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Ph.D. THESIS

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**Investigation of the source size and strong interaction with
the femtoscopic correlations of baryons and antibaryons in
heavy-ion collisions registered by ALICE**

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Abstract

The strong interaction is one of the four fundamental forces of nature. It binds together quarks inside protons and neutrons (which are example of baryons - particles composed of three quarks) and assures the stability of the atomic nucleus. Parameters describing the strong potential are also crucial for the neutron stars models used in astrophysics. What is more, a precise study of strongly interacting particles may help to better understand the process of baryon annihilation.

The current knowledge of the strong interactions between baryons other than nucleons is limited - there exist only a few measurements of the cross sections for pairs of (anti)baryons. The reason is that in many cases it is not possible to perform scattering experiments with beams of particles and antiparticles, as the exotic matter (such as Λ , Ξ or Σ baryons) is very short-living. This issue can be solved thanks to the recent particle colliders like the Large Hadron Collider and experiments dedicated to study the heavy-ion collisions, such as ALICE (A Large Ion Collider Experiment). In high-energy heavy-ion collisions thousands of particles are produced, including very massive ones. Moreover, similar amount of matter and antimatter is created. This provides a unique environment for the study of strong interaction of pairs of (anti)baryons at low relative momenta.

Measurement of correlation in momenta of (anti)baryons produced in the heavy-ion collisions may allow to determine parameters describing the strong potential. In this thesis the focus is put on the development, verification and application of a method based on such correlations. For the purpose of this thesis, theoretical in-depth study of such method was performed. Next, an application was developed to allow for simultaneous fit of multiple pair types. The procedure of fitting baryon-(anti)baryon correlation functions was tested on the model data. Then, correlation functions were calculated using the data collected by ALICE. Finally, the newly-developed procedure was applied to the experimental data, taking into account a series of effects (so-called residual correlations, correction of the non-femtoscopic background, momentum resolution). Strong interaction parameters were then extracted from the fit for different baryon-antibaryon pairs and systematic uncertainties were estimated.

Keywords: femtoscopy, correlations, baryons, strong interaction, antimatter

Streszczenie

Oddziaływanie silne jest jednym z czterech oddziaływań podstawowych. Spaja ono kwarki wewnątrz protonów i neutronów (będących przykładami barionów - cząstek złożonych z trzech kwarków), a także odpowiada za stabilność jądra atomowego. Parametry opisujące potencjał oddziaływania silnego są kluczowe dla modeli astrofizycznych opisujących gwiazdy neutronowe. Co więcej, precyzyjne badania silnie oddziałujących cząstek mogą pomóc w zrozumieniu procesu anihilacji barionów.

Obecna wiedza dotycząca oddziaływania silnego barionów innych niż nukleony jest ograniczona - istnieje zaledwie kilka pomiarów przekrojów czynnych na oddziaływanie par (anty)barionów. W wielu przypadkach jest to spowodowane trudnościami technicznymi z przeprowadzeniem eksperymentów bezpośrednio na wiązkach cięższych (anty)cząstek (takich jak bariony Λ , Ξ czy Σ), ze względu na ich krótki czas życia. Problem ten może być jednak rozwiązany dzięki współczesnym zderzaczom cząstek takim jak Wielki Zderzacz Hadronów oraz eksperymentom zaprojektowanym do badania zderzeń ciężkich jonów, np. ALICE (A Large Ion Collider Experiment). W relatywistycznych zderzeniach ciężkich jonów produkowane są tysiące cząstek, w tym podobna ilość materii i antymaterii, co zapewnia unikalne warunki do badania oddziaływania silnego par (anty)barionów przy niskich pędach względnych.

Pomiar korelacji pędowych (anty)barionów produkowanych w zderzeniach ciężkich jonów może umożliwić określenie parametrów opisujących oddziaływanie silne. W niniejszej pracy nacisk został położony na rozwój, weryfikację oraz zastosowanie metody bazującej na korelacjach pędowych. Przeprowadzono szczegółową analizę teoretyczną metody. Stworzono aplikację umożliwiającą jednoczesne dopasowanie funkcji korelacyjnych dla wielu różnych par barionów, która została przetestowana na danych modelowych. Następnie policzone zostały funkcje korelacyjne dla danych zebranych przez ALICE, do których zastosowano nowo opracowaną procedurę. Uwzględniono przy tym szereg dodatkowych efektów, takich jak: korelacje rezydualne, korekcja tła, ograniczona rozdzielczość pędowa detektora. Parametry oddziaływania silnego wraz z niepewnościami systematycznymi zostały wyznaczone dla różnych par barion-antybarion.

Słowa kluczowe: femtoskopia, korelacje, bariony, oddziaływanie silne, antymateria

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Chapter 1

Introduction

Particle colliders are one of the most powerful experimental devices in the modern physics. They are capable of increasing the energy of particles to values thousands times higher than their rest masses (for example, mass of the proton $m_p \approx 1 \text{ GeV}$, while the center-of-mass energy in collisions at LHC $\sqrt{s} = 13000 \text{ GeV}$). This creates a unique environment in which a wide range of physical phenomena can be studied. Colliders may operate on different particle types, delivering for example electron-positron, proton-proton or heavy-ion reactions. In this thesis, the focus will be put on the latter one, where after the collision a relatively big and complex system is being formed. Processes taking place in such a system lead to the production of thousands of new particles, which carry the information about phenomena physicists want to investigate. Parameters such as trajectory, energy or velocity are registered by sophisticated detectors, which allow for a reconstruction of the complete information about produced particles (momentum, charge, type of the particle).

Study of the correlations between different properties of particles produced in relativistic heavy-ion collisions is a great tool to gain knowledge about physical phenomena taking place under extreme conditions (very high temperatures and energy densities) as well as on very small scales (of the order of the nucleus size). In this thesis the focus is put on the correlations in momenta of baryons and antibaryons - particles composed of three (anti)quarks. The measurement and the detailed study of the so-called correlation function for different pairs of (anti)baryons provides an information about the initial conditions and further interactions between those particles. In case of (anti)baryons, the influence of the strong interaction on the shape of the correlation function is clearly visible, in addition to the Coulomb interaction and Quantum Statistics.

What makes this type of studies especially interesting is the similarity between the state of matter after the collision to the one which existed shortly after the Big Bang. In both cases temperature and energy density is extremely high, which causes formation of the

Quark-Gluon Plasma (QGP), in which quarks and gluons are moving freely, being no longer bound in more complex particles. Measuring properties of the QGP formed after the collision of heavy-ions one can study processes taking place in the very early Universe.

An interesting issue which can be studied with use of the baryon-antibaryon correlation functions is the matter-antimatter symmetry and the process of annihilation. Similar amount of matter and antimatter is produced in the collision, which then interacts and gets registered in the detector. The annihilation channel of baryon-antibaryon scattering will influence the shape of the correlation function, which can be used to study this process. Comparison of baryon-baryon correlations with those of baryon-antibaryon pairs allows in turn for the investigation of similarities or differences between matter and antimatter.

The measurement of the parameters describing strong interaction of baryons may also have an influence on the models describing processes occurring inside of neutron stars - a super dense and relatively small stars formed after the collapse of massive stars. The nuclear force is closely related with the strong interaction of baryons as well, thus its knowledge is also crucial for understanding the internal structure of the nucleus and its stability. Those two examples show a direct link with cosmology and physics of the atomic nucleus, which confirms the importance of this type of measurements.

In this thesis a possibility of extracting parameters describing strong interaction using newly developed procedure based on the simultaneous analysis of correlation functions for a variety of baryon-(anti)baryon pairs will be tested. A detailed theoretical validation of the method used to study these functions will be demonstrated (Chap.6). Then, results of the method's tests on the model data will be shown (Chap.7). Next, the procedure will be applied to the experimental data and results, approved as ALICE preliminary, of the measurement of scattering lengths and effective ranges for different baryon-antibaryon pairs will be presented (Chap.8). Finally, various tests of the fitting procedure will be shown to demonstrate how different modifications influence the obtained results (Chap.9).

Chapter 2

Relativistic heavy-ion collisions

In this chapter, the physics of relativistic heavy-ion collisions will be described, starting from the Standard Model of Particle Physics and the Quantum Chromodynamics (QCD), through the characterisation of the course of a typical event and experimental searches of the Quark-Gluon Plasma, to the modelling of processes taking place just after the collision.

2.1 Standard Model of Particle Physics

The Standard Model of Particle Physics is a theory describing the fundamental constituents of matter and interactions between them using a small set of elementary particles (see Fig.2.1). The particles can be divided into two groups: fermions (with half-integer spin) and bosons (with integer spin). Fermions can be seen as the building blocks of matter, while bosons are the particles mediating interactions between those blocks. The first group can be further divided into quarks (six particles building more complex systems, such as protons and neutrons) and leptons (for example an electron or a neutrino). The second group contains photons (carriers of the electromagnetic interaction), gluons (responsible for the strong interaction), bosons W^+ , W^- and Z^0 (associated with the weak interaction) and the recently discovered Higgs boson. Hypothetically, there should also exist a graviton, which would be classified as a boson (transferring the gravitational interaction), however its existence has not yet been confirmed experimentally.

All the particles presented above also have their antimatter counterparts. Different (anti)quarks can build more complex systems, for example a proton, which is composed of three so-called valence quarks (two up quarks and one down quark) or neutron (two down quarks and one up quark). Many other combinations of (anti)quarks are allowed and have been experimentally confirmed, including mesons (containing two quarks), baryons (three quarks) and recently also pentaquarks (five quarks). The name *hadron* is used as a general description of a system build of (anti)quarks.

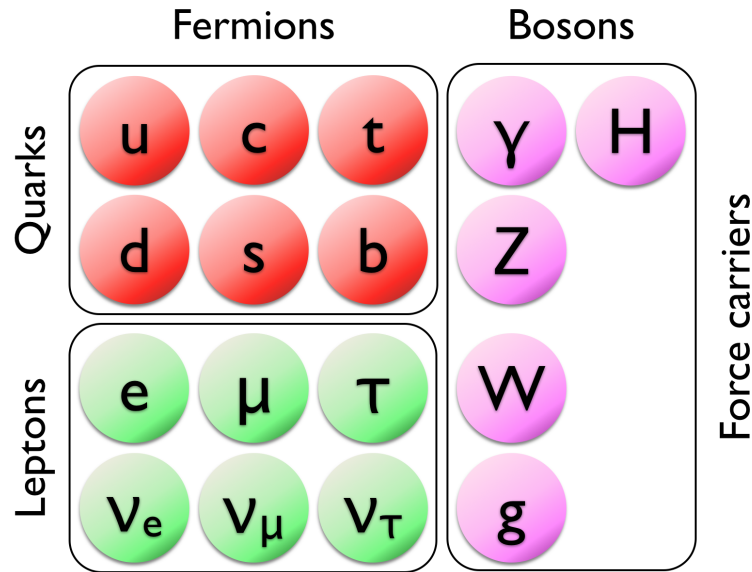


Figure 2.1: The Standard Model of Particle Physics.

2.2 Quantum Chromodynamics

The Quantum Chromodynamics (QCD) is a quantum field theory of the strong interaction between quarks. The process is described by the exchange of gluons, which are the strong interaction carriers. The strong interaction has some interesting properties - unlike the electromagnetism or gravity, the strong force becomes more prominent when interacting particles are pulled apart. Because of that, the more energy is put into the system, the more difficult it becomes to separate its constituents. In addition, at some point the binding energy becomes high enough to spontaneously create a quark-antiquark pair. These newly created quarks will become bound together with those which one tried to separate, forming two new particles. This phenomenon is called confinement and is the reason why a free quark cannot be observed.

Free quarks cannot be registered in detectors, however properties of strongly interacting matter can be studied if one accumulates a big energy in a small volume ($15 - 30 \text{ GeV}/fm^3$ in the collisions at the LHC [1]), under extremely high temperature (in the LHC the initial temperature is about $310 - 370 \text{ MeV}$ [1][2][3], five orders of magnitude higher than in the core of the Sun). Such conditions can be obtained in a high-energy collisions of heavy-ions. Then, one can observe the last step of the evolution of such a system in regular detectors (registering produced particles), to deduce the properties and learn about the behaviour of quarks when they are not bound in hadrons.

2.3 The course of the event

In this section the typical heavy-ion collision will be described, from beams acceleration, through the collision itself, to final interactions of produced particles.

2.3.1 Beams acceleration

To make the collision of heavy-ions possible, particle colliders such as the Large Hadron Collider (LHC) or the Relativistic Heavy Ion Collider (RHIC) are used. More detailed description of the LHC will be given in Sec.3.2. The collider has four main operation phases: acceleration, keeping beams in their paths, focusing and adjusting beams' positions. To accelerate positively charged ions RF cavity resonators are used, in which standing electromagnetic waves cause an overall force acting on passing particles and moving them along the accelerator's axis. To maintain the circular trajectory of the beams, superconducting electromagnets are used. Next, beams of particles are squeezed by quadrupole magnets to reduce their transverse size and therefore increase the frequency of collisions. Finally, steering magnets adjust the positions of the beams and make it possible for ions moving with nearly the speed of light to collide.

2.3.2 Collision

In case of the LHC, the colliding beams consist of lead ions accelerated to the energy of the order of a few TeV per nucleons' pair. For a reference, the mass of a nucleon is of the order of 1 GeV, which means that the energy carried by each colliding particle is up to a few thousands times higher than its rest mass. The evolution of the system after the collision is presented in Fig.2.2. Just after the collision, the system is extremely hot and dense - after about $1 fm/c$ (around $10^{-24}s$ [4]), which is needed to achieve thermal equilibrium of constituents of the medium, the Quark-Gluon Plasma is formed (see Sec.2.3.3). Then the system gradually expands and cools down and quarks start to bound back together to form hadrons (see Sec.2.3.4). The moment when all inelastic processes cease and all quarks are confined back again is called the chemical freeze-out (resonances may decay also after this time). Next, the hadron-gas phase takes place, in which hadrons may scatter multiple times in elastic collisions, due to the Coulomb and strong interactions, which are

the main interest of this thesis. The moment when the mean free path becomes larger than the size of the system (thus in general there are no more elastic collisions between hadrons) is called the thermal freeze-out. After this moment the system contains mainly hadrons with well defined momenta, which will then be measured in the detector.

The aforementioned resonances are the particles, which may decay after the chemical and thermal freeze-out through the strong interaction. Their lifetimes are comparable with the lifetime of the QGP, which is of the order of 10^{-23} s [5].

Particles can also decay through the weak interaction. In such case, the flavour of the particle may change, for instance, the Λ baryon is composed of (uds) quarks and weakly decays into proton (uud) and pion ($d\bar{u}$). In case of the Λ baryon, the lifetime is many orders of magnitude longer than for resonances (10^{-10} s) and it can travel a few centimeters before it decays [6]. This means that momenta of particles coming from weak decays and being registered by the detector will be defined long after the freeze-outs.

2.3.3 Quark - Gluon Plasma

The Quark-Gluon Plasma (QGP) is an extremely hot state of matter (about $1.8 \cdot 10^{12}$ K, ten thousand times hotter than in the center of the Sun [1][2][3]) with extremely high energy density, in which systems of quarks and gluons can no longer be treated as separate hadrons, but rather as a collection of freely moving (but still strongly interacting) entities. There are many ways to create a QGP, either by increasing the temperature of the system, or increasing the so-called baryochemical potential μ_B , which describes how the energy of the system changes with the changing number of baryons.

This is illustrated in Fig.2.3, which is referred to as the QCD phase diagram. The high-energy collisions start from the region typical for a regular baryonic matter (small temperatures and baryochemical potential of about 1 GeV). Then μ_B decreases and temperature increases (reaching conditions necessary to produce the QGP), to finally cool down and come back to the region of regular matter. Colliders such as the LHC or RHIC reproduce conditions similar to those existing in the early Universe, where the temperature was extremely high and the net baryon density was close or equal to zero, which means that there was almost the same amount of matter and antimatter. On the contrary, neutron stars have relatively small temperature (order of keV), but very high baryochemical

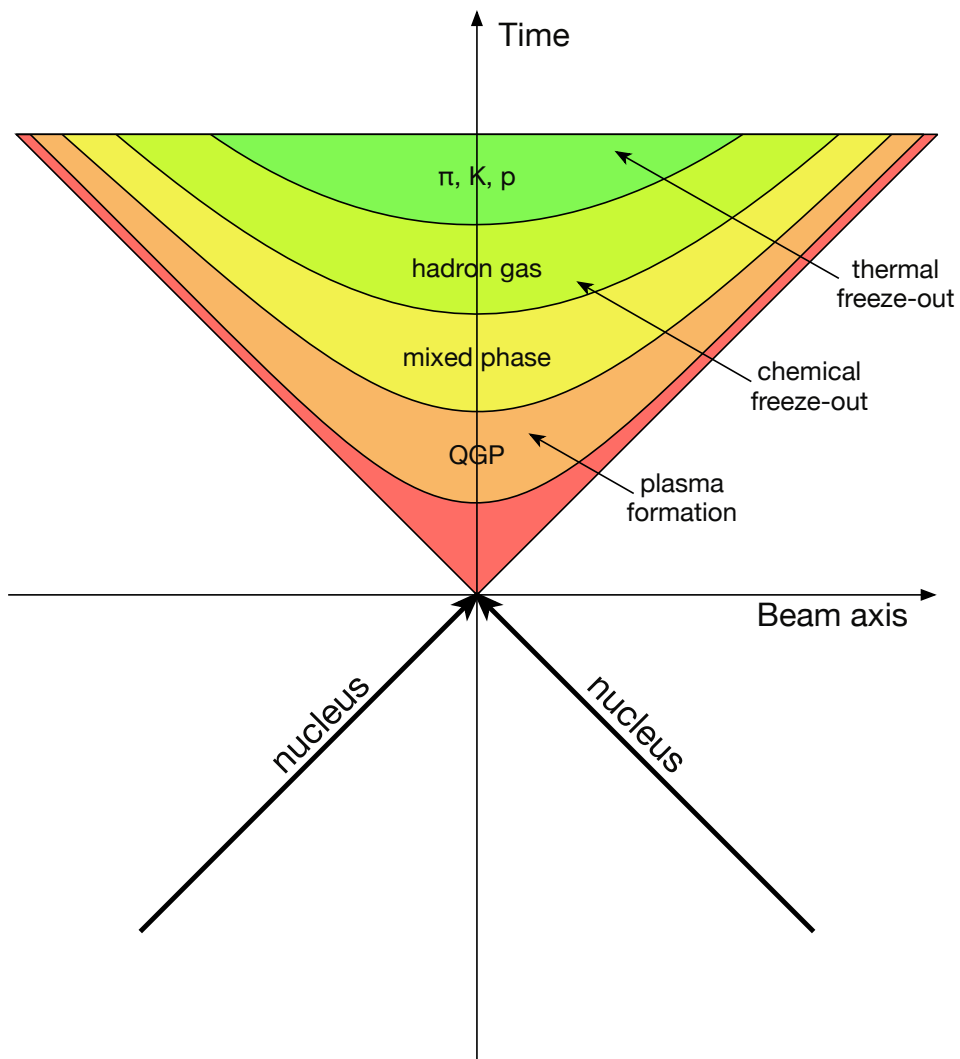


Figure 2.2: Space-time evolution of the QGP in the heavy-ion collision.

potential, which may also lead to the creation of QGP in the core of such stars [7].

2.3.4 Hadronization

As described in Sec.2.3.2, hadronization is a process of forming hadrons from quarks and gluons. The example can be found in heavy-ion collisions, where quarks and gluons composing the QGP blob undergo this process. Hadronization is a part of Quantum Chromodynamics, however direct calculations cannot be performed, as the perturbative theory can only be used for very high energies. Because of that, phenomenological models are used to describe the process (for example Therminator 2 [8]).

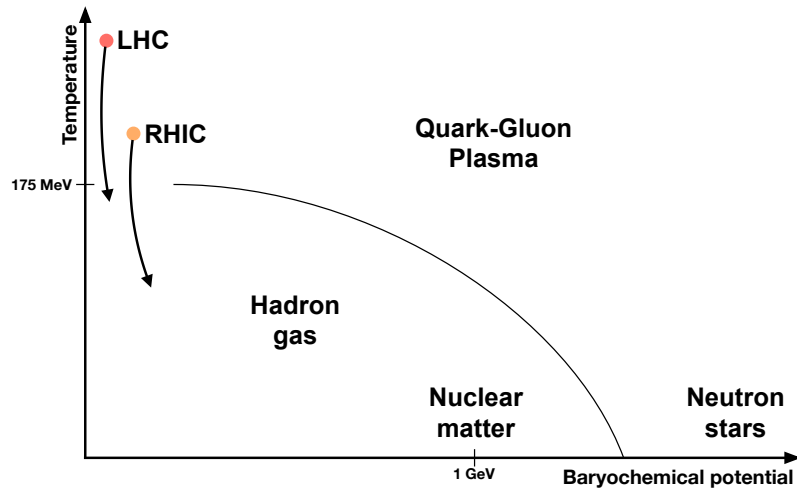


Figure 2.3: The QCD phase diagram.

2.3.5 Baryon Interactions

Finally, after the hadronization, baryons and other particles interact with each other and reach the detector. The more detailed theoretical description of baryon interactions will be given in Sec.4.1.2. It is important to notice, that interactions at this stage will determine kinematic properties of particles registered in the experiment. As a consequence, one can analyse measured observables (such as momenta or angles of trajectories) to investigate strong interaction between baryons, which is a basic underlying principle of the study described in this thesis.

2.4 Importance and experimental signatures of QGP

The QGP most probably existed just after the Big Bang. This fact shows the importance of the studies of this state of matter. Producing a sample of QGP in the laboratory would allow to analyse in detail the evolution of the system built of it and this knowledge can be transferred to the field of study of the very early Universe.

Precise measurements of the parameters (such as momenta) of particles produced from the hadronization of the QGP may lead to the designation of the strong interaction parameters. Those parameters are the constants describing the scattering at low relative momenta, they are crucial for the understanding of the nuclear force (binding protons

and neutrons inside the nuclei) and are needed for models describing neutron stars. This demonstrates another direct link with cosmology.

Neutron stars models depend strongly on the knowledge of the nuclear forces. As shown in [9], slight differences in characteristic of the strong interaction at the density of regular matter may lead to significant differences in extreme densities such as those in the neutron stars. This makes precise measurements of the strong interaction parameters crucial for the deeper understanding of the processes taking place in the neutron stars.

Finally, when the QGP produced in the relativistic heavy-ion collision hadronizes, similar amount of matter and antimatter is created. This in turn is useful for in-depth studies of the matter-antimatter symmetry. Precise measurements of parameters describing initial state and further interactions of matter and antimatter can shed a light on one of the most fundamental questions in modern physics - why do we observe more matter than antimatter in the Universe. Heavy-ion collisions create a unique environment in which particles and antiparticles are created together and directly interact with each other before detection. The number of combinations of different particles and antiparticles even in a single collision is unparalleled in other types of experiments.

All the above examples show that studies of QGP have a great scientific value. There are several experimental signatures of the QGP and the most important ones will be described in the following sections. As shown in Sec.2.4.2 and Sec.2.4.3, the density and temperature of the medium produced in heavy-ion collisions match expectations for the QGP formation. As can be seen in Sec.2.4.1 measurements also suggest a free movement of quarks in this medium. All these measurements together build a consistent picture showing that the Quark-Gluon Plasma is indeed produced in collisions at RHIC and can be further studied at LHC. One of the possible ways of conducting such study is through the femtoscopic correlations of (anti)baryons (see Chap.4), which are sensitive to the size of the emission source. The size of the source in turn depends on the properties of QGP, for example if hydrodynamical description of the medium produced in the collision is valid, one would expect that the source size for different pairs of particles depends on their average transverse mass, which is a direct consequence of the collective flow [10][11][12]. Thus analysis of baryon-(anti)baryon correlation functions provides an information about the nature of the QGP.

2.4.1 Elliptic flow

One of the most convincing signatures of QGP is the elliptic flow. This phenomenon is based on the fact, that only in perfectly head-on ions collision the region of particles' production is spherical. In all other cases, where the collision is not perfectly central, an almond-shaped source will be formed. Due to the anisotropy in the azimuthal angle, the pressure gradient will not be uniform and thus neither will be the angular distribution of particles (see Fig.2.4). The expected strength of this collective flow is different for the regular matter and a medium as dense and hot as QGP. The mathematical description of the phenomenon is based on the Fourier expansion of the azimuthal momentum distribution:

$$\frac{Ed^3N}{dp^3} = \frac{1}{2\pi p_T} \frac{d^2N}{dp_T dy} (1 + 2v_1 \cos(\phi - \Phi_R) + 2v_2 \cos[2(\phi - \Phi_R)] + \dots), \quad (2.1)$$

where $\phi - \Phi_R$ is the difference between the Reaction Plane angle and the azimuthal angle of each particle. The Reaction Plane is defined by the beam axis and the impact parameter b , where b is a distance between centers of colliding nuclei. An illustration of the collision geometry and the elliptic flow is shown in Fig.2.4. Coefficients v_n are given by:

$$v_n = \langle \cos[n(\phi - \Phi_R)] \rangle. \quad (2.2)$$

The coefficient of the second term in the above equation (v_2) is called the elliptic flow and has been measured by experiments at RHIC and LHC (see Fig.2.5 and Fig.2.6). As can be seen, values of v_2 measured at RHIC agree perfectly for mesons and baryons if one scales them by the transverse energy E_T and then by the number of quarks [13]. The later measurements performed by ALICE at LHC demonstrate that the same trend is observed for a different system colliding with higher energy [14].

This can be explained under the assumption that the hadron flow is a combination of the flow of quarks, which is a clear evidence for pseudo-free quarks in the matter produced after the collision, supporting the QGP hypothesis. These results are consistent with hydrodynamical description of the system formed after heavy-ion collision and support the coalescence models [15], which assume that hadrons are formed from QGP through the recombination of quarks and jet fragmentation [16].

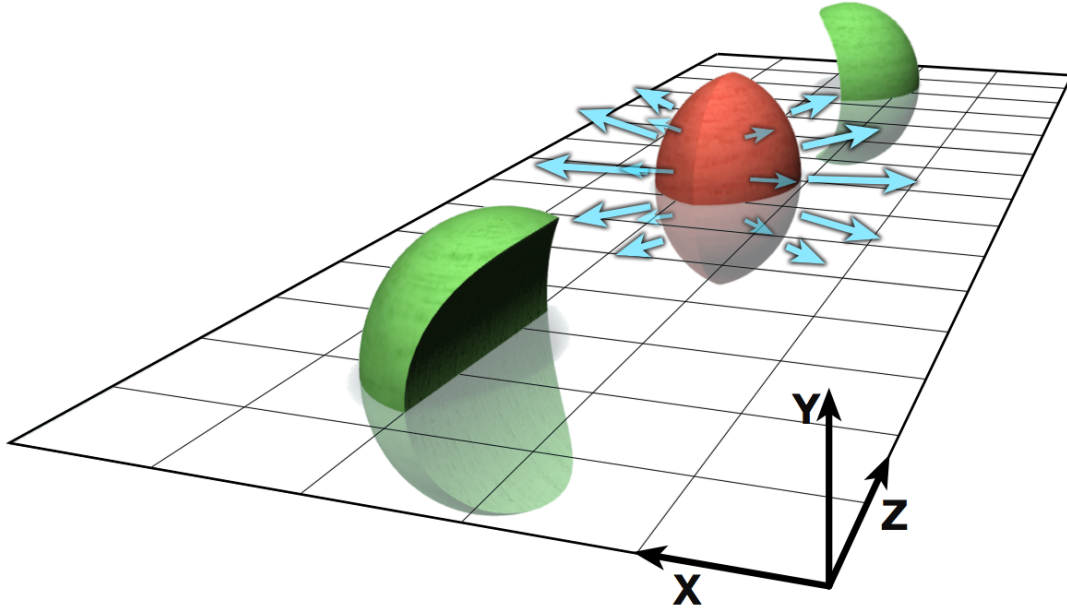


Figure 2.4: Elliptic flow illustration. The grid in XZ plane represents the Reaction Plane of the collision.

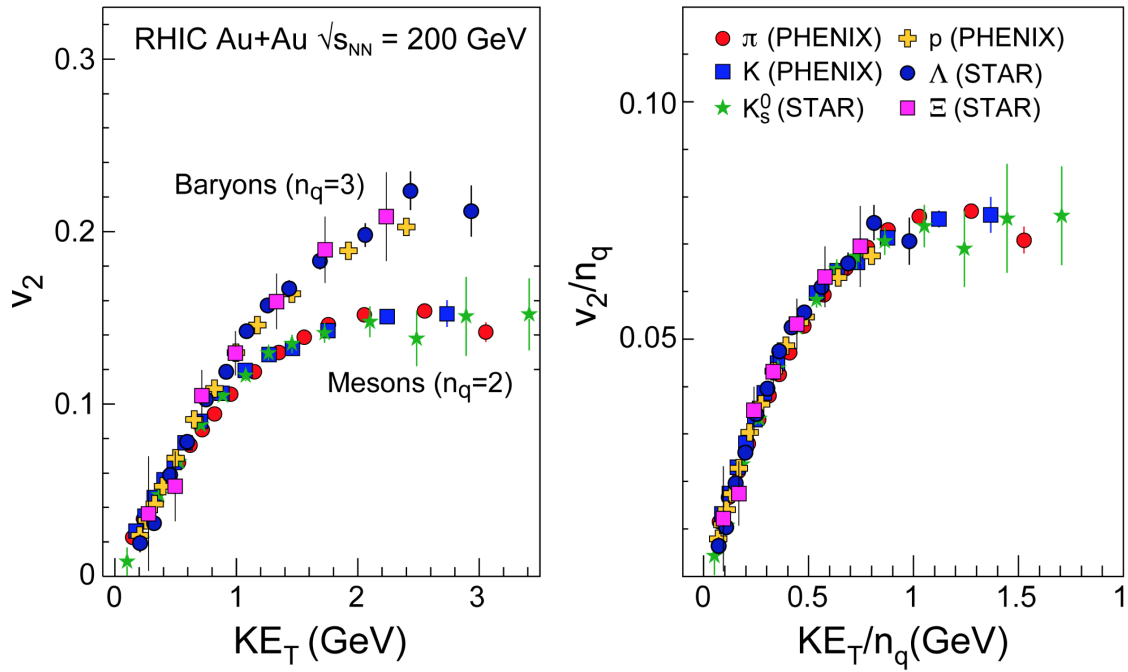
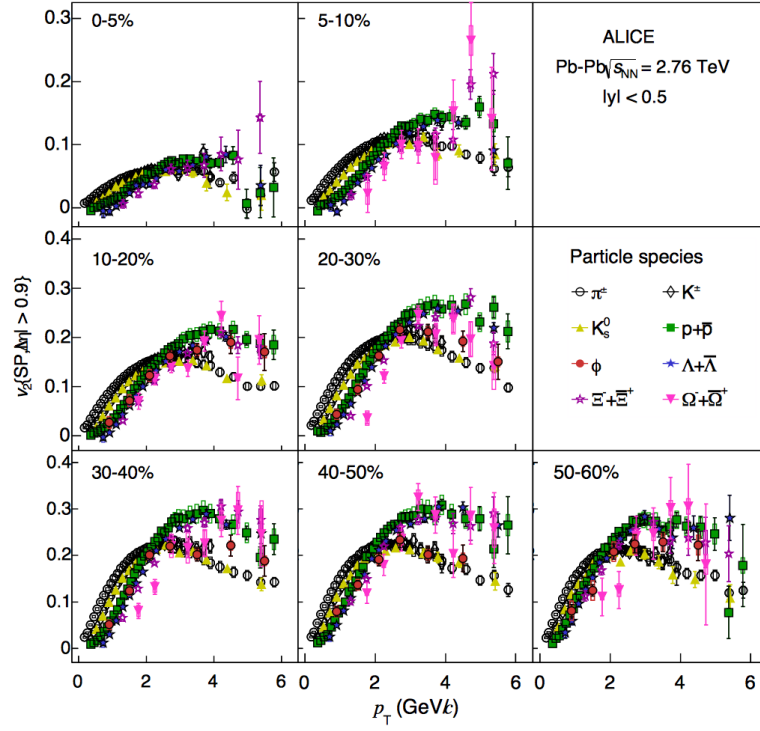
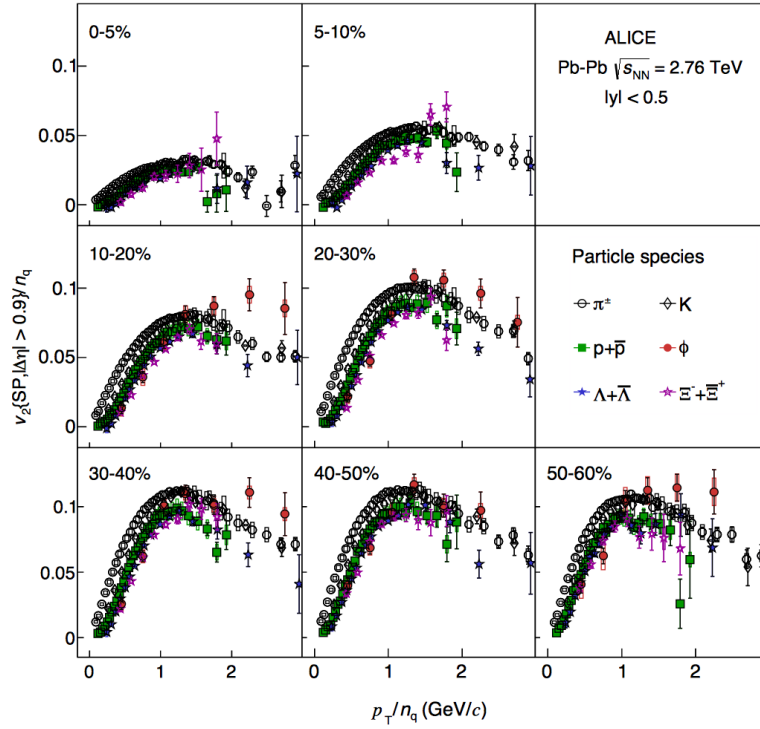


Figure 2.5: v_2 measurements from RHIC. In the right plot values are scaled by the number of quarks [13].



(a) No scaling



(b) Scaled by number of quarks

 Figure 2.6: v_2 measurements from ALICE at the LHC [14].

2.4.2 Jet quenching

Another signature of the QGP is the so-called jet quenching. Jets are streams of high energy particles produced after the collision originating from two partons (quarks or gluons) going in the opposite directions. If the energy of those partons is big enough, they will produce a new quark-antiquark pair, which will in turn emit gluons. The process may occur many times, as long as there is still enough energy in this system to produce new pairs. The collection of particles coming from such process is identified as a jet. One can predict how jets will propagate depending on the density and taking into account the size of the medium, concluding that in a dense medium like QGP, jets should be suppressed comparing to dilute or no medium at all.

One of the observables sensitive to jet quenching is the nuclear modification factor R_{AA} , which is the ratio of the number of particles with given momenta in heavy-ion collisions to the one measured in proton-proton collisions scaled by the average number of binary collisions of nucleons. In the first case it is expected that the QGP is formed and thus strong suppression of highly energetic particles from jets should be observed. In the second case, there should be almost no medium for jets to interact with. The results of measurements of R_{AA} can be seen in Fig.2.7. The measured ratio clearly shows a strong jet suppression in heavy-ion collisions, both at RHIC and LHC.

For the measurement of jet quenching another technique based on the angular two-particle correlations can be used [18]. In this method, the difference in the azimuthal angle $\Delta\varphi$ is measured for each pair of particles produced in the collision. The particles coming from the same jet will go in almost the same direction, so that $\Delta\varphi \approx 0$, while for particles from two back-to-back jets $\Delta\varphi \approx \pi$. The measurement of the angular correlations is presented in Fig.2.8 - for p-p and d-Au collisions there are two peaks in zero and π corresponding to particles from jets. However, in Au-Au collisions the second peak is not present. This is consistent with the assumption of the QGP being a dense medium strongly interacting with the particles forming a jet. If two back-to-back jets are produced the one which has shorter distance to traverse in the QGP may survive as a collimated stream of high momenta particles. Then the other one will undergo many interactions with the medium and thus will get scattered, causing smearing of the angular distribution around $\Delta\varphi = \pi$ and decreasing the peak.

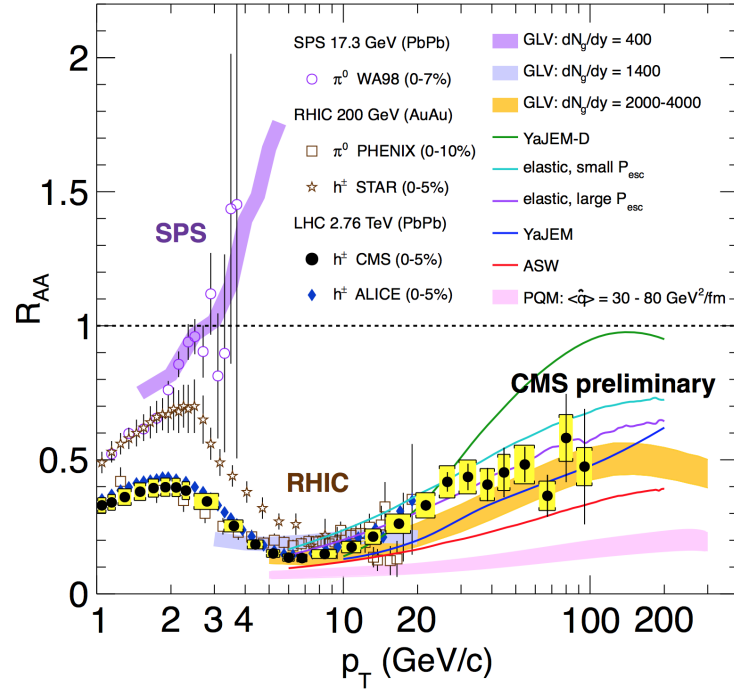


Figure 2.7: Nuclear modification factor measurements in SPS, RHIC and LHC [17].

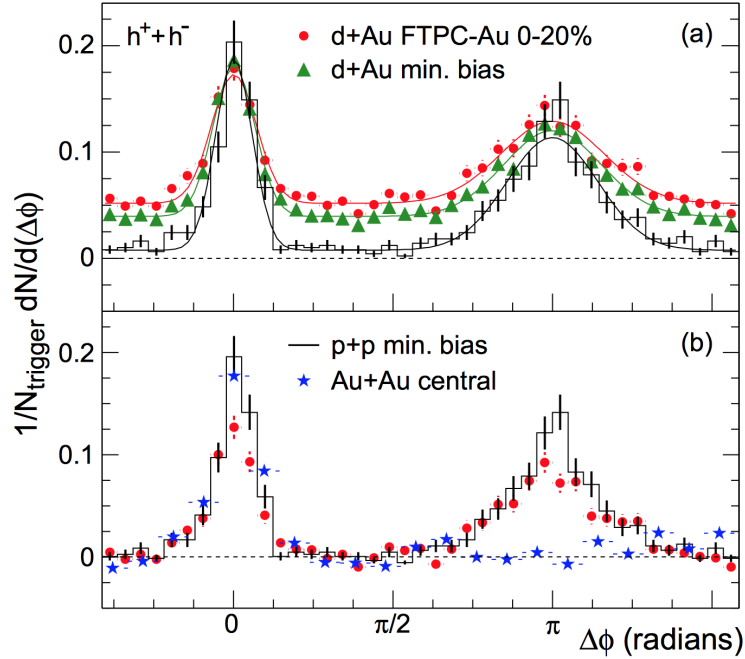


Figure 2.8: Correlations in azimuthal angle measured by STAR experiment [18].

