

*To Ludwig Tauscher with many
thanks for a fruitful collaboration*
Staffan Carius

π^0 , γ , AND η SPECTROSCOPY OF $p\bar{p}$
ANNIHILATIONS AT REST

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Thesis for the degree of Doctor of Technology

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Abstract

The experiment PS182 performed at the Low Energy Antiproton Ring at CERN is presented. It was designed to search for exotic states (narrow) produced in proton-antiproton annihilations at rest. A spectrometer consisting of two sets of bismuth germanium oxide (BGO) crystals and a lead-glass array has been used to measure the π^0 and η inclusive momentum spectra and the inclusive photon energy spectrum produced in $p\bar{p}$ annihilations at rest. The tests of the BGO sets in an electron beam of energies from 50 to 450 MeV is described, as well as procedures to extract the energy and position resolutions for detected photons. Also discussed are the methods of calibration and reconstruction of impact position of photons detected by the lead-glass array

For the production rate of narrow exotic states, an upper limit is given as a function of the momentum of the emitted π^0 -meson or as a function of the energy of the emitted photon, respectively. The measured inclusive spectra show structures of ordinary mesonic resonances, which allows deduction of the branching ratios for the two neutral annihilation channels $p\bar{p} \rightarrow \pi^0 \omega$ and $p\bar{p} \rightarrow \eta \omega$. From the absence of signals upper limits are extracted for the neutral two-mesonic annihilation channels $p\bar{p} \rightarrow \pi^0 \eta$, 2η , $\eta\eta'$. By measuring coincidence events of three photons, the branching ratios of the rare annihilation channels $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0 \gamma$ are measured with precision, as well as an upper limit for the channel $p\bar{p} \rightarrow 3\pi^0$. Discriminating events of two high-energetic back-to-back photons resulted in an upper limit for $p\bar{p} \rightarrow 2\gamma$.

To be presented, with the permission of the Faculty of Applied Physics and Mathematics of the Royal Institute of Technology, for public criticism at the Research Institute of Physics (main auditorium) on May 30 1986 at 10.00.

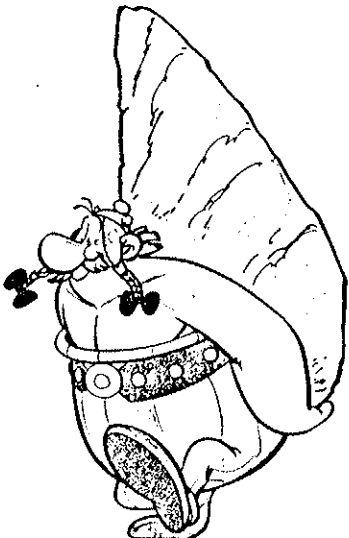
Preface

This thesis is a detailed description of the experiment PS182 performed at the Low Energy Antiproton Ring (LEAR) at the European particle accelerator laboratory CERN, situated outside Geneva, Switzerland. It was proposed to study rare annihilation channels of the $p\bar{p}$ system through the inclusive π^0 -momentum spectrum. The proposal was presented to the Research Board and the experiment was approved in May 1981. The first run period, a two days test run was in August 1983. Since then there have been four periods of run time, viz. in December 1983, in May and August 1984, and in May 1985. I will discuss data and results from them all. Before the chapter presenting the evaluation and interpretation of the data, there is a chapter describing the experimental apparatus and machine environment, and including an extensive discussion of the performances of the different detectors. The results from the detector investigations have been published [4] in Nuclear Instruments and Methods in Physics Research, section A.

I joined the group in mid-1981 when the experiment was in a progressed stage of preparation. The detector had already been designed and ordered, but the data acquisition system was not yet settled. During the design stage of the experiment several alternative solutions to various hardware aspects of the experiment were attempted before making a final choice. For instance, I worked half a year on developing an interface between CAMAC and a Motorola microcomputer [99], which eventually was too slow an alternative for the data acquisition system and will therefore not be mentioned.

Experiments in high-energy physics are tending to grow in size and are performed by large collaborations containing many people and usually several students. It can then be hard to separate off a particular well-defined task for each graduate student — as was the case in this experiment. I am, in this thesis, emphasizing work I have done myself or taken part in. Nevertheless I have, for completeness, also to mention work entirely done by colleagues. In these cases I refer to a private communication. After the introduction to the subject of the thesis I present a chapter that brings up a few theoretical topics concerning $p\bar{p}$ annihilation. This chapter should not be considered as being a part of the scientific work behind this thesis, but only as an information to the reader.

The discussion may sometimes seem unnecessarily detailed for the purposes of this thesis, but since I decided not to adopt the style of the author of an article to be published in a scientific journal, I allowed myself to analyse in depth a few subjects I regard as interesting physics, and which can be illustrative of useful methods pertaining to experimental elementary particle physics. Further, I have given extensive references to further reading material.



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CHAPTER 1

INTRODUCTION

The experiment being presented in this thesis belongs to the first generation of experiments performed at the $p\bar{p}$ -facility at CERN named LEAR (Low Energy Antiproton Ring), which was unique at the time of being built (1983). The experiment has the code name "PS182" and the title "Investigations on Baryonium and Other Rare $p\bar{p}$ Annihilation Modes Using High-resolution π^0 Spectrometers" [138]. When LEAR came in operation it was the first time in history high-intensity antiproton beams became available. The proton-antiproton annihilation mechanism could now be studied at several orders of magnitude more precision. In addition, the quality of the LEAR beam is much higher as compared to the conversion beams used before LEAR. The beam momentum is lower at LEAR and defined to the accuracy of a tenth of a percent, and the beam is also completely free from other contaminating particles. Several groups proposed experiments to investigate different features of the $p\bar{p}$ -annihilation process. Although the gross behaviour had been thoroughly measured in bubble chambers, details about this complex reaction were only poorly known, in particular for the completely neutral annihilation channels.

1.1. PHYSICS MOTIVES

We know from electron-positron processes, that annihilation channels are an exceedingly fruitful way of studying the hadron spectrum. The e^+e^- cross-section revealed the "charmed" and "beautiful" quark spectra. The states directly accessible from e^+e^- annihilation must, however, have the quantum numbers of the photon. Baryon-antibaryon ($B\bar{B}$) annihilation, on the other hand, has no such restriction and should thus exhibit a far richer spectrum. For each spin and isospin combination there are four types of states, having even and odd parity, even and odd C -parity, that can couple to $B\bar{B}$ channels, of which three are accessible to nucleon-antinucleon ($N\bar{N}$) processes.

Already in 1956 an article [5] put forward that the $p\bar{p}$ -annihilation process could be structure dependent in contrast to the e^+e^- annihilation which is a point-like reaction. It was then noticed that the phenomenological "strong" NN potential, describing the nuclear forces, when transformed into an $N\bar{N}$ potential turns out to be "strongly" attractive. Imaginably the $p\bar{p}$ annihilation at rest could proceed stepwise from level to level down the potential well. Later, after the *quark* concept [66] was invented in 1964, the baryons were understood as being composite objects comprising three quark constituents (q). Similarly the antiprotons possess three antiquarks ($\bar{q}$). These hypothetical constituents were supposed to carry a "strong" kind of charge named *colour*. The nucleons were then constructed from two different types of quarks having different so called *flavour*, which is a feature connected to the electric charge and the earlier concept of nucleon property, isotopic spin. By analogy with the electromagnetic theory, "Quantum Electro Dynamics" (QED), that renders the electromagnetic interaction as a consequence of photon exchange between electric charges, the theory for strong interaction between quarks, "Quantum Chromo Dynamics" (QCD), explains the interaction as a result of *gluon* exchange between colour charges. In contrast to QED however, where the exchange particle (the photon) is neutral, the gluons (g) in QCD carry colour charge themselves. This fact implies that the gluons interact with each other, which has some quite remarkable consequences. Possibly this circumstance accounts for the QCD coupling constant being so large, which in its turn causes the large strength of nuclear forces. It is also indicated by QCD that quarks do not appear freely in real life, but only in clusters that are colourless to the outer world. This phenomenon has been named *confinement*, and the physical quark combinations are referred to as colour *singlets*, because they are described by the singlet representation of the QCD gauge-group $SU(3)$. Such colour singlets are the ordinary mesons $|q\bar{q}\rangle$ and baryons $|qqq\rangle$ but also states like $|qqq\bar{q}\rangle$, $|q\bar{q}g\rangle$, and $|gg\rangle$. Of these singlet states the mesons and the baryons have

been well understood, but it has not been possible to identify any other kind of singlet state, from the present knowledge point of view that is. Although some pre-LEAR experiments claim they have observed signals that indicate the existence of these kind of objects, the experimental situation is not clear at all. Perhaps because these states should occur but are elusive they have been called *exotic*. Several experiments of the first LEAR era were motivated by the search for these exotic states. It certainly appears somewhat obscure that they would not exist at all, as they are **not** principally forbidden. Of course they might be very short-lived and therefore produce a spectrum too broad to be utilizable for identification, or there could be many states lying so close that they form a continuum (fig. 1.1).

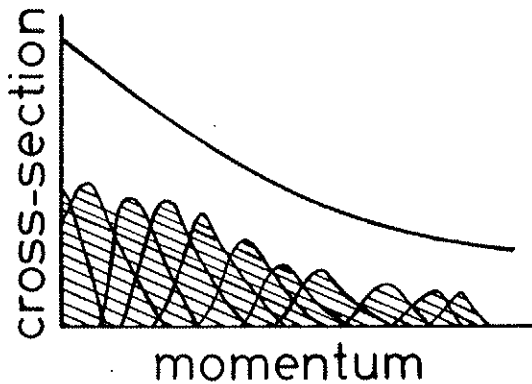


Fig. 1.1: Many exotic states form a continuous cross-section.

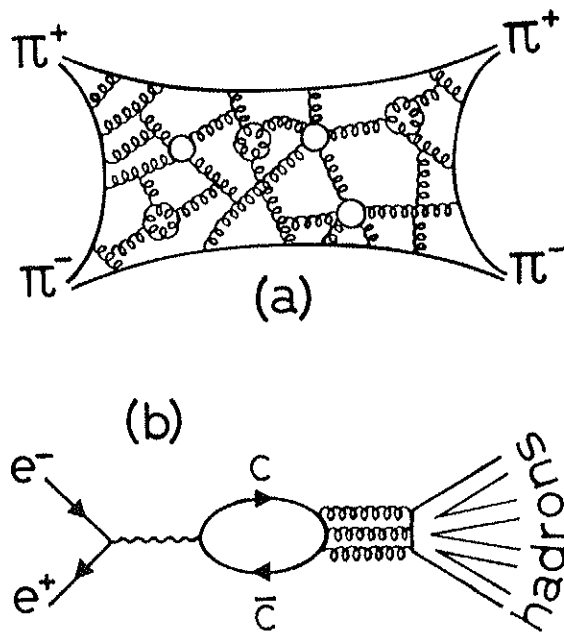


Fig. 1.2: A low-energy QCD process (a) is a complex mess of quark-gluon loops in contrast to a high-energy process as J/ψ -production in e^+e^- annihilations (b).

Unfortunately it is very difficult to compute masses and lifetimes of bound states in QCD. Because of the large coupling constant QCD does not allow for a perturbation expansion at low energies. Higher order processes seem to be important at spectroscopic energies, i.e. one can anticipate an avalanche of quark-gluon loops as illustrated by the pattern in fig. 1.2a. The same holds for scattering at low energies. Though $p\bar{p}$ scattering has a much more comprehensive spectrum than e^+e^- , having six sets of quantum numbers for every spin, it is considerably more complicated to analyse. For while any experimentalist can calculate the dominant Feynman diagram for J/ψ production and decay (fig. 1.2b) almost nobody can compute a strong interaction process

at low energies, since they are a complex mess of quark-gluon interactions (fig. 1.2a). Though these interactions are not computable as yet, there are rules that tell us that in the low-energy regime the dynamics is controlled by the flow of quantum numbers in the s -channel. For $\pi^+\pi^-$ scattering this means that of a $q\bar{q}$ pair. Thus, in the low-energy domain we see well-separated resonant states $\rho, f, g, h, \dots$ (fig. 1.3a). As the energy increases the resonances overlap and collectively build a smooth cross-section, which is most easily described in terms of t -channel $q\bar{q}$ exchange (fig. 1.3a tilted 90°). All this is well-known, as the fact is that there is every evidence that the same t -channel $q\bar{q}$ exchanges, as in meson-meson and meson-baryon scattering, occur in $B\bar{B}$ processes too. Therefore, when the appropriate "duality" diagram for the $B\bar{B}$ process is drawn (fig. 1.3b), we see that one quark in a baryon bag annihilates with an antiquark in the antibaryon forming a diquark-antidiquark ($q^2\bar{q}^2$) state [125]. Such a state has a special relationship to $B\bar{B}$ channels, because it can readily create a

$q\bar{q}$ pair in the joining string and fall apart into a baryon and an antibaryon. To disintegrate into mesons is much more difficult requiring a complete rearrangement of colour, since a diquark is not a colour singlet. Hence such states have generally come to be called *baryonium* [37]. This idea is given cre-

dence by the knowledge that much of baryon spectroscopy points to an asymmetric spatial distribution of quarks in a baryon. Two of the quarks often cluster forming a diquark which makes the process of fig. 1.3b very likely. However difficult to compute this kind of process, several partially phenomenological models, using different approaches, have been developed to overcome the shortcomings of QCD in the low-energy region. In chapter 2 three of these approaches are reviewed. Chapter 2 also includes some other aspects of the $p\bar{p}$ -annihilation process.

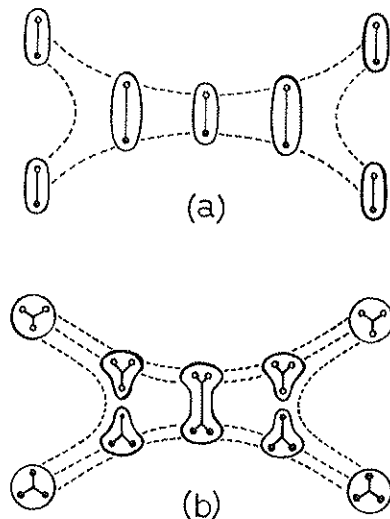


Fig. 1.3: In low energy $\pi^+\pi^-$ scattering (a) one observes well-separated resonant states $\rho, f, g, \dots$. The similar should hold for baryon-antibaryon scattering (b).

Some of the observed and established resonances fitting into the spectrum of ordinary $q\bar{q}$ -mesons have been interpreted [82] as being states of the kind $|qq\bar{q}\bar{q}\rangle$. Others have been suggested as candidates for states $|gg\rangle$ of gluons, frequently referred to as *glueballs*. Though the gluon itself is massless, bound gluon states generate mass. Even if there is much circumstantial evidence for these resonances to be exotic by nature they could still perfectly well be ordinary mesonic ones. Many conceivable $|qq\bar{q}\bar{q}\rangle$ states can be reached from $p\bar{p}$ annihilation at rest, i.e. from an atomic $p\bar{p}$ state. From the quark-picture point of view the $p\bar{p}$ annihilation is ideal for production of states like $|qq\bar{q}\bar{q}\rangle$ as the Zweig-rule [114] is not violated. The ultimate requirement for experiments that make spectroscopic measurements like PS182 is that at least some exotic states are narrow and arise above the background.

1.2. INDICATIONS FROM EARLIER EXPERIMENTS

The so called "S-meson" as found by Carroll et al. [33] and Brückner et al. [28] was a pointer to baryonium spectroscopy. A narrow peak was observed in the total cross-section and in the annihilation cross-section obtained from $p\bar{p}$ and $p\bar{d}$ scattering. However, attempts to deduce the quantum numbers for the states were confused. Moreover, it sat on a background ten times larger than the bump itself, the constitution of this background being unclear. Another place to look for baryonium states is in πN -production experiments. Thus, Benkheiri et al. [18] presented a spectrum with claims of two clear states at 2.02 and 2.20 GeV/c^2 . These discoveries delighted the colour chemists of the time who predicted [76] several states in this energy region. Even more excitement was induced [77] by a state at 2.95 GeV/c^2 reported by Evangelista et al. [59]. All these experiments, low energy $p\bar{p}$ scattering and πN -production experiments, have been repeated [70] [39] and nothing has been observed, at least with upper limits far below those of the original observations. Even the S-resonance is a shadow of its former self as reported by Hamilton et al. [70].

Later other groups [123] [134] investigated the inclusive photon spectrum produced from $p\bar{p}$ annihilations at rest. The spectra obtained by Richter et al. showed structure, and the authors concluded that three observed lines at 108, 183, and 216 MeV, respectively, were present as they appeared at the same positions with similar yields in three independent measurements. One of these lines (108 MeV) was possibly confirmed by Smith et al. [134] although the yields quoted by the two groups were not in agreement.

The pre-LEAR situation¹ was indeed confused. Nevertheless there existed some distinct promise in the promiscuous data flow thanks to an experiment by Eisenhandler et al. [57] that measured $p\bar{p} \rightarrow \pi^+\pi^-$ in the 0.9–2 GeV/c momentum range. The differential cross-section measured in that experiment is seen to be full of structure indicating the presence of high spin waves — in fact waves up to spin five. Recognizing that an amplitude analysis is not possible with just unpolarized data, this same experimental group measured the cross-section with a polarized target [34] at several momenta in

¹ There is a good review article by Pennington [117].

essentially the same range. The way to analyse such data is well-defined [17]. Thus, from each of the 1024 equally possible partial-wave amplitudes describing the $p\bar{p} \rightarrow \pi^+\pi^-$ data, it can be predicted what the observables should be for the pure isospin zero channel $p\bar{p} \rightarrow \pi^0\pi^0$. This differential cross-section has also been measured in the same momentum range [55]. These allow to discriminate between the many possible amplitudes — allowing in fact just two [94]. The dominant odd-spin waves are full of structure, even though they build a bumpy but rather smooth reaction cross-section. A mere inspection of the total cross-section can therefore be very misleading. To study whether the structures correspond to resonances, one must make a sophisticated phase-shift analysis and look at the behaviour of the phases. All the relative phases are determined, only the overall forward phase is not. Choosing this one in line with "causality" — the outgoing wave cannot leave the scattering centre before the incoming wave has arrived — all the partial-wave² amplitudes [94] (with common notations)

$$a_l = \frac{1}{2i} (\eta_l e^{2i\delta_l} - 1), \quad (1.1)$$

from the partial-wave expansion

$$f(k, \theta) = \frac{1}{k} \sum_l (2l+1) a_l P_l(\cos\theta) \quad (1.2)$$

are seen to rotate anticlockwise in a resonant-like fashion in the relevant Argand plot³. This is an evident indication that the $p\bar{p}$ annihilation might hide multitudinous secrets about a variety of resonances of a new kind.

1.3. CREDIT TO LEAR

Apparently, measuring the total or elastic $p\bar{p}$ cross-sections yields limited information. Similarly an observed signal in the inclusive energy spectrum, obtained from a spectroscopic measurement of a particular kind of particle emitted from $p\bar{p}$ annihilations at rest, can never prove the existence of an exotic state, as the possibility that it originates from an ordinary mesonic state cannot be ruled out. A more strict proof requires the ability to perform an amplitude analysis as was pointed out in the previous section. Since spin is an essential complication in such analyses, these require measurement of, not only the $N\bar{N}$ total and elastic cross-sections, but also the differential cross-section with unpolarized and polarized beam and target too, not to mention the polarization state of the final $N\bar{N}$ pair, a task that requires years of high precision, high statistics work and with a dedicated machine. *LEAR* is just such a facility. Even so, this is a daunting task. Valuable insight can however be gained, as we saw in the last section, by looking at simpler final states with spinless particles, like $\pi\pi$, $K\bar{K}$, $\pi\eta$ etc. Of the twelve sets of quantum numbers with isospin 0, 1 accessible to $N\bar{N}$

Fig. 1.4: (Taken from ref. [117]).



² For an introduction to the partial-wave description see for instance the book by Perkins [118].

³ In the complex plane the amplitude a_l will move around the unitary circle anticlockwise for a perfectly elastic resonance.

interactions, such processes pick out simply the four natural parity channels to be explored in turn. Three-body final states are needed to expose the unnatural parity sector [94], though none of the experiments of the first era of LEAR has addressed itself to such exploration. Nevertheless $p\bar{p} \rightarrow \pi^+\pi^-$ is present [29]. As was argued in section 1.2 the way to a unique amplitude, a unique set of nonperturbative solutions to QCD, is to measure the unknown phases of the amplitudes by constraining supplementary data on $N\bar{N} \rightarrow \pi\pi$ with well-established results on its $\pi N \rightarrow \pi N$ crossed channels. Polarized beam experiments at LEAR can make this a reality. Recall this is just the spectrum coupling to both $N\bar{N}$ and $\pi\pi$. One therefore expects six times as many states to occur in $p\bar{p} \rightarrow p\bar{p}$ with its six times more accessible quantum numbers. Small wonder the "S-meson" may be a very complicated object and not the single S -matrix pole assumed in references [33] [28] [70].

The analysis outlined in section 1.1 determines for a resonance (M) the product of its branching ratios to $N\bar{N}$ and $\pi\pi$, for the dominant $J=1,3$ states, to be

$$R(M \rightarrow N\bar{N}) \cdot R(M \rightarrow \pi\pi) \sim 0.02 \quad (1.3)$$

Note that for a $q\bar{q}$ -meson which is a daughter of the ρ - f trajectory we would anticipate

$$R(M \rightarrow \pi\pi) \sim 10\%, \quad (1.4)$$

and if such states have coupling of similar strength to $N\bar{N}$ as to $\pi\pi$ then

$$R(M \rightarrow N\bar{N}) \sim 1\% \text{ to } 3\%, \quad (1.5)$$

as a result of the smaller phase space related to $N\bar{N}$ than related to $\pi\pi$. Thus the product in eq. (1.3) would be between 0.001 and 0.003, i.e. an order of magnitude smaller than observed. Since the $\pi\pi$ coupling is unlikely to be very much larger when there are so many other meson channels accessible, we must conclude that for such states to show up so strongly in $N\bar{N} \rightarrow \pi\pi$, their coupling to $N\bar{N}$ must be substantially larger than to $\pi\pi$. This may indicate that what has been observed are indeed mesons with a larger $|qq\bar{q}\bar{q}\rangle$ component. To "prove" this statement one has to insist on an equivalent analysis of the same spin states in $N\bar{N} \rightarrow N\bar{N}$ scattering, and LEAR enables this possibility. There is much work

for LEAR to do for many years ahead. With isospin 0 and 1 there are sixteen types of meson states to study, of these four are closed to the $N\bar{N}$ channel. LEAR will allow access to this "Mesonic Hall"⁴. While in the $N\bar{N} \rightarrow B\bar{B}$ channel everything is seen at once, the simple two-body annihilation channels only allow peering in through the windows one at a time. From the glimpses already seen, through looking at pre-LEAR data on the $p\bar{p} \rightarrow \pi\pi$ channel, a rich spectrum of hadron states is apparent; a rich spectrum deserving of further investigation. There may well exist a whole new family of mesons coupling strongly to baryon-antibaryon channels. Though these are less spectacular, producing only broad structures and smooth cross-sections like that for $p\bar{p} \rightarrow \pi\pi$, rather than ψ -like peaks obtained in e^+e^- annihilation, they may teach us just as much about the force holding quarks together and the nature of confinement. The $p\bar{p}$ channel allows a unique insight into the "Mesonic Hall", where each room is potentially packed with states, not only those made of two or more quarks, but also glueballs and hemaphrodites. Knowledge of these will shed vital light on the nature of QCD. Indeed, if QCD is part of an even grander theory, then its spectroscopy, with supersymmetric partners for example, may be revealed too.

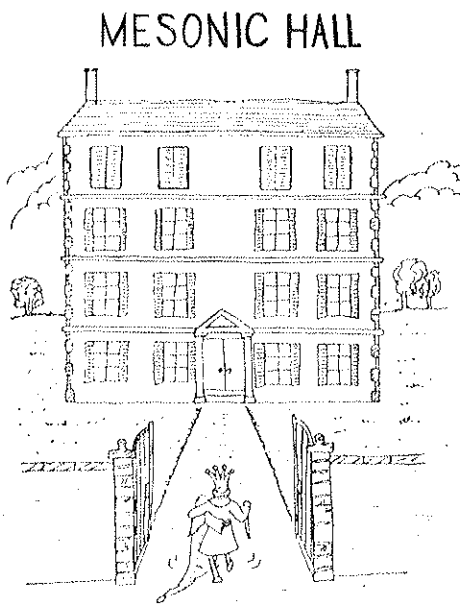


Fig. 1.5: (Taken from ref. [117]).

⁴ Quotation by M.R. Pennington.

1.4. PROFIT FROM EXPERIMENT PS182

It was pointed out in the above section that an experiment making only spectroscopic measurements, like this experiment PS182, will not be able to uniquely define the internal quark structure of the state that caused a presumed signal. However, several of the proposed experiments at LEAR are of this kind, and the more elaborate experiments able to do a complete partial-wave analysis are yet to be done. Experiments of the spectroscopic kind, that are much simpler to carry out and that require a less complicated detector, can nevertheless serve as a useful guidance in the search. Several of the experiments in the first era of LEAR have achieved their first results by now. The experiment PS182 that is presented and analysed in the present work resulted so far in three different basic measurements, viz. the three inclusive momentum spectra of, respectively, π^0 -mesons, η -mesons, and photons, emitted from $p\bar{p}$ annihilations produced experimentally by an antiproton beam stopping in liquid hydrogen. The π^0 and the photon spectra are used to derive upper limits for the population of exotic states, which are about an order of magnitude below those of earlier data. The spectra show, however, in addition several signals from ordinary mesonic resonances. Thus, the two neutral two-body annihilation channels $p\bar{p} \rightarrow \pi^0\omega$ and $p\bar{p} \rightarrow \eta\omega$, are directly measurable from the spectra. It is also observed that signals from the decay channels $p\bar{p} \rightarrow \pi^0\eta$, $p\bar{p} \rightarrow \eta\eta$, and $p\bar{p} \rightarrow \eta\eta'$ are absent. These numbers might help the understanding of the $p\bar{p}$ -annihilation mechanism as these decay channels cannot simply be explained by a pure quark rearrangement mechanism. The $p\bar{p}$ -annihilation process was extensively studied with bubble chamber techniques during the 1960's [11] [65]. With these techniques it is not easy to discern neutral particles, and subsequently the data concerning decay modes that comprises two or more neutrals are generally poor.

The principle of π^0 -spectroscopy is to measure coincidence events of two photons, because the π^0 immediately (compared to detector dimensions) after production decays into two photons. Hence, the detector is constructed from γ -calorimeters. In principle one is then able to perform spectroscopy of any particle that decays into two photons, but apart from the π^0 it is only the η -meson that does so at a sufficient rate for practical use. By removing the coincidence condition in the trigger the inclusive photon spectrum is obtained. The detector used by PS182 is extensively described in chapter 3. The evaluation of events, and the methods to interpret the spectra, are then discussed in detail in chapter 4.

An interesting data set consists of triple photon coincidences. This event sample allows us to measure the rare decays modes $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0\gamma$ with good precision. If the strength of the $2\pi^0$ channel is compared to the strength of the $\pi^+\pi^-$ channel measured in bubble chambers, information can be gained of the relative contributions from P -wave and S -wave, respectively, to $p\bar{p}$ annihilations at rest in liquid hydrogen, something that might have implications when correctly judging bubble chamber data. Earlier experimental results [25] [47] on the $2\pi^0$ channel are in disagreement. By measuring events of two high-energetic back-to-back photons we obtain an upper limit for the decay $p\bar{p} \rightarrow 2\gamma$. This rare annihilation channel and the channel $p\bar{p} \rightarrow \pi^0\gamma$ are imagined to proceed via vector dominance [45], which is a mechanism that well explains the cross-section for e^-p elastic scattering. However, this model has some difficulties when applied to e^+e^- annihilation. The $\pi^0\gamma$ and 2γ channels are yet another check of the Vector Dominance Model, which will be further discussed in the next chapter.

CHAPTER 2

THEORETICAL ASPECTS ON $p\bar{p}$ ANNIHILATIONS

A proton-antiproton annihilation is a complex process involving three quarks and three antiquarks, and analysis of the $p\bar{p}$ annihilation is further complicated by the fact that it is governed by strong interactions. The difficulty with QCD (Quantum Chromo Dynamics), the theory for strong interaction, is evident from an expression of the strong coupling constant α_{QCD} , which is the parameter occurring to increasing orders in a perturbation expansion. Let us compare the situation in QED (Quantum Electro Dynamics), which is a well known and well working theory, and governs e^+e^- annihilation for instance. The electromagnetic coupling constant α_{QED} can be expressed as

$$\alpha_{\text{QED}}(q^2) = \frac{\alpha_{\text{QED}}(\mu^2)}{1 - [\alpha_{\text{QED}}(\mu^2)/\pi] \ln(q^2/\mu^2)},$$

where μ is the "renormalized mass" ($= m_e$). Because $\alpha_{\text{QED}}(q^2)$ decreases with decreasing $|q^2|$, a perturbation expansion will converge faster for small values of $|q^2|$, and processes are dominated by first order diagrams. The corresponding expression for α_{QCD} , the strong coupling constant, is

$$\alpha_{\text{QCD}}(q^2) = \frac{\alpha_{\text{QCD}}(\mu^2)}{1 + B [\alpha_{\text{QCD}}(\mu^2)/4\pi] \ln(q^2/\mu^2)},$$

where μ again is a "renormalized mass", and $B = (11 - 2n/3)4\pi$, n being the number of quark flavours. At present we know only of six flavours. We see that if $n < 16$ the strong coupling constant $\alpha_{\text{QCD}}(q^2)$ **increases** with decreasing $|q^2|$, which means that a perturbation expansion will converge worse for smaller $|q^2|$, and better for larger $|q^2|$ — a fact known as "asymptotic freedom". This means that strong interaction processes at small momentum transfers $|q^2|$, as $p\bar{p}$ annihilation at rest for instance, are not dominated by the first order diagram, but higher order diagrams might be as important, which makes calculations complicated and maybe endless. Experimental results indicate that calculations according to the Feynmann rules are valid for $|q^2| > (2\text{GeV})^2$. Even if QCD gives a good interpretation of the dynamics in a strong interaction process, spectroscopical⁵ quantities are difficult to compute. Large efforts have been invested by many people to get around this deficiency of QCD, and several approaches have been tried. In this chapter three of these models are briefly presented (section 2.5). The situation for $p\bar{p}$ annihilation at rest becomes even harder by the fact that the reaction seldom takes place in vacuum but usually in liquid hydrogen which has an impact on the quantum numbers of the initial $p\bar{p}$ state. The first section of this chapter gives a survey of the atomic situation prior to the actual annihilation process. The two following sections 2.2 and 2.3 then takes up approaches to the understanding of $p\bar{p}$ annihilations into final states that only contain ordinary mesons, which is the case for the vast amount of all annihilations. Section 2.4 discusses some aspects on annihilations into two-body final states. If these two final objects are ordinary mesons, the process involves an annihilation of a quark-antiquark pair, and the process cannot simply be explained by a rearrangement of quarks and antiquarks into three mesons. Another possibility could be that one of the objects in the final state is a quasi-nuclear state or a state composed of two quarks and two antiquarks. Such latter states are frequently referred to as "baryonium".

⁵ Some spectroscopical properties of QCD can be derived from group-theoretical considerations. For a treatment of the subject see for instance the book by Close [38].

2.1. PROTONIUM

An antiproton enters the target of the PS182 experiment with a momentum around 200 or 300 MeV/c. It is slowed down by Coulomb interactions with the atomic electrons in the liquid hydrogen. This interaction mechanism is well described by the Bethe-Bloch formula (see further section 3.3.2). When brought to a stop the antiproton is captured by a proton and will replace the electron in the hydrogen atom. To remain bound, after having knocked out the electron, and form a proton-antiproton system, the antiproton must have an energy E_n roughly equal to the binding energy $E_B = 13.6$ eV of the electron. This hydrogen-like $p\bar{p}$ -atom is called "protonium". Its energy levels are thus given by the usual eigenvalues to the Schrödinger equation:

$$E_n = \frac{Ze^2}{2a_B n^2}, \quad (2.1)$$

where the Bohr radius is

$$a_B = \frac{a_0}{Z} = \frac{\hbar^2}{Z\mu e^2} \quad (2.2)$$

with the reduced mass

$$\mu = \frac{mm_p}{m+m_p}, \quad (m=m_e, m_p). \quad (2.3)$$

Thus the antiproton is captured in an energy level having a principle quantum number n given by

$$a_B(\bar{p}) \cdot n^2 = a_B(e^-) \Rightarrow \frac{n^2}{\mu(\bar{p})} = \frac{1}{\mu(e^-)} \Rightarrow n^2 = \frac{m_e + m_p}{2m_e}, \quad n \approx 30. \quad (2.4)$$

If the annihilation takes place in "vacuum", almost gaseous hydrogen, there follows a cascade process of de-excitations (fig. 2.1). The lifetime τ_n of a level n can be found by computing the transition probability w_n with classical harmonic time-dependent perturbation theory, that is using Fermi's Golden Rule⁶ ($\hbar\omega = \Delta E = E_n - E_{n-1}$)

$$w_n = \frac{1}{2J+1} \sum_{m'} \sum_m w[nlm \rightarrow n'l'm'] = \frac{1}{2J+1} \cdot \frac{4\alpha}{3} \cdot \frac{\omega^3}{c^2} \cdot \frac{l}{2l+1} \left| \int_0^\infty R_{n'l'}(r) R_{nl}(r) r^3 dr \right|^2, \quad (2.5)$$

where R_{nl} and $R_{n'l'}$ are the radial wave functions of a hydrogen-like atom. For instance, the lifetime of the 2p level is, using

$$R_{nl} = \frac{1}{\sqrt{24}a_B^3} \frac{r}{a_B} e^{-r/2a_B}, \quad R_{n'l'} = \frac{2}{\sqrt{a_B^3}} e^{-r/a_B}, \quad (2.6)$$

obtained to be

$$\tau_{2p} = \tau(2p \rightarrow 1s) = 1.6 \cdot 10^{-14} \text{ s} \quad (2.7)$$

⁶ See for instance the book by Sakurai [128].

In general the lifetime of an excited hydrogen-like atom goes up with increasing n , roughly as n^3 for a fixed value of l , and as

$$\tau_n \sim \left(\sum_l \frac{(2l+1)}{n^5} \right)^{-1} \sim n^{4.5} \quad (2.8)$$

for the average over l . Summing from $n=2$ to $n=30$ we get

$$\tau_{p\bar{p}} \cong \frac{\tau_{2p}}{2^{4.5}} \sum_{n=2}^{30} n^{4.5} \approx 1.87 \cdot 10^{-8} \text{s}. \quad (2.9)$$

In liquid hydrogen, however, the history of protonium is altered due to the higher density of the environment. Because of the lifetime of protonium, it is very likely to collide with a hydrogen molecule before annihilation starts. The average time $\bar{t}_c$ between successive collisions can be computed from the mean path length ℓ and the expectation value $\langle v \rangle$ of their velocity distribution. For the velocity distribution one can use a classical Maxwell-distribution

$$f(v) d^3v = N \left(\frac{M}{2\pi kT} \right)^{3/2} e^{-Mv^2/2kT} d^3v, \quad (2.10)$$

where N is the total number of molecules of mass M , and T is the temperature in degrees absolute (K), k being Boltzmann's constant. We get for the expectation value of the velocity:

$$\langle v \rangle = \frac{1}{N} \int v f(v) d^3v = \sqrt{\frac{8kT}{\pi M}}. \quad (2.11)$$

For liquid hydrogen is $M = 2m_p$, and that has a temperature of $T = 26$ K, we then get

$$\langle v \rangle_{\text{LH}_2} = c \sqrt{\frac{8k}{\pi} \cdot \frac{26}{2m_p}} \approx 523 \text{ m/s}. \quad (2.12)$$

The classical treatment of liquid hydrogen can be justified by comparing the "thermal wave length" λ_T being the de Broglie wave length of the hydrogen molecules taken for the expectation value of the momentum in a certain direction:

$$\lambda_T = \frac{\hbar}{(Mv_x)} = \frac{h}{\sqrt{2\pi MkT}} = \frac{2\pi\hbar c}{\sqrt{4\pi m_p k \cdot 26}} \approx 2.4 \text{ \AA}. \quad (2.13)$$

The thermal wave length should then be compared to the mean distance a between the molecules in liquid hydrogen:

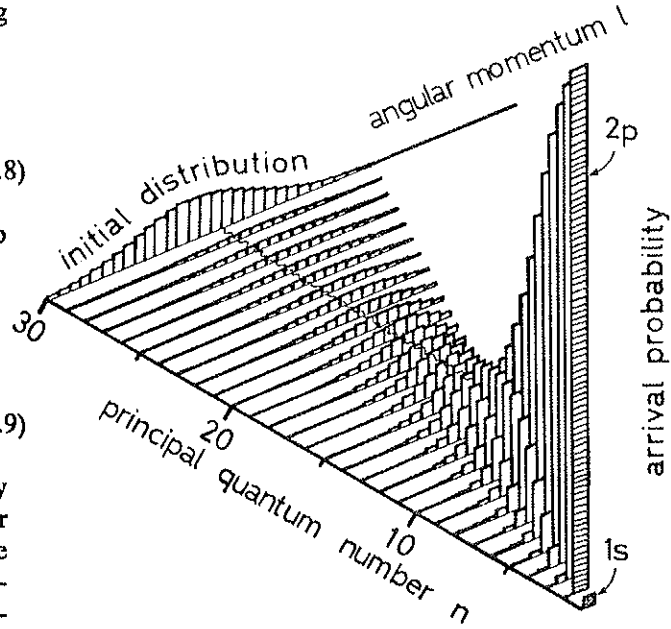


Fig. 2.1: Cascade de-excitation of protonium.

$$a \cong \sqrt[3]{\frac{A}{N_A \rho}} = 3.6 \cdot 10^{-8} \text{ cm} \approx 360 \text{ \AA} \gg \lambda_T. \quad (2.14)$$

The mean free path can be found from considering the total number of collisions z_c per unit time

$$z_c = \frac{1}{2} n^2 \sigma \left(\frac{M}{2\pi kT} \right)^3 \int_{-\infty}^{\infty} e^{-Mv_1^2/2kT} e^{-Mv_2^2/2kT} |\mathbf{v}_1 - \mathbf{v}_2| d^3v_1 d^3v_2 = 2n^2 \sigma \sqrt{\frac{kT}{\pi M}}, \quad (2.15)$$

where n is the molecular density

$$n = \frac{N_A \rho}{A} = 2.1 \cdot 10^{22} \text{ cm}^{-3}, \quad (2.16)$$

and σ is the total collision cross-section

$$\sigma \cong 9\pi a_d^2 = 1.5 \cdot 10^{-15} \text{ cm}^2, \quad (2.17)$$

$a_d = 0.74 \text{ \AA}$ being the equilibrium separation distance between the hydrogen atoms in a molecule. Finally we arrive at the average time $\bar{t}_c$ between two successive collisions

$$\bar{t}_c = \frac{\ell}{\langle v \rangle} = \frac{N}{2z_c} = \frac{1}{\sqrt{2N\sigma\langle v \rangle}} = \frac{1}{4N\sigma} \sqrt{\frac{\pi M}{kT}} \approx 4.2 \cdot 10^{-13} \text{ s}. \quad (2.18)$$

We can now compare this time $\bar{t}_c$ between collisions with hydrogen molecules with the lifetime of protonium $\tau_{p\bar{p}}$ [eq. (2.9)], and it is clear that the $p\bar{p}$ -atom will meet many hydrogen molecules before the proton and the antiproton has come close enough to annihilate.

A "collision" between a $p\bar{p}$ -atom and a hydrogen molecule is, however, not so drastic. Mostly the $p\bar{p}$ -atom will just pass through the hydrogen molecule (fig. 2.2). The radius of the $p\bar{p}$ -atom is

$$a_B(\bar{p}) \cong a_B(e^-) \frac{2m_e}{m_p} \approx 100 \text{ fm} \ll a_d(\text{H}_2) \sim 1 \text{ \AA}, \quad (2.19)$$

wherefore the $p\bar{p}$ -atom appears to be a small neutral object to the hydrogen molecule. The $p\bar{p}$ -atom would hardly notice the presence of a hydrogen molecule except for the strong electrostatic field E between the protons in a hydrogen molecule. This will induce Stark effect in the protonium atom, i.e. splitting of the energy levels due to the external electric field E . The phenomenon can be treated in the Hamiltonian of the protonium atom as a small perturbation

$$H' = eZ \quad (2.20)$$

if the $p\bar{p}$ -atom is considered as a small dipole oriented along the z -axis. We then solve the Schrödinger equation

$$(H + H')\phi_i = (E + E_i)\phi_i, \quad (2.21)$$

where the wave functions are assumed to be linear combinations of the ordinary hydrogen wave functions ψ_k ($k \triangleq nlm$):

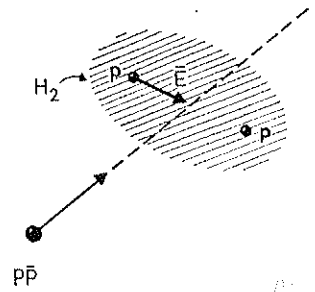


Fig. 2.2: When the small $p\bar{p}$ -atom passes through a hydrogen molecule it is exposed to the strong electric field E inside the molecule.

Mathematicians are rather like Frenchmen. If you speak to them they translate it into their own language, and then it is immediately turned into something entirely different.

(Goethe, 1749–1832)

$$\phi_l = \sum_k a_{lk} \psi_k. \quad (2.22)$$

The trivial but somewhat tedious calculation results in the four solutions

$$\begin{aligned} E_1' &= 3a_0 eE, \quad E_2' = -3a_0 eE, \\ E_3' &= E_4' = 0, \end{aligned} \quad (2.23)$$

and the corresponding eigenfunctions

$$\begin{aligned}\phi_1 &= [\psi_{200} + \psi_{210}]/\sqrt{2}, \\ \phi_2 &= [\psi_{200} - \psi_{210}]/\sqrt{2}, \\ \phi_3 &= [\psi_{211} + \psi_{21-1}]/\sqrt{2}, \\ \phi_4 &= [\psi_{211} - \psi_{21-1}]/\sqrt{2}.\end{aligned}\quad (2.24)$$

These so called *hybrid* wave functions mix orbitals of the $p\bar{p}$ -atom that have the same principle quantum number n , but have different angular momenta l . In the ordinary level scheme (fig. 2.3) this corresponds to horizontal "sliding". Notice that this is **not** some kind of radiative transition. This "Stark mixing" subsequently leads to an admixture of an high S -orbital, i.e. $l=0$, which has a relative large overlap with the proton at the origin, and does not have any centrifugal barrier that prevents the proton and the antiproton from annihilating.

The conclusion of the above considerations is that most frequently the $p\bar{p}$ annihilation in liquid hydrogen takes place from a high S -level. A cascade computation by Borie [24] indicates a fraction as large as 70% to 90%.

2.2. ANNIHILATION AT REST

The experimental data presented in table 2.1 was measured with bubble chambers, i.e. annihilations in liquid hydrogen [11] [65] [68]. Most commonly the final state consists of only pions. The relative occurrence of different multiplicities can be explained by a simple statistical model developed by Orfanidis and Rittenberg [113]. This model requires two phenomenologically determined parameters as input, viz. the average multiplicity of charged pions and the total average multiplicity. It works well in the energy region up to 4 GeV.

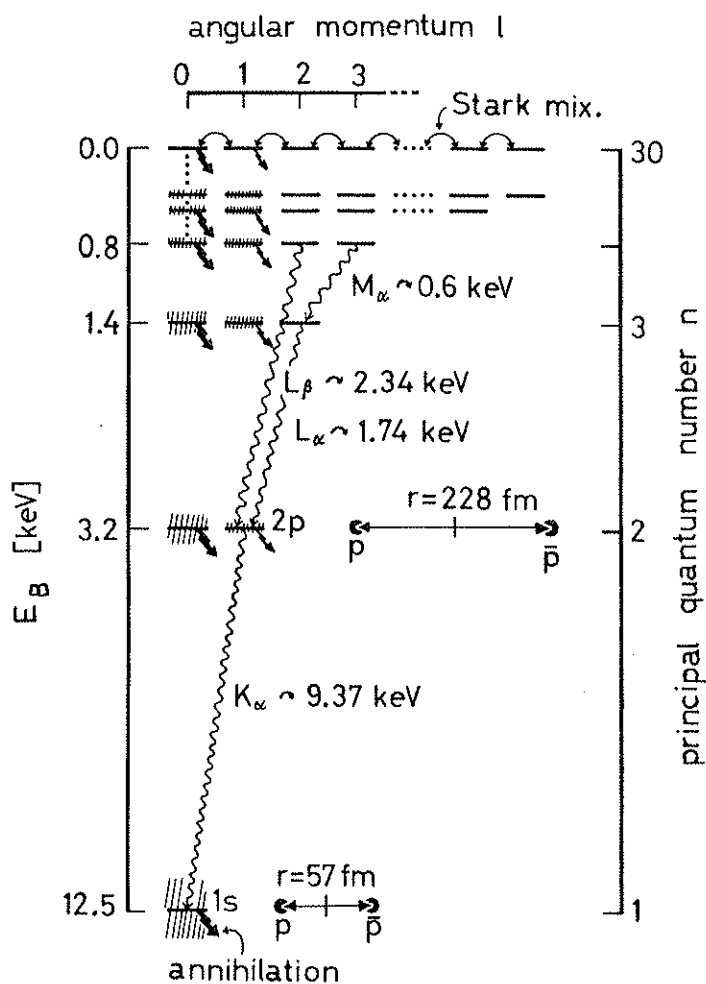


Fig. 2.3: Energy levels of protonium.

In consideration of the quark concept [66] a natural annihilation mechanism appear to be a rearrangement of the three quarks of the proton and the three antiquarks of the antiproton into three mesons, i.e. three $[q\bar{q}]$ -states. An approach to explain the $p\bar{p}$ annihilation inspired by this picture is the so called "Rearrangement Model", which is viewed in subsection 2.2.1. Several other models have been developed to imagine the $p\bar{p}$ annihilation process [69]. Many authors have explored the possibilities of using the phenomenological nuclear potential to explain features of the $p\bar{p}$ annihilation. A potential model, however, predicts only two-body disintegration modes, though many-body final states can be constructed from two-body states of weakly bound resonances.

2.2.1 Quark Rearrangement

In the view of the quark picture of hadrons the rearrangement into three mesons appear to be the straight forward annihilation process of the $p\bar{p}$ system (fig. 2.4). A model based on this idea was presented 1966 by H. Rubinstein [127]. This model explains annihilations into three or more mesons. More than three occur owing to the primary being resonances, i.e. excited mesonic states that decay further. No strange mesons can appear with a pure rearrangement mechanism. Therefore ϕ production should also be suppressed, since⁷ it is 99% $s\bar{s}$.

Table 2.1: Contributions to $p\bar{p}$ annihilations at rest in liquid hydrogen. This table is taken from ref. [12] and updates the one published by R. Armenteros and B. French [11] with data published later by the CERN-Collège de France Collaboration [49]. The table has been complemented with data from sources: (1) from ref. [68], and (2) from theoretical estimations from ref. [113]. Channels marked "MC" are used for a Monte Carlo simulation.

Final state	Intermediate resonant state	Percentage of all annihilations CERN	Percentage of all annihilations Columbia	Source	Comment
All neutrals		4.1±0.4	3.2±0.5		
	$3\pi^0$		0.8	(2)	
	$4\pi^0$		1.2	(2)	
	$5\pi^0$		0.7	(2)	
$\pi^+\pi^-$		0.37±0.03	0.32±0.03		
$\pi^+\pi^-\pi^0$	all	6.9±0.35	7.8±0.9		MC
	$\rho\pi$	5.8±0.3	4.1±0.4		
	$f^0\pi^0$	0.24±0.07			
$\pi^+\pi^-(\text{MM})$		35.8±0.8	34.5±1.2		
	$\eta\pi^+\pi^-$	0.8±0.1			
	$\eta\rho^0$	0.22±0.17			
$\pi^+\pi^-2\pi^0$			9.3±3.0	(1)	MC
$\pi^+\pi^-3\pi^0$			23.3±3.0	(1)	MC
$\pi^+\pi^-4\pi^0$			2.8±0.7	(1)	MC
$2\pi^+2\pi^-$		6.9±0.6	5.8±0.3		
	$A^{\pm}_2\pi^{\mp}$	2.0±0.3			
	$\omega\rho^0\pi^{\pm}$				
	ρ^0f^0	0.90±0.2			
	$\rho^0\pi^+\pi^-$	1.50±0.3			
	$\rho^0\rho^0$	0.12±0.12	0.4±0.3		
$2\pi^+2\pi^-\pi^0$	all	19.6±0.7	18.7±0.9		MC
	$\omega^0\pi^+\pi^-$	3.0±0.3	3.3±0.4		
	$\omega^0\rho^0$	2.1±0.2	0.7±0.3		
	ω^0f^0	1.7±0.2			
	$\rho^0\pi^+\pi^-\pi^0$	13.7±0.6	7.3±1.7		
	$\rho^{\pm}\pi^{\mp}\pi^+\pi^-$		6.4±1.8		
	$B^{\pm}\pi^{\mp}$	0.7±0.1			
	$\omega\pi^{\pm}$				
	$\eta\pi^+\pi^-$	0.35±0.04	0.34±0.1		
	$A^{\pm}_2\pi^{\mp}$	0.13±0.03			
	$\omega\pi^{\pm}$				
$2\pi^+2\pi^-(\text{MM})$		20.8±0.7	21.3±1.1		
	$\eta'\pi^+\pi^-$	0.11±0.02			
$2\pi^+2\pi^-2\pi^0$			16.6±1.0	(1)	MC
$2\pi^+2\pi^-3\pi^0$			4.2±1.0	(1)	MC
$3\pi^+3\pi^-$		2.1±0.2	1.9±0.2		
$3\pi^+3\pi^-\pi^0$	all	1.9±0.2	1.6±0.3		MC
	$\omega 2\pi^+2\pi^-$	1.3±0.3			
	$\eta 2\pi^+2\pi^-$	0.17±0.07			
	$\eta'\pi^+\pi^-$	0.04±0.01			
$3\pi^+3\pi^-(\text{MM})$			0.3±0.1		
$K\bar{K}m(m\geq 0)$		6.86±0.25			

⁷ ϕ is a mixture of the singlet representation $|1\rangle$ and the octet representation $|8\rangle$ of $SU_f(3)$:

$$\phi = \cos\theta|1\rangle + \sin\theta|8\rangle \quad |1\rangle = 1/\sqrt{3}[u\bar{u} + d\bar{d} + s\bar{s}] \quad (2.25)$$

$$\omega = -\sin\theta|1\rangle + \cos\theta|8\rangle \quad |8\rangle = 1/\sqrt{6}[u\bar{u} + d\bar{d} - 2s\bar{s}]$$

which taken together gives

$$\phi = \left[\frac{\cos\theta}{\sqrt{3}} + \frac{\sin\theta}{\sqrt{6}} \right] (u\bar{u} + d\bar{d}) + \left[\frac{\cos\theta}{\sqrt{3}} - \frac{2\sin\theta}{\sqrt{6}} \right] s\bar{s} \quad (2.26)$$

Experimental results gives

$$\tan^2\theta = \frac{m_\omega}{m_\phi} \frac{R(\phi \rightarrow e^+e^-)}{R(\omega \rightarrow e^+e^-)} = 1.66 \Rightarrow \theta = \pm 48.5^\circ. \quad (2.27)$$

The observed ϕ -mass m_ϕ is consistent with the negative value, implying

$$\phi = 0.01(u\bar{u} + d\bar{d}) + 0.99s\bar{s}. \quad (2.28)$$

The Rearrangement Model fits 80% to 90% of the experimental data. The model was later improved by M. Maryama and T. Ueda [95]. The basic assumptions of the model are:

- hadrons are synthesized of quarks in a relative S -state and belong to $SU(6)$;
- hadrons move in relative S -waves;
- there is no final state interaction that exchanges spin, isospin, or strangeness.

