T sum of tracks in a cone with a variable cone size $\Delta R < 0.3$ around the interested object. At high energies different objects can be very close to each other so the cone size gets smaller for larger transverse momentum:

$$\Delta R = \min\left(\frac{10\text{GeV}}{p_T}; 0.3\right).$$

The working points definition is given in Tab. A.1 below.

	Calo isolation	Track isolation
Loose_VarRad (e)	$E_T^{\text{cone20}}/p_T(e) < 0.2$	$p_T^{\text{varcone30}}/p_T(e) < 0.15$
FixedCutLoose (γ)	$E_T^{\text{cone20}}/p_T(\gamma) < 0.065$	$p_T^{\text{cone20}}/p_T(\gamma) < 0.05$
PflowLoose_FixedRad (μ)	$(p_T^{\text{varcone30}} + 0.4E_{\text{neflow}}^{\text{cone20}})/p_T(\mu) < 0.16$ for $p_T(\mu) < 50\text{GeV}$ $(p_T^{\text{cone20}} + 0.4E_{\text{neflow}}^{\text{cone20}})/p_T(\mu) < 0.16$ for $p_T(\mu) > 50\text{GeV}$	

Table A.1 : Definitions of the working points used for the $ZZ\gamma$ signal region

Appendix B

ATLAS recommended significance formula

The formula described here comes from Ref. [52].

In the following, s represents the mean expected number of signal events, b the mean expected number of background events with an uncertainty σ and the mean total event count is assumed to be $\langle n \rangle = s + b$. Hence, for the background-only hypothesis: $\langle n \rangle = b$. The recommended formula to estimate the significance Z for observing n events is:

$$Z = \begin{cases} +\sqrt{2\left(n\ln\left[\frac{n(b+\sigma^2)}{b^2+n\sigma^2}\right] - \frac{b^2}{\sigma^2}\ln\left[1 + \frac{\sigma^2(n-b)}{b(b+\sigma^2)}\right]\right)} & \text{if } n \geq b \\ -\sqrt{2\left(n\ln\left[\frac{n(b+\sigma^2)}{b^2+n\sigma^2}\right] - \frac{b^2}{\sigma^2}\ln\left[1 + \frac{\sigma^2(n-b)}{b(b+\sigma^2)}\right]\right)} & \text{if } n < b. \end{cases} \quad (\text{B.1})$$

In this thesis, Z is calculated by taking $n = s + b$, and therefore only the $n \geq b$ version is needed.

Appendix C

Pile-up background estimation

Pile-up photon background event rate

Consider two independent processes with production cross-sections σ_A and σ_B . The total number of events of type A or B produced for integrated luminosity $\mathcal{L}$ is:

$$N_A = \mathcal{L} \sigma_A, \quad N_B = \mathcal{L} \sigma_B. \quad (\text{C.1})$$

Similarly, the total number of inelastic pp interactions (with cross-section σ_{inel}) produced is:

$$N_{\text{inel}} = \mathcal{L} \sigma_{\text{inel}}. \quad (\text{C.2})$$

This number can also be expressed as a function of $\langle \mu \rangle$, the average number of inelastic collisions per bunch crossing, and N_{bc} , the total number of bunch-crossings as:

$$N_{\text{inel}} = \langle \mu \rangle N_{\text{bc}}. \quad (\text{C.3})$$

It is assumed that $\sigma_A \ll \sigma_{\text{inel}}$ and $\sigma_B \ll \sigma_{\text{inel}}$. The probability f_A and f_B that a given bunch-crossing contains an event of type A or B is:

$$f_A = \frac{N_A}{N_{\text{bc}}}, \quad f_B = \frac{N_B}{N_{\text{bc}}}.$$

The probability that a given bunch-crossing contains an event of type A and, simultaneously, an event of type B is therefore:

$$f_{AB} = f_A f_B = \frac{N_A N_B}{N_{\text{bc}}^2}.$$

Thus, the number of bunch-crossings containing simultaneously an event due to process A and an event due to process B is:

$$N_{AB} = f_{AB} N_{\text{bc}} = \frac{N_A N_B}{N_{\text{bc}}}.$$

Using the expressions of N_A and N_B from Equation C.1 and the two expressions of N_{inel} from Equations C.2 and C.3, N_{AB} becomes:

$$N_{AB} = \frac{N_A N_B}{N_{\text{bc}}} = \mathcal{L} \sigma_A \cdot \mathcal{L} \sigma_B \cdot \frac{\langle \mu \rangle}{\mathcal{L} \sigma_{\text{inel}}} = \langle \mu \rangle \mathcal{L} \frac{\sigma_A \sigma_B}{\sigma_{\text{inel}}}.$$