2.4.3 Direct photon energy distribution

Next signature of the QGP is the slope of the energy distribution of direct photons, which are those that do not come from the hadronic decays. Thus direct photons mainly originate from the Compton effect ($qg \rightarrow \gamma q$), annihilation ($q\bar{q} \rightarrow \gamma\gamma$) or braking radiation ($q \rightarrow \gamma q$). These photons are very useful for measurements of the properties of the matter soon after the collision, as they are unlikely to interact with other particles and thus they may carry the information about the temperature of the QGP.

The measurement of the transverse momentum (p_T) spectrum of direct photons in heavy-ion collisions is presented in Fig.2.9 together with fitted curves predicted by several models, assuming different initial temperatures. As can be seen, the measured temperature is about 240 MeV for RHIC energies and more than 300 MeV for the LHC energies, which is above the minimum predicted as a condition for the QGP to form (150-170 MeV [2][3]).

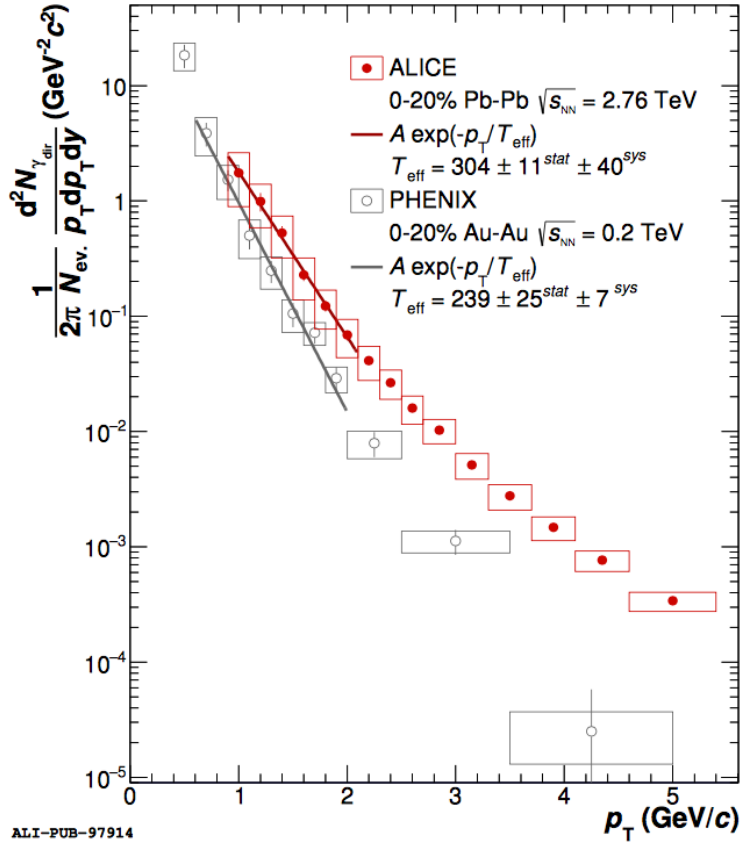


Figure 2.9: Direct photons p_T spectra from PHENIX and ALICE [19].

2.5 Therminator model

As explained in Sec.2.3.4, the analysis of hadron formation from the Quark-Gluon Plasma cannot be based on the direct calculations from the Quantum Chromodynamics. To handle this issue, several phenomenological models describing hadronization have been developed. One of them, which was used for the purpose of this thesis, is the Therminator 2 (THERMal heavy IoN generATOR) [8].

Therminator is an event generator based on the statistical hadronization. The input of the application is the so-called freeze-out hypersurface, which describes a probability of the emission of particle with given momentum from a given space-time location. The model assumes a single freeze-out, which means that the time of chemical freeze-out is assumed to be the same as for the thermal freeze-out.

Freeze-out hypersurfaces can be calculated theoretically and tuned to correspond to the conditions met in the existing colliders. By default Therminator 2 provides input files suitable for the analysis of RHIC Au-Au collisions at $\sqrt{s_{NN}} = 200 \text{ GeV}$ and LHC Pb-Pb collisions at $\sqrt{s_{NN}} = 5.5 \text{ TeV}$. In this thesis, the following hypersurface was used:

- colliding system: lead-lead,
- collision energy: 5.5 TeV per nucleon pair,
- initial central temperature of 500 MeV ,
- freeze-out temperature of 145 MeV ,
- centrality bins: 0-5%, 5-10%, 10-20%, 20-30%, 30-40%, 40-50%, 50-60%, 60-70%.

The algorithm uses the Monte Carlo method to integrate the provided emission function. The Monte Carlo routine is also used to perform decays of the particles. It should be emphasised that in this model all known resonances are taken into account, as well as weakly decaying particles (such as Λ baryon), which is crucial for the residual correlations' studies (see Chap.5). As an output the application generates ROOT files [20] containing information about particles grouped into events, which is similar to the files usually used in the analysis of the experimental data.

Chapter 3

Experiment

The experimental data analysis presented in this thesis is based on the measurements performed by the ALICE collaboration. In this chapter CERN, the LHC and ALICE will be described.

3.1 CERN

The European Organisation for Nuclear Research (CERN) [21] is the largest particle physics laboratory in the world, in which thousands of physicists and engineers work together on the experiments designed to study some of the most interesting problems in the modern science. A few examples of the issues which can be investigated using CERN's facilities are the matter-antimatter asymmetry, the Higgs mechanism, the evolution of the early Universe or the stability and properties of heavy nuclei.

3.2 LHC

The most complex research instrument (being also the most complex machine ever built by humans) located at CERN is the Large Hadron Collider (LHC). This 27 km long collider is the last part of the CERN's accelerators complex, preceded by the Super Proton Synchrotron (SPS), Proton Synchrotron (PS), Low Energy Ion Ring (LEIR), BOOSTER, LINAC 2 and LINAC 3.

LINAC 2 is a liner accelerator which is dedicated to preform initial protons acceleration to 50 *MeV*. Protons are then injected into BOOSTER, which accelerates them to 1.4 *GeV*. LINAC 3 is a linear accelerator which is used for the acceleration of ions to 4.2 *MeV* per nucleon. Then, ions are sent to LEIR, which further accelerates them to 72 *MeV* per nucleon. Next, protons or heavy-ions are injected to the Proton Synchrotron, which can increase energy of beams up to 25 *GeV*. The next step is an acceleration in the SPS, which can reach up to 450 *GeV* before finally sending beams to the LHC. The LHC has

so far delivered p-p collisions with center of mass energies of 900 GeV , 2.36 TeV , 7 TeV , 8 TeV and 13 TeV , p-Pb collisions at 5.02 TeV and Pb-Pb at 2.76 TeV and 5.02 TeV per nucleon pair. To make acceleration of beams to such energies possible, LHC operates in ultra-high vacuum which reduces probability of random beam-gas collisions. Moreover, 1232 superconducting bending magnets cooled down to $-271.75\text{ }^{\circ}\text{C}$ (1.4 K) are used to keep particles on their trajectories [22]. As a result the LHC can provide up to 600 million collisions per second. To register products of these collisions, specialised detectors were built in four caverns along the beams' pipes: ALICE, ATLAS, CMS and LHCb.

3.3 ALICE

A Large Ion Collider Experiment (ALICE)[23] is one of the four main experiments at the LHC. It is focused on studies of the heavy-ion collisions, mainly to investigate properties of the Quark-Gluon Plasma and fundamental interactions between particles. In the relativistic heavy-ion collision, thousands of new particles are produced and need to be registered. ALICE consists of 19 specialised detectors:

- Silicon Pixel Detector (SPD)
- Silicon Drift Detector (SDD)
- Silicon Strip Detector (SSD)
- Time Projection Chamber (TPC)
- Transition Radiation Detector (TRD)
- Time Of Flight (TOF)
- Photon Spectrometer (PHOS)
- Electro-magnetic Calorimeter (EMCal)
- High Momentum Particle Identification Detector (HMPID)
- A Cosmic Ray Detector (ACORDE)
- Photon Multiplicity Detector (PMD)
- Forward Multiplicity Detector (FMD)
- Fast Scintillator Detector (V0)
- Fast Quartz Detector (T0)
- Zero Degree Calorimeter (ZDC)

- Muon Chamber (MCH)
- Muon Trigger (MTR)
- ALICE Diffractive (AD)
- Charged Particle Veto (CPV)

The schematic view of the ALICE detector is presented in Fig.3.1. In the next section, the most important sub-detectors from the point of view of this thesis will be described in detail.

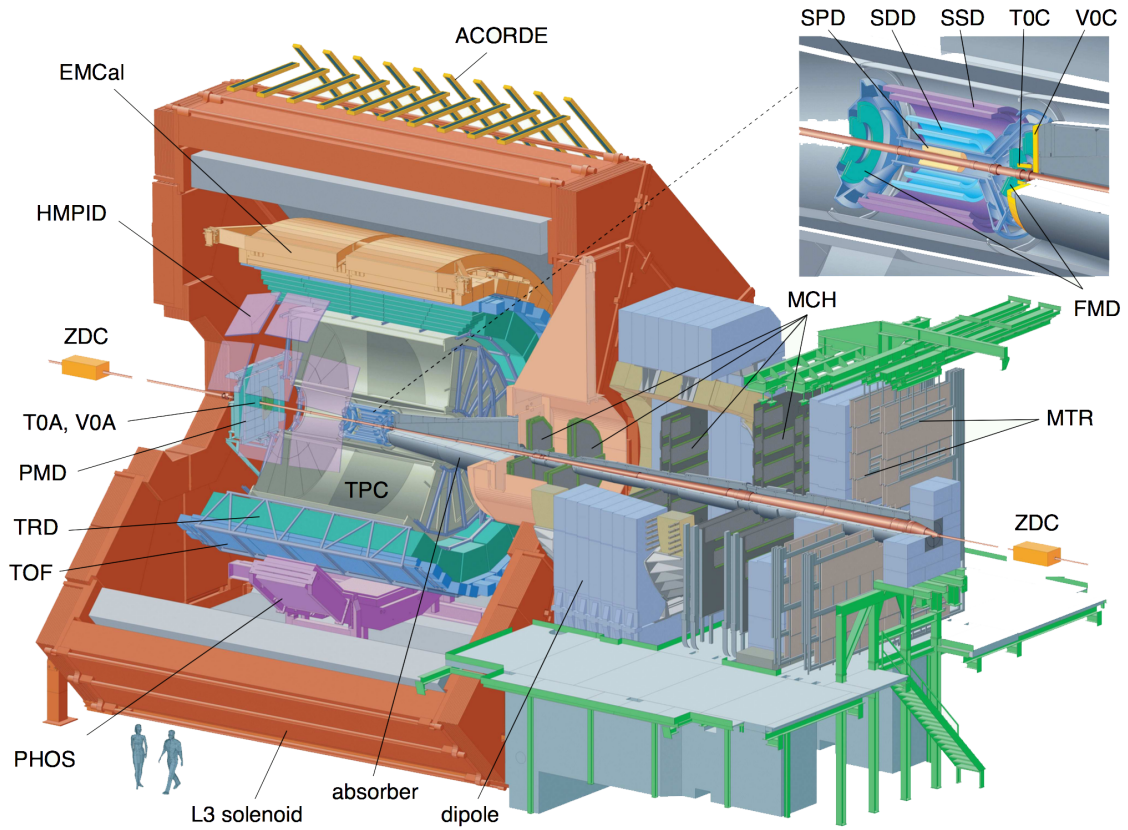


Figure 3.1: The schematic view of ALICE detector [23].

3.3.1 Sub-detectors description

For the study described in this thesis, the most important is the particle identification and measurement of momenta of baryons and products of their decays. These measurements are usually done by combining readouts of different detectors, especially the Inner Tracking System (ITS), Time Projection Chamber (TPC) and Time Of Flight (TOF), which are described below. All these detectors are placed inside a solenoid magnet capable of

producing magnetic field of up to 0.5 T [24], which is used to bend trajectories of charged particles and thus help to identify them based on the track curvature.

3.3.1.1 ITS

The Inner Tracking System (ITS) is a set of three detectors: SPD, SDD and SSD (see Fig.3.2). Each of them is a silicon semi-conducting detector arranged in two barrel-shaped layers located close to the collision point.

First two layers (SPD) are composed of pixel chips, each fragmented into 8192 pixels of the size of $50\text{ }\mu\text{m} \times 425\text{ }\mu\text{m}$ [25]. Knowing the position of each pixel one can get three-dimensional coordinates of the point of interaction between charged particle produced in the collision and the material of the detector.

Layers 3 and 4 (SDD) are split into strips in which charge generated by the passing particle travels to the readout electrode with a known velocity, which is sufficient to calculate three-dimensional space location of the interaction between detector's silicon and the particle being measured.

Last two layers (SSD) are the strips of semi-conductor forming an overlapping grid. The particle leaving a trace in the detector fires strips in both layers. Combining the information about their locations one can deduce a full three-dimensional location.

The construction of the Inner Tracking System allows to register six points per particle along its path with high spatial precision: $12\text{ }\mu\text{m}$ for SPD, $35\text{ }\mu\text{m}$ for SDD and $20\text{ }\mu\text{m}$ for the SSD [26]. This in turn is crucial for the precise reconstruction of the primary vertex (the collision point) and particles' trajectories in the most dense region, which is close to the vertex.

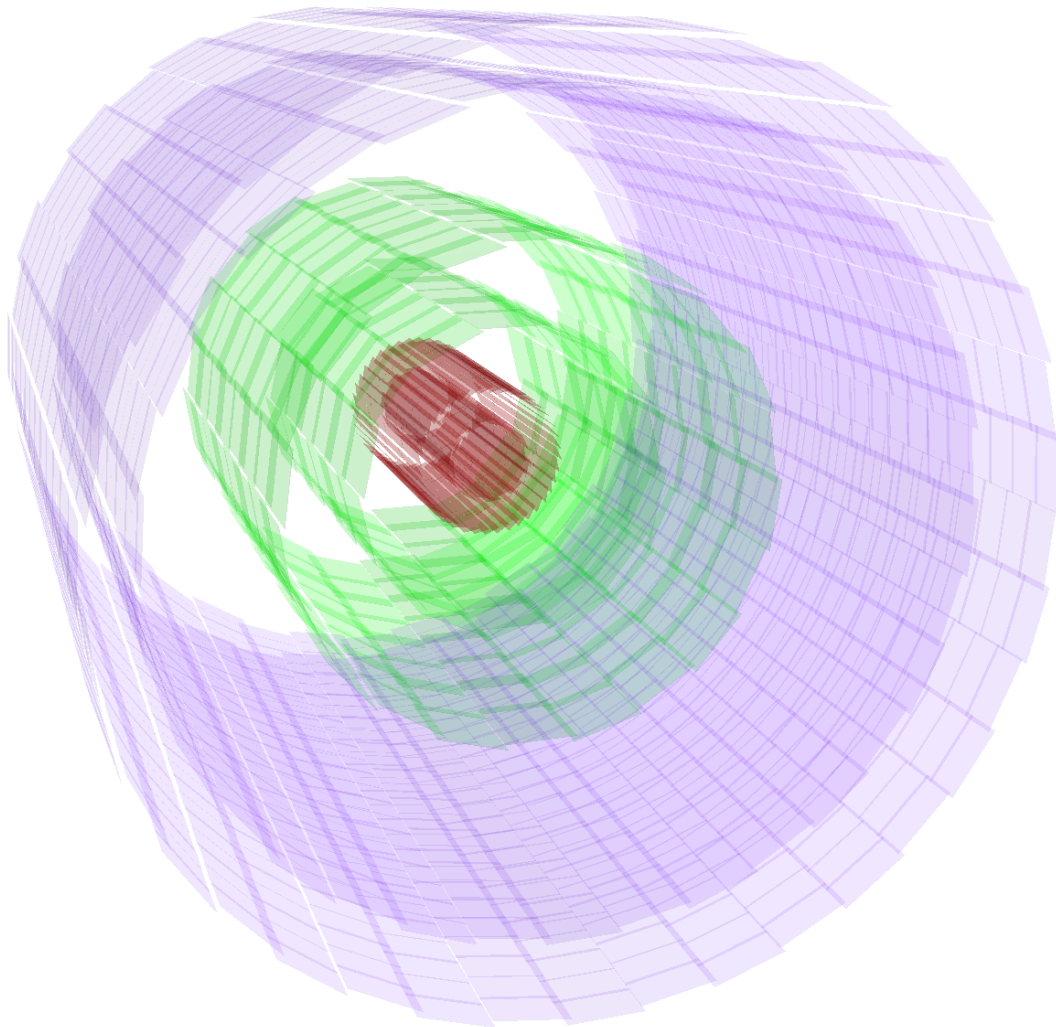


Figure 3.2: Visualisation of the ITS prepared in the ALICE Event Display (see Sec.3.3.3). Red - SPD, green - SDD and blue - SSD.

3.3.1.2 TPC

The Time Projection Chamber (TPC) [27] is a 5 *m* long barrel which is currently filled with a mixture of 90% of neon and 10% of CO_2 (the gas mixture may change between different periods of data taking). In the middle and on the two ends of the detector the high-voltage (up to 100 kV) electrodes are placed. Each of the end caps consists of hundreds of thousands of readout pads (570 000 in total).

When a charged particle crosses the active part of TPC, atoms of the gas get ionised along its path and electrons freed in the process are being attracted to the positively

charged readout plate. As the movement of the electrons is parallel to the detector's z axis, electric impulse will be generated on the readout pad in the location corresponding to the location of the interaction point between particle coming from the collision and the detector's active material. The third component of the location can be calculated thanks to the known electrons' drift velocity in the gas mixture and the measurement of the time between the collision and the readout. As the drift velocity of electrons strongly depends on the gas conditions, the temperature and pressure of the gas has to be maintained with a high precision. The spatial resolution of points reconstructed from the TPC is between 0.4 mm and 2.0 mm , depending on the distance from the end cap and the angle of the track [28].

The visualisation of the TPC is shown in the Fig.3.3.

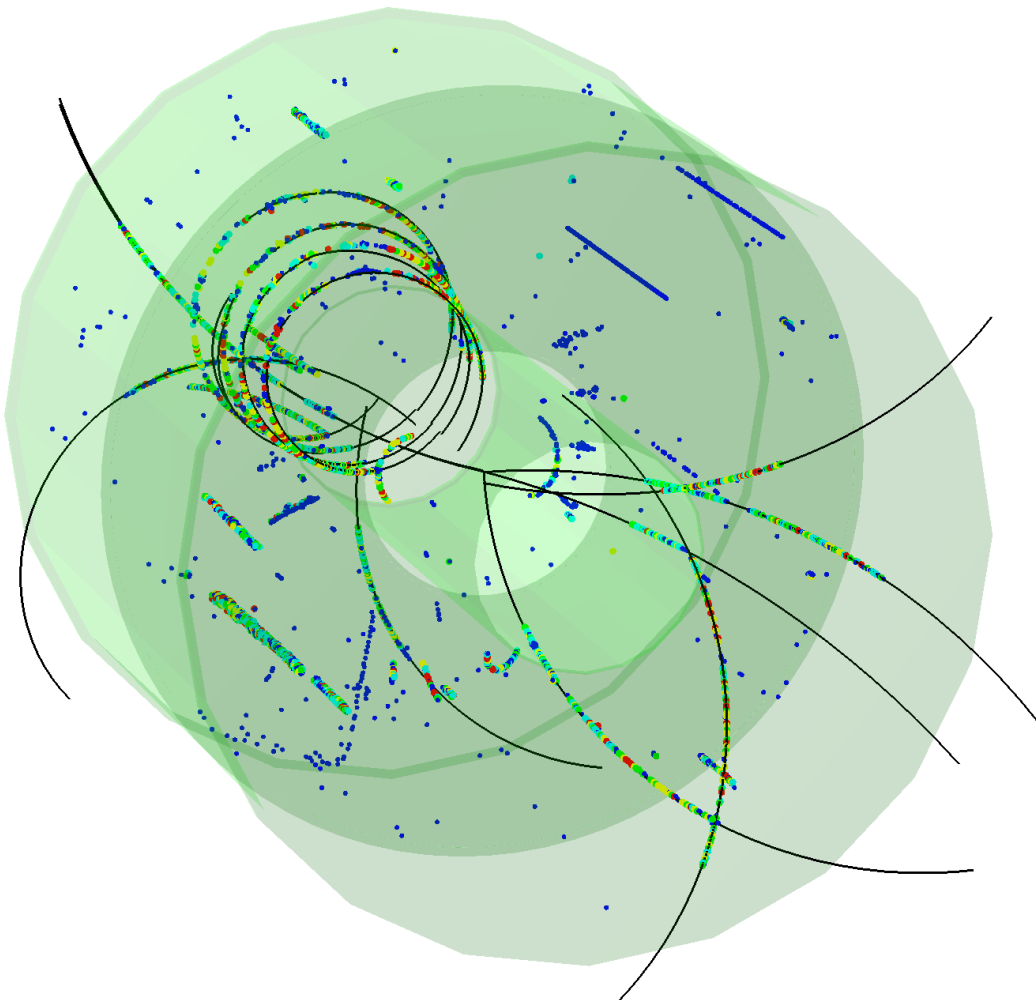


Figure 3.3: Visualisation of the TPC with reconstructed clusters and fitted particle tracks. The picture comes from the ALICE Event Display (see Sec.3.3.3).

3.3.1.3 TOF

The Time Of Flight (TOF) [29] is a cylindrical detector dedicated to measure time between the collision and the moment when particles approach the outer area of the ALICE detector. TOF covers the full azimuthal angle and polar angles from -45° to 45° , covering the total area of 160 m^2 and consisting of 157248 readout pads. The detector is divided into 18 modules, which consist of 1638 Multigap Resistive Plate Chambers (MRPC). The MRPC is built of several thin glass sheets separated by layers of gas. The high-voltage is applied to the sides of the MRPC. When a charged particle goes through the detector, it causes an ionisation of the gas in MRPC, which provokes an electrons avalanche. This stream of electrons can be then read in form of the electric impulse on the readout plate. TOF provides a precise time measurement with the resolution of 80 ps [30], which (knowing the path of the particle) can be used to calculate particle's velocity. The information from TOF is usually combined with the measurements from other detectors in order to determine particles' masses and thus to perform identification.

3.3.2 Event reconstruction

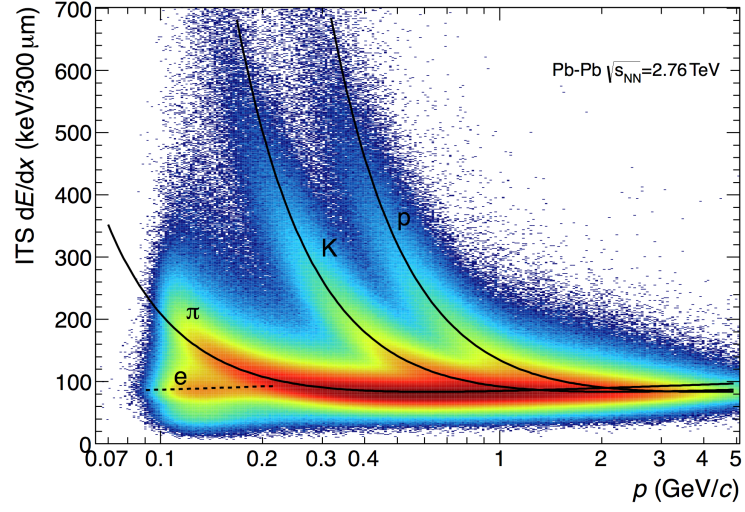
Reconstruction is a process in which raw information registered by detectors is transformed into physical parameters of the observed particles. It is performed in several steps. First, the electronic readout is translated into information about 3D coordinates of the points of interaction with the detector material. Each point, so-called cluster, may have additional parameters assigned (usually energy deposit in given point). Next, pattern recognitions algorithms are applied to assign clusters to track candidates. Finally, dedicated algorithm fits trajectories to optimally match reconstructed clusters, which leads to designation of tracks (see Fig.3.3).

In addition to the measurement of particle trajectory (track) it is crucial for the physics analysis to correctly identify types of particles. This information can be gained combining measurements from different detectors. Both ITS and TPC provide an information about the energy loss dE/dx , which is connected to the parameters of the particle, helping to identify them. The energy loss of charged particles caused by the Coulomb scattering with the electrons in atoms of the medium is described by the Bethe-Bloch formula [31]:

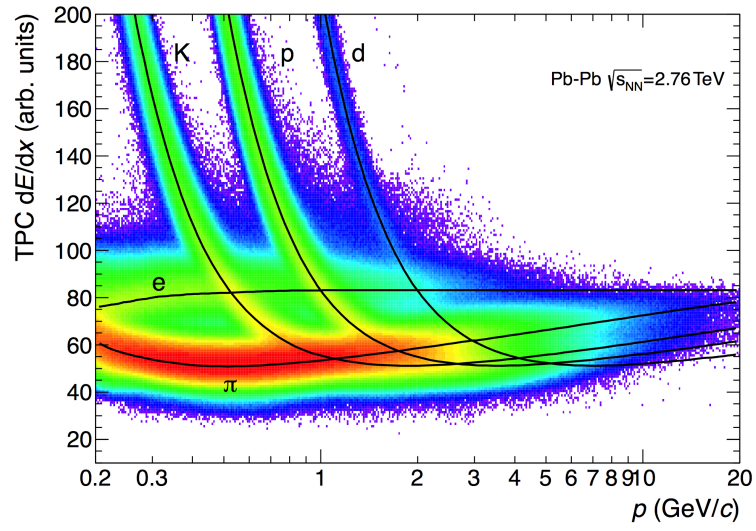
$$-\frac{dE}{dx} = \kappa z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \gamma^2 \beta^2}{I^2} E_{kin}^{max} - \beta^2 - \frac{\delta}{2} \right] \quad (3.1)$$

where κ is a constant, z is the charge of the particle, Z stands for the atomic number of the medium, A is a mass number of the medium, β is the velocity, m_e is the mass of the electron, c is the speed of light, γ is the Lorentz factor, I is the mean excitation energy of the material, E_{kin}^{max} is the maximum energy transfer in a binary collision and δ is a factor related to the possible polarization of the medium, which decreases probability of the interaction and thus also the energy loss.

Measurements of dE/dx can be seen in Fig.3.4. Particles can be identified effectively in the regions of momentum where bands do not overlap significantly. To improve identification in regions where separation is not clear enough, one can include measurement of the velocity from TOF. An example of the β measurements as well as the combination of the dE/dx measurement from TPC with the β measurement from TOF can be found in Fig.3.5.

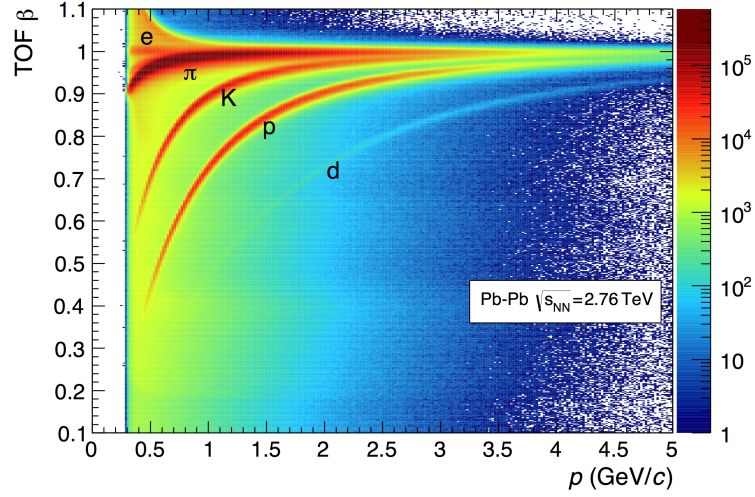


(a) ITS

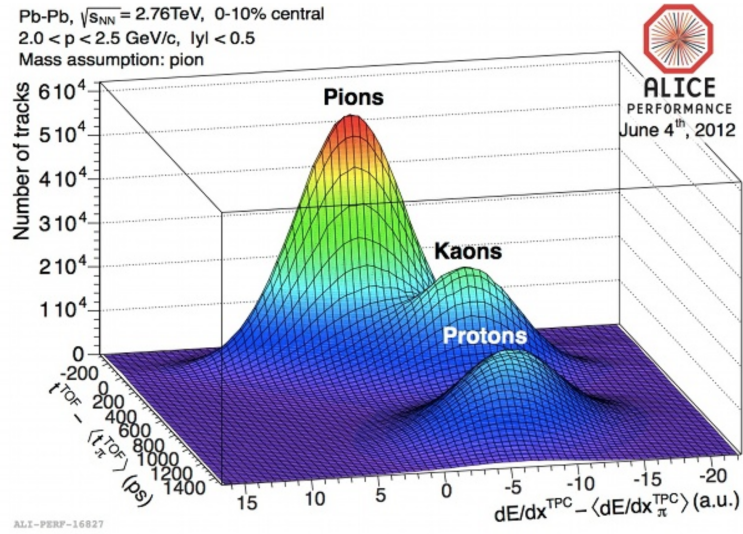


(b) TPC

Figure 3.4: Energy loss dE/dx measured by ALICE detectors [23].



(a) TOF



(b) TPC+TOF

Figure 3.5: TOF used for particle identification. (a) TOF measurement of particles' velocity β [23]. (b) Combination of TPC dE/dx with TOF β measurements for particle identification [32].

Once reconstruction of the parameters of particles registered directly by ALICE detector is done, the so-called secondary vertices (the place where unstable particle decays) can be found thanks to the known decay modes of heavier particles. For example, the Λ baryon decays into proton and pion, which can be observed directly in the detector. Then, for each pair of such particles one can check a hypothesis that they come from the decay of a Λ baryon, looking at the distance of closest approach of the two tracks and measuring the invariant mass of the particles' pair. This process can be repeated to find even heavier particles, which decay in cascades, for example $\Xi^- \rightarrow \Lambda^0 + \pi^- \rightarrow p + \pi^- + \pi^-$. The illustration of the secondary particles reconstruction is presented in Fig.3.6.

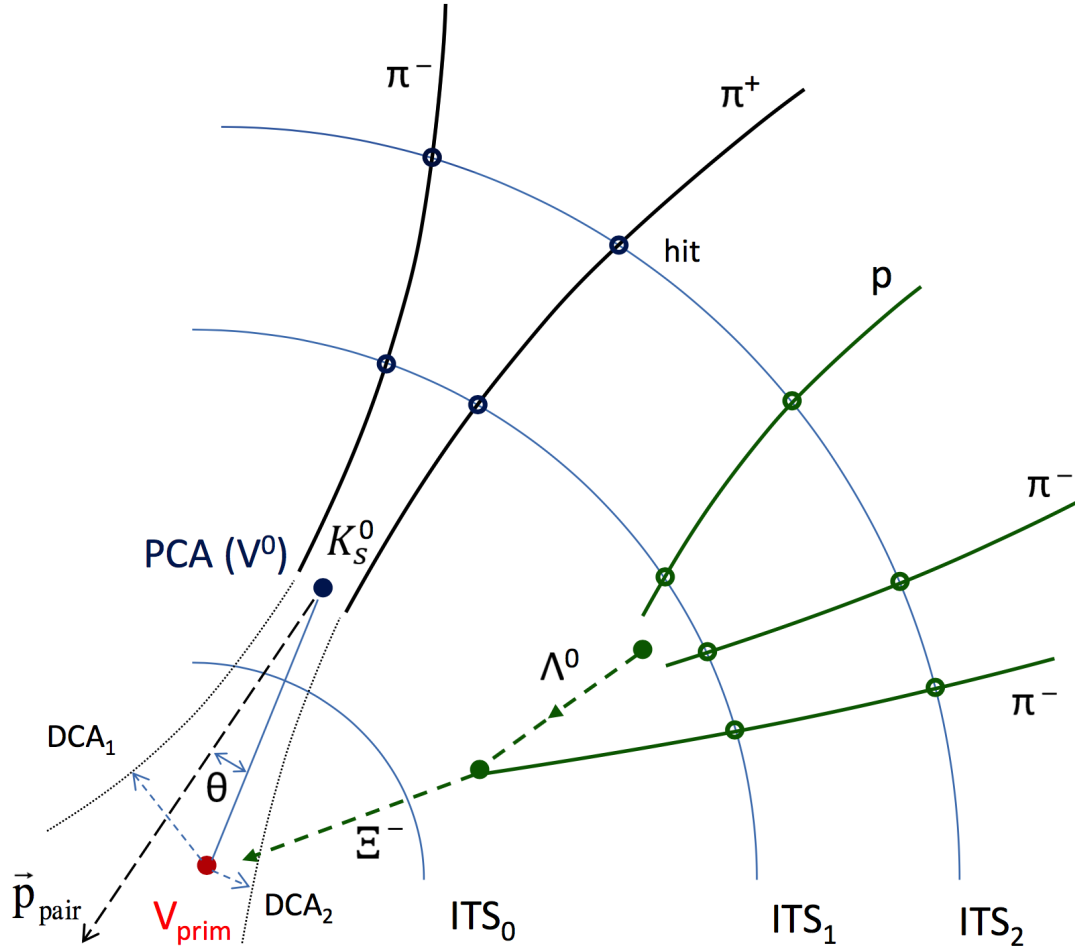


Figure 3.6: Principle of the secondary vertex reconstruction [23].

3.3.3 Event visualisation

As the output of all the detectors is purely electronic, a dedicated system has to be used in order to visualise measurements done by the experiments. This is crucial not only to ease the perception of what happens after the collision, but also as an early check of the correctness of the reconstruction.

An important part of the work performed during my PhD studies was the development and maintenance of the event visualisation system of ALICE. Detailed description of the system as well as projects related to the visualisation can be found in Appendix A. An example of the proton-proton event visualisation is shown in Fig.3.7 and for lead-lead collision in Fig.3.8

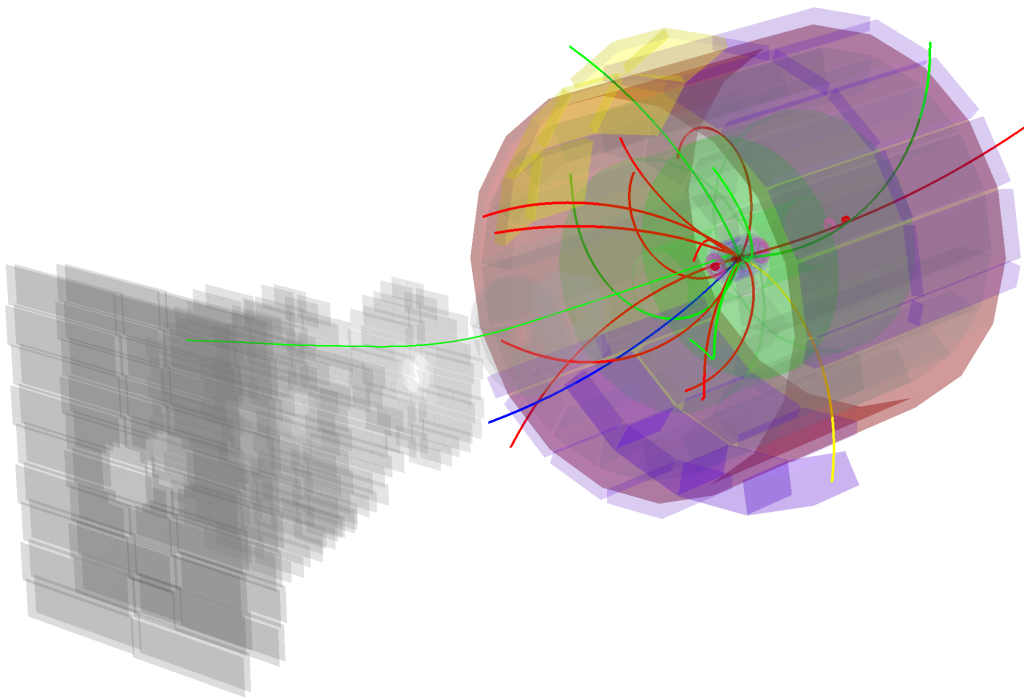


Figure 3.7: An example of the proton-proton event visualisation with ALICE Event Visualisation Environment.

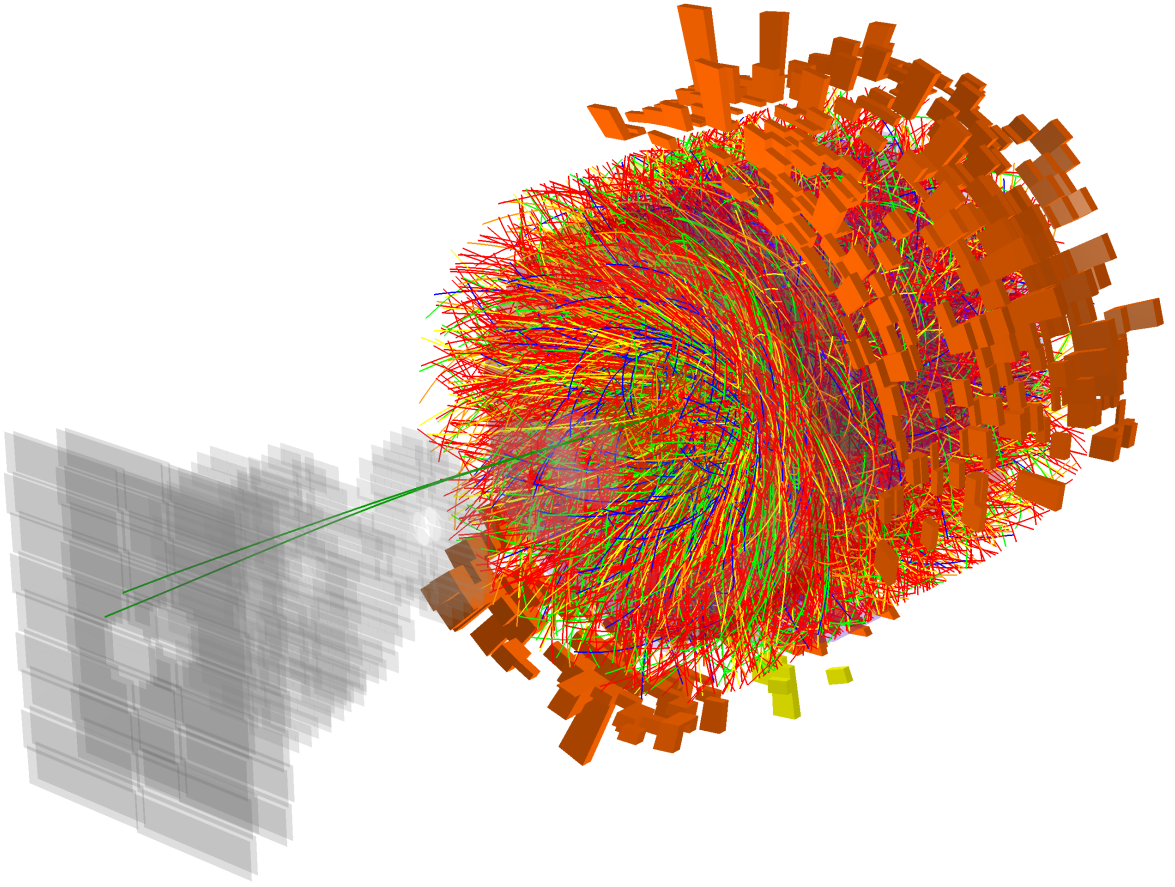


Figure 3.8: An example of the Pb-Pb event visualisation with ALICE Event Visualisation Environment.