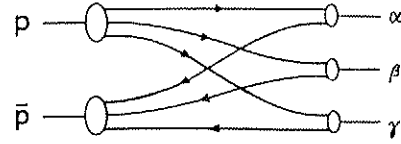


Fig. 2.4: Quark rearrangement from a $p\bar{p}$ -state into three mesons.

The probability for a $p\bar{p}$ state of spin S and isospin I to decay into three mesons α , β , and γ is written as

$$P(I, S) = b(I, S) \cdot C(I, S, \alpha, \beta, \gamma) \cdot g_\alpha \cdot g_\beta \cdot g_\gamma \cdot V(\alpha, \beta, \gamma, E), \quad (2.29)$$

where the functions have the following meanings:

- $b(I, S)$: spin and isospin dependence due to the initial $p\bar{p}$ interaction;
 $C(I, S, \alpha, \beta, \gamma)$: the overlap square of the spin-isospin wavefunctions of initial and final states;
 $V(\alpha, \beta, \gamma, E)$: the phase space volume of the three final mesons;
 $g_\alpha, g_\beta, g_\gamma$: the strength for a $q\bar{q}$ -pair to combine into mesons α , β , and γ , respectively.

In table 2.2 numbers predicted by this rearrangement model are listed. In this model annihilations into two mesons are forbidden. An attempt to allow for such processes inside the model was made by Dover [54] and is further mentioned in section 2.4

2.3. ANNIHILATION IN FLIGHT

Before the antiproton is brought to a stop in the liquid hydrogen target of PS182 it penetrates between 5 and 15 cm of the medium. During its flight through the liquid hydrogen there is a finite probability that the antiproton collides with a proton and annihilates. The total annihilation cross-section can be parameterized [52] as

$$\sigma_{an} [\text{mb}] = 38 + 35/p_{\bar{p}} [\text{GeV}], \quad (2.30)$$

in the momentum region between 200 MeV/c and 2 GeV/c. In this momentum region the annihilation cross-section is about 55% to 60% of the total cross-section, the rest being mainly due to elastic scattering (35% to 40%) and charge exchange $p\bar{p} \rightarrow n\bar{n}$. It is of interest to know the ratio of the number of annihilations taking place in flight and the number of annihilations occurring at rest, because the different partial waves are expected to contribute differently for the two cases. The relative contributions from the different partial waves can be qualitatively estimated [23] [61] and are shown in fig. 2.5 as a function of the $\bar{p}$ -momentum. In this so called "Boundary Condition Model" it is assumed that an impact

Table 2.2: Comparison between the predicted branching ratios in percent of various $p\bar{p}$ annihilation channels and experimental values. (Taken from ref. [95]).

Channel	Experiment	
$\pi^0 \pi^0 \pi^0$	1.03 ± 0.34	0.93
$\eta^0 \pi^0 \pi^0$	0.35 ± 0.11	0.2
$m\pi^0$ ($m \geq 4$)	2.2 ± 0.5	2.31
$\pi^+ \pi^- \pi^0$	1.0 (2.0, 3.6)	0.70
$\pi^+ \pi^- 2\pi^0$	7.3 ± 3.0	3.84
$\pi^+ \pi^- 3\pi^0$	23.2 ± 3.0	20.5
$\pi^+ \pi^- 4\pi^0$	2.8 ± 0.7	1.89
$\pi^+ \pi^- m\pi^0$ ($m \geq 5$)	—	4.20
$2\pi^+ 2\pi^-$	5.4 ± 0.3	6.35
$2\pi^+ 2\pi^- \pi^0$	18.0 ± 0.9	18.8
$\omega^0 \pi^+ \pi^-$	3.4 ± 0.4	3.40
$\rho^0 \pi^+ \pi^- \pi^0$	7.3 ± 1.7	7.80
$\rho^+ \pi^- \pi^+ \pi^-$	6.4 ± 1.8	6.22
$\eta^0 \pi^+ \pi^-$	0.28 ± 0.07	0.20
others	—	1.16
$2\pi^+ 2\pi^- 2\pi^0$	15.5 ± 1.0	15.1
$2\pi^+ 2\pi^- 3\pi^0$	4.2 ± 1.0	4.92
$2\pi^+ 2\pi^- m\pi^0$ ($m \geq 4$)	—	1.11
$3\pi^+ 3\pi^-$	1.9 ± 0.2	1.12
$3\pi^+ 3\pi^- \pi^0$	1.6 ± 0.3	0.87
$\omega^0 2\pi^+ 2\pi^-$	1.2 ± 0.3	0.27
$\eta^0 2\pi^+ 2\pi^-$	0.14 ± 0.05	0.50
$3\pi^+ 3\pi^- m\pi^0$ ($m \geq 2$)	0.3 ± 0.1	0.28

parameter less than 1 fm always results in a bounded $p\bar{p}$ state that subsequently annihilates. Below momentum 200 MeV/c P -wave annihilations contributes less than 10^{-4} , according to these predictions. There are, however, other predictions [106] which show that P -wave annihilations could be (at least) as important as S -wave annihilations down to even lower momenta.

2.4. TWO-BODY FINAL STATES

From table 2.1 we can see that there are roughly 10% of all annihilations that result in a two-body final state. The annihilation into two mesons cannot be described by a pure rearrangement mechanism. To cure this problem Dover introduced [54] the possibility of terminating a pair of $q\bar{q}$ -lines into the vacuum (fig. 2.6). The vacuum is then assigned the quantum numbers $^{11}L_S = ^{13}P_0$. The coupling to the vacuum is, however, of a strength that has to be determined phenomenologically. An interesting result is the predicted ratio between the branching ratios for $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow 2\eta$

$$\frac{R(p\bar{p} \rightarrow 2\eta)}{R(p\bar{p} \rightarrow 2\pi^0)} \approx \frac{1}{8}. \quad (2.31)$$

The same ratio was predicted by Genz [67] to be a factor of 2 larger. Genz also gave the relative production of $\eta\eta'$ as

$$\frac{R(p\bar{p} \rightarrow \eta\eta')}{R(p\bar{p} \rightarrow 2\pi^0)} < \frac{1}{6}. \quad (2.32)$$

A special case of two-body annihilations are the channels $p\bar{p} \rightarrow \pi^0\gamma$ and $p\bar{p} \rightarrow 2\gamma$. A mechanism that has been conventionally adopted to imagine these reactions is the one of "vector meson dominance". In this model one assumes the photon to originate from the conversion of an off-shell ρ -meson or ω -meson (fig. 2.9). Thus, the decay channel $p\bar{p} \rightarrow \pi^0\gamma$ can be related to $p\bar{p} \rightarrow \pi^0\rho^0$ and $p\bar{p} \rightarrow \pi^0\omega$. Analogously $p\bar{p} \rightarrow 2\gamma$ can be related to $p\bar{p} \rightarrow 2\rho^0$, $p\bar{p} \rightarrow \rho^0\omega$, and $p\bar{p} \rightarrow 2\omega$. The branching ratio $R_{\pi^0\gamma}$ for $p\bar{p} \rightarrow \pi^0\gamma$ from an initial $^3S_1$ -state can then be expressed as [45]

$$R_{\pi^0\gamma} = \left(\frac{eg_{\gamma\rho}}{m_\rho^2} \right) \left(\frac{k_{\pi^0\gamma}}{k_{\pi^0\rho}} \right)^3 \left[R_{\pi^0\rho} + \left(\frac{g_{\gamma\omega} m_\rho^2}{g_{\gamma\rho} m_\omega^2} \right)^2 R_{\pi^0\omega} + 2 \frac{g_{\gamma\omega} m_\rho^2}{g_{\gamma\rho} m_\omega^2} \cos\varphi \sqrt{R_{\pi^0\rho} R_{\pi^0\omega}} \right]. \quad (2.33)$$

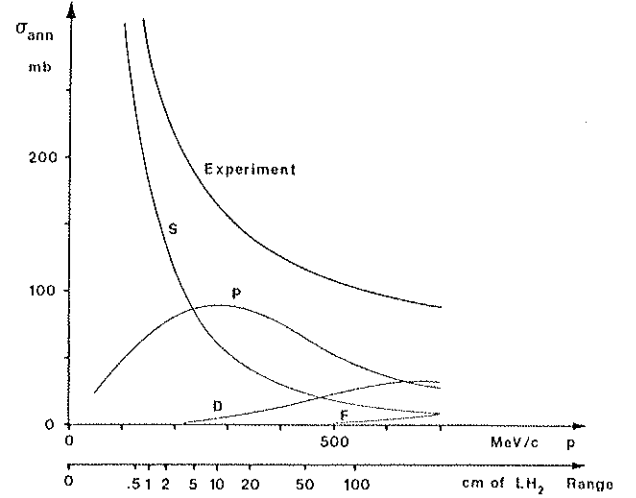


Fig. 2.5: Relative contributions from different partial waves to the $p\bar{p}$ annihilation cross-section as a function of the $\bar{p}$ -momentum p . The lower bottom axis is the range of a $\bar{p}$ -beam of momentum p in liquid hydrogen. (Adapted from ref. [23]).

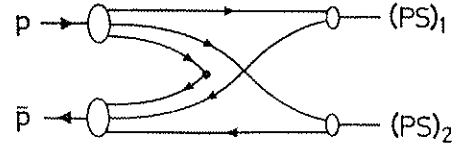


Fig. 2.6: Quark rearrangement of $p\bar{p}$ into two pseudo-scalar mesons requires an annihilation vertex.

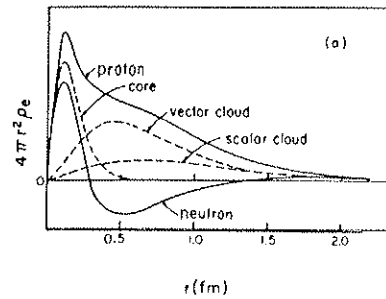


Fig. 2.7: Contributions to the spatial charge distribution for proton and neutron. (Taken from ref. [78]).

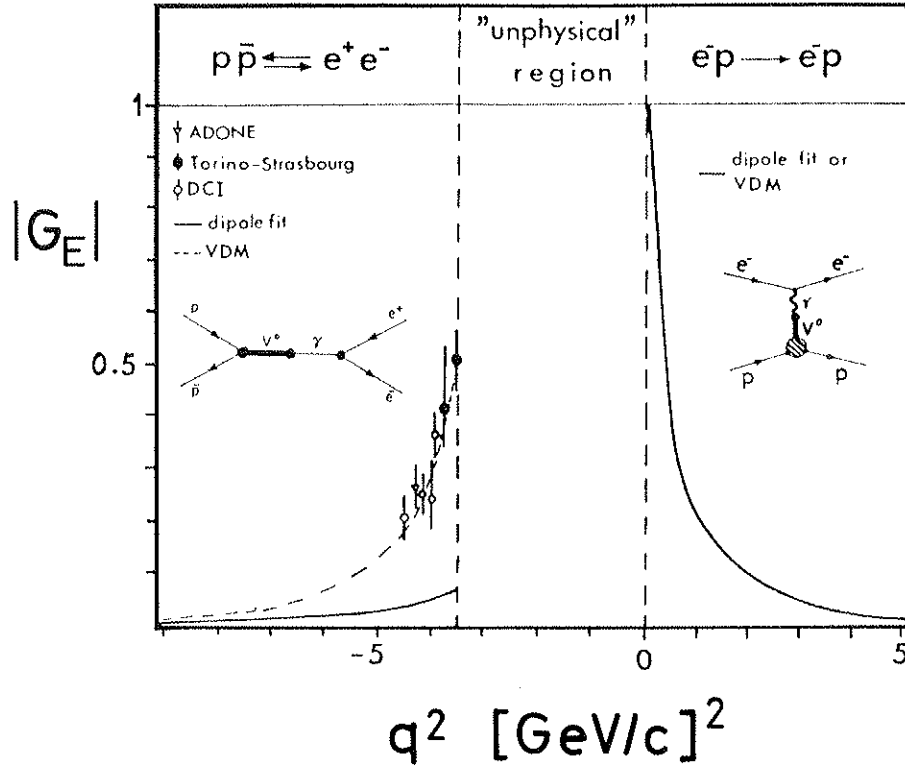


Fig. 2.8: The observed proton form factor G_E is explained by vector dominance.

This formula is derived under three assumptions:

- the intermediate state is dominated by a single virtual meson of isospin $I = 0, 1$;
- ϕ coupling is omitted;
- $SU(3)$ symmetry $\Rightarrow g_{\gamma\omega} = g_{\gamma\rho}/3$.

The coupling constant $g_{\gamma\rho}$ is measured in e^-p -scattering:

$$\frac{eg_{\gamma\rho}}{m_\rho^2} = 0.055. \quad (2.34)$$

The phase space factor is

$$\left(\frac{k_{\pi^0\gamma}}{k_{\pi^0\rho}} \right)^3 = 1.755. \quad (2.35)$$

Putting these numbers into eq. (2.36) then gives

$$R_{\pi^0\gamma} = 5.3 \cdot 10^{-3} \cdot [R_{\pi^0\rho} + 0.11 R_{\pi^0\omega} + 0.67 \cos\varphi \sqrt{R_{\pi^0\rho} R_{\pi^0\omega}}] . \quad (2.36)$$

The decay channel $p\bar{p} \rightarrow \pi^0\rho^0$ is accurately measured in bubble chambers [65] and has a strength of $R_{\pi^0\rho^0} = (1.9 \pm 0.2)\%$. The yield of $p\bar{p} \rightarrow \pi^0\omega$ is measured in this experiment to be $R_{\pi^0\omega} = (0.5 \pm 0.3)\%$. Introducing these numbers into eq. (2.36) we obtain

$$R_{\pi^0\gamma} = (1.04 + 0.35 \cos\varphi) \cdot 10^{-4} \approx 1.4 \cdot 10^{-4}, \quad (2.37)$$

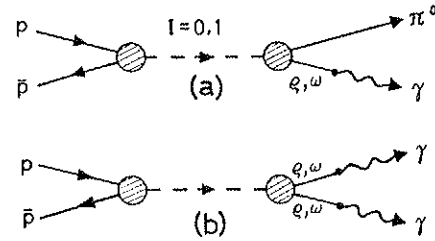


Fig. 2.9: $p\bar{p}$ annihilation (a) into $\pi^0\gamma$ and (b) into 2γ in the Vector Dominance Model.

where the strong phase φ determines the interference between the isospin 0 and 1 intermediate states. If $SU(3)$ symmetry was exact we would have $\varphi \equiv 0$.

In a similar manner the branching ratio $R_{\gamma\gamma}$ for $p\bar{p} \rightarrow 2\gamma$ can be estimated [115] from

$$R_{2\gamma} = \left(\frac{eg_{\gamma\rho}}{m_\rho^2} \right)^4 \left(\frac{k_{\gamma\gamma}}{k_{\rho\rho}} \right)^3 \left[1.2R_{\rho\rho} + 0.22R_{\rho\omega} + 0.52\cos\varphi' \sqrt{R_{\rho\rho}R_{\rho\omega}} \right], \quad (2.38)$$

where the phase space factor in this case is

$$\left(\frac{k_{\gamma\gamma}}{k_{\rho\rho}} \right)^3 = 5.316, \quad (2.39)$$

and assuming $R_{\rho\rho} \approx R_{\rho\omega}$, justified by $SU(3)$ symmetry. We then rely on measurements of $p\bar{p} \rightarrow 2\rho^0$ and $p\bar{p} \rightarrow \rho^0\omega$. Numbers given in ref. [14] are $R_{\rho\rho} = (4 \pm 3) \cdot 10^{-3}$ and $R_{\rho\omega} = (7 \pm 3) \cdot 10^{-3}$. With these numbers put into eq. (2.38) we obtain

$$R_{2\gamma} = (3.1 + 1.3\cos\varphi') \cdot 10^{-6} \approx 4 \cdot 10^{-6}, \quad (2.40)$$

where we again have assumed the strong phase $\varphi' \approx 1$.

Another possibility of two-body final states, especially appealing to this experiment, is when one of the objects in the final state is an exotic particle X or a quasi-nuclear state $(N\bar{N})^*$. The interesting reactions for this experiment is when this object is neutral, so that the other particle emitted is a photon or a π^0 -meson. The branching ratios

$$R_\gamma^* = \Gamma_\gamma / \Gamma_{tot}, \quad R_{\pi^0}^* = \Gamma_{\pi^0} / \Gamma_{tot}, \quad \Gamma_{tot} = \Gamma_{an(N\bar{N})^*} (\Gamma_{\pi^0} + \Gamma_\gamma), \quad (2.41)$$

for the two reactions $p\bar{p} \rightarrow \gamma + (N\bar{N})^*$ and $p\bar{p} \rightarrow \pi^0 + (N\bar{N})^*$ have been estimated by Dover [53] using an optical potential model. Assuming the range of the potential to be $r_0 = 0.82$ fm they obtain the number $R_{\pi^0}^* \sim 10^{-2}$ and $R_\gamma^* \sim 0.1 R_{\pi^0}^*$. Others [97], however, have obtained $R_{\pi^0}^* \approx 10^{-3}$. The production rate of exotic states X of the type $|qq\bar{q}\bar{q}\rangle$, via the decay $p\bar{p} \rightarrow \gamma X$ is computed by Ader [3] in the so called "Bag Model" (subsection 2.5.2) to be $R_{\gamma X} \sim 10^{-4}$ with photon energies between 200 and 400 MeV. These estimations together with the argument that the π^0 -yield should be an order of magnitude larger than the photon yield due to the absence of the electromagnetic vertex, was a basic motivation for this experiment PS182.

Despite model-dependent quantities such as branching ratios, masses of exotic states and their widths, there are certain restrictions [43] from conservation of fundamental quantum numbers put on the states possible to reach from $p\bar{p}$ annihilation. These quantum numbers are the total spin J , parity P , charge-parity C , and G -parity. If subscript X refers to quantum numbers of the exotic state, and subscript i labels initial state, and l_f is the relative angular momentum between the two objects in the final state, the quantum numbers of the object X are given by

$$\begin{aligned} J_X + J + l_f &= l_i, \\ P_X P (-1)^{l_f} &= (-1)^{l_i+1}, \\ C_X C &= (-1)^{l_i+S_i}, \\ G_X G &= (-1)^{l_i+S_i+I_i}, \end{aligned} \quad (2.42)$$

where $(J, P, C, G) = (0, -, +, -)$ for a π^0 -associated decay mode, and $(J, P, C, G) = (0, -, +, +)$

for an η -associated decay mode. The quantum numbers for the case of photon emission⁸ are, according to the Vector Dominance Model, the same as the quantum numbers of the ρ -meson or the ω -meson: $(J, P, C, G) = (1, -, -, \pm)$. In the second relation of (2.42) the fact that $P_{p\bar{p}} = -1$ is used, and the third relation holds if the two particles in the final state are neutral and therefore eigenstates of the charge-parity operator. Tables 2.3 and 2.4 give the quantum numbers of the final states reached from different initial $p\bar{p}$ states for the two cases that the decay is associated with the emission of a π^0 -meson or an η -meson, respectively.

2.5. EXOTIC FINAL STATES

Several models have been developed to overcome the problems of QCD at low momentum transfers, and to avoid calculations based on perturbation expansions. All these models have, however, to use phenomenology of some kind as input [79]. Generally one can say about these models that they all depend on parameters that have to be adjusted to experimental data, and that they predict a large number of bound states. Rarely, however, do masses, line widths, or quantum numbers of different models agree. In general they do, on the other hand, agree "reasonably" well with the measured meson spectrum. A review of the field is given in the article by Rossi and Veneziano [142]. In subsection 2.5.1 the Potential Model is presented, which is historically the first one. It is based on the $N\bar{N}$ -potential derived from G -parity transformation of the phenomenological "One-Boson-Exchange Potential" (OBEP) valid for nucleon-nucleon interactions. This model does not consider the particles to be composite objects and is therefore questionable when concerning phenomena that occur on a distance scale that is of the size of the nucleons themselves. A model that takes the quark content of the nucleons into account is the so called "Bag Model". In this model one has imposed confinement of the quark fields by introducing a linear term in the QCD Lagrangian. This ad hoc measure can be interpreted as a kind of "vacuum pressure". The existence of confinement is indicated by QCD. It can be understood from the appearance of gluon self-interactions, i.e. the coupling of gluons to each other, which is, from a mathematical point of view, a consequence of QCD being based on a non-Abelian gauge-group⁹. Since α_{QCD} grows when $|q^2|$ approaches zero, higher order diagrams will become more and more important, and there might be an "avalanche" of quark-gluon loops. Imagine there is a repulsive force

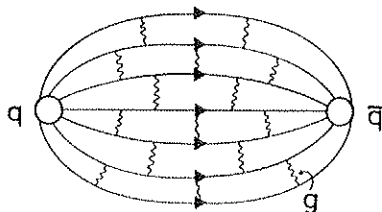


Fig. 2.10: The gluon-gluon self-interaction imagined as a repulsion between colour flux lines.

between the colour flux lines which terminate on colour charges, i.e. quarks. Stretching the quarks apart forces the colour flux lines to come closer, which then results in an increase of energy (fig. 2.10). In the Bag Model the quark wave functions are forced to zero outside a spherical cavity but are otherwise unconstrained. A more elaborated model [80] — not presented here — called the "Flux Tube Model", introduces additional degrees of freedom (gluons) in the form of elastic band vibrations between the quarks. Decays are made possible by having a finite probability of breaking a band.

In 1974 a non-perturbative method of renormalizing QCD was proposed by Wilson [144]. The new method is based on a discretizing of space-time into a four-dimensional cubic lattice. This "Lattice QCD"¹⁰ has had several successes, as, e.g., the "numerical evidence" of confinement.

⁸ The selection rules for a pure electromagnetic decay $p\bar{p} \rightarrow 2\gamma$ are the same as for a positronium decay; see for instance the book by Sakurai [128].

⁹ See for instance the book by Achison and Hey [2].

¹⁰ There is a good introductory article on the topic by Schierholz [129].

Table 2.3: Quantum numbers of the final states X reachable from different initial $p\bar{p}$ -states through the reaction $p\bar{p} \rightarrow \pi^0 X$. l_f is the relative angular momentum between the π^0 and the state X. Ordinary mesonic states are indicated.

Initial $p\bar{p}$ -state		Final state $I_X^{G_X} J_X^{P_X C_X}$		
$I_i^{G_i} J_i^{P_i C_i}$	$2S_i+1 L_i J_i$	$l_f=0$	$l_f=1$	$l_f=2$
$0^+ 0^{-+}$	$^1S_0$	$1^- 0^{++} \delta^0$	$1^- 1^{-+}$	$1^- 2^{++} A_2^0$
$1^- 0^{-+}$	$^1S_0$	$0^+ 0^{++} S^{*,\epsilon}$	$0^+ 1^{-+}$	$0^+ 2^{++} f, f', \Theta$
$0^- 1^{--}$	$^3S_1$	$1^+ 1^{+-} B^0$	$1^+ 0^{--}$ $1^+ 1^{--} \rho^0, \rho^{0'}$ $1^+ 2^{--}$ $1^{--} \gamma$	$1^+ 1^{+-} B^0$ $1^+ 2^{+-}$
$1^+ 1^{--}$	$^3S_1$	$0^- 1^{+-} H$	$0^- 0^{--}$ $0^- 1^{--} \omega, \phi, \phi'$ $0^- 2^{--}$ $1^{--} \gamma$	$0^- 1^{+-} H$ $0^- 2^{+-}$
$0^- 1^{+-}$	$^1P_1$	$1^+ 1^{--} \rho^0, \rho^{0'}$ $1^{--} \gamma$	$1^+ 0^{+-}$ $1^+ 1^{+-} B^0$ $1^+ 2^{+-}$	$1^+ 1^{--} \rho^0, \rho^{0'}$ $1^+ 2^{--}$ $1^{--} \gamma$
$1^+ 1^{+-}$	$^1P_1$	$0^- 1^{--} \omega, \phi, \phi'$ $1^{--} \gamma$	$0^- 0^{+-}$ $0^- 1^{+-} H$ $0^- 2^{+-}$	$0^- 1^{--} \omega, \phi, \phi'$ $0^- 2^{--}$ $1^{--} \gamma$
$0^+ 0^{++}$	$^3P_0$	$1^- 0^{-+} \pi^0$	$1^- 1^{++} A_1^0$	$1^- 2^{-+} A_3^0$
$0^+ 1^{++}$	$^3P_1$	$1^- 1^{-+}$	$1^- 0^{++} \delta^0$ $1^- 1^{++} A_1^0$ $1^- 2^{++} A_2^0$	$1^- 1^{-+}$ $1^- 2^{-+} A_3^0$
$0^+ 2^{++}$	$^3P_2$	$1^- 2^{-+} A_3^0$	$1^- 1^{++} A_1^0$ $1^- 2^{++} A_2^0$	$1^- 0^{-+} \pi^0$ $1^- 1^{-+}$ $1^- 2^{-+} A_3^0$
$1^- 0^{++}$	$^3P_0$	$0^+ 0^{-+} \eta, \eta', \iota$	$0^+ 1^{++} D, E$	$0^+ 2^{-+}$
$1^- 1^{++}$	$^3P_1$	$0^+ 1^{-+}$	$0^+ 0^{++} S^{*,\epsilon}$ $0^+ 1^{++} D, H$ $0^+ 2^{++} f, f', \Theta$	$0^+ 1^{-+}$ $0^+ 2^{-+}$
$1^- 2^{++}$	$^3P_2$	$0^+ 2^{-+}$	$0^+ 1^{++} D, E$ $0^+ 2^{++} f, f', \Theta$	$0^+ 0^{-+} \eta, \eta', \iota$ $0^+ 1^{-+}$ $0^+ 2^{-+}$

Table 2.4: Quantum numbers of the final states X reachable from different initial $p\bar{p}$ -states through the reaction $p\bar{p} \rightarrow \eta X$. l_f is the relative angular momentum between the π^0 and the state X. Ordinary mesonic states are indicated.

Initial $p\bar{p}$ -state		Final state $I_X^{G_X} J_X^{P_X C_X}$		
$I_i^{G_i} J_i^{P_i C_i}$	$2S_i+1 L_i J_i$	$l_f=0$	$l_f=1$	$l_f=2$
$0^+ 0^{-+}$	$^1S_0$	$0^+ 0^{++} \quad S^{*,\epsilon}$	$0^+ 1^{-+}$	$0^+ 2^{++} \quad f, f', \Theta$
$1^- 0^{-+}$	$^1S_0$	$1^- 0^{++} \quad \delta^0$	$1^- 1^{-+}$	$1^- 2^{++} \quad A_2^0$
$0^- 1^{--}$	$^3S_1$	$0^- 1^{+-} \quad H$	$0^- 0^{--}$ $0^- 1^{--} \quad \omega, \phi, \phi'$ $0^- 2^{--}$ $1^{--} \quad \gamma$	$0^- 1^{+-} \quad H$ $0^- 2^{+-}$
$1^+ 1^{--}$	$^3S_1$	$1^+ 1^{+-} \quad B^0$	$1^+ 0^{--}$ $1^+ 1^{--} \quad \rho^0, \rho'^0$ $1^+ 2^{--}$ $1^{--} \quad \gamma$	$1^+ 1^{+-} \quad B^0$ $1^+ 2^{+-}$
$0^- 1^{+-}$	$^1P_1$	$0^- 1^{--} \quad \omega, \phi, \phi'$ $1^{--} \quad \gamma$	$0^+ 0^{+-}$ $0^+ 1^{+-} \quad H$ $0^+ 2^{+-}$	$0^- 1^{--} \quad \omega, \phi, \phi'$ $0^+ 2^{--}$ $1^{--} \quad \gamma$
$1^+ 1^{+-}$	$^1P_1$	$1^+ 1^{--} \quad \rho^0, \rho'^0$ $1^{--} \quad \gamma$	$1^+ 0^{+-}$ $1^+ 1^{+-} \quad B^0$ $1^+ 2^{+-}$	$1^+ 1^{--} \quad \rho^0, \rho'^0$ $1^+ 2^{--}$ $1^{--} \quad \gamma$
$0^+ 0^{++}$	$^3P_0$	$0^+ 0^{-+} \quad \eta, \eta', \iota$	$0^+ 1^{++} \quad D, E$	$0^+ 2^{-+}$
$0^+ 1^{++}$	$^3P_1$	$0^+ 1^{-+}$	$0^+ 0^{++} \quad S^{*,\epsilon}$ $0^+ 1^{++} \quad D, E$ $0^+ 2^{++} \quad f, f', \Theta$	$0^+ 1^{-+}$ $0^+ 2^{-+} \quad A_3^0$
$0^+ 2^{++}$	$^3P_2$	$0^+ 2^{-+} \quad A_3^0$	$0^+ 1^{++} \quad D, E$ $0^+ 2^{++} \quad f, f', \Theta$	$0^+ 0^{-+} \quad \eta, \eta', \iota$ $0^+ 1^{-+}$ $0^+ 2^{-+}$
$1^- 0^{++}$	$^3P_0$	$1^- 0^{-+} \quad \pi^0$	$1^- 1^{++} \quad A_1^0$	$1^- 2^{-+} \quad A_3^0$
$1^- 1^{++}$	$^3P_1$	$1^- 1^{-+}$	$1^- 0^{++} \quad \delta^0$ $1^- 1^{++} \quad A_1^0$ $1^- 2^{++} \quad A_2^0$	$1^- 1^{-+}$ $1^- 2^{-+} \quad A_3^0$
$1^- 2^{++}$	$^3P_2$	$1^- 2^{-+} \quad A_3^0$	$1^- 1^{++} \quad A_1^0$ $1^- 2^{++} \quad A_2^0$	$1^- 0^{-+} \quad \pi^0$ $1^- 1^{-+}$ $1^- 2^{-+} \quad A_3^0$

In computing QCD on the lattice several people have been able to derive masses of exotic states such as glueballs $[gg]$. The concept of Lattice QCD will be viewed at an elementary level in subsection 2.5.3.

2.5.1 The $N\bar{N}$ -potential for Bound States

A striking feature of the $N\bar{N}$ -potential, that was early observed [5] and later explored by Shapiro [132], is that it is strongly attractive. The repulsive "hard core" of the NN-potential (mainly due to exchange of ω -mesons according to OBEM) is turned into a deep potential well by G -parity transformation. The possibilities for this well to house any bound quasi-nuclear states was investigated by Shapiro [132]. Later work used the interpretation of the nuclear potential to be caused by exchange of virtual mesons. This OBEP can be transformed into an $N\bar{N}$ -potential by term-wise applying G -parity transformation [51]:

$$V_{NN} = \sum_r V_{NN}^{(r)}, \quad (2.43)$$

$$V_{N\bar{N}}^t = \sum_r G_r (V_{NN}^{(r)})^*,$$

where r labels one of the exchange-mesons $\pi, \rho, \omega, \phi, \sigma_1, \sigma_0$, or σ_c , and G is the G -parity of respective mesons. The mesons σ_1 and σ_0 are identified with the ϵ - and the δ -resonances, respectively, while the entirely hypothetical σ_c -meson is to allow for nonresonant two-pion contributions¹¹. Since the ω has negative G -parity the corresponding term will change sign in $V_{N\bar{N}}^t(r)$, and subsequently the repulsive core of the NN-potential is turned into an attractive well (fig. 2.11).

At some radial distance the annihilation process must become important. This possibility can be opened by adding an "annihilation potential" V_{an} containing an imaginary part W_0 [52]:

$$V_{N\bar{N}}(r) = V_{N\bar{N}}^t(r) + V_{an}(r), \quad (2.44)$$

$$V_{an}(r) = \frac{V_0 + iW_0}{1 + e^{-(r-R)/a}}.$$

Comparing experimental data indicates that $V_0 \approx W_0 \approx 20$ GeV, $R \approx 0$, and $a \approx 0.2$ fm. The result that the range of the annihilation region is only ~ 0.1 fm, whereas the range of the nuclear force is ~ 1 fm, thus indicates there should be room for possible bound states. Calculations using the so called "Paris Potential" [30] yield a rich spectrum of levels close to, and above, the threshold $2m_p$. The energy scale of this level scheme is, however, strongly dependent on the range r_0 of the hard core which is difficult to determine experimentally. A promising attempt to improve the Potential Model was lately made by Shapiro [42]. Good agreement with results from scattering experiments is achieved. Masses of quasi-nuclear states predicted by this improved model are compared in fig. 2.11 with predicted values

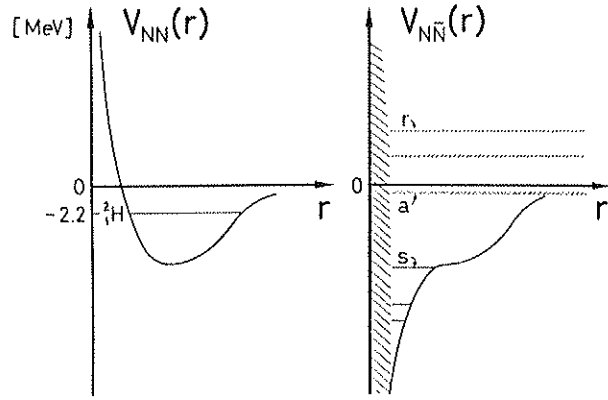


Fig. 2.11: When the NN-potential is transformed into an $N\bar{N}$ -potential the repulsive core is turned into an attractive potential well that possibly could house strongly bound $N\bar{N}$ -states.

¹¹ See for instance the book by Hornyak [78]. Later the $^3P_0$ -mesons have been incorporated to explain the small rise of the nuclear density observed at the centre of a nucleus.

obtained with the Paris Potential, with mass values predicted by the Bag Model.

Of interest for any experiment searching for exotic or quasi-nuclear states is the possible lifetime that determines the line width Γ . A suggestion [132] is that Γ can be computed as

$$\Gamma = 10[\text{mb}] \times \int R^2(r) V(r) dr, \quad (2.45)$$

where $R(r)$ is the radial wave function. This would give a width of 10 to 50 MeV/ c^2 for states and about 50 MeV/ c^2 below the threshold $2m_p$. [30].

An annihilation potential deduced from rearrangement considerations [106] predicts low-lying states with masses around 500 MeV/ c^2 and with widths Γ of only 10 to 20 MeV/ c^2 .

2.5.2 The Bag Model

Confinement can be introduced in a model by hand, so to say, by adding a linear term b (the "bag" parameter) to the QCD Lagrangian [82]:

$$\mathcal{L}(r) = (\mathcal{L}_{\text{QCD}} - b) \theta(r) - \frac{1}{2} \bar{\psi} \psi \delta_s, \quad (2.46)$$

$$\mathcal{L}_{\text{QCD}} = i\bar{\psi} (\gamma_\mu \partial^\mu + ig_s \frac{\lambda_a}{2} \gamma^\mu A_\mu^a) \psi - m_q \bar{\psi} \psi - \frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu}, \quad (2.47)$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c, \quad (2.48)$$

This has the consequence of forcing the quark fields to zero outside a certain radius and "confining" the fields to the spherical cavity set by the step function $\theta(r)$. A totally antisymmetric wave function representing whatever quark content considered, is then assumed to compute the eigenstates under the boundary conditions imposed by the cavity [82]. The energy eigenvalues $E(r)$ will then be dependent on the radius r of the cavity. Masses of the states are obtained by minimizing $E(r)$ with respect to r . An attractive result of this model is its ability of roughly reproducing the correct masses of the observed hadrons. In this Bag Model it is then possible to compute the masses of other colour-singlet combinations for the bag content, as baryonium for instance. The resulting $|qq\bar{q}\bar{q}\rangle$ -states can be classified according to quantum numbers. States for which the quarks and the antiquarks move in a relative S -wave have a large width into mesonic channels and are therefore not commonly referred to as "baryonium". The masses of states are predominantly the kinetic energy of the constituents, and the model predicts the ratio 2:3:4 $\approx$ 700:1100:1500 MeV/ c^2 for masses of mesons, baryons, and baryonium, respectively. A more promising configuration of the quarks would be a diquark q^2 and an anti-diquark $\bar{q}^2$ rotating around each other with a relative angular momentum $l \geq 1$ in an elongated dumb-bell-shaped bag structure. Because a diquark is not a colour singlet the object cannot disintegrate unless substantial rearrangement takes place. And if the object is of the type $|q^2 \bar{q}^2\rangle$ the angular momentum between the q^2 and the $\bar{q}^2$ produces a centrifugal barrier that prohibits such rearrangement.

Concerning the colour structure of such objects one can distinguish between what has been called "true baryonium" and "mock-baryonium". The diquark consisting of two identical fermions has to be in a totally antisymmetric state. Suppose the quarks move in a relative S -wave (which is symmetric). Their spins couple either to a symmetric spin 0 singlet or to a spin 1 antisymmetric triplet. Furthermore, quarks are members of the flavour triplet representation 3 of $SU_f(3)$ as well as of the colour triplet representation of $SU_c(3)$. Coupling two quarks to a diquark then gives

$$3 \otimes 3 = \bar{3} \oplus 6, \quad (2.49)$$

i.e. the diquark can either belong to the antisymmetric triplet $\bar{3}$ or to the symmetric sextet 6. To have an overall antisymmetric wave function the diquark can then only be one of the four types ψ, ξ, ζ , or η listed in table 2.5. If we consider baryonium produced from $p\bar{p}$ annihilations at rest the diquark

world is **not**. The theory needs to be regularized which involves the introduction of a cutoff to remove "high frequency" components of the fields. These are then reintroduced while couplings, field scales, masses, etc., are varied to keep the low energy physics unchanged — i.e. removing the cutoff by re-normalization. This introduces a mass scale $\Lambda_c \sim \exp(-C/g^2(a))$, with an essential singularity for $g(a)=0$ (a is the lattice parameter), which means there is no perturbative expansion [129]. This means that the calculation of confinement radius, hadron masses, etc., amounts to a nonperturbative problem, requiring a nonperturbative regularization scheme. The lattice regularization [144] is such a scheme. It consists of formulating the theory on a discrete lattice of points in Euclidian space-time. The bonus of this formulation is (if the space-time volume is kept finite to proceed only later to the infinite volume limit) that the originally field-theory problem becomes one of statistical mechanics, which invites using methods developed for solving statistical-mechanical systems.

The mathematical problem to compute the expectation value of an observable O is formulated as

$$\langle O \rangle = Z^{-1} \int [dU_{x,\mu}] O(U) e^{-S(U)}, \quad (2.51)$$

where $S(U)$ is the lattice action [144], and Z is the lattice partition function, and the "link variables" $U_{x,\mu}$ are associated with the gauge fields $A_\mu(x)$. Compare the field-theoretical case with the Euclidian action:

$$S_E = \int_{-\infty}^{+\infty} dt \int d^3x \mathcal{L}_E(\phi, \dot{\phi}), \quad (2.52)$$

where $\mathcal{L}_E$ is the Lagrangian in Euclidian space. The partition function is then in the field-theoretical description ($\tau \equiv 1/kT$)

$$Z = \text{Tr}(e^{-\tau H}) = \sum_{\phi} \langle \phi | e^{-\tau H} | \phi \rangle = \int [d\phi]_0^\tau e^{-\int_0^\tau dt \int d^3x \mathcal{L}_E(\phi, \dot{\phi})}, \quad (2.53)$$

and the expectation of the observable O is written:

$$\langle O \rangle = Z^{-1} \int [d\phi]_0^\tau O e^{-\int_0^\tau dt \int d^3x \mathcal{L}_E(\phi, \dot{\phi})}. \quad (2.54)$$

The results from several calculations of glueball masses are converging [129] [120]. A glueball mass is given by the observable $O = \phi(t)\phi(0)$ where $\phi(t)$ is a so called "zero-momentum-operator" obtained by summing over all points x and the three rotations of a 2×2 loop (fig. 2.13)

$$\phi = \sum_{J,m} \alpha_{Jm} \phi(J, m), \quad (2.55)$$

where $\phi(J, m)$ is the perimeter of the loop, obtained by multiplying the U matrices along the links:

$$\phi(J, m) = \text{Tr}(U_{x,\mu} \dots U_{x,\nu}^{-1}). \quad (2.56)$$

Mass values are presented in fig. 2.14, where also masses predicted by the Bag Model and the Flux Tube Model are shown for comparison. A couple of the observed resonances have been suggested [16] as glueball candidates, viz. the $\iota(1440; 0^{-+})$ and the $\theta(1690; 2^{++})$.

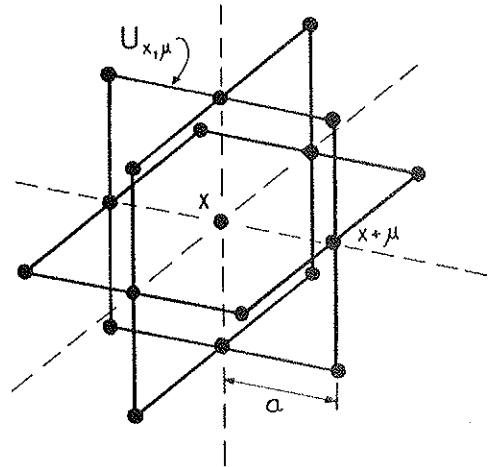


Fig. 2.13: Pictorial QCD lattice.

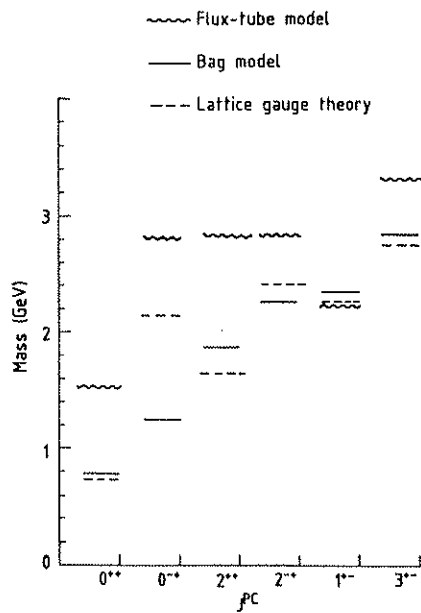


Fig. 2.14: Mass spectrum of $|gg\rangle$ -states (glueballs) as predicted by three different models. (Adapted from refs. [31] [80]).

*What admits no doubt in my mind is
that the Creator must have known a great
deal of wave mechanics and solid state
physics, and must have applied them.*

(A. Szent-Györgyi)

CHAPTER 3

THE LEAR EXPERIMENT PS182

The main purpose of the LEAR experiment PS182 was, as already mentioned in chapter 1, to investigate the occurrence of rare decay channels of the $p\bar{p}$ system by means of the inclusive π^0 spectrum. If an intermediate exotic state is formed in the annihilation process, and is associated with the emission of a π^0 -meson the momentum of this π^0 will be well defined. When studying a large number of $p\bar{p}$ annihilations these π^0 's will eventually build a peak in the inclusive spectrum. Even though a π^0 will attain a well-defined momentum in each particular reaction, as it is a two-body decay, its momentum differs between reactions due to the finite lifetime of the exotic state, i.e. the width of its mass distribution. Thus the observed π^0 peak might have a substantial width. Theories, as learned from the previous chapter, seldom predict the width of an exotic state with accuracy, and they could very well be extremely short-lived and consequently broad. This fact is maybe also the explanation why these states have been so hard to identify. In order to prevent the detector from rendering a possible peak even more jaded, it is desirable for an experiment to make the momentum resolution of the detector as good as possible. At least it should be smaller than the width of an ordinary mesonic resonance — as the ρ for instance. The different issues of this task will be extensively discussed in this chapter. However, before going into details about the capabilities of the apparatus used in PS182, I will somewhat less circumstantially describe the layout of the experiment (section 3.2), the beam and the target (subsections 3.3.2 and 3.3.3), and the machine environment (subsection 3.3.1).

The experiment PS182 was designed to measure the momentum of π^0 -mesons ejected from annihilating $p\bar{p}$ systems in a liquid hydrogen target. Because of the short mean lifetime of the π^0 , being $\tau \approx 8 \cdot 10^{-17}$ s, the maximum π^0 range is only $c\tau = 24$ nm before it decays into two photons, which it does with 99% probability. Thus, it is in reality two coinciding photons that have to be detected. The π^0 momentum is then calculated from the sum of the two photon energies. It is evidently necessary to assure that the detected photon pair in fact was emitted from one and the same π^0 -meson as it very often happens that several π^0 's are produced in a $p\bar{p}$ annihilation. Hence, one firstly computes the invariant mass of a detected photon pair, which requires in addition knowledge about the angle between the directions in which the photons radiated. Therefore the apparatus has to be able to measure both photon energies and angles, i.e. the impact positions of the photons. And of course it is desirable to measure as accurately as possible to finally achieve optimal π^0 -momentum resolution. To meet these requirements the detector was constructed from a large array of lead glass and two smaller arrays made from bismuth germanium oxide (BGO), that is a relatively new detector material in high-energy physics. The properties of the BGO calorimeters (that have been extensively tested) are discussed in detail in section 3.4. Section 3.5 describes the methods of calibrating and determination of the resolution of the lead-glass array. The last section of this chapter deals with the special features of the apparatus that are of importance for π^0 spectroscopy, i.e. photon coincidence measurements. Before presenting the experiment, however, a few words should be said about the technique of π^0 spectroscopy, since this is what the experiment is devised for.

3.1. PRINCIPLES OF π^0 SPECTROSCOPY

The π^0 -meson decays with 99% probability into two photons. From two photons radiated from the same vertex there are three quantities available for measurement in the lab frame. These are the two energies of the photons and the angle between them. In the π^0 rest frame the two γ -rays are colinear and the decay is completely isotropic since the π^0 is spinless. When transforming the kinematics to the lab frame the photons will be subject to relativistic aberration and acquire a Lorentz boost. The magnitude of the boost depends then on the

angle of the colinear decay to the π^0 direction. Hence, the laboratory opening angle between the two photons is uniquely determined from the energy sharing between them. A relation between the photon energies and the opening angle in the lab frame can be derived from the Lorentz-invariant inner product $P_1^\mu P_{2\mu}$ of the 4-momenta of photon 1 and 2, respectively (see fig. 3.1). Evaluating it in the lab frame gives then with notations from fig. 3.1:

$$P_1^\mu P_{2\mu} = (E_1, \mathbf{p}_1)(E_2, \mathbf{p}_2) = E_1 E_2 - \mathbf{p}_1 \mathbf{p}_2 = E_1 E_2 (1 - \cos \phi) . \quad (3.1)$$

And evaluating it in the π^0 rest frame gives

$$P_1^\mu P_{2\mu}' = \left(\frac{m_{\pi^0}}{2}, \mathbf{p}_1' \right) \left(\frac{m_{\pi^0}}{2}, \mathbf{p}_2' \right) = \left(\frac{m_{\pi^0}^2}{4} \right) (1 - \cos \pi) = \frac{m_{\pi^0}^2}{2} . \quad (3.2)$$

Since the value of any inner product is unchanged under Lorentz transformation we have

$$m_{\pi^0}^2 = 2E_1 E_2 (1 - \cos \phi) , \quad (3.3)$$

where $\phi = \theta_1 + \theta_2$ is the opening angle between the photons in the lab frame, and θ_1, θ_2 are the angles between γ -ray 1 and 2, respectively, and the π^0 direction. For a certain π^0 energy the amount of energy acquired by one photon is then a function of the opening angle:

$$E_\gamma = \frac{E_{\pi^0}}{2} \left[1 \pm \sqrt{1 - \left(\frac{m_{\pi^0}}{E_{\pi^0} \sin(\phi/2)} \right)^2} \right] . \quad (3.4)$$

The distribution of the decays in the E_γ -versus- ϕ plane is plotted in fig. 3.2a for π^0 's with 550 MeV/c momentum. The projections of the distribution onto the energy axis and the angle axis are shown in figs. 3.2b and 3.2c, respectively. Fig. 3.2d shows the distribution of the angle θ_1 between one photon and the π^0 direction in the lab frame, which can be derived in the same way as the photon energy distribution. This is done in Appendix A. The energy distribution turns out to be uniform between the maximum and the minimum energies, i.e. the forward and backward boosted cases, respectively. Fig. 3.3 shows the same distributions for π^0 's of 150 MeV/c momentum.

As seen from the figures the opening-angle distribution increases sharply towards a minimum angle, which is given by

$$\phi_{min} = 2 \arctan \left(\frac{m_{\pi^0}}{p_{\pi^0}} \right) . \quad (3.5)$$

The minimum angle in the lab frame occurs when the decay is at 90° centre-of-mass angle to the π^0 direction. In this case the two photons attain the same boost and have subsequently the same energy in the lab frame, i.e. $E_1 = E_2$ (cf. fig. 3.2a). We call this a "symmetric" decay, which is the most usual case as demonstrated by the angular correlation function (fig. 3.2b) of the decay photons. The θ distributions do not, however, peak at $\phi_{min}/2$ (see fig. 3.2d), but at a smaller angle due to the Lorentz contraction of space. Because most of the decays occur close to the minimum angle, and this is proportional to the π^0 momentum, there is a strong correlation between the observed opening angle and the momentum of the π^0 . The derivation of this so called Jacobian peak distribution is also compiled in Appendix A.

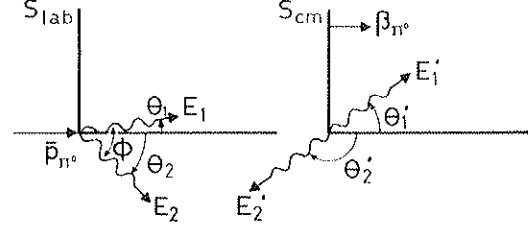


Fig. 3.1: Kinematics of a π^0 -decay in the lab frame S_{lab} and the centre-of-mass system S_{CM} .