Thus, the number of pile-up events $N_{\text{PU}} \equiv N_{AB}$, *i.e.* the number of events where two processes happen simultaneously in the same-bunch-crossing but in two different pp interactions, is:

$$N_{\text{PU}} = \langle \mu \rangle \mathcal{L} \frac{\sigma_A \sigma_B}{\sigma_{\text{inel}}}. \quad (\text{C.4})$$

In the pile-up photon background for $ZZ\gamma$ production, the two processes A and B are the production of a single photon and the production of a pair of Z bosons.

Inclusive photon cross-section

The ATLAS paper Ref. [45] gives the measured fiducial cross-section for inclusive isolated photon production as:

$$\sigma_{\text{fid}}^{\gamma}(p_{\text{T}}^{\gamma} > 125 \text{ GeV}) = 399 \pm 13(\text{exp.}) \pm 8(\text{lumi.}) \text{ pb},$$

where the fiducial region is defined by the photon transverse energy $p_{\text{T}}^{\gamma} > 125 \text{ GeV}$, pseudorapidity $|\eta^{\gamma}| < 2.37$ excluding $1.37 < |\eta^{\gamma}| < 1.56$ and isolation $E_{\text{T}}^{\text{iso}} < 4.8 + (4.2 \times 10^{-3}) p_{\text{T}}^{\gamma} [\text{GeV}]$, $E_{\text{T}}^{\text{iso}}$ being the amount of transverse energy inside a cone of size $\Delta R = 0.4$ in the $(\eta; \phi)$ plane around the photon candidate, excluding an area of size $\Delta\eta \times \Delta\phi = 0.125 \times 0.175$ centred on the photon.

For simplification, the photon production cross-section is taken as:

$$\sigma^{\gamma}(p_{\text{T}}^{\gamma} > 125 \text{ GeV}) = 400 \text{ pb}.$$

Assuming that the inclusive photon cross-section depends on the minimum photon transverse energy as $\sigma^{\gamma}(p_{\text{T}}^{\gamma} > x) \propto x^{-3}$, it can be extrapolated to lower $p_{\text{T}}^{\gamma 1}$:

$$\sigma(p_{\text{T}}^{\gamma} > 15 \text{ GeV}) = (400 \text{ pb}) \times \left(\frac{125}{15} \right)^3 = 231 \text{ nb}. \quad (\text{C.5})$$

$$\sigma(p_{\text{T}}^{\gamma} > 20 \text{ GeV}) = (400 \text{ pb}) \times \left(\frac{125}{20} \right)^3 = 98 \text{ nb}. \quad (\text{C.6})$$

¹By considering the predicted photon production cross-section in different p_{T}^{γ} regions, one can observe an approximate dependence on x^{-3} [47].

ZZ cross-section

In the ATLAS $ZZ \rightarrow 4\ell$ paper Ref. [12], Table V gives the measured cross-section:

$$\sigma(ZZ \rightarrow \ell\ell\ell\ell) = 46.2^{+2.5}_{-2.3} [\pm 1.5(\text{stat})^{+1.2}_{-1.1}(\text{syst})^{+1.6}_{-1.4}(\text{lumi})] \text{ fb} . \quad (\text{C.7})$$

This cross-section was measured for the three Z decay channels $4e$, 4μ and $2e2\mu$, for a fiducial region dominated by on-shell ZZ production and defined in Table I. The fiducial region includes the requirements on lepton transverse momentum $p_T^\ell > 20, 15, 10, 5$ GeV, mass of SFOS pairs $m_{\text{SFOS}} > 5\text{GeV}$, mass of lepton pairs $66 < m_{\ell\ell} < 116\text{GeV}$ and lepton separation $\Delta R(\ell, \ell) > 0.2(0.1)$ for different flavour (same flavour) leptons. It uses dressed leptons with pseudorapidity $|\eta^\ell| < 2.7$.