Chapter 4

Two-particle correlations at low relative momentum

Correlations of the momenta of particles produced in relativistic heavy-ion collisions can provide unique information about the processes taking place at the sub-nucleus scale. In this chapter a theoretical introduction to the method based on the correlations in momenta of the pairs of (anti)baryons will be given. Then, it will be shown how one can measure the correlation function in the experiment.

4.1 Baryon correlations

In case of (anti)baryon pairs there are mainly three sources of correlations in the resulting correlation function: Quantum Statistics (QS), strong interaction and Coulomb interaction [33][34].

The QS will be present for identical baryons, thus it should be taken into account when calculating for example $\Lambda\Lambda$ correlation function. The strong interaction will significantly affect function's shape for all baryon pairs. The Coulomb interaction should be included in calculations for pairs of charged particles, such as $p\Xi^-$ or $\Sigma^+\Xi^-$. In some cases, several effects have to be combined - as an example one can take the $\Sigma^+\Sigma^+$ pair, where all three sources of correlations are present.

The theoretical overview of the Quantum Statistics will be shown in Sec.4.1.1, while the influence of the strong and Coulomb interaction will be presented in Sec.4.2.

4.1.1 Quantum Statistics

4.1.1.1 Hanbury-Brown Twiss effect

To describe the influence of the Quantum Statistics on the two-particle correlation function one should first move back to its origin in astronomy. The original method was proposed in 1954 by Robert Hanbury-Brown and Richard Twiss for the measurements of the sizes of stars based on the intensity interferometry of the electromagnetic waves which they emit [35]. It was named after its inventors the Hanbury-Brown Twiss effect (HBT).

The illustration of the method is shown in Fig. 4.1. Two particles (photons in this case) are emitted from two points x_1 and x_2 in the star at a distance L . They are then registered in detectors A and B at locations x_a and x_b respectively. The amplitude of the signal measured by detectors A and B is a superposition of the emitted waves and depends on the detectors' separation $d = |x_a - x_b|$. Performing a measurement of the momenta of photons and combining it with the measurement of the distance to the star L , one can derive the average separation R in the emission sources.

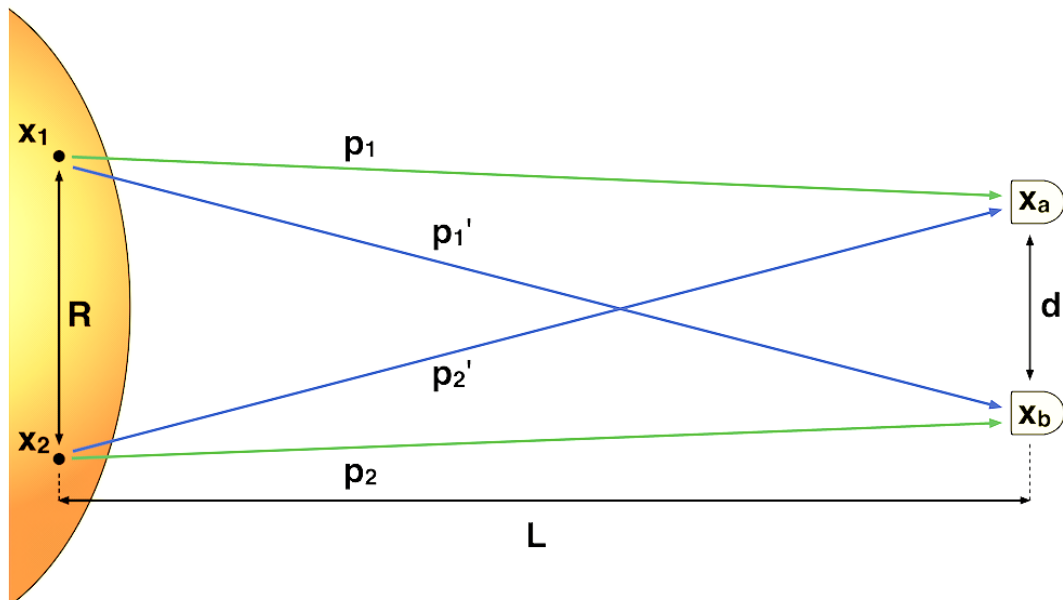


Figure 4.1: Illustration of the HBT measurement principle.

4.1.1.2 Intensity interferometry in heavy-ion collisions

The modified version of the HBT method can be also applied to particles produced in the heavy-ion collisions. From the point of view of Quantum Mechanics, all particles, such as baryons and mesons, are described by the wave functions, which interfere similarly to photons in the HBT measurement. In case of the identical particles, it is not possible to distinguish the following scenarios:

- particle with momentum p_1 is emitted from location x_1 and particle with momentum p_2 is emitted from location x_2 ,
- particle with momentum p_1 is emitted from location x_2 and particle with momentum p_2 is emitted from location x_1 ,

Because of that, the pair wave function should be written as a superposition of these two, equally probable cases:

$$\Psi_{12} = \frac{1}{\sqrt{2}} \left(e^{ip_1 x_1 + ip_2 x_2} \pm e^{ip_1 x_2 + ip_2 x_1} \right) \quad (4.1)$$

Depending on the type of particles, the wave function should be symmetrized or antisymmetrized. For integer spin bosons, the (+) sign should be taken for the symmetrization of the wave function to obey the Bose-Einstein statistics. The (-) sign corresponds to antisymmetrization due to the Fermi-Dirac statistics. In case of a pair of half-spin fermions both symmetrization and antisymmetrization take place, depending on the configuration of spins (see next section).

One can define a general form of the two-particle correlation function:

$$C(k^*) = \frac{\int S_{4D}(\bar{r}^*) \left| \Psi(\bar{k}^*, \bar{r}^*) \right|^2 d^4 \bar{r}^*}{\int S_{4D}(\bar{r}^*) d^4 \bar{r}^*} \quad (4.2)$$

where $\bar{r}^*$ denotes 4D space-time separation and $S(\bar{r}^*)$ is a four-dimensional source emission function of a pair, which describes the probability density of the emission of particles with given space-time separation. The $\bar{k}^*$ is a vector describing a momentum of one of the particles in the pair rest frame (PRF). $\Psi(\bar{k}^*, \bar{r}^*)$ is a pair wave function, describing all the interactions between particles [33]. Detailed description of the theoretical correlation function will be presented in Sec.4.2.

4.1.1.3 Pair of baryons

In case of two half-integer spin particles, such as two baryons, the resulting spin of the pair can be either $S = 0$ or $S = 1$. There are three possible combinations resulting in $S = 1$, which is the so-called triplet state:

$$|\uparrow\uparrow\rangle \quad (4.3)$$

$$\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle \quad (4.4)$$

$$|\downarrow\downarrow\rangle \quad (4.5)$$

One combination results in $S = 0$, which is referred to as a singlet-state:

$$\frac{1}{\sqrt{2}}|\uparrow\downarrow\rangle - \frac{1}{\sqrt{2}}|\downarrow\uparrow\rangle \quad (4.6)$$

Depending on the resulting spin, the pair-wave function is either anti-symmetric (triplet states) or symmetric (singlet state). This has to be taken into account while constructing a correlation function, which will be a combination of functions of particles interacting in singlet and triplet states:

$$C(\mathbf{x}) = \frac{3}{4}C^t(\mathbf{x}) + \frac{1}{4}C^s(\mathbf{x}) \quad (4.7)$$

where $\mathbf{x}$ is a vector of variables, $C(\mathbf{x})$ is a full correlation function and $C^t(\mathbf{x})$ and $C^s(\mathbf{x})$ are correlation functions in triplet and singlet state respectively. Factor $\frac{3}{4}$ corresponds to the probability of the triplet state and $\frac{1}{4}$ is a probability of the singlet state in the unpolarized case.

4.1.2 Scattering theory

The scattering theory [36] is usually used to describe the interaction of particles from a beam with particles of the target or another beam. The basic quantity used to describe this type of processes is the differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{I_S}{I_E} \quad (4.8)$$

where I_S is the intensity of beam scattered on the target per unit solid angle per unit time:

$$I_S = \frac{\text{number of particles}}{\text{angle} \times \text{time}} \quad (4.9)$$

and I_E is the intensity of incoming beam per unit area angle per unit time:

$$I_E = \frac{\text{number of particles}}{\text{area} \times \text{time}} \quad (4.10)$$

The visual representation of the concept described above can be seen in Fig. 4.2. Usually another value is used to describe scattering processes, which is a total cross section, defined as the integral of the differential cross section over all solid angles:

$$\sigma_t = \int \frac{d\sigma}{d\Omega} d\Omega \quad (4.11)$$

Another useful quantity is the elastic cross section σ_e , which is defined as a cross section measured for particles scattered only in the elastic processes (so with no energy change during the process).

4.1.2.1 Effective range approximation

In the low-energy limit the description of the scattering can be significantly simplified using so-called effective range approximation. The low-energy region in quantum mechanics corresponds to the big wavelengths. If scattering particle has a big wavelength, one can say that it will not recognise the internal structure of the potential it is scattering on and thus what matters is only the magnitude of the potential as seen from a large distance.

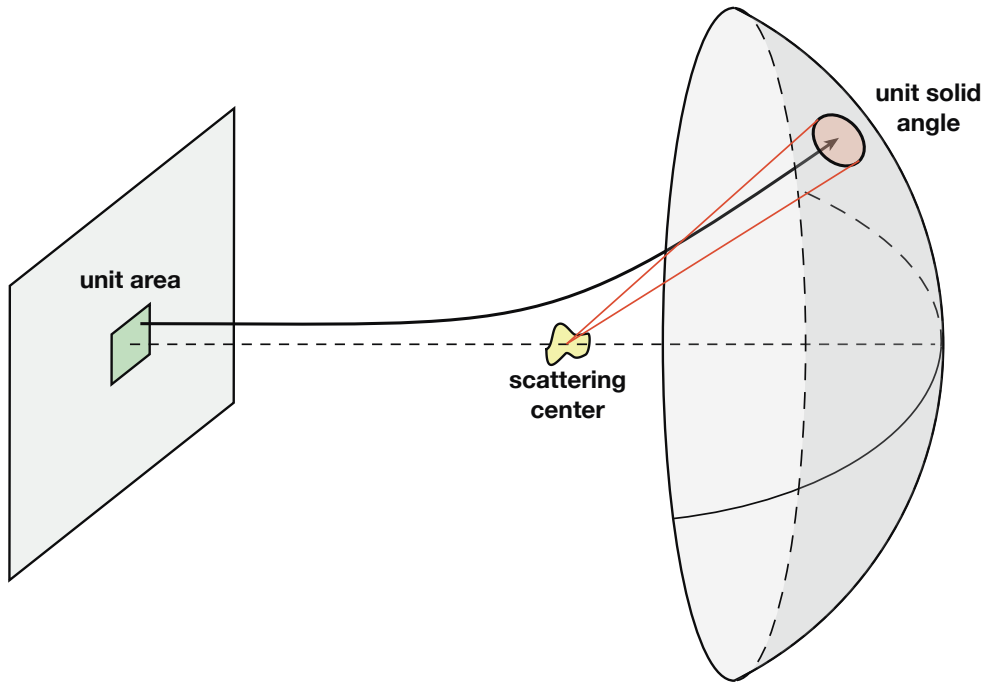


Figure 4.2: Illustration of the scattering theory basic concepts.

This allows to neglect the higher order components of the outgoing wave (angular momentum $l > 0$) and simplifies the form of the scattered wave to the symmetric s-wave ($l = 0$).

Under assumption of the spherically symmetric potential and the small relative momenta of the scattering particles the wave function can be defined as the sum of the incoming plane wave and the outgoing spherical s-wave [37]:

$$\Psi(k^*, r^*) = e^{-i\vec{k}^* \vec{r}^*} + f(k^*) \frac{e^{ik^* r^*}}{r^*} \quad (4.12)$$

where $f(k^*)$ is the scattering amplitude, which in the effective range approximation (small $r_0 k^*$, where r_0 is the length scale of the potential) is of the form [33]:

$$f(k^*) = \left[\frac{1}{f_0} + \frac{d_0 k^{*2}}{2} - ik^* \right]^{-1} \quad (4.13)$$

where f_0 (scattering length) and d_0 (effective range) are the complex numbers representing the effective scattering parameters. The total cross section is by definition the integral of

the square of the scattering amplitude over solid angles:

$$\sigma_t = \int |f(k^*)|^2 d\Omega \quad (4.14)$$

which can also be expressed as a series:

$$\int |f(k^*)|^2 d\Omega = \frac{4\pi}{k^{*2}} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l(k^*) \quad (4.15)$$

The first term in the low-energy limit equals:

$$\sigma_t \stackrel{l=0}{=} \frac{4\pi \sin^2 \delta_0(k^*)}{k^{*2}} \stackrel{k \rightarrow 0}{=} 4\pi |f(k^*)|^2 \quad (4.16)$$

The latter equation shows the relation between the introduced interaction parameters and the fundamental quantity of the scattering theory, which is the total cross section. Measuring those parameters one actually measures the low-energy region of the total scattering cross section.

4.1.2.2 Inelastic scattering

From the general form of the scattering amplitude (Eq.4.13) one can re-write Eq.4.16 in the following way:

$$\sigma_t = \frac{4\pi}{k^*} \Im f(k^*) \quad (4.17)$$

which for elastic scattering only leads to:

$$\Im f(k^*) = k^* |f(k^*)|^2 \quad (4.18)$$

This demonstrates, that in the low relative momentum region (where $k^* \rightarrow 0$), for elastic scattering, the imaginary part of the scattering amplitude vanishes, and so does the imaginary part of the scattering length f_0 .

If there exists an inelastic channel, in which momentum after scattering is $\tilde{k}^*$, the imaginary part of the scattering amplitude will be a sum of the elastic and inelastic components:

$$\Im f(k^*) = k^* |f(k^*)|^2 + \tilde{k}^* |f(\tilde{k}^*)|^2 \quad (4.19)$$

In such case, the relative momentum after scattering will not be zero, thus also the imaginary part of the scattering length will be positive ($\Im f_0 > 0$). This fact is important and will be used in further parts of this thesis, as in case of interactions of baryon and antibaryon created in a heavy-ion collision the main inelastic channel is through the annihilation. This means that measuring the imaginary part of the scattering length allows to understand the low-energy region of the baryon-antibaryon annihilation cross-section.

4.1.2.3 Status of knowledge of baryon interactions

Strong interaction of baryons and antibaryons is not well measured and understood. The Particle Data Group [6] collects all the measurement of properties of elementary particles and their interactions. In Fig. 4.3 and Fig. 4.4 all the measured cross sections for (anti)baryon pairs are shown. As can be seen, interaction between (anti)baryons have been measured only for a few systems: pn , pd , $\bar{p}n$, $\bar{p}d$, $p\Lambda$ and $p\Sigma^-$. One can also notice, that uncertainties for proton-lambda measurements are significant.

There are no measurements of the interaction cross section for many baryon-baryon pairs. The knowledge of the baryon-antibaryon interactions is even smaller and there are no measurements of those except $\bar{p}n$, $\bar{p}d$ and $\bar{p}p$.

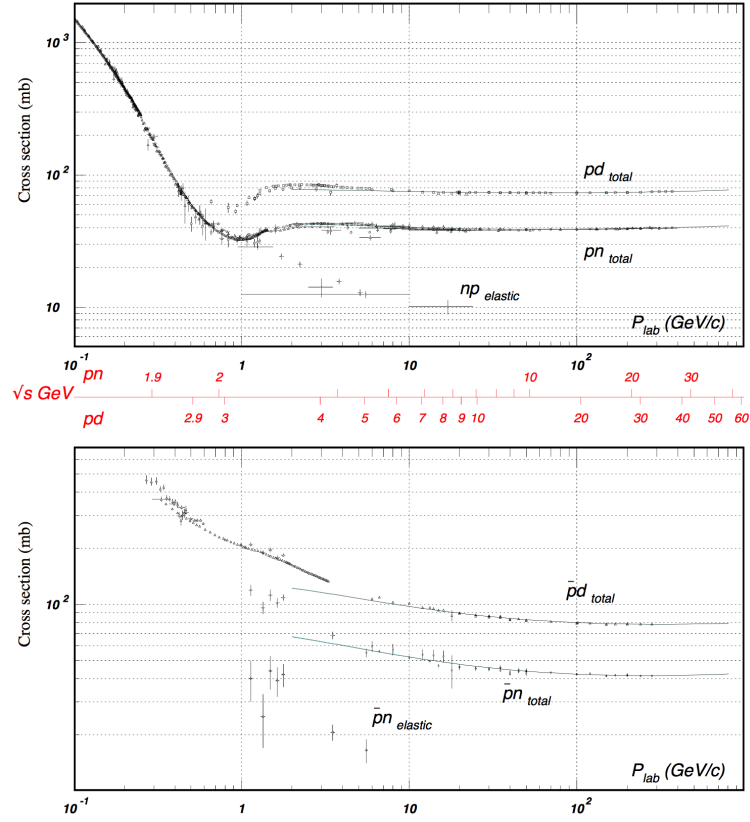


Figure 4.3: Measured cross sections of (anti)protons, neutrons and deuterons [6].

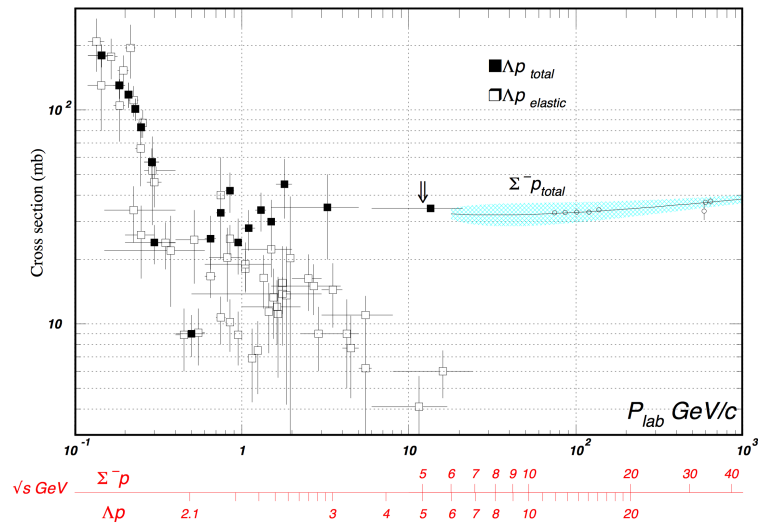


Figure 4.4: Measured cross sections for heavier baryons [6].

4.1.2.4 Extended-soft-core Model

Theoretical description of the interaction between two baryons is complicated, which is a consequence of the complex internal structure (valence quarks, gluons exchanging strong interaction and constant creation and annihilation of the sea quarks), as well as many possible scattering channels. One of the models which attempts to describe and predict parameters of the strong interaction for various baryon-baryon pairs is the Nijmegen Extended-soft-core Model (ESC) [38][39][40][41][42].

The Nijmegen models for baryon-baryon interactions have been developed since 1990s, gradually including more and more effects and taking into account the most recent advancements in the baryon interactions theory. It was also found to be the most complete model in terms of number of baryon combinations for which it gives predictions of the strong scattering parameters. The version which will be used in this thesis is denoted as ESC08c.

In general, Nijmegen models use the strategy of dividing strong potential into three ranges [38]:

- distances $r > 2 \text{ fm}$ (region-I), where potential is attractive,
- $r < 1 \text{ fm}$ (region-III) in which strong repulsion is present,
- the intermediate region-II, in which transition between repulsion and attraction occurs.

To describe the potential in different ranges, in ESC08c a number of interaction channels is considered. Here only a few examples:

- one-pion exchange to describe potential in region-I (the Yukawa interaction [43]),
- one-boson exchange and meson-pair exchange in the intermediate region-II,
- pomeron and odderon exchange (even and odd number of gluons exchanged) for the short-range region-III,
- Pauli exclusion principle for the core-quarks, affecting the potential in region-III.

Detailed description of the model for nucleon-nucleon scattering can be found in [39]. For baryons containing strange quarks (such as the Λ baryon), one can be referred to [40], [41] and [42]. The values of scattering parameters predicted by the model will be used for the data analysis presented in Chap.8 to reduce the number of free fit parameters.

4.1.2.5 Previous measurements of $p\bar{\Lambda}$ interaction parameters

A measurement of the proton-antilambda interaction parameters was first performed by STAR experiment [44]. The obtained results yield negative real part of the scattering length $\Re f_0 \approx -2 \text{ fm}$ and positive imaginary part $\Im f_0 \approx 1 \text{ fm}$.

The effective range d_0 was fixed to zero in [44], to reduce a number of free fit parameters. This could be improved by adding other systems which are linked with proton-antilambda through the residual correlations (see Chap.5).

In addition, in the aforementioned work, the extracted source size for proton-antilambda is about two times smaller than this obtained for proton-lambda. This unexpected result was addressed in [45], where it was found that one can obtain similar source sizes for proton-lambda and proton-antilambda if all residual correlations (see Chap.5) are taken into account. The obtained results are $\Re f_0 = 0.49 \pm 0.21 \text{ fm}$ and $\Im f_0 = 1.00 \pm 0.21 \text{ fm}$.

In the analysis presented in [45], $d_0 = 0 \text{ fm}$ is assumed and there is no binning in centrality. It was also assumed, that the source size is the same for all residual components.

The analysis of proton-antilambda correlations functions was also performed in [46]. In this case functions were measured in different centrality bins and residual correlations were taken into account. However, source size is still assumed to be the same for all residual components and d_0 is fixed to zero. Extracted real part of the scattering length is consistent with zero and its imaginary part is close to 1.5 fm .

In this thesis, correlation functions will be analysed in centrality bins and residual components will be taken into account. In addition, several improvements will be made:

- it will be taken into account that residual components do not have the same source radius as the primary pair,
- different combinations of (anti)protons and (anti)lambda baryons will be analysed together to increase statistics and decrease uncertainties,
- the effective range will be treated as a free parameter of the fit.

4.2 Theoretical correlation function

The theoretical correlation function as defined in Eq.4.2 depends on the source emission function and the pair-wave function. In this section detailed description of both components will be presented.

4.2.1 Source emission function

The 3D source emission function for a pair of particles can be defined as [47]:

$$S(\bar{r}^*) = \exp \left[-\frac{r_{out}^{*2} + r_{side}^{*2} + r_{long}^{*2}}{4r_0^2} \right] \quad (4.20)$$

where r_{out}^* , r_{side}^* and r_{long}^* are separations in the directions of the Longitudinally Co-Moving System (LCMS), where for each pair of particles a reference frame is selected in a way that the resultant longitudinal momentum vanishes. The out direction is parallel to the transverse momentum of the pair, which is a projection of the pair-momentum vector on the transverse plane. Thus side is the direction perpendicular to out and long. The r_0 is the width of the distribution and from now on and will be referred to as the source size or the source radius. In the experiment a symbol R_{inv} is used for the source radius to make a distinction between theoretical parameter and the measurement. In further chapters of this thesis the latter symbol will be used.

The usually used simplification is to transform the source function from 3D coordinates to 1D by averaging over all angles. In such case replacing the 3+1 dimensional source function in Eq.4.2 by spherically-symmetric one-dimensional function requires change of coordinates and thus the Jacobian of the transformation should be put under the integral:

$$C(k^*) = \frac{\int |J(r^*, \phi, \theta)| S(r^*) |\Psi(\bar{k}^*, \bar{r}^*)|^2 dr^* d\phi d\theta}{\int |J(r^*, \phi, \theta)| S(r^*) dr^* d\phi d\theta} \quad (4.21)$$

The Jacobian of the transformation from cartesian to the spherical coordinate system is $r^2 \sin \phi$. The source function and the pair-wave function do not depend on the ϕ angle, so the integral over ϕ can be reduced from nominator and denominator. All normalisation factors from the source function will also vanish, as they are the same in the nominator and denominator. This leads to the final form of the two-particle correlation function with

one-dimensional gaussian source:

$$C(k^*) = \frac{\int r^{*2} \exp\left(\frac{-r^{*2}}{4r_0^2}\right) |\Psi(\bar{k}^*, \bar{r}^*)|^2 dr^* d\theta}{\int r^{*2} \exp\left(\frac{-r^{*2}}{4r_0^2}\right) dr^* d\theta} \quad (4.22)$$

4.2.2 Pair-wave function

4.2.2.1 Strong interaction

In case of two baryons interacting with strong force only, the scattering amplitude is given by Eq.4.13. For the calculation of the correlation function the squared modulus of the amplitude is needed. It is convenient to first write separately real and imaginary parts of the scattering amplitude:

$$\Re \Psi(\bar{k}^*, \bar{r}^*)^S = \frac{f_1 \cos(k^* r^*) + f_2 \sin(k^* r^*)}{r^*(f_1^2 + f_2^2)} + \cos(\bar{k}^* \bar{r}^*) \quad (4.23)$$

$$\Im \Psi(\bar{k}^*, \bar{r}^*)^S = \frac{f_1 \sin(k^* r^*) - f_2 \cos(k^* r^*)}{r^*(f_1^2 + f_2^2)} - \sin(\bar{k}^* \bar{r}^*) \quad (4.24)$$

where f_0 and d_0 are the scattering parameters introduced in Sec.4.1.2.1 and:

$$f_1 = \frac{\Re f_0}{|f_0|^2} + \frac{d_0 k^{*2}}{2} \quad f_2 = -\frac{\Im f_0}{|f_0|^2} - k^* \quad (4.25)$$

Then the resulting pair-weight takes the following form:

$$|\Psi(k^*, r^*)^S|^2 = 1 + \frac{1 + 2r^* [f_1 \cos(\zeta) + f_2 \sin(\zeta)]}{r^{*2}(f_1^2 + f_2^2)} \quad (4.26)$$

where $\zeta = k^* r^* (1 + \cos \theta^*)$ and θ^* is the angle between $\bar{k}^*$ and $\bar{r}^*$.

4.2.2.2 Strong and Coulomb interactions

In case where two particles interact both with strong and Coulomb interactions, the form of the pair wave function changes [34]:

$$\Psi(\bar{k}^*, \bar{r}^*) = \sqrt{A_C(\eta)} \left[e^{-i\bar{k}^* \bar{r}^*} F(-i\eta, 1, i\zeta) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right], \quad (4.27)$$

with a modified scattering amplitude:

$$f_C(k^*) = \left[\frac{1}{f_0} + \frac{d_0 k^{*2}}{2} - i k^* A_C(k^*) - \frac{2h(\eta)}{a_C} \right]^{-1}, \quad (4.28)$$

where:

- $A_C(\eta)$ is a Gamow factor depending on the masses of particles and k^* ,
- $\eta = (k^* a_C)^{-1}$ (a_C is a Bohr radius of the pair),
- F is a confluent hypergeometric function,
- $\rho = k^* r^*$,
- $\zeta = \rho(1 + \cos\theta^*)$ (where θ^* is an angle between k^* and r^*),
- $f_C(k^*)$ is a strong scattering amplitude including Coulomb interaction,
- $\tilde{G}$ is a known function which combines regular and singular s-wave Coulomb functions,
- $h(\eta)$ is a known function.

The Gamow factor is defined as:

$$A_C(\eta) = 2\pi\eta [\exp(\pm 2\pi\eta) - 1]^{-1} \quad (4.29)$$

The confluent hypergeometric function is given by:

$$F(\alpha, 1, z) = 1 + \frac{\alpha z}{1!^2} + \frac{\alpha(\alpha+1)z^2}{2!^2} + \dots \quad (4.30)$$

$\tilde{G}$ function takes the following form:

$$\tilde{G}(\rho, \eta) = P(\rho, \eta) + 2\eta\rho [\ln|2\eta\rho| + 2C - 1 + \chi\eta] B(\rho, \eta) \quad (4.31)$$

where:

$$P(\rho, \eta) = \sum_{s=0}^{\infty} P_s \quad P_0 = 1 \quad P_1 = 0 \quad \dots \quad (4.32)$$

$$B(\rho, \eta) = \sum_{s=0}^{\infty} B_s \quad B_0 = 1 \quad B_1 = \eta\rho \quad \dots \quad (4.33)$$

and the following terms are given by:

$$n(n+1)P_{n+1} = 2\eta\rho P_n - \rho^2 P_{n-1} - (2n-1)2\eta\rho B_n \quad (4.34)$$

$$(n+1)(n+2)B_{n+1} = 2\eta\rho B_n - \rho^2 B_{n-1} \quad (4.35)$$

The $h(\eta)$ function is defined as:

$$h(\eta) = \frac{1}{\eta^2} \sum_{n=1}^{\infty} \frac{1}{n(n^2 + \eta^{-2})} - C + \ln |\eta| \quad (4.36)$$

where $C = 0.5772$ is the Euler constant.

Regardless of whether the Coulomb interaction is present or not, the resulting correlation function depends only on the space-momentum properties of the particles and the strong interaction parameters f_0 and d_0 .

4.2.3 (Anti)symmetrization of the pair-wave function

In case of identical baryons (for example $\Lambda\Lambda$) one should take into account symmetrization and anti-symmetrization of the pair-wave function. Space-(anti)symmetrization of the pair-wave function comes from the fact that two particles from the pair can be exchanged and the modulus of the wave function should not change its value. This can be written as follows:

$$\Psi(\bar{k}^*, -\bar{r}^*) = \Psi(\bar{k}^*, \bar{r}^*) \quad (4.37)$$

or

$$\Psi(\bar{k}^*, -\bar{r}^*) = -\Psi(\bar{k}^*, \bar{r}^*) \quad (4.38)$$

where first case is referred to as symmetric and second one as antisymmetric. To achieve (anti)symmetrization one should write a wave function as a superposition of particle registered by detector A emitted from location x_1 and the one registered by detector B and emitted from x_2 with the inverted case (see Sec.4.1.1):

$$\Psi(\bar{k}^*, \bar{r}^*)_{sym} = \frac{1}{\sqrt{2}} \Psi(\bar{k}^*, \bar{r}^*)_{AB} \pm \frac{1}{\sqrt{2}} \Psi(\bar{k}^*, \bar{r}^*)_{BA} \quad (4.39)$$

where one takes $+$ sign for symmetrization and $-$ sign for antisymmetrization. When two particles are exchanged, the direction of $\bar{r}^*$ vector changes and so does the angle between

k^* and r^* (denoted as θ^*). The angle after exchange of particles will be $\theta_{ex}^* = 180^\circ - \theta^*$. The illustration of the two cases is shown in Fig.4.5.

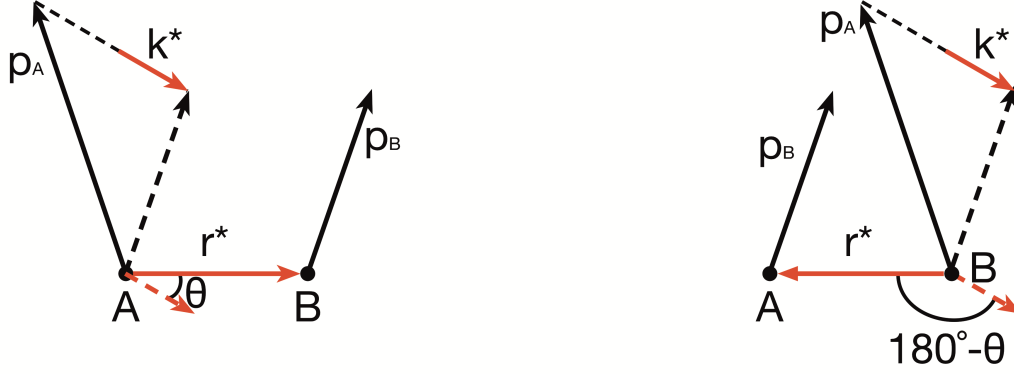


Figure 4.5: Illustration of the particles exchange and its impact on θ^* angle.

4.2.3.1 Strong interaction - symmetrization

The symmetrized version (see Sec.4.2.3) of the pair-wave function with only the strong interaction can be written as follows:

$$\Psi(\bar{k}^*, \bar{r}^*)_{sym}^S = \frac{1}{\sqrt{2}} \left[e^{-i\bar{k}^* \bar{r}^*} + f(k^*) \frac{e^{ik^* r^*}}{r^*} \right] + \frac{1}{\sqrt{2}} \left[e^{i\bar{k}^* \bar{r}^*} + f(k^*) \frac{e^{ik^* r^*}}{r^*} \right] \quad (4.40)$$

In the second part of the above equation minus sign before vector product vanishes, because angle after exchange of particles is $\theta_{ex}^* = 180^\circ - \theta^*$, and thus $\cos \theta_{ex}^* \rightarrow -\cos \theta^*$. Then the equation can be further evaluated to:

$$\Psi(\bar{k}^*, \bar{r}^*)_{sym}^S = \sqrt{2} \left[\cos(\bar{k}^* \bar{r}^*) + f(k^*) \frac{e^{ik^* r^*}}{r^*} \right] \quad (4.41)$$

The part with the scattering amplitude $f(k^*)$ does not depend on r^* , the denominator is a value of separation which does not change under exchange of particles and the cosine is by definition symmetric.

For numeric calculations it is convenient to write real and imaginary part of the above wave function separately. For the form of the scattering amplitude introduced in Eq.4.13

the real part takes form:

$$\Re \Psi(\bar{k}^*, \bar{r}^*)_{sym}^S = \sqrt{2} \left[\frac{f_f \cos(k^* r^*) - k^* \sin(k^* r^*)}{r^* (f_f^2 + k^{*2})} + \cos(\bar{k}^* \bar{r}^*) \right] \quad (4.42)$$

and imaginary part is:

$$\Im \Psi(\bar{k}^*, \bar{r}^*)_{sym}^S = \sqrt{2} \frac{f_f \sin(k^* r^*) + k^* \cos(k^* r^*)}{r^* (f_f^2 + k^{*2})} \quad (4.43)$$

where f_f is defined as:

$$f_f = \frac{1}{f_0} + \frac{d_0 k^{*2}}{2} \quad (4.44)$$

This finally leads to the squared modulus of the pair-wave function where Quantum Statistics and strong interactions are present:

$$\left| \Psi(\bar{k}^*, \bar{r}^*)_{sym}^S \right|^2 = 4 \frac{f_f \cos(k^* r^*) - k^* \sin(k^* r^*)}{r^* (f_f^2 + k^{*2})} \cos(\bar{k}^* \bar{r}^*) + \frac{2}{r^{*2} (f_f^2 + k^{*2})} + 2 \cos^2(\bar{k}^* \bar{r}^*) \quad (4.45)$$

The above formula can be used directly in Eq.4.2 for singlet state in case there is only strong interaction and Quantum Statistics.

4.2.3.2 Strong interaction - antisymmetrization

In case of antisymmetrization (see Sec.4.2.3) the pair-wave function with only the strong interaction takes the following form:

$$\Psi(\bar{k}^*, \bar{r}^*)_{asym}^S = \frac{1}{\sqrt{2}} \left[e^{-i\bar{k}^* \bar{r}^*} + f(k^*) \frac{e^{ik^* r^*}}{r^*} \right] - \frac{1}{\sqrt{2}} \left[e^{i\bar{k}^* \bar{r}^*} + f(k^*) \frac{e^{ik^* r^*}}{r^*} \right] \quad (4.46)$$

As angle after exchange of particles is $\theta_{ex}^* = 180^\circ - \theta^*$, and thus $\cos \theta_{ex}^* \rightarrow -\cos \theta^*$, the minus sign before vector product vanishes in the second part of the above equation. This can be further evaluated to:

$$\Psi(\bar{k}^*, \bar{r}^*)_{asym}^S = -i \sqrt{2} \sin(\bar{k}^* \bar{r}^*) \quad (4.47)$$

Then one can easily calculate the squared modulus of the pair-wave function where Quantum Statistics and strong interactions are present:

$$\left| \Psi(\bar{k}^*, \bar{r}^*)_{asym}^S \right|^2 = 2 \sin^2(\bar{k}^* \bar{r}^*) \quad (4.48)$$

Which can be used in Eq.4.2 for triplet states in case there is only strong interaction and Quantum Statistics.

4.2.3.3 Strong interaction and Quantum Statistics for a pair of baryons

To write down a full correlation function for a pair of identical baryons, one could use Eq.4.7 and calculate singlet and triplet components according to Eq.4.45 and Eq.4.48 respectively. However, according to [33], this can be simplified to the following form:

$$C(k^*)_{identical} = -\frac{1}{2} \exp(-2k^{*2}r_0^2) + C^S(k^*) \quad (4.49)$$

where $C^S(k^*)$ is a correlation function where only strong interaction is present. A comparison of both methods of calculating correlation function for identical, strongly-interacting baryons, as well as singlet and triplet components, are presented in Fig.4.6. Functions drawn with points are calculated by numerical integration of the gaussian source with wave functions presented in Sec.4.2.3.1 and Sec.4.2.3.2. Lines represent analytical function given by the Eq.4.49. As can be seen, analytical formula fits perfectly to the red points representing numerical integration of the correlation function. As full numerical integration is much more resource-demanding, the analytical formula will be used in further parts of this thesis where Quantum Statistics and the strong interactions are present in the correlation function.