Of all the observed photon pairs the computed invariant mass will only rarely coincide with the true π^0 mass. Firstly, the two photons might not at all come from the same π^0 decay, but be just an accidental coincidence. Secondly, the measured values of the three quantities have experimental errors due to detector resolution and imperfections. We therefore introduce a variable m_0 being the computed invariant mass of the two detected photons. If we denote the measured photon energies by E_1 and E_2 and the observed opening angle between their directions by ϕ , we write eq. 3.3 in form that will be frequently referred to:

$$m_0^2 = 4E_1E_2\sin^2(\phi/2) \quad (3.6)$$

How the experimental errors in the measured quantities $x = (E_1, E_2, \phi)$ influences the invariant mass m_0 is given by the formula¹ for error propagation:

$$\sigma_{m_0}^2 = \sum_{l=1}^3 \left(\frac{\partial m_0}{\partial x_l} \right)^2 \sigma_l^2 \quad (3.7)$$

Evaluating the partial derivatives we get:

$$\left(\frac{\sigma_{m_0}}{m_0} \right)^2 = \left(\frac{\sigma_{E_1}}{2E_1} \right)^2 + \left(\frac{\sigma_{E_2}}{2E_2} \right)^2 + \left(\frac{\sigma_\phi}{2\tan(\phi/2)} \right)^2 \quad (3.8)$$

For small π^0 -momenta, i.e. a large minimum angles, the last term of eq. (3.8) can be neglected as the coefficient vanishes and makes it inconsequential even with poor angular resolution. The invariant mass resolution is then determined by the energy resolutions. At higher π^0 -momenta the angular resolution term becomes dominant as the coefficient approaches infinity when the opening angle goes to zero.

In principle the π^0 -momentum is fully determined from the two photon energies

$$p_{\pi^0} = \sqrt{(E_1 + E_2)^2 - m_{\pi^0}^2} \quad (3.9)$$

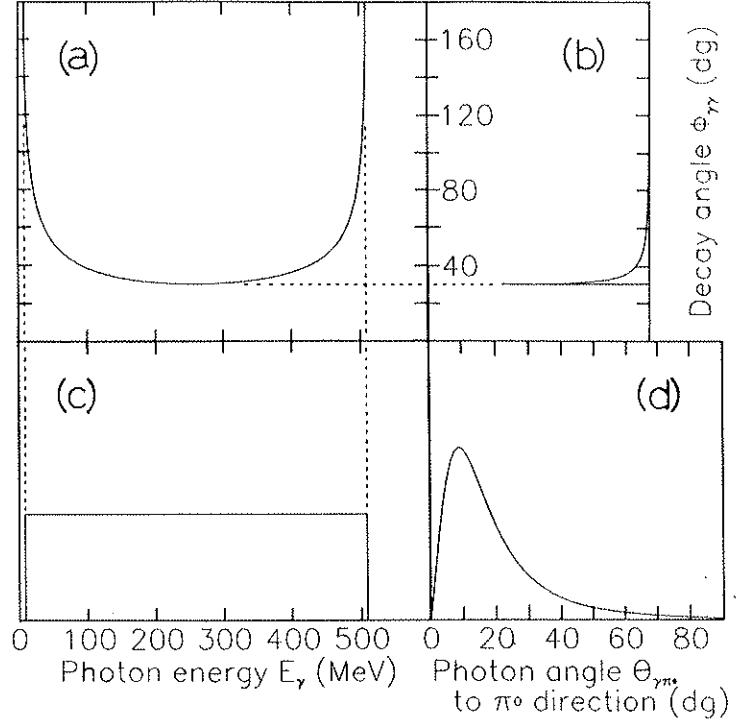


Fig. 3.2: The photon energy E_γ is a function of the opening angle ϕ between the directions of the two decay photons (a). The projection of the distribution onto the energy axis (c) is uniform, whereas the projection onto the ϕ_γ -axis (b) shows a sharp peak of the minimum angle. The distribution (d) of decay angles θ_γ between the π^0 -direction and a photon direction also peaks at a small angle, but, due to Lorentz contraction of space, this angle is not half the minimum opening angle ϕ_{min} .

¹ See for instance the book by Bevington [21].

where m_{π^0} is the true π^0 -mass and **not** the measured invariant mass m_0 . However, by measuring the decay angle as well one can improve the momentum resolution by fitting the kinematics to reproduce the correct invariant mass. The idea of such a fitting procedure is to adjust the measured values according to the relative uncertainties of the measurements so that they combine to m_{π^0} after adjustment. To be sure that the individual measurements are improved one has to know the experimental errors, because the fit weights the corrections with the inverse of respective error. The final π^0 -momentum resolution will then depend on the different resolutions of the apparatus in a complicated way. The π^0 -momentum resolution σ_p can with good approximation be expressed as

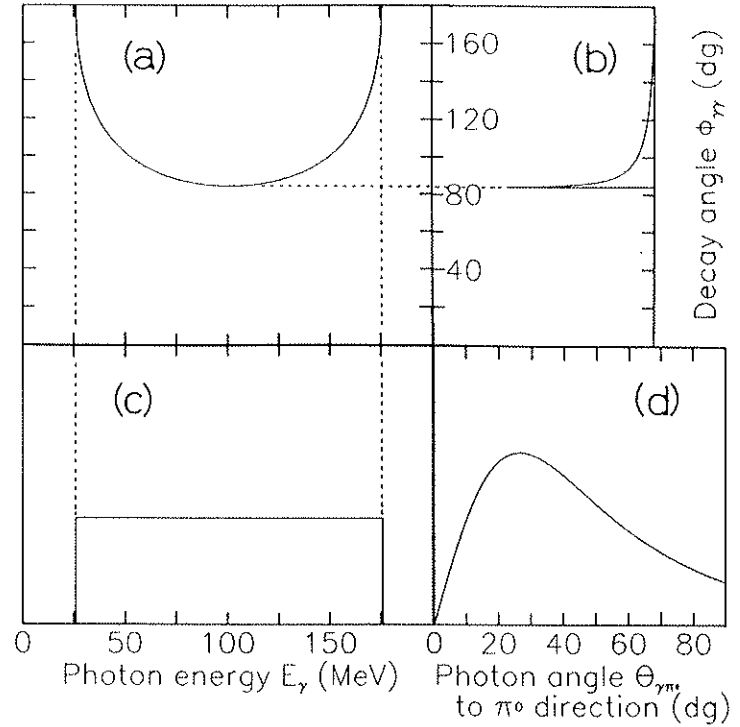


Fig. 3.3: Same as fig. 3.2 but for π^0 's of momentum 150 MeV/c.

$$\left(\frac{\sigma_p}{p}\right)^2 = \frac{1}{\beta^2} \left[\frac{1}{4} \frac{\beta^2}{(1-\beta^2)(1-x^2)} \sigma_\phi^2 + \frac{x^2}{(1+x)^2(1-x^2)} \frac{\sigma_{E_1}^2}{E^2} + \frac{x^2}{(1-x)^2(1-x^2)} \frac{\sigma_{E_2}^2}{E^2} + \right. \\ \left. + \frac{3}{4} \frac{(1-2x)^2}{(1+x)^4(1-x^2)} \frac{\sigma_{E_1}^4}{E^4} + \frac{3}{4} \frac{(1+2x)^2}{(1-x)^4(1-x^2)} \frac{\sigma_{E_2}^4}{E^4} + \frac{1}{(1-x^2)^3} \frac{\sigma_{E_1}^2 \sigma_{E_2}^2}{E^4} \right] \quad (3.10)$$

where E is the π^0 energy, and x is the asymmetry between the two photon energies:

$$x = \frac{E_1 - E_2}{E_1 + E_2} \quad (3.11)$$

*I like work; it fascinates me.
I can sit and look at it for hours.
I love to keep it by me;
the idea of getting rid of it
nearly breaks my heart.*

(Jerome K. Jerome)

3.2. LAYOUT OF THE EXPERIMENT

The detector "dream" for the purpose of this experiment would be a "crystal-ball" calorimeter covering 4π solid angle and made from a material providing good energy resolution, as for instance NaI(Tl) or BGO that offers a few percents energy resolution at 1 GeV. Maximal sensitivity is obtained when all radiation directions in space are covered, and no γ -ray escapes detection. Furthermore, the angular resolution has to be good. It can be improved by increasing the inner radius of the "crystal ball" so that each crystal element subtends a smaller angle. However, the detector volume blows up roughly as the square of the radius. The beam height at LEAR would allow for a "crystal ball" made from BGO that has a radius of 1 m. The price of such a detector would, however, become fabulous. At the time this experiment was proposed (in 1981) the BGO price was around $\$5/\text{cm}^3$, and with 20 cm long crystals the total cost (of the crystals only) would exceed $\$15\text{M}$, which of course was a completely unrealistic cost for the experiment.

To compromise and reasonably meet the requirements with the budget available the detector was constructed from a big array of lead-glass blocks complemented with two smaller arrays of BGO crystals. As lead glass is a Cherenkov calorimeter the energy resolution (at the current energies) will be about 10%. This might seem somewhat poor but can be compensated by a good angular resolution owing to the kinematic fitting procedure. The detector setup is presented in fig. 3.4. The two sets of seven BGO crystals each are mounted on a ring surrounding the target. They can be moved to any azimuthal angle relative to the lead-glass array and are adjustable in the radial direction between 25 cm and 80 cm, to enable variation of solid angle and angular resolution. The lead-glass array is rigidly installed below the beam line at a radius of 80 cm and subtends an azimuthal angle of 120° . It extends 25.6 cm in the beam direction and covers a total solid angle of 5.3%.

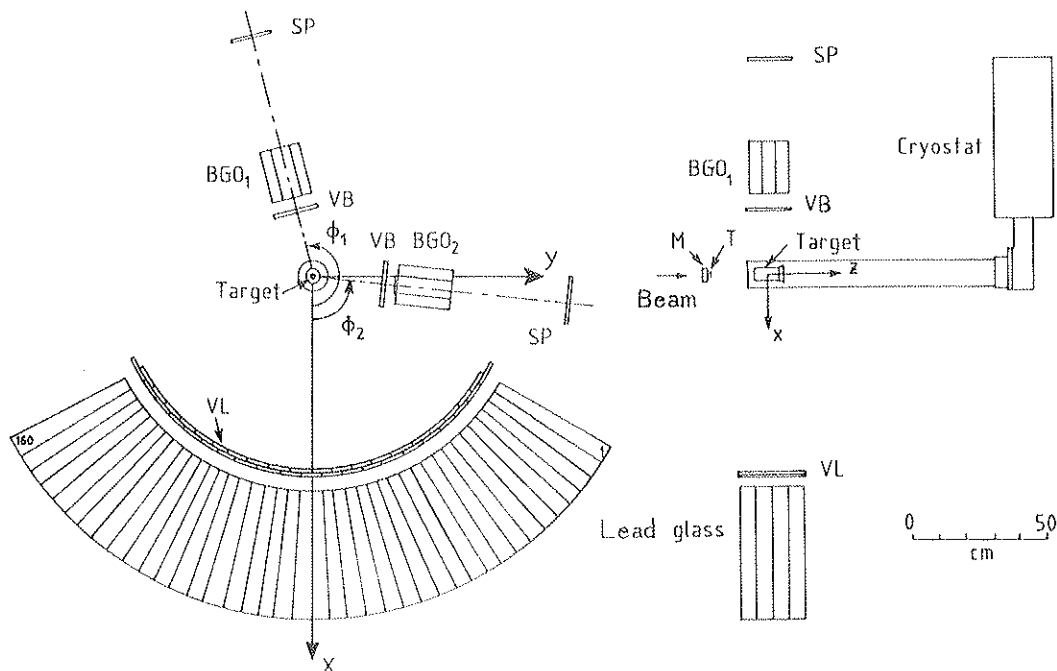


Fig. 3.4: The experimental setup of PS182. Except the γ -calorimeters the scintillators VB and VL are shown which serve as veto counters for charged pions, and the scintillators SP for triggering on charged pions that passed through a BGO set. The telescope scintillator T and the beam moderator M are also shown.

The positioning of the BGO sets was different during different run periods. The detector configuration shown in fig. 3.4 was arranged for the first run period that took place in December 1983. The geometrical parameters of the individual run periods are summarized in table 3.1. Note the symbols D83, M84, A84, and M85 referring to the different run periods, respectively, which will be frequently used further on. In addition to the calorimeters the setup comprises a number of plastic scintillators. In front of the γ -calorimeters there are 1 cm thick scintillators², labeled "VB" and "VL" in fig. 3.4, respectively. These will recognize charged pions³ and will only with negligible probability yield a signal from passing photons. Hence, they serve as veto counters to reject charged pion events. The veto counter VL before the lead-glass array consists of a double layer of 41 scintillator strips, each covering eight lead-glass blocks, i.e. two columns across the four rows. Each layer is half a centimeter thick, and they are displaced relative one to another by half a unit so as not to leave any dead space inbetween them. Behind the BGO sets there are also 1 cm thick scintillators mounted, labeled "SP" in the figure. These provide the possibility to select events of charged pions that passed through the whole BGO set, by applying a coincidence condition with the front scintillator. These events do not belong to the actual measurement but are used for calibration of the BGO crystals. Around the target immediately outside the shielding there was a cylindrical counter mounted during the last run period in May 1985 (not shown in fig. 3.4). It has an inner radius of 6.4 cm and covers 92% solid angle. It is assembled of 20 strips of dimensions $0.5 \times 2.0 \times 30 \text{ cm}^3$. With these scintillators the charged multiplicity of a reaction can be measured, and also their spatial distributions.

Table 3.1: Geometrical parameters from the four different run periods in December 1983, in May 1984, in August 1984, and in May 1985, respectively. (Note the mnemonics for denoting the different run periods.)

Run period:	D83	M84	A84	M85
Angle of BGO ₁ [°]	195.0	122.0	180.0	180.0
Angle of BGO ₂ [°]	85.0	286.9	286.9	286.9
Distance from target [cm]				
of BGO ₁	31.2	30.0	30.0	50.0
of BGO ₂	30.2	35.0	30.0	52.5
No. of hodoscopes				
of BGO ₁	0	0	0	0
of BGO ₂	1	0	2	0

The liquid hydrogen target of 5 cm diameter had different length during different run periods. During period D83 in December 1983 it was only 10 cm long. However, the range of the 300 MeV/c $\bar{p}$ -beam would be around 11 to 12 cm, thus a few millimeters of plexiglass was inserted in the beam-line to serve as a moderator (labeled "M" in fig. 3.4). Immediately before the target window the telescope scintillator (labeled "T") is positioned, which tags an entering antiproton. This scintillator is only 1 mm thick and has a diameter of 2 cm, although, the $\bar{p}$ -beam was never broader than a centimeter.

² These are of the type NE102A from Nuclear Enterprise.

³ Due to the large lifetime of the charged pions that is $\tau_{\pi^{\pm}} = 2.6 \cdot 10^{-8} \text{ s}$ they will have a range of $c\tau_{\pi^{\pm}} = 7.8 \text{ m}$ in contrast to the neutral pion range of 24 nm.

3.3. BEAM AND TARGET

The LEAR machine at CERN at which the experiment PS182 was conducted generates a $\bar{p}$ -beam of extremely high quality. The small dimensions of the beam and the well-defined momentum produces a small and confined interaction region in the target that makes the vertex definition of the γ -pairs accurate. It also allows for the target dimensions to be of the order of a few centimeters. With such a small target volume the probability for secondary reactions induced by mesons produced in the $p\bar{p}$ annihilations interacting with another proton is brought down to an acceptable level. The LEAR machine, which possesses some unique, quite remarkable capabilities, will be shortly surveyed in the following subsection. In subsection 3.3.2 the external beamline where PS182 was performed is described. Subsection 3.3.3 then discusses the moderation of the beam and the resulting annihilation region in the target.

3.3.1 LEAR $\bar{p}$ Factory

The Low Energy Antiproton Ring is a small storage ring to generate intensive and uncontaminated $\bar{p}$ -beams of extremely low momenta. It receives the antiprotons from another machine called the Antiproton Accumulator (AA) where the antiprotons are produced and stacked until high luminosity is achieved. The interconnections of these machines via the Proton Synchrotron (PS) at CERN is illustrated in fig. 3.5.

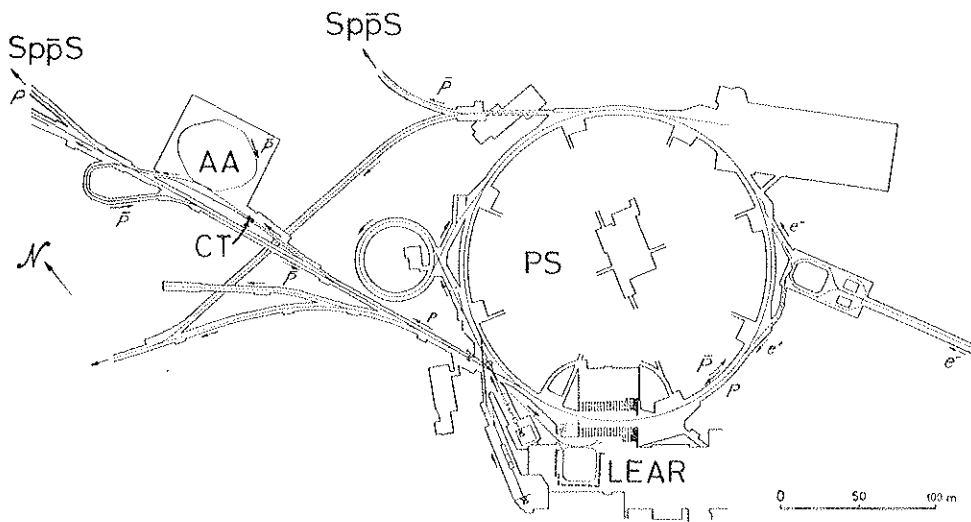


Fig. 3.5: The CERN PS-complex. Antiprotons are produced in the conversion target CT by protons accelerated in the proton synchrotron PS. The antiprotons are then collected and stacked in the antiproton accumulator AA. When a stack is full the antiprotons are transferred to the PS and can, from there, be distributed to LEAR or the $p\bar{p}$ -collider Sp $\bar{p}$ S.

The production of the antiproton beam starts with the focusing of 26 GeV/c protons from the PS onto a conversion target (labeled "CT" in fig. 3.5) made from copper and being 11 cm long and 3 cm diameter. The antiprotons are produced through the reaction

$$p + p \rightarrow p + p + p + \bar{p}$$

and emerge from the target at various angles and momenta (fig. 3.6) [44]. The $\bar{p}$ yield has a flat maximum at momenta between 3 and 4 GeV/c. The converted beam is collimated and subsequently col-

lected and stacked in the Antiproton Accumulator (AA), which is designed to accept $\bar{p}$'s with momenta in the range $3.57 \text{ GeV}/c \pm 0.75\%$ and angles contained inside an acceptance of $100\pi \cdot 10^{-6} \text{ m} \cdot \text{rad}$ in both the transverse planes [85].

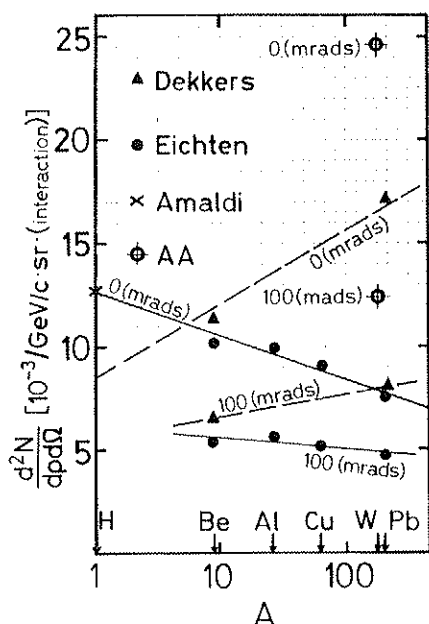


Fig. 3.6: Antiproton yield for various materials for 23.1 GeV/c protons and 4 GeV/c antiprotons. The AA points are extrapolated for 26 GeV/c protons. (Adapted from ref. [85]).

Protons arrive at, and antiprotons leave, the conversion target in bursts of 500 ns duration every 2.4 seconds. Stacks of antiprotons are accumulated during days to weeks with an accumulation rate of about $3.5 \cdot 10^9 \bar{p}$'s/hour, provided the full PS rate of $6 \cdot 10^{13}$ protons was delivered onto the conversion target. The possibility of stacking $\bar{p}$'s and successively build up a more and more intense beam is enabled by the technique named "stochastic cooling". The idea of this technique is to reduce the transverse and longitudinal momentum spread of the beam, "calm" it down so to say, in order to make it denser and subsequently more intense [102]. When viewed from a reference system travelling along with the beam and having a velocity equal to the average particle velocity, the large momentum spread of the particles resembles a hot gas. Hence, the concept "cooling" of an accelerator beam when the momentum spread is being reduced⁴. The cooling has also the effect of filtering out contaminating particles from the beam. The consequence of the diminished momentum spread is that the beam will shrink down in dimensions, and thus the edges of the phase space volume preserved by the machine is emptied of particles and made available for additional antiprotons to be appended.

The AA provides both the Sp $\bar{p}$ S collider and LEAR with antiprotons. When the system is operating in 100% LEAR mode about $10^9 \bar{p}$'s, known as a "fill", are transferred from the AA via the PS into LEAR over a couple of minutes. The antiprotons are injected

in LEAR at a momentum of 600 MeV/c which generally is more than wanted by the experiments at LEAR. The beam is therefore decelerated in LEAR. However, as the beam is decelerated the emittance E will increase. The emittance is defined as the area of the ellipse that represents the beam in phase space. With z along the beam direction the equation of motion can be written

$$m_p \frac{d^2x}{dt^2} + vGx = 0, \quad (3.12)$$

where G is the gradient of the magnetic field. By choosing z instead as independent variable it becomes, with $x' \equiv dx/dz$

⁴ There exists basically two different techniques of cooling. The one named stochastic cooling works with an electronical recoupling system; the position of a small sample of the beam is probed with a "pick-up", and a signal is sent across the storage ring to a "kicker" that will correct the displacement of the sample to the beam average. This technique reaches a saturation point due to that the pick-up signal vanishes as the beam becomes cooler. The other method is electron cooling which works with a well controlled electron beam that is travelling along with the $\bar{p}$ -beam for a short distance. The $\bar{p}$'s are then acquiring the order of the electron beam by Coulomb interaction. This method works more efficient the cooler the $\bar{p}$ -beam is, and is therefore complementary to the stochastic cooling.

$$x'' + \frac{G}{p}x = 0$$

or

$$\left(\frac{x'}{G/p}\right)^2 + x^2 = a^2 = \text{const.}, \quad (3.13)$$

which shows that the phase space has an elliptic shape and is spanned by x' and x . The emittance is then proportional to the inverse of the momentum p :

$$E = \pi a^2 \frac{G}{p} [\text{m} \cdot \text{rad}]. \quad (3.14)$$

Therefore, the beam has again to be cooled so that antiprotons not are peeled off and to achieve optimal values. To minimize losses the deceleration is made stepwise and alternated with cooling intervals. In the first step the momentum is taken down to 300 MeV/c, which, together with the subsequent cooling period requires some 10 minutes. Fig. 3.7 shows the improvement of momentum spread achieved by the cooling technique in LEAR [103]. Fig. 3.8 shows a drawing of LEAR with the placing of the kicker magnets and pickups of the cooling system and the interconnecting waveguides. The LEAR beam has, at 300 MeV/c, an optimal cross-section of about 1 mm² in the machine and a longitudinal momentum spread $\sim 10^{-4}$ at an intensity of 10^9 $\bar{p}$'s [91]. It is extracted during 50 minutes which yields an extraction rate of roughly $3 \cdot 10^5$ $\bar{p}$ /s to be shared among three external beamlines operating simultaneously. The overall cycle time of the system is thus about an hour.

3.3.2 External Beam C1

The experiment PS182 is situated at branch C1 of the central beamline of the three external beams running out from LEAR (see fig. 3.9). Two different beam momenta were delivered, viz. 200 MeV/c and 300 MeV/c. The rate were in the best cases between 10^4 and 10^5 $\bar{p}$'s/s. The typical intensity variation of a spill is shown in fig. 3.10.

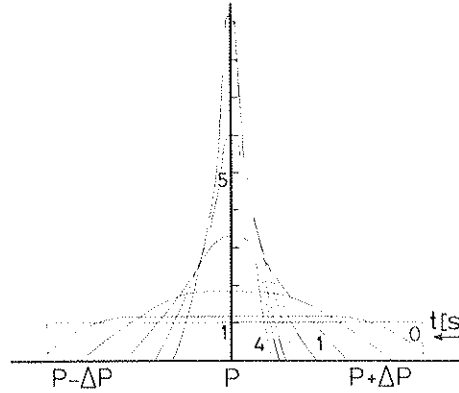


Fig. 3.7: The improvement of momentum spread achieved by the stochastic cooling technique installed in LEAR.

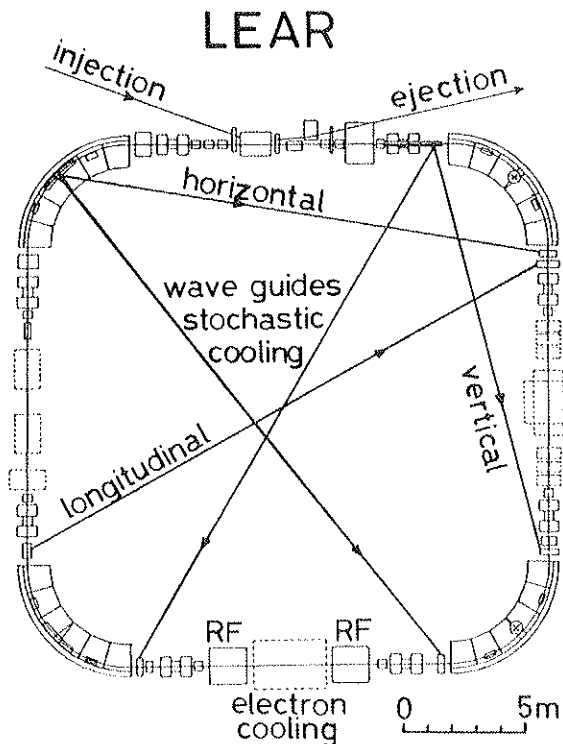


Fig. 3.8: The LEAR machine at CERN with the waveguides of the stochastic cooling system. The beam is decelerated in the RF cavities. Dashed lines show future devices, as the electron cooling for instance.

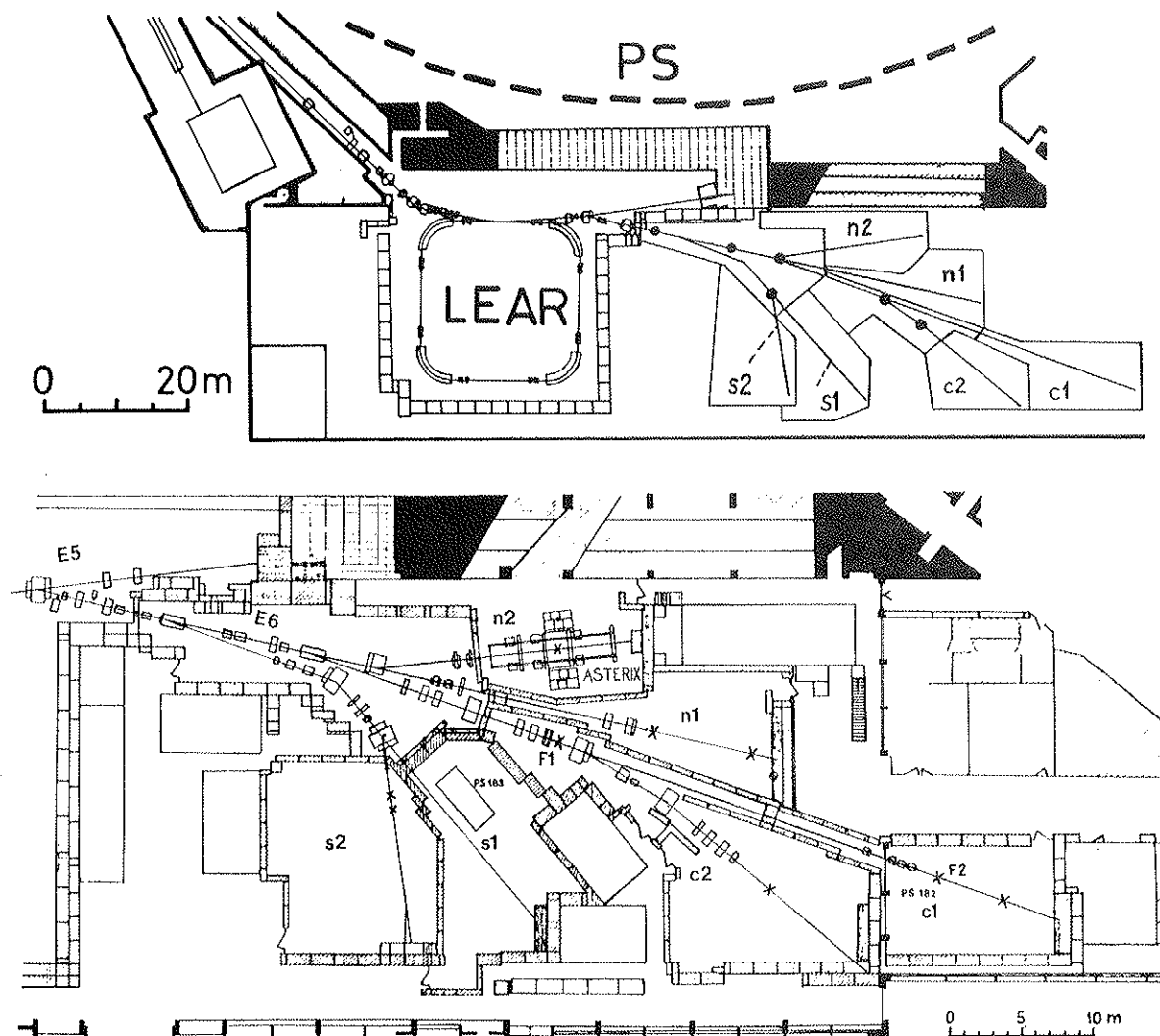


Fig. 3.9: The experimental area in connection to LEAR. The top view shows the location of the experimental area relative LEAR and the proton synchrotron (PS). The experiment PS182 is situated at beamline C1 in the lower right corner of the experimental area. Note the location of the primary focus F1.

In the branch C1 there is a primary focus ("F1" in fig. 3.9) half-way down the beamline serving another experiment. Because of this focus there has to be a half-meter gap in the beam pipe, and the beam has consequently to pass through two 100 μm thick windows about 25 m upstream of our target, one of aluminum and one of mylar. This will naturally introduce a longitudinal momentum spread and blow up the lateral size of the beam until it arrives at the secondary focus and the target of PS182. The beam cross-section was measured just upstream of our target with a $7 \times 7 \text{ cm}^2$ wire chamber. A monitor picture of this measurement is shown in fig. 3.11. The wire spacing is 0.4 cm implying a beam cross-section of $0.79 \times 0.76 \text{ cm}^2$ full-width at half-maximum.

The longitudinal momentum spread induced by the energy straggling when passing through the windows could, however, not be directly measured but only estimated by a calculation. The distributional shape of the energy straggling of a particle passing through matter is mainly due to Coulomb

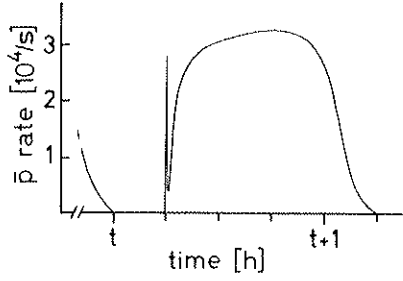


Fig. 3.10: Intensity of the external beam C1 from LEAR as a function of time.

given by a Vavilov distribution [141] [136] that has the probability density function

$$f(\lambda) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \phi(s) e^{\lambda s} ds, \quad (3.15)$$

where c is an arbitrary real positive constant, and

$$\phi(s) = C e^{\psi(s)}, \quad (3.16)$$

$$C = e^{\kappa(1+\gamma\beta^2)}, \quad (3.17)$$

$$\psi(s) = s \ln(\kappa) - \kappa e^{-s/\kappa} + (s + \kappa\beta^2) \left[\int_0^1 \frac{1}{t} (1 - e^{-st/\kappa}) dt - \gamma \right], \quad (3.18)$$

where $\gamma = 0.5772\dots$ is Euler's constant. The variable λ is the parameter introduced by Landau [90] that contains the actual energy loss ε_{loss} :

$$\lambda = \frac{\varepsilon_{loss} - \varepsilon_{av}}{\xi} - 1 - \beta^2 + \gamma - \ln(\kappa), \quad (3.19)$$

where ε_{av} is the average energy loss, and

$$\xi = 0.30058 \frac{m_e c^2}{\beta^2} \cdot \frac{Z}{A} \cdot s, \quad (3.20)$$

with s being the path length in g/cm^2 . The shape of the Vavilov distribution — that actually is a whole series of distributions — is shown in fig. 3.12 and is determined by the parameter κ which is proportional to the ratio of the mean energy loss over the path length of the particle to the largest energy transfer possible in a single collision with an atomic electron:

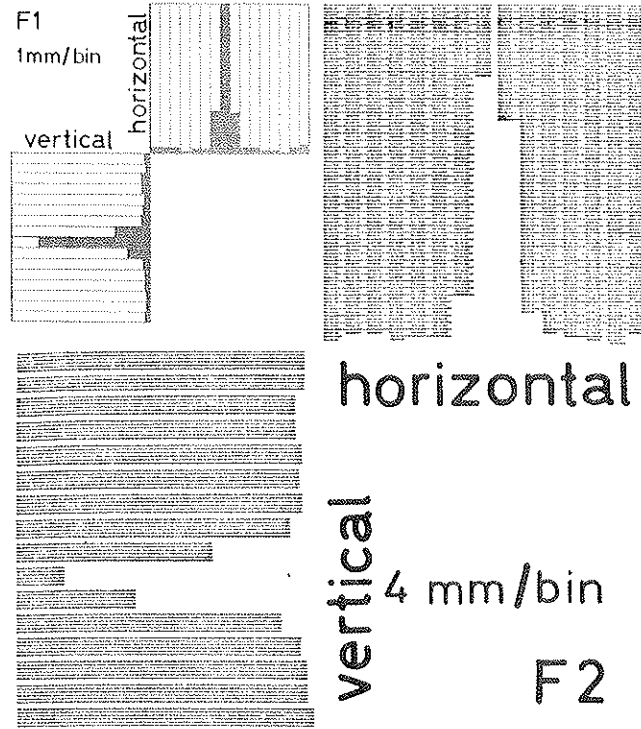


Fig. 3.11: A monitor picture of a measurement of the cross-section of the external beam C1 at the target of PS182. The inset in the upper left corner is a similar measurement of the cross-section of the same beam 25 m upstreams at the primary focus F1 (cf. fig. 3.9).

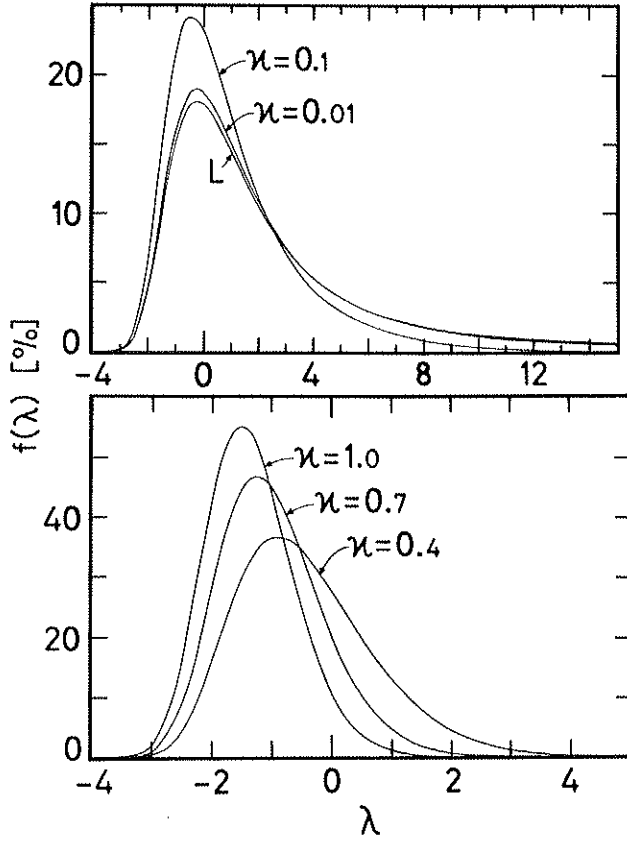


Fig. 3.12: Vavilov distributions for different values of κ . For small values of κ (~ 0.01) it approaches the Landau distribution L , and for large κ 's (> 10) it can be approximated by a Gaussian. See the text for definition of the energy-loss parameter λ . (Adapted from ref. [131]).

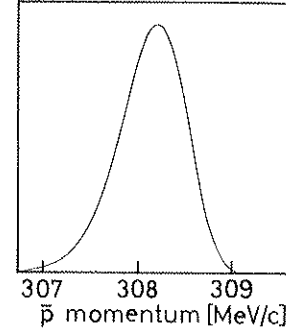


Fig. 3.13: Estimated momentum distribution of the $\bar{p}$ -beam upon arrival at PS182.

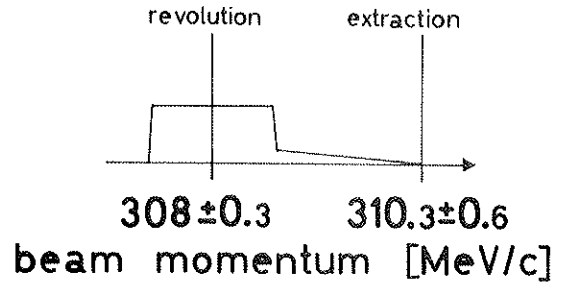


Fig. 3.14: Momentum distribution of the LEAR $\bar{p}$ -beam as circulating in the machine. The average momentum is defined by the revolution frequency. When kicked out of the machine the $\bar{p}$'s acquire a slightly higher momentum so that the momentum of the extracted $\bar{p}$ -beam is 310 MeV/c.

$$\kappa = \frac{\xi}{\varepsilon_{max}}, \quad (3.21)$$

$$\varepsilon_{max} = \frac{2m_e c^2 \beta^2}{1 - \beta^2} \left[1 + \frac{2m_e}{m_p} \frac{1}{\sqrt{1 - \beta^2}} + \left(\frac{m_e}{m_p} \right)^2 \right]^{-1}. \quad (3.22)$$

In the high-energy limit, when $\kappa \rightarrow 0$, the Vavilov distribution approaches the Landau distribution, and for $\kappa < 0.01$ it can be replaced by the Landau distribution [141]. When $\kappa \geq 10.0$ the Vavilov distribution may, with good approximation, be replaced by a Gaussian distribution $G(\mu, \sigma)$ [131] with the mean value and the standard deviation according to

$$\begin{aligned} \mu &= \gamma - 1 - \beta^2 - \ln(\kappa) , \\ \sigma^2 &= (2 - \beta^2)/2\kappa . \end{aligned} \quad (3.23)$$

In our case we have 300 MeV/c $\bar{p}$'s penetrating 100 μm of aluminum, half a meter of air, and then 200 μm of mylar. Thus, κ is in our case

$$\kappa = \frac{\xi}{e_{\max}} = \frac{0.872 \text{ MeV}}{0.112 \text{ MeV}} = 0.781 , \quad (3.24)$$

which shows that we are just between the two limits. The relevant Vavilov shape has then to be folded with the momentum spread of the beam prior to the windows. The result is displayed in fig. 3.13, where the mean value ε_{av} is given by the beam momentum upon extraction (fig. 3.14) together with the Bethe-Bloch formula [20]:

$$\varepsilon_{av} = \frac{dE}{ds} = \xi \left[\ln \frac{2m_e c^2 \beta^2 \varepsilon_{\max}}{I^2(1-\beta^2)} - 2\beta^2 - \delta - U \right] , \quad (3.25)$$

where I is the mean excitation energy of the stopping medium (fig. 3.15). The last two terms in the brackets are corrections, whose effects are appreciable only over limited energy regions. The correction δ is due to the density effect, i.e. polarization of the medium, and $U = 2C/Z$ is the shell-correction term due to non-participation of the inner shells ($K, L, \dots$) for very low velocities of the incident particle.

The density effect, which can be understood as a screening of the incident particle at large impact parameters by the polarization of the medium, can be expressed as (Steinheimer's equation) [135]

$$\begin{aligned} X < X_0 & \quad \delta = 0 \\ X_0 \leq X \leq X_1 & \quad \delta = 4.606X + D + a(X_1 - X)^m \\ X > X_1 & \quad \delta = 4.606X + D \end{aligned} \quad (3.26)$$

$$X = \log_{10} \frac{p}{m_p c} = \frac{1}{2} \log_{10} \frac{\beta^2}{1-\beta^2} ,$$

Table 3.2: Parameters for Steinheimer's equation. (Taken from ref. [60]).

Z	D	a	m	X_1	X_0
4	2.83	0.413	2.82	2.0	-0.10
13	4.21	0.0906	3.51	3.0	0.05
29	4.38	0.107	3.39	3.0	0.17
47	5.09	0.183	3.05	3.0	0.02
82	6.16	0.344	2.66	3.0	0.39
92	5.89	0.318	2.66	3.0	0.20

where the parameters a, m, D, X_0 , and X_1 depend on the dielectric constant of the medium and can be interpolated from table 3.2. The shell-correction term is a more delicate task to handle [60]. Roughly speaking, one may say that the electrons of the inner shells $K, L, \dots$ cease to contribute to the stopping power in succession as the incident particle's velocity decreases. The appropriate value of U can be interpolated from fig. 3.16. We therefore have enough information to use eq. (3.25) to calculate the range.

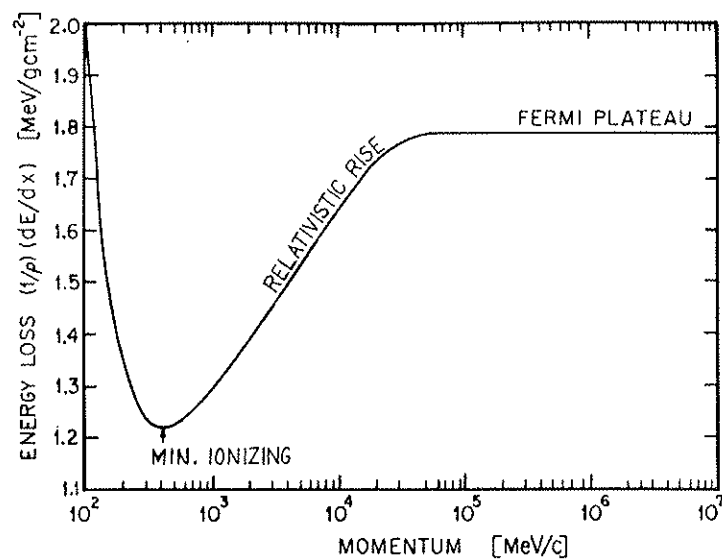


Fig. 3.15: Differential energy loss of a charged particle when passing through matter. (Taken from ref. [145]).

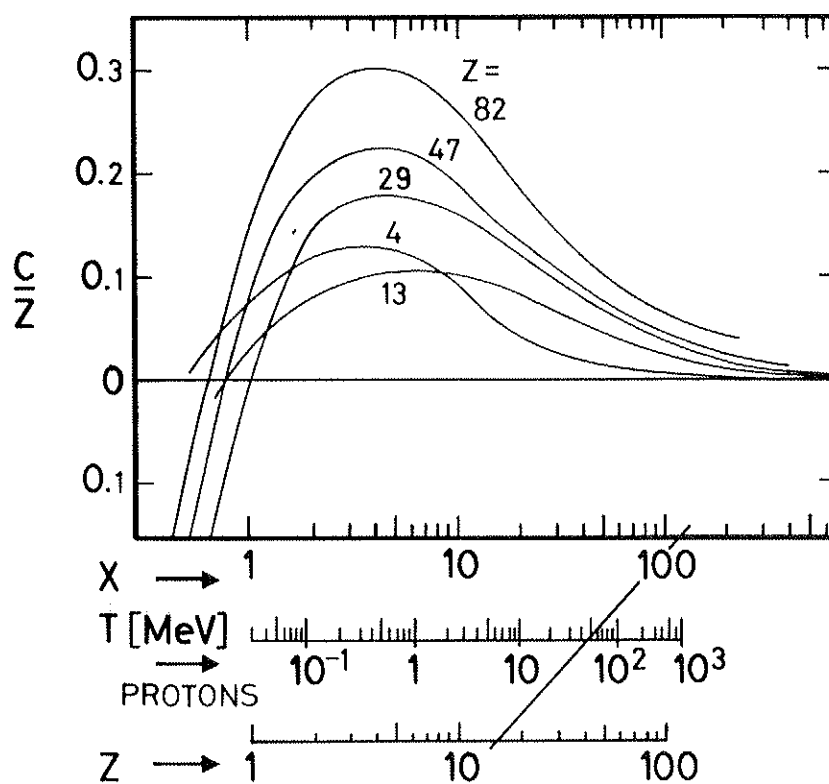


Fig. 3.16: The shell-correction C/Z to the Bethe-Bloch formula as a function of the parameter $x = \beta^2 137^2 / Z$, which can be bypassed by using the nomogram, where T is kinematic energy of protons. (Adapted from ref. [60]).

3.3.3 Liquid Hydrogen Target

The target and telescope configuration is shown in fig. 3.17 as it was arranged during the D83 run period. Depending on the beam momentum and the target length there has to be a small piece of moderating material inserted into the beamline. In other run periods either the beam momentum was lower or the target was longer so that any moderator was redundant. The thickness of the moderator "M", that was made from plexiglass, is determined by computing the rough range of the entering

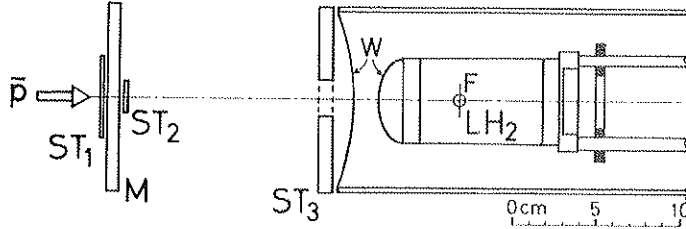


Fig. 3.17: Target-telescope setup of PS182 as it was arranged in the run period D83. Three telescope scintillators ST were present; ST₁ was to collimate the beam after possible scattering in the moderator M. This was, however, not a problem and subsequently only scintillator ST₂ was kept in later run periods. Windows W are made from mylar. F is the beam focus at the centre of the target volume of liquid hydrogen, LH₂.

$\bar{p}$ -beam without passing through any moderating material and comparing the range obtained with additional matter. The range is deduced by, firstly, expressing the total thickness of each type of material in g/cm², and then converting to carbon-equivalents by scaling with the ratio of the stopping powers [58]. One then obtains a hydrogen-equivalent total thickness t in g/cm² of the material before the target, as e.g. windows, scintillator, and its encapsulation. How deep d the antiprotons will penetrate the target hydrogen is then given by the difference

$$R_0 - t = d, \quad (3.27)$$

where R_0 is the range of 308 MeV/c antiprotons in liquid hydrogen. R_0 can be calculated by integrating eq. (3.25); T_0 is the kinetic energy upon incidence:

$$R_0(T) = \int_0^{T_0} \frac{\rho dT}{-dE/ds} = \rho \int_{T_0}^0 \frac{dT}{dE/ds} \sim m^{1-2/n} p^{2/n}, \quad (3.28)$$

where $n < 0.6$ for a nonrelativistic particle⁵. This procedure was performed in the Monte Carlo program mentioned below, which was used to simulate the annihilation region. For a quick estimate, however, the range is most easily looked up in a range diagram (see fig. 3.18). The range estimation resulted in a range of 4.8 cm in liquid hydrogen with 0.7 cm of plexiglass as a moderator.

The size of the annihilation region, i.e. the stopping region, is necessary to be known for determination of the resolution of the angle between two detected γ -rays. Because the detectors are placed in a plane perpendicular to the beamline the transverse beam dimensions are most important. These are caused by multiple scattering and the emittance of the external beam. The longitudinal dimension is not that crucial for the angular resolution, but it is nevertheless important that the stopping distribution is well contained in the hydrogen volume. The longitudinal extent is given by range straggling and the momentum spread of the incoming $\bar{p}$'s. The range straggling is a Gaussian distribution given by [145]

$$P(R)dR = \frac{1}{\sigma\sqrt{\pi}} e^{-(R-R_0)^2/\sigma^2} dR \quad (3.29)$$

with $\sigma^2 = 2 \int P(R)(R-R_0)^2 dR$. The straggling parameter S is defined as

⁵ See for instance the book by Ritson [124].

$$S^2 \equiv \frac{1}{4} \pi \sigma^2 = \frac{1}{2} (R - R_0)_{av}^2 = 2\pi^2 \alpha^2 (\hbar c)^2 N_A \frac{Z}{A} \int_0^{T_0} \left(\frac{1}{\rho} \frac{dE}{ds} \right) dT, \quad (3.30)$$

where α is the fine structure constant and N_A is Avogadro's number. The full-width at half-maximum γ of the range-straggling distribution can be deduced from the straggling parameter:

$$\gamma = 2S \sqrt{\frac{2}{\pi}} \ln 2 \quad (3.31)$$

By folding the range straggling γ with the momentum spread one obtains an approximate longitudinal width of the stopping distribution:

$$\Gamma^2 = \gamma^2 + \left(\frac{\Delta p}{p} \right)^2 R_0^2 \approx 1 \text{ cm}. \quad (3.32)$$

The range curve was measured in the beginning of the D83 run period with the setup shown in fig. 3.19. The target was for D83 only 10 cm long, thus the range in liquid hydrogen should not be longer than 5 cm. The target was replaced by the scintillator S_2 complemented by a copper plate as thick as needed to obtain the equivalent thickness to 5 cm of liquid hydrogen. The trigger condition required coinciding signals from ST_1 , ST_2 , and S_1 , but S_2 was coupled in anticoincidence. When varying the moderator thickness we observed the curve displayed in fig. 3.20. Assuming a beam momentum spread of 10^{-3} at 300 MeV/c the range straggling is unfolded to be

$$\gamma = 1.2 \text{ cm},$$

which is a perfectly acceptable longitudinal dimension of the annihilation region.