Estimate for $ZZ\gamma$ pile-up background

Equation C.4 is used with the cross-section values given in Equations C.5 and C.6. In ATLAS, the average pile-up is equal to $\langle\mu\rangle = 33.7$. An assumed efficiency factor $C = 0.3^2$ to go from particle level to detector level is included in addition. Therefore, an estimate for $(ZZ) + (\gamma)$ pile-up background summed over the $4e$, 4μ and $2e2\mu$ channels for $p_T^\gamma > 15$ GeV and $p_T^\gamma > 20$ GeV respectively is given by:

$$N_{\text{PU}}^{ZZ\gamma}(p_T^\gamma > 15 \text{ GeV}) = 33.7 \times (139\text{fb}^{-1}) \times 0.3 \times \frac{(46.2\text{fb}) \times (231\text{nb})}{80\text{mb}} = 0.19 ,$$

$$N_{\text{PU}}^{ZZ\gamma}(p_T^\gamma > 20 \text{ GeV}) = 33.7 \times (139\text{fb}^{-1}) \times 0.3 \times \frac{(46.2\text{fb}) \times (98\text{nb})}{80\text{mb}} = 0.08 .$$

Estimate for $ZZ\gamma$ double-parton scattering background

Another similar background to pile-up is double-parton scattering (DPS) [48] where all three γ and Z bosons are produced in the same pp collision in two different parton-parton interactions. In this specific case, two channels must be considered, one where the two Z bosons are produced in a different parton-parton interaction than the photon and one where a Z and a photon are produced separately from the second Z . The average pile-up $\langle\mu\rangle$ is not taken into account in Equation C.4 as only one pp collision is considered. The inelastic pp interaction cross-section σ_{inel} is replaced by an effective cross-section σ_{DPS} determined empirically to be approximately 15 mb [48]. For double-parton scattering $(ZZ) + (\gamma)$ and for sum of $4e$, 4μ , $2e2\mu$ channels:

$$N_{\text{DPS}}^{ZZ\gamma}(p_T^\gamma > 15 \text{ GeV}) = (139\text{fb}^{-1}) \times 0.3 \times \frac{(46.2\text{fb}) \times (231\text{nb})}{15\text{mb}} = 0.030 ,$$

$$N_{\text{DPS}}^{ZZ\gamma}(p_T^\gamma > 20 \text{ GeV}) = (139\text{fb}^{-1}) \times 0.3 \times \frac{(46.2\text{fb}) \times (98\text{nb})}{15\text{mb}} = 0.013 .$$

²This is a typical value for an efficiency factor for this kind of ATLAS analysis.

The Z boson production cross-section is $\sigma_Z = 736.2$ pb [54] and the $Z\gamma$ production cross-section for $p_T^\gamma > 15$ GeV is $\sigma_{Z\gamma} = 1507$ fb [6] (assuming a similar p_T^γ dependence of the $\sigma_{Z\gamma}$ cross-section as in Equations C.5 and C.6, the cross-section is extrapolated to be 639 fb for $p_T^\gamma > 20$ GeV). Thus, for double-parton scattering ($Z\gamma$) + (Z) for each decay channel:

$$N_{\text{DPS}}^{ZZ\gamma}(p_T^\gamma > 15 \text{ GeV}) = (139\text{fb}^{-1}) \times 0.3 \times \frac{(1507\text{fb}) \times (736.2\text{pb})}{15\text{mb}} = 0.003 ,$$

$$N_{\text{DPS}}^{ZZ\gamma}(p_T^\gamma > 20 \text{ GeV}) = (139\text{fb}^{-1}) \times 0.3 \times \frac{(639\text{fb}) \times (736.2\text{pb})}{15\text{mb}} = 0.001 .$$

Multiplying by a factor four to account for the $4e$, 4μ , $2e2\mu$ channels gives 0.012 events for $p_T^\gamma > 15$ GeV and 0.004 events for $p_T^\gamma > 20$ GeV. Summing the two contributions gives an overall estimate of DPS background of ~ 0.04 events for $p_T^\gamma > 15$ GeV and ~ 0.02 events for $p_T^\gamma > 20$ GeV.