4.2.3.4 Strong and Coulomb interactions - (anti)symmetrization

For the pair where both strong and Coulomb interactions are present, the ζ variable is sensitive on the exchange of particles, as its general form is:

$$\zeta = k^* r^* + \bar{k}^* \bar{r}^* \quad (4.50)$$

When the direction of $\bar{r}^*$ changes, ζ will also change. For convenience $\zeta_{ex} = k^* r^* (1 - \cos \theta^*)$ will be used to denote this argument when particles are exchanged. In this case, the (anti)symmetrized pair wave function takes the following form:

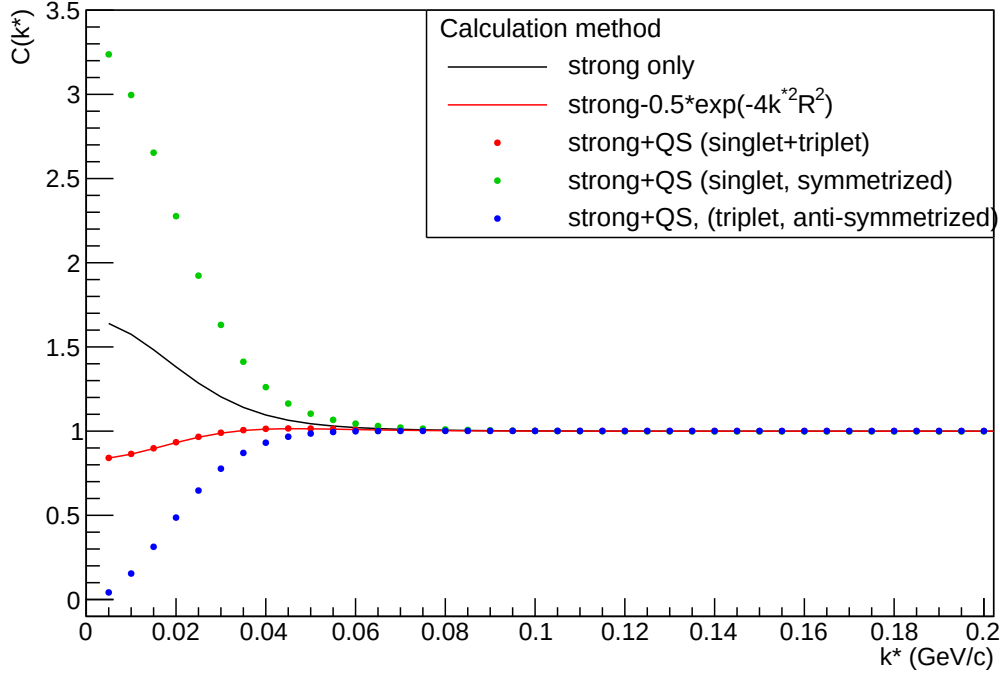


Figure 4.6: Comparison of two methods of calculating correlation function with strong interaction and Quantum Statistics.

$$\begin{aligned} \Psi(k^*, r^*)_{sym}^{SC} = & \frac{\sqrt{A_C(\eta)}}{\sqrt{2}} \left[e^{-i\vec{k}^* \vec{r}^*} F(-i\eta, 1, i\zeta) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right] \\ & \pm \frac{\sqrt{A_C(\eta)}}{\sqrt{2}} \left[e^{i\vec{k}^* \vec{r}^*} F(-i\eta, 1, i\zeta_{ex}) + f_C(k^*) \frac{\tilde{G}(\rho, \eta)}{r^*} \right] \end{aligned} \quad (4.51)$$

where (+) sign should be taken for symmetrization and (−) sign for antisymmetrization. In case where strong interaction, Coulomb interaction and Quantum Statistics are present, there is no an easy way to analytically calculate the modulus of pair-wave function, so one has to perform the numerical calculation.

4.2.4 Lednický - Lyuboshitz model

The Lednický - Lyuboshitz model [33] is a way to analytically integrate Eq.4.22 in case only strong interaction is present, so that it can be used for fast fits of the experimental data.

Assuming the gaussian shape of the source function, for source sizes $r_0 > 1.5 \text{ fm}$ (outside of the potential), the following formula can be derived to describe the two-particle

correlation function with only the strong interaction:

$$C(k^*) = 1 + \sum_S \rho_S \left[\frac{1}{2} \left| \frac{f(k^*)}{r_0} \right|^2 \left(1 - \frac{d_0^S}{2\sqrt{\pi}r_0} \right) + \frac{2\Re f(k^*)}{\sqrt{\pi}r_0} F_1(2k^*r_0) - \frac{\Im f(k^*)}{r_0} F_2(2k^*r_0) \right] \quad (4.52)$$

where the sum is put to account for singlet or triplet spin state and F_1 and F_2 are known functions defined as follows:

$$F_1(z) = \int_0^z \frac{e^{x^2-z^2}}{z} dx \quad F_2(z) = \frac{1 - e^{-z^2}}{z} \quad (4.53)$$

Formula 4.52 is very useful for fitting the experimental data, as the computational time is much smaller than for the numerical integration of the Eq.4.22. On the other hand, the assumptions made to derive this formula may introduce some significant differences when comparing to the theoretical function. A detailed analysis of the possible differences will be presented in Sec.6.1.

Unfortunately, similar formula taking into account Coulomb interactions does not exist, which means that numerical integration of the pair-wave function with the source emission function has to be performed to obtain values of the correlation function in this case, which significantly slows down the fitting procedure.

4.3 Experimental approach to the correlation function

Two-particle correlation functions for baryon pairs can be measured directly in heavy-ion experiments like ALICE. It can be defined as:

$$C(p_1, p_2) = \frac{P(p_1, p_2)}{P(p_1)P(p_2)} \quad (4.54)$$

where $P(p_1, p_2)$ in the nominator is the probability of observing particles with momenta p_1 and p_2 in the same event, while $P(p_1)$ and $P(p_2)$ in the denominator represent probabilities of registering particles with those momenta independently, i.e. in two different events.

The momentum distribution in the nominator will be affected by random combinations of momenta, the detector effects (such as limited acceptance), but it will be also slightly modified by the interactions between particles. The term in the denominator will also contain distribution of momenta affected by the detector effects, but it will not include interactions, as particles contributing to this part come from different events and thus could not interact.

Defining the correlation function in the way shown above allows to get rid of some of the detector effects and to divide out the statistical distribution of the momenta, while leaving the two-particle effects resulting from the fundamental interactions.

Experimental correlation functions and fits using theoretical formula or Lednicky - Lyuboshitz model will be presented in Chap.8.

Chapter 5

Interlinks between all baryon correlations (residual correlations)

One of the most important effects one should take into account when analysing experimental correlation functions are so-called residual correlations. Heavy particles are known to decay into lighter ones, for example, the Λ baryon may decay into proton-pion pair.

One can imagine a situation in which a primary proton and a Λ baryon are produced in the collision and interact with each other. Then, the Λ baryon decays and proton and pion are registered in the detector. What will be seen in the analysis are two protons (the primary one and the one coming from the decay) and a pion. Building a proton-proton correlation function one will in fact observe effects of the interaction between proton and Λ baryon instead of desired proton-proton interaction. The influence coming from this type of correlations is called residual correlations and is illustrated in Fig.5.1.

Residual correlations are present for almost all systems one may want to study in heavy-ion collisions. It should not be treated as a contamination that needs to be removed. Residual correlations are an integral part of the correlation functions analysis. One can actually notice, that the presence of residual correlations can be beneficial - measuring the correlation function of baryons which can be easily registered in the experiment one can simultaneously measure some effects coming from the interaction of heavier baryons which cannot be directly observed in the detector. In other words, residual correlations are a visible manifestation of the interactions between particles which otherwise cannot be measured in the experiment.

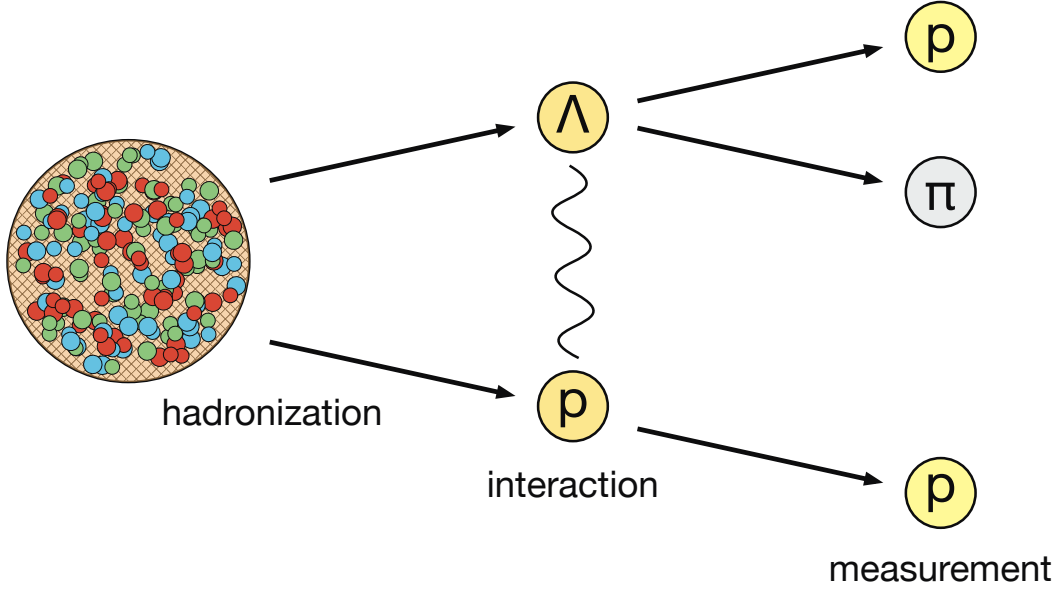


Figure 5.1: Illustration of the residual correlations mechanism.

In this thesis mainly correlation functions of (anti)protons and (anti)lambdas will be studied. The most common decays containing one of them are:

- $\Lambda \rightarrow p\pi^-$
- $\Sigma^+ \rightarrow p\pi^0$
- $\Sigma^0 \rightarrow \Lambda\gamma$
- $\Xi^0 \rightarrow \Lambda\pi^0$
- $\Xi^- \rightarrow \Lambda\pi^-$
- $\Omega^- \rightarrow \Lambda K^-$

When studying a correlation function, one should take into account decays leading to both particles in question. The measured correlation function will be a weighted sum of all contributions, which can be formally written as [45]:

$$C_{xy}(k^*) = 1 + \sum_i \lambda_i [C_i(k^*) - 1] \quad (5.1)$$

where xy is a measured system (for example $p\Lambda$), i denotes residual pairs influencing system xy and λ_i is a fraction of xy pairs coming from the decay of pair i . In the experiment some particles can be incorrectly identified, which means that part of the calculated

correlation function will come from particles other than the studied ones and those which decay into them. Also for this reason, fractions of residual components will not sum up to unity. This is connected with purity, which defines how many particles reconstructed as x were real, primary particles of type x . One should either correct the correlation function for the purity of the sample or take this effect into account while calculating fractions.

All main residual contributions to the pp , $p\Lambda$ and $\Lambda\Lambda$ functions are listed in Tab.5.1. One can notice, that some pairs of particles contribute via residual correlation to more than one system, which is also illustrated in the table. For example, correlation coming from $\Lambda\Lambda$ interaction is a primary component of $\Lambda\Lambda$ correlation function, but at the same time it is a residual component of $p\Lambda$ function and pp function as well. Because of that, one should treat residual correlations as a net of interlinks between various systems and take advantage of this fact when fitting experimental functions rather than applying a top-down approach.

pp residual components	$p\Lambda$ residual components	$\Lambda\Lambda$ residual components
pp	$p\Lambda$	$\Lambda\Lambda$
$p\Lambda$	$\Lambda\Lambda$	$\Lambda\Sigma^0$
$\Lambda\Lambda$	$\Lambda\Sigma^0$	$\Lambda\Xi^0$
$p\Sigma^+$	$\Lambda\Xi^0$	$\Lambda\Xi^-$
$\Lambda\Sigma^+$	$\Lambda\Xi^-$	$\Lambda\Omega^-$
$\Sigma^+\Sigma^+$	$\Lambda\Omega^-$	$\Xi^0\Xi^-$
	$p\Sigma^0$	$\Xi^0\Xi^0$
	$p\Xi^0$	$\Xi^-\Xi^-$
	$p\Xi^-$	$\Xi^-\Sigma^0$
	$\Lambda\Sigma^+$	$\Xi^0\Sigma^0$
	$\Xi^0\Sigma^+$	$\Sigma^0\Sigma^0$
	$\Xi^-\Sigma^+$	
	$\Sigma^0\Sigma^+$	

Table 5.1: List of residual components of pp , $p\Lambda$ and $\Lambda\Lambda$ correlation functions. Different colors illustrate interlinks between residual correlations of different systems. Pink: shared by all systems, orange: the same for pp and $p\Lambda$, blue: the same for $p\Lambda$ and $\Lambda\Lambda$.

Chapter 6

Analysis of the method

The method which is proposed to be used to extract spatial dimensions of the emission sources of baryons and the scattering parameters of the strong interactions acting between them is basically to fit a theoretical form of the correlation function to the functions measured experimentally and extract those parameters from the fit.

In case no Coulomb interaction is present, the Lednicky - Lyuboshitz model described in Sec.4.2.4 can be applied. However, authors of the model make a few assumptions and approximations, which may influence the correlation function shape in some regions. To determine whether the method can be useful to extract these parameters, one should also check the sensitivity of the function to those parameters.

In this chapter, both aspects will be studied in detail and results of the theoretical analysis of the method will be presented. Possibility of using spin-averaged scattering parameters for baryon pairs will be investigated as well.

6.1 Analytical and numerical integration comparison

The general form defining the two-particle correlation function is described by Eq.4.22. In the Lednicky - Lyuboshitz model (see Sec.4.2.4) the source emission function is assumed to be gaussian and then the analytical integration (under some assumptions) leads to the Eq.4.52.

To check the consistency between these two formulas one can perform a numerical integration of the gaussian source emission function with the pair-wave function and compare obtained values with those from the model.

Another aspect which should be taken into account is a condition that spatial separation of particles should not be smaller than $r^* = 1 \text{ fm}$, because then the internal structure of the potential becomes important and one should take into account higher

order components of the scattered wave. In the numerical integration one can turn on or off a cut on r^* , assuming the correlation function to be equal 1 if $r^* \leq 1$ (thus no correlation), while using Lednický - Lyuboshitz model this assumption is embedded into the final formula.

The check was performed for two cases: with imaginary part of the scattering length f_0 being equal zero (which corresponds to baryon-baryon correlation function) and $\Im f_0$ different than zero (for baryon-antibaryon). In both cases several values of the real part of the scattering length were tested, with and without the cut on r^* . Also a value of the effective range d_0 was set either to zero or some positive value, to verify the assumption made in previous analyses of proton-antilambda correlation functions (see Sec.4.1.2.5).

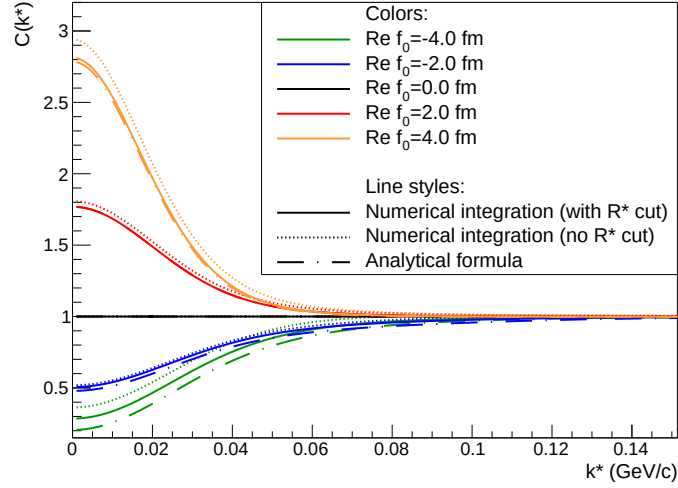
Results of the check for baryon-baryon case are presented in Fig.6.1. Looking at the plots one can draw several observations:

- for $d_0 \neq 0.0 \text{ fm}$ analytical formula is similar to the numerical integration with a cut in r^* turned on. What is more, the further f_0 value from zero, the less accurate analytical integration becomes,
- for $d_0 = 0.0 \text{ fm}$ analytical formula overlaps perfectly with numerical integration without r^* cut.

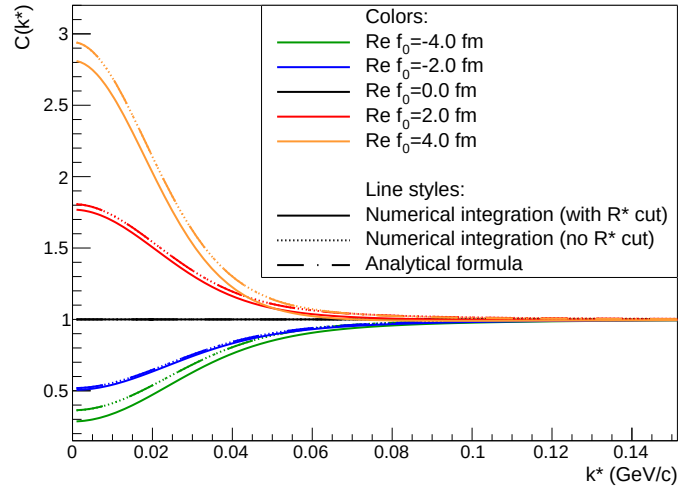
Results for the baryon-antibaryon pairs can be seen in Fig.6.2. One can observe the same effects for baryon-antibaryon as for baryon-baryon pairs.

To conclude the check of consistency between numerical and analytical integration, there are some small differences between the two in some cases. The influence of the cut on r^* below 1.0 fm is not obvious, as it depends on the value of d_0 - for effective range equal to zero analytical formula is actually identical to the numerical integration without a cut, while for effective range different than zero, it fits better the function with the cut.

The discrepancy depends on the values of parameters, however it is rather small in all the cases. It is also significant mainly for higher values of real part of the scattering lengths ($\Re f_0 > 2.0 \text{ fm}$), while in this analysis rather small values ($\Re f_0 < 2.0$) are expected. This confirms that the Lednický - Lyuboshitz model can be used for the analysis of the experimental data.



(a) $d_0 = 3.0$ fm



(b) $d_0 = 0.0$ fm

Figure 6.1: Analytical vs. numerical integration of the correlation function for baryon-baryon pairs. Value of $\Re f_0$ changes from -4.0 fm to 4.0 fm, $\Im f_0 = 0.0$ fm and $R_{inv} = 3.5$ fm.

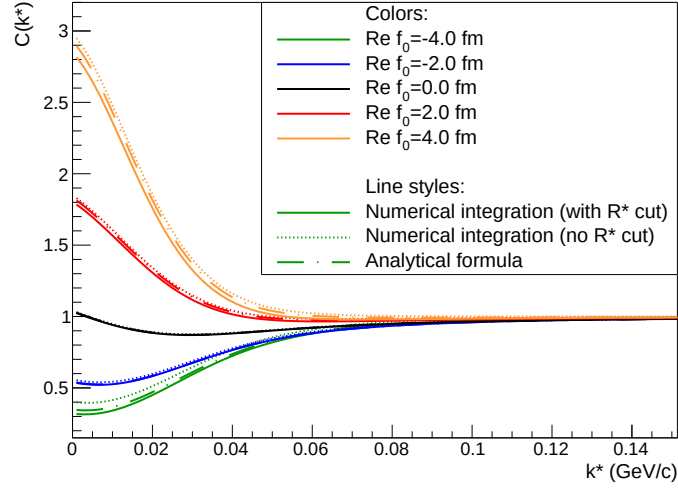
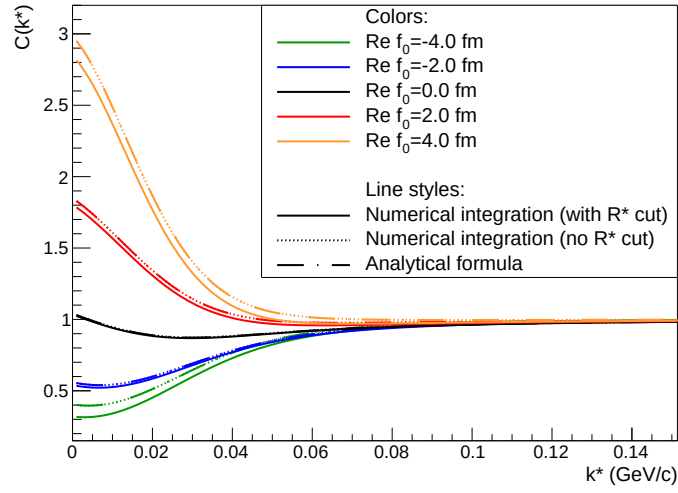

 (a) $d_0 = 1.0$ fm

 (b) $d_0 = 0.0$ fm

Figure 6.2: Analytical vs. numerical integration of the correlation function for baryon-antibaryon pairs. $\Re f_0$ changes from -4.0 fm to 4.0 fm, $\Im f_0 = 1.0$ fm and $R_{inv} = 3.5$ fm.

6.2 Influence of the source size on the discrepancy between numerical and analytical correlation function

Another assumption that was made by Lednický and Lyuboshitz while building their model was that the source size r_0 should not be smaller than 1.5 fm , as below this limit the internal shape of the potential becomes significant. In this section, the influence of the source size on the discrepancy between analytical and numerical integration of the correlation function will be presented.

The results for baryon-baryon pairs are shown in Fig.6.3. As can be observed, for bigger source size $R_{inv} = 3.5 \text{ fm}$ all three functions (analytical integration and numerical integration with and without the cut on r^*) agree very well. For the source size on the limit of 1.5 fm the numerical integration without a cut diverges significantly from the other two.

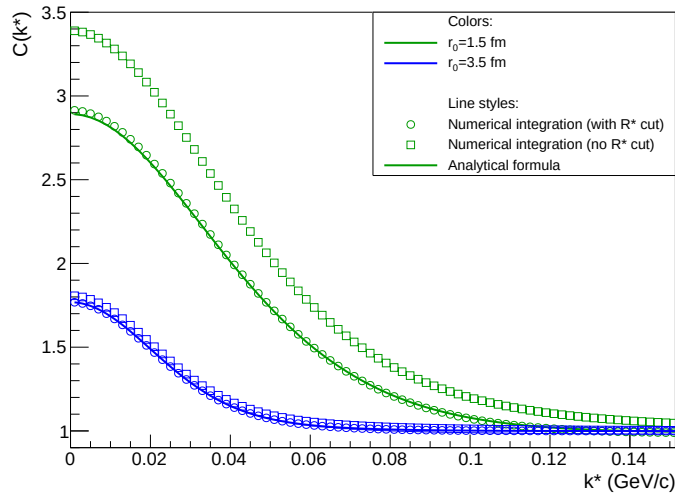


Figure 6.3: Numerical and analytical integration of the correlation function for different source sizes ($R_{inv} = 1.5 \text{ fm}$ and 3.5 fm). $\Re f_0 = 2.0 \text{ fm}$, $\Im f_0 = 0.0 \text{ fm}$ and $d_0 = 3.0 \text{ fm}$.

Corresponding results for example baryon-antibaryon parameters can be seen in Fig.6.4. In this case one can also see that for smaller source sizes there is a significant discrepancy which cannot be observed for bigger sizes. However, in this case the analytical integration matches numerical one without r^* cut, unlike in case of baryon-baryon pairs.

6.3. SENSITIVITY OF THE FUNCTION ON THE STRONG INTERACTION PARAMETERS

Results presented in this section show, that for source sizes of the order of $3 - 4 \text{ fm}$ Lednický - Lyuboshitz model reproduces exact numerical integration precisely. This means that for source sizes expected for baryon-(anti)baryon pairs and thus for the purpose of this thesis the aforementioned model can be used.

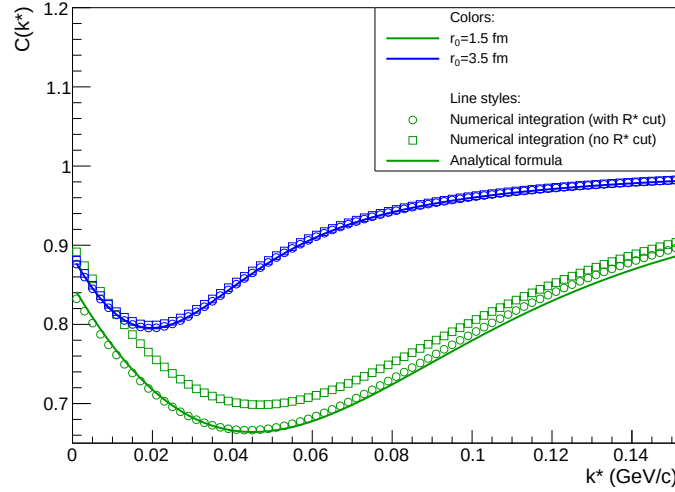


Figure 6.4: Numerical and analytical integration of the correlation function for different source sizes ($R_{inv} = 1.5 \text{ fm}$ and 3.5 fm). $\Re f_0 = -0.5 \text{ fm}$, $\Im f_0 = 1.0 \text{ fm}$ and $d_0 = 1.0 \text{ fm}$.

6.3 Sensitivity of the function on the strong interaction parameters

Fitting a theoretical formula to the measured correlation function of (anti)baryon pairs is a promising method of finding the strong interaction parameters. Before applying the procedure, one should check how these parameters influence the function, to verify whether or not it is possible to extract them from the fit.

The sensitivity of the function on the real part of the scattering length was already presented in Fig.6.1 and Fig.6.2. One can see that the change of this parameter significantly changes the shape of the correlation function. The same results for changing imaginary part of the scattering length can be found in Fig.6.5. Also in this case a strong influence on the parameter value can be observed.

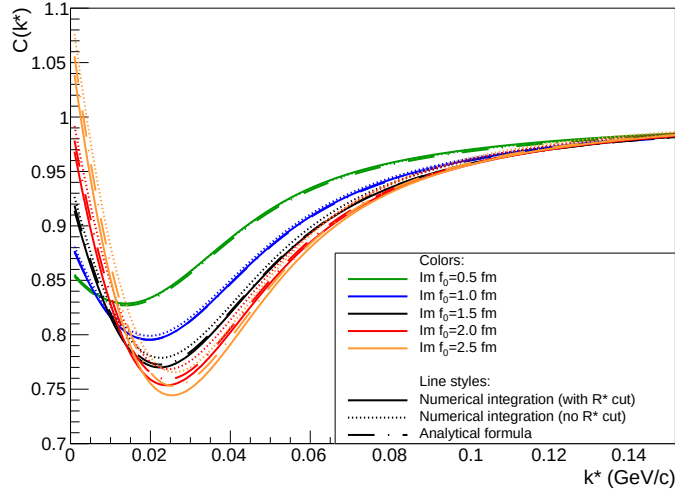


Figure 6.5: Influence of the $\Im f_0$ on the correlation function (changing from 0.5 fm to 2.5 fm). $\Re f_0 = -0.5 \text{ fm}$, $d_0 = 1.0 \text{ fm}$ and $R_{inv} = 3.5 \text{ fm}$.

In separate analysis the value of the effective range has been changed from -4.0 fm to 24.0 fm . The results for baryon-baryon and baryon-antibaryon can be seen in Fig.6.6 and Fig.6.7 respectively. In this case of baryon-baryon, function does not change much for different values of the parameter when numerical integration is performed. However, for analytical integration the shape of the correlation function strongly depends on the value of d_0 parameter. In case of baryon-antibaryon a significant change of the function shape can be observed even in case of the numerical integration.

The analysis presented in this section shows, that both baryon-baryon and baryon-antibaryon correlation functions are sensitive on the real and imaginary part of the scattering length. The effective range has a significant influence on the baryon-antibaryon correlation function, however the impact on baryon-baryon function is small. This shows that in principle it should be possible to extract scattering lengths for both baryon-baryon and baryon-antibaryon and the effective ranges for baryon-antibaryon pairs. The possibility of extracting effective range for baryon-baryon systems is questionable.

6.3. SENSITIVITY OF THE FUNCTION ON THE STRONG INTERACTION PARAMETERS

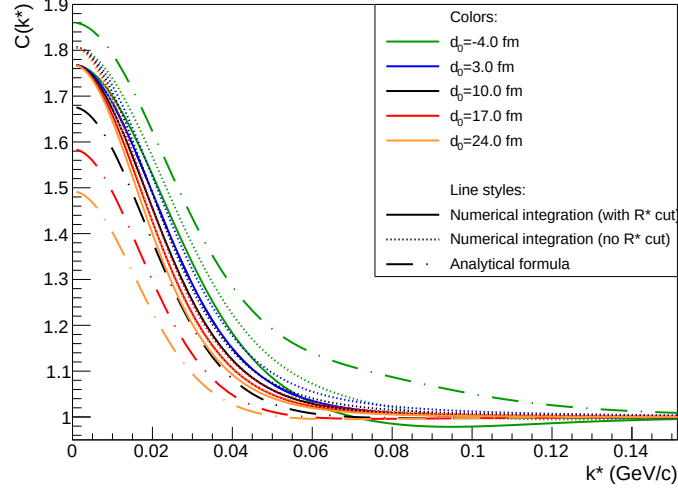


Figure 6.6: Influence of the d_0 on the bb correlation function (changing from -4.0 fm to 24.0 fm). $\Re f_0 = 2.0$ fm, $\Im f_0 = 0.0$ fm and $R_{inv} = 3.5$ fm.

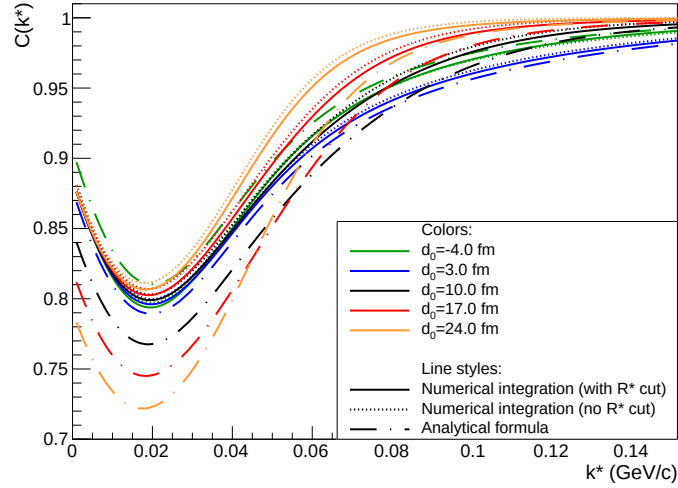


Figure 6.7: Influence of the d_0 on the $b\bar{b}$ correlation function (changing from -4.0 fm to 24.0 fm). $\Re f_0 = -0.5$ fm, $\Im f_0 = 1.0$ fm and $R_{inv} = 3.5$ fm.

6.4 Spin-averaged scattering parameters

As explained in Sec.4.1.1.3, a pair of baryons can interact in singlet or triplet state and parameters of the strong interaction (see Sec.4.1.2.1) are different in these states. However, the widely used simplification is to assume that there exist a set of parameters f_0^A and d_0^A such that:

$$C(k^*, f_0^A, d_0^A) \approx \frac{3}{4}C(k^*, f_0^t, d_0^t) + \frac{1}{4}C(k^*, f_0^s, d_0^s) \quad (6.1)$$

where superscript s stands for singlet-state parameters, t is for triplet-state parameters and A are the spin-averaged parameters. This simplification will be used in this thesis to reduce number of the free parameters of the fit.

6.4.1 Verification by fitting

To verify whether it is in fact possible to approximate singlet and triplet parameters by some averaged values, one can calculate theoretical correlation functions built from both singlet and triplet parts and then fit to it a function with only one set of parameters. Such analysis was performed in this thesis for four different sets of parameters, both for baryon-baryon and baryon-antibaryon. Parameters were chosen to be similar to the predictions for different baryon pairs from the Extended-soft-core Model (see Sec.4.1.2.4). In case of baryon-antibaryon the same parameters were taken, but the imaginary part of f_0 was set to $1 fm$.

The results for bb and $b\bar{b}$ are presented in Fig.6.8 and Fig.6.9 respectively. As can be seen, spin-averaged scattering parameters can describe the shape of the full correlation function very well. There are some small differences visible for example for $p\Sigma$ and $\bar{\Xi}\Xi$. However, they seem to be negligible and thus in all fits performed for the purpose of this thesis a spin-averaged interaction parameters will be used to reduce number of free fit parameters.

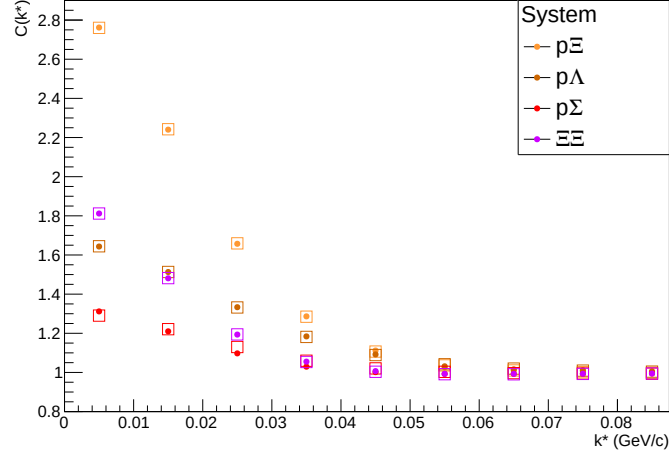


Figure 6.8: Fit with spin-averaged strong interaction parameters to theoretical baryon-baryon correlation function. Round points - theoretical function, squares - fit.

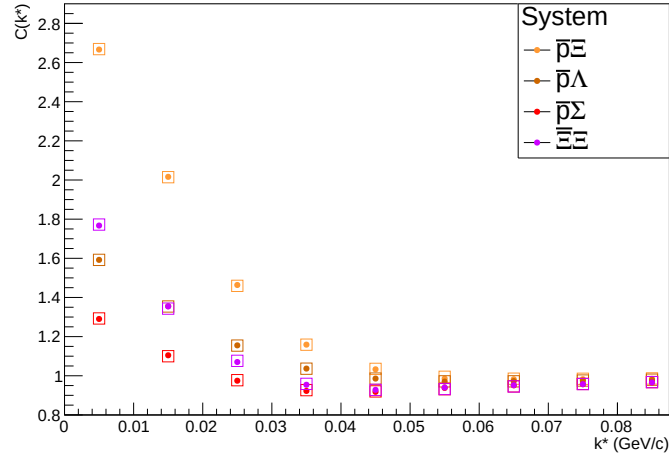


Figure 6.9: Fit with spin-averaged strong interaction parameters to theoretical baryon-antibaryon correlation function. Round points - theoretical function, squares - fit.

6.4.2 Analytical derivation

Another way to verify if spin-averaged scattering parameters can be used is an approximated analytical derivation. One can insert analytical form of the correlation function from the Lednický - Lyuboshitz model (see Sec.4.2.4) into Eq.6.1. Then, the resulting equation can be split into a set of equations:

$$\begin{cases} |f^A(k^*)|^2 \left(1 - \frac{d_0^A}{2\sqrt{\pi}r_0}\right) = \frac{1}{4} [|f^s(k^*)|^2 \left(1 - \frac{d_0^s}{2\sqrt{\pi}r_0}\right)] + \frac{3}{4} [|f^t(k^*)|^2 \left(1 - \frac{d_0^t}{2\sqrt{\pi}r_0}\right)] \\ \Re f_0^A = \frac{1}{4} \Re f_0^s + \frac{3}{4} \Re f_0^t \\ \Im f_0^A = \frac{1}{4} \Im f_0^s + \frac{3}{4} \Im f_0^t \end{cases} \quad (6.2)$$

which automatically shows what values of $\Re f_0$ and $\Im f_0$ could be used for spin-averaged parameters. Then one can transform the first equation to calculate spin-averaged value of d_0 :

$$d_0^A = 2\sqrt{\pi}r_0 - \frac{\frac{1}{4} |f(k^*)^s|^2 (2\sqrt{\pi}r_0 - d_0^s) + \frac{3}{4} |f(k^*)^t|^2 (2\sqrt{\pi}r_0 - d_0^t)}{|f(k^*)^A|^2} \quad (6.3)$$

where squared modulus of the scattering amplitude can be written as:

$$|f(k^*)|^2 = \frac{4|f_0|^2}{4 + 4\Re f_0 d_0 k^* + 8\Im f_0 k^* + |f_0|^2 d_0^2 k^* + 4|f_0|^2 k^{*2}} \quad (6.4)$$

As can be seen, the averaged value of d_0 depends on k^* . In case this dependance is not very strong, one could simplify the above formula for $k^* \rightarrow 0$:

$$|f(k^*)|^2 = |f_0|^2 \quad (6.5)$$

and use it to calculate spin-averaged value of the effective range using Eq.6.3.

The spin-averaged parameters were calculated using the aforementioned formalism for a few sample values of spin-dependant parameters. The values were chosen to be similar to the ones of existing baryon-baryon pairs. The comparison of a full, spin-dependant correlation functions with the approximated, spin-averaged ones is shown in Fig.6.10. The same results are presented in Fig.6.11 for correlation functions with the $\Im f_0$ set to 1 fm to mimic baryon-antibaryon pairs. For each system, the d_0^A dependance on k^* is also shown in a bottom plot. The spin-averaged correlation function was calculated for

a few different values of $d_0^A(k^*)$ and lines of the same color were plotted (but they usually overlap perfectly).

What can be observed in Fig.6.10 and Fig.6.11 is that spin-averaged correlation functions are not sensitive to the change of d_0^A values due to its k^* dependance and that this dependance itself is not very strong. Thus Eq.6.5 indeed can be used to calculate spin-averaged effective range. Next conclusion is that for some values of scattering parameters the proposed way of calculating spin-averaged values works very well (for example $p\Lambda$ and $p\Xi$), while for others there are clear differences (especially for $\Xi\Xi$).

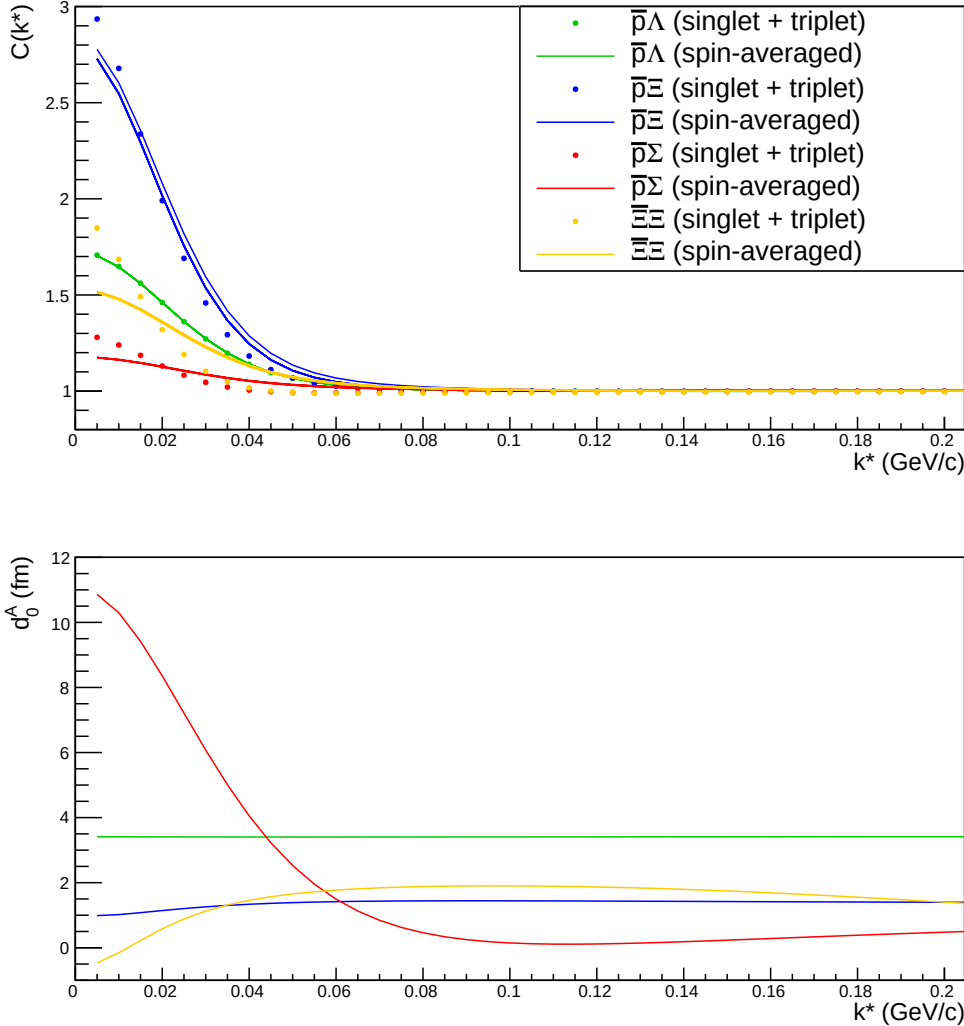


Figure 6.10: Top: Baryon-baryon correlation functions with analytically derived spin-averaged strong interaction parameters on top of theoretical baryon-baryon correlation function with singlet and triplet components included. Bottom: d_0^A dependance on k^* .