The transverse size of the stopping region is to a large extent governed by multiple scattering and cannot be estimated straight forwardly but has to be done in a cascade calculation. A Monte Carlo program was developed to simulate the blow-up of the beam before it came to a stop in the liquid hydrogen. The princi-

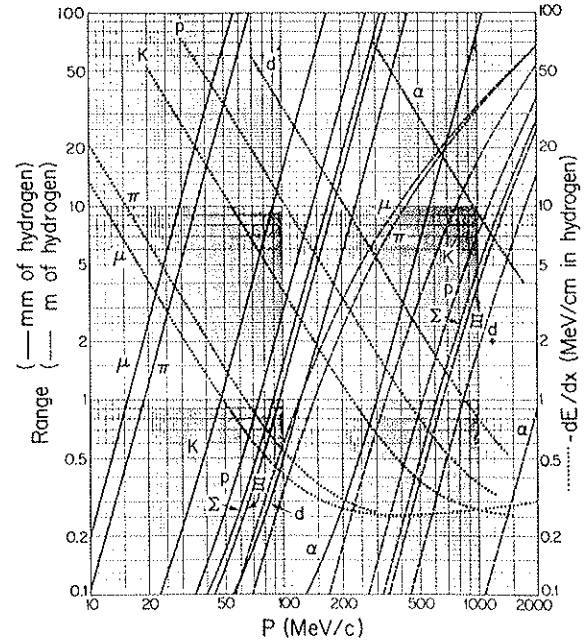


Fig. 3.18: Ranges of different particles in liquid hydrogen as a function of incident particle momentum p . (Taken from ref. [122]).

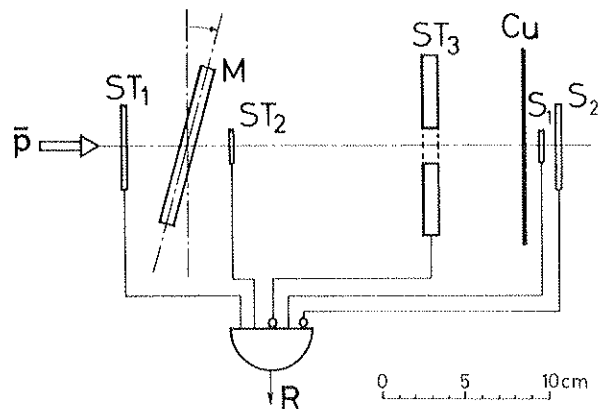


Fig. 3.19: Setup to measure the range curve. 5 cm of hydrogen is replaced by the equivalent thickness of copper and the scintillator S_1 . The signal R gives the rate of $\bar{p}$'s that reach S_2 , i.e. that would penetrate more than 5 cm of LH_2 . The range curve is obtained by measuring the rate $R/N_{\bar{p}}$ for different thicknesses of the moderator M . The thickness is varied by turning M as indicated.

ple of the program is to "pick" a $\bar{p}$ with a 3-momentum p_0 according to the beam specifications, let it successively pass through consecutive slabs of material, and after each step compute the energy loss and the scattered angle. Each slab is chosen thin enough that the formula (3.25) gives a good approximation for the average energy loss E_{av} . The actual energy loss E_{loss} for a particular case is then obtained by throwing a random number according to the appropriate Vavilov, Landau, or Gaussian distribution (cf. section 3.3.2). If p_i is the magnitude of the momentum of the $\bar{p}$ when it enters slab i , the momentum on exit will be

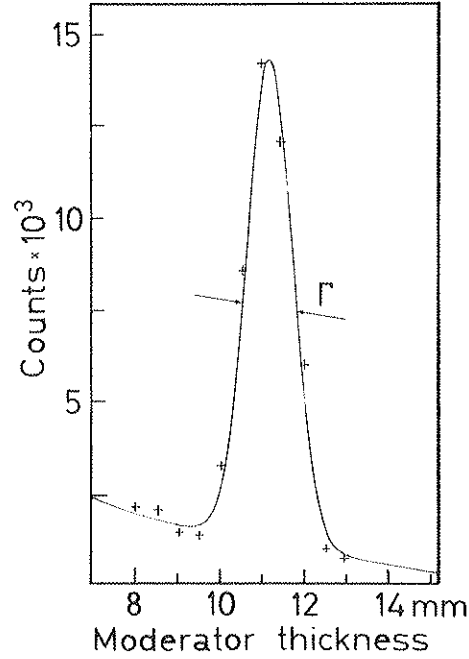


Fig. 3.20: Range curve obtained with the setup shown in fig. 3.19. The width Γ acquires contributions from both the range straggling and the momentum spread of the beam.

$$p_{l+1} = \left[p_l \left(p_l - \frac{2E_{loss}}{\beta_l} \right) - E_{loss}^2 \right]^{1/2}. \quad (3.33)$$

After penetration of the slab, also the direction $\hat{p}_l$ of the momentum is modified due to multiple scattering. The probability that the $\bar{p}$ is scattered an angle θ is given by the Scott-Snyder-Molière theory, and can be expressed as

$$P(\theta)d\theta = \frac{2\theta}{\langle\theta^2\rangle} e^{-\theta^2/\langle\theta^2\rangle} d\theta, \quad (3.34)$$

where $\langle\theta^2\rangle$ is the mean-square value of θ given by [20] [145]

$$\langle\theta^2\rangle = 2\theta_1^2 \ln \left(\frac{\theta_1}{\theta_{min}} \right) = \theta_1^2 \ln \left[4\pi Z^{4/3} \frac{N_A \rho}{A} s \left(\frac{\hbar}{m\beta c} \right)^2 \right]. \quad (3.35)$$

Here θ_{min} is the minimum angle of scattering in a single encounter, $\theta_{min} \cong \lambda/a$, where λ is the de Broglie wavelength of the particle and a is the radius of the atom of the scatterer, $a \sim a_H Z^{-1/3}$. The angle θ_1 is the angle for which there is, on the average, only one collision with $\theta > \theta_1$ throughout the distance s :

$$\theta_1^2 = 4\pi\alpha^2(\hbar c)^2 s N_A \rho \frac{Z(Z+1)}{A} \cdot \frac{1}{(p\beta c)^2}. \quad (3.36)$$

A random number θ is thrown according to the distribution (3.34) to obtain the angular deviation from the incoming trajectory to the outgoing trajectory (see fig. 3.21). The azimuthal angle around the incoming trajectory is taken uniformly.

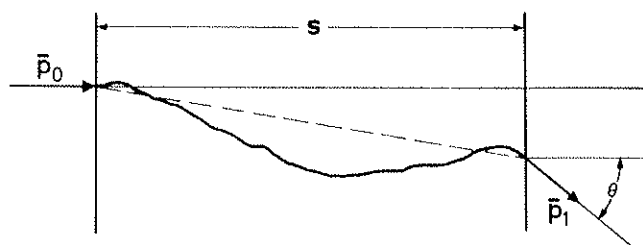


Fig. 3.21: A particle penetrating matter is scattered by Coulomb interactions with the electrons of the medium and will, after a large number of relatively small scattering angles, be deflected an angle θ depending on the thickness s of the penetrated material.

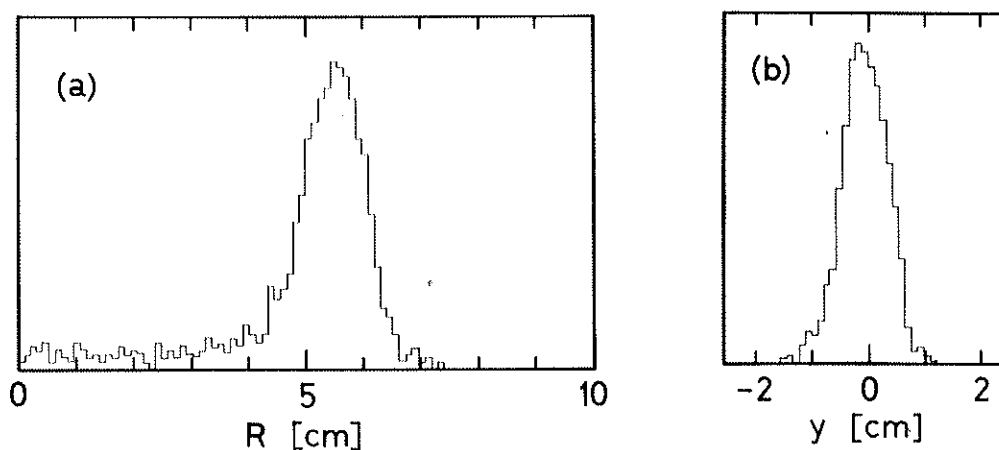


Fig. 3.22: The stopping distribution of the $\bar{p}$ -beam in the liquid hydrogen target as simulated with Monte Carlo techniques for the run period D83. The longitudinal projection (a) shows the tail towards the entrance window of the target produced by anti-protons annihilating in flight. The transverse projection (b) shows that the annihilation region is well confined in the hydrogen volume.

Neanderthalaren är den äldste av alla talare
och borde vara ett föredöme för alla andra
för att han har hållit mun i 500 000 år.

(Cello)

Because energy is lost continuously, the slab thickness has to shrink along the particle path. So as not to run into a disastrous loop of "infinitesimal" dimensions a cut-off has to be introduced. Thus, the $\bar{p}$ is put to a stop, uniformly chosen, within the next slab that is $100\ \mu\text{m}$ thin when its kinetic energy T is only $720\ \text{keV}$. This cut-off is chosen because the range of a $720\ \text{keV}$ $\bar{p}$ is just $100\ \mu\text{m}$ in liquid hydrogen, and $100\ \mu\text{m}$ is the accuracy that corresponds to that required in this simulation.

The result of the simulation for a beam momentum of $300\ \text{MeV}/c$ is shown in fig. 3.22a–b. We see that the range straggling is in well agreement with the measured range curve. However, the calculated average range is slightly larger than that measured. This result is in fact not surprising as the simulation approximates the integral [eq. (3.28)] of the Bethe-Bloch formula with a series of linear approximations, i.e. in each step i the energy loss is computed as if the antiproton preserved the same initial energy E_i throughout the whole slab, whereas in reality it loses energy continuously. This has an impact on the multiple scattering as well; one can anticipate a slightly narrower stopping region than is shown in fig. 3.22b. The simulation approximates a $\bar{p}$ -track with a polyline as illustrated in fig. 3.23 where the kinks are on the actual track. To estimate the correction to the width of the stopping distribution we scale down each slab thickness according to the ratio between the simulated range and the measured one, and recalculate the multiple-scattering angles θ_i for each step i . The extreme trajectory T_{ex} is then replaced by the corrected trajectory T_{cex} that produces an annihilation spot that has a full-width at half-maximum of about $8.5\ \text{mm}$, which is roughly 9% wider than the measured beam cross-section.

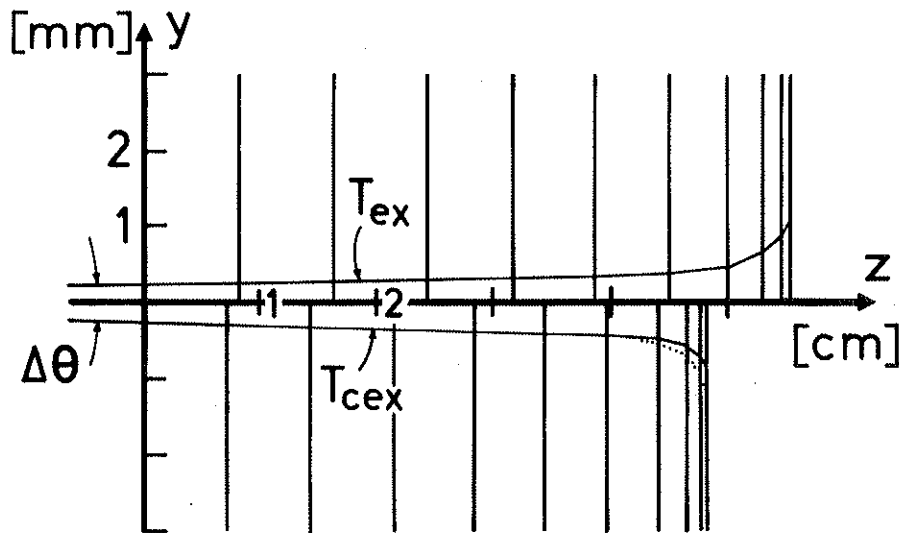


Fig. 3.23: The simulated $\bar{p}$ trajectory T_{ex} obtained for maximum deflection in each slab of liquid hydrogen. When the slab thicknesses are scaled down according to the measured range this trajectory is modified to T_{cex} which yields a slightly narrower stopping distribution.

3.4. THE BGO CALORIMETERS

A detector is generally called a "calorimeter" if it is used to determine the energy of a particle by total absorption. Any quantity that is proportional to the absorbed energy may be measured. Such quantities are for instance scintillation light, Cherenkov light, and ionization. Scintillation calorimeters are usually blocks of transparent material in which the energy of a particle is converted and degraded to detectable atomic ionizations and excitations. For one of the detectors in this experiment these blocks were crystals grown from bismuth germanium oxide $\text{Bi}_4\text{Ge}_3\text{O}_{12}$ (or simply BGO). BGO was first grown in small crystals by Nitsche [111] in 1965 as a replacement for bismuth silicon enlytite $\text{Bi}_4\text{Si}_3\text{O}_{12}$, a rare natural mineral, in laser host material. The luminescence properties were studied in detail by Weber and Monchamp [143]. Small crystals have been used extensively for some time in nuclear medicine, particular in computer tomography where its short radiation length allows an improvement of the spatial resolution as compared to NaI(Tl) . The PS182 collaboration was one of the first groups in high energy physics that studied the use of large BGO crystals for γ -calorimeters [116]. In the first subsection 3.4.1 some general properties are discussed of BGO as a detector material in high-energy physics experiments.

Before the BGO detectors were operated as γ -calorimeters in this experiment they were tested with electrons of energies from 50 to 450 MeV at the SIN⁶ cyclotron. Results from this test run will be mentioned together with the results obtained during experimental runs. The test results are thus presented in their relevant context.

In subsection 3.4.2 the geometry of the BGO detectors used in this experiment is described, and subsection 3.4.3 deals in detail with the performance and the relevant parameters of the detectors.

3.4.1 BGO As Detector Material

The properties of BGO are well documented elsewhere [143] [50] [89]. Here only its main features are summarized and compared with NaI(Tl) , the material with the most understood energy resolution. The properties are collected in table 3.3. In contrast to NaI(Tl) BGO is nonhygroscopic and is less delicate to handle due to its mechanical and thermal stability. It is chemically stable against most cleaning agents and is soluble in HCl. BGO is a cubic crystal grown from a melt of roughly 1050°C (fig. 3.24). It can be grown in crystalline form from two different stoichiometric mixtures [143] in the ratios 2:3 or 6:1 of Bi_2O_3 and GeO_2 . The fluorescence of $\text{Bi}_{12}\text{GeO}_{20}$ is, however, very poor, and it has found no application in γ -spectroscopy. When grown from highly pure materials BGO is completely transparent. For small impurity admixtures the crystals have a faint yellowish colour and tend to have small voids, which are caused by ions of significantly different size than Bi or Ge ions. The crystals can easily be machined and polished.

Fig. 3.25 shows a compilation of some optical data for BGO. The fluorescence is attributed to the $^3\text{P}_1 \rightarrow ^1\text{S}_0$ transition of the Bi_3^+ ion (fig. 3.26). Due to the large Stokes' shift⁷ the fluorescence occurs

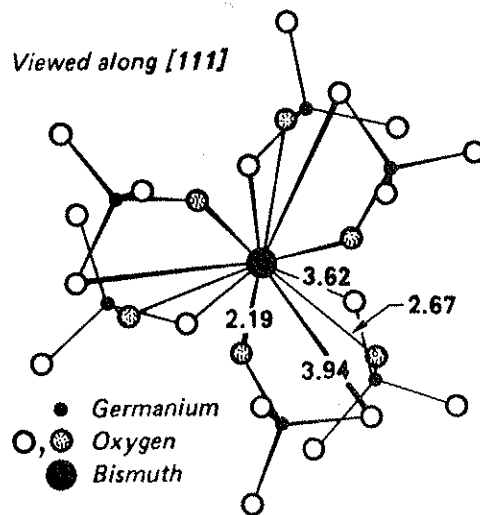


Fig. 3.24: Crystal structure of BGO. (Taken from ref. [143]).

⁶ Schweizerische Institut für Nuklearforschung, Villigen, Switzerland.

⁷ This energy shift is due to the interaction with the vibrational modes of the crystal. It can be considered as the photon emits or absorbs a phonon quantum. For more details see for instance the book by Kittel [88] or the one by Omar [112].

in a region where the crystal is fully transparent. Because there is no overlap between the tails of the excitation and fluorescence spectra (fig. 3.25a) partial self absorption is absent. Thus, one can expect

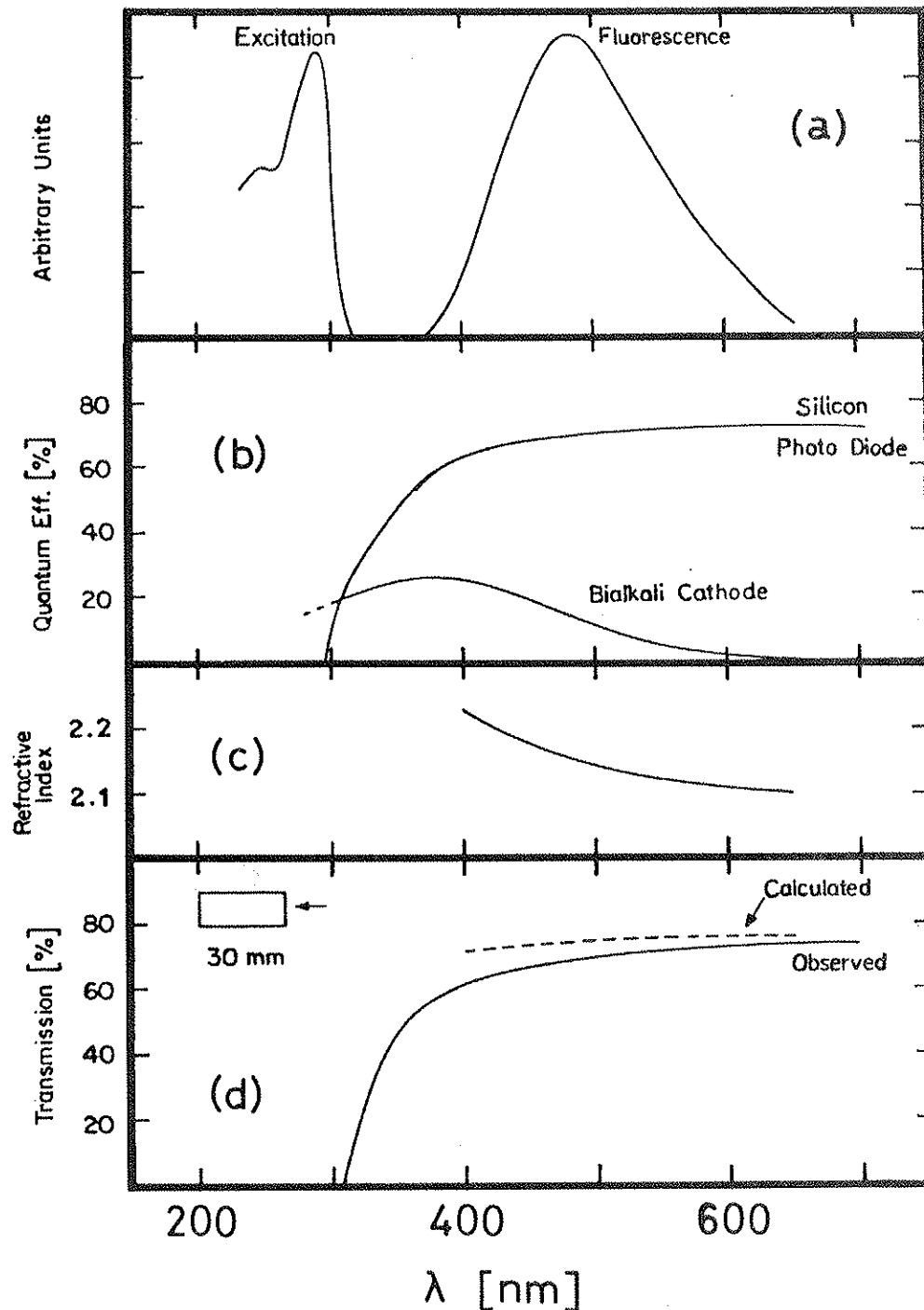


Fig. 3.25: Optical properties of BGO: (a) excitation and fluorescence spectra compared to the quantum efficiency (b) of a photo cathode and a photo diode, (c) the refractive index of BGO, and (d) transmission. (Adapted from ref. [50]).

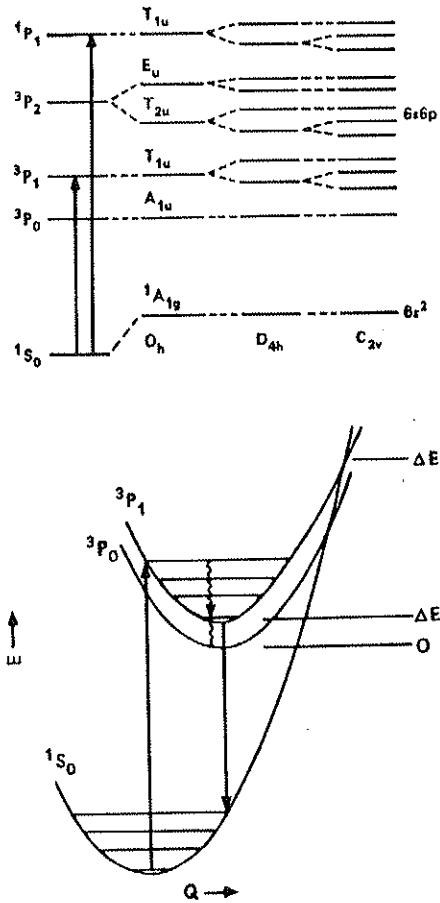


Fig. 3.26: Relaxation of the Bi^{3+} -ion in BGO. The energy levels of Bi^{3+} (left-hand side of top view) are split by the influence of the crystal field (right-hand side of top view); the $(2J+1)$ -fold degeneracy is reduced. The luminescence is due to the radiative transition from the $^3\text{P}_1$ state to the $^1\text{S}_0$ state. The $^3\text{P}_1$ state is thermally populated if the energy difference $\Delta E = E(^3\text{P}_1) - E(^3\text{P}_0) < kT$. The $^3\text{P}_0$ state is reached via nonradiative decays (wavy symbol) from the $^3\text{P}_1$ or $^1\text{P}_1$ states excited by optical transitions. Notice the crossover energy $\Delta E'$ that enables nonradiative relaxation at high temperatures. Notice also the Stokes shift between absorption and emission wave lengths. (Taken from ref. [143]).

Table 3.3: Properties of BGO. (Taken from ref. [50]).

Density	7.13	[g/cm ³]
Eff. atomic number	74	
Rad. length	1.12	[cm]
Hygroscopic	no	
Mech. stab.	good	
Emission peak	480	[nm]
Fluor. dec. time	350	[ns]
Refr. index	2.15	
$dE/dx(\text{min})$	9.2	[MeV/cm]

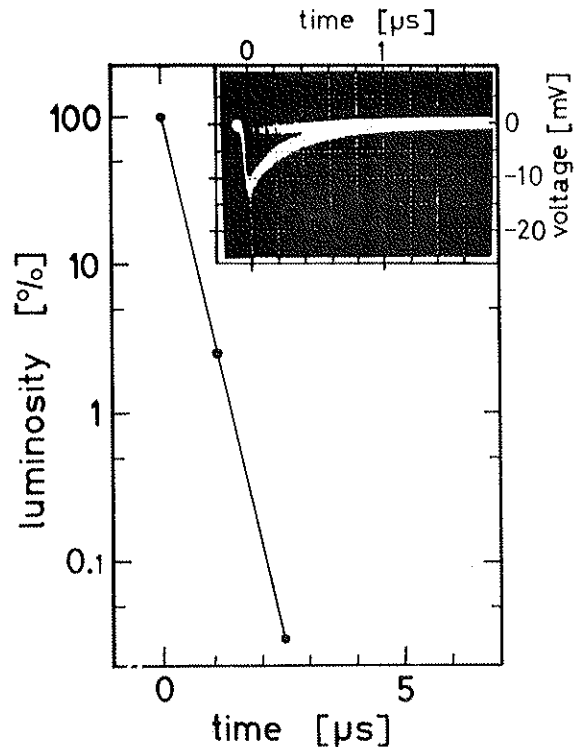


Fig. 3.27: The light output from BGO as function of time after X-ray source is switched of. (Adapted from ref. [73]). The inset shows the oscilloscope pulse obtained with one of the BGO crystals used in PS182.

high uniformity in crystal response along its length. This question will be further discussed in section 3.4.3.2. For comparison fig. 3.25b shows the typical quantum efficiency of a photomultiplier cathode. A disadvantage of BGO compared to NaI(Tl) is that its scintillation conversion efficiency, or "sensitivity", is not more than 10% of the sensitivity of NaI(Tl). It is, on the other hand, much more stable against radiation damage than NaI(Tl). After an exposure of as much as 1500 R the material recovers within a few hours [137].

The fluorescence decay time of BGO is approximately 350 ns at 20°C (fig. 3.27) and has a strong temperature dependence [50] of $\alpha(20^\circ) = -4 \text{ ns/dg}$ (fig. 3.28). The inset in fig. 3.27 shows the BGO pulse as measured with the setup used in the experiment PS182. The light output per unit energy also shows a strong temperature dependence [50]: linear between 14–50°C with 1%/dg normalized to 20°C (fig. 3.28).

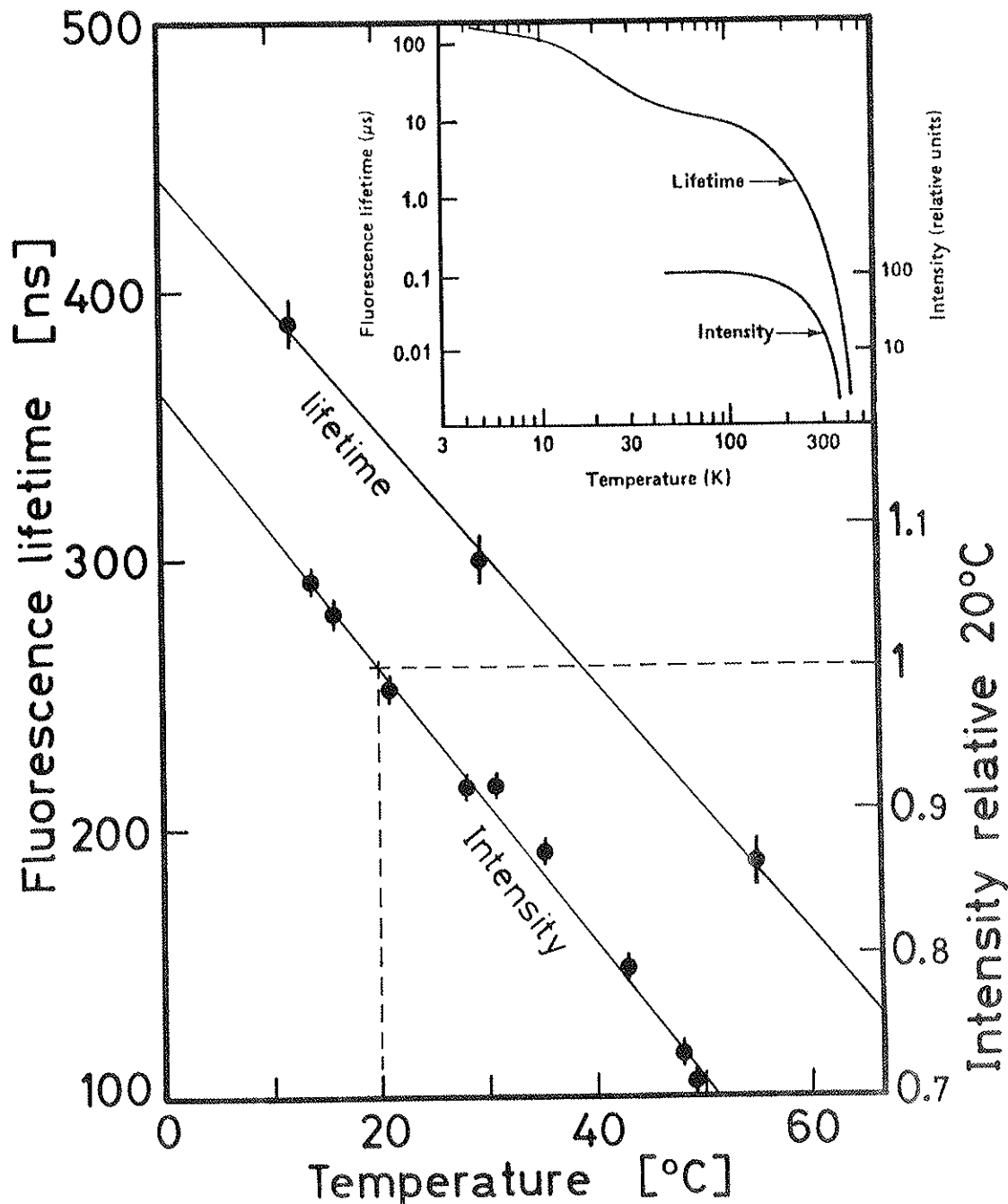


Fig. 3.28: Temperature dependence of the light constant and the light output from BGO. (Adapted from ref. [50], inset adapted from ref. [143]).

BGO is a primary scintillator and its light output increases with cooling. NaI(Tl) shows the inverse effect. For NaI(Tl) the scintillation mechanism is much more complicated and depends strongly on the activation concentration. As a consequence of the strong temperature dependence of BGO a stabilisation of the calorimeter temperature to within 1°C and a precise calibration system would be desirable to make full use of the high energy resolution of the material⁸.

A Monte Carlo simulation using the SLAC-EGS program [64] has been performed by Dietl et al. [50] to define the needed specifications of a BGO γ -calorimeter. Cross-sections for BGO are shown in fig. 3.29. Some results from this simulation are represented in fig. 3.30a – c. Fig. 3.30a shows the longitudinal energy deposition for various incident energies. At least 20 radiation lengths (X_0) are necessary to absorb 98% of the shower for energies up to 1 GeV. Fig. 3.30b shows the fraction of front and rear leakage for a $20X_0$ long BGO crystal of infinite width. Beside material inhomogeneity the fluctuations of these leakages will finally determine the resolution. Fig. 3.30c shows the fraction of contained shower energy as a function of the crystal width for a crystal with square cross-section.

3.4.2 Geometry

The BGO detectors used in this experiment consist each of a set of seven crystals with the shape of hexagonal prisms. These crystal elements⁹ are 20.3 cm ($\sim 20X_0$) long and 5.4 cm between parallel faces (see fig. 3.31). Each set is equipped with a hodoscope consisting of seven small BGO bars of dimensions $1.0 \times 1.2 \times 5.0 \text{ cm}^3$ placed in front of the hexagonal crystals. The crystals are covered with $100 \mu\text{m}$ thick aluminum foil. The scintillation light is detected by Hamamatsu

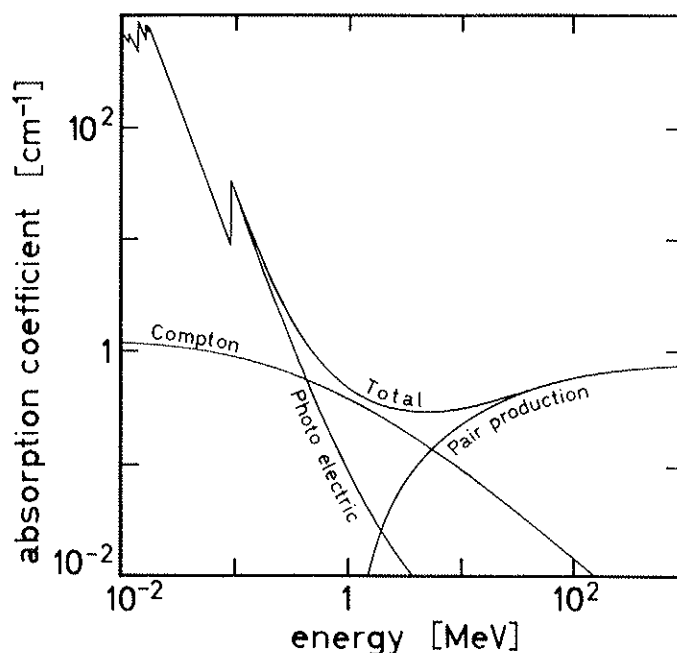


Fig. 3.29: BGO light absorption coefficients energy. (Adapted from ref. [73]).

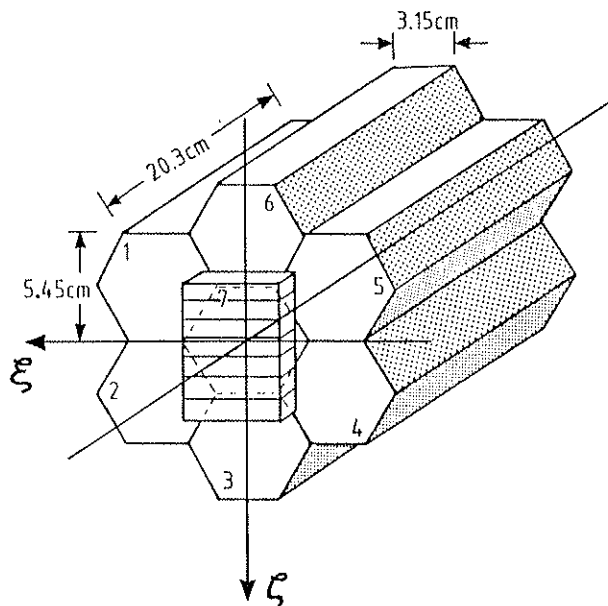


Fig. 3.31: View and dimensions of the BGO-detector system used in PS182. The front hodoscope consists of seven BGO bars of dimensions $5.0 \times 1.0 \times 1.2 \text{ cm}^3$.

⁸ The calibration of the crystals in PS182 could be reproduced every ten minutes, which made the temperature stabilization less crucial.

⁹ Manufactured by Harshaw Chemical Co., Solon, Ohio, USA.

photomultipliers (type R 1306-01) glued onto the rear surface of the crystals. The BGO assemblies are housed in plastic boxes designed as a shield against rapid temperature variations in the experimental hall; as mentioned above BGO has a large temperature coefficient for light production.

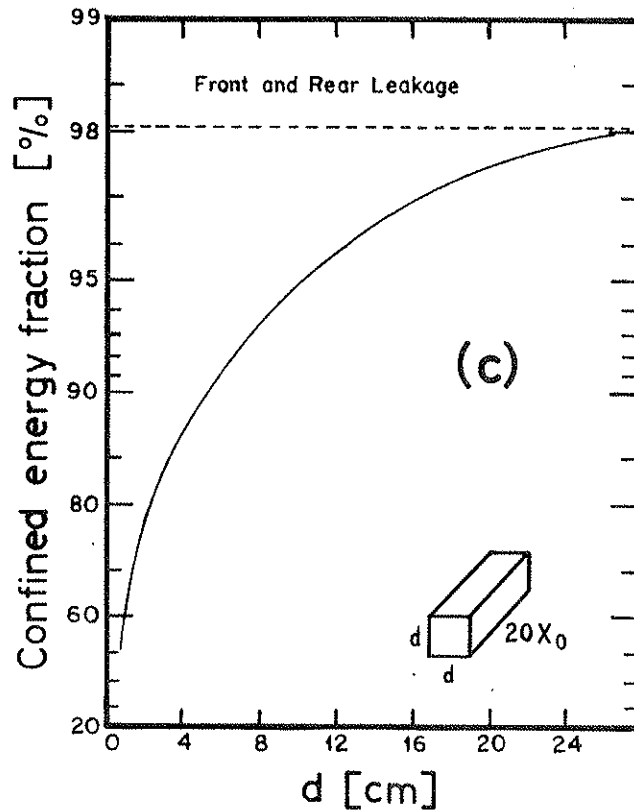
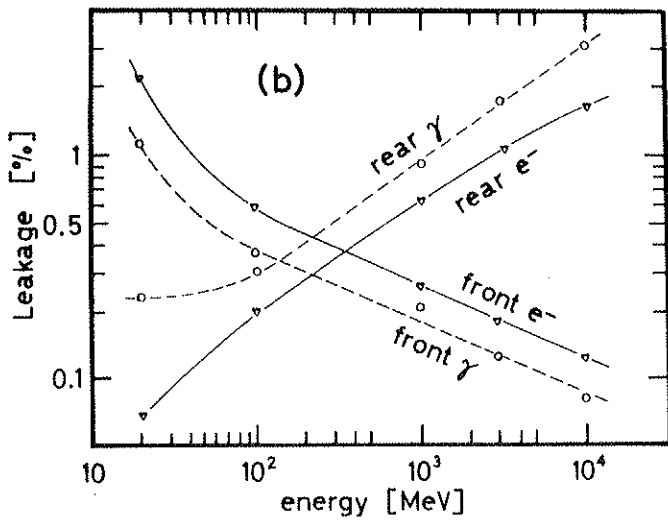
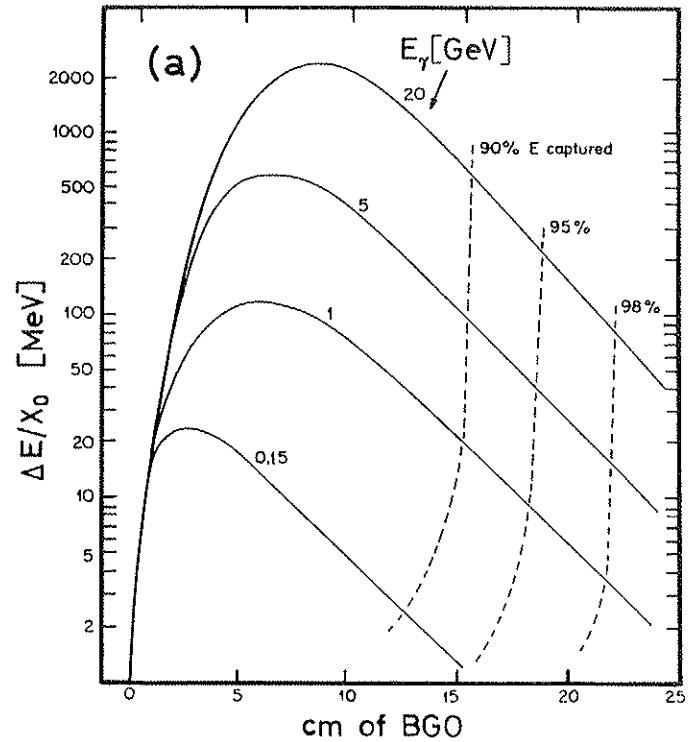


Fig. 3.30: Results from a Monte Carlo simulation by Dietl et al.: (a) differential energy absorption $\Delta E/X_0$ as a function of crystal thickness, (b) rear and front leakages as a function of impact energy, and (c) fraction of shower energy contained in a crystal with square cross-section of width d and 20 radiation lengths long. (Adapted from ref. [50]).

3.4.3 Performance of the System

The performance of a calorimeter, which generally improves with increasing energy, is limited both by statistical fluctuations of the elementary phenomena through which the energy is converted and degraded to the level of detectable atomic ionizations and excitations, and by the technique chosen to measure the final products of the cascade process. When a high energy photon or electron enters the calorimeter it initiates a multiplicative process of bremsstrahlung and pair production. This cascade phenomenon is called an electromagnetic shower (see Appendix B). In this experiment we want to use the calorimeter to measure the energy of photons and the spatial coordinates of their impact. Of interest are then the energy resolution of the detector and the accuracy with which the impact position can be reconstructed. Of course also the detection efficiency is of interest, which is a function of the impact position. In this subsection the properties of the BGO calorimeters used in this experiment will be discussed in detail, as well as the procedures used to investigate them. There is a sub-subsection devoted to each property. The subject here is confined to those functions that concern the actual detector, and the treatment of functions that depend on electronics or software manipulations will be left until the next chapter.

Several results concerning the performance of our BGO detectors were obtained from the test run at SIN. This test measurement was conducted at the π M3-beam which has a momentum spread of less than 1%. During the measurements in this beam contaminating muons and pions were vetoed by a time-of-flight condition. The BGO detector was generally placed with the beam along the symmetry axis of the central crystal. A $1 \times 1 \text{ cm}^2$ trigger scintillator defined the impact position of the electrons. Sometimes the detector was displaced or tilted relative to the beam axis in order to see the influence of this on the energy resolution. Before discussing energy resolution, impact-position reconstruction, and efficiency the first sub-subsection concerns the energy response function and procedures for calibrating the crystals.

3.4.3.1 Linearity And Calibration

It is desirable that the energy response function of the detector is linear to facilitate absolute calibration and data analysis. This means that the charge of the photomultiplier pulse is directly proportional to the energy deposited in the crystal. This so called "linearity" was checked with the electron beam at SIN. By adjusting the beam for various electron momenta ($\sim$ energies), the charge output can be plotted versus deposited energy. This is done in fig. 3.32. The deviation from a linear behaviour, expressed by the ratio $|\sqrt{a_2/a_1}|$ after having fitted the second-order polynomial $E'(E) = a_0 + a_1 E + a_2 E^2$, amounts to $|\sqrt{a_2/a_1}| = 4.5 \cdot 10^{-3}$.

During the test run at SIN the seven crystals of a BGO set were calibrated by using the signals from stopped muons and pions of 300 MeV/c momentum, which correspond to the deposited amounts of energy¹⁰ 212 MeV and 191 MeV, respectively. This method, however, could not be used during the experiment at LEAR. Instead different methods were used [130]. A primary calibration of the crystals was done with a ^{228}Th source before each run period. The observed peaks from the 2.6 MeV line (fig. 3.33) were matched by adjusting the high-voltage of the photomultipliers. This rough

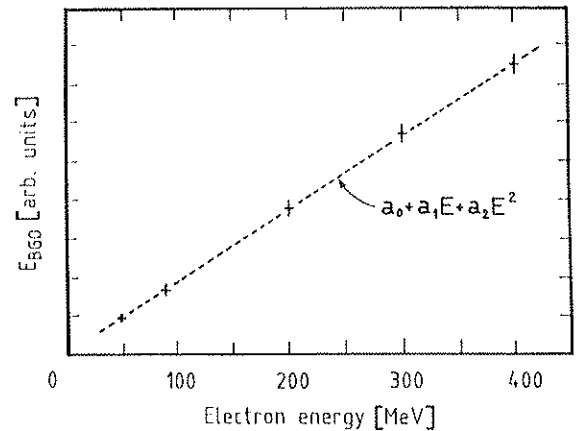


Fig. 3.32: Response E_{BGO} of a BGO calorimeter to a monochromatic electron beam versus beam momentum.

¹⁰ The energy released by the secondary electron from the muon decay is neglected due to the long lifetime of the muon thus the ADC gate was set to 600 ns. The energy released by the secondary muon from the pion decay yields a tail towards higher energies due to a different path length in the crystal.

calibration was accurate to the order of 10%. One method used for the final calibration employed charged pions that penetrated the full length of the BGO crystals. Such pions were continuously monitored throughout the experimental runs. The corresponding events were defined by simultaneous signals from the plastic scintillators in front of and behind the BGO set (see fig. 3.4), and energy deposited in the BGO. Only those events where only one single BGO crystal contained energy were selected in an offline analysis. The energy released by these pions gives a peak at 198 MeV in the spectrum from a BGO hexagon (fig. 3.34). These peaks were used for calibration and correction of the calibration time drifts. For a LEAR run period, that typically extends over a few days, the calibration of a crystal drifted not more than 0.5%. The inclusive γ -spectra from each individual crystal was also used for calibration by fitting a parabolic expression to the high-energy part of the spectra, and adjusting the end points to one another. This method was consistent with the one employing charged pions but less accurate (see further section 4.1.1.1).

When the BGO hodoscope was used in front of the hexagonal crystals this had also to be calibrated. The hodoscope bars, as the hexagonal crystals, were roughly calibrated before the run periods with the ^{228}Th source (fig. 3.35). The final calibration for the hodoscope was also based on the peaks observed from the through-going charged pions (fig. 3.36). Because the peaks are dominated by pions around 400 MeV/c momentum (i.e. where the pion spectrum from $p\bar{p}$ annihilations at rest peaks), their position were adjusted to 10.7 MeV [eq. (3.25)]. An accuracy of about 10% could be achieved.

A bad calibration has the effect of deteriorating the energy resolution of the detector. In fact the energy resolution achieved during experimental runs was never as good as it was during the test run at SIN. This might seem strange, since the charged pion method is very accurate and efficient, and it can be frequently repeated. The reason for this will be further discussed in the following section.

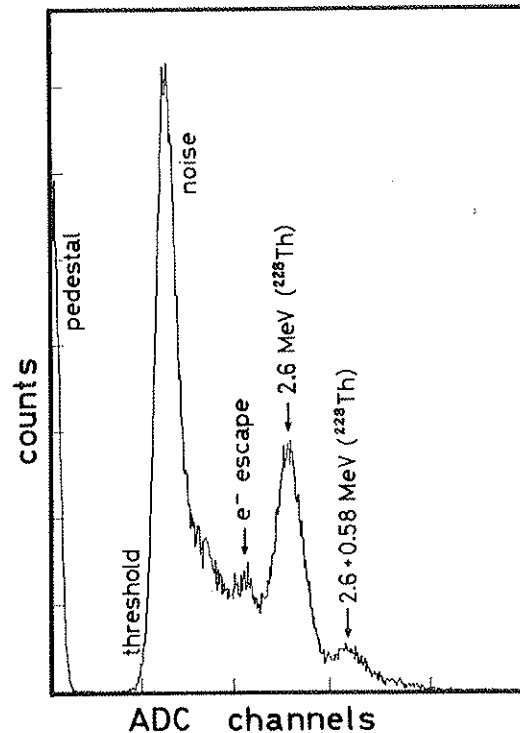


Fig. 3.33: Spectrum from a ^{228}Th source as measured with one of the hexagonal BGO crystals.

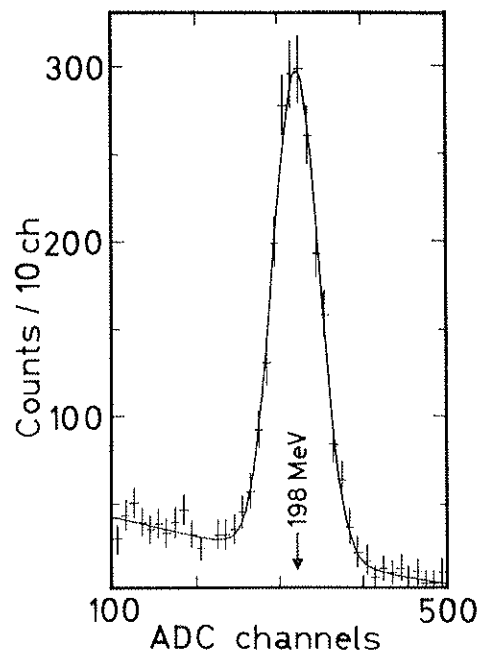


Fig. 3.34: Spectrum obtained from charged pions that passed through one of the hexagonal BGO crystals.

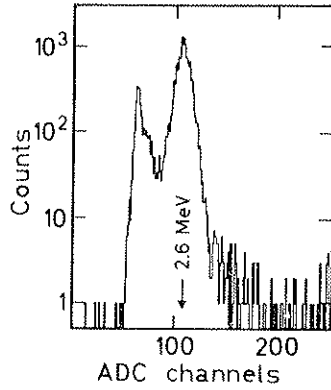


Fig. 3.35: Spectrum from a ^{228}Th source as measured with a BGO hodoscope bar.

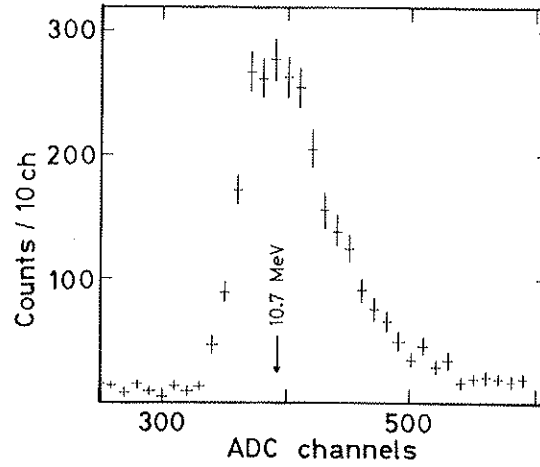


Fig. 3.36: Spectrum obtained from charged pions that passed through a BGO hodoscope bar.

3.4.3.2 Energy Resolution

The energy resolution in the energy region below 3 MeV was investigated with γ -rays from the radio-active sources ^{203}Hg , ^{22}Na , ^{137}Cs , ^{88}Y , and ^{228}Th . The source was attached to the front face of the crystal. The result, after having quadratically subtracted the amplifier noise (~ 200 keV), is shown in fig. 3.37a. The energy dependence is fitted by the expression

$$\frac{\Gamma}{E} = \frac{0.49\%}{\sqrt{E [\text{GeV}]}} \quad (3.37)$$

where Γ is the full-width at half-maximum. In this low-energy domain the resolution is limited by statistical fluctuations in the number of photo-electrons. The number $n = m/\mu$ of photo-electrons per MeV that reach the first dynode can be deduced if we assume an energy μ for which the total number of photo-electrons $m \gg 15$. In that case the Poisson distribution $Po(m)$ of the photo-electrons can be approximated by a Gaussian $G(\mu, \sigma)$ with $m = \mu^2/\sigma^2$. We then have that the number of photo-electrons reaching the first dynode is $n = 230 \text{ MeV}^{-1}$. If we assume 10% transmission efficiency of the BGO crystal and 25% quantum efficiency of the photo cathode of the photomultiplier, we get an estimated number of scintillation photons primarily emitted of about 5000 per MeV of shower energy. Fig. 3.37a also displays the resolution at higher energies. There the resolution was determined as a function of the energy of the electron beam at SIN. At higher energies the energy dependence is fitted by the expression

$$\frac{\Gamma}{E} = \frac{2.65\%}{\sqrt[4]{E [\text{GeV}]}} \quad (3.38)$$

where Γ again is the full-width at half-maximum. The resolution here is dominated by shower leakage outside the active detector volume.

Common sense is the most fairly distributed thing in the world, for each one thinks he is so well-endowed with it that even those who are hardest to satisfy in all other matters are not in the habit of desiring more of it than they already have.