Appendix D

Comparison between $Z\gamma$ cross-check signal and background numbers and numbers found during the $Z\gamma$ analysis

In order to cross-check the number of $Z\gamma$ fake photons used in Equation 6.7, the numbers of $Z\gamma$ observed and background events are recalculated in the same region used as signal region during the Run 2 ATLAS $Z\gamma$ analysis [11]. The numbers found are compared to those obtained during the $ZZ\gamma$ analysis. The results are summarised in Tab. D.1 below.

The numbers on the first line are equal as the same data samples made in 2018 are used. However, the identification and isolation working points have been updated which explains the differences between numbers of background events obtained with recent MC simulations and numbers obtained during the $Z\gamma$ analysis. All the numbers agree within uncertainty.

	recalculated $Z\gamma$ numbers	$Z\gamma$ analysis numbers
N_{obs}	95,756	95,756
$N_{t\bar{t}\gamma}$	$3,666 \pm 16 \pm 550$	$3,635 \pm 14 \pm 387$
N_{WZ}	$493 \pm 6 \pm 145$	$453 \pm 4 \pm 97$
$N_{WW\gamma}$	$221 \pm 3 \pm 66$	$204 \pm 3 \pm 44$
N_{ZZ}	$205 \pm 3 \pm 62$	$166 \pm 1 \pm 36$

Table D.1 : Comparison between recalculated observed number of events and estimated number of background events and numbers obtained during the ATLAS Run 2 $Z\gamma$ analysis. The first error is the statistical uncertainty and the second error is the systematic uncertainty.

Appendix E

List of input variables used in TMVA processes

The exhaustive list of all 18 TMVA input variables is given in Tab. E.1. They are listed according to their BDT ranking in Case 2 as established by TMVA. The variable importance computed by ROOT is given in the last column.

Variable	Description	Importance
mll2	invariant mass of the lightest lepton pair	0.112
mll1	invariant mass of the heaviest lepton pair	0.105
mll γ 2	invariant mass of the lightest $\ell\ell\gamma$ system	0.088
pt γ 1	transverse momentum of the photon	0.060
dR γ 12 γ	separation between second lepton and photon	0.056
dR γ 13 γ	separation between third lepton and photon	0.056
dR γ 14 γ	separation between fourth lepton and photon	0.055
dR γ 11 γ	separation between first lepton and photon	0.052
dPhi γ 12 γ	difference between lightest lepton pair and photon ϕ angles	0.051
m1 γ 1	mass of the first lepton-photon pair	0.047
l1pt	transverse momentum of the first lepton	0.043
l2pt	transverse momentum of the second lepton	0.041
mll γ 1	invariant mass of the heaviest $\ell\ell\gamma$ system	0.040
m4 γ 1	mass of the fourth lepton-photon pair	0.040
l4pt	transverse momentum of the fourth lepton	0.040
m2 γ 1	mass of the second lepton-photon pair	0.039
m3 γ 1	mass of the third lepton-photon pair	0.039
l3pt	transverse momentum of the third lepton	0.037

Table E.1 : Description of all input variables used for the BDT analysis in Chapter 7. The variables are ranked according to the TMVA calculated importance.

Below are shown all the input variables signal and background distributions in Case 2, *i.e.* after the SR1 cuts have been applied. The name of each variable is written at the top of each plot and their definitions can be seen in the list above.