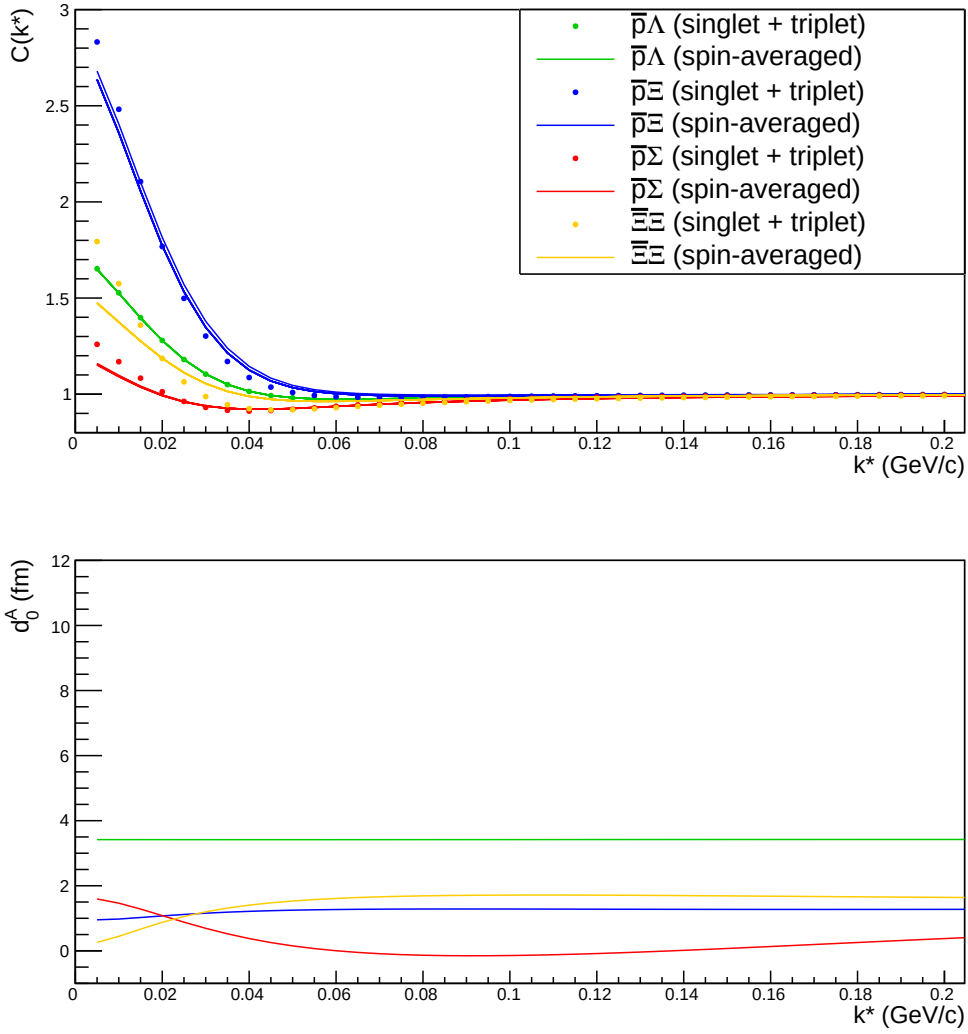


Figure 6.11: Top: Baryon-antibaryon correlation functions with analytically derived spin-averaged strong interaction parameters on top of theoretical baryon-antibaryon correlation function with singlet and triplet components included. Bottom: d_0^A dependance on k^* .

The analysis presented in this and previous section shows that spin-averaged parameters can be used to describe the correlation function which is normally composed of the singlet and triplet part. In some cases explicit analytical formulas can be used. Otherwise, as verified by fitting spin-averaged correlation functions, there still exists a set of parameters which will reproduce the original shape sufficiently well.

Chapter 7

Test of the method on model data

In the previous chapter a theoretical basis of the method of extracting source sizes and scattering parameters based on the two-particle correlation function was presented. Before applying the procedure to the experimental data, one could test it on the model data, where a full control of the input parameters and monitoring of the space and momentum distributions of the particles is possible.

In this chapter the focus will be put on the extraction of the source radius and a comparison of results obtained from the fit with those from the space distributions from Therminator model.

All functions analysed in this chapter were generated using Therminator model (see Sec.2.5). The software used to generate final correlation functions takes into account strong and Coulomb interactions, as well as the Quantum Statistics when applicable. Moreover, one can control which residual correlations will be included and whether or not particles from strong decays should be taken into account.

7.1 Including residual correlation in the fit

In case of experimental data one cannot (and should not) get rid of the residual correlations applying special procedures during data analysis (see Chap.5). They should be included in the fit using the Eq.5.1. One should also take into account, that the correlation between particles from residual pairs is measured as a function of the k^* of the primary pair, while theoretical values are calculated for the k^* of the residual pair (Chap.5). Thus before adding a residual component with weight λ , one needs to apply a transformation from k_{parent}^* to $k_{children}^*$. This can be achieved using transformation matrices, which represent probability that the residual pair with relative momentum of parent particles k_{parent}^* will decay to children particles that will contribute to the final correlation function with given $k_{children}^*$.

7.1.1 Transformation matrices

For the purpose of this thesis, transformation matrices were generated from the Therminator model for all residual components of all analysed systems. The complete list of all pairs taken into account can be found in Tab.5.1. Three systems were analysed: pp , $p\Lambda$ and $\Lambda\Lambda$. For each of them, each residual component was considered - for example $\Lambda\Sigma^0$ residual component of $p\Lambda$ correlation function. This required to loop over all Λ baryons decaying into protons and all Σ^0 baryons decaying into Λ , calculating k^* of the pair before and after the decay.

For each set of particles a histogram was filled with given values of $k_{children}^*$ and $k_{parents}^*$, which resulted in the creation of matrices representing unnormalized probability that registered pair with momentum $k_{children}^*$ comes from the decay of particles with momentum $k_{parents}^*$. An example of those matrices is shown in Fig.7.1

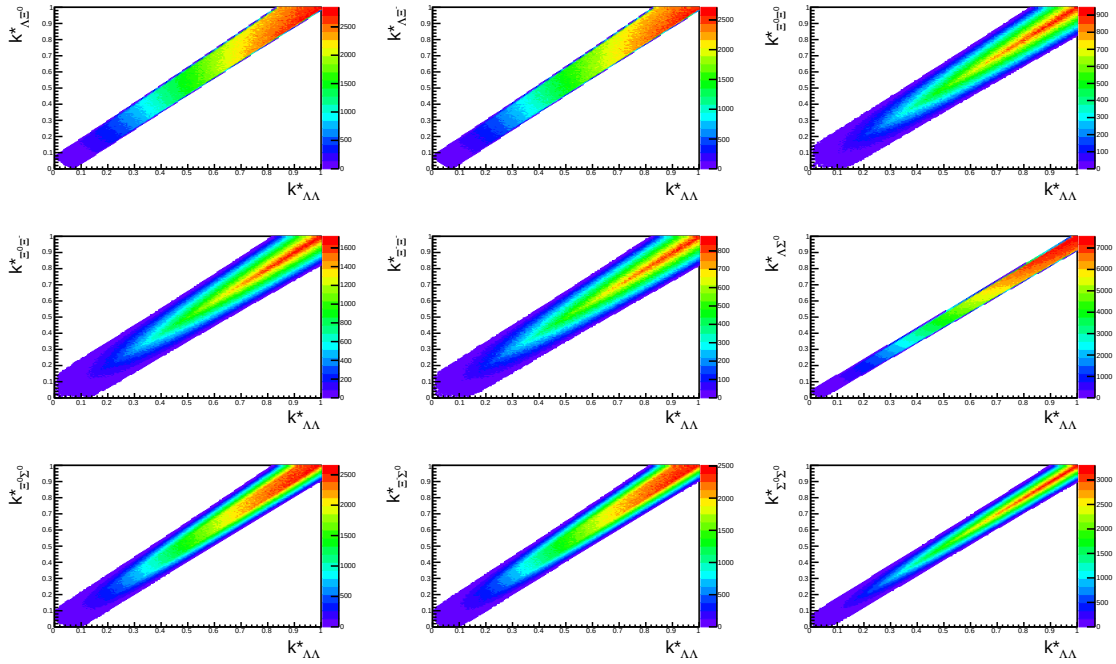


Figure 7.1: Transformation matrices for residual components of $\Lambda\Lambda$ correlation function.

The correlation function can be transformed from the momentum space of the parent (interacting) particles to the momentum space of the children (observed) particles, according to the formula [46]:

$$C(k_{children}^*) = \frac{\sum_{k_{parents}^*} C(k_{parents}^*) W(k_{children}^*, k_{parents}^*)}{\sum_{k_{parents}^*} W(k_{children}^*, k_{parents}^*)} \quad (7.1)$$

where $W(k_{children}^*, k_{parents}^*)$ is the element of the momentum transformation matrix for given values of $k_{children}^*$ and $k_{parents}^*$.

7.1.2 Generating correlation function from model data

To generate correlation functions from Terminator data, first an appropriate set of particles was selected. This means that from all the particles from events generated from the model not only those studied were selected, but also all the particles decaying into them. Depending on the purpose of the function, sometimes particles contributing to the residual correlations were excluded, while in other cases they were taken into account.

Then, for each pair of selected particles a weight (modulus of the pair-wave function) was calculated. The appropriate form of the function is chosen, depending on what combination of strong, Coulomb and Quantum Statistics was present for this pair. For example, for $\Lambda\Lambda$ pair there is no need to include Coulomb interaction, as this baryon has no electric charge. However, in addition to the strong interaction, one needs to include Quantum Statistics in this case. In case of $p\Lambda$ only the strong interaction is included, while for pp all three effects must be taken into account, as particles of such pairs interact strongly, with Coulomb interaction and they obey the Quantum Statistics, as these are identical baryons. Equations used for all the cases can be found in Chap.4.

Next, for each pair the k^* value was calculated and an entry with previously calculated weight was added to the histogram. The same procedure was repeated twice - in one case only particles coming from the same event were taken into account. Then, first particle was taken from one event and the second particle was taken from a different event (the so-called event mixing) and added to histogram with weight equal to unity (no interaction). The final correlation function was calculated as a ratio of functions generated from same events to the one from mixed events. Thanks to this procedure all correlations which do not come from the interaction between particles will be divided out.

The procedure described above is similar to the experimental approach to the correlation functions described in Sec.4.3. However, weights are calculated from theoretical formulas, which gives the full control and knowledge of the input strong interaction parameters.

An example of a correlation function generated from Therminator data can be seen in Fig.7.2 - what can be seen in this case is a $\bar{p}\Lambda$ correlation function, excluding all residual correlations.

7.2 Extracting source radii from the fit

To extract source radii from the fit, one should apply minimisation algorithms, changing values of the free fit parameters, calculating and optimising χ^2 value. For the purpose of this thesis, a MIGRAD algorithm implemented in the ROOT framework was used [20][48].

The form of the fit function was selected for each system separately, taking into account strong interaction, Coulomb force, Quantum Statistics (see Chap.4) and influence from the residual correlations (see Chap.5).

As a first step, the fitting application was run on the model data with all parameters fixed to known values. The result is presented in Fig.7.2. In this case residual correlations were excluded at the level of correlation function building (so the correlation coming from weakly - decaying baryons are removed from the final correlation function). However, particles decaying into protons and Λ baryons very soon after the collision (up to $10^8 fm/c$ [6]) via the strong decays were included in the fit. The strong interaction parameters were fixed to the values set while generating the function. In order to determine the source size, distributions of the initial spatial separation for each pair of particles were generated. Then, a gaussian fit was performed and the the source size was extracted as a width of the fitted distribution. An example of such distribution together with fit can be seen in Fig.7.3.

As can be seen, the theoretical correlation function does not exactly match the model data. This is a consequence of the fact that the emission source is not perfectly gaussian and that there are some small differences between analytical and numerical integration of the correlation function (as shown in Chap.6).

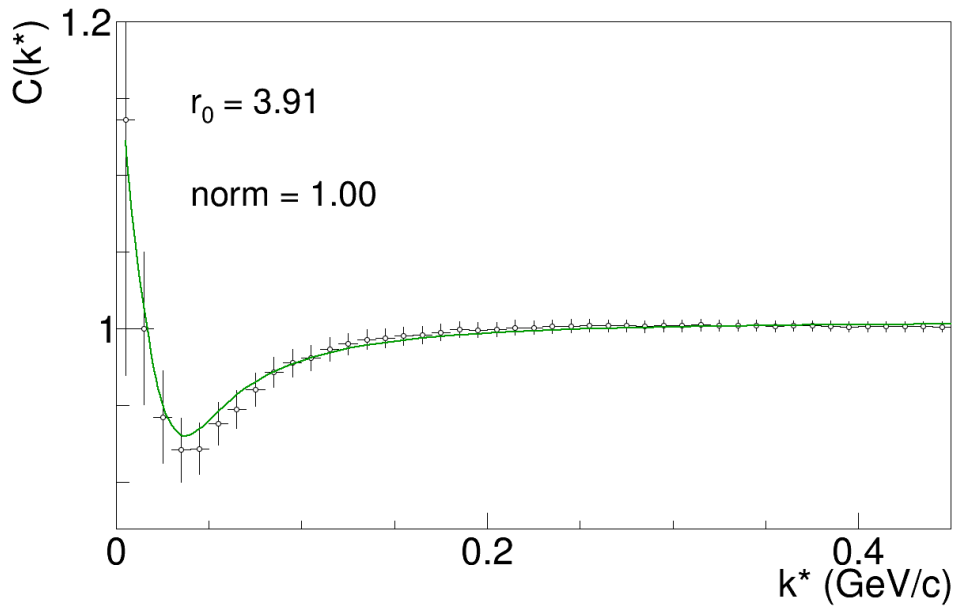


Figure 7.2: Theoretical $p\bar{\Lambda}$ correlation function with fixed parameters (green line) drawn on top of the function from Therminator (points).

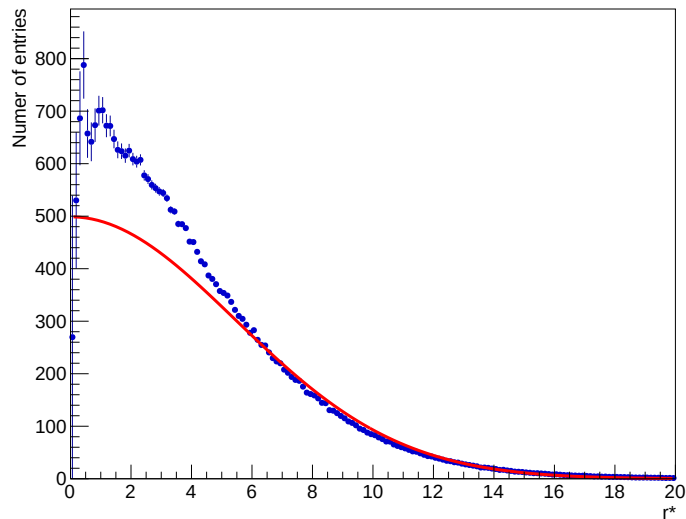


Figure 7.3: Distribution of the initial space separation of $p\Lambda$ pairs in the most central (0-5%) events from Therminator together with a gaussian fit.

One can ask if source radii extracted from fits will be correct despite these effects. To address this question, a series of fits was performed, for both $p\Lambda$ and $\bar{p}\Lambda$, for several impact parameters (corresponding to different centralities), excluding or including particles coming from strong decays. Then, radii were extracted from each fit and compared to what can be obtained from the spatial distributions (as described in the previous paragraph). The results of this comparison are presented in Fig.7.4, the black line represents $R_{fit} = R_{dist}$. As can be seen, radii extracted from the fit match those from the distributions in all cases.

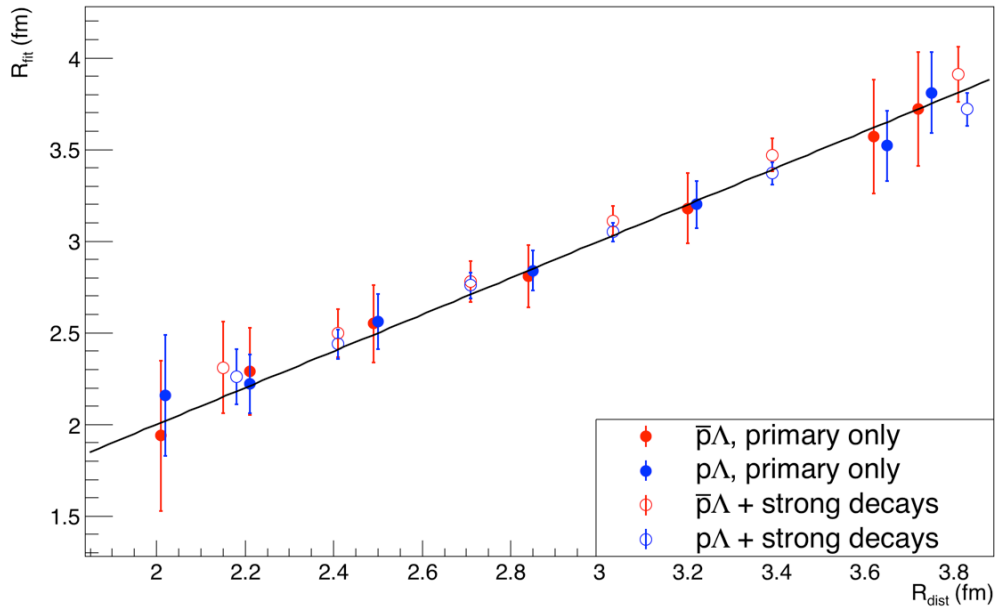


Figure 7.4: Radii from fit compared to radii from spatial distributions.

7.3 Fitting single function with residual correlations

The $\bar{p}\Lambda$ correlation function from Therminator including all residual correlations (taking into account decay kinematics using transformation matrices - see Sec.7.1.1), accounting for Quantum Statistics and Coulomb interactions where it was needed, was fitted. All parameters except $b\bar{b}$ scattering length were fixed to known values. The result of the fit is shown in Fig.7.5. As can be seen, fitting procedure including residual correlations works very well.

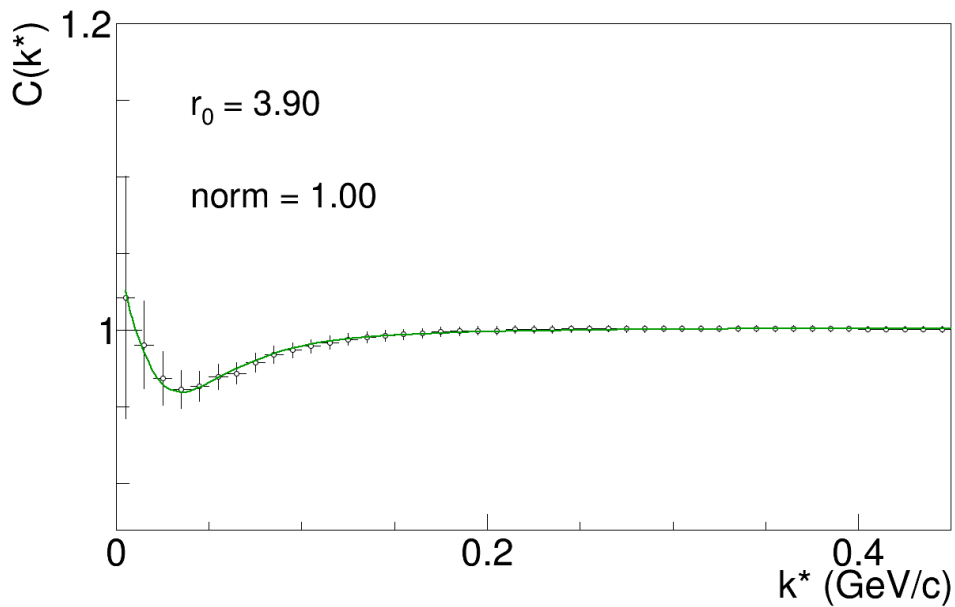


Figure 7.5: $p\bar{\Lambda}$ correlation function taking into account residual correlations (green line) fitted to the function from Therminator (points).

7.4 Simultaneous fit in several centrality bins

The emission source size should generally depend on the initial size of the system, which is in turn related to the impact parameter b , being a distance between centers of the colliding nuclei. If there would be some relation between the impact parameter and the source size, one could perform a simultaneous fit in several centrality bins, fitting radius only in one of them and then scaling it for other centralities. This would significantly reduce number of free parameters and thus the statistical uncertainties, improving the reliability of the fit.

7.4.1 Modelling radius dependance on the impact parameter

The assumption that the emission source has a gaussian shape is an approximation simplifying analysis of the correlations. In fact, the emission source has more complex, three-dimensional structure. A space distribution from the Therminator model for central and peripheral collisions is shown in Fig.7.6. The *out* and *side* are defined in the Longitudinally Co-Moving System (LCMS) - see Sec.4.2.1. One can notice that indeed the source is bigger in central collisions and that it has a shape that is not spherical.

The assumption made in this thesis is that the source shape depends directly on the initial geometry of the overlapping region of the colliding nuclei. The geometry of the collision is shown in Fig.7.7, where b is the impact parameter and R_x and R_y are the dimensions of the overlapping region. Then it is assumed that the 1D source radius is a geometrical combination of three-dimensional sizes of the overlapping part of nuclei, which can be written as follows:

$$R_{inv}^2(b) = \frac{R_x(b)^2}{a_x^2} + \frac{R_y(b)^2}{a_y^2} + \frac{R_z(b)^2}{a_z^2} \quad (7.2)$$

where R_{inv} is the one-dimensional radius and a_x , a_y , a_z are the scaling factors for each dimension. The R_x dimension is taken twice on purpose, as R_x and R_z are equal for the intersection of two overlapping spheres. The above formula resembles the equation of ellipsoid, in which case the R_{inv} is a factor which scales the ellipsoid in all three dimensions. Then, using purely geometrical relations, one can find a dependance of the R_x and R_y on

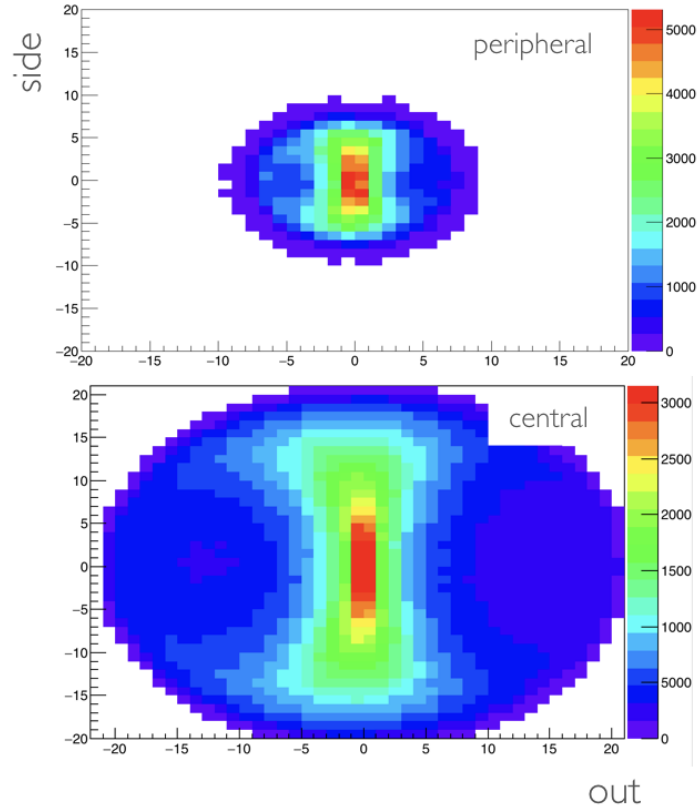


Figure 7.6: Emission source shape from Therminator model.

the nucleus size and the impact parameter:

$$R_x(b) = R_{Pb} - \frac{b}{2} \quad R_y(b) = \sqrt{R_{Pb}^2 - \left(\frac{b}{2}\right)^2} \quad (7.3)$$

where R_{Pb} is the radius of the nucleus, in this case lead, which can be approximated using a droplet-model [49] to be about 7 fm.

The proposed function was fitted to radii extracted from Therminator for $\bar{p}\Lambda$ pairs. The result is presented in Fig.7.8. As can be seen, the function fits data very well and thus could be used for simultaneous fit of several centrality bins. However, one should remember, that using such relation introduces new parameters which must be assumed or designated from other measurements.

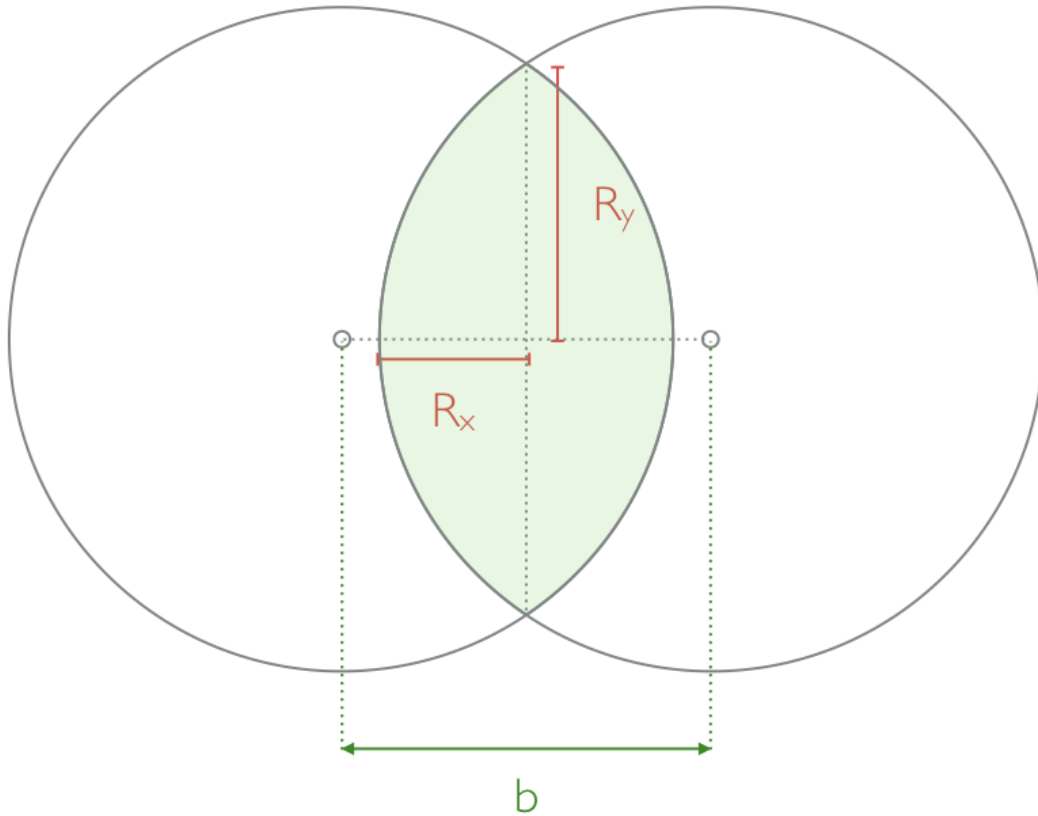


Figure 7.7: Illustration of the collision geometry.

Finally, the correlation functions from Therminator were fitted in different centrality bins (for primary pairs, with strong decays included). The scattering parameters were fixed to known values, the radius for the first centrality bin was a free fit parameter and other radii were calculated using Eq.7.2. All fits are presented in Fig.7.9. As can be seen, the procedure works fine and qualitatively all fits match the data.

Comparison of the results from fit and radii from distributions with and without the use of the $R_{inv}(b)$ relation is shown in Fig.7.10. What can be seen is that the statistical uncertainties are indeed significantly reduced and radii extracted from fit match those from distributions very well.

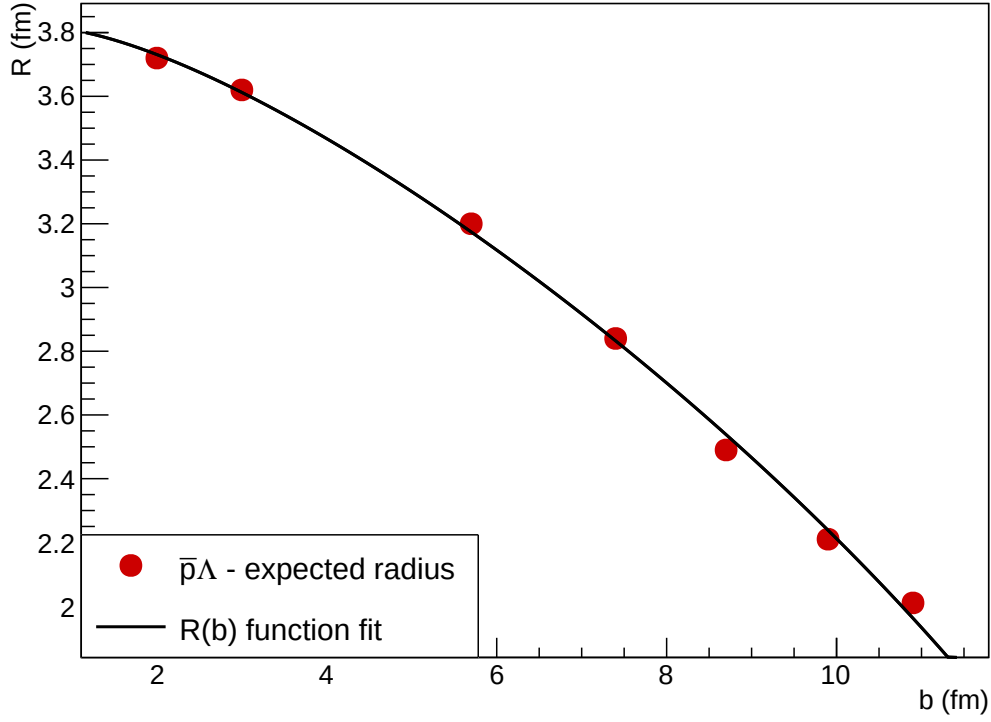


Figure 7.8: Fit of proposed $R_{inv}(b)$ function to radii from Therminator.

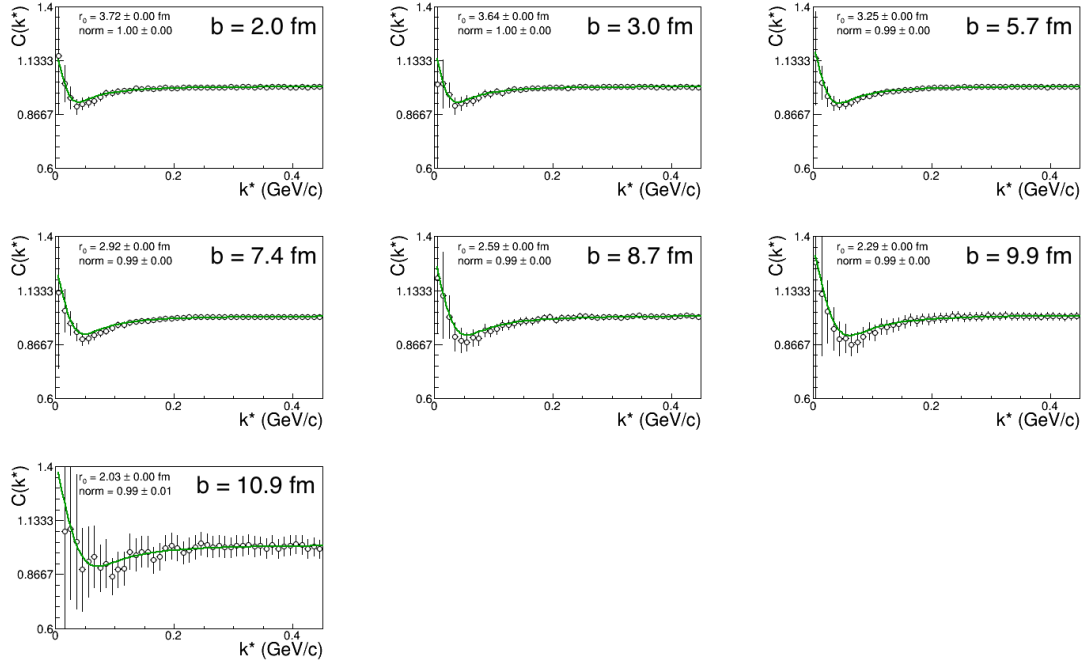


Figure 7.9: Simultaneous fit of functions from Therminator in different centrality bins using proposed $R_{inv}(b)$ relation.

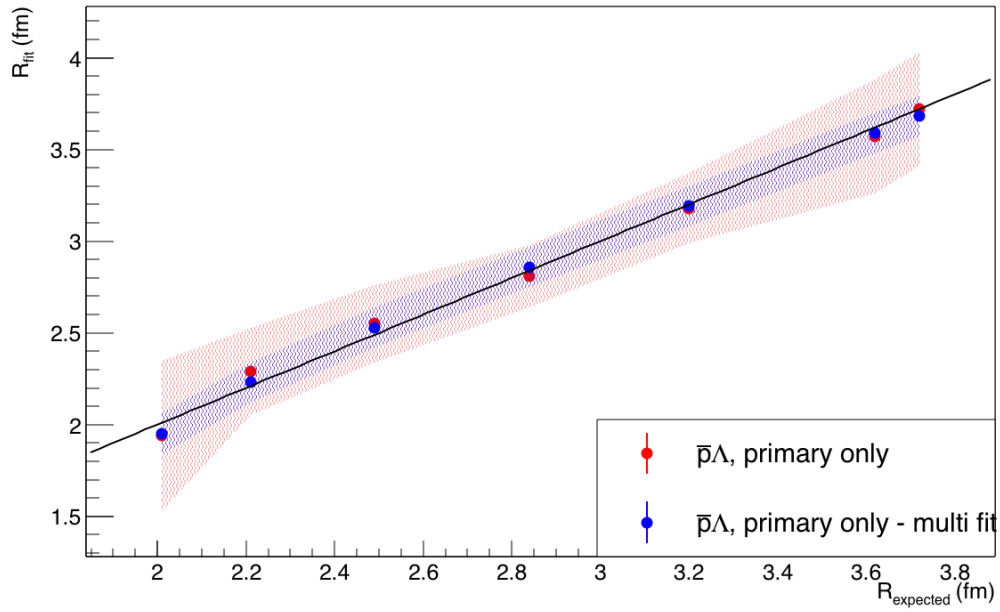


Figure 7.10: Radii extracted from fit vs. radii from spatial distributions analysed independently for each impact parameter (red) and together for all functions, using $R_{\text{inv}}(b)$ relation (blue).

7.4.2 Radius dependence on multiplicity

The impact parameter cannot be measured directly in the experiment. This makes it necessary to introduce a relation between source size and some value which is closely related to the impact parameter, but can be measured in the experiment. A quantity which is usually used in this case is the charged multiplicity N_{ch} (the total number of charged particles reconstructed in the event).

It is usually assumed, that the one-dimensional source radius scales linearly with the cube root of multiplicity:

$$R_{inv} = \alpha \sqrt[3]{N_{ch}} + \beta \quad (7.4)$$

as multiplicity depends on the volume of the crossing part of the colliding nuclei [50]. This comes from the fact, that particle production is proportional to the volume of the medium from which they are produced. If the radius of the source is given by R_{inv} , then the volume is proportional to R_{inv}^3 and so is the multiplicity N_{ch} . Inverting the last relation one obtains the above equation.

This relation can be used to fit many centrality bins from experimental data, where information about the impact parameter is not accessible. What can be noticed is a limitation of the values of parameters α and β , as for a single proton the source size is equal to the size of a proton and the resulting multiplicity is 1. This leads to the relation: $R_p = \alpha + \beta$, where $R_p \approx 0.88 \text{ fm}$ [6].

One can also try to use the geometrical relations described in Sec.7.4.1. Assuming that multiplicity is proportional to the volume of the crossing part of the nuclei:

$$N_{ch} \propto V(b) \quad (7.5)$$

where the volume $V(b)$ can be also described using geometrical relations:

$$V(b) = \frac{\pi}{12} (4R_{Pb} + b) \cdot (2R_{Pb} - b)^2 \quad (7.6)$$

Next step is to invert this function to get $b(V)$ relation, which will be needed in further steps:

$$b(V) = -2R_{Pb} \left(\cos \frac{\theta}{3} - \sqrt{3} \sin \frac{\theta}{3} \right) \quad (7.7)$$

where θ is defined as:

$$\theta(V) = -\arctan \frac{\sqrt{24\pi R_{Pb}^3 V - 9V^2}}{3V - 4\pi R_{Pb}^3} \quad (7.8)$$

Finally, all the aforementioned functions can be combined to get a relation $R_{inv}(a \cdot N_{ch})$, where a is a scaling factor between volume and multiplicity. If multiplicity scales linearly with the volume (which is assumed), one can designate a as a ratio of the volume of the lead nuclei to the maximum multiplicity.

The proposed relation was fitted to the radii from Therminator plotted as a function of multiplicity. Also simpler Eq.7.4 was fitted to the same data. The results are shown in Fig.7.11. As can be seen, both functions describe radius dependence on multiplicity very well. The more complex formula depends on more free parameters and has to be tuned to the model data. As the results obtained with both methods are similar, in further chapters of this thesis the simpler formula will be used for radii scaling with multiplicity.