(René Descartes, 1596–1650)

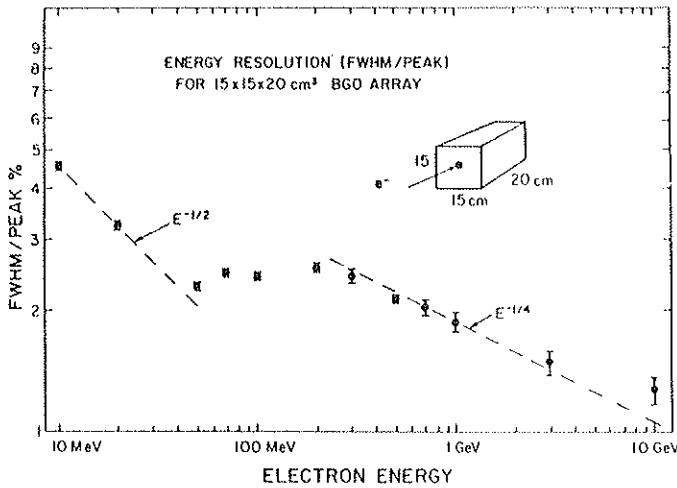
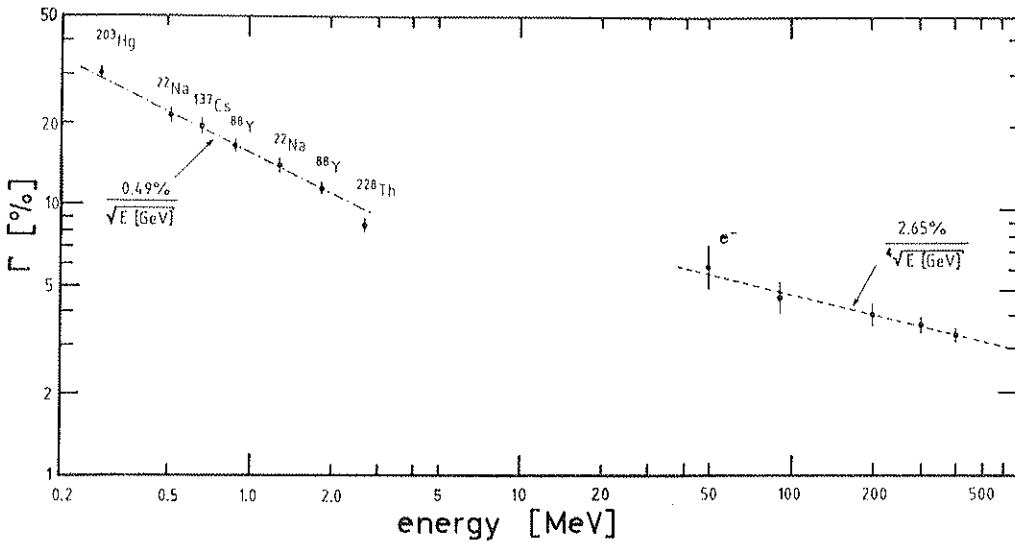


Fig. 3.37: Energy resolution full-width at half-maximum Γ of a BGO calorimeter (a) measured at high energies with an electron beam and at low energies with the indicated sources. The simulated resolution (b) for a crystal volume as indicated shows the similar dependence. ((b) adapted from ref. [35]).

Fig. 3.37b is a Monte Carlo simulation by Cavalli using the SLAC-EGS program [35]. To find a one-over-fourth-root law valid for BGO as has been found to fit NaI(Tl) data, might seem somewhat astonishing, since this behaviour was believed to be caused by the complicated scintillation mechanism present in NaI(Tl). It now rather seems to be connected to the occurrence of leakage. Other groups [35] have found the one-over-square-root law valid up to 11 MeV. It certainly looks like the energy resolution becomes worse again somewhere between 11 and 30 MeV before it decreases and flattens out. In fact, a similar energy dependence has been observed for NaI(Tl) [32]. The behaviour can be explained [32] by employing an expression for the energy resolution of a single crystal of the type (E in MeV):

$$\left(\frac{\Gamma}{E}\right)^2 = c_0 + c_1 E^{-1} + c_2 E^{-2}, \quad (3.39)$$

where the respective terms can be attributed to the following effects:

- *constant term*: systematic fluctuations in the response of each crystal can occur in several ways [22] [27], primarily associated with efficiency in the conversion to photo electrons due to, for example, nonuniformities in the scintillation yield [119] throughout the crystal volume, or nonisotropies of the optical efficiency of the cavity, or nonuniformities in the conversion efficiency of the photo cathode or the capture efficiency of the first dynode;

- *second term*: mainly due to statistical fluctuations in the number of photons generated as discussed earlier;
- *third term*: reflects the fixed channel width of the ADC and the occurrence of electronic noise.

The fact that shower leakage plays a more dramatic role at energies around 200 MeV than around 450 MeV seems likely from comparing the lineshape at different energies (fig. 3.38). The tail towards zero is clearly more pronounced at the lower energy. This circumstance is also understandable when recalling that the shower is relatively broader at lower energies (cf. Appendix B).

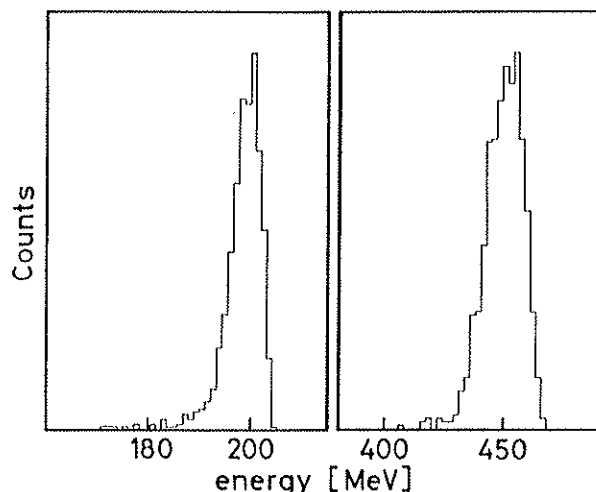


Fig. 3.38: Observed line shapes from a BGO calorimeter for (a) 200 MeV electrons and (b) for 450 MeV electrons. The tail towards zero seems to be relatively more pronounced at lower energies, indicating that fluctuations play a more dramatic role.

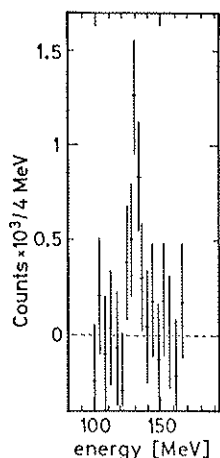


Fig. 3.39: The signal in the inclusive photon energy spectrum from the Panofski reaction $\pi^- p \rightarrow \gamma n$, measured with a BGO calorimeter reveals the detector resolution at low photon energies.

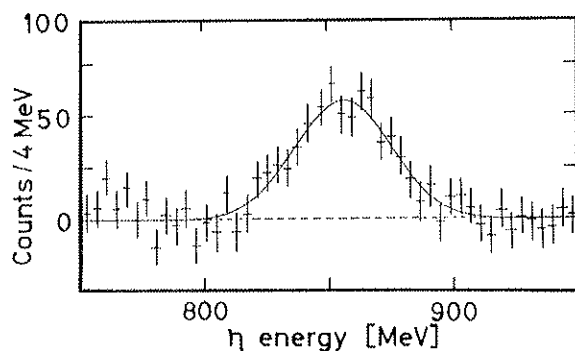


Fig. 3.40: The signal in the inclusive η -energy spectrum from the reaction $p\bar{p} \rightarrow \eta\omega$, is utilized to deduce the energy resolution of a BGO calorimeter at high energies.

The resolution achieved during the experimental runs at LEAR can be deduced from the width of a peak from a reaction appearing in the γ -spectrum. Because the resolution obtained during these runs is caused by γ -showers, whereas the resolution measured during the SIN test run was caused by electron showers, there is no a priori reason why they should be the same. Also the run conditions were somewhat different as well as the calibration method employed. There are two spectral peaks available for deduction of the energy resolution. In the γ -spectrum one finds the Panofski peak at 129 MeV (fig. 3.39) coming from the secondary reaction $\pi^- p \rightarrow \gamma n$ that arises from π^- -mesons produced in the $p\bar{p}$ annihilations. The width of the peak is about 9 MeV indicating a resolution at 1 GeV of as much as 4% full-width at half-maximum. This value, however, is suspected to be somewhat too large as these reactions occur away from the target centre. The other peak that can be used is a signal in the energy spectrum from measured η -mesons (fig. 3.40). The signal stems from the two-body reaction $p\bar{p} \rightarrow \eta\omega$. If no kinematic fitting is performed the peak width directly reflects the energy resolution for detected photons. Events used for the spectrum in fig. 3.40 are the most ($x \leq 20\%$) symmetric η -decays [eq. (3.11)]. In this case the photon energy is half of the η -energy. The energy resolution full-width at half-maximum is thus the width of the peak quadratically subtracted with the contribution from the other photon, which was measured with the lead-glass detector (see next section 3.5). As a result the energy resolution at 428 MeV is 18 MeV which indicates a resolution of 3% at 1 GeV according to eq. (3.38).

Several reasons can be imagined for the energy resolution to be worse during the experimental LEAR runs. One difference to the SIN test run is the method of calibration. During the test run the calibration was accomplished by using stopped muons and pions, while through-going charged pions were utilized during the experimental runs. As they are through-going they release energy throughout the whole crystal contrary to the case if they stop in the crystal. These charged particles do not produce an electromagnetic shower as photons and electrons, but release most of their energy through ionization and excitation in the immediate vicinity of their track. This means the energy is deposited in a narrower region than for showers. The efficiency factor used to convert energy deposited in the crystal to number of photo electrons hitting the first dynode of the photomultiplier must be a function of the position in the crystal where the energy is deposited. The position dependence arises, e.g., from the nonuniformity of scintillation efficiency and transparency of the crystal. Because about 90% of the energy of an electromagnetic shower is deposited in the first 10 radiation lengths of the crystal, one would expect the calibration method that uses stopped charged particles to give a better result as it, so to say, calibrates the relevant part of the crystal.

Nonuniformities should be mainly longitudinal, but also these can be expected to cause small effects since there is no self-absorption present as already mentioned. Furthermore, due to the very high refractive index of BGO, one does not anticipate variations from different reflection modes in the crystal cavity. The largest contribution to longitudinal nonuniformity can thus be expected from local deficiencies in transparency. If there is a local region of inferior transparency one would anticipate a nonuniformity curve that is constant in the beginning of the crystal and then suddenly rises and becomes constant again in the latter part of the crystal closer to the photomultiplier (cf. ref. [116]). The BGO crystals were optically compensated to give a longitudinally uniform light output. According to Harshaw the compensation was accomplished by partially wrapping with optically diffuse paper — the crystals were delivered encapsulated and equipped with photomultipliers. It seems, however, unlikely that this is the only measure undertaken, since most of the light remains trapped inside the crystal owing to the large refractive index of BGO. An investigation performed by Dietle et al. [50], in fact, also shows that the signal output is completely insensitive to the wrapping of the faces orthogonal to the surface viewed by the photomultiplier. A guess could rather be that the surfaces have been partially roughened. The longitudinal nonuniformity in light output from a crystal used in the experiment was measured with two methods. Firstly, a beam of 800 MeV/c pions was used.

The beam traversed the crystal perpendicular to its symmetry axis at different distances from the front face. Secondly, a ^{137}Cs source ($E_\gamma = 662$ keV) illuminated the crystal through a 5 mm hole in a 5 cm thick lead collimator. The two methods gave consistent results: the nonuniformity of the crystals was found to be typically $\pm 2\%$ in the first 18 cm of their length. Fig. 3.41 displays the nonuniformity of the worst and the best crystals.

When using through-going charged pions to calibrate the crystals one is sensitive to the integral over the whole nonuniformity curve. For the case of an electromagnetic shower one is only sensitive up to a certain depth. This means that discrepancies between the seven crystals in the last parts (closest to the

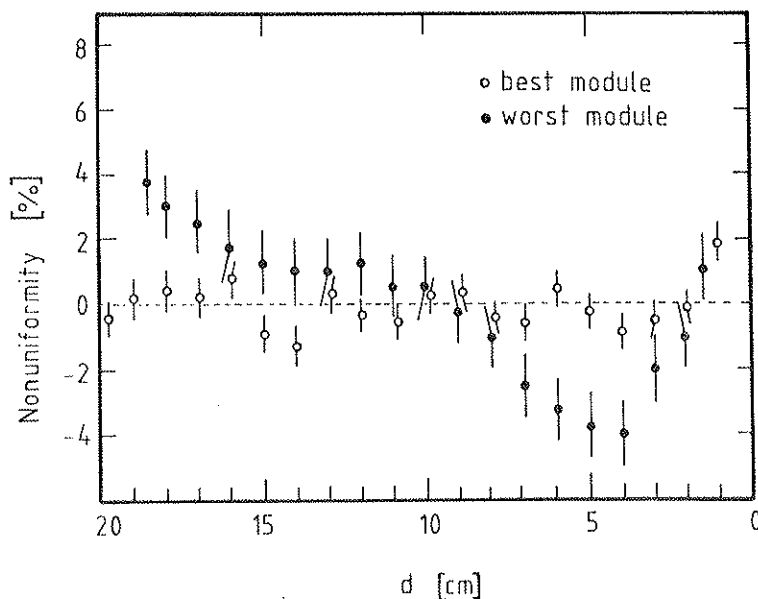


Fig. 3.41: Longitudinal nonuniformity in light output from two of the hexagonal BGO crystals as a function of the distance d from the photomultiplier; data for the most-uniform and the least-uniform hexagonal crystals obtained with a ^{137}Cs source.

photomultiplier) of their nonuniformity curves, will cause the calibration to be worse for showers than for the charged pions. In this context it must be better to use charged particles that stop before they reach the back of the crystals. Another circumstance that discriminates between the experimental runs and the test run is the fact that a shower initiated by an electron always starts practically at the same position, viz. immediately on entry of the crystal, whereas a photon shower only starts at the point the photon converts into an electron-positron pair (cf. Appendix B). As much as ~20% of the incoming photons around 200 MeV do not convert until after one radiation length — 200 MeV is the mean value of the photon spectrum from $p\bar{p}$ annihilations. This means that photon showers, having the same energy, probe different parts of the nonuniformity curve, in contrast to an electron-initiated shower that always affects the same range. Further differences: during the SIN test run the incoming electron was always triggered by the $1 \times 1 \text{ cm}^2$ telescope scintillator, which caused all showers to be well centred and symmetric around the symmetry axis of the detector. The photon showers measured during experimental LEAR runs, on the other hand, could start at any transversal position in the central crystal (and sometimes even somewhat outside it). These photon showers cannot be expected to be symmetric around the detector axis, as the γ -rays originated from the target that was as close as 30 cm — compare to the 5.4 cm diameter of the central crystal — and the emission source is not infinitely small either. All these circumstances, that the photon showers were neither as well centred, nor as aligned to the detector, nor as well longitudinally defined as the electron showers must contribute to the deterioration of the energy resolution observed in the experimental runs.

The Panofski peak in the inclusive γ -spectrum from the experimental data run in May 1985 indicates an energy resolution as bad as 10.2 MeV at 129 MeV. It is hard to understand the underlying reason for this fact. It could have been an indirect consequence of the high $\bar{p}$ -rate present during that run period. The high rate seems to have caused "pile-up" effects in the data taking system, although not pile-up of light in the BGO crystals themselves. Fairly often an ADC gave 0 output during this run period, which is something that should normally happen very rarely, as the "pedestal", i.e. the ADC output corresponding to zero input, was set to 40. The pedestal is measured by feeding the ADC with a gate signal that is uncorrelated to the signals from the BGO crystals. A typical spectrum obtained from such a measurement is shown in fig. 3.42. It shows that electronical noise fluctuations are an order of magnitude smaller than 40. The $\bar{p}$ -rate did, during the May run in 1985, reach values of as much as 40 000 $\bar{p}/s$. Unfortunately the LEAR beam up to now has had a micro duty cycle correlated to the 50 Hz frequency of the electrical-power network, which has caused the instantaneous rate to sometimes be twice as high, i.e. 80 000 $\bar{p}/s$. On the average this will give a BGO event every 5th ms, since the BGO solid angle was 0.25%. However, this 5 ms is the mean value μ of a Poisson distribution $Po(t;\mu)$ which implies that 0.5% of the antiprotons, i.e. 40 per second, arrive already after 500 μs . This time period is not in conflict with the BGO light constant of 350 ns, but might well be with the time constant on the input of the ADC, which is a few hundred microseconds. If the time interval between two consecutive signals is so short that the previous pulse has not completely ebbed out yet when the next one arrives, the interpretation will be that the pedestal has shifted. If the pedestal walk-around is the reason for the worse energy resolution during that run period, it would give a relatively smaller contribution to the resolution at higher energies, which also seems to be the case from studying the upper end-points of the γ -spectra. Unfortunately it is not easy to discover problems like this and undertake measures during a LEAR run period, which only lasts for about 48 hours and a new magnetic tape has to be mounted every ten minutes.

When using the hodoscope in front of the hexagonal BGO crystals the

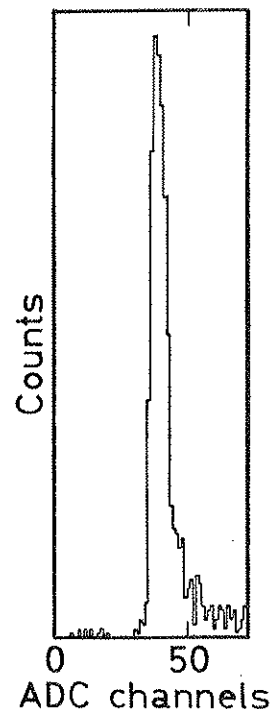


Fig. 3.42: The "zero-point" signal put out by an ADC when there is no energy deposited in the BGO crystal, the so called "pedestal", is typically a few ADC channels wide.

*Handla inte mot andra
som du skulle vilja
att de handlade mot dig
Ni har kanske inte samma smak.*

(Bernard Shaw)

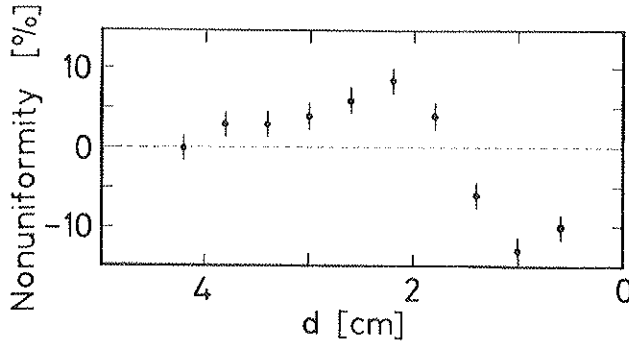


Fig. 3.43: Longitudinal nonuniformity in light output from one of the BGO hodoscope bars as a function of distance d from the photomultiplier.

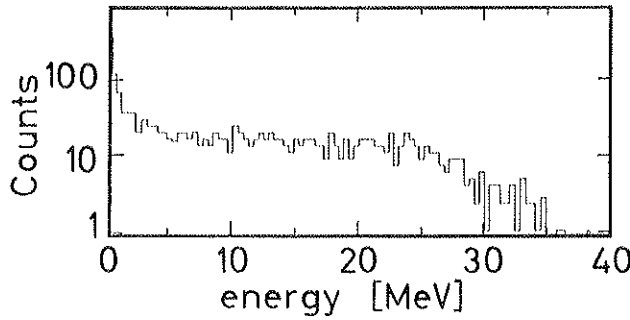


Fig. 3.44: Photon energy spectrum as measured by one of the BGO hodoscope bars.

energy resolution of the system deteriorates. For those photons that convert and start a shower in a hodoscope bar this fraction of the energy has to be added to that measured in the hexagonal crystals. Thus, the hodoscope slabs have to be calibrated relative to the hexagonal crystals. As mentioned in the previous section 3.4.3.1 this could only be done within ten percent. Because the photons cross a bar in the transverse direction, only a very limited region of the crystal is illuminated, and thus the nonuniformity will play a relatively more important role in this case than it does when the shower develops along the crystals as in the hexagonal ones. The nonuniformity of the hodoscopes was measured with a ^{137}Cs source; a result is shown in fig. 3.43. The contribution to the energy resolution from the hodoscope is most crucial at low photon energies, when quite a significant fraction of the shower energy can be deposited in the hodoscope. Fig. 3.44 shows the photon spectrum measured with a hodoscope bar, and reveals that it can contain as much as 30 MeV. How much the hodoscope

contributes to the resolution can be estimated by comparing the different widths of the " $\pi^0\rho$ -peak" as it appears in the three different run periods. This π^0 -signal originates from the two-body reaction $p\bar{p} \rightarrow \pi^0\rho^0$. In December 1983 one hodoscope was mounted in front of BGO_2 , in May 1984 no hodoscope was used, and in August 1984 two hodoscopes were assembled in front of each other and in front of BGO_2 . Assuming the additional width λ of the December peak compared to the May peak is due to the hodoscope, it would be expected that the August peak is 2λ broader than the May peak. This is indeed the fact:

$$\begin{aligned} \Gamma^2(\text{Dec}) &= \Gamma^2(\text{May}) + \lambda^2 & \Rightarrow \lambda &= 11.2 \text{ MeV} \\ \Gamma^2(\text{Aug}) &= \Gamma^2(\text{May}) + 4\lambda^2 & \Rightarrow \lambda &= 12.5 \text{ MeV} \end{aligned} \quad (3.40)$$

This result indicates that the hodoscope contributes at an energy E of 393 MeV a resolution full-width at half-maximum of $\lambda/E = 3.0\%$, compared to a 3.8% contribution from the big hexagonal crystals. This large contribution from the hodoscope is worse than expected. Assuming the calibration of the hodoscope contributes 10%, nonuniformity 20%, and intrinsic BGO resolution of 6% at maximal energy deposition in a hodoscope bar, the worst energy jitter produced by the hodoscope would be 7 MeV. The origin of the additional 4 to 5 MeV is not understood. Although the cross-section for pair production becomes smaller for lower energies, a larger fraction of the shower will be deposited in the hodoscope so the relative influence on the energy resolution of the combined detector is suspected to become worse at lower photon energies.

3.4.3.3 Reconstruction of the Impact Position

Photons striking a BGO detector result in two types of events. For those photons that convert in the hodoscope, the relevant coordinate (ξ) of the impact point is provided within a centimeter by the hodoscope information (the coordinate system is defined in fig. 3.31). The error distribution is then a square distribution of width $b=1$ cm and standard deviation $\sigma_b = b/\sqrt{12}=0.59$ cm. In fact it is not a perfect square but has rounded corners, because the photons do not irradiate perpendicularly to the hodoscope entrance face. Close to a boundary between two slabs a photon will not penetrate the full thickness of the slab, but will continue into the next one. Thus fewer photons will be registered close to the boundary as the conversion probability is proportional to the penetration depth. Furthermore, the part of the shower encountered by the neighbouring slab has to be smaller otherwise the photon is considered to have entered that one instead. This effect becomes of course more pronounced for the outer slabs, that also will show a small systematic shift towards the centre. Since one wants an error distribution for the estimated impact position independent of which slab was hit, the distribution for the individual slabs have to be shifted and added. The resultant distribution looks like a Gaussian with a flat top¹¹. The fact that the shape is not Gaussian has an impact on the kinematic fitting of the π^0 -momentum, which will be further mentioned in section 3.6.

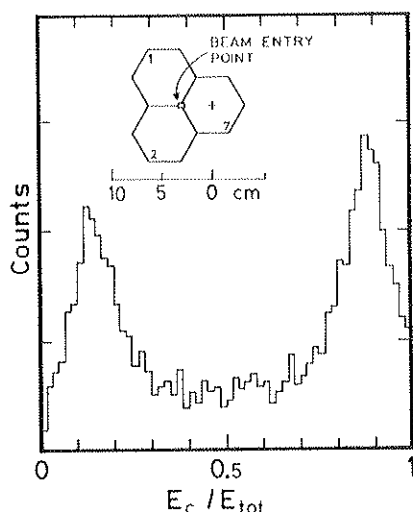


Fig. 3.45: The distribution of the ratio E_c/E_{tot} , i.e. between the energy contained in the central crystal and the total energy registered by the complete detector, obtained for a 200 MeV electron beam entering the detector at the indicated position between three crystals.

During the run period in August 1984 there were two hodoscopes mounted in front of the BGO₂ set. By combining the information from the outer and the inner hodoscopes one can decide in which half of the outer hodoscope bar the photon converted. The distribution will in this case then have a width $b=0.5$ cm and a standard deviation $\sigma_b = b/\sqrt{12}=0.29$ cm. This is of course only valid for the sample of photons that convert in the outer hodoscope.

For the photons that do not convert in any hodoscope element, the impact position (ξ, ζ) is estimated from the observed division of energy among the hexagonal crystals. The position resolution that can be obtained in this way has been discussed by several authors for arrays of lead-glass modules [15] [133] [75] [8] [86], and also for NaI(Tl) arrays [32]. The simplest estimate of the impact position (ξ, ζ) is provided by associating the energy E_i observed in each crystal $i=1, \dots, 7$ with the centre coordinates (ξ_i, ζ_i) on the face of the crystal, and by computing the centre of gravity of the observed distribution. The accuracy of this estimate is limited by the cross-sectional size of each crystal relative to the transverse dimensions of the electromagnetic shower (cf. Appendix B) and especially at low energies below 100 MeV, by fluctuations in the transverse development of the shower. The effects of shower fluctuations are evident from the test run at SIN (fig. 3.45). For this particular test 100 MeV electrons were injected at normal incidence to the detector front face at the position indicated in the figure. The impact position was defined by a 2×2 mm² trigger scintillator. In the figure is

plotted the ratio between the energy E_c deposited in the central crystal and the energy E_{tot} deposited in the complete set. It is evident that quite frequently shower fluctuations result in the deposition of substantial energy in only one or the other of the crystals. Only rarely is the energy approximately equally shared between the two crystals. A major deficiency of the simple centre-of-gravity algorithm in the present application is that it cannot adequately reflect the actual transverse development of the electromagnetic shower. The method, in effect, assumes that the energy deposition has a linear transverse

¹¹ Such a shape can easily be parameterized by a "hyper-Gaussian", in which the exponent is raised to a number $\beta > 2$ instead.

dependence which is in contrast to reality (see fig. B.4 in Appendix B). Consequently the procedure introduces considerable spatial distortion.

The radial distribution of an electromagnetic shower in BGO as computed with the EGS Monte Carlo program, referred to previously in this section, can be used to calculate the fractional energy deposited in the central crystal as a function of the impact position. The result is plotted in fig. 3.46 and shows that very little energy escapes from the central crystal if the shower axis is displaced by less than 1.5 cm. This suggests that the position resolution may be improved by associating extra weight to the small energies that are deposited in the outer crystals. This can be achieved by computing the moments of the observed energy distribution weighted by a fractional power α of the energy [32]. This gives preferentially more weight to energies observed further from the shower axis. Accordingly, the coordinates (ξ, ζ) of the impact point can be estimated by

$$\xi = \frac{1}{N_E} \sum_{l=1}^7 \xi_l E_l^\alpha, \quad \zeta = \frac{1}{N_E} \sum_{l=1}^7 \zeta_l E_l^\alpha, \quad N_E = \sum_{l=1}^7 E_l^\alpha, \quad (3.41)$$

where E_l are the observed energies, (ξ_l, ζ_l) are the coordinates of the crystal axes, and the exponent α remains to be optimized. The parameter α was fixed by comparing values of $|\xi|$ to data for which the impact position had been measured with the BGO hodoscope. The optimal value was $\alpha = 0.308$. The position resolution obtained with this improved algorithm can be measured for 428 MeV photons by, also for this purpose, utilizing the signal from the reaction $p\bar{p} \rightarrow \eta\omega$ which was used to deduce the energy resolution, but this time the relevant peak is in the corresponding distribution of opening angles $\phi_{\gamma\gamma}$ between the γ -rays observed from the decays of the η -mesons. Fig. 3.47a shows a fit of the peak, and the inset is a display of the full spectrum. The shape of this peak is originally, before smeared by detector resolution, a Jacobian peak distribution (see section 3.1). Thus the fitted function is a convolution of a Gaussian

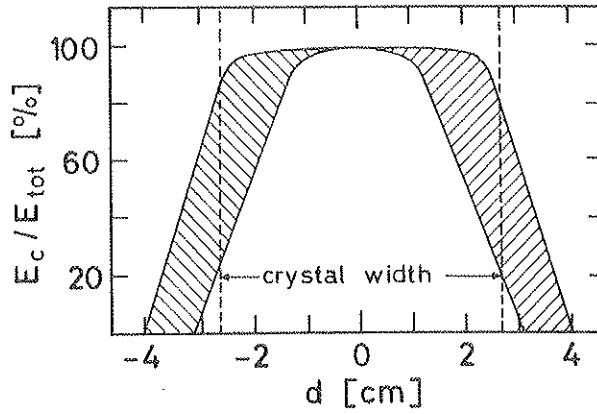


Fig. 3.46: Simulated fraction of energy contained in the central crystal as a function of the displacement d from the symmetry axis of the detector. The striped area indicates the influence of fluctuations.

and the appropriate Jacobian. The resultant shape is again displayed in fig. 3.47b together with the unfolded Gaussian and Jacobian¹². The width of the Gaussian embraces three contributions, viz. 0.34 cm from the finite vertex definition (i.e. the width of the annihilation spot in the target), from the lead-glass resolution (entering via the other γ -ray), and from the BGO position resolution. Subtracting the two former quadratically gives a root-mean-square value of 0.55 cm for the BGO resolution.

Because the formulae (3.41) are based on the shower spread, the accuracy of the position (ξ, ζ) will clearly be dependent on the shower energy. This centre-of-gravity method works well for higher photon energies, as demonstrated by the resolution at 428 MeV. At lower energies the shower spread is not as large, and fluctuations are more dramatic, reducing the utility of the centre-of-gravity method. At energies of 10 to 20 MeV the complete shower is almost always contained in a single crystal. Any algorithm in this case cannot yield better result than a square distribution of width $b = 5.4$ cm, the diameter of a crystal, that yields a standard deviation $\sigma_b = b/\sqrt{12} = 1.6$ cm.

¹² Notice the shift of the peak off the minimal angle.

For the fitting program of the π^0 (or η) kinematics, the best estimation of the position resolution of every photon energy is desired. Knowing the energy resolution as a function of shower energy [eqs. (3.37) and (3.38)], a heuristic formula is adopted for the position resolution as calculated according to eq. (3.41) versus the shower energy E [GeV]:

$$\sigma_{\xi} = \sigma_{\zeta} = \begin{cases} 5.4 \text{ cm}/\sqrt{12} = 1.6 \text{ cm} & E < 20 \text{ MeV} \\ E^{-\beta} \cdot 0.4 \text{ cm} & E > 20 \text{ MeV} \end{cases} \quad (3.42)$$

where β is a free parameter. The value $\beta = 0.35$ fitted the observed angular-resolution values. When no hodoscope is used both the position resolution in the ξ and in the ζ directions are given by eq. (3.42). On the other hand, when the hodoscope is operating the σ_{ζ} is given by the values discussed earlier in this section, and these are **independent** of the shower energy. The σ_{ξ} has, however, still to be estimated with eq. (3.42). However, the ξ -coordinate along the beam axis is not that crucial when measuring the angle between the γ -rays as the detectors are placed in the xy -plane of the detector setup (see fig. 3.4).

The improvement in overall spatial resolution provided by the modified formulae (3.41) over the plain centre-of-gravity algorithm ($\alpha \equiv 1$) is illustrated more specifically in fig. 3.48. The figure shows how the estimated impact position typically depends on the true impact point for these types of algorithm [32]. There is a severe biasing of the estimated impact point towards the crystal centre by the plain centre-of-gravity estimate. This effect is reduced by the modified algorithm ($\alpha < 1$) — but still present — implying that it bears a closer resemblance to the true development of a shower.

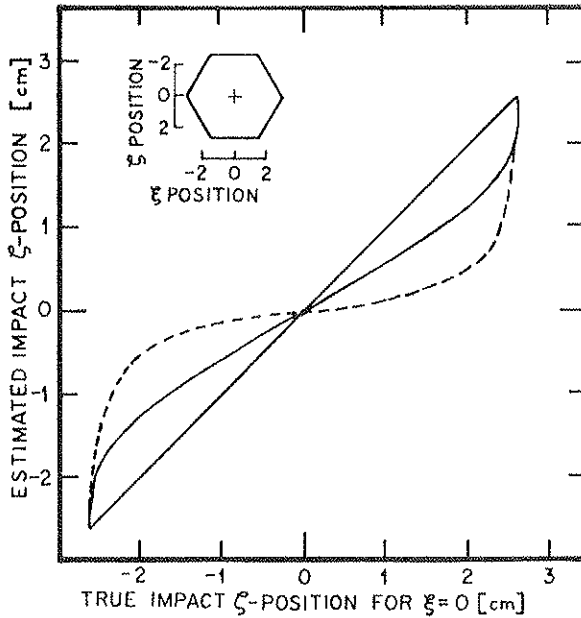


Fig. 3.48: Reconstructed photon-impact position versus the true impact position and as estimated with the centre-of gravity algorithm (dashed line), and with the weighted centre-of-gravity algorithm (solid line). (Adapted from ref. [32]).

centre-of-gravity algorithm was thus made, using this information. This "new algorithm" [100] reads the three largest energies among the seven BGO hexagons, viz. E_c from the central crystal and E_i, E_j from the adjacent peripheral crystals i and j (only a small fraction of all events lead to non-adjacent i, j). From the software conditions for accepting a BGO event (see next chapter), and from the symmetry of the system, we can always write $E_c > E_i > E_j$. From these three numbers, two ratios

One could conceive several approaches to obtain better position resolution. An immediate idea is to divert the value of α for different shower energies. For the extreme case when α is a well-calibrated function $\alpha(E)$ of the shower energy it seems plausible that the systematic "disease" could be almost cured. The best result would be achieved with a method based on a calibration experiment in which a well-collimated beam of γ -rays is directed sequentially at many points on the face of the BGO detector, and the response pattern of the detector is recorded. This information could then be used to develop a probabilistic (average) relationship between the energy division and the impact point. This relationship may be of a look-up-table type that consists of an extensive library of showers, their energy topology and impact point. This approach was not pursued, since such detailed experimental information has not been obtained.

Some experimental information on the location of the photon impact point is available for those runs where coincidences between elements of the BGO hodoscope and the main detector were recorded. An attempt to improve the

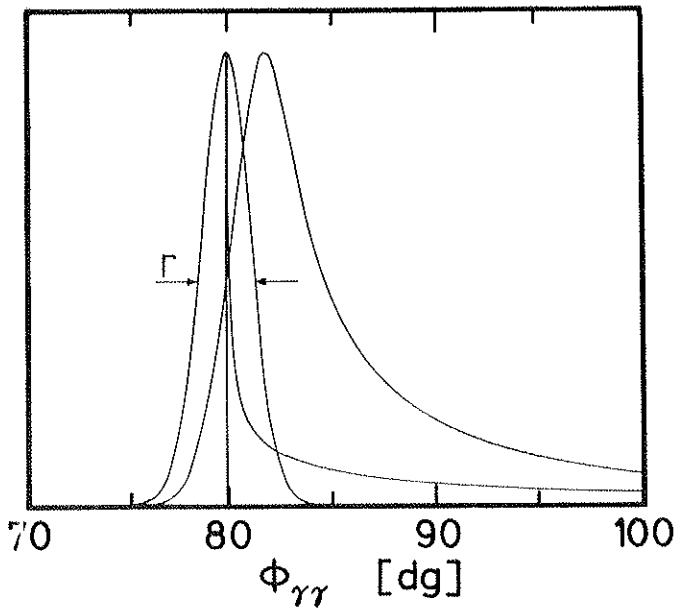
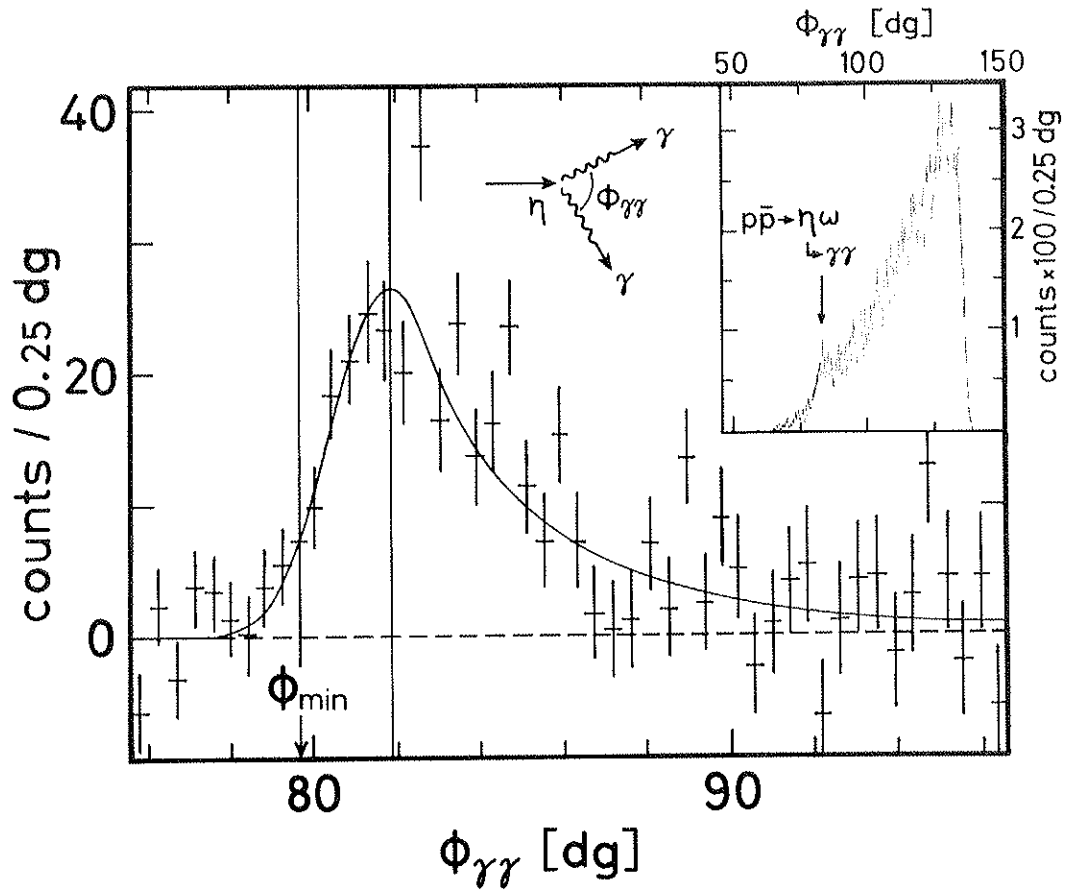


Fig. 3.47: The peak from $p\bar{p} \rightarrow \eta\omega$ in the spectrum of opening angles between photon pairs observed from η -decays (a). The inset shows the full spectrum. The fitted function is a folding of the Jacobian peak distribution with the detector resolution Γ being a Gaussian (b).

$$\begin{aligned} A &\equiv \ln E_c - \ln E_i \\ B &\equiv \ln E_i - \ln E_j \end{aligned} \quad (3.43)$$

are formed. The quantities A and B are then histogrammed for those events where a coincidence with hodoscope element $k = 1, \dots, 7$ is observed, i.e. where the impact position lies inside a known region in ξ and ζ . From these histograms an average relation between (ξ, ζ) and (A, B) is derived. A crude dependence on the total shower energy was obtained by carrying out the above analysis for several energy intervals. The resulting position algorithm shows a marked improvement in the ζ direction (direction in which the binning information from the hodoscope is available) as can be seen from the comparison in fig. 3.49.

3.4.3.4 Detection Efficiency

Not all events registered by a BGO calorimeter will be accepted for the data analysis. Some photons that hit a BGO crystal are never registered, because they deposit too little energy in the detector to give a signal above the discriminator threshold, or there was a veto signal present from the front scintillation counter caused by a charged particle. If a photon enters the BGO detector close to the edge of the front face a substantial part of the shower energy escapes out of the active detector volume and is lost. The correlation between the measured shower energy and the original energy of the photon is then poor. Thus, it is desirable to reject photons that enter the calorimeter so far from the symmetry axis that a significant fraction of their shower energy systematically leaks out of the detector. This rejection is attended to by requiring a certain fraction of the total shower energy to be contained in the central crystal. The purpose of this constraint is to confine accepted events to a central region on the entrance surface. Hence, the effective solid angle of a BGO detector, i.e. its overall detection efficiency, is determined by the software cuts applied to the energy division among the seven crystals (see further next chapter).

According to Appendix B the transverse shower spread has a Gaussian shaped core. Hence, one would expect the detection efficiency to be 100% and nearly uniform as a function of radial distance from the symmetry axis, out to a radius slightly larger than the central crystal, and then to drop off according to an integrated Gaussian. Since a so called Woods-Saxon shape¹³ is a simple two-parameter description of such a dependence we have assumed that the efficiency versus radial distance from the symmetry axis may be approximated by

$$\omega(r) = \frac{\omega_0}{1 + e^{(r-R)/a}} \quad (3.44)$$

with

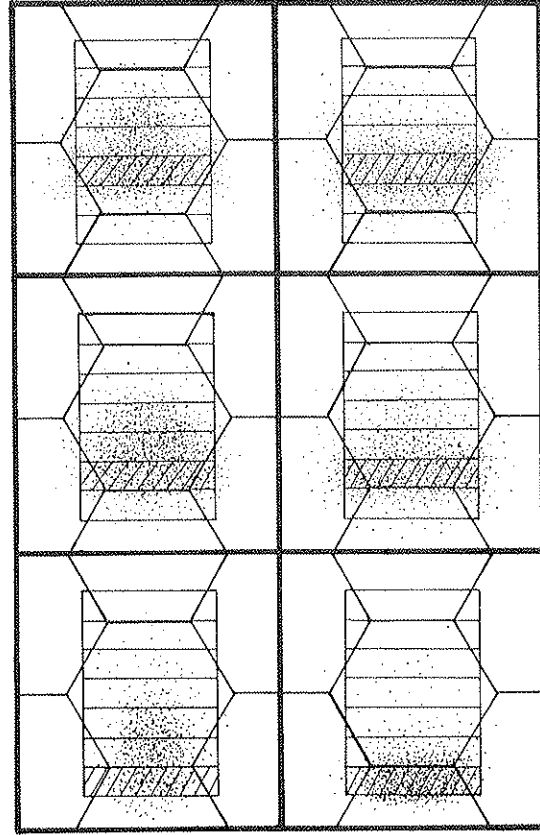


Fig. 3.49: Comparison between the reconstructed photon-impact position estimated with the weighted centre-of-gravity algorithm (left-hand side), and with the "new algorithm" (right-hand side). The true impact position is within the striped hodoscope bar.

¹³ This shape has been used to describe the density of nuclear matter [78].

$$\omega_0 = \frac{3N}{4\pi R^2 [1 + (\pi a/R)^2]} , \quad (3.45)$$

where N is the total number of accepted events, R is the radius at half height and $a = s/4\ln(3)$, s being the diffuseness (see fig. 3.50). The values of the two parameters R and s can be determined by utilizing information from the hodoscope again. The event sample consists of those photons that both convert in the hodoscope and yield such showers in the hexagonal crystals that satisfy the topological demands. If the photons accepted have a density distribution according to eq. (3.44), the number n of conversions in hodoscope slab i can be calculated as

$$n_i = \int_{-2.5}^{2.5} \int_{\xi_i}^{\xi_{i+1}} \omega(\xi, \zeta) c(\xi, \zeta) d\xi d\zeta , \quad (3.46)$$

where $\omega(\xi, \zeta) = \omega(\sqrt{\xi^2 + \zeta^2}) = \omega(r)$ given by eq. (3.44). The second function $c(\xi, \zeta)$ is the conversion probability in the hodoscope, which also has to be dependent on ξ and ζ as the photons do not irradiate parallel to the surface normal. However, a geometrical estimation of how much c varies indicates that it is less than 1%. Thus, it can with good approximation be set constant, $c_0 \equiv c(\xi, \zeta)$, which yields an unknown normalization that cancels when computing ratios n_i/n_j . Because the situation is symmetric around slab four we calculate the three ratios $(N_1 + N_7)/2N_4$, $(N_2 + N_6)/2N_4$, and $(N_3 + N_5)/2N_4$. They are plotted versus R in fig. 3.51 with s as an incremented parameter. The horizontal lines represent the values deduced from experimental data, the surrounding striped regions being the statistical errors. These striped areas can be transformed into the R -versus- s plane (fig. 3.52). The over-lap area shows that $R = 2.9$ cm and $s = 1.3$ cm for photons around 200 MeV, where the photon spectrum has its mean, whereas they are $R = 2.7$ cm and $s = 1.6$ cm for high-energy photons above 500 MeV.

To obtain an effective solid angle Ω to have an equivalent geometrical acceptance the radius R^* of a step function has to be calculated such that

$$\pi R^{*2} = 2\pi \int_0^\infty \omega(r) r dr , \quad (3.47)$$

which yields an equivalent radius R^* about 1.5% larger than R which is roughly the statistical error. With the BGO calorimeter placed at 30 cm distance from the target centre the effective solid angle is 2.4‰ for 200 MeV photons and 2.1‰ for 500 MeV photons.

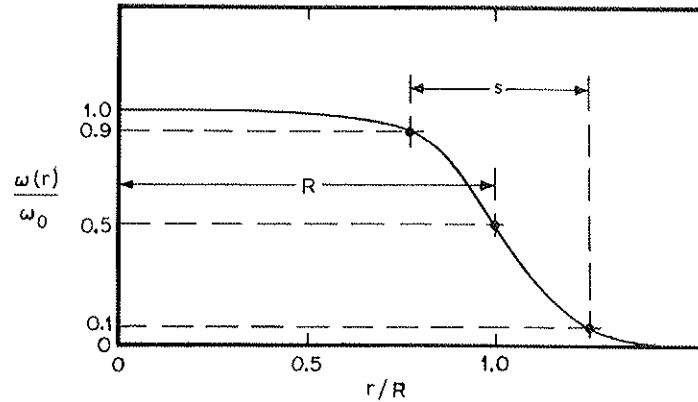


Fig. 3.50: Parameters of a Woods-Saxon shape $\omega(r)$ (see further text).

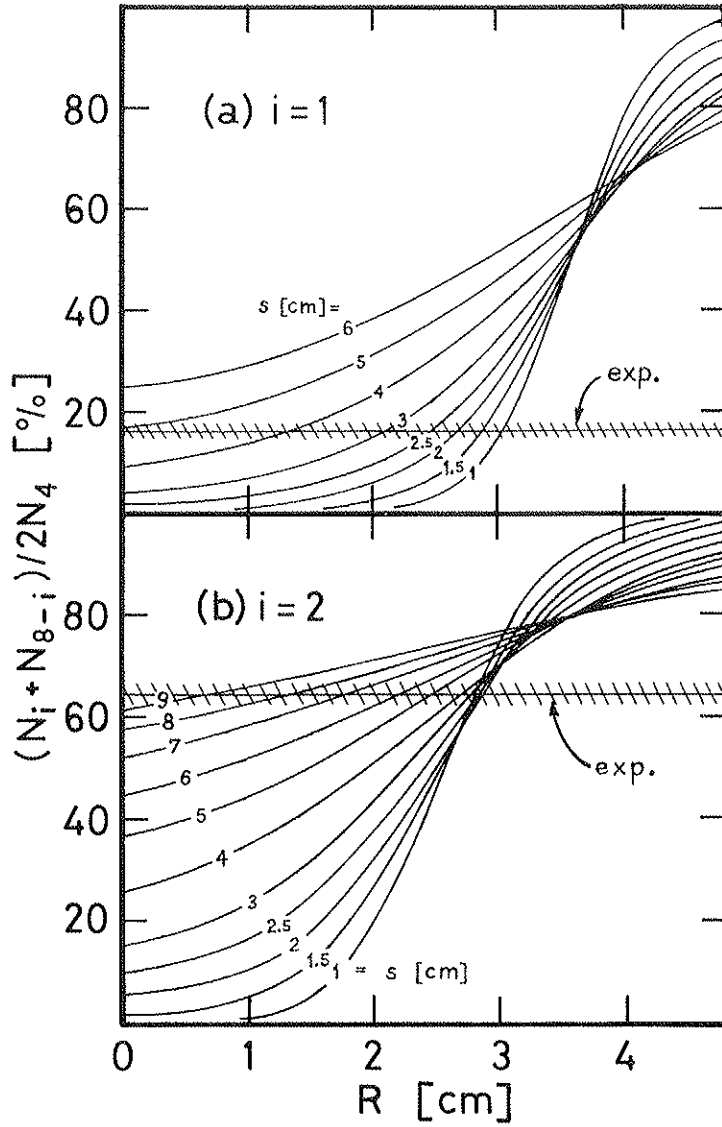


Fig. 3.51: Expected ratios $(N_i + N_{8-i}) / (2N_4)$ of count rates N_i as recorded by hodoscope bars i versus the parameter R of the Woods-Saxon-shaped distribution of accepted events. The measured values are indicated.

Of course R^* depends on how large a fraction $\eta = E_c / E_{tot}$ of the shower is required to be contained in the central crystal. This was $\eta \geq 15\%$ except for a particular data sample of triple photon coincidences for which it was $\eta \geq 50\%$ (see further next chapter). For this latter case the equivalent radius is $R^* = 2.6$ cm for the photons above 500 MeV, which are the interesting ones in this sample. The corresponding effective solid angle is 1.9% (with the detector at 30 cm distance from the target).

Using the hodoscope has the drawback of decreasing the overall detector efficiency, mainly because of the limited conversion probability. The total efficiency Ω_H , of the system is in this case determined by, on one hand, the solid angle of the overlap area of the hodoscope and the effective area of acceptance on the front face of the hexagonal crystals, and on the other hand, by the fraction c_0 of passing photons that actually convert in the hodoscope:

$$\Omega_H = c_0 \int_{-2.5}^{2.5} \int_{-3.5}^{3.5} \omega(\xi, \xi') d\xi d\xi' . \quad (3.48)$$

The photon conversion probability is, at 200 MeV (which is the mean value of the photon spectrum), dominated by the cross-section for pair production, since the cross-section for photo-electric effect drops as $1/E^3$, and for Compton scattering it drops as $1/E$ making these processes inconsequential above 10 MeV. Around 200 MeV the pair-production cross-section is, however, energy dependent, and the asymptotic value given by the equations in Appendix B is attained only above 1 GeV. Since the effective atomic number of BGO is $Z \approx 30$, and thus $2\pi Z/137\beta \approx 1.3$, the value obtained with the Born approximation should be corrected. Carrying out the calculation results in a conversion probability c_0 in the hodoscope of roughly 70%, which can be checked by comparing the trigger rates from the two hodoscopes, respectively, that was mounted one in front of the other during the A84 run period. This indicates that $c_0 = (76 \pm 12)\%$. The total efficiency obtained with a hodoscope is then

$$\Omega_H = 0.7 \cdot 2.0\% = 1.4\%. \quad (3.49)$$

To summarize, the hodoscope diminishes the overall detector efficiency from 2.1% to 1.4% which can seem a little bit unjustified in an experiment where extremely high sensitivity is most desirable and the narrowness of the exotic quark-states under consideration is doubtful indeed. However, the method of kinematic fitting — which was not realized until after the design — reduced the need of the good position resolution offered by the hodoscope, and it was for this reason the hodoscope was removed after the first run period.

3.5. THE LEAD-GLASS DETECTOR

For many years now lead-glass detectors have been used for Cherenkov calorimetry. It has become a common practice to designate as “calorimeters” such particle detectors that measure the particle’s energy through total absorption by stopping it and yielding a signal approximately proportional to its energy. Like in BGO a photon entering lead glass will initiate a cascade process of alternating pair production and bremsstrahlung reactions, called an electromagnetic “shower”. In Appendix B the relevant features of electromagnetic showers are briefly reviewed in a utilitarian way. Parameters used here are defined in Appendix B, and some of the formulae pertaining to Cherenkov detection are quoted in this section. After the first subsection that recalls the physics of Cherenkov calorimetry, there is a subsection discussing the designing of the geometry. The last subsection 3.5.3 deals with the performance of the lead-glass detector used in PS182.