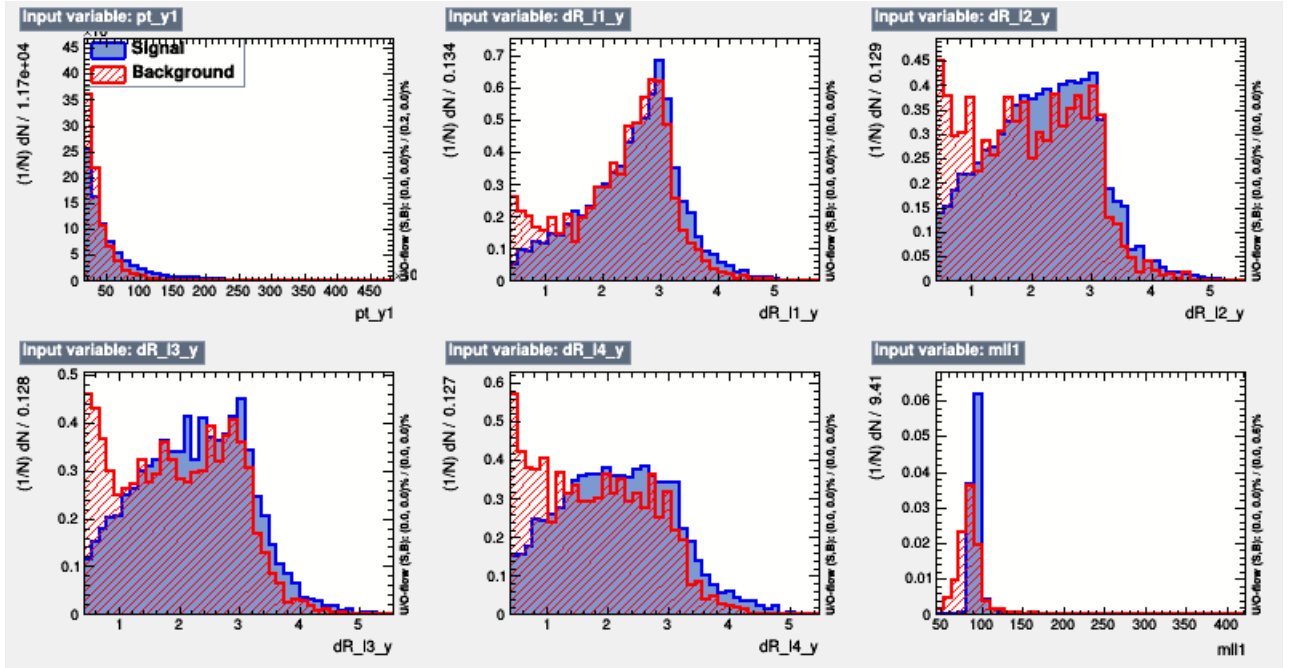


Fig. E.1 : Input variables signal (blue) and background (red) distributions.