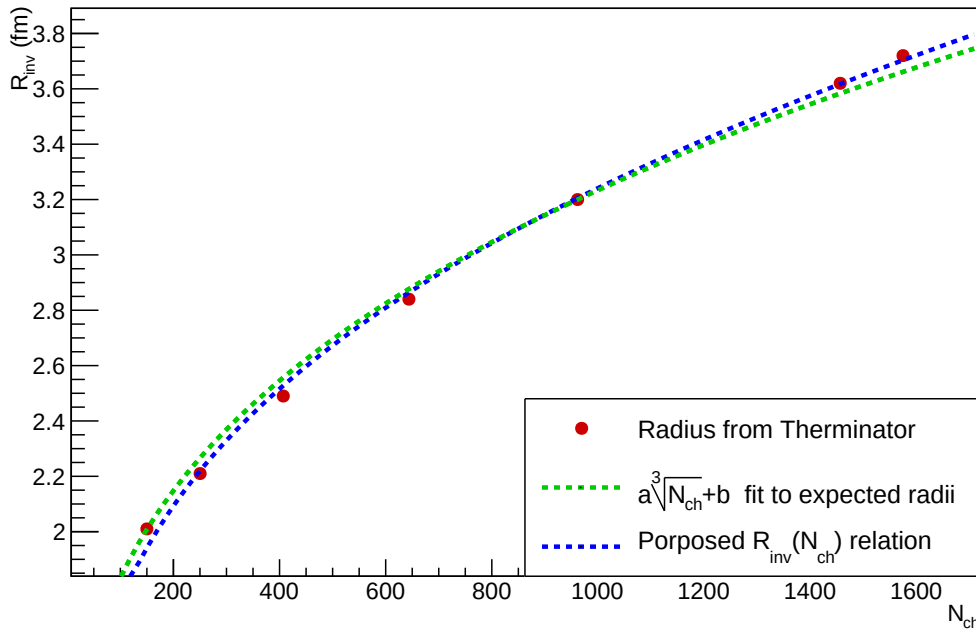


Figure 7.11: Radius vs. multiplicity with two functions describing the relation between them.

Chapter 8

ALICE data analysis and fit

The LHC operates in a few-year long periods called runs. The so-called Run 1 started in 2009 and ended in 2013, while Run 2 began in 2015 and is planned to be continued until 2018. ALICE has been detecting and registering products of proton-proton, proton-lead and lead-lead collisions during both Run 1 and Run 2.

In this chapter selected correlation functions obtained for the purpose of this thesis from Run 1 and Run 2 data of ALICE will be presented. They come from collisions of lead ions at $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ and $\sqrt{s_{NN}} = 5.02 \text{ TeV}$ respectively. The results of applying the fitting procedure described in Chap.7 will be shown as well. All remaining correlation functions measured and fitted during this analysis can be found in Appendix B.

For the measurement performed for the purpose of this thesis, an internal ALICE analysis note was created, of which I am the main author. A part of this thesis coincides with the content of the analysis note. The obtained results were presented as a paper proposal to the ALICE Physics Forum, which was accepted. Selected figures containing correlation functions and extracted fit parameters were approved by ALICE Collaboration as preliminary results. Work on the publication is ongoing.

8.1 Data selection

The correlation functions presented in this chapter come from the Run 1 and Run 2 data registered by ALICE and denoted as LHC11h and LHC15o respectively. A series of event, single-particle and two-particle cuts was applied to the data. The primary vertex for an event was requested to be from -8 cm to 8 cm from the center of the detector, which allows to reduce number of particles which do not originate from the event of interest. No further event cuts were applied.

As detailed analysis of cuts has already been performed by other collaborators, event, particle and pair selection in this thesis is taken directly from these analyses [51] [52] [53] [54]. These results were discussed and approved within the collaboration. The most important single-particle and pair cuts will be listed and explained in the next sections.

8.1.1 Single-particle cuts

A series of single-particle cuts was applied during this analysis. In this section all these cuts will be explained and their established values will be presented.

The pseudorapidity is a function of the angle between the beam axis and the particle's initial momentum vector:

$$\eta = -\ln \left[\tan \frac{\theta}{2} \right] \quad (8.1)$$

The cut on η is applied to remove detector effects connected with a limited acceptance of the TPC.

Next, to assure good quality of tracks reconstruction and to make sure that their origin is close to the primary vertex, number of clusters registered per track in TPC was required to exceed 80. A maximum value of χ^2 (which is a measure of average distance between given points and the fitted curve) should be smaller than 4.0 per TPC cluster. A TPC cluster represents a point in space where an interaction with the active material of the detector took place - usually as a result of charged particle passing through it.

The transverse momentum p_T is the projection of the momentum vector to the XY plane, parallel to the beam axis. The particle identification quality depends on this parameter and its proper limitation is needed to assure good purity of the sample. To make sure that the sample is not significantly contaminated by secondary particles coming from the weak decays and particle-detector interactions, one can also apply a cut on the Distance of Closest Approach (DCA) to the primary vertex.

The following particle cuts were applied for protons:

- pseudorapidity $|\eta| < 0.8$,
- minimum 80 clusters in TPC,
- maximum allowed χ^2 per TPC cluster of 4.0,

- transverse momentum $0.7 < p_T < 4.0 \text{ GeV}/c$ for protons and $0.7 < p_T < 5.0 \text{ GeV}/c$ for antiprotons,
- Distance of Closest Approach (DCA) in transverse plane $DCA_{XY} < 2.8 \text{ cm}$, in longitudinal plane $DCA_Z < 3.2 \text{ cm}$.

The Λ baryons decay into (anti)proton and $\pi^{-(+)}$. In this case, additional cuts can be applied to reduce probability that two random tracks (proton and pion) will be reconstructed as Λ . In this analysis decay length (distance traveled by particle before the decay) and the cosine of pointing angle θ (the angle between the momentum of the particles pair and the line segment connecting primary and secondary vertices) were used to limit these random proton-pion associations. In case of unstable particles, one can apply a cut on the minimal distance between daughter particles, as they should originate from the same point in space. Finally, reconstructed invariant mass of the decaying particle was required to be close to the measured mass of the Λ baryon $m_\Lambda = 1115.683 \text{ GeV}/c^2$ [6].

For Λ baryons cuts were the following:

- pseudorapidity $|\eta| < 0.8$,
- minimum 80 clusters in TPC for each daughter,
- transverse momentum of Λ baryon $0.5 < p_T < 5.0 \text{ GeV}/c$,
- transverse momentum of daughter proton $0.5 < p_T < 4.0 \text{ GeV}/c$,
- transverse momentum of daughter antiproton $0.3 < p_T < 4.0 \text{ GeV}/c$,
- transverse momentum of daughter pion $0.16 < p_T < 4.0 \text{ GeV}/c$,
- DCA between daughter particles (proton and π) less than 0.4 cm ,
- DCA of the reconstructed Λ baryon to the primary vertex less than 0.5 cm ,
- maximum decay length of 60.0 cm ,
- cosine of the pointing angle $\cos\theta > 0.9993$,
- invariant mass of Λ in range $m_\Lambda - 0.0038 < m_\Lambda < m_\Lambda + 0.0043$,
- invariant mass of $\bar{\Lambda}$ in range $m_{\bar{\Lambda}} - 0.0036 < m_{\bar{\Lambda}} < m_{\bar{\Lambda}} + 0.0041$.

8.1.2 Two-particle cuts

In addition to single-particle cuts which are used to assure good quality of reconstruction and particle identification, several two-particle cuts were applied. The main problems met in this type of analysis are merging and splitting of tracks. Merging is an effect where two

tracks being very close to each other are reconstructed as one track, while splitting means the trace of single particle is reconstructed as two separate tracks.

First procedure which can be applied is a cut on the separation of the tracks: on the entrance to TPC, on the exit of TPC and the average separation along the tracks. Next, one can limit a fraction of clusters shared by both reconstructed tracks. In some cases it is also useful to set minimum required difference in pseudorapidity or in azimuthal angle at some distance from the center of TPC (as described in [55]):

$$\Delta\varphi^* = \varphi_1 - \varphi_2 + \arcsin\frac{eBR}{2p_{T1}} - \arcsin\frac{eBR}{2p_{T2}} \quad (8.2)$$

where φ_1 and φ_2 are the azimuthal angles, p_{T1} and p_{T2} are the transverse momenta, e is the elementary charge, B is the external magnetic field in z direction in which the detector is placed and R is the distance from the center of TPC.

The following criteria were set for (anti)proton-(anti)proton:

- $|\Delta\varphi^*| > 0.012$,
- $|\Delta\theta| > 0.017$,
- fraction of shared clusters smaller than 5%,
- minimum average separation in TPC: 5.0 cm ,
- no cuts on the separation at entrance and exit of TPC.

For combinations of Λ and $\bar{\Lambda}$ baryons:

- minimum average separation: 5.5 cm for $p\pi^+$, $\bar{p}\pi^-$ and $p\bar{p}$ pairs,
- no cuts on the separation at entrance and exit of TPC.

For combinations of (anti)protons with (anti)lambda baryons:

- fraction of clusters shared between (anti)protons (where one is the primary track and second comes from the (anti)lambda decay) smaller than 5%,
- minimum average separation: 5.0 cm for $p\bar{p}$ pairs,
- minimum average separation: 8.0 cm for $p\pi^+$ and $\bar{p}\pi^-$ pairs,
- no cuts on the separation at entrance and exit of TPC.

8.1.3 Centrality and multiplicity

The analysis was performed in six centrality bins. Ranges of centrality bins together with corresponding average multiplicities for 2011 and 2015 ALICE data are listed in Tab.8.1 [56][57].

Centrality	Multiplicity LHC11h	Multiplicity LHC15o
0-5%	1601	1923
5-10%	1294	1586
10-20%	966	1180
20-30%	649	786
30-40%	426	512
40-50%	261	318

Table 8.1: Centrality bins and corresponding average multiplicities for 2011 and 2015 ALICE data [56][57].

8.2 Fitting procedure description

Fitting of experimental data requires a series of corrections, procedures and assumptions. In this section, procedure of including residual correlation in the fit will be described. Then, assumptions about scattering parameters and source sizes scaling will be listed. Finally, corrections of the non-femtoscopic background and limited momentum resolution of the detector will be explained.

8.2.1 Residual correlations

8.2.1.1 Pair fractions

To account for the residual correlations, fractions for each pair were calculated based on the AMPT model [58]. The data used are the Monte Carlo productions in which AMPT was tuned to model the Pb-Pb collisions registered by ALICE.

The procedure to obtain fractions was the following: identical cuts that are applied to generate experimental correlation functions were applied to particles and pairs coming

from the model data. Then, for each mixed pair (each particle comes from different event) which would normally enter the denominator of the correlation function, the type of mother particle was saved. Finally, fractions were calculated as a ratio of number of pairs coming from given decay to the total number of pairs. The results of fractions calculation are shown in Tab.8.2. It was verified that the statistical uncertainties of obtained fractions are negligible. These fractions estimate how many reconstructed pairs come from primary particles and what is the percentage of those coming from given decay. They also take into account other impurities resulting from misidentification or detector effects. In addition, it was verified that fractions for baryon-antibaryon pairs are the same as for corresponding baryon-baryon pairs.

Pair	λ	Pair	λ	Pair	λ
pp	0.25	$p\Lambda$	0.29	$\Lambda\Lambda$	0.37
$p\Lambda$	0.12	$\Lambda\Lambda$	0.08	$\Lambda\Xi^-$	0.04
$p\Sigma^+$	0.04	$\Lambda\Sigma^+$	0.03	$\Lambda\Xi^0$	0.03
$\Lambda\Lambda$	0.02	$p\Xi^{0/-}$	0.02	$\Lambda\Sigma^0$	0.00
$\Lambda\Sigma^+$	0.01	$p\Sigma^0$	0.00	$\Sigma^0\Sigma^0$	0.00
$\Sigma^+\Sigma^+$	0.00	$\Lambda\Sigma^0$	0.00	$\Xi^-\Sigma^0$	0.00
		$\Lambda\Xi^{0/-}$	0.00	$\Xi^0\Sigma^0$	0.00
		$\Sigma^+\Sigma^0$	0.00	$\Xi^{0/-}\Xi^{0/-}$	0.00
		$\Xi^{0/-}\Sigma^+$	0.00		

Table 8.2: Fractions from AMPT for residual components of pp , $p\Lambda$, $\Lambda\Lambda$ correlation functions.

8.2.1.2 Momentum transformation

The momentum transformation from the k^* frame of parent particles to the frame of daughter particles was taken into account using transformation matrices generated from the Therminator model. The details of the procedure as well as the examples of matrices can be found in Sec.7.1.

8.2.1.3 Radii and m_T scaling

Source sizes for residual pairs are assumed to follow relation [10]:

$$R(m_T) = \frac{a}{\sqrt{m_T}} \quad (8.3)$$

The scaling factor a is designated when radius of the primary pair is set and then for each residual pair this radius is scaled using the above relation. The transverse masses m_T were generated from HIJING model [59] (see Tab.8.3) using distributions of the transverse momentum k_T :

$$\langle m_T \rangle = \sqrt{\langle k_T \rangle^2 + (2\mu)^2} \quad (8.4)$$

where μ is the reduced mass of the pair.

Pair	$p\Xi^0$	$\Lambda\Xi^0$	$p\Xi^-$	$\Lambda\Xi^-$	$\Xi^0\Xi^0$	$\Xi^0\Xi^-$	$\Xi^-\Xi^-$	$p\Sigma^+$	$\Lambda\Sigma^+$
$\langle m_T \rangle$	1.538	1.726	1.300	1.515	1.547	1.859	1.584	1.265	1.471
Pair	$\Xi^0\Sigma^+$	$\Xi^-\Sigma^+$	$\Sigma^+\Sigma^+$	$\Lambda\Sigma^0$	$\Xi^0\Sigma^0$	$\Xi^-\Sigma^0$	$\Sigma^0\Sigma^0$	$\Sigma^0\Sigma^+$	$p\Sigma^0$
$\langle m_T \rangle$	1.836	1.538	1.494	1.475	1.756	1.550	1.507	1.503	1.275

Table 8.3: Mean transverse masses calculated from the HIJING model [59].

8.2.2 Strong interaction parameters and source sizes

The radii scaling with multiplicity was performed using Eq. 7.4 with multiplicities listed in the previous section. It was assumed that radius for given baryon-baryon pair is the same as for corresponding baryon-antibaryon pair.

Interaction parameters for all baryon-baryon pairs were taken from the ESC08c model (see Sec.4.1.2.4) and are listed in Tab.8.4. For baryon-antibaryon pairs parameters are unknown and were free parameters of the fit (three sets of parameters, for proton-antilambda, lambda-antilambda and all other baryon-antibaryon pairs). In case of baryon-antibaryon, the spin-averaged interaction parameters were used to reduce number of free parameters of the fit (see Sec.6.4).

Pair	$f_0^s (fm)$	$d_0^s (fm)$	$f_0^t (fm)$	$d_0^t (fm)$
$p\Lambda$	2.46	3.14	1.73	3.55
$\Lambda\Lambda$	-	-	0.85	5.13
$p\Xi^{0/-}$	-0.56	-2.52	5.36	1.43
$\Lambda\Xi^{0/-}$	9.83	2.38	12.90	2.00
$\Xi^{0/-}\Xi^{0/-}$	7.25	2.00	-0.53	1.63
$p\Sigma^+$	3.91	3.41	-0.61	-2.35
$\Xi^{0/-}\Sigma^{+/0}$	2.80	2.46	10.90	1.92
$\Sigma^+\Sigma^+$	-	-	0.65	19.97
$\Lambda\Sigma^{+/0}$	from $\Lambda\Lambda$			
$\Sigma^0\Sigma^0$	from $\Lambda\Lambda$			
$p\Sigma^0$	from $p\Lambda$			
$\Sigma^0\Sigma^+$	from $\Lambda\Lambda$			

Table 8.4: Baryon-baryon effective scattering parameters used for fitting in this analysis (based on the ESC08c model - see Sec.4.1.2.4).

8.2.3 Momentum resolution

Another effect that should be taken into account while analysing correlation functions is the momentum resolution. A momentum of a particle deduced during the reconstruction process is always determined with some uncertainty, which will also cause uncertainty of k^* calculated for each pair of particles. As a consequence, the final correlation function will be smeared, which should also be corrected during the fitting procedure.

One possible method of correcting the momentum resolution is to apply reconstruction algorithms to the model data and compare generated pair momentum difference $q = 2 \cdot k^*$ with the reconstructed one. Then one can build 2D matrix to store reconstruction output for all q values from given range. An example of such matrix for proton-proton generated with HIJING is shown in Fig.8.1.

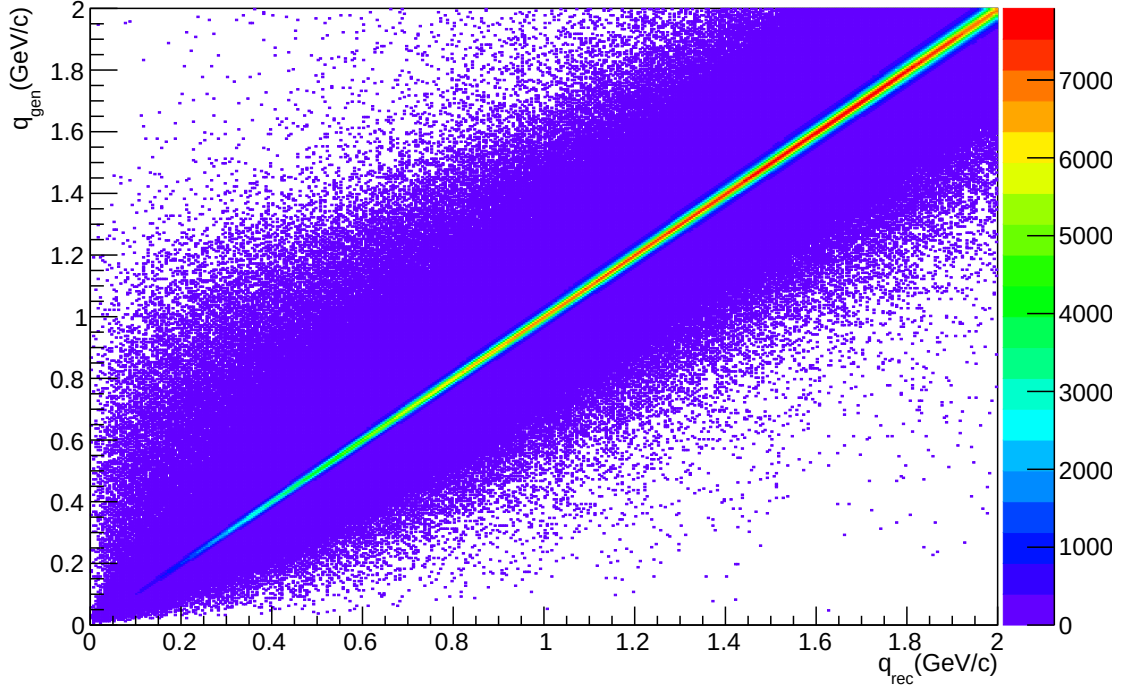


Figure 8.1: Momentum resolution matrix for proton-proton for Run 2 ALICE data.

Next, one can make projections of the momentum resolution matrix for each bin of reconstructed q_{rec} to see which generated values q_{gen} have which probability to give such momentum difference as a result. To simplify the fitting procedure, each slice of the matrix was fitted with a gaussian function. As can be seen in Fig.8.2, width of the distribution σ_{mr} is almost constant within the q range of interest. Thanks to this, one can further simplify correction for the momentum resolution, smearing the formula used in the fit with a gaussian distribution of constant width, using the following formula:

$$C(k_{mr}^*) = \frac{\int_{k_{mr}^*-5\sigma_{mr}}^{k_{mr}^*+5\sigma_{mr}} C(|k_{true}^*|) k_{true}^{*2} \exp\left[-\frac{(k_{mr}^*-k_{true}^*)^2}{2\sigma_{mr}^2}\right] dk_{true}^*}{\int_{k_{mr}^*-5\sigma_{mr}}^{k_{mr}^*+5\sigma_{mr}} k_{true}^{*2} \exp\left[-\frac{(k_{mr}^*-k_{true}^*)^2}{2\sigma_{mr}^2}\right] dk_{true}^*} \quad (8.5)$$

The average widths of the distributions were calculated with the method described above. The results for 2011 and 2015 data were the same. The matrices and widths of the fitted gaussian were generated and calculated as a function of the pair momentum difference q . The correlation functions were measured as a function of the momentum of one of the particles in the pair-rest frame $k^* = q/2$. To be able to use calculated widths for

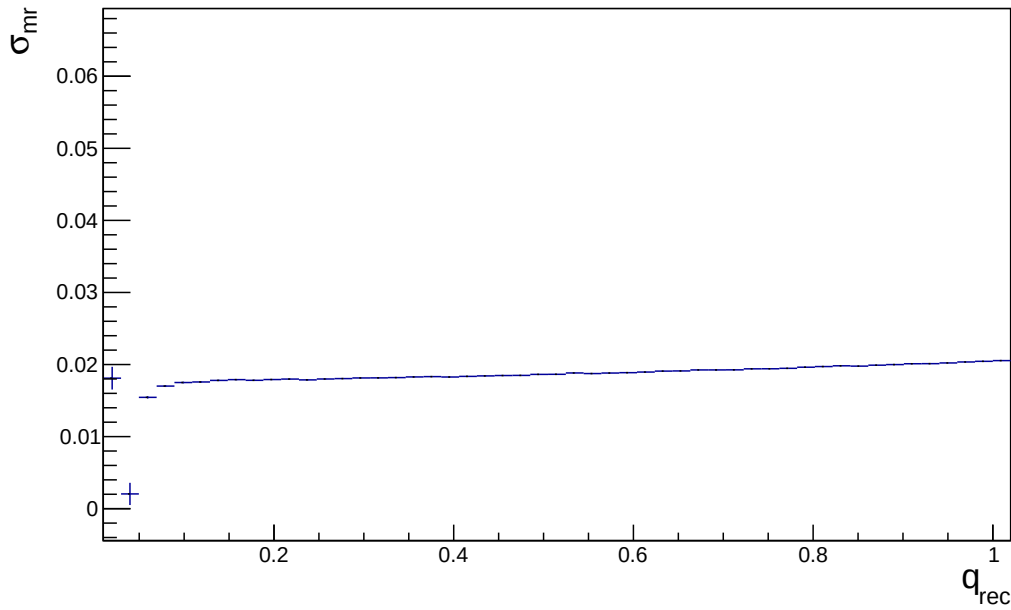


Figure 8.2: Width of the gaussian fit to the momentum resolution projections as a function of q_{rec} . HIJING data tuned to model Run 2 lead-lead collisions registered by ALICE.

smearing of the measured correlation functions, they were transformed to k^* . The results for different pairs are the following:

- pp and $p\bar{p}$: 0.010 GeV/c,
- $p\Lambda$ and $p\bar{\Lambda}$: 0.010 GeV/c,
- $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$: 0.009 GeV/c,

The example of how momentum resolution smearing affects the theoretical function is shown in Fig.8.3

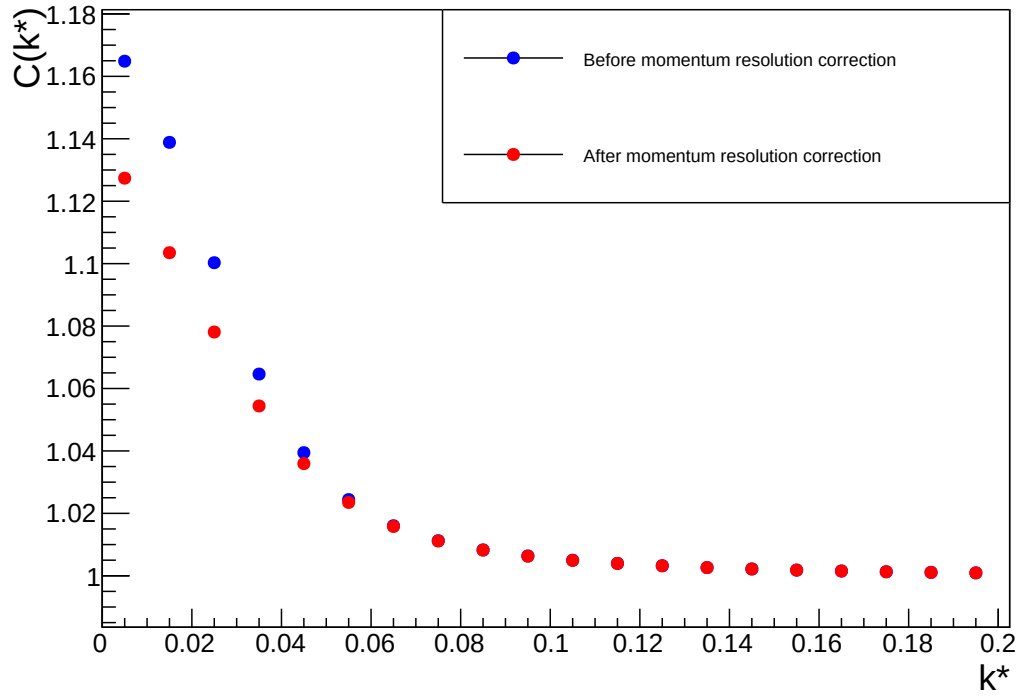


Figure 8.3: Theoretical $p\Lambda$ -like correlation function: only strong interaction, $\Re f_0 = 2.5 \text{ fm}$, $\Im f_0 = 0.0 \text{ fm}$, $d_0 = 3.0 \text{ fm}$. Plots before (blue) and after (red) momentum resolution correction with distribution width $\sigma = 0.010 \text{ GeV}/c$.

8.2.4 Background and normalisation correction

The experimental correlation function contains a non-femtoscopic background coming from the collective flow (see Sec.2.4.1). To remove this background, data from Therminator model was used. Different examples of correlation functions were calculated in a way described in Sec.7.1.2, but not including two-particle interactions. This allowed to study the influence of the non-femtoscopic background on the correlation function shape. An example of such function, together with linear and quadratic function fits, is shown in Fig.8.4.

In this analysis, it was chosen to parametrize background with a quadratic function. For each experimental function a quadratic fit was performed in a k^* region where femtoscopic effects are not prominent and then estimated background was subtracted from the function. Then, function was normalized just after the region of the femtoscopic correlations. The background and normalization ranges for all systems are listed in Tab.8.5.

An example of the correlation functions before and after applying this procedure are presented in Fig.8.5. As can be seen, the procedure flattens the function for higher k^* and the slope is bigger for more peripheral collisions, which is consistent with the elliptic flow, as it should be more prominent for peripheral collisions and less visible for the central ones [14].

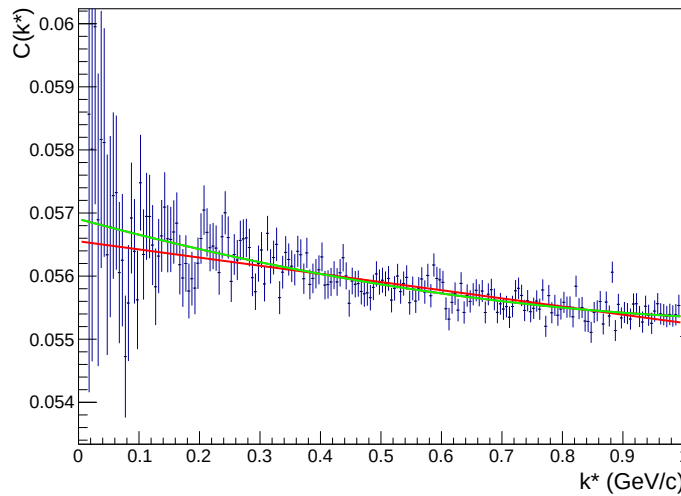


Figure 8.4: Non-femtoscopic background estimation from Therminator data. Red line - linear fit, green line - quadratic function fit.

Pair	Background		Normalisation	
	$\min k^*(GeV/c)$	$\max k^*(GeV/c)$	$\min k^*(GeV/c)$	$\max k^*(GeV/c)$
pp	0.20	0.80	0.20	0.30
$p\bar{p}$	0.15	0.80	0.20	0.30
$p\Lambda$	0.20	0.80	0.20	0.30
$p\bar{\Lambda}$	0.30	0.80	0.30	0.40
$\Lambda\Lambda$	0.20	0.80	0.20	0.30
$\Lambda\bar{\Lambda}$	0.30	0.80	0.30	0.40

Table 8.5: Background fit and normalisation ranges.

8.3 Simultaneous fit of different systems

In this section correlation functions for all combinations of (anti)proton and (anti)lambda baryons obtained in this analysis from ALICE data of 2011 and 2015 will be shown. Additional assumptions made to perform the fit will be discussed. All 72 functions generated and fitted for the purpose of this thesis (6 systems, 6 centrality bins, 2 collision energies) can be found in Appendix B.

For the purpose of this thesis, an application fitting theoretical correlation functions to the experimental data was developed. It allows to perform a complicated fit:

- for many different systems, which may share some fit parameters, for example scattering lengths and effective radii,
- including all residual correlations, taking into account strong and Coulomb interactions, as well as Quantum Statistics,
- scaling radii of residual components with some assumed relation (in this case Eq.8.3),
- in several centrality bins, assuming some form of scaling with multiplicity (for example Eq.7.4),
- apply momentum resolution correction at the level of the fit,

The simultaneous fit of several baryon-(anti)baryon systems has never been performed before. The results of such fit will be presented in this section.

Systematic uncertainties of the correlation functions presented in this section and in Appendix B were previously estimated by other collaborators. They include mainly

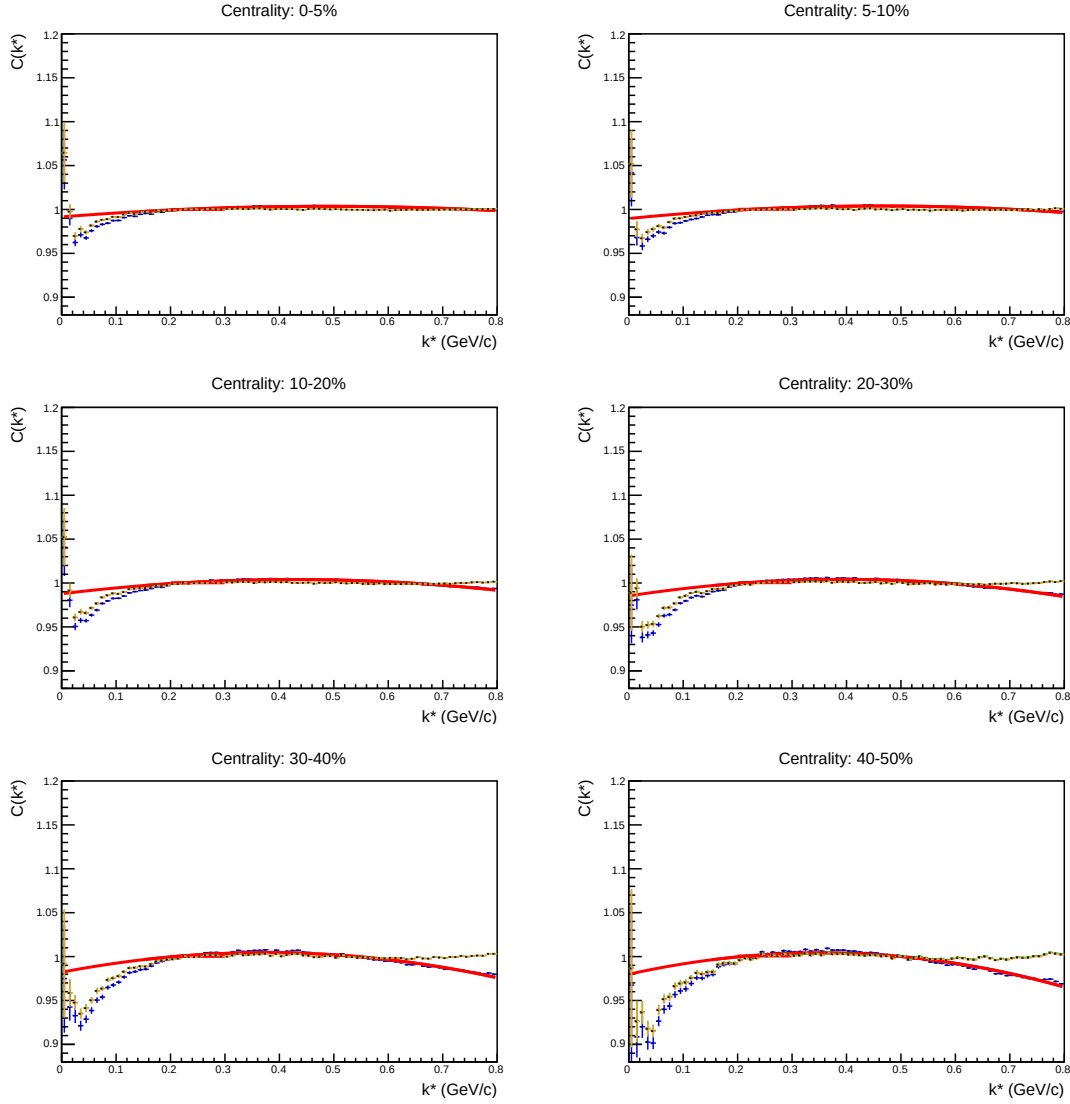


Figure 8.5: Example of background removing procedure for $\bar{p}\Lambda$ correlation function calculated from Run 2 ALICE data. Blue points - original function, red line - linear fit, orange points - normalised function with background removed.

uncertainties coming from two-track effects, such as splitting and merging (explained previously in this chapter). Different cuts' settings may influence the shape of the measured correlation function and this is reflected in the systematic uncertainty.

Additional test was performed to verify if systematic uncertainties of the measured functions may affect values of parameters extracted from the fit - the fit function was allowed to shift within systematic uncertainties. The results obtained with this modification were practically unaffected and the total χ^2 remains almost the same.

8.3.1 Assumed source sizes

To reduce number of free parameters of the fit and allow for proper statistical uncertainty calculation, radii for all systems were fixed. Values for different baryon pairs were calculated based on the previous measurements of ALICE for $\pi^\pm\pi^\pm$, $K^\pm K^\pm$, $K_s^0 K_s^0$, pp and $\bar{p}\bar{p}$ pairs [60]. It was assumed that source size follows the relation given by Eq.8.3

A fit to the ALICE data was performed and transverse masses for pp , $p\Lambda$ and $\Lambda\Lambda$ for 2011 and 2015 data were designated from the data. Then, the source sizes were extracted from function fitted to radii from previous measurements. The fit is shown in Fig.8.6, the extracted radii are listed in Tab.8.6.

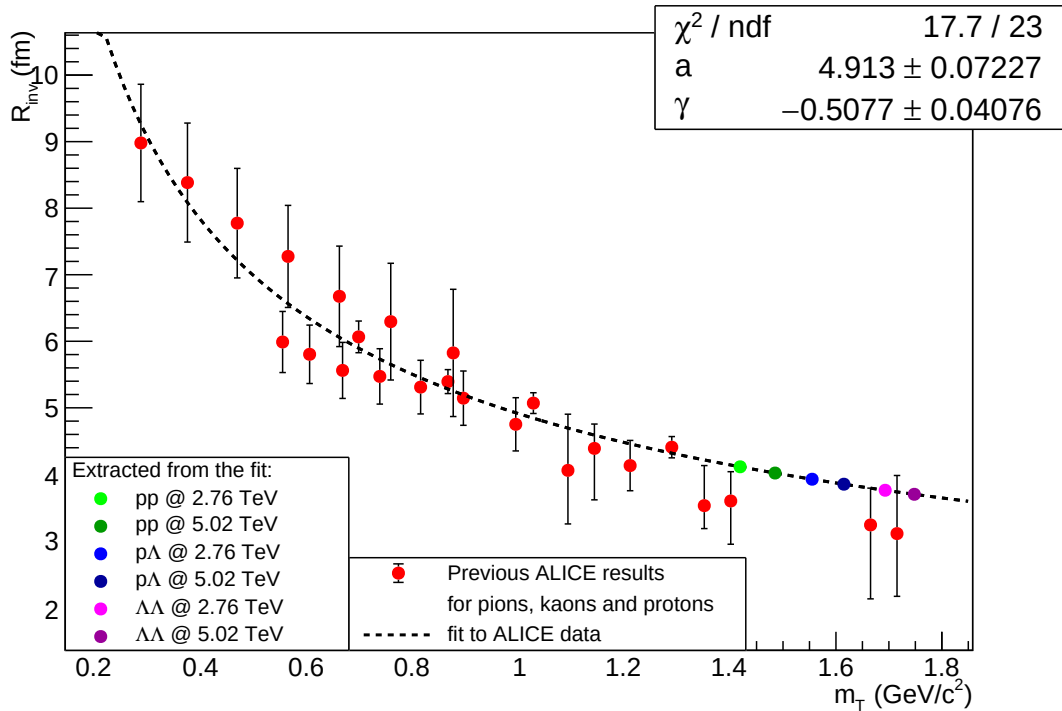


Figure 8.6: Fit to radii of $\pi^\pm\pi^\pm$, $K^\pm K^\pm$, $K_s^0 K_s^0$, pp and $\bar{p}\bar{p}$ measured by ALICE [60].

Pair	$R_{inv}^{0-5\%} (fm)$
pp (2.76 TeV)	4.23
pp (5.02 TeV)	4.12
$p\Lambda$ (2.76 TeV)	4.04
$p\Lambda$ (5.02 TeV)	3.95
$\Lambda\Lambda$ (2.76 TeV)	3.87
$\Lambda\Lambda$ (5.02 TeV)	3.79

Table 8.6: Radii calculated for pairs of identical baryons studied in this analysis.

8.3.2 Correlation functions with fits

Correlation functions were measured and fitted for all combinations of (anti)proton and (anti)lambda baryons. Whole analysis was performed for two collision energies (2.76 TeV and 5.02 TeV) in six centrality bins, which gives in total 72 correlation functions.

An example of calculated and fitted proton-proton correlation functions can be found in Fig.8.7 and different baryon-antibaryon correlation functions are presented in Fig.8.8. The referenced figures prepared for the purpose of this thesis were approved as preliminary results by the ALICE Collaboration.

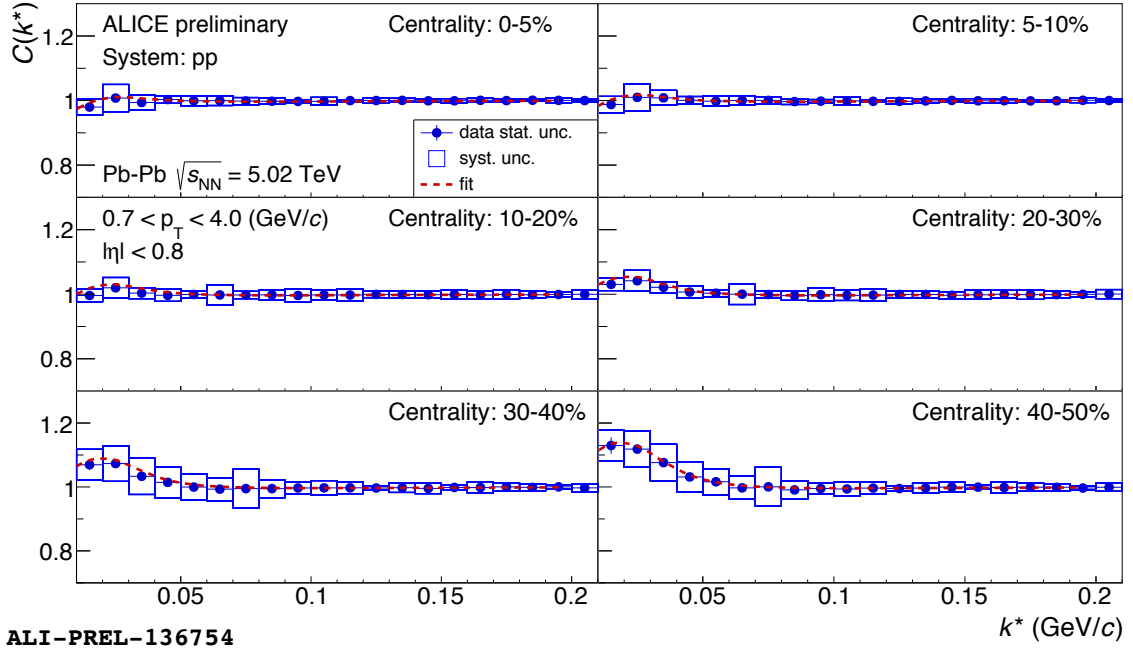


Figure 8.7: Proton-proton correlation functions from lead-lead collisions at 5.02 TeV together with fit.