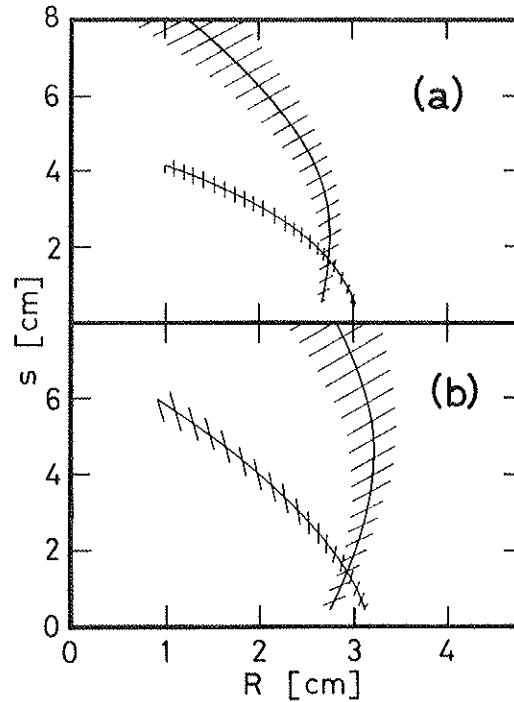


Fig. 3.52: Allowed values of the radial parameter R and the diffuseness s of the distribution of accepted events as given by the measured count rates of hodoscope bars 1, 7 and the two bars 2, 6, respectively, (a) for photon energies around 500 MeV and (b) around 200 MeV.

3.5.1 Lead Glass — a Cherenkov Calorimeter

There are some differences in the case of using lead glass compared to the BGO situation. Firstly, the shower will extend larger distances in lead glass than it would in BGO for the same photon energy, because of the larger radiation length in lead glass. Secondly, the amount of light produced in lead glass is much less than what is produced in BGO. The light produced in lead glass is of Cherenkov type in contrast to BGO where it is produced through scintillation. The Cherenkov radiation appears from electrons and positrons moving faster than the speed of light in the medium. Due to their small rest mass this happens already at an energy of about 1 MeV. The Cherenkov photon emission is given by the number N of photons emitted per unit track length [83]:

$$N = \alpha/c \cdot \int [1 - 1/(n^2\beta^2)] 2\pi d\nu, \quad (3.50)$$

where n is the refractive index and $\beta = v/c$, v being the particle's velocity. For the visible spectrum where lead glass is not opaque

$$N = 500 \sin^2\theta_c [\text{cm}^{-1}] \text{ (visible spectrum)}, \quad (3.51)$$

where θ_c is the Cherenkov angle (see fig. 3.53) given by

$$\theta_c = \arcsin [1/(n\beta)]. \quad (3.52)$$

With $n=1.62$ for the lead-glass and $\beta \approx 1$, the Cherenkov angle is $\theta_c = 56^\circ$, implying the number of photons emitted per unit track length is

$$N = 500 \cdot (0.85)^2 = 350 \text{ cm}^{-1}. \quad (3.53)$$

Since β is close to one already for a few MeV of electron energy, the number of Cherenkov photons is virtually constant for all $e^\pm$ energies, until well below the critical energy ϵ , beneath which cascades do not propagate. Because of the small number of photons produced, one would anticipate the energy resolution to be governed by photon statistics. In this context it is desirable to have as large overall track length T as possible, which would be favoured by a large radiation length — since $N=350T$. The total track length can be written (see Appendix B)

$$T = F(z)X_0E/\epsilon, \quad (\text{B.11})$$

where E is the initial photon energy, ϵ is the critical energy, X_0 is the radiation length, and $F(z)$ is approximately given by

$$F(z) = e^z [1 + z \ln(z/1.53)]$$

(B.12)

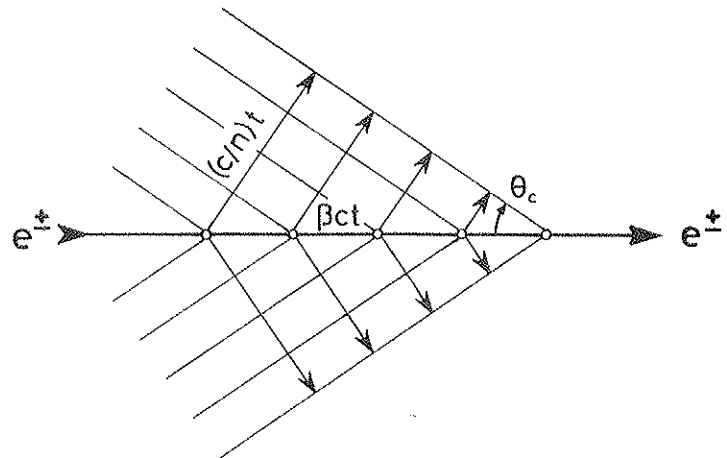


Fig. 3.53: When the velocity βc of a particle exceeds the light speed c/n of the medium it penetrates there occurs coherent light due to polarization of the medium, so called Cherenkov radiation.

Gud existerade,
hur skulle jag föredraga tanken
att jag inte vore Gud.

(Friedrich Nietzsche)

with

$$z = 4.58(Z/A)(E_c/\varepsilon) ,$$

where the cutoff energy E_c , below which the electrons and positrons become invisible, depends on the detection method employed; for Cherenkov detection it is roughly 1 MeV. The lead-glass blocks used in this experiment are fabricated from type F2 glass¹⁴. The basic properties of the F2-melt are listed in table 3.4. The effective Z of the glass is 60, and the average atomic mass number is $A = 140$. With these values we can calculate z and subsequently $F(z)$ to obtain the total track length

$$T = 0.72X_0E/\varepsilon = 0.24E[\text{cm/MeV}]. \quad (3.54)$$

A 200 MeV γ -ray will therefore yield a total track length of $T = 47$ cm, resulting in a total number of $N = 17 \cdot 10^3$ emitted Cherenkov photons. Of these, however, only a limited fraction will ever reach the photo cathode of the photomultiplier due to transmission inefficiencies.

The lead-glass detector in this experiment is designed to simultaneously measure the two photons from decays of π^0 -mesons. As was pointed out in section 3.1 it is necessary to measure also the decay angle between the γ -rays, in order to identify the π^0 . Thus, the detector has to be devised to measure also the photon impact positions. The maximum π^0 -energy $\hat{E}_{\pi^0}$ obtained from $p\bar{p}$ annihilations at rest occurs for a two-body disintegration mode of the $p\bar{p}$ system. The π^0 -energy is then

$$\hat{E}_{\pi^0} = (1/4m_p)(4m_p^2 + m_{\pi^0}^2 - m^2), \quad (3.55)$$

where m is the rest mass of the other particle in the final state. Obviously the maximum π^0 -energy is acquired for $m = 0$, i.e. for the process $p\bar{p} \rightarrow \pi^0\gamma$, which gives $\hat{E}_{\pi^0} = 933$ MeV and a minimum decay angle of 16.5° . It is of course necessary to place the detector far enough from the target so that showers from two such photons do not overlap. The distance from the target depends then on the transverse spread of the showers. A typical containment parameter is (cf. Appendix B)

$$R_{95} \cong 2R_M \cong 14A/Z [\text{g/cm}^2], \quad (3.56)$$

where the Molière radius is $R_M \cong 2X_0$ for high- Z nuclei. Taking the numbers from table 3.4 the 95% containment radius is $R_{95} = 9.1$ cm. Some further contemplation might be appropriate for this problem.

It can be realistic to assume that R_{95} is a too large gauge of the lateral development of a shower in the context of the Cherenkov technique. A more relevant measure is possibly the radial dimension of the central component of the shower (cf. fig. B.4), which comes at fixed direction and velocity. This is not because the particles in the broad halo around the central radiative core of the shower, do **not** produce Cherenkov light at all, but that **that** light might have more difficulties to reach the photo cathode. Because the slow electron component in the soft halo (which is largely due to photon propagation around and below the critical energy ε) is closer to isotropy in angular distribution (cf. fig. B.5), a bigger fraction of the subsequent Cherenkov light will be lost as it will not be totally internally reflected. In fact, the situation in a lead-glass block is close to critical when it is irradiated longitudinally and having the photomultiplier viewing the smallest surface, which is opposite the entrance one. The geometrical situation with a γ -ray coming in parallel to the longitudinal axis is illustrated in fig. 3.54. Because the critical angle for total reflection is

$$\theta_{tr} = \arcsin(1/n), \quad (3.57)$$

it is exactly the complementary angle to the Cherenkov angle θ_c , i.e. $\theta_{tr} + \theta_c = 90^\circ$. Hence, the light emitted in a plane perpendicular to the reflecting surfaces will have an irradiation angle just on the limit to total reflection. However, this is only the case for a part of the light as the Cherenkov light is emitted in a cone around the track. The largest irradiation angle occurs for that γ -ray that points to the

¹⁴ Manufactured by Jenaer Glaswerk Schott & Gen, Mainz, FRG.

corner as indicated in the fig. 3.54. With the dimensions that correspond to the blocks used in this experiment the largest angle is 17° . From the considerations above it is plausible that, for tracks deviating from the longitudinal direction, an increasing part of the emitted light will never reach the photocathode. It then appears reasonable to assume that the dimension of the central component of the shower is a more relevant number for the lateral spread in the context of Cherenkov detection. This dimension is roughly one Molière radius

$$R_M \cong 2X_0 = 6.1 \text{ cm.} \quad (3.58)$$

Consequently, the distance between the two photons on the entrance surface of the lead-glass array should be at least twice this number in order to avoid overlap of the two showers. In practice one needs some additional space between to have at least one lead-glass block without energy separating the two showers. The reason is to be able to decide unambiguously in the offline analysis that the event consists of two disjunct showers (see further next chapter). The width of a lead-glass block has to be typically at least a couple of centimeters for mechanical reasons and limits set by the dimensions of available photomultipliers. Thus, the lead-glass detector has to be placed with its entrance face at least about 60 cm from the target.

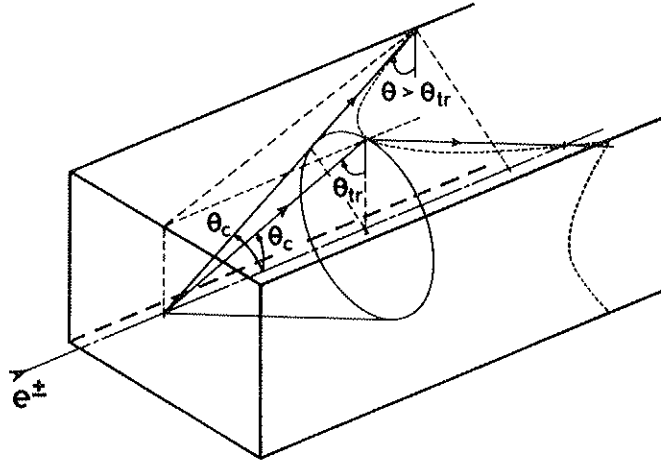


Fig. 3.54: Cherenkov light is emitted in a cone around the particle track with half-top angle θ , which is exactly the complement to the critical angle for total internal reflection θ_{tr} if the particle track is parallel to the cavity walls.

Table 3.4: Properties of lead glass type F2.

Density	3.61	[g/cm ³]
Eff. atomic number	60	
Rad. length	3.05	[cm]
Refr. index	1.62	

3.5.2 Geometry

The distance away from the target is limited in this experiment by the beam height above the floor as the lead-glass detector is installed below the target (see fig. 3.4). Another requirement, that competes with this limitation, is that of having a block length of enough radiation lengths to contain most of the shower energy; at least 15 radiation lengths are desirable.

As discussed in the previous section for the BGO case and as will be below for the lead glass, it is of interest to have an accurate estimation of the impact position of the γ -ray on the entrance surface of the detector. The ultimate reason for this is the wish of having the best possible angle measurement in

*How long should a man's legs be ?
They ought to be long enough to reach the ground.
(Abraham Lincoln, 1809-1865)*

order to improve the π^0 -momentum resolution as mentioned in section 3.1. Good angular resolution can be achieved by having a fine modularity of the detector and/or by positioning the detector far from the target. Since we are limited by the beam height and some space has to be left between the detector and the floor for photomultipliers, bases, and support constructions, we are left with the possibility of increasing the modularity. On the other hand, however, one does not want to make the blocks too thin, as this decreases the ratio between active detector volume and so called "dead space", i.e. for instance wrapping and support materials between the blocks, that absorb energy but do not yield any signal. To compromise and reasonably meet all these requirements the lead-glass blocks were made from type F2 glass and in the shape of blunt wedges, 50 cm long and with front faces of $4.2 \times 6.4 \text{ cm}^2$ and rear faces of $6.4 \times 6.4 \text{ cm}^2$ (see fig. 3.55). The complete detector consists of an array of 160 elements that are stacked in four rows of 40 transverse to the beam. The four rows extend totally 25.6 cm in the direction of the beam. The distance from the target centre to the front face of each block is 80 cm. At this distance each block subtends an angle of 3° , implying there is at minimum four blocks between two shower maxima from a π^0 -decay. Because a shower rarely extends more than three blocks, this means that there usually is a block without energy between the two showers, as was required.

The lead-glass block are wrapped in $20 \mu\text{m}$ thick aluminized mylar foil and covered with black PVC sheet. The photomultipliers used were EMI-9839B with a two inch diameter covering only 50% of the rear area; it is glued onto the surface with the epoxy Epotek¹⁵ 301-2, that has a refractive index of 1.54. When transmitted from the lead glass to the photo cathode, the light has to pass two totally reflecting junctions, because the refractive index of the lead glass ($n_1 = 1.62$) is larger than the refractive index of the epoxy ($n_2 = 1.54$) which, in its turn, is larger than the refractive index of the glass window of the photomultiplier ($n_3 = 1.45$). In fig. 3.56 the geometry is illustrated for a light beam travelling through the three optical media. The criterion for transmission out of the glass window of the photomultiplier into the photo cathode is thus set by the refractive index of the lead glass:

$$\sin\theta < (n_2/n_1)(n_3/n_2)(1/n_3) = 0.62 \Rightarrow \theta < 38^\circ. \quad (3.59)$$

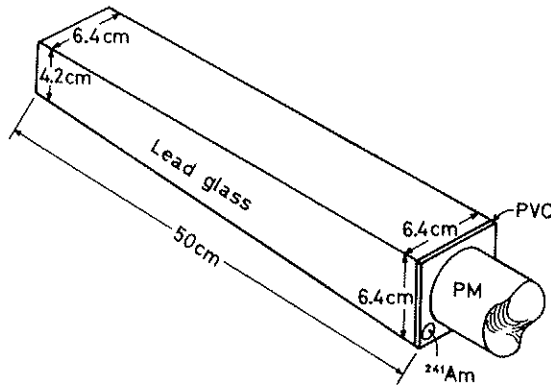
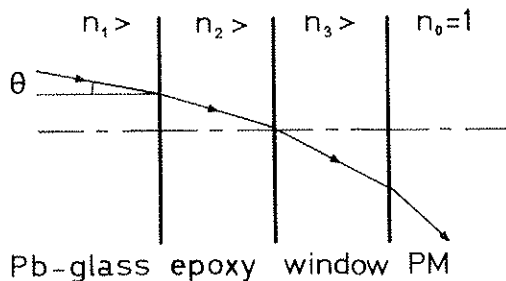


Fig. 3.55: View and dimensions of one of the elements of the lead-glass array.



Because the Cherenkov light will reach the rear face with irradiation angles up to 56° , the transmission criterion will most certainly cause a significant part of the light to be trapped inside the block cavity and unable to reach the photo cathode.

The part of the rear face surrounding the photomultiplier is covered by a 5 mm thick PVC plate. This plate is supplied to provide a larger distance between the mylar foil covering the block and the graphite shielding of the photomultiplier. Because the blocks are supported by a metallic frame the electrical potential of the mylar foil is at ground. If the distance between the mylar foil and the graphite, that has a negative high voltage (-2kV), is too short sparks will occur with subsequential discharge of capacities of the photomultiplier base electronics (cf. section 3.7), which might generate spurious signals.

Fig. 3.56: Light beam passing from the lead glass to the photo cathode.

¹⁵ Epotecny, Vélizy, France.

3.5.3 Performance of the Array

In order to monitor the gain and functioning of the individual photomultipliers each lead-glass block was equipped with an ^{241}Am α -source mounted on a small piece of plastic scintillator (2 mm $\times$ 8 mm diameter). The scintillator was glued onto the rear end of the block close to the photo multiplier as indicated in fig. 3.55. The light pulse produced was equivalent to a shower of between 20 and 60 MeV. Variations occurred from block to block due to differences in light coupling of the source scintillator. The spectral peaks obtained from these sources are so broad that they are scarcely useful for calibration. Other methods were employed for the calibration as described in the following sub-subsection. The next sub-subsection 3.5.3.2 discusses the reconstruction of the impact position of γ -rays, while sub-subsection 3.5.3.3 investigates the energy resolution, and the last sub-subsection concerns the detection efficiency of the detector.

3.5.3.1 Linearity and Calibration

To be able to monitor the response of the lead-glass system as a function of energy of the detected photon one needs a γ -source of well defined energy. Such a source was never available. Another way to probe the level of linearity of the system is to compute the invariant mass m_0 of two-photon coincidences [eq. (3.6)] as a function of the energy E_1 of one photon, while the energy E_2 of the other photon is kept roughly constant. In practice, E_2 is selected to be in the interval 200 to 300 MeV, in order to achieve enough statistics. The mean value m_0 of the π^0 invariant mass peak should then be independent of the energy E_1 if the system responds linearly to the energy E_1 . As fig. 3.57a demonstrates the observed dependence reveals a non-linear behaviour. However, before any conclusions are drawn from this plot some precautions should be made; one can imagine another circumstance to cause the observed dependence. When computing m_0 versus E_1 we have implicitly assumed that the decay angle ϕ is harmless and perfectly measured. In fact this angle is deduced from the estimated impact positions on the entrance surfaces of the two hit blocks, plus a multiple of the block width corresponding to the number of blocks in-between. This results in a certain distance on a cylindrical surface with a radius of 80 cm. Suppose now that the target is not

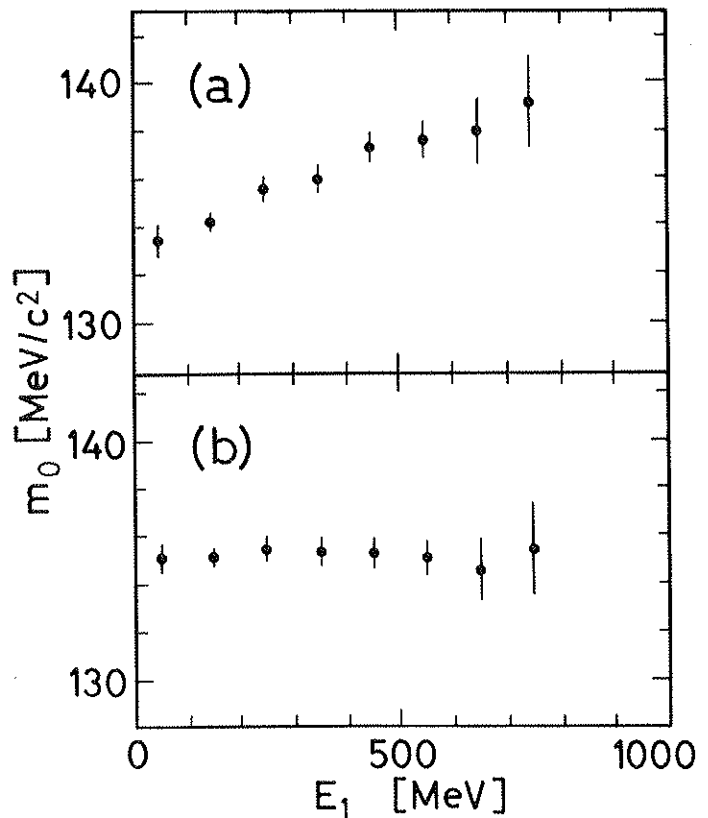


Fig. 3.57: Position m_0 of the π^0 invariant mass peak versus the energy E_1 of one decay photon. The energy of the other photon is kept between 200 and 300 MeV/c: (a) before any corrections, and (b) after correcting for non-linearities.

perfectly centred so that the radius is not exactly 80 cm. This would cause the angle subtended by each block to deviate from 3° , and subsequently introduce a systematic error into the measurement of ϕ so that the real decay angle φ always is

$$\varphi = \phi + \Delta\phi(\varphi) = \phi + \varphi \cdot \Delta\phi(3^\circ)/3^\circ. \quad (3.60)$$

The target could not be more accurately centred than ± 7 mm, which is a 1% error on the radius introducing a 1% error in the angle. Because $m_0 \sim \sin(\phi/2)$ and $d[\sin(\phi/2)] = 1/2 \cdot \cos(\phi/2) d\phi$, this systematically growing error in ϕ with ϕ will be enhanced in m_0 by the trigonometric function. Remembering that the strong correlation between the π^0 -energy $E_{\pi^0} = E_1 + E_2$ and the decay angle φ through the Jacobian peak distribution and that larger energies E_{π^0} ($\sim E_1$) correspond to smaller minimal angles ϕ , it then becomes clear that a wrongly defined geometry could result in a similar dependence of m_0 versus E_1 . It could, however, never account for all the effect observed in fig. 3.57a. Moreover, the angle can be quite well calibrated by measuring the width of the peak appearing in the spectrum of decay angles observed from η -mesons. This peak originates from the reaction $p\bar{p} \rightarrow \eta\omega$ and was already used in section 3.4.3.3 to check the position resolution of the BGO's. Fig. 3.57 shows, in fact, the result after having done this, and the effect remains.

There can be several reasons for this non-linear characteristic. It might be intrinsic in the FERA ADCs from LeCroy who cite a worst integral non-linearity of 1.7%, which is in the same order of magnitude as the observed effect, or it could be caused by the home-made electronics device serving as splitter/amplifier (see section 3.7), or perhaps it could even arise from the wedge shape of the lead-glass blocks themselves. Regardless of the reason for this non-linear performance the effect is corrected by the following expression (E_γ in MeV):

$$E'_\gamma = E_\gamma - (5.4 \cdot 10^{-3} \cdot E_\gamma)^2 + 1.8. \quad (3.61)$$

The dependence of m_0 versus E_1 after the correction is presented in fig. 3.57b.

The preliminary relative calibration of the 160 blocks of the lead-glass array was performed by using cosmic rays. Two scintillators of dimensions $1 \times 20 \times 30$ cm³ were placed horizontally in the position of the target (see fig. 3.4), and a cosmic ray event was defined by coinciding signals from one of these scintillators and a lead-glass block. Under this condition the cosmic particle is likely to pass through the block longitudinally, which is of importance because of the deposited energy being directly proportional to the track length. Only events with energy in one single block were selected in this procedure. The type of spectra obtained is shown in fig. 3.58. Particles traversing only a part of the block or stopping in it yield a continuum whereas the through-going ones appear in the peak. The energy deposited by the particles causing the peak is, on the average, equivalent to a 500 MeV shower. Depending on the orientation of the block it took between 12 and 48 hours to accumulate a peak with 60 counts, which is the minimum needed. This method is limited by statistics; relative calibration of the

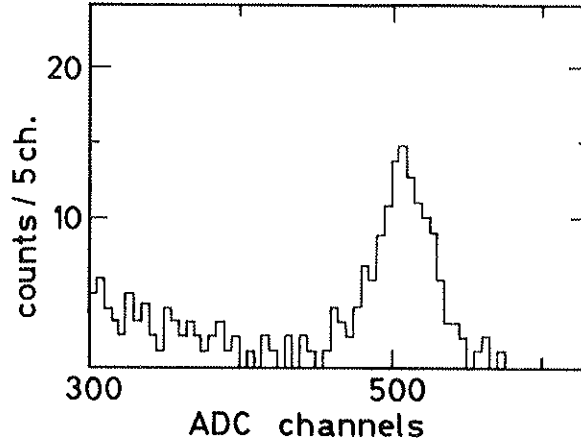


Fig. 3.58: Spectrum from cosmic rays as measured with a lead-glass block.

order of 12% was achieved this way. A secondary method employed a data sample obtained during an experimental run. A loose trigger condition was used so the array of 160 blocks should be uniformly illuminated with γ -rays. This trigger consisted of the signal from the telescope counter, informing about an incoming antiproton, and more than 10 MeV energy deposited in the lead-glass array. For each block an exponential was fitted to the upper end of the measured spectrum (fig. 3.59), and calibration constants were derived by equalizing the exponents. With this method the relative calibration could be made better than 7% around 500 MeV.

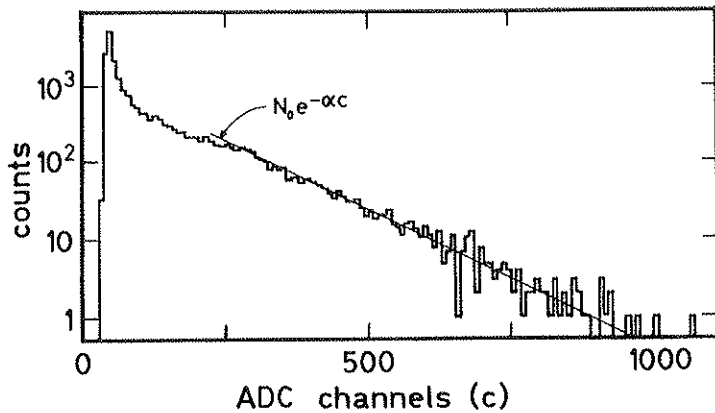


Fig. 3.59: The photon spectrum as measured with a lead-glass block shows an exponential dependence at the upper end.

During the offline analysis of experimental data the π^0 -data itself could be utilized to further adjust the calibration by creating an invariant mass plot for each lead-glass block. When a coincidence of two photons is encountered the invariant mass m_0 is computed and filled into the two histograms that correspond to the two struck lead-glass blocks where the photons entered. The events selected were restricted to those with two (or more) photons detected by the lead-glass array, and those with at least 60% of the shower energy contained in the element being plotted. Fig. 3.60 shows the position of the π^0 invariant mass peak for each of the 160 blocks. The bottom view in the figure is the projection onto the mass axis which shows how much the finite relative calibration may contribute to the energy resolution. Because

$$\frac{\delta E}{E} = 2 \frac{\delta m_0}{m_0}, \quad (3.62)$$

the calibration constants are derived easily. The peak positions after calibration give the projection onto the m_0 -axis shown in fig. 3.61. The distribution has a full-width at half-maximum of 1.8% implying a contribution to the energy resolution of 3.6%. Repeating this procedure for following spills, i.e. allowing for about an hour's drift time, shows that the calibration for an individual block typically fluctuates within $\pm 1.4\%$. In fig. 3.62 the time evolution of a block in an inner row is displayed. The blocks in the outer rows generally show somewhat larger drifts, but these are less important as they mostly contain a small fraction of the total shower energy.

*Det är mycket nedslående att leva i en tid,
där det är lättare att spränga en atom
än en fördom.*

(Albert Einstein)

Fig. 3.61: Same as fig. 3.60 after calibration.

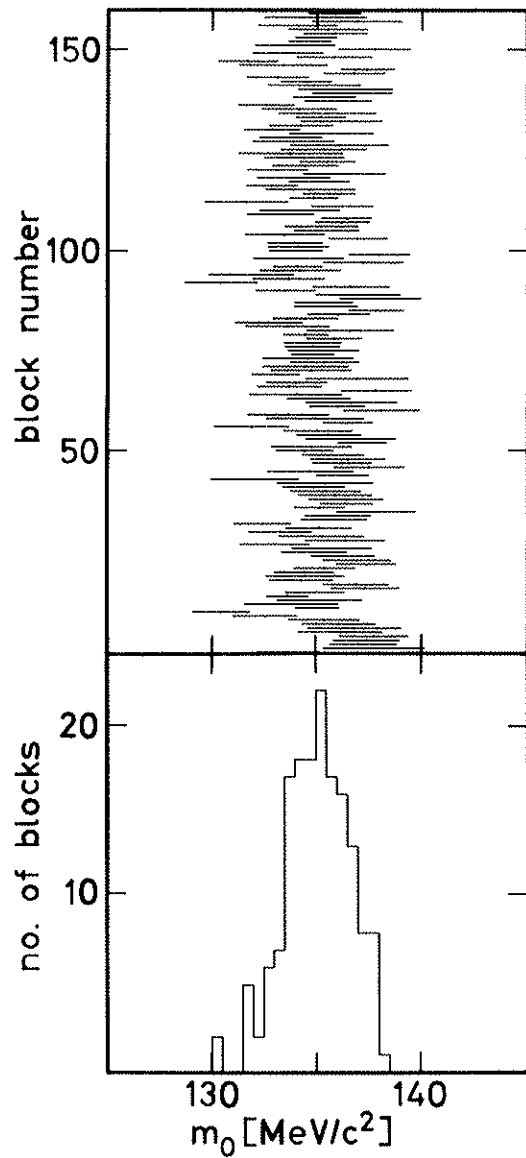


Fig. 3.60: Positions m_0 of the π^0 invariant mass peaks for the 160 individual lead-glass blocks. The projections onto the mass axis is also shown.

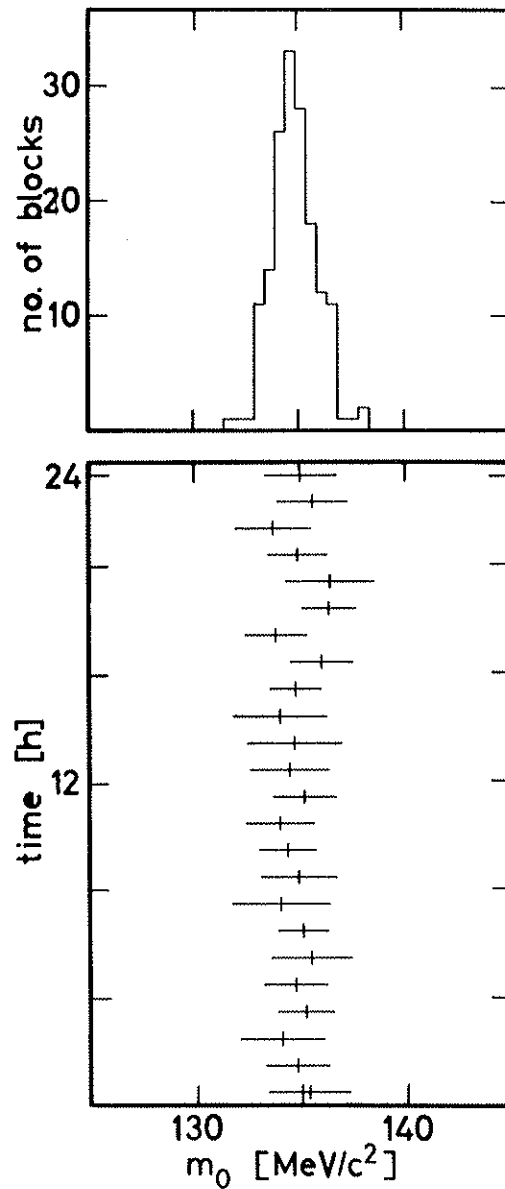


Fig. 3.62: Position m_0 of the π^0 invariant mass peak for one of the blocks in the lead-glass array versus time.

*Error is boundless.
Nor hope nor doubt,
Though both be groundless,
Will average out.*

(J.V. Cunningham)

3.5.3.2 Reconstruction of the Impact Position

The roughest estimation of the impact point of a γ -ray is just to take the centre of the block containing the maximum energy. As was demonstrated in section 3.4.3.3, a better estimation can be achieved by computing the centre of gravity of the energy sharing between neighbouring blocks surrounding the one with maximum energy. In fig. 3.63 is plotted the average number of blocks involved in a shower as a function of the sum energy of the cluster, i.e. the registered shower energy. Considering this plot, one would anticipate a position algorithm based on the centre of gravity of the energy measurement to give no better result below 100 MeV than the width of a block. The algorithm used for the lead glass is not identical to the one used for the BGOs, but still based on the same principle. The idea is simple: the impact position is displaced from the centre of the block containing the maximum energy with a fraction of the distance to the centre of the next block, that is proportional to the ratio of the energies deposited in the blocks, i.e. a linear weighting of the displacement from the centre according to the transversal energy spread. Because the shower spread is not linear (cf. Appendix B) the so obtained position is subsequently mapped according to a phenomenologically deduced lookup table. The stage of deriving the centre of gravity considers a matrix of nine lead-glass blocks covering the cluster, where the middle one contains the maximum energy. Block energies are summed by rows and by columns (the geometry is illustrated in fig. 3.64)

$$E_j^{(s)} = \sum_{i=1}^3 E_{ij}, \quad E_i^{(z)} = \sum_{j=1}^3 E_{ij}, \quad (3.63)$$

where $i, j = 1, 2, 3$. The preliminary coordinates (s' , z') of the impact point are then derived as

$$s' = s_k + \frac{E_3(s) - E_1(s)}{E_2(s)} b_s, \quad z' = z_k + \frac{E_1(z) - E_3(z)}{E_2(z)} b_z \quad (3.64)$$

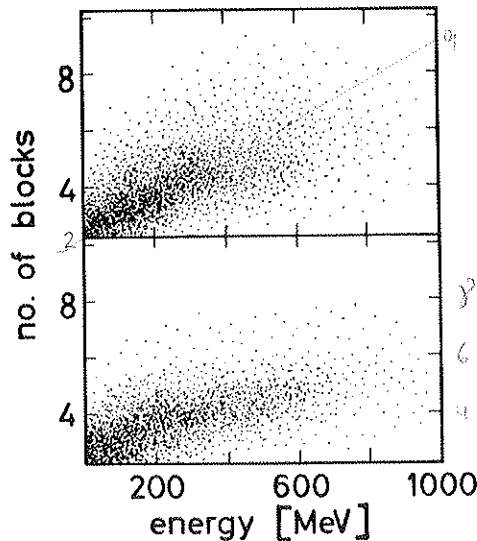


Fig. 3.63: Number of lead-glass blocks involved in a shower as a function of shower energy with a lower cutoff at 1 MeV (top view), and at 3 MeV (bottom view).

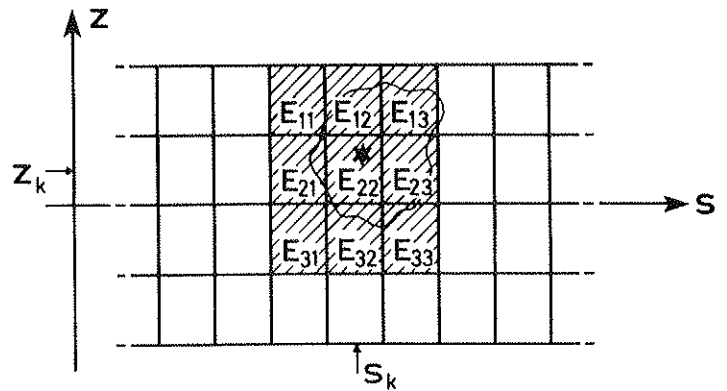


Fig. 3.64: The reconstruction algorithm for the photon impact position considers a matrix of nine lead-glass blocks around the block containing the maximum energy.

where (s_k, z_k) are the centre coordinates of block $k = 1, \dots, 160$ and $b_s = 4.2$ cm, $b_z = 6.4$ cm.

As more extensively discussed in the BGO section an algorithm of this type suffers from severe shortcomings. The problem that the estimated impact point systematically lies closer to the centre than the real one is not equally pronounced for all photon energies. The effect is most visible for the lowest energies. To try to correct for this phenomenon the final coordinates (s, z) of the estimated impact point are mapped according to weight functions $W_s(k, E)$ and $W_z(k, E)$ which depend on the shower energy E and on the block number $k = 1, \dots, 160$. The energy dependence of W_s and W_z should then emulate the nonlinear function of deposited energy versus transverse distance from the shower axis, and map the calculated impact position away from the centre. In this way the disability of the centre-of-gravity estimate to find the real impact might be improved, at least to some extent.

The weights W_s and W_z are taken from a large lookup table that has a phenomenological basis. It was established during the experimental data run when the loose trigger condition was in operation. This trigger, as already mentioned, consisted solely of the beam-telescope signal and the requirement that more than 10 MeV be deposited somewhere in the lead-glass array. The electronics device called the "Logic Unit" (see section 3.7) was, however, functioning to reject charged particles. The purpose of this trigger was that the lead-glass array should be uniformly illuminated with γ -rays. The goal was hence to construct the lookup table such that a flat distribution was obtained as response. The weights W_s and W_z have obviously to depend on which block was hit, i.e. where the struck block is situated in the array; if it is in the two central rows or in one of the edge rows. If the maximum energy was deposited in an edge block, one of the adjacent rows of the nine-block matrix (see fig. 3.64) is absent, and consequently the other energies have to be weighted differently.

In contrast to the BGO case there was no opportunity to monitor the performance of the lead-glass algorithm in an electron or photon beam. Some plots can be made from the experimental $\bar{p}$ -runs that demonstrate the collective performance of the procedure. Otherwise we are left with investigations based on simulated showers produced by Monte Carlo technique. The response induced by real γ -showers can be visualized in two plots of estimated impact points, one being a projection in s -direction, i.e. ϕ -direction, the other in the z -direction. They are presented in fig. 3.65 separated into two pairs, one for each of the energy intervals: 50–250 MeV, and 500–700 MeV. It is evident from these plots that the ability of estimating

the true impact point is defective also for this algorithm. In fact, the inherent quality of enhancing the central region of each block module is tremendously difficult to get rid of, unless the elements of the array are made in very small dimensions compared to the size of a shower. For the block size used in this experiment there will always be a considerable probability that almost all shower energy is deposited in a relatively narrower region and hardly spreads over several blocks. This class of events will of course be interpreted as having started at the centre of a block.

The spatial resolution achieved with this position-reconstruction procedure, including the systematic malfunctioning, can be observed when histogramming the angles between the γ -rays from decaying η -mesons. Because only the relative angle between the photons enters this measurement, the systematic error in each estimated impact position will randomize, and only the average appear in the resolution. This angle spectrum is similar to the one used in the BGO case but this time both photons are re-

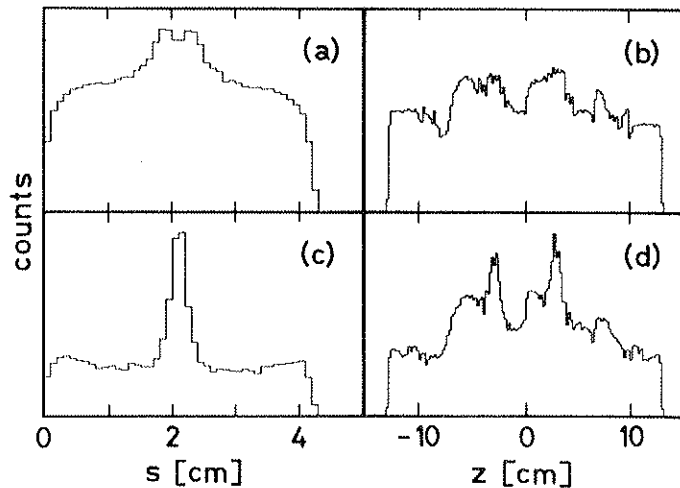


Fig. 3.65: Distributions of reconstructed coordinates s and z of the photon impact point for photon energies (a, b) of 50–250 MeV and (c, d) of 500–700 MeV.

corded by the lead-glass detector. The width of the peak from the reaction $p\bar{p} \rightarrow \eta\omega$ does therefore only contain the position resolution of the lead glass and the uncertainty in vertex definition from the finite annihilation spot. As in the BGO case (cf. section 3.4.3.3), the angle resolution full-width at half-maximum $\Gamma_\phi = 1.7^\circ$ is obtained by unfolding the Gaussian part of the peak shape (fig. 3.66). Subtracting the vertex-definition contribution quadratically, and dividing by $\sqrt{2}$ then yields the root-mean-square position resolution σ_s of the lead glass at 428 MeV:

$$\sigma_s = \frac{1}{4\sqrt{\ln 2}} \left[\left(\frac{\pi \Gamma_\phi}{180^\circ} \right)^2 (80 \text{ cm})^2 - \Gamma_x^2 \right]^{1/2} = 0.67 \text{ cm} . \quad (3.65)$$

As for the BGO, for the lead glass an ad hoc formula is assumed to describe the position resolution as a function of the photon energy [cf. eq. (3.42), on page 60]. The justification is as follows: the position reconstruction is ultimately based on energy measurements that follow a square-root law, and is given to first order by eqs. (3.63) and (3.64), which implies the errors propagate according to

$$\begin{aligned} \left(\frac{ds}{b_s} \right)^2 &= \frac{1}{E_2^2} \left[dE_1^2 + dE_3^2 \left(\frac{E_1 - E_3}{E_2} \right)^2 dE_2^2 \right] \approx \frac{1}{E_2^2} (dE_1^2 + dE_3^2) \approx 2f^2 \left(\frac{dE_1}{E_1} \right)^2 \\ \Rightarrow ds &\approx \frac{dE_1}{E_1} f b_s \sqrt{2} , \end{aligned} \quad (3.66)$$

where $f = E_1/E_2$ only varies slowly with energy since the lateral spread of a shower stays fairly constant [eq. (3.56)] with energy (cf. Appendix B). In this context it appears plausible that the function of position resolution versus energy can also show an inverse square-root dependence (E in GeV):

$$\sigma_s = \zeta \sqrt{E} = 0.44 \text{ cm} / \sqrt{E} , \quad (3.67)$$

where the second equal sign introduces the fitted value $\zeta = 0.44$ cm resulting from the measured point $\sigma_s = 0.67$ cm at the photon energy $E = 0.428$ GeV.

It is presumed, however, that the position resolution can never be worse than what is defined by the block width $b_s = 4.2$ cm, i.e. a standard deviation of $\sigma_s = b_s/\sqrt{12} = 1.2$ cm. Thus, the energy dependence is set constant up to 134 MeV and then falls off according to the inverse square-root law:

$$\sigma_s = \begin{cases} 1.2 \text{ cm} & E < 134 \text{ MeV} \\ 0.44 \text{ cm} / \sqrt{E [\text{GeV}]} & E > 134 \text{ MeV} \end{cases} \quad (3.68)$$

$\approx \frac{0.055 \text{ rad}}{\sqrt{E [\text{GeV}]}}$

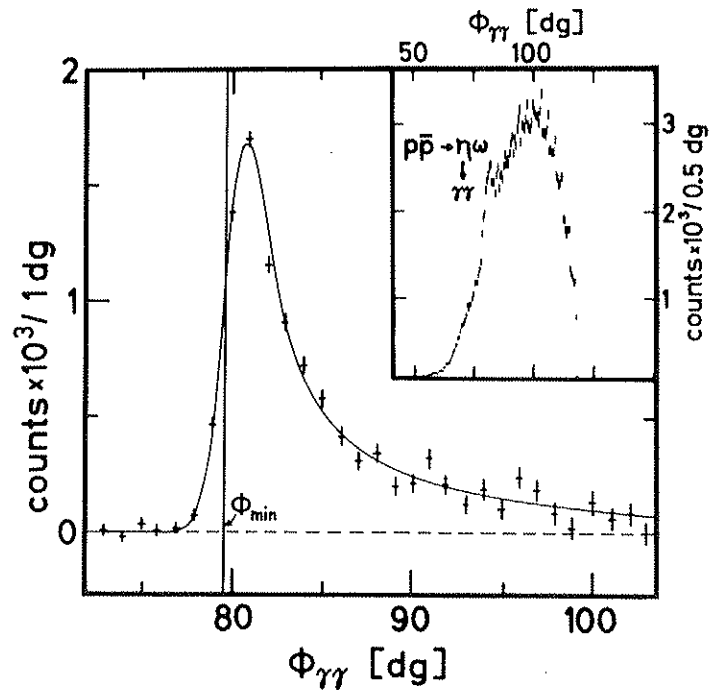


Fig. 3.66: The peak from $p\bar{p} \rightarrow \eta\omega$ in the spectrum of opening angles between photon pairs observed from η -decays. The inset shows the full spectrum. The fitted function is a folding of the Jacobian peak distribution with the detector resolution.

Whether this algorithm possesses any tendency of **global** systematic shifting, i.e. migration of events across block borders, could be investigated by covering the array, or a part of it, with lead just leaving a slit at a certain distance from the symmetry line. Such a measurement has not been conducted (but cf. also sub-subsection 3.5.3.4). Nevertheless the response could be checked for Monte Carlo simulated showers. These were produced with a comprehensive Monte Carlo program used to simulate the complete experiment¹⁶. The result is presented in fig. 3.67. According to this Monte Carlo test the position algorithm does not seem to suffer from any global shortcomings. Notice, however, the tiny tail towards the boundary of the array appearing outside $|z| = 8$ cm.

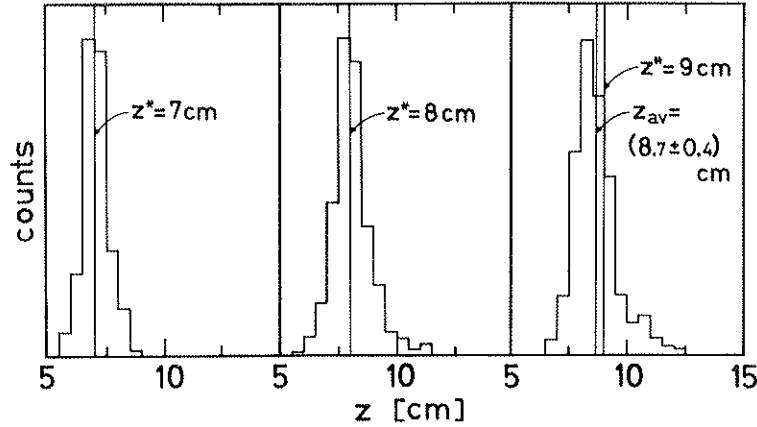


Fig. 3.67: The reconstructed z -coordinates of photon impact points z^* as resulting from Monte Carlo simulated showers.

3.5.3.3 Energy Resolution

As mentioned in the beginning of this section the energy resolution of a Cherenkov calorimeter is governed by photon statistics. It was also said that the number of photons produced per unit track length is constant, and that the total track length is directly proportional to the photon energy [eqs. (3.53) and (3.54)]. Because of the statistical phenomena that the error is inversely proportional to the square-root of the statistics, a resolution function is employed that is characterized by

$$\frac{\Gamma}{E} = \frac{\lambda}{\sqrt{E [\text{GeV}]}} \quad , \quad (3.69)$$

where Γ is the full-width at half-maximum and λ is the resolution parameter to be determined. However, when considering the resolution of a spectrum composed of entries from any lead-glass block there has also to be a constant term present, Λ , that is related to the finite (relative) calibration of the lead-glass blocks. This contribution was estimated in section 3.5.3.1, but is necessarily not as large as observed from fig. 3.61. Because a shower generally spreads over several blocks, λ will receive contributions from several blocks and is thereby also susceptible to the calibration and absorbs a part from Λ so to say. When fitting the resolution full-width at half-maximum to observed data it can therefore be reasonable to try a function of the kind

$$\frac{\Gamma}{E} = \frac{\lambda}{\sqrt{E [\text{GeV}]}} + \Lambda \quad , \quad (3.70)$$

¹⁶ This Monte Carlo program includes a subroutine [41] that simulates the transverse spread of a shower by throwing a random number according to a double exponential dependence.

where Λ and λ are to be fitted.

A quantity that is easy to measure and closely related to the energy resolution is the width of the π^0 invariant mass peak. It can be measured as a function of the π^0 -momentum. If we apply an asymmetry cut $x \leq 10\%$ between the photon energies [eq. (3.11)], we have that the π^0 -energy is $E \cong 2E_1 \cong 2E_2$ and $\Gamma_{E_1} \cong \Gamma_{E_2}$. Furthermore the decay angle is $\phi \cong \phi_{min} = 2\arctan(m/p)$. Thus, we get from eq. (3.8)

$$\left(\frac{\Gamma_{m_0}}{m_0}\right) = \left(\frac{2\Gamma_{E_1}}{E}\right)\left(\frac{\Gamma_\phi}{2}\right)\left(\frac{p}{m}\right), \quad (3.71)$$

or

$$\Gamma_{m_0}^2 = 4\Gamma_{E_1}^2(1-\beta^2) + \frac{1}{4}\Gamma_\phi^2 p^2. \quad (3.72)$$

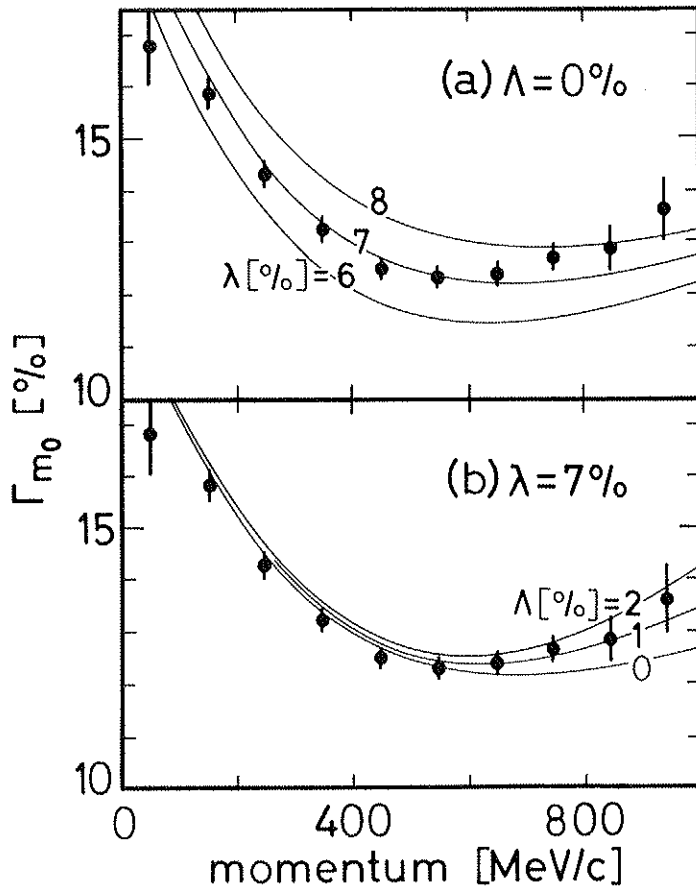


Fig. 3.68: Estimated full-width at half-maximum Γ_{m_0} for the π^0 invariant mass peak of the LL detector combination for different parameter values of the energy resolution (see further text).