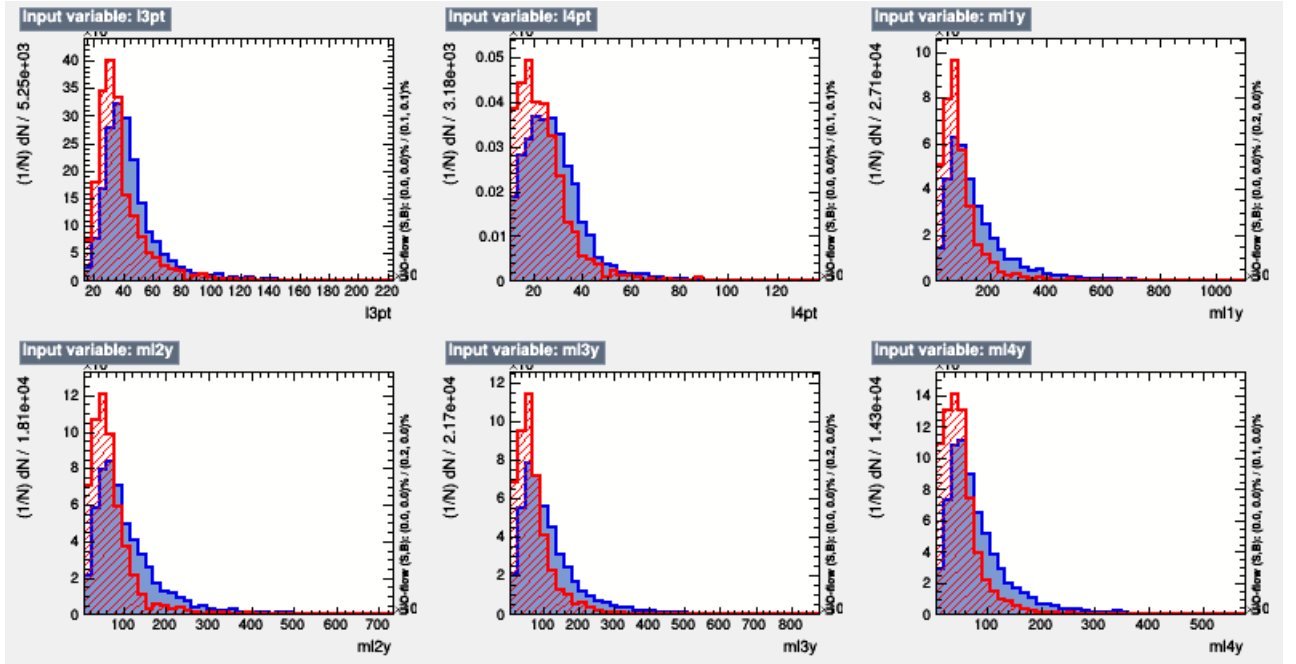


Fig. E.2 : Input variables signal (blue) and background (red) distributions.

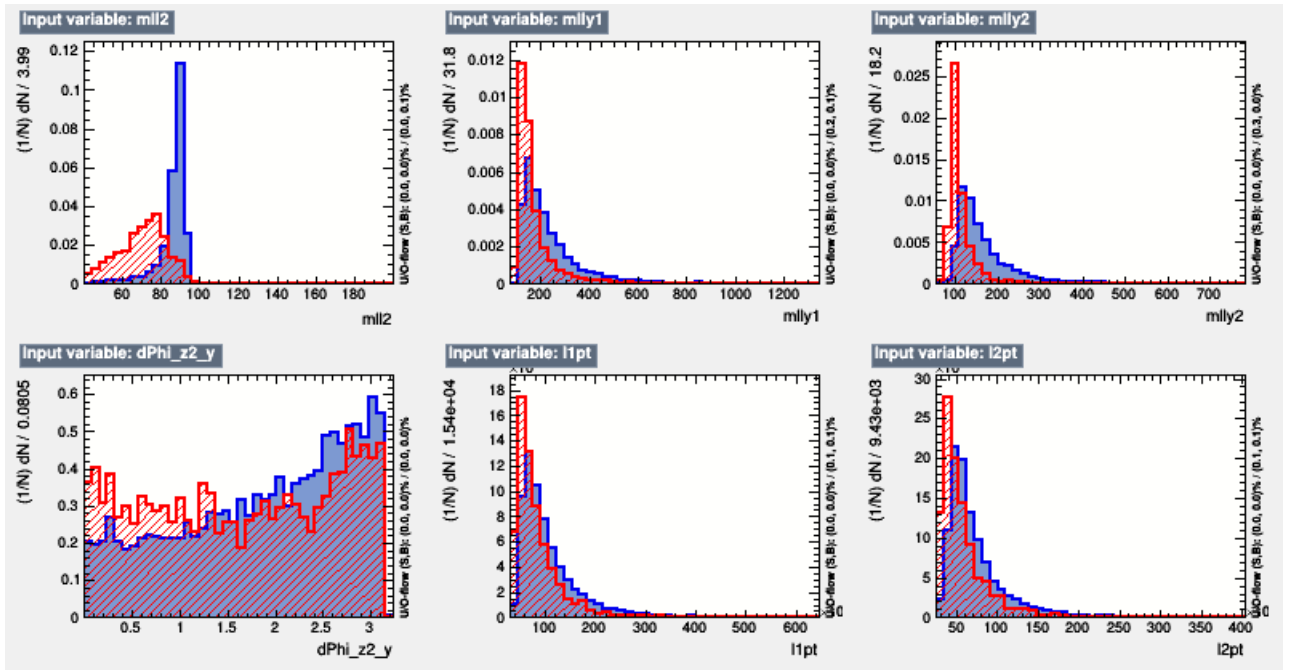


Fig. E.3 : Input variables signal (blue) and background (red) distributions.