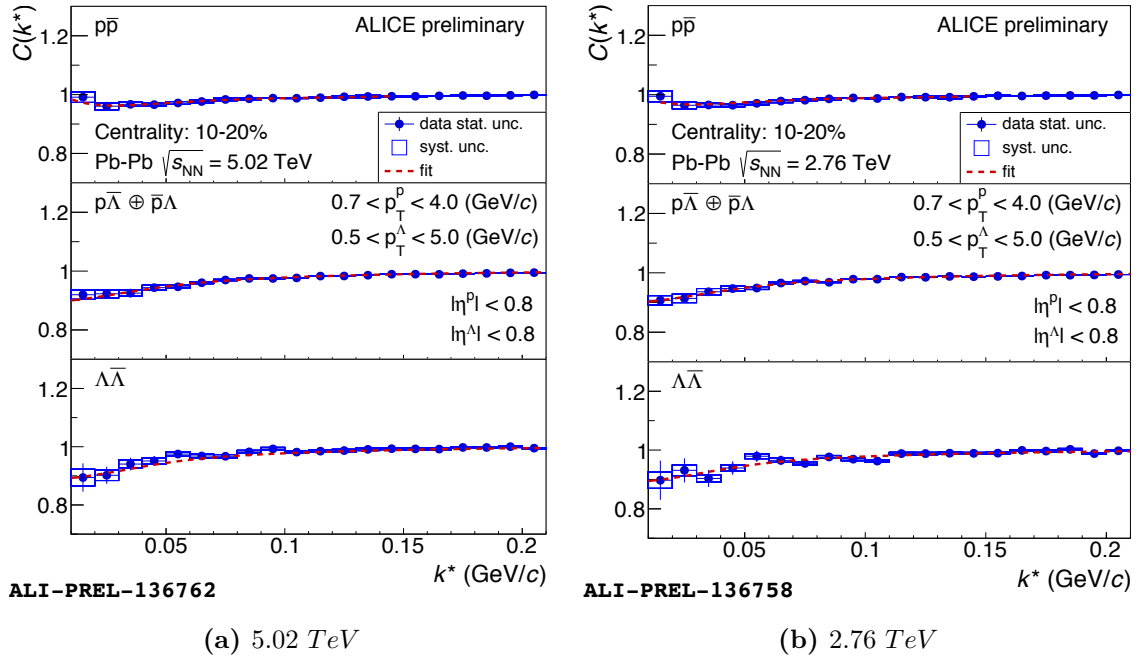


Figure 8.8: Examples of baryon-antibaryon correlation functions from the lead-lead collisions at 2.76 TeV and 5.02 TeV in 10 – 20% centrality bin together with fit.

8.4 Systematic uncertainties

To account for the systematic uncertainties of extracted fit parameters coming from the method, a series of modified fits was performed. The following components were included in the study:

- changing pairs' fractions by $\pm 10\%$,
- modifying values of baryon-baryon scattering parameters from ESC08c model by $\pm 10\%$,
- changing momentum resolution sigma by $\pm 30\%$,

The systematic uncertainty was calculated for all parameters as:

$$\Delta X_{syst}^2 = \sum_{i=1}^N (\bar{X} - X_i)^2 \quad (8.6)$$

where ΔX_{syst} is the systematic uncertainty, N is the number of modifications, $\bar{X}$ is the base value of the parameter and X_i are the values obtained from fits with the modifications.

8.5 Final fit results

The procedure of extracting source radii and strong interaction parameters was tested in detail in Chap.6 and Chap.7. Then it was applied to the correlation functions calculated for the purpose of this thesis from data gathered by ALICE in 2011 and 2015. For the extraction of baryon-antibaryon scattering parameters all measured baryon-antibaryon systems were used and fit was performed simultaneously for 36 correlation functions (3 systems, 6 centrality bins, 2 collision energies).

The results of the simultaneous fit of baryon-antibaryon systems ($p\bar{p}$, $\bar{p}\Lambda \oplus p\bar{\Lambda}$ and $\Lambda\bar{\Lambda}$) are summarised in Tab.8.7. The final global χ^2 was 1.57 (pdf).

Parameter	$b\bar{b}$	$\bar{p}\Lambda \oplus p\bar{\Lambda}$	$\Lambda\bar{\Lambda}$
$\Re f_0$ (fm)	$-1.04^{+0.24(syst)}_{\pm 0.15(stat)}$	$-1.11^{+0.18(syst)}_{\pm 0.04(stat)}$	$-0.96^{+0.16(syst)}_{\pm 0.06(stat)}$
$\Im f_0$ (fm)	$0.51^{+0.30(syst)}_{\pm 0.19(stat)}$	$0.63^{+0.15(syst)}_{\pm 0.05(stat)}$	$0.50^{+0.13(syst)}_{\pm 0.08(stat)}$
d_0 (fm)	$2.96^{+0.52(syst)}_{\pm 0.73(stat)}$	$2.56^{+0.55(syst)}_{\pm 0.14(stat)}$	$2.71^{+0.26(syst)}_{\pm 0.31(stat)}$

Table 8.7: Summary of the results of simultaneous fit of six systems in six centrality bins from two different collision energies.

The strong interaction parameters for baryon-antibaryon pairs are plotted in Fig.8.9. The ellipses represent systematic uncertainties and dashed lines show statistical uncertainties of the extracted parameters. Parameters of $p\bar{\Lambda}$ and $\Lambda\bar{\Lambda}$ were fitted independently, while for all other $b\bar{b}$ pairs it was assumed that these parameters have the same values.

In Fig.8.10 the results are plotted in comparison with previous measurements for $p\Lambda$ [61], $\Lambda\Lambda$ [62], $\eta'p$ [63], $p\bar{p}$ [64] and predictions for different baryon-baryon pairs from ESC08c model (see Sec.4.1.2.4). This helps to put measurement performed in this thesis in context of other measurements and to visualise its relative precision.

The results obtained in this analysis show that all $b\bar{b}$ pairs have non-zero imaginary part of the scattering length. This is an interesting and important result, as $\Im f_0$ is associated with the annihilation processes (see Sec.4.1.2.2), which sheds a light on how this phenomenon applies to pairs of different (anti)baryons. The imaginary part of the scattering length vanishes if there is no annihilation channel present. The result shows that even for non-identical $b\bar{b}$ pairs, such as $p\bar{\Lambda}$, the imaginary part has a non-zero value, which shows that there exists an annihilation channel.

The real part of the scattering length is negative for all pairs, which corresponds to the repulsive interaction or the existence of the bound state [65]. This is also a very important result, as currently it is not even known if the strong interaction is attractive or repulsive for $b\bar{b}$ pairs. The second scenario is also very interesting, pointing to possible creation of proton-antilambda or lambda-antilambda bound states, which could be searched for in the experiments in the future.

All the figures shown in this section were presented to the ALICE Collaboration and approved as preliminary results.

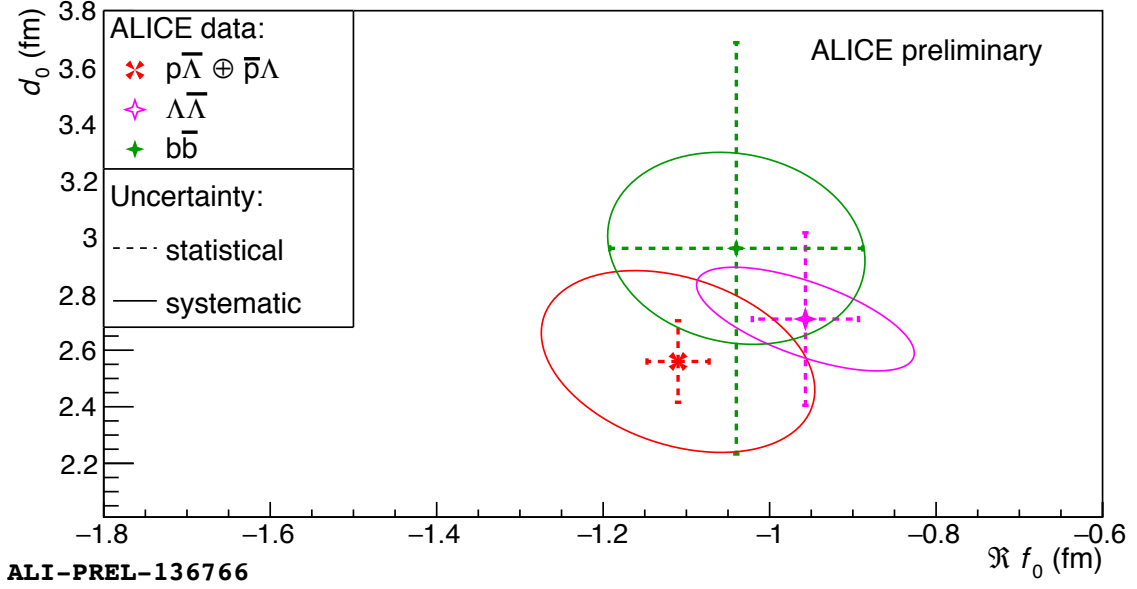
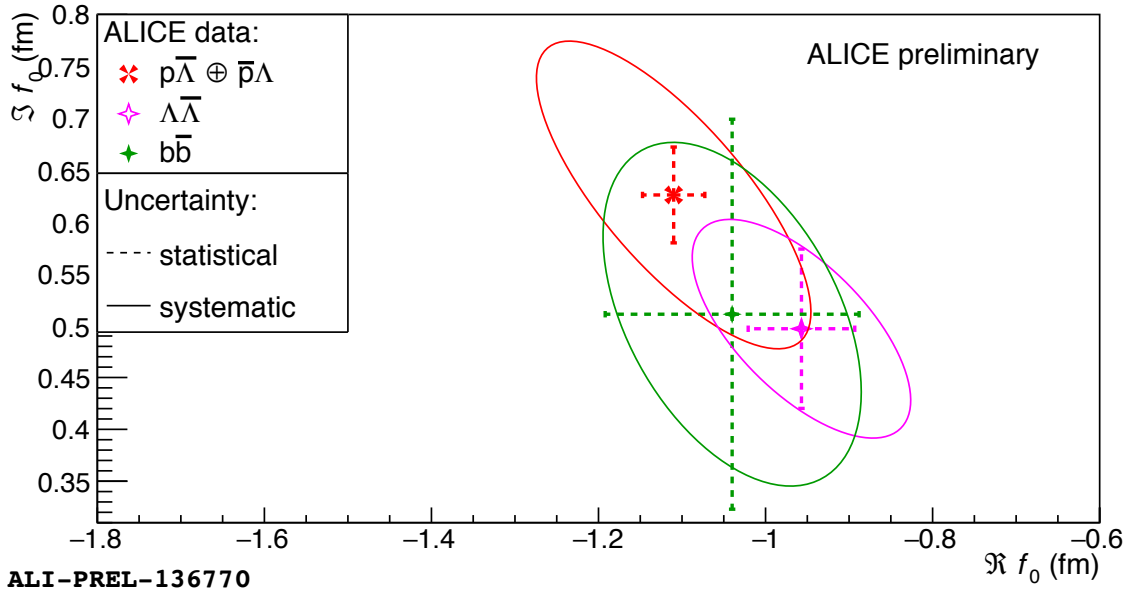

 (a) d_0 vs. $\Re f_0$

 (b) $\Im f_0$ vs. $\Re f_0$

Figure 8.9: Baryon-antibaryon strong interaction parameters extracted from fit to 2011 and 2015 ALICE data.

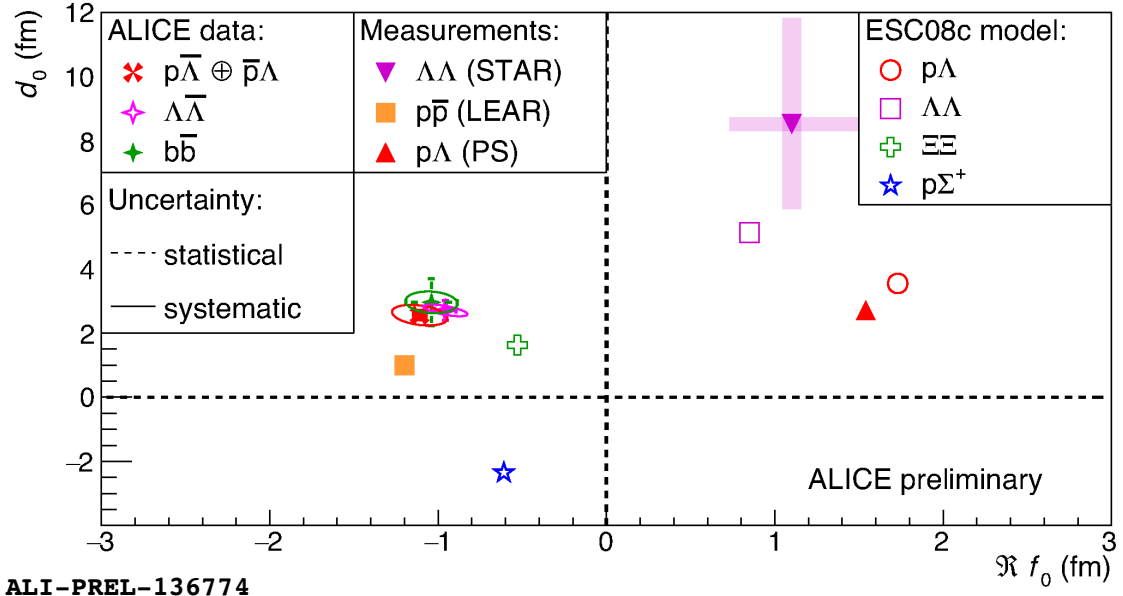
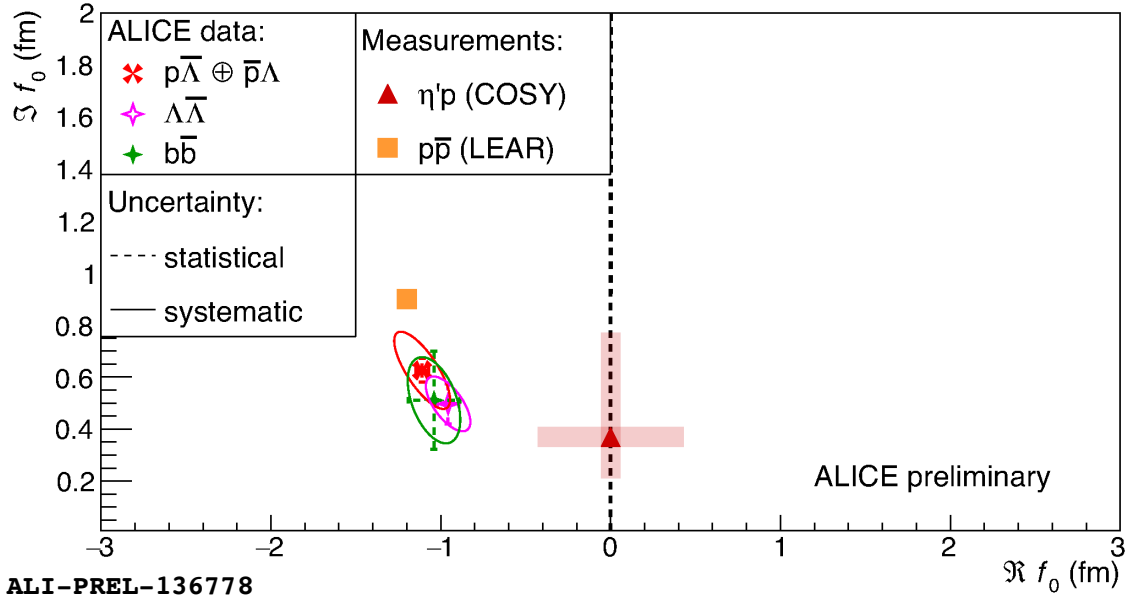

 (a) d_0 vs. $\Re f_0$

 (b) $\Im f_0$ vs. $\Re f_0$

Figure 8.10: Baryon-antibaryon strong interaction parameters extracted from fit to 2011 and 2015 ALICE data in comparison with previous measurements and ESC08c model predictions for baryon-baryon pairs.

Chapter 9

In-depth analysis of the fit

The simultaneous fit of all studied baryon-antibaryon systems was performed and results were presented in Chap.8. In this chapter, a detailed analysis of the results including several modifications of the fitting procedure will be presented. The goal of this study is to check what is the impact of different assumptions on the fit results.

9.1 Datasets comparison

The analysis was performed on two different datasets: LHC11h and LHC15o, which come from lead-lead collisions with two different energies and were collected in 2011 and 2015. To verify that results are consistent throughout both datasets, the fit was performed for each of them separately.

The comparison of the results can be seen in Fig.9.1. Obtained results are consistent, which confirms that both datasets can be fitted together.

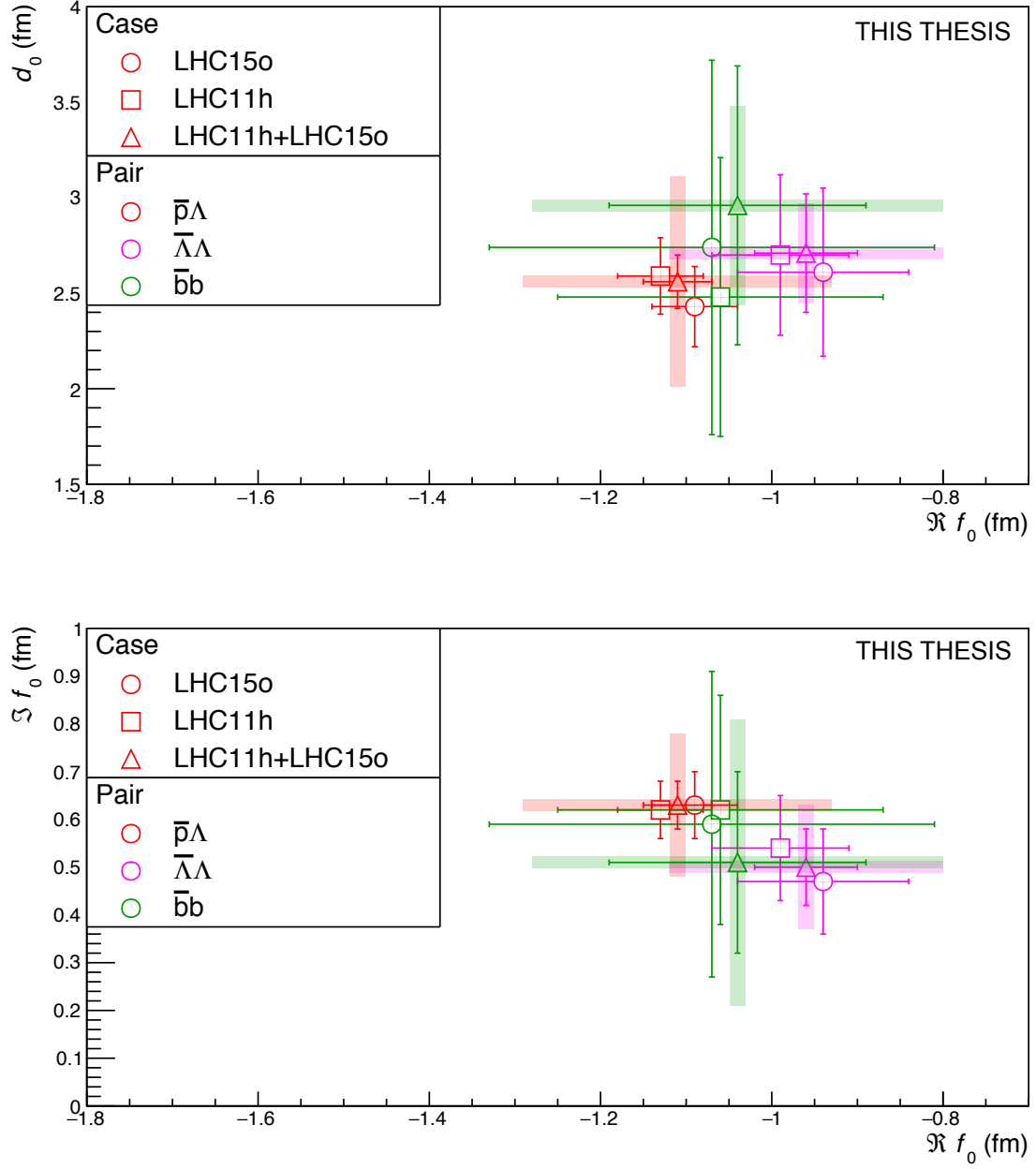


Figure 9.1: Comparison of fit results from LHC11h and LHC15o datasets.

9.2 Assumption of equal parameters for all $b\bar{b}$ pairs

As the simultaneous fit yealds similar values of the scattering parameters for all $b\bar{b}$ pairs, one can perform a fit assuming that fit parameters are the same for all systems. The results of such fit are presented in Fig.9.2.

What can be noticed is that this modification does not influence the fit results significantly. The global χ^2 remains the same: 1.57 (pdf). This suggests that assumption of equal scattering lengths and effective ranges for all $b\bar{b}$ pairs is reasonable.

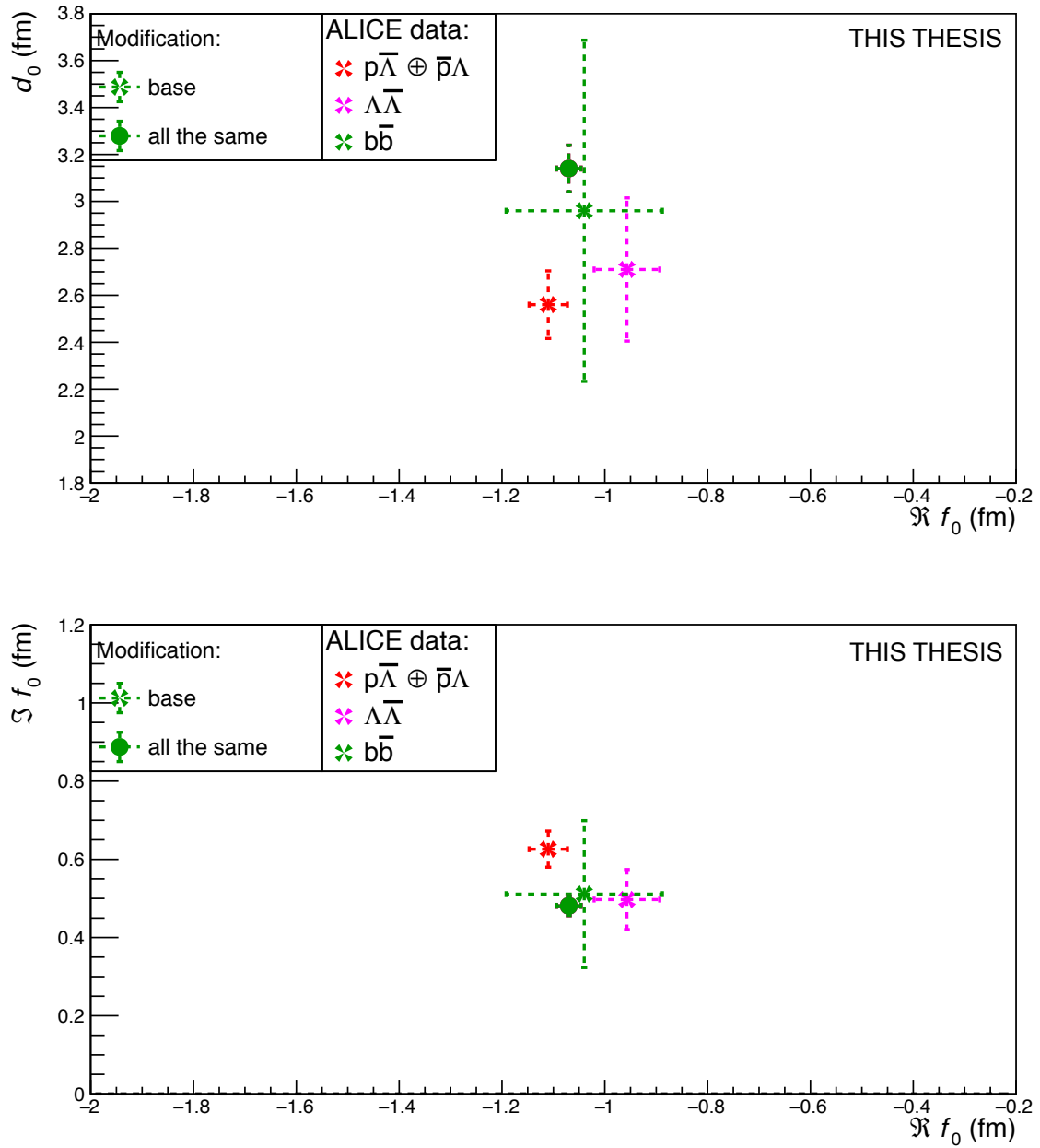


Figure 9.2: Fit results with assumption of the same parameters for all $b\bar{b}$ pairs.

9.3 Importance of residual correlations

In this section examples for all residual components for each fitted system will be presented. For this demonstration, data of 2015 was taken with centrality 10–20%. However, the overall picture does not depend on the dataset nor the centrality chosen.

The results for pp and $p\bar{p}$ are shown in Fig.9.3, for $p\Lambda$ and $p\bar{\Lambda}$ in Fig.9.4, for $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ in Fig.9.5. The residual components in the plots are scaled according to fractions calculated from Monte Carlo data (see Tab.8.2) and transformed to the k^* frame of the primary pair (see Sec.7.1). The blue dashed line shows a sum of all components and the black dashed line is the final fit function, after applying correction for the momentum resolution (see Sec.8.2.3).

A few conclusions can be drawn from these plots. The $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ functions, apart from the primary part, are influenced by $\Lambda\Xi^{0/-}$ components. The $p\Lambda$ correlation functions is dominated by the primary part. The $p\bar{\Lambda}$ function is significantly affected by $\Lambda\bar{\Lambda}$ residual component. In the pp and $p\bar{p}$ functions the $p\Lambda$ and $p\bar{\Lambda}$ components make an important contribution to the final function shape.

The momentum resolution effect is strongly affecting all the functions, which demonstrates how important is its correction to obtain proper results.

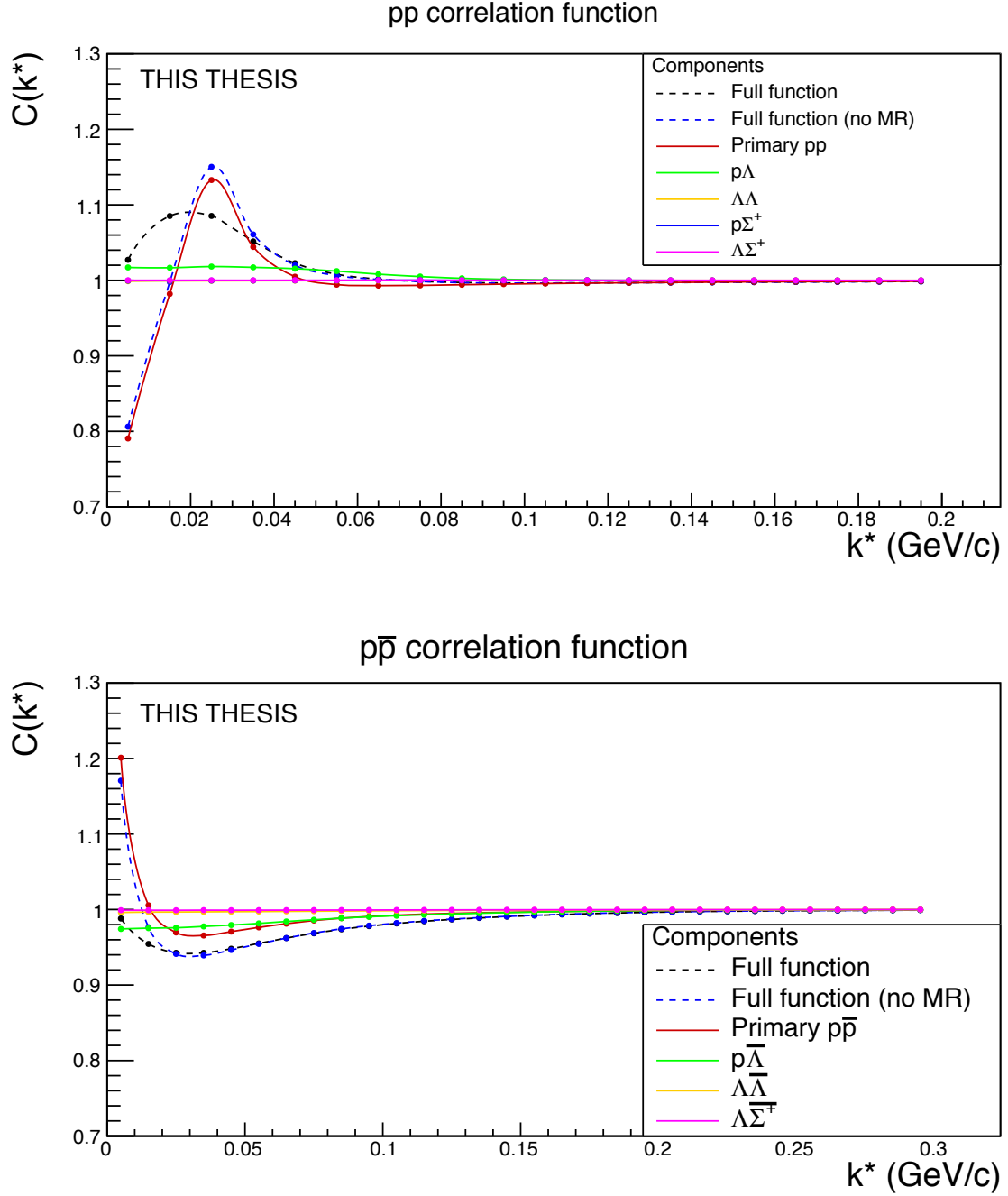


Figure 9.3: Residual components of pp and $p\bar{p}$ correlation functions fitted to ALICE data of 2015.

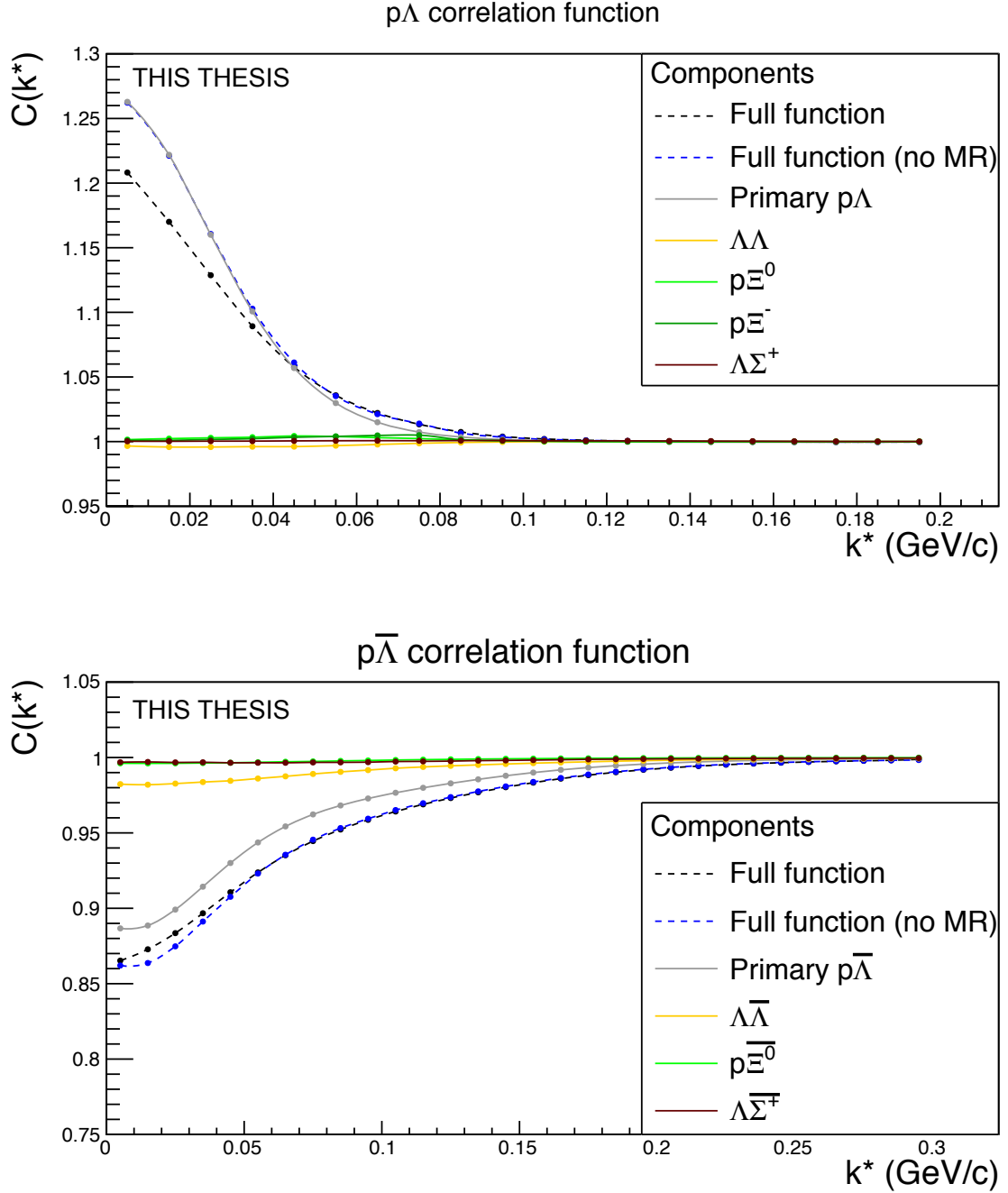


Figure 9.4: Residual components of $p\Lambda$ and $p\bar{\Lambda}$ correlation functions fitted to ALICE data of 2015.

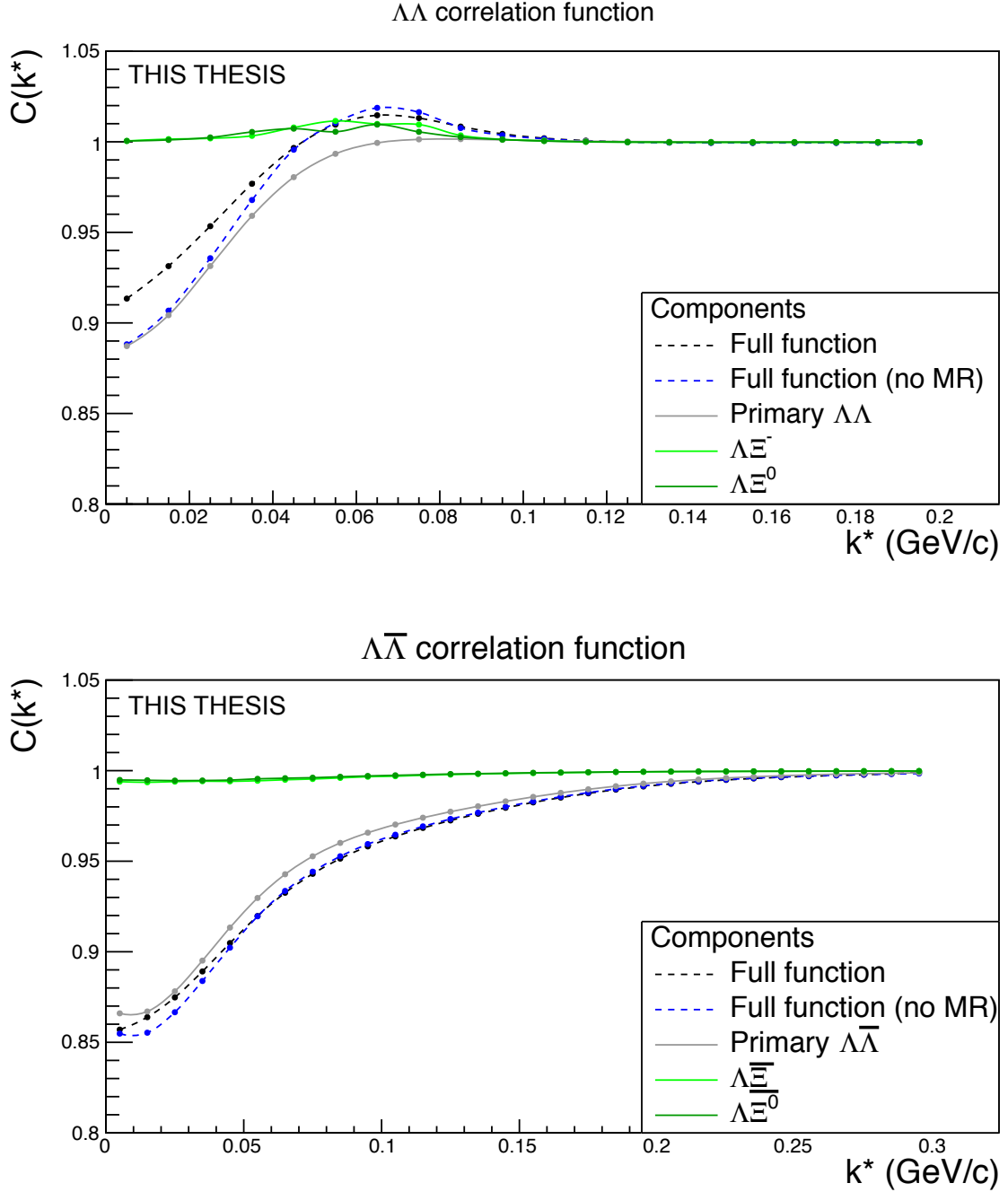


Figure 9.5: Residual components of $\Lambda\Lambda$ and $\Lambda\bar{\Lambda}$ correlation functions fitted to ALICE data of 2015.

Chapter 10

Conclusions

The focus of this thesis was a measurement of the interaction parameters for various baryon-(anti)baryon pairs. The femtoscopic correlations were proposed as a tool to perform such measurement.