If we now introduce the energy dependences of Γ_{E_1} [eq. (3.70)] and infer Γ_ϕ from the position resolution, we can plot Γ_{m_0} versus π^0 -momentum and compare to the measured values. However, to fit this analytic expression is a very tricky and tedious task, especially if the asymmetry cut is omitted when it involves an integration over the Jacobian distribution. The Monte Carlo technique comes to the rescue, and a simulation is performed of the expected width of the π^0 invariant mass peak as a function of the π^0 -momentum. Fig. 3.68a presents three curves fitted to the Monte Carlo results for three different values of the parameter λ of 6%, 7%, and 8%, respectively, Λ being equal to zero for all three cases. A comparison with the experimental data points demonstrates that λ is 6.7%. It should be emphasized that the data points from lower-momentum π^0 's are most influenced by the energy resolution [cf. eq. (3.72)]. Fig. 3.68b shows also three curves fitted to the Monte Carlo results for $\lambda = 7.0\%$ and Λ taking three different values, viz. 0%, 1%, and 2%, respectively. It seems clear that Λ cannot be larger than 2%.

The resolution enables us to estimate the number of emitted Cherenkov photons, in the same way as the number of scintillation photons were estimated in the BGO case (see page 54) The mean

value of the Poisson distribution $Po(m)$ of the photo electrons that reach the first dynode of the photomultiplier is $m = (\mu/\sigma)^2$. If we consider a 200 MeV photon we have $\mu = 200$ MeV and $\sigma = 13$ MeV, given by eq. (3.70) with the fitted parameters. Thus, the number of photo electrons is in average $m = 240$ at 200 MeV (~ 1.2 per MeV). If we assume a transmission efficiency of the Cherenkov light of 10% and a quantum efficiency of the photo cathode of 10%, the total number of Cherenkov photons obtained from a 200 MeV γ -ray is estimated to be around 10^4 . The theoretically calculated number on page SYSPAGE. is a factor of 1.7 larger but the transmission efficiency of 10% assumed above could very well be smaller by the same factor.

3.5.3.4 Detection Efficiency

The solid angle of the lead-glass array is 5.3%, but this will not be the effective solid angle as the efficiency drops at the edge of the detector. The z -dependence of the fall-off could not be investigated. A first order approximation must be that the efficiency is proportional to the depth penetrated by the γ -ray. Because the γ -rays emit from a few millimeters wide region at the target centre, they will enter with an irradiation angle around 9° close to the boundary. With the radiation length being 3.05 cm photons hitting closer than 1.5 cm to the edge will only penetrate three radiation lengths then. In addition, if the photon converts, a substantial part of the shower is lost systematically due to leakage — the event will then be rejected later by the fitting to the π^0 -decay kinematics (see further next chapter). Not to be exposed to these boundary conditions, only those photons are accepted in the offline analysis that hit further away from the edge than 3.8 cm, i.e. $|z| \leq 9$ cm (see fig. 3.4). If this 9 cm cut was sharp the solid angle would be 3.7%. It has however to be based on the reconstructed impact position which unavoidably introduces some uncertainty. To judge from the Monte Carlo investigation of the position algorithm the shifting across the 9 cm border does not seem to cause a serious effect. But nevertheless there is a hint of systematic diffusing of estimated impacts towards the edges. In addition the π^0 -efficiency ω might be diminished at larger z by the fitting of the π^0 -decay kinematics, which might systematically reject an increasing part of the events detected further off-centre in the lead-glass array.

If we presume the whole array being uniformly illuminated with γ -rays, a study can be carried out to examine the overall π^0 -efficiency regardless of any reason, but first we pay attention to the detection efficiency of single photons. For this purpose we only consider one half of the lead-glass array at a time. This means that we record only those showers firing a cluster of lead-glass blocks for which the block containing the maximum energy is found on that side of the xy -plane under consideration (fig. 3.69). For these events we compile two quantities: the energy E_{inner} being the sum of the block energies in the inner row closest to the symmetry plane, and the energy E_{outer} being the sum of block energies in the outer row at the edge. Events are then histogrammed versus the ratio

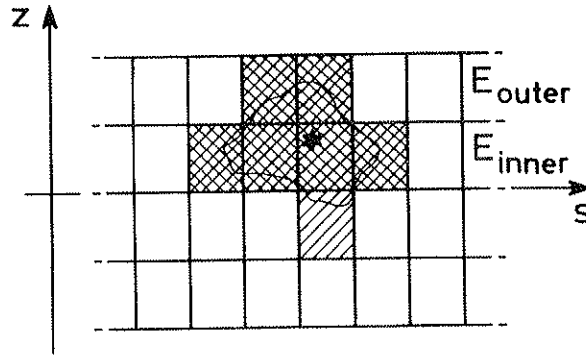


Fig. 3.69: Detection efficiency may be monitored by considering the energy of the shower deposited only in the two rows on one side of the lead-glass array

$$\xi = \frac{E_{outer}}{E_{inner} + E_{outer}}. \quad (3.73)$$

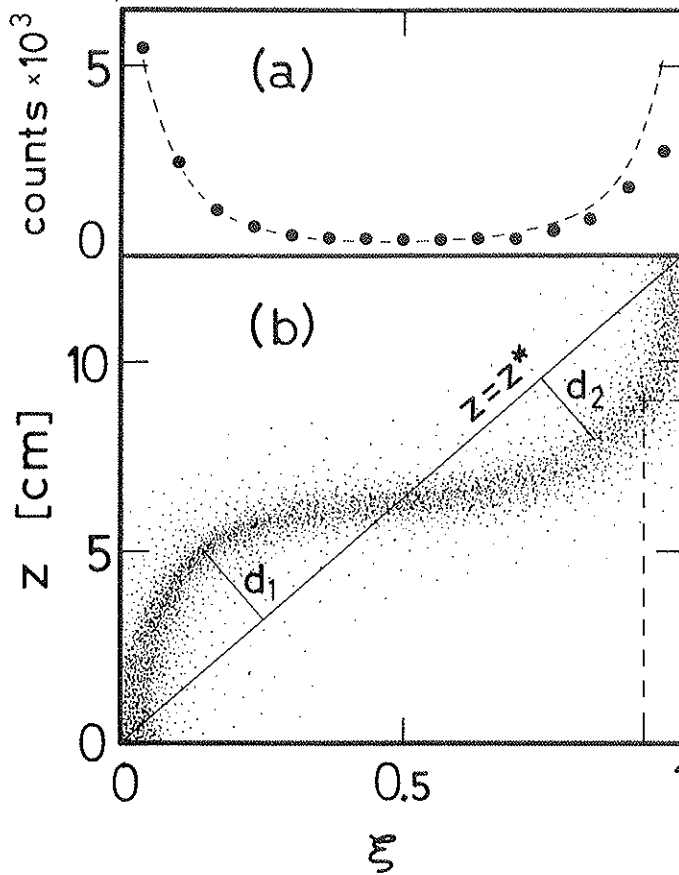


Fig. 3.70: Distribution of (b) reconstructed z -coordinates of photon impact points versus the ratio $\xi = E_{\text{outer}} / (E_{\text{inner}} + E_{\text{outer}})$ (see further text). The projection (a) onto the ξ -axis shows an asymmetry.

The result for total cluster energies between 50 and 250 MeV is presented in fig. 3.70a. Showers depositing roughly equal amount of energy in both rows are likely to have been initiated in the very vicinity of the interface between the rows. Hence $\xi = 0.5$, should on the average, correspond to the junction $|z| = 6.4$ cm, $\xi = 0$ to the symmetry line, and $\xi = 1$ to the edge. The dashed line on the right side of the histogram is a mirroring of the left half. Because the array was presumed to be uniformly illuminated, the discrepancy between the dashed line and the count rate at the upper-end part of the histogram must be due to detection inefficiency. The second question is then what value of ξ corresponds to $|z| = 9$ cm. Fig. 3.70b shows a scatter plot of the event sample in the ξ -versus- z plane. As demonstrated by the plot the distance $|z| = 9$ cm corresponds to the ratio $\xi = 0.9$. There certainly seems to be a slight tendency of shuffling events toward the edge, since the deviations d_1 and d_2 from the straight line $z = z^*$,

the true impact are, slightly different. This is, however, not significant inside $|z| \leq 9$ cm, and nor is the detection efficiency significantly different from 100% yet.

We now come to the detection efficiency for π^0 's. This efficiency depends on software cut conditions, because the measured kinematics of a registered two-photon coincidence have to be analysed and fitted to the π^0 -decay kinematics. A decision has to be taken whether the event is to be regarded as a π^0 -decay and subsequently accepted, otherwise it should be rejected. Thus, it is required that the χ^2 -value¹⁷ returned from the fit should be less than 3.8. It is expected that a larger fraction of the shower is lost when the photon impinges closer to the edge. Thus one (or both) of the photon energies entering the invariant mass m_0 [eq. (3.6)] will be too small, and consequently the invariant mass m_0 will be too small, even if the photons were correlated to the same π^0 . The π^0 -peak in the invariant mass spectrum would therefore attain a tail toward zero if events of larger $|z|$ are selected. In fig. 3.71 the π^0 invariant mass peaks for four different z -intervals, viz. $3 \leq |z| \leq 5$ cm, $5 \leq |z| \leq 7$ cm, $7 \leq |z| \leq 9$, and $9 \leq |z| \leq 11$ are compared. Apparently the peak develops a tail toward zero at larger $|z|$. The immediate consequence of this distortion of the shape is that a larger fraction of the true π^0 -decay events will be refused by the χ^2 -cut at 3.8 than what is theoretically true for a Gaussian shape, which is 5%. By

¹⁷ Topics concerning the software analysis are treated in chapter 4, but this item is anyhow mentioned here because the lead-glass detector was to a large extent used to measure coincidences of two photons.

recording the relative count rate after the χ^2 -cut as a function of $|z|$ we can monitor the size of the unfortunate lost fraction of events as a function of $|z|$, and thereby also the detection efficiency $\omega(z)$. The result is shown in fig. 3.72a for π^0 -momenta between 200 and 400 MeV/c, where the momentum distribution culminates. We see that the rejected fraction do not deviate significantly from the theoretical 5%. However, it can be suspected that this is too large a number for higher momenta, since the showers become longer for higher energies. Replaying the game for π^0 -momenta between 700 and 900 MeV/c results in the z -dependence displayed in fig. 3.72b. For these momenta it seems as additional events are lost already for $|z| > 8$ cm, which, in fact, quite accurately corresponds to what is expected from geometry if the shower penetrates the full depth of the lead-glass blocks. A calculation of the π^0 detection efficiency $\omega(p)$ including the geometrical acceptance, gives the momentum dependence presented in fig. 3.73. The vertical axis is the fraction of π^0 's detected and accepted by the χ^2 -cut relative the number produced in the target.

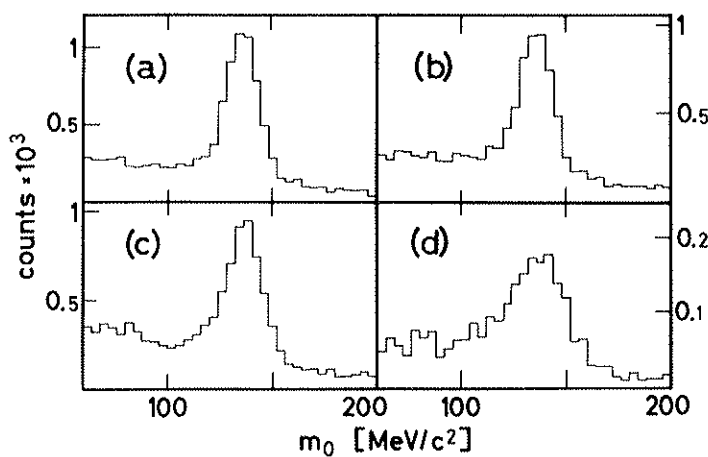


Fig. 3.71: The π^0 invariant mass peak for four different z -intervals: (a) 3–5 cm, (b) 5–7 cm, (c) 7–9 cm, and (d) 9–11 cm.

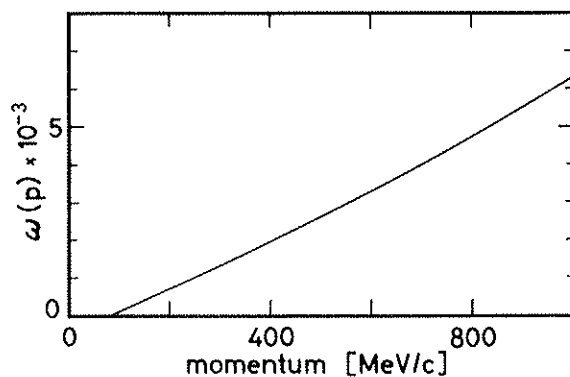


Fig. 3.73: The π^0 -detection efficiency $\omega(p)$ as a function of the π^0 -momentum.

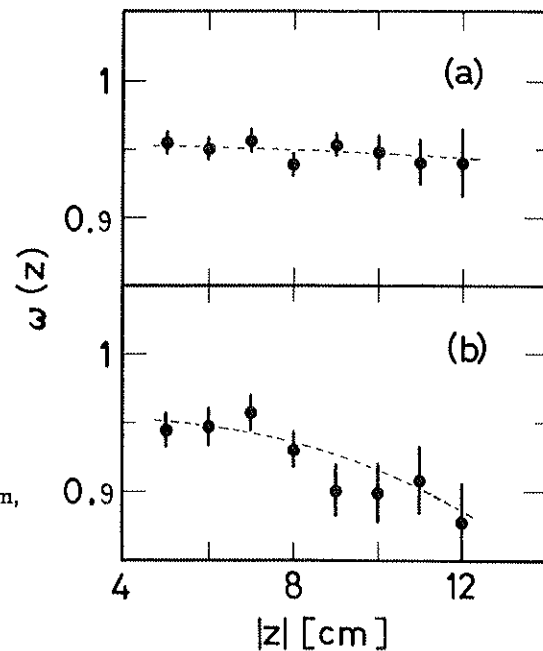


Fig. 3.72: The π^0 -detection efficiency $\omega(z)$, including a χ^2 -cut at 3.8, as a function of z -position across the lead-glass array, (a) for π^0 -momenta of 200–400 MeV/c, and (b) for π^0 -momenta above 700 MeV/c.

3.6. SPECTROSCOPY OF π^0 MESONS

There are several properties of the experimental apparatus that become important when using it for coincidence measurement of photon pairs. Such a property is, for instance, the geometry of the detector, besides the energy and angle resolutions that finally will determine the π^0 -momentum resolution. When the spectrometer covers considerably less solid angle than 4π the detectable π^0 -momentum range and the sensitivity for π^0 's, will to a large extent be determined from its geometry. Since the detector in this experiment consists of three separate parts, there are three different possibilities to detect a photon pair. However, the lead-glass array is large enough to distinguish several shower clusters which brings an additional combination. The performances of these different detector combinations are presented in the first subsection. Another important task when using the detector as a pair spectrometer is the relative positioning of the separate parts. This issue is dealt in the second subsection. The last subsection discusses the different kinds of resolution, and how they influence each other.

3.6.1 Detector Combinations

To identify a π^0 it is necessary to detect both photons from its decay, i.e. two (at least) detector signals have to be registered. By combining different detector signals, events can be classified according to logical types¹⁸ of coincidences:

- 1: LL, both photons detected by the lead-glass array;
- 2: B₁L, one photon detected by BGO₁, and the other photon detected by the lead-glass array;
- 3: B₁LL, two photons detected by the lead-glass array and identified as a π^0 -decay, and a third one detected by BGO₁;
- 4: B₂L, analogous to case 2;
- 5: BB, one photon detected by BGO₁ and the other by BGO₂.

These combinations exhibit some quite different characteristics. In particular the π^0 -momentum ranges covered are different. Also the number of π^0 's relative to the amount of background is different.

3.6.1.1 Signal-to-background Ratios

In as much as 85% of all $p\bar{p}$ annihilations the final state consists of a π^0 -meson and more than two other particles. These annihilation channels produce a continuous momentum spectrum from π^0 -mesons. In 55% of all annihilations the final state contains more than two photons. For these the possibility arises of detecting a coincidence of two photons that do not originate from the same π^0 -meson. These "fake" events of, so called, "uncorrelated coincidences" do also yield a continuous spectrum. The rate of true π^0 -coincidences recorded by a particular detector combination relative to the rate of uncorrelated coincidences is profoundly determined by the geometry, i.e. the angular interval subtended.

In the procedure to extract the true π^0 -events the first restriction applied is to require the invariant mass m_0 of the photon pair [eq. (3.6)] to be inside a window centred at the π^0 rest mass m_{π^0} . One would anticipate the relative contribution to the count rate from uncorrelated coincidences to be lower at smaller coincidence angles, since uncorrelated photons are more likely to occur with a larger relative angle, in particular when they have high energies. They will then generate an unacceptable high invariant mass [cf. eq (3.6)]. There are in addition, however, other circumstances that might influence the two count rates differently, e.g. there must not be any $\pi^\pm$ too close to detected photons (cf. the two former sections) although it is hard to imagine any obvious reason why this limitation should not

¹⁸ Note the abbreviations for the different combinations which will be frequently used further on.

strike similarly the two types of coincidences. A constraint that should not concern the true coincidences significantly, but could well affect the uncorrelated ones, is the demand of having at least three lead-glass blocks between the two maxima of two consecutive clusters — correlated photon pairs do not occur narrower than 16.5° from $p\bar{p}$ annihilations at rest.

In figs. 3.74a–c the invariant mass spectra are presented for three detector configurations, namely LL(M85), B₂L(M84), and B₁L(D83), respectively. The inset in fig. 3.74a is a magnification of the tail to visualize the peak from η -mesons. These spectra are computed according to eq. (3.6). There are the two peaks in the invariant mass spectra that arise from decays of, respectively, π^0 -mesons and η -mesons; the decay rate into the 2γ -channel being 99% and 40%, respectively. As already expected from the considerations above the configuration that covers the smallest decay angles (LL) has the smallest relative contribution from uncorrelated background.

In the offline analysis (described in chapter 4) a cut is applied in the invariant mass spectra. Only events within the windows between $\pm 3\sigma$ around the masses m_{π^0} and m_η are sustained for further evaluation; σ being the root-mean-square of the respective peak. These windows are marked as striped areas on the plots. A cut at $\pm 3\sigma$ will preserve about¹⁹ 99.8% of the true π^0 -coincidences — i.e. the events in the peak. The 2% of the good events that are rejected by this cut corresponds in numbers to about a few thousand per data sample.

A number that is of interest later is the signal-to-background ratio S/B of an event sample inside a window, both for understanding the continuous background in the momentum spectrum, as well as for the reason that the amount of uncorrelated background in a momentum histogram will reduce the sensitivity for a structure in the true π^0 -spectrum to appear significantly. To derive the signal-to-background ratios a wider interval of $\pm 7\sigma$ around the mass values is considered. By fitting a function that is a sum of a Gaussian and a second-order polynomial the ratio between the areas above and below the polynomial and between the $\pm 3\sigma$ ordinates can be calculated. Fig. 3.75 illustrates an example [LL(M85) spectrum] of this fitting procedure.

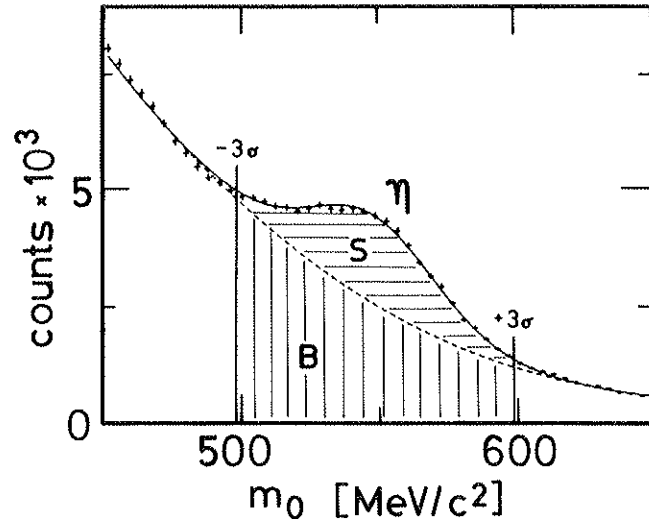


Fig. 3.75: By fitting a Gaussian plus a polynomial to the $\pm 7\sigma$ region around an invariant mass peak the signal-to-background ratio S/B is determined for the event sample inside the 3σ -limits.

¹⁹ It is "about" 99.8% because the peaks are not perfect Gaussians, but are slightly asymmetric as was pointed out in the former section. The skewness is typically only $\xi = -5\%$ which does not alter the preserved number of events. I then use the definition [26] of the skewness as the third moment with respect to the mean value normalized to the standard deviation to the third power:

$$\xi = 1/\sigma^3 \int (x - \bar{x})^3 f(x) dx, \quad (3.74)$$

which in this case becomes

$$\xi = 1/\sigma^3 \int (m_0 - m_{\pi^0})^3 G(m_0) dm_0, \quad (3.75)$$

where $G(m_0)$ is a fitted function composed of two halves of different Gaussians $G_1(m_{\pi^0}, \sigma_1)$ and $G_2(m_{\pi^0}, \sigma_2)$, having the same mean value and scale, but of different width.

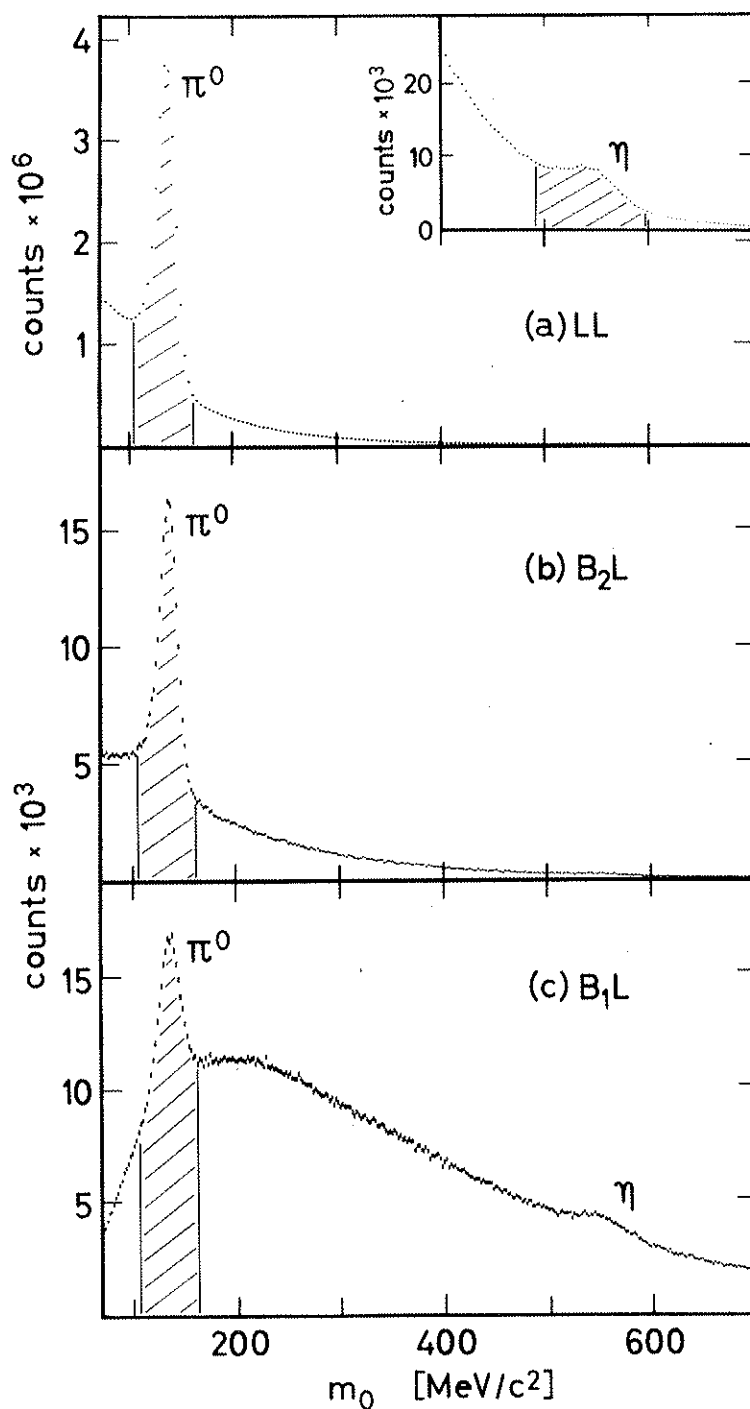


Fig. 3.74: Invariant mass spectra from the three detector combinations (a) LL, (b) B_2L , and (c) B_1L . The inset in (a) is a magnification of the tail.

3.6.1.2 Momentum Ranges

Another important consequence of the detector geometry is the maximum and minimum momenta that are possible to observe. When referring to the momentum "range" of a particular detector combination we understand the momentum interval for which the minimum angle, given by eq. (3.5), is subtended. Higher energy π^0 's outside the actual range can, however, be detected with lower efficiency owing to the tail of the Jacobian peak distribution that extends to 180° . When looking at the π^0 -momentum spectra (presented in the next chapter) the tail of higher momentum π^0 's can seem quite large. This is to a large extent due to uncorrelated events, which of course do not have to preserve the same kinematical restrictions. In table 3.5 the momentum ranges of the different detector configurations arranged in different run periods are listed. The BGO calorimeters are mounted on a ring surrounding the target and can be moved to any angle relative to the lead-glass array. They were placed differently in the different run periods to optimize for different π^0 -momenta.

As pointed out earlier it is also possible to identify η -mesons through their decay into two photons (40% branching ratio). Subsequently the momentum spectra of the η 's can be displayed. Analogous to the π^0 -momentum range of a detector configuration there is an η -momentum range of that same configuration. The η -range will obviously not cover the same momenta, but lie at higher momenta due to the larger rest mass of the η -meson. The η -ranges are also listed in table 3.5 for the relevant detector combinations. Not all combinations are useful for η -detection, because the efficiency for that momentum range is too small to yield significant statistics in a momentum display, or there might hardly be any η -mesons produced at those momenta.

Table 3.5: Momentum ranges for π^0 's and η 's of different detector combinations.

Det. comb.	Run per.	π^0		η	
		min	max	min	max
LL	A84,M85	82	1025	336	4168
B_2L	D83	44	573	180	2330
	M84,A84	60	1053	246	4284
B_1L	D83	0	100	0	410
	M84	0	216	0	880
	A84	0	75	0	307

Cuts in the offline analysis diminish the azimuthal angle accepted by the lead-glass array by roughly half a block at each end of the array. Thus, the ranges in table 3.5 are calculated with an effective azimuthal angle of the lead glass running from -58.5° through $+58.5^\circ$.

3.6.2 Alignment and Angle Calibration

The angle ϕ between two photons detected by the lead-glass array is derived by measuring the arc elongation between the estimated impact positions. The azimuthal angle is then obtained from the radius of the cylindrical surface the lead glass follows (see further section 4.1.2.1.2). All geometrical distances are accurately measured and do not introduce significant uncertainties. Also the target and the telescope scintillator can be well centred in the setup. The telescope scintillator is, however, significantly larger than the beam diameter which is why the annihilation region, i.e. the γ -source, really is defined by the beam location. Because the quadrupoles that focus the beam are adjusted before each run period, and the beam spot is not measured after that, the beam can in fact pass the telescope scintillator at an unknown position off the centre. But it is of course confined by the size of the telescope scintillator which is 2 cm in diameter. This means that the annihilation region, that has a full-width at half-maximum of 9 mm, can perfectly possibly be displaced by something like 5 to 7 mm from the centre of the target without the trigger rate going down, which would introduce a systematic error in the angle measurement. How large error this systematic shift of the emission source of the photons introduces in the measurement of the decay angle ϕ , depends on the orientation of the displacement relative to the directions of the photons. Thus, it will not be sufficient to check one single calibration point to uncover this systematic error.

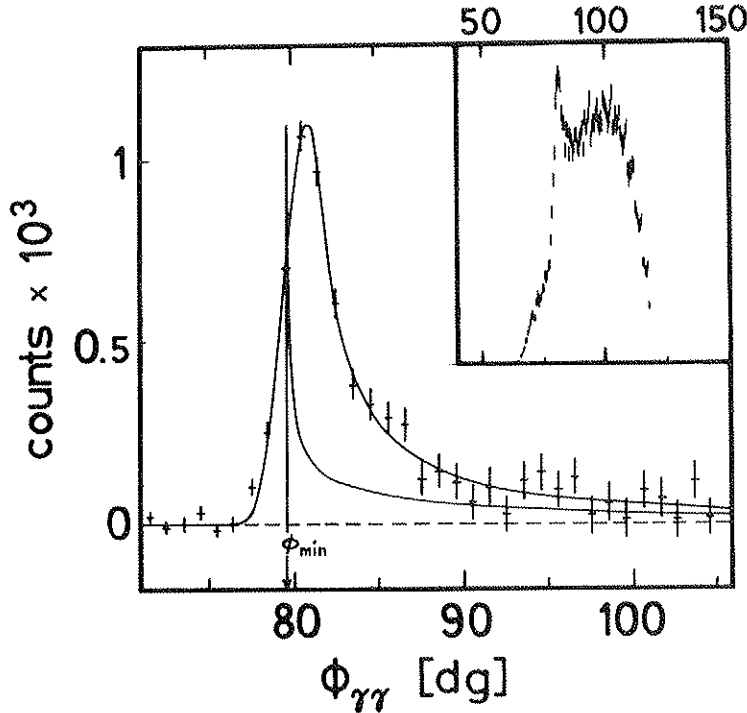


Fig. 3.76: The peak from $p\bar{p} \rightarrow \eta\omega$ in the spectrum of opening angles between photon pairs observed from η -decays after fitting of the π^0 -decay kinematics. The inset shows the full spectrum. The fitted function is a folding of the Jacobian peak distribution with the detector resolution.

The scale can be fixed at 79.8° by investigating the spectrum of decay angles from η -mesons. In this spectrum there is a sharp peak originating from monoenergetic η 's produced in $p\bar{p}$ annihilations through the reaction $p\bar{p} \rightarrow \eta\omega$ which yields η 's with momentum $p_\eta = 656$ MeV/c and a minimum opening angle $\phi_{min} = 79.8^\circ$. Fig. 3.76 shows this peak with a fitted function, which shows that the peak position can be determined to an accuracy of a few tenth of a degree. Notice that the detector resolution shifts the peak position. If there is any systematic error in the measurement of the kind outlined above, it could be revealed by comparing the position of the π^0 invariant mass peaks produced from different event samples separated by angular intervals. It is, however, not possible to slice up the spectrum of decay angles, since events in one and the same bin then can have been detected at different places in space. The invariant mass spectra displayed in fig. 3.77a are produced from π^0 -events for which γ_1 was entering the lead glass at $\varphi_1 \in [-8^\circ, 8^\circ]$, i.e. in the middle of the ar-

ray, and γ_2 struck at $\varphi_2 \in I_i$ ($i = 1, 2, 3$), where $I_1 = [15^\circ, 20^\circ]$, $I_2 = [35^\circ, 40^\circ]$, and $I_3 = [50^\circ, 55^\circ]$. The effect is most contrasting when the influence from the energy resolution is minimum, which is the case when the two photons share the π^0 -energy equally. Therefore, these plots were produced under the condition $x = (E_1 - E_2)/E_{\pi^0} \leq 10\%$. No systematic effect is observed.

When it comes to the B_1L coincidences this method is not so applicable, whilst still being so for B_2L events. Because BGO_1 was placed at a large angle relative to the lead glass, the invariant mass will be almost insensitive to a small discrepancy in angle owing to the coefficient $[2\tan(\phi/2)]^{-2}$ in eq. (3.8) that vanishes as ϕ grows. Another procedure was adopted which employs the event sample of triple coincidences B_1LL . By detecting a π^0 in the lead glass and a third photon opposite in BGO_1 one is sensitive to the reactions $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0\gamma$. One is also exposed to other processes, but when requiring the π^0 -momentum to be larger than 850 MeV/c and the γ -energy in the BGO_1 to be above 850 MeV, only these two types of decays should survive, and from the $2\pi^0$ decays only those when the BGO_1 photon was extremely forward boosted. The largest decay angle θ_1 to the π^0 -direction is for $E_1 = 850$ MeV:

$$\theta_1 = \arccos \left\{ (1/p_{\pi^0}) [E_{\pi^0} - m_{\pi^0}^2/(2E_1)] \right\} = 2.6^\circ. \quad (3.76)$$

The alignment of BGO_1 relative to the lead-glass array appears to be good as shown by fig. 3.78, where the position on the lead-glass surface towards which the π^0 -direction points is plotted.

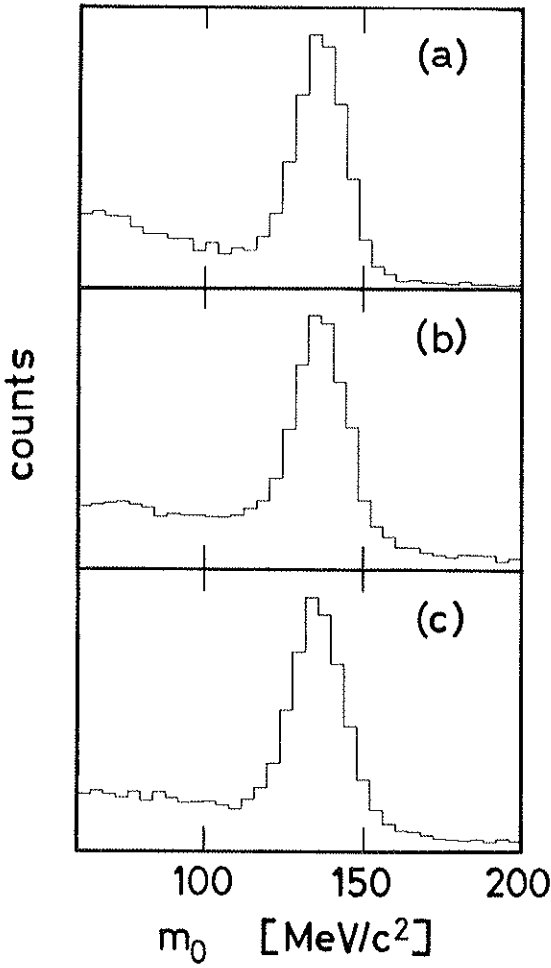


Fig. 3.77: π^0 invariant mass peaks for three different azimuthal-angle intervals: (a) $15^\circ - 20^\circ$, (b) $35^\circ - 40^\circ$, and (c) $50^\circ - 55^\circ$.

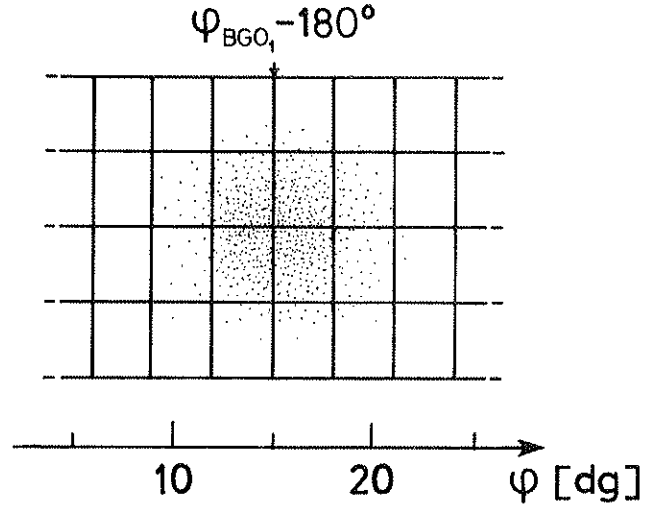


Fig. 3.78: Distribution of positions in the lead-glass surface appointed by π^0 's for which a third photon was detected by BGO₁ in the opposite direction.

3.6.3 Momentum Resolution

The π^0 -momentum spectra are extracted by a least squares fit of the kinematics. In this procedure the three measured quantities $x = (E_1, E_2, \phi)$ are fitted to combine to the correct invariant mass m_{π^0} as the invariant mass of a photon pair generally differs from this value:

$$4E_1E_2\sin^2(\phi/2) = m_0^2 \neq m_{\pi^0}^2. \quad (3.77)$$

Also for correlated photons this is mostly the case due to detector resolutions. The fitting program therefore adds a small correction to the observed values

$$\begin{aligned} E_1' &= E_1 + \Delta E_1 \\ E_2' &= E_2 + \Delta E_2 \\ \phi' &= \phi + \Delta\phi. \end{aligned} \quad (3.78)$$

The sizes of the corrections Δx depend on the respective errors attached to the different numbers i.e. the resolutions. A number that is accurately measured, e.g. a large angle ($\sim 1\%$), will be relatively less

adjusted in fitting it to yield $m_0 = m_{\pi^0}$ than a worse measured quantity as, e.g., a small photon energy in the lead glass ($\sim 20\%$). More precisely, Δx_i does not depend only on the resolution σ_i of that particular distribution, but rather on the relation between the $\{\sigma_i\}_1^3$, i.e. $\Delta_i = \Delta_i(\sigma_1, \sigma_2, \sigma_3)$. So to obtain the best possible fit, i.e. the best estimation of the real values of the energies E_1 and E_2 before resolution smearing, one needs precise knowledge about the errors $\sigma = (\sigma_1, \sigma_2, \sigma)$ of which the inverse σ^{-1} are used in weighting the measurements x . The fitted numbers $x' = (E_1', E_2', \phi')$ are then used to compute the π^0 -momentum $p = \sqrt{(E_1' + E_2')^2 - m_{\pi^0}^2}$. If the evaluation concerns η 's, the fit is of course to $m_0 = m_\eta$ instead.

Energy resolutions for any detector and any photon energy are well known from the former sections and will not be further mentioned (refer to sections 3.4.3.2 and 3.5.3.3). The angular resolution, however, needs some further comment. It is determined from the uncertainty in defining the directions of the two photons. Their directions are given by the emission point and their respective impact points on the detectors entrance surfaces. Because of the short lifetime of the π^0 its range is completely negligible compared to the dimensions of the annihilation region. The photon emission source is then the annihilation region in the target and thus has a root-mean-square width of 0.34 cm in the plane transverse to the beam. The accuracy with which the impact point of a photon can be determined depends on how it was detected. When a photon is detected by the lead-glass array the location μ of its impact is undefined within an assumed Gaussian distribution $G(\mu, \sigma)$, where the standard deviation $\sigma \approx 0.7$ cm. The width of the Gaussian varies somewhat with the photon's energy, but is about 0.7 cm for the spectral maximum. Similarly, when the γ -ray is detected by a BGO calorimeter the error in impact position is around 0.5 cm (σ) at the spectral maximum, but is in fact also energy dependent. For the photons that are detected by a BGO hodoscope the impact position is more accurately estimated²⁰ with a σ of only 0.3 cm. An important difference is that the hodoscope measures the impact equally accurately for any photon energy. The resolution yielded by the centre-of-gravity algorithm is on the other hand nearly a factor of 2 larger at low photon energies around 100 MeV. When estimating the importance of the impact definition one has also to take into account the shape of the γ -spectrum, i.e. at which energies most photons occur from $p\bar{p}$ annihilations at rest, which is at low energies where the centre-of-gravity method is inferior to the hodoscope measurement.

There are, however, other concerns that might be more important to the π^0 -measurement, because what finally is wanted, besides good momentum resolution, is to achieve high sensitivity in the inclusive π^0 -momentum spectrum. For this purpose the lower efficiency of the hodoscope (cf. section 3.4.3.4), which decreases the statistics, can be of major effect. One can also fear that the profit from the accurate hodoscope position is, at least partially, wasted by its negative influence on the energy resolution of the system.

The data sample that remained after the cut imposed on the invariant mass distribution, was subsequently processed by a program that calculated a fit of the kinematics to the hypothesis that it should render the right invariant mass. To execute this procedure in a proper way the correct errors in each particular measured quantity have to be provided. Thus, much emphasis has been put on this point; see further section 4.1.2.1. Here I will make an outline of how the experimental uncertainties interlace in the kinematic fitting to yield the final resolution of the π^0 -momentum, which is computed from the adjusted photon energies and the known rest mass $m_{\pi^0} = 134.96$ MeV. Exactly how the different errors affect each other is hard to penetrate, but the improvement of the momentum resolution is apparent from a comparison of figs. 3.79a and 3.79b, where it is striking how much sharper the " $\rho\omega$ -peak" has become after the fitting. The improvement can amount to as much as a factor of 3 for LL coincidences relative to just using the raw measured photon energies. Instead of trying to make a strict mathematical derivation, I will cite a few typical cases and some results from Monte Carlo investigations. In these cases, below, it is assumed that the π^0 decays symmetrically, i.e. $E_1 = E_2$:

²⁰ A precaution should be made for the kinematic fitting, because of the different shape of the hodoscope distribution, which is quite square-like, i.e. almost 100% of the distribution is confined to the full-width at half-maximum $\Gamma_b = b = 1.0$ cm, whereas only 83% of a Gaussian distribution is within $\Gamma_G = 2\sqrt{2\ln 2} \sigma = 1.3$ cm. This can cause a larger tail of the χ^2 -distribution than theoretically expected. This could, however, not be resolved experimentally from the background.

- case 1: a small angle LL event has $\Delta E/E \approx 0.51\Delta\phi/\tan(\phi/2)$ which implies that the angle is slightly improved;
- case 2: a large angle LL event has $\Delta E/E \approx 5.3\Delta\phi/\tan(\phi/2)$ which implies that the energies are improved;
- case 3: a small angle B₂L event has $\Delta E_1/E_1 \approx 0.16\Delta\phi/\tan(\phi/2)$ and $\Delta E_2/E_2 \approx 0.45\Delta\phi/\tan(\phi/2)$ which implies that the angle and the lead-glass energy are slightly improved;
- case 4: a large angle B₁L event has $\Delta E_1/E_1 \approx 3.4\Delta\phi/\tan(\phi/2)$ and $\Delta E_2/E_2 \approx 15.11$ which implies that the lead-glass energy is substantially improved.

By assuming energy resolutions according to eqs. (3.36) and (3.70), and by inferring angular resolutions from the heuristic eqs. (3.42) and (3.68) the response of the kinematic fitting can be monitored as a function of the decay angle ϕ between the two photons. The decay angle is $\theta = \vartheta_1 + \vartheta_2$ where ϑ_1 and ϑ_2 are the angles between respective photon and the π^0 -direction measured in the lab frame which are given by relativistic kinematics

$$\cos \vartheta_{1,2} = (1/p_{\pi^0}) [E_{\pi^0} - m_{\pi^0}^2 / (2E_{1,2})] \quad (3.79)$$

For a particular decay angle θ the two energies E_1 and E_2 are uniquely defined:

$$E_1 = (E_{\pi^0}/2) [1 + \sqrt{1 - (m_{\pi^0}/E_{\pi^0} \sin \theta/2)^2}] , \quad (3.80.)$$

where we conventionally have chosen γ_1 to be the forward boosted photon. The energy of the other photon is then given by energy conservation $E_2 = E_{\pi^0} - E_1$. The four calculated variables $\bar{x} = (E_1, E_2, \varphi_1, \varphi_2)$, where φ_1 and φ_2 are the azimuthal angles in the coordinate system defined in fig. 3.4, $\phi = \theta = |\varphi_1 - \varphi_2|$ being the angular elongation between the photons, are then "smeared" according to a Gaussian distribution $G(\bar{x}_i, \sigma_i)$, where $\bar{x}_i$ is the computed number and σ_i the appropriate standard deviation. The smearing is carried out by throwing a random number²¹ R_U uniformly between 0 and 1 and numerically calculating the Gaussian random number R_G as

$$R_G = F^{-1}(R_U), \quad (3.81)$$

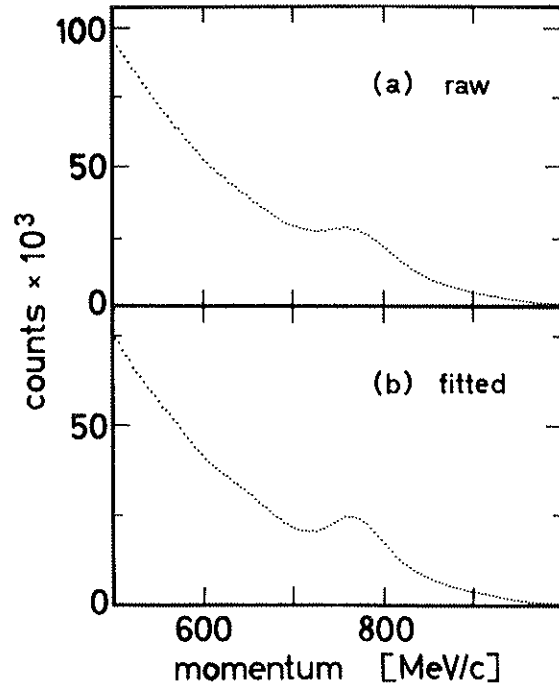


Fig. 3.79: Comparison of the peaks produced from π^0 's recoiling from ρ^0 -mesons and ω -mesons as obtained (a) directly from the measured photon energies, and (b) from the energy values given by the fitting of the π^0 -kinematics.

²¹ The IBM computer random-number generator produces a well scattered series of uniformly distributed pseudo-random numbers with a high repetition frequency of the order of 10^{38} .

where $F(R_G)$ is the cumulative distribution²²

$$F(R_G) = (1/\sqrt{2\pi}) \int_{-\infty}^{R_G} e^{-(x-\bar{x})^2/2\sigma_1^2} dx. \quad (3.82)$$

The smeared variables x' are subsequently subject to the kinematical fitting, and the impaired numbers are again amended to the output variables x'' . From these the two energies E_1'' and E_2'' are used to calculate the π^0 -momentum

$$p_{\pi^0}'' = \sqrt{(E_1'' + E_2'')^2 - m_{\pi^0}^2}. \quad (3.83)$$

These momentum values will build up a distribution to which a Gaussian $G(p_{\pi^0}'', \sigma_p'')$ can be fitted. The standard deviation $\sigma_p'' = \sigma_p''(\phi)$ of this fitted function is then compared to the standard deviation σ_p' of the "raw" π^0 -momentum computed from the smeared energies E_1' and E_2' . In a similar manner the amelioration of the π^0 -momentum by fitting the kinematics can be investigated for the two detector configuration B₂L and B₁L. The result of this investigation is that the largest improvement is obtained for large angle LL events and B₁L events. For LL coincidences the improvement is between a factor of 2 and 3 over its entire momentum range. The improvement for B₂L coincidences is not significant.

Taking the root-mean-square of the function $\sigma_p''(\phi)$ with the integral extending over the angle interval $[\varphi_1, \varphi_2]$ subtended by the considered detector configuration, and with the Jacobian peak distribution $J(\phi)$ as a weight function then gives the momentum resolution independent of opening angle ϕ :

$$\sigma_p'' = \frac{\int_{\varphi_1}^{\varphi_2} \phi \sigma_p''(\phi) J(\phi) d\phi}{\int_{\varphi_1}^{\varphi_2} \sigma_p''(\phi) J(\phi) d\phi}. \quad (3.84)$$

By using the Monte Carlo technique above, the momentum resolutions for π^0 's and η 's has been simulated as functions of the momentum. The result for LL coincidences is shown in fig. 3.80. The momentum resolutions for the other coincidence types are easier to estimate due to the influence of the energy resolution of the BGO. The curves in fig. 3.80 can be compared to the signals from the reactions $p\bar{p} \rightarrow \pi^0 + (\rho^0, \omega)$ and $p\bar{p} \rightarrow \eta + (\rho^0, \omega)$. The $\rho\omega$ -peak in the π^0 -spectrum is dominated by recoils from ρ^0 -mesons which gives π^0 's of momentum 773 MeV/c, while the $\rho\omega$ -peak in the η -spectrum is predominantly arising from ω events that yield η 's of 656 MeV/c momentum. The full-width at half-maximum of this latter peak is 6% which directly gives the momentum resolution because the ω -resonance is narrow (9 MeV/c²). The peak in the π^0 -spectrum is, however, substantially broadened by the intrinsic width of the ρ -resonance (154 MeV/c²). If we assume a classical²³ Breit-Wigner shape

²² If, in another application, F^{-1} would not exist or be too complicated, but F could be split into n terms F_l which are invertible, a useful trick to compute a random number R_F that will generate the correct frequency function is to calculate R_F from each term F_l at a time. Which l to choose is then also generated randomly, and the frequency with which each l occurs is proportional to the coefficient c_l in the sum:

$$F = \sum_{l=1}^n c_l F_l$$

If this possibility would not be feasible either, one can always use the "hit-or-miss" Monte Carlo technique, i.e. throwing a position uniformly inside the smallest square that covers the whole function F , and reject the number if it is above the curve and keep it if it is below.

²³ Assuming a classical Breit-Wigner is not entirely correct, but is sufficient here as it gives significantly the same width.

for the ρ^0 rest-mass distribution $m \in BW(m_\rho, \Gamma_\rho) = BW(769, 154)$ [MeV/c²], the momentum spread of the recoiling pion can be estimated

$$\Delta p/p \sim \Gamma_\rho/(2m_\rho) \Rightarrow \Delta p \approx 77 \text{ MeV}/c. \quad (3.85)$$

The contribution to the peak width from the experimental resolution can hence be unfolded by fitting a function being a convolution of a Gaussian $G(p, \sigma)$ representing the experimental resolution, and a Breit-Wigner $BW(773, 77)$, representing the line shape. The resultant full-width at half-maximum of the resolution is 7%.

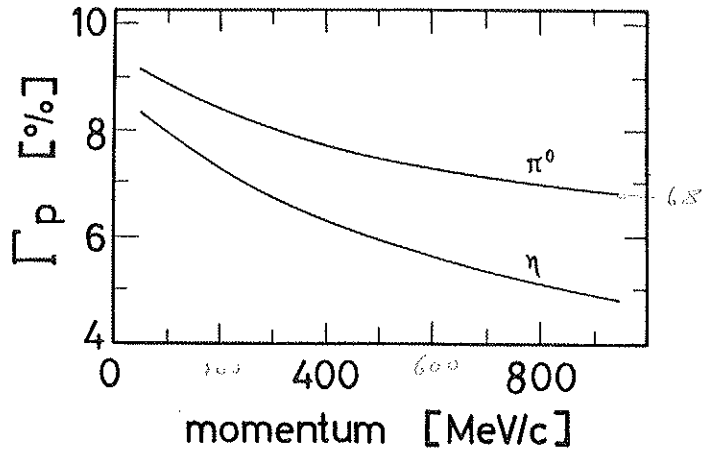


Fig. 3.80: Computed full-width at half-maximum Γ_p of the momentum resolution for π^0 's and η 's detected by the lead-glass array as a function of momentum, respectively.