In Chap.6 the detailed study of the femtoscopic correlations formalism was done. It was shown that analytical and numerical integration of the correlation function gives slightly different results, which may become an issue for some specific combination of the function parameters. The sensitivity of the function was also studied, showing that the scattering length influences the shape of the correlation function significantly, while effective range causes notable changes for baryon-antibaryon but subtly changes baryon-baryon correlation functions. Finally, it was shown that using spin-averaged scattering parameters allows to achieve very similar shape of the correlation function as in case of two different sets for singlet and triplet states. Based on the results presented in this chapter it was established, that the procedure based on the analytical integration of the correlation function, with a single set of spin-averaged scattering parameters can be used to measure all of them for baryon-antibaryon pairs.

Next, a test of the method was performed in Chap.7 on the correlation functions generated from the Thermanator 2 model. Source sizes were extracted from the fit to demonstrate that correct values can be obtained using the method based on the femtoscopic correlations. Momentum transformation matrices were calculated for all studied pairs to account for the kinematics of the heavier baryons decays. The procedure was then tested in presence of the residual correlations. Finally, it was extended to allow for a fit in several centrality/multiplicity bins simultaneously.

In Chap.8 correlation functions calculated from the data collected by ALICE were presented. A short summary of experimental cuts was shown. A series of corrections was explained and applied, such as limited momentum resolution of the detector or background

and normalisation corrections. All the assumptions concerning source sizes were discussed. Finally, fit results together with statistical and systematic uncertainties were presented.

In the last chapter, a few additional tests were performed, showing that the same set of parameters could be used for all studied baryon-antibaryon pairs. The importance of residual correlations was also discussed.

To sum up, the main outcome of this thesis was a measurement of the strong interaction parameters for $\bar{p}\Lambda \oplus p\bar{\Lambda}$, $\Lambda\bar{\Lambda}$ and effective parameters for heavier $b\bar{b}$ pairs. The most interesting conclusion is, that the real part of the scattering length for all studied pairs is negative, which corresponds to either strong repulsion between those pairs, or the existence of bound states of baryons and antibaryons. Another observation is that the imaginary part of the scattering length is positive for all studied pairs, which is related to the annihilation channels. This shows that all baryon-antibaryon pairs annihilate, including pairs which are not exact matter-antimatter counterparts, such as proton-antilambda.

Presented results open new research opportunities, either in the direction of searches for new particles (baryon-antibaryon bound states), or measurement of possible subtle spatial separation of matter and antimatter in the heavy-ion collisions due to the repulsive strong interactions.

Correlation functions and extracted values of the strong interaction parameters obtained from the analysis performed for the purpose of this thesis were presented to the ALICE Collaboration. Selected figures were approved as preliminary ALICE results (shown in Chap.8). The analysis note was created and the paper proposal was accepted by ALICE Physics Forum and ALICE Physics Board. Work on the publication is currently ongoing.

Appendices

Appendix A

Event visualisation in ALICE

A significant part of the work done during my PhD studies was focused on the maintenance and development of the visualisation system of ALICE (see Sec.3.3). In this chapter the most important components of the system and related projects will be described.

A.1 Online visualisation system

The current scheme of the online visualisation system of ALICE is presented in Fig.A.1. The flow of the data starts from the detector, registering collisions of protons or lead nuclei and sending raw data to the Data Center, where it is stored for analysis. In further sections different modules of the visualisation system will be described.

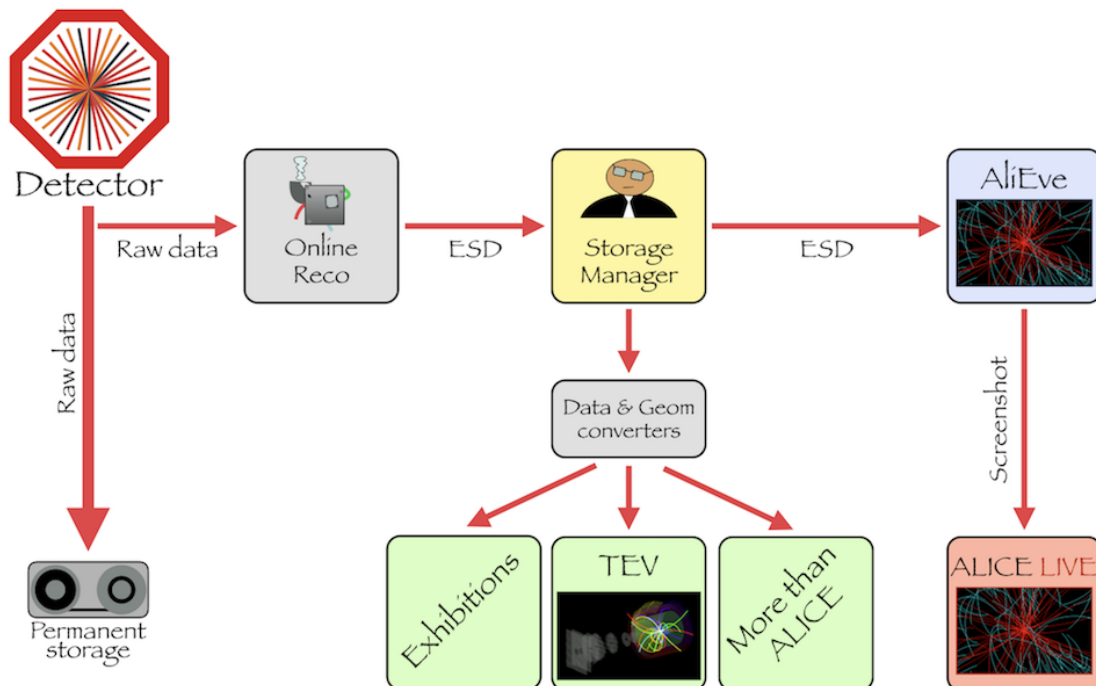


Figure A.1: Scheme of the online visualisation system of ALICE.

A.1.1 Online reconstruction

The online reconstruction takes a sample of raw data and reconstructs physical properties of particles produced in the collision, such as trajectory, initial momentum, type of the particle or its electric charge. This module allows for an early check of the quality of the data - if reconstruction cannot be performed online, this may indicate that also an offline reconstruction will not be possible, which makes the data unusable for physics analysis.

A.1.2 Storage Manager

The Storage Manager is a module responsible for starting online reconstruction when new run starts. Then it collects reconstructed events (so-called ESD objects, which stands for Event Summary Data) and saves them in the dedicated storage space. For each event an entry in the database is added to allow fast search of interesting events. Storage Manager also communicates with other modules and provides them with events fulfilling specified conditions (such as beam types, energy, multiplicity).

This module was created from scratch to meet the requirements of ALICE Run Coordination and needs of other collaborators. During three years of the studentship it evolved from a concept to fully-functioning application running 24/7 in the production.

A.1.3 AliEve

This module is an application responsible for the visualisation of reconstructed data. It is based on the ROOT framework [20] and allows for 3D visualisation of the detector geometry as well as particle tracks, clusters and calorimeters readout. It allows for an easy navigation between events, switching on and off different parts of geometry and particles visualisation, manipulation of the 3D model and 2D projections. It was also extended to help to produce screenshots which are ready to be used in publications and presentations. Examples of the visualisation from AliEve were presented in Fig.3.7 and Fig.3.8.

A.2 External applications

Beyond the internal system of visualisation there are several outreach projects in ALICE, whose aim is to share the latest results and educate the wide audience. In order to achieve this goal, an additional step is required - one has to translate geometry and data of ALICE to an external format, which can be read and understood by everyone. This in turn demonstrates a need to develop data and geometry converters.

The universal geometry converter was developed as a part of my doctoral studies. It reads geometrical shapes as defined within ROOT framework (so-called TGeoShapes) and converts them to Collada, which is a widely used format based on the XML.

The data converter was written by a bachelor student from Warsaw University of Technology working under my supervision. It extracts information which is crucial from the visualisation point of view (trajectories, particle types, information about decays) from the ESD objects and provides JSON or XML files as an output.

Thanks to the geometry and data converters, external applications gain access to ALICE data and can implement their own visualisation. A few examples of such projects will be presented in next sections.

A.2.1 Total Event Visualiser

Total Event Visualiser (TEV) is a tool developed by the CERN Media Lab and meant to be an universal visualisation tool for all LHC experiments. It is based on the game engine called Unity, which gives a big advantage over other possible solutions - the application can be deployed for any platform, including mobile and web platforms. It also guarantees modularity, which makes it relatively easy to include TEV visualisation in other projects (this is already in use in the CERN Microcosm exhibition).

Thanks to data and geometry converters ALICE is now included in the Total Event Visualiser.

A.2.2 More than ALICE

More than ALICE is a project aimed at using augmented reality to entertain and educate ALICE visitors. It was created by three students from Warsaw University of Technology

as a part of their bachelor theses preparation and further developed by two participants of the CERN Summer Students program, working under my supervision. The application is capable of tracking ALICE detector (while visitors are in the experimental cavern) or its paper model. Then it overlays informations about different sub-detectors on the image of mobile device camera. This helps to understand where different modules are located and how they work.

It is also possible to display visualisation of particles on top of the image of the detector or its paper model, which helps to imagine what happens during the collision. Thanks to the design of the visualisation system of ALICE, the reconstructed data can be visualised online. The planned scheme is presented in Fig.A.2 - More than ALICE tracks the paper model of the detector. Total Event Visualiser is embedded into the application and communicates with the Storage Manager, which in turn has an access to recently reconstructed online events. It uses converters to make the data understandable by external applications.

The user of the application can see online collisions displayed on his own mobile device on top of the camera image of the paper model, which one can assemble himself. The whole process allows to involve visitors and make it easier to understand the basic concepts of the high-energy physics.

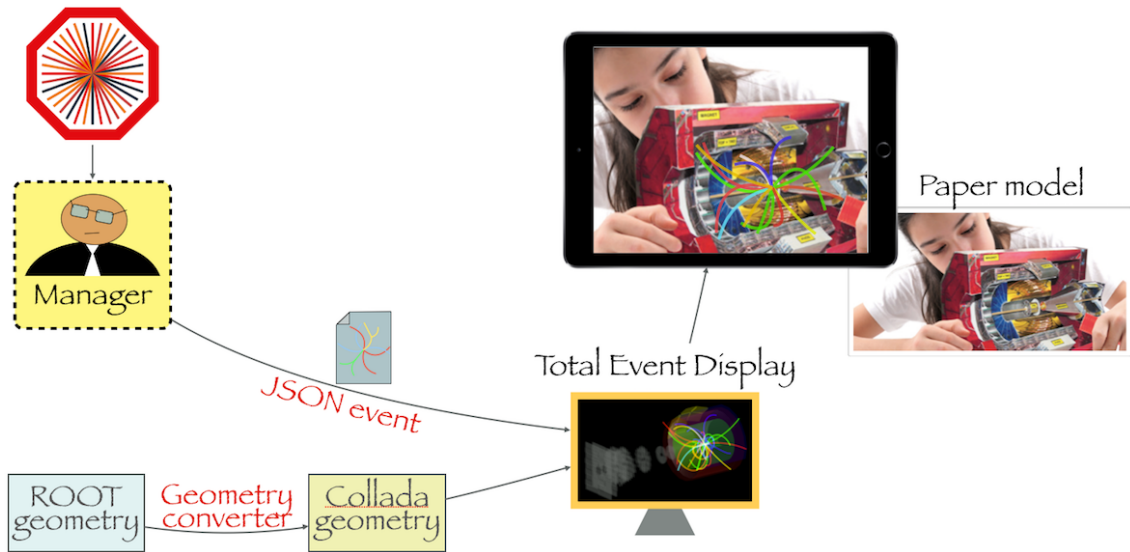


Figure A.2: Scheme of More than ALICE incorporated in the visualisation system.

A.2.3 Little Bang Theory

Visualisation is also willingly used by artists. One of the artistic projects closely related to the visualisation system of ALICE is the Little Bang Theory. It is an untypical representation of the products of particles collisions, which uses two beams of lasers scattered from semi-spherical mirrors.

Thanks to the ALICE data converter, the angle of scattered rays represents the initial direction of particles produced in the collision, while color represents particles type of momentum. Unlike in ordinary visualisation, trajectories cannot be curved, which reduces information contained in this representation, but also makes it unique. In addition, momenta of particles read from JSON files are being sonified and played along with the visual show.

Appendix B

Measured and fitted correlation functions

In this chapter all correlation functions measured for the purpose of this thesis together with fits will be presented.

B.1 Baryon-baryon

Proton-proton correlation functions are presented in Fig.B.1 and Fig.B.2 for energy of 2.76 TeV and 5.02 TeV per nucleon pair respectively. Proton-lambda functions are shown in Fig.B.3 for 2.76 TeV and Fig.B.4 for 5.02 TeV . Fig.B.5 and Fig.B.6 present lambda-lambda correlation functions at 2.76 TeV and 5.02 TeV respectively.

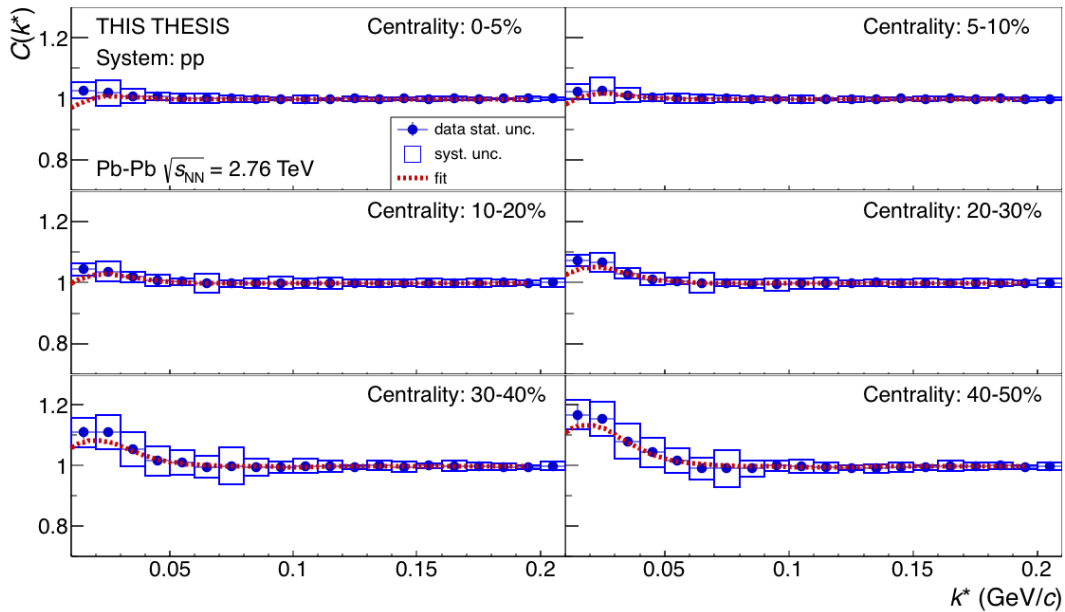


Figure B.1: Measured pp correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

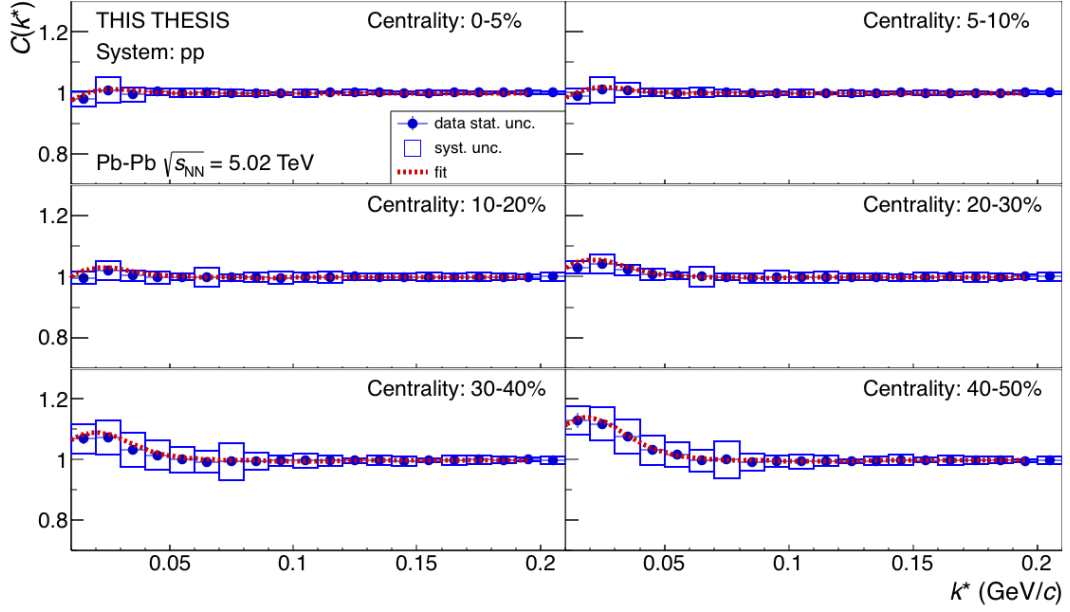


Figure B.2: Measured pp correlation function at energy of 5.02 TeV per nucleon pair together with fit.

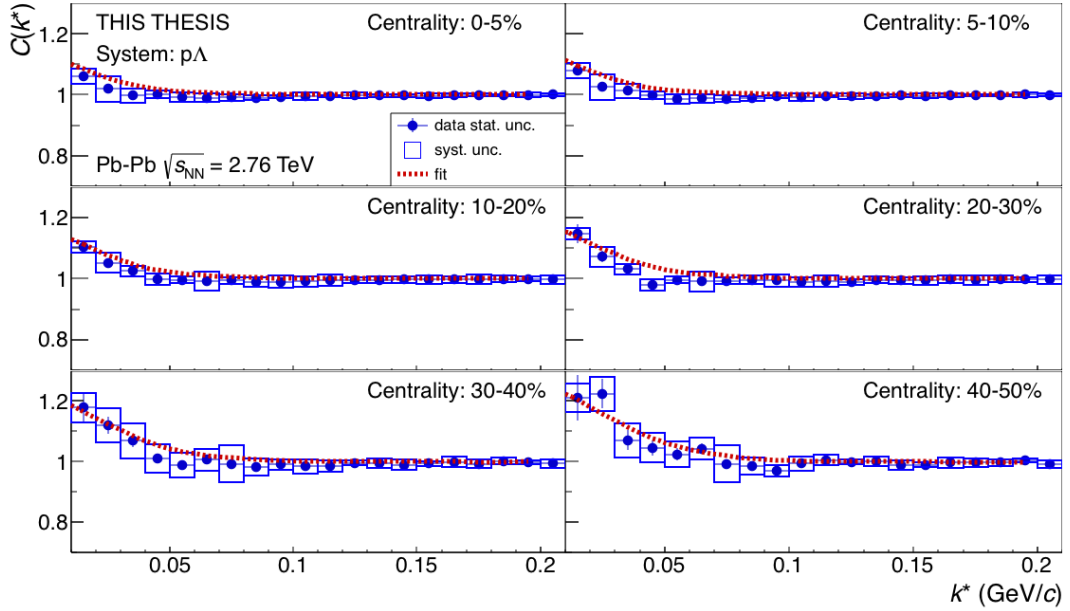


Figure B.3: Measured $p\Lambda$ correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

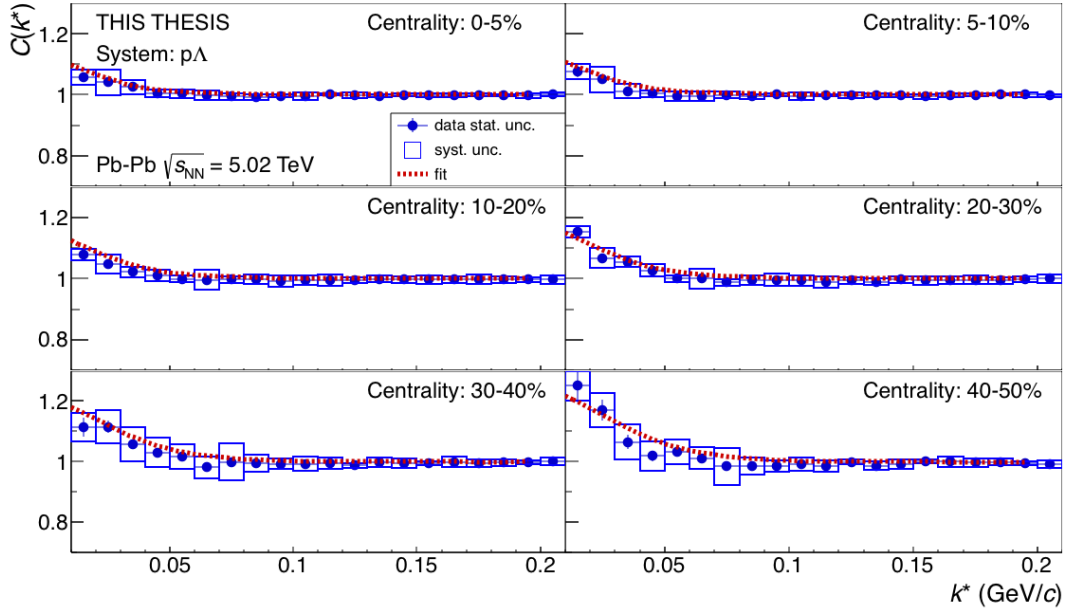


Figure B.4: Measured $p\Lambda$ correlation function at energy of 5.02 TeV per nucleon pair together with fit.

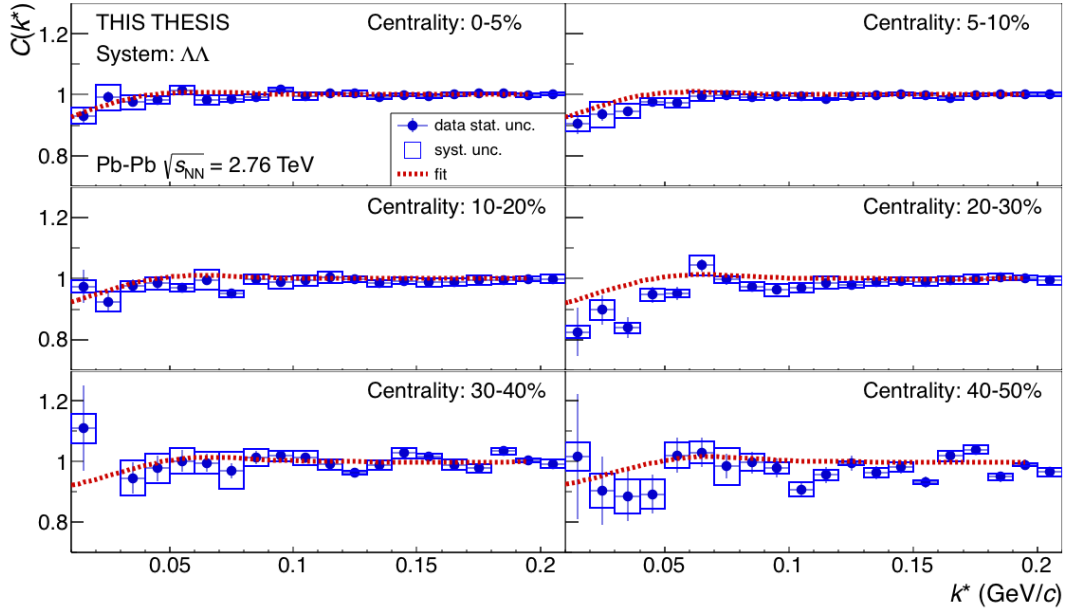


Figure B.5: Measured $\Lambda\Lambda$ correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

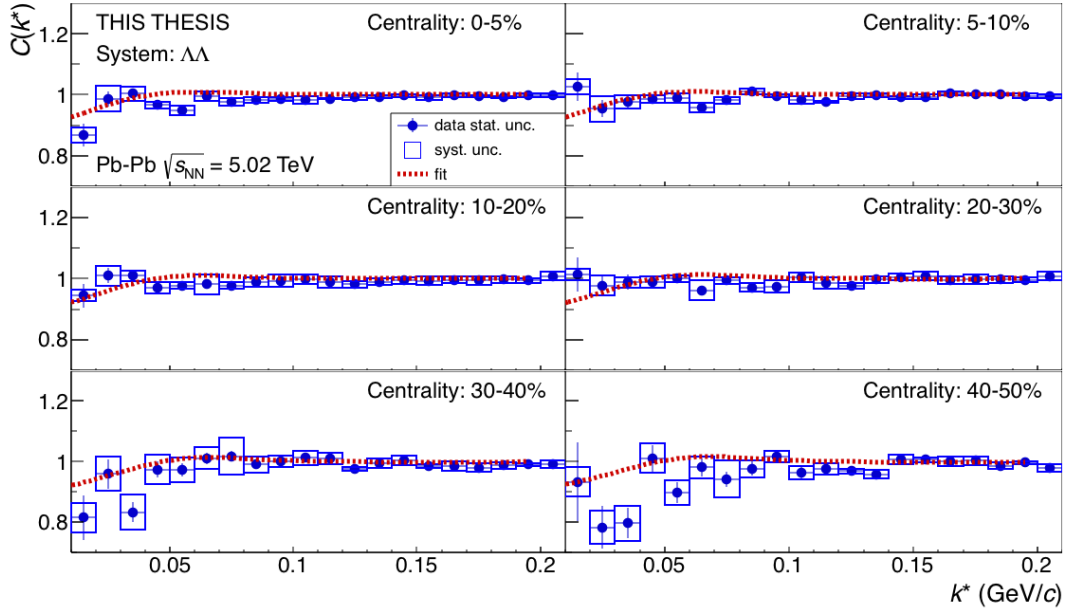


Figure B.6: Measured $\Lambda\Lambda$ correlation function at energy of 5.02 TeV per nucleon pair together with fit.

B.2 Baryon-antibaryon

Correlation functions for $p\bar{p}$ are shown in Fig.B.7 and Fig.B.8 for energy of 2.76 TeV and 5.02 TeV per nucleon pair respectively. $p\bar{\Lambda}$ functions are presented in Fig.B.9 for 2.76 TeV and Fig.B.10 for 5.02 TeV . Fig.B.11 and Fig.B.12 contain $\Lambda\bar{\Lambda}$ correlation functions at 2.76 TeV and 5.02 TeV respectively.

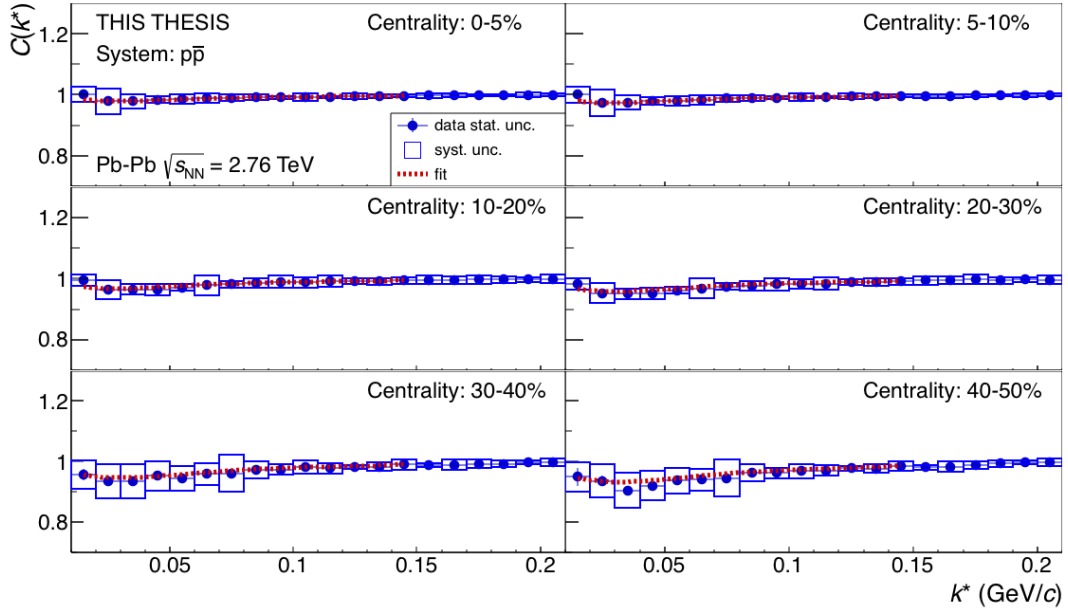


Figure B.7: Measured $p\bar{p}$ correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

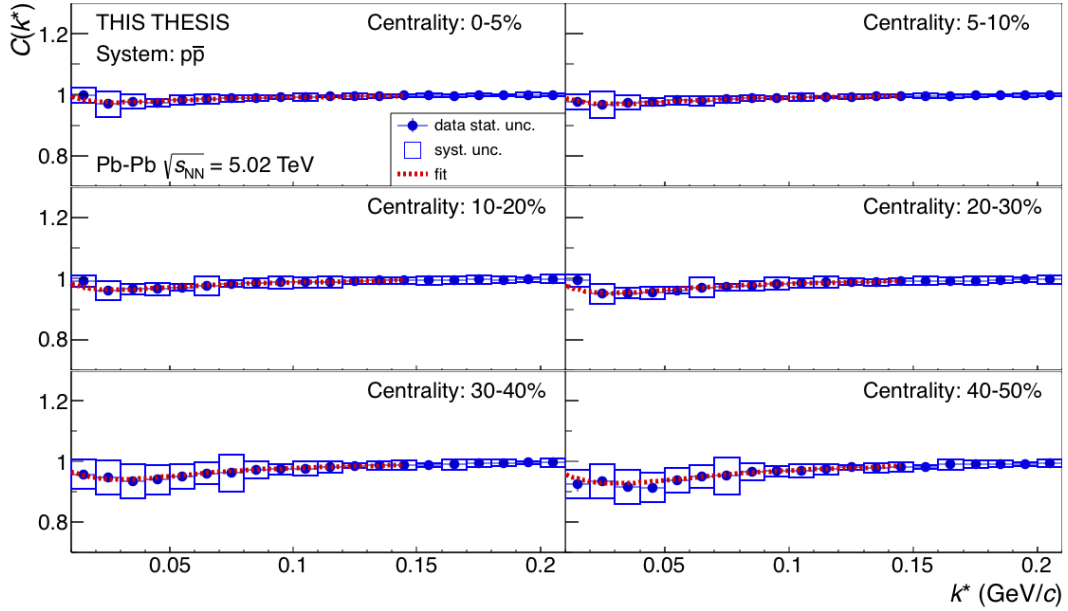


Figure B.8: Measured $p\bar{p}$ correlation function at energy of 5.02 TeV per nucleon pair together with fit.

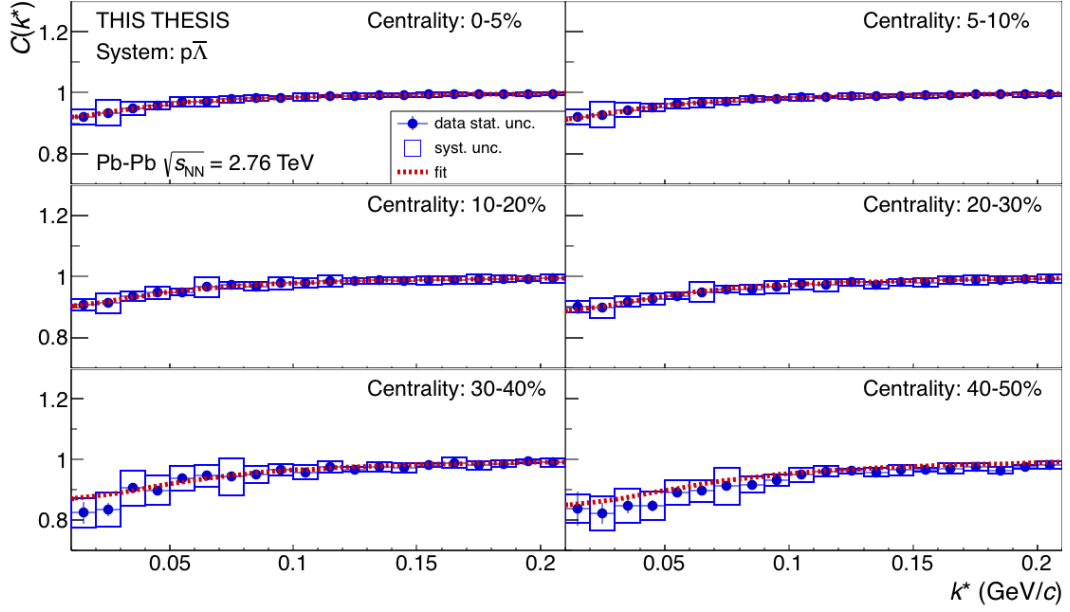


Figure B.9: Measured $p\bar{\Lambda}$ correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

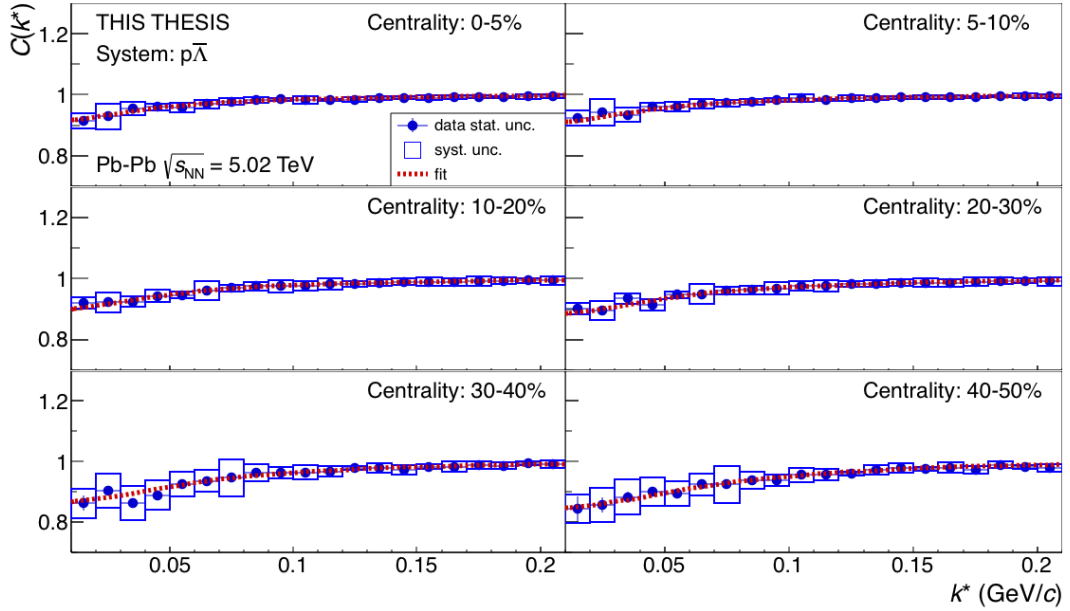


Figure B.10: Measured $p\bar{\Lambda}$ correlation function at energy of 5.02 TeV per nucleon pair together with fit.

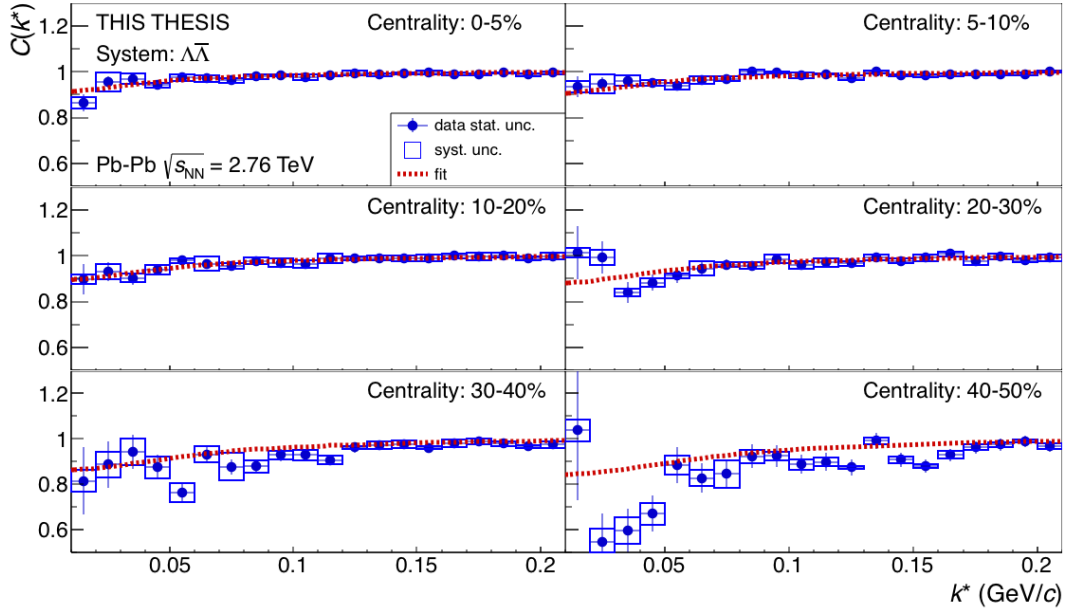


Figure B.11: Measured $\Lambda\bar{\Lambda}$ correlation functions at energy of 2.76 TeV per nucleon pair together with fit.

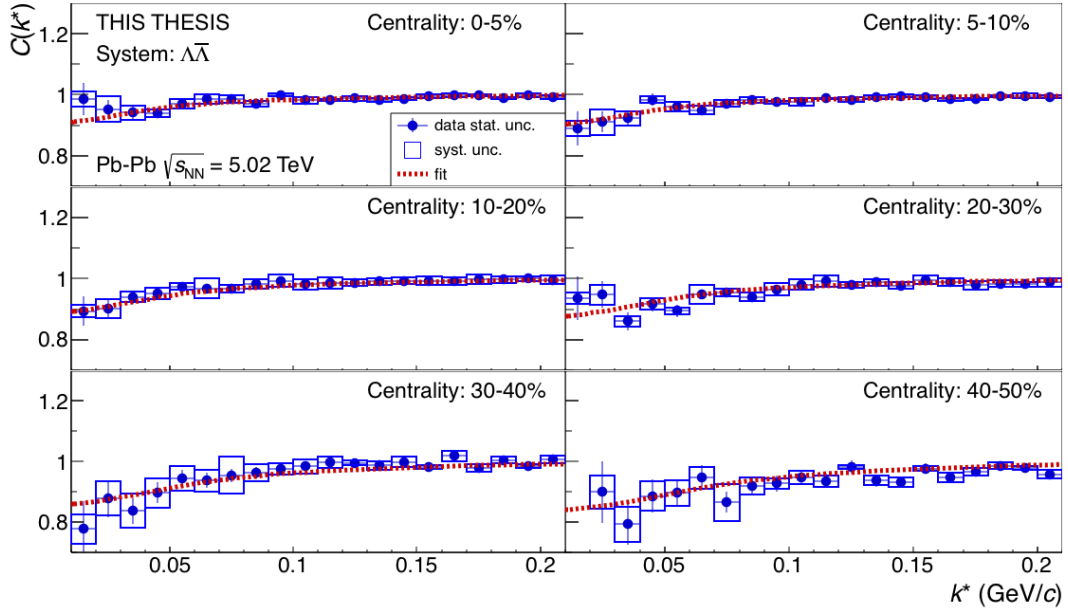


Figure B.12: Measured $\Lambda\bar{\Lambda}$ correlation function at energy of 5.02 TeV per nucleon pair together with fit.

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