3.6.4 Detection Efficiency

A variety of mono-energetic π^0 -decays appear isotropically in the centre-of-mass system, but are characterized by eq. (A.4) in the lab frame. Thus, the laboratory acceptance of π^0 's having a particular momentum cannot be derived from the solid angles of the detectors in any simple way. The detector efficiencies as functions of impact position have been deduced in sections 3.4 and 3.5, and subsequently the equivalent geometrical acceptances were estimated. If we adopt these dimensions, the "geometrical acceptance" for π^0 's and η 's can be computed by using Monte Carlo techniques. The result is shown in figs. 3.81a–f. As already indicated in the introduction of this section the acceptances do certainly depend on the geometry. The vertical axis is the percentage of π^0 's or η 's detected relative to the number produced in the target. The branching ratio $R(\eta \rightarrow 2\gamma) = 0.4$ is not included in these acceptances. The rapid drop-off some of the detector combinations show towards lower momenta is due to decays going out of range, i.e. the minimum decay angle is not

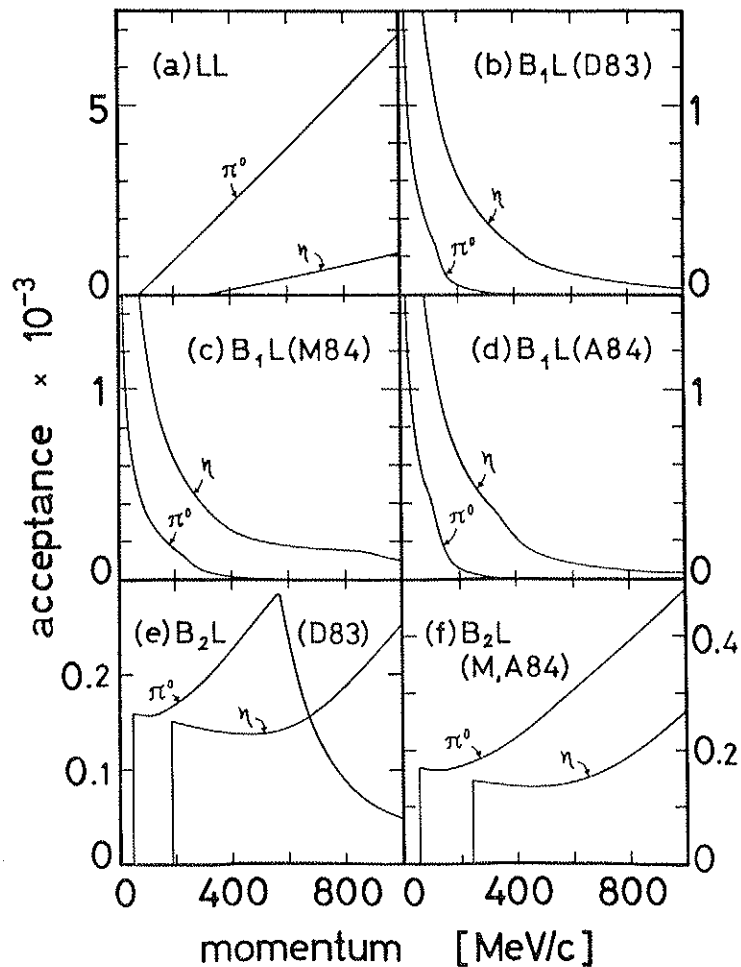


Fig. 3.81: Geometrical acceptances for detection of π^0 's and η 's by different detector combinations.

subtended any more. There is also a minimum angle covered by the combination, producing the less steep fall-off towards higher momenta, where the accepted decays occur in the tail of the Jacobian peak distribution. Caution: it should be emphasized here that the geometrical acceptances of π^0 's and η 's, presented here, are strictly valid only for "single-particle" events (and perfect detection) when there are no other particles present that can cause veto signals or evoke other quenching phenomena that might deplete the π^0/η yields preserved by cut conditions.

3.7. DATA ACQUISITION SYSTEM

In this section the data-acquisition system is presented, basically in the form of flow-chart diagrams. The general structure of the data-taking system is presented in the first subsection, and then follows two subsections that describe, in somewhat more detail, the BGO subsystem and the lead-glass subsystem, respectively.

General Architecture

The general organization of the system is presented in fig. 3.82. The main principles of the information flow are as follows. Electrical signals are generated by photomultipliers and attached base circuitry in the experimental area and sent through about 30 m long coaxial cables (type BNC) to the counting room. Signals are there split (at least) two-fold, one part is going to an ADC via 160 ns delay cables while the other part is fed to the fast trigger electronics. The trigger logic (fig. 3.83) is assembled from standard electronics units, except for a couple of devices that are home-built. One of these is the primary splitter/amplifier for the 160 lead-glass signals, the other is a unit named the "Logic Unit" (LU) which determines the number of neutral showers in the lead-glass array. After the signals have been digitized by ADCs, situated in CAMAC²⁴ crates, the data words are read out by a dedicated CAMAC crate controller denoted "ICC". This device is one part of a readout system that was specially developed for this experiment [140].

The ICC operates a memory buffer of FIFO type that is situated in a separate crate. Thus, the ICC communicates with the host computer, a VAX-11/750 that is the final station of the data-acquisition system, via a bus to the FIFO memory which is CAMAC external. The ICC, which is programmable from the host computer, puts the information read in from ADCs, TDCs, and PUs together with a few additional words into a defined event format (fig. 3.84), which typically embraces 30 to 40 16-bit words. This "event" is then transferred to, and buffered in, the FIFO memory, which can hold in total 65,000 16-bit words of information. When the FIFO memory is filled to 67% it sends an "interrupt" signal via CAMAC to the host computer, i.e. the VAX-11/750. It takes about 1 ms before the VAX responds to an interrupt and has established a DMA transfer. The FIFO buffer is then emptied at a rate of 1 μ s per word, which is done without disabling the external connection to the ICC.

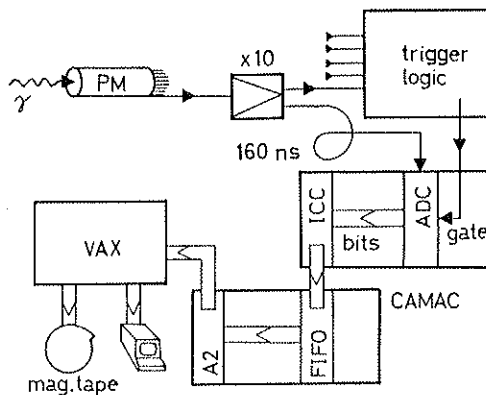


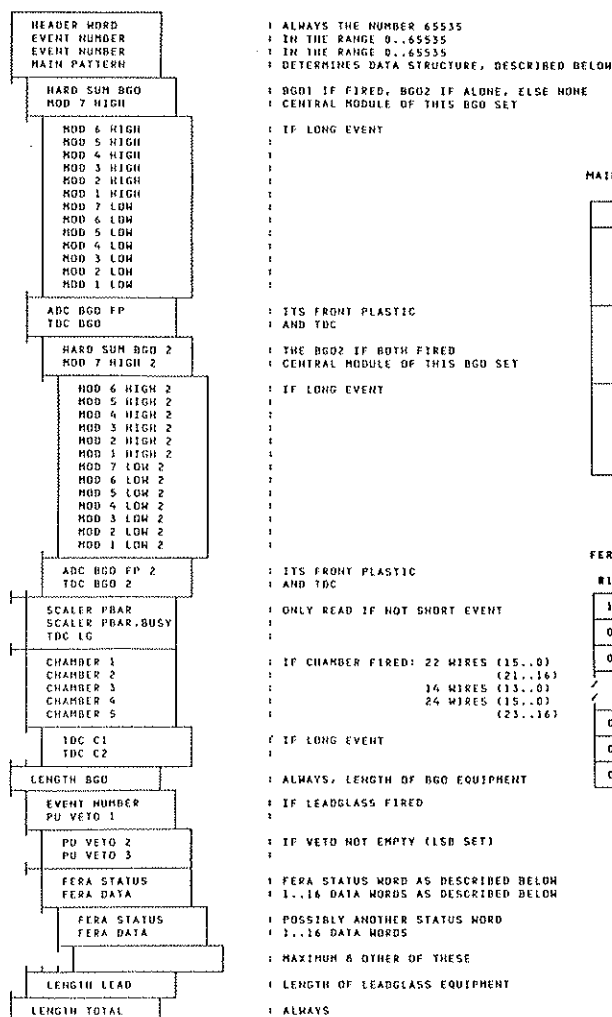
Fig. 3.82: General organisation of the data acquisition system.

²⁴ The CAMAC standard is defined in EURATOM Reports EUR 4100e, 4600e, and 6500e.

This readout system is extremely efficient in the sense that it eliminates the deadtime the VAX interrupt would have caused if it was carried out for each event. At rates of about a thousand events per second, which was expected for PS182, this would just have resulted in a tremendous deadtime. The deadtime in reality was typically about 10%, measured as the ratio between the rates of the signals denoted " $\bar{p}$ " and " $\bar{p}$ -busy" (cf. fig. 3.83a), being the antiproton rate measured by the telescope scintillator and the rate of recorded events, respectively. The signal named "busy" is a veto signal generated by the VAX when it does not accept interrupts and thus disables the system for data taking. The 10% deadtime was mainly caused by the unfortunate micro-duty cycle of the beam originating from the 50 Hz frequency of the electrical power net-work, which resulted in a twice as high instantaneous rate as the average rate of about 30,000 $\bar{p}$ /s. The VAX, after having read in the content from the FIFO memory, copies the event buffer to magnetic tape, and sorts out a subset of the events for online monitoring.

EVENT STRUCTURE OF PS182 DATA

THIS SHEET IS VALID AS FROM 10-APR-1984
AND REPLACES THE PREVIOUS ONES



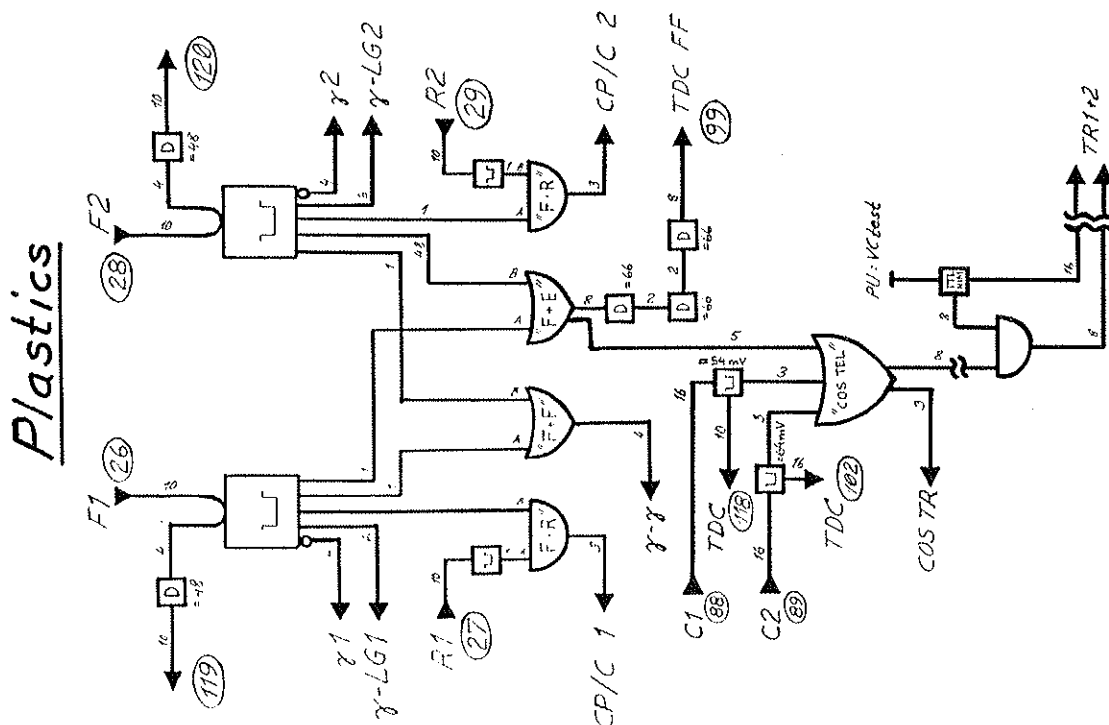
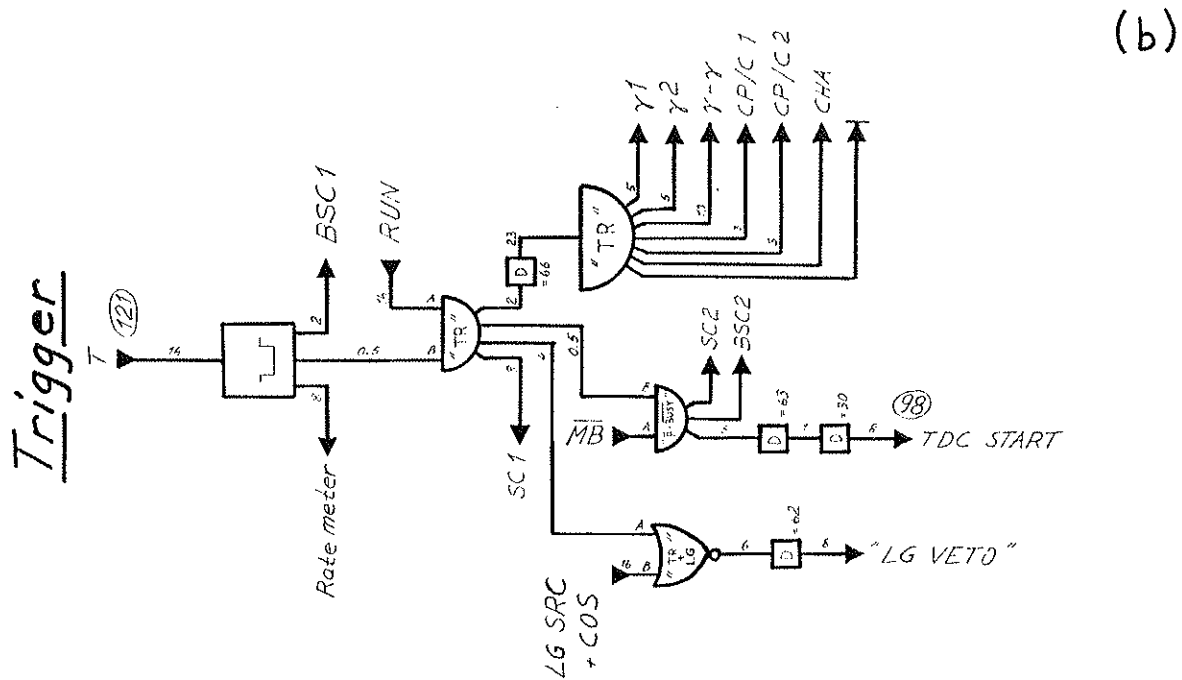
MAIN PATTERN

BIT	MEANING
0	BGO1 HAS FIRED
1	BGO2 HAS FIRED
2	THIS IS NOT A SHORT EVENT
3	CHAMBER HAS FIRED
4	LEADGLASS HAS FIRED
5	
6	
7	
8	
9	
10	LU 2 GAMMA
11	SINGLE BGO1
12	SINGLE BGO2
13	PEDESTAL TRIGGER
14	FRONTPLASTIC1, REARPLASTIC
15	FRONTPLASTIC2, REARPLASTIC

FERA STATUS

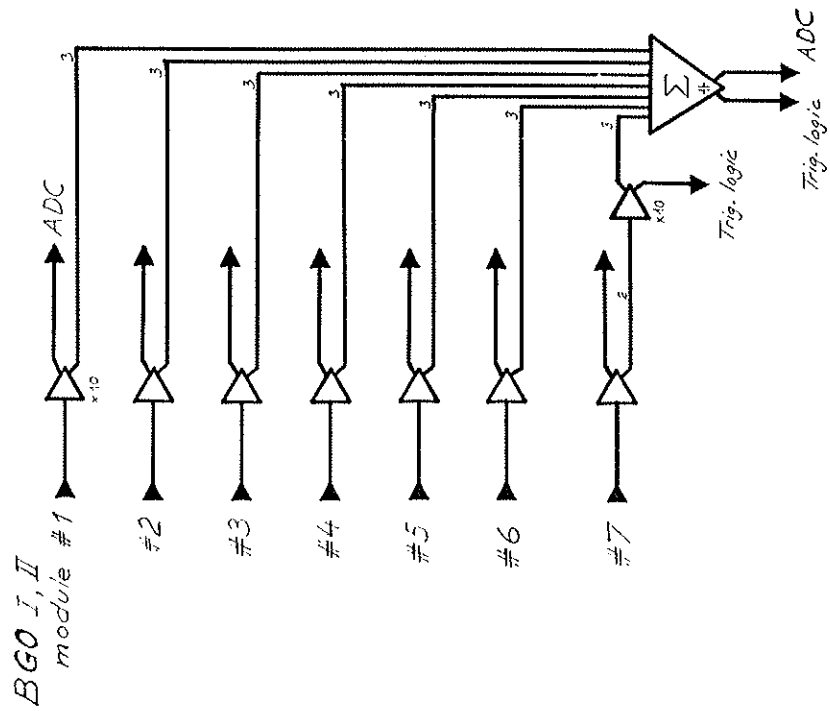
R16		R11			R 8		R
1	MC = 0 TO 15	0	0	0	VIRTUAL STATION NUMBER		
0	FIRST SA	DATA					
0	SA	DATA					
/							
/							
0	SA	DATA					
0	SA	DATA					
0	LAST SA	DATA					

Fig. 3.84: Structuring of datawords from one event; meaning of the bits of the "Main Pattern word"; bit structure of the information from the FERA ADCs.

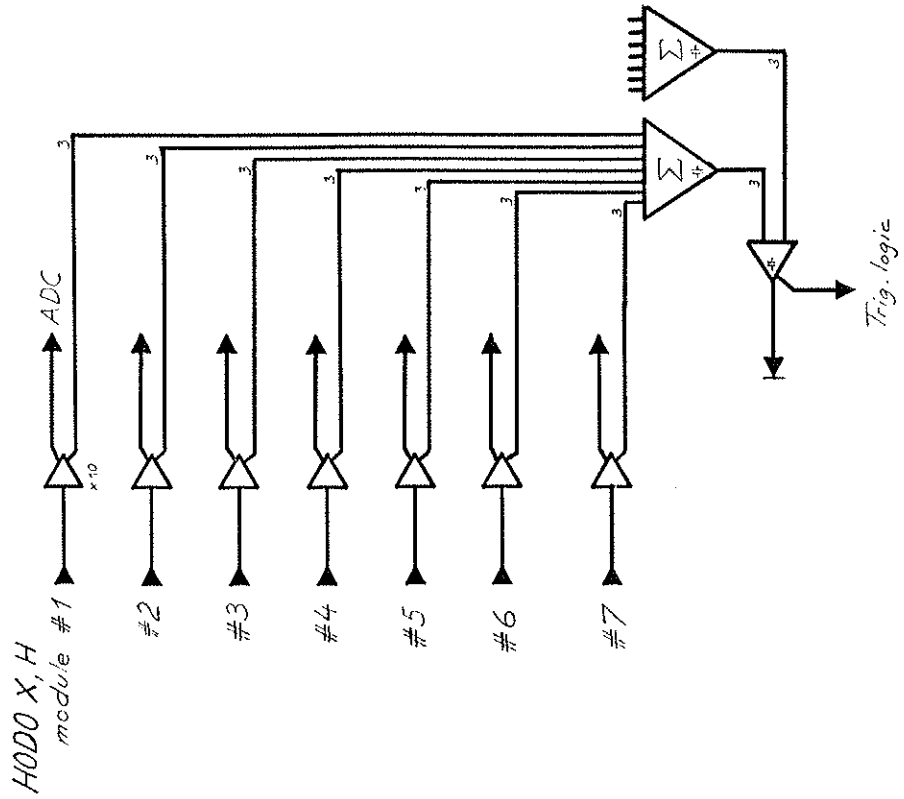


CAVE

hexagons

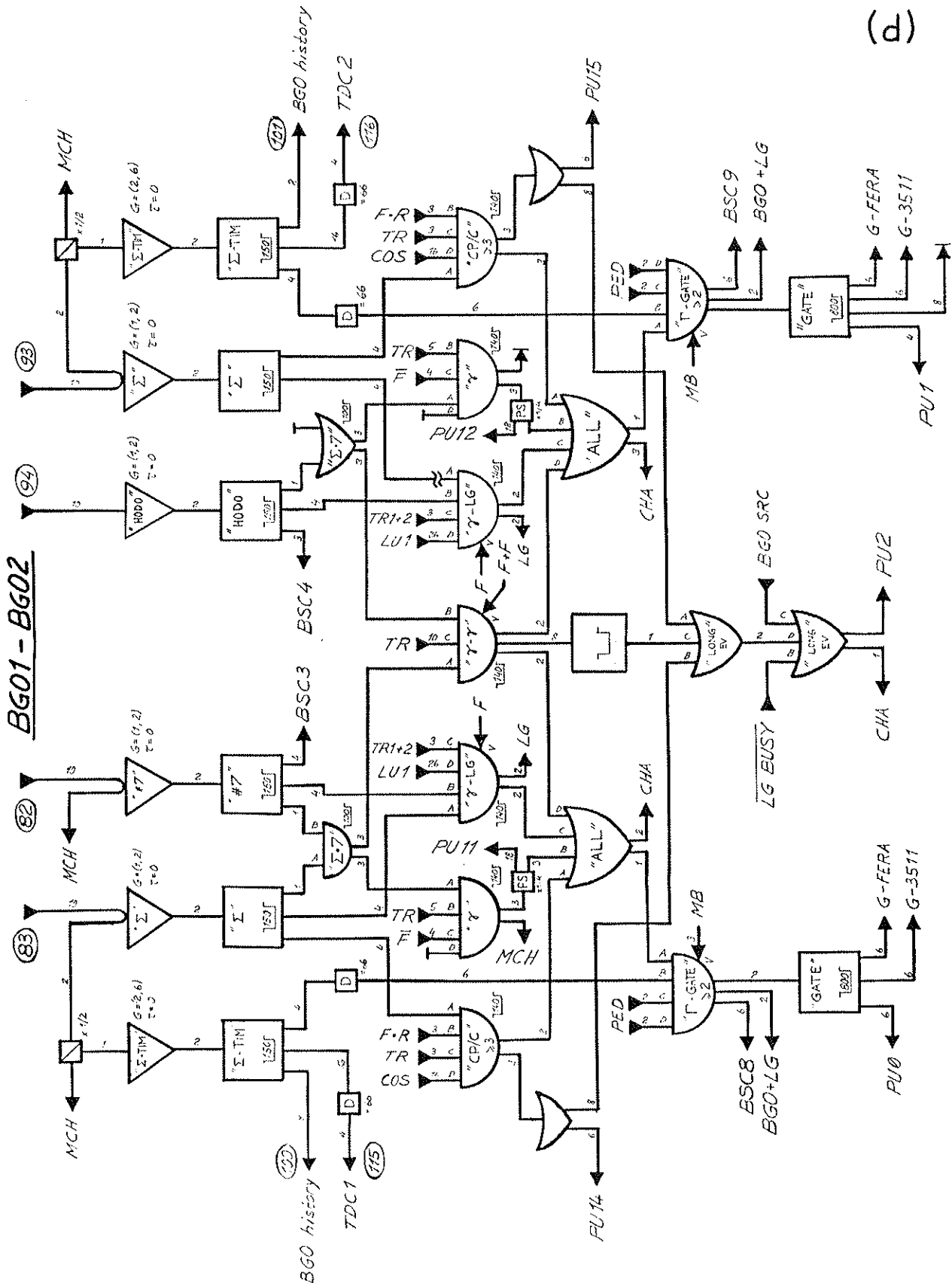


hodoscope



(c)

(d)

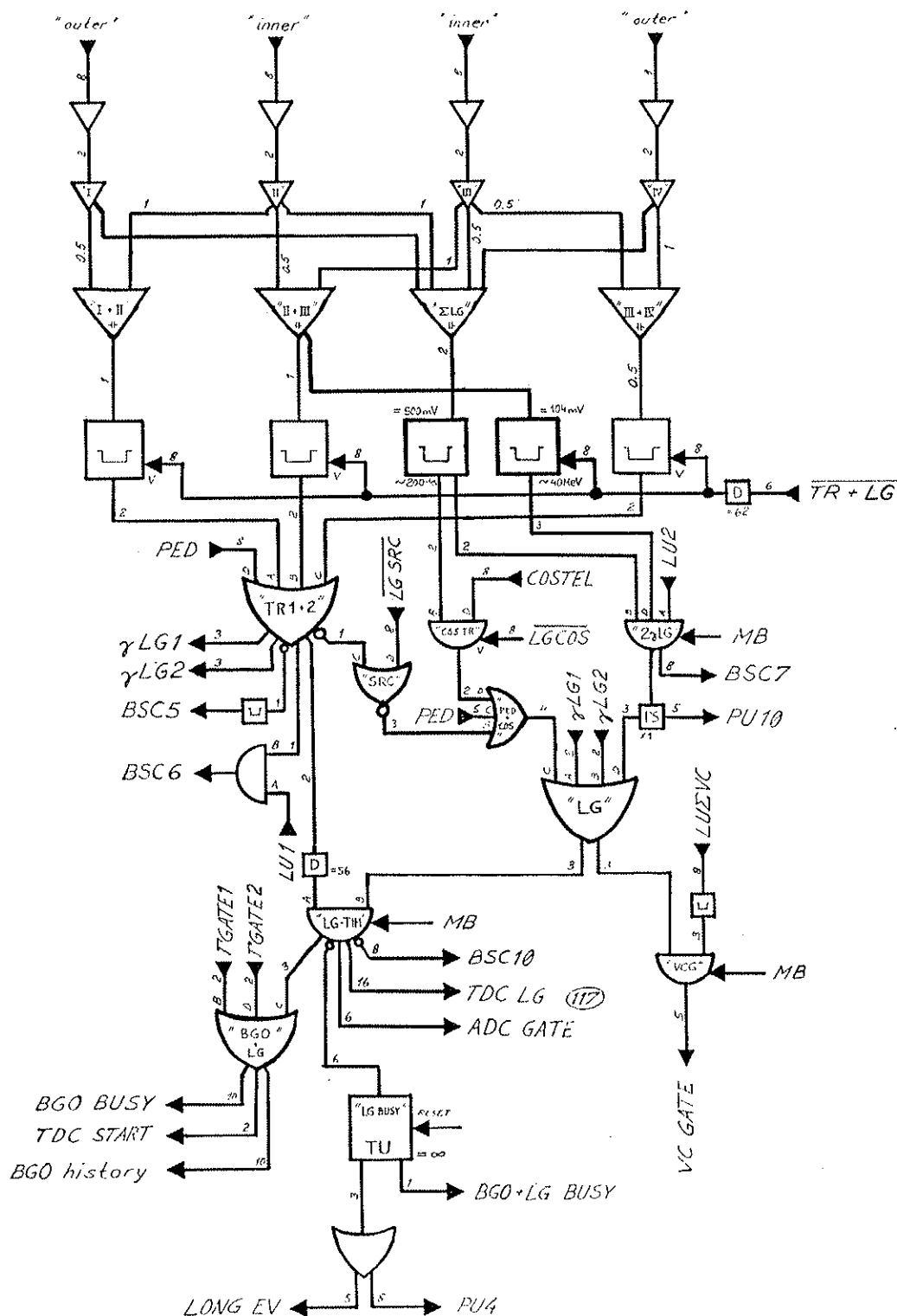


One machine can do the work of fifty ordinary men. No machine can do the work of one extraordinary man.

(Elbert Hubbard, 1856-1915)

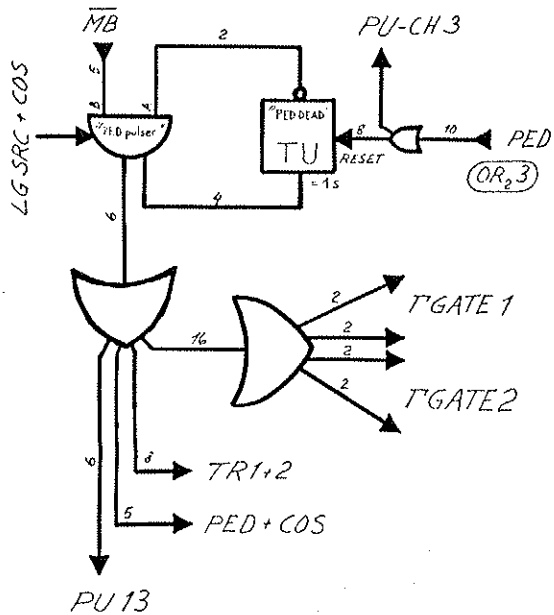
Pb - glass

(e)

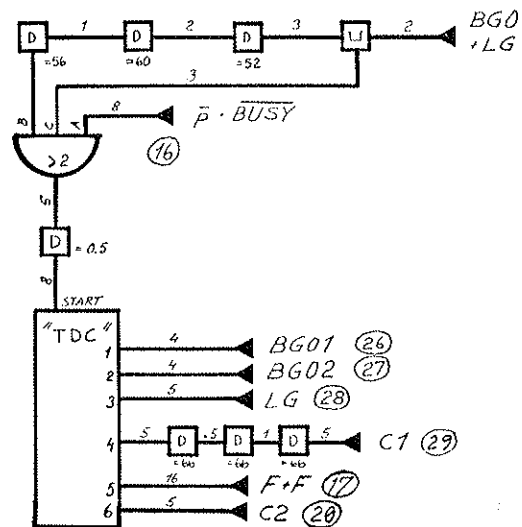


(f)

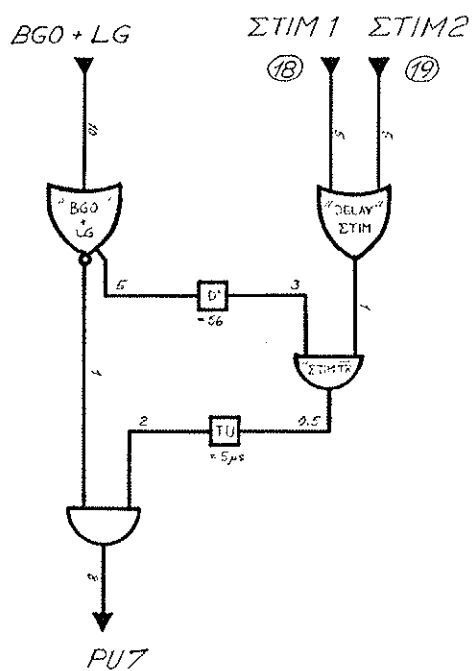
PED pulser



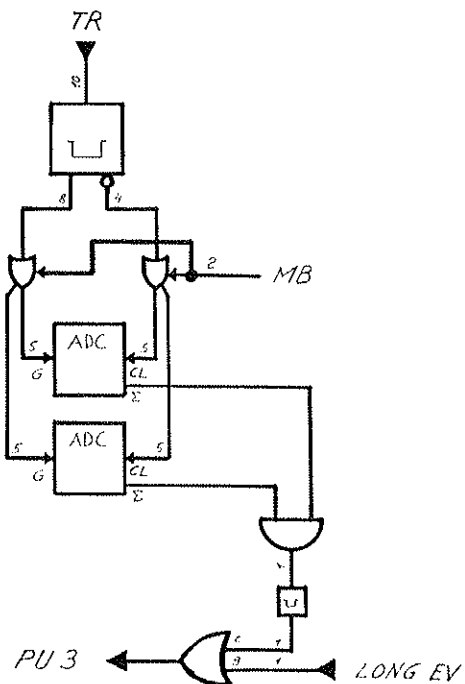
TDC



BGO history



Chamber



The BGO Subsystem

The hexagonal BGO crystals were equipped with 2" photomultipliers²⁵ type 1306-01 from Hamamatsu. These photomultipliers, that have a low noise level, were supported with a base circuitry that consisted of a, so called, "progressive" resistor chain (fig. 3.85a), i.e. the resistances between the dynodes are not equal. Thus, the amplification was brought down an order of magnitude, as compared to what is achieved with a "linear" resistor chain (all resistance are equal), and saturation was thus avoided for energies deposited in a BGO crystal that are less than 1.4 GeV; more then 1 GeV was not expected in this experiment. Running the photomultipliers on negative high-voltage allows the signal to be coupled out without a DC-isolating capacity, and thus avoiding the problem of zero-point fluctuations (which is particular risky at high signal-rates).

The BGO hodoscope crystals were equipped with photomultipliers²⁶ type R647-1 from Hamamatsu. These photomultipliers were supported by a linear resistor chain (fig. 3.85b) and run on positive high-voltage. Because of the tight mechanical construction, the sensitive photocathode was not put on negative high-voltage but the anode on positive instead.

All signals from the BGO systems were, in the experimental area, immediately amplified ten times and split two-fold. One part of the signals were then fed to an analog summing unit (fig. 3.86e), while the other parts were sent to the counting room, and reached the ADCs after about 350 ns. Meanwhile the signals from the summer and the central hexagonal crystal (in A84 also the hodoscope summed signal) were sent together with the signals from the veto counters (cf. fig. 3.86) to the counting room via 90 ns cables, and were there fed to the trigger electronics (fig. 3.86d). The parallel discriminator branch in fig. 3.86d, denoted " Σ -TIM", was there to minimize the so called "time-walk", i.e. the relative timing between the discriminated signal and the original analog signal being dependent on the size of the analog signal. Thus, the signal was amplified to get a steeper rising edge to base the timing on, while the other discriminator branch, denoted " Σ " took the logical trigger decision based on the signal height; the discriminator threshold was proportional to 1 MeV of deposited energy. The two pieces of information were then combined in the coincidence unit, denoted "T-GATE", that released the gate signal to the ADCs. The types of ADCs used changed between run periods, but were always of a charge sensitive type from LeCroy²⁷. During D83 the type used were 2249W, and during the other run periods they were of type FERA 4300. For the sum signals the ADCs always used were spectroscopic ADCs of type 3511. The trigger thresholds in effect during the different run periods are compiled in table 3.6.

Table 3.6: Trigger thresholds and conditions in effect during different run periods.

Run period:	D83	M84	A84	M85
Hodoscope in trigger			×	
Lead glass in trigger	×	×	×	×
LU in trigger		×	×	×
Thresholds [MeV]				
hardware				
BGO ₁ -7	25	2.2	2.74	2.74
BGO ₁ Σ	18	13.8	10.4	10.4
BGO ₂ -7	25	2.2	—	2.74
BGO ₂ Σ	18	13.8	10.4	10.4
hodoscope	0	0	0.6 + 0.8	0
Pb-gl. block	2	2	2	2
Pb-gl. Σ	5	10	5	5
software				
BGO ₁ Σ	10	20	20	20
BGO ₂ Σ	10	20	20	20

²⁵ Data sheet No. PM-231-1, Hamamatsu TV Co., Hamamatsu, Japan

²⁶ Data sheet Cat. No. CPMT-186-02, Hamamatsu TV Co., Hamamatsu, Japan

²⁷ LeCroy, Spring Valley, New York, USA.

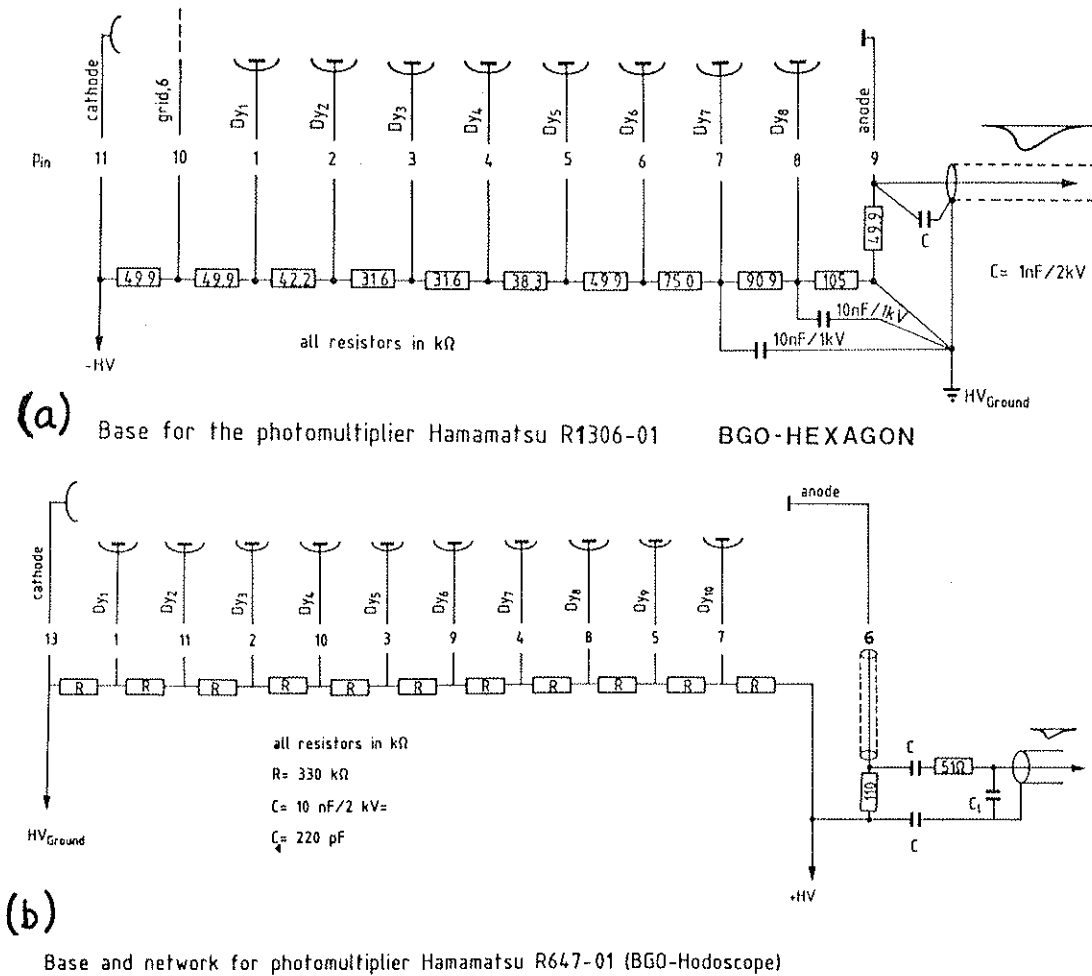


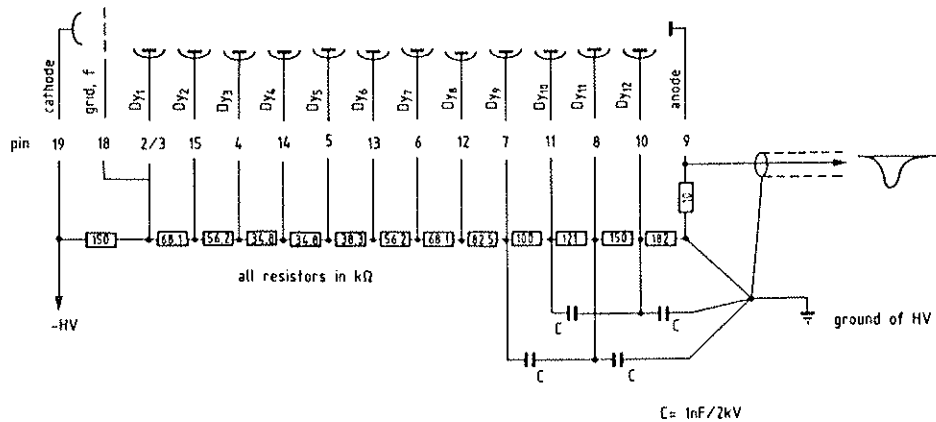
Fig. 3.85: Resistor support chain for the photomultipliers for (a) the hexagonal BGO crystals, and (b) for the BGO hodoscope bars.

The Lead-glass Subsystem

The lead-glass blocks were equipped with 2" photomultipliers of type EMI-9839B, which were supported by a progressive resistor chain (fig. 3.87) and run on negative high-voltage. Because the light produced in lead glass is of Cherenkov type, it comes during a short time period as compared to the time-constant of the photomultiplier (viz. about 10 ns) and thus can cause large momentaneous currents. The progressive resistor chain should prohibit saturation for deposited energies less than 1.4 GeV.

The 160 lead-glass signals were sent directly to the counting room together with the signals from the 41 veto counters via 30 m long cables of type BNC. On arrival in the counting room the lead-glass signals were fed to a home-built amplifier/splitter/summer. One part of the signals were then discriminated and fed to another home-built device named "Logic Unit" (LU). This electronics device, that also had access to the 41 discriminated signals from the veto counters, decided within 35 ns whether the energy deposited in the lead-glass array originated from a charged particle or not. It generates two output signals that were fed to the trigger logic, one if there was exactly one neutral cluster present, and the other one if there were (at least) two neutral clusters. The LU recognized a number of fired lead-glass blocks as a "neutral cluster" if none of the veto counters in front of these blocks had fired. Two clusters were distinguished if there was a column of four lead-glass blocks in-between that contained less than 20 MeV deposited energy. The discriminator threshold for each block corresponded to 2 MeV deposited energy.

The other parts of the 160 lead-glass signals were sent via 160 ns delay cables to ADCs. Meanwhile four sum signals were fed to the fast trigger electronics (fig. 3.83f), one sum signal for each row of 40 lead-glass blocks. The ADCs were of type FERA 4300 from LeCroy. These 11-bit ADCs have a short conversion time of about 10 ns, and were operated in the sparse-readout, pedestal-subtraction mode. Data was read out via the LeCroy ECL bus at a rate of 100 ns per word. This is an external bus that allows access to several modules from one single CAMAC station holding a drive unit.



Base for the photomultiplier EMI 9839B (leadglass)

Fig. 3.86: Resistor support chain for the photomultipliers for the lead-glass blocks.

*...The morality of art consists
in the perfect use of an
imperfect medium.*

(Oscar Wilde)

Truth is never pure, and rarely simple. *(Oscar Wilde)*

*Every single truth can be expressed in an almost unlimited number of ways. For the time
being at least, it is necessary to make a choice. What should guide us in this choice ?*

(Henri Poincaré, 1854-1912)

CHAPTER 4

MEASUREMENTS

In the last section of the previous chapter we saw how the signals from the different detectors were guided via the trigger electronics to ADCs and other CAMAC units. The information was subsequently stored on magnetic tapes by the online VAX computer. These data tapes were then after each run period analysed, and histograms were generated. This procedure is described in the first section of this chapter. Section 4.2 presents the methods of interpreting the different spectra, and numerical results are finally collected in the last section 4.3.

4.1. OFFLINE ANALYSIS

This section will follow what happens to the data from the moment on it is recorded online as ADC bit patterns to the point it can be displayed as histograms of physical parameters. The magnetic data tapes recorded online were firstly read by a computer program that performed the relative calibration of the BGO crystals, among other things. It also made a rough absolute calibration and converted the ADC numbers to energies in MeV. This first step of the evaluation is called the "pre-analysis" and is described in subsection 4.1.1. Each event was then rewritten by the same program, in a condensed format, to a second generation of tapes. These tapes, that are considerably fewer in number, were in their turn read by another computer program that sorted out the different types of events, made reconstruction of impact positions, calculated angles, performed the kinematic fitting, and finally filled the relevant histograms. This second part of the evaluation is treated in the second subsection 4.1.2.

4.1.1 Pre-evaluation of Data Tapes

The data tapes, that online were written on a VAX-11/750 computer, were after the run periods transported to the CERN computer centre and analysed by the pre-evaluation program that is described throughout this section. The analysis was carried out on the IBM 3090/Siemens 7890S or the Cyber 875 computers at the computer centre, because these machines are considerably faster than the VAX-11/750. A LEAR run period, that typically extends over a few days, results in some 10^9 recorded events. This amount of data requires between 50 and 100 magnetic tapes of storage.

4.1.1.1 BGO Calibration

The pre-evaluation program starts by searching for the charged-pion events in the BGOs which are marked with bits 14 and 15 in the "Main Pattern Word" (cf. section 3.7 and fig. 3.84). These events are quite rare in the event stream on the tapes; roughly one event in a thousand. A histogram is filled for each BGO crystal with these charged-pion events. In fig. 3.34 is shown an example of such a histogram. The through-going charged pions produce a sharp peak that is used to calibrate the crystals. The positions of these peaks are taken to be the mean values μ_i of a fitted Gaussian. In practise a Gaussian plus a second order polynomial is fitted [130] because there is a small background under the peak. The error of the mean values obtained from the fit is typically $\Delta\mu/\mu_i = 0.5\%$. A table of calibration constants $\{c_i\}_1^7$ for each BGO set is derived by putting

$$c_i = \frac{\mu_7}{\mu_i} , \quad (4.1)$$

so that the energy registered by crystal i can be expressed as

$$E_i = ac_i A_i, \quad (4.2)$$

where a is an absolute calibration constant still to be deduced and the numbers A_i are the ADC channel numbers. The contribution from finite relative calibration to the energy resolution of a BGO set can now be estimated. The error in each crystal energy is

$$\frac{\Delta E_i}{E_i} = \frac{\Delta c_i}{c_i} = 2 \frac{\Delta \mu_i}{\mu_i} = 1\%, \quad (4.3)$$

and they combine to yield an error in the total energy of

$$\Delta E_{tot} = \sum \Delta E_i = 1\% \cdot a \sum A_i = 1\% E_{tot}. \quad (4.4)$$

This error is, at all energies, smaller than the other contributions to the resolution (see further section 3.4.3.2).

Another method to calibrate the BGO crystals relative to each other, that was investigated [130], utilized the inclusive photon spectra recorded by each individual crystal. By fitting the square-root expression

$$N(E) = N_i \sqrt{E_i - E} \quad (4.5)$$

to the upper-end part of the spectrum from each crystal i and equalizing the end points E_i calibration constants could be extracted. This method gave results consistent with the charged-pion method, but was less efficient as it takes more run time to acquire enough statistics. The histograms can therefore not be reproduced with as short time intervals as the charged-pion ones can, and hence cannot produce as good time stability. Even if photon events in general are much more frequent than the charged-pion events, so are high-energetic photons quite rare, and these are the events that define the upper end-point of the spectra. With the charged-pion method a peak of a few hundred counts can be collected within ten minutes of run time, which is enough for calibration. The calibration drifts can thus be kept below 1%, which is of the same size as the calibration itself.

The complete sets, of seven crystals each, have also to be calibrated on an absolute scale. The final absolute calibration is left to be done in the second step of the analysis. Nevertheless a rough calibration can be carried out already during the pre-evaluation. By making a charged-pion plot after a BGO set has been absolutely calibrated (see next section), the energy released by these pions is seen to be 198 MeV¹. In the pre-evaluation the energies E_i are therefore set to

$$E_i = ac_i A_i = 198 \text{ MeV} \cdot A_i / \mu_i, \quad (4.6)$$

where $a = 198 \text{ MeV}/\mu_7$.

4.1.1.2 Goodness of BGO Events

When a photon strikes a BGO set too close to the boundary of the detector, a significant part of the shower energy systematically leaks out of the active detector volume. For such events it is impossible to determine the original photon energy. In order to avoid these situations one applies constraints to the energy division among the crystals. This is most appropriately done in the offline analysis after the relative calibration has been performed, rather than applying hardware conditions in the trigger logic online. For instance, requiring at least 50% of the total energy to be deposited in the central crystal, would reject photons that hit outside the central crystal (except maybe a few low-energetic ones being

¹ This indicates that these pions are not really minimum ionizing. Minimum ionizing pions deposit 9 MeV per radiation length in BGO. Since our crystals are 20 cm long minimum-ionizing pions are expected to release about 163 MeV.

subject to extreme shower fluctuations). It is of interest to accept as many events as possible in order to have maximum detector efficiency. In order to decide at which value a cut in the ratio $\eta = E_c / E_{tot}$ should be applied, one must know the lateral and longitudinal extent of electromagnetic showers. The development of electromagnetic showers in BGO has been simulated by Dietl et al. [50] using the SLAC-EGS program [64]. Their results were presented in section 3.4. One could allow photons to enter as close as 3 to 4 cm to the edge of the assembly, without systematic loss of shower energy. The energy E_c deposited in the central crystal could for such a case be as little as 15% of the total shower energy E_{tot} . Accordingly, not to throw away unnecessarily many events, we ask that

$$\eta = \frac{E_c}{E_{tot}} \geq 15\% . \quad (4.7)$$

This condition does, however, not alone completely assure that leakages were not significant for all showers. If the photon enters close to the border between two peripheral crystals, the central crystal may contain 15% of the overall registered energy, and still a substantial part of the shower energy can have leaked out of the system. To withdraw the boundary of accepted events from the inner corners it is in addition required that, if the maximum energy E_{max} was registered by a peripheral crystal, the majority of the remaining energy $E_{tot} - E_{max}$ is contained in the central crystal, i.e.

$$\eta < 50\% \Rightarrow E_c \geq (E_{tot} - E_{max})/2. \quad (4.8)$$

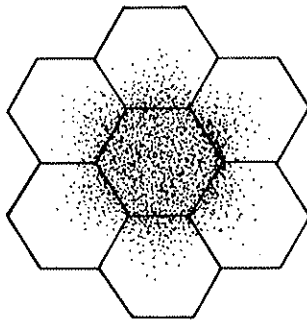


Fig. 4.1: Distribution of the impact positions of accepted photons.

Fig. 4.1 shows the location of accepted events. The effect of these cuts can now be monitored by plotting events in the η -versus-energy plane (fig. 4.2). It is an immediate impression that the cuts are indeed energy dependent. What was said above, that a cut at 50% would reject hits outside the central crystal, is not fully true. Due to the fact that photons entering the detector off the central axis always strike with an impact angle different from zero, these showers develop in direction towards the lateral boundary of the assembly. Hence, one would also expect photons that enter just inside the edge of the central crystal to be rejected by such a cut. One can assume this effect to be more pronounced for higher energies, since shower dimensions grow faster with energy in the longitudinal direction than in the transverse direction.

4.1.1.3 Definition of Lead-glass Clusters

Another important task carried out by the pre-evaluation program is the recognition of showers in the lead glass. This includes determination of the number of showers and where in the array they are located. This procedure starts by summing block energies four by four so that an array of forty energies is obtained, one for each column of four blocks in the z -direction (beam direction). The program then searches for local maxima in this array of forty energies. A shower is identified with a cluster of fired lead-glass blocks around these maxima if the following criteria are satisfied:

- 1°) two adjacent maxima must not be any closer such that there are at least three columns left in-between the maxima;
- 2°) if two clusters overlap, they are considered as being one cluster;
- 3°) clusters were than five columns wide are rejected;
- 4°) the sum cluster energy must exceed 10 MeV;

- 5°) scintillators in front of lead-glass blocks containing energy must not have fired, otherwise the cluster is considered as being caused by a charged pion and subsequently rejected.

The reason for "three columns" under 1°), is the fact that the smallest π^0 -decay angle occurring from $p\bar{p}$ annihilations at rest is 16.5°, and a column of lead-glass blocks subtends 3°. The 10 MeV limit under 4°) is reasonable to apply already here because the lead-glass blocks have been quite accurately calibrated before the run period (cf. section 3.5.3.1); the final calibration of the lead-glass array being done in the second part of the analysis described in the next section.

4.1.1.4 BGO Hodoscopes and Scalars

During the run period in August 1984, when BGO₂ was equipped with two hodoscopes, the hodoscope information was treated by the pre-evaluation program. The two hodoscopes were, during this run period, placed one in front of the other, and both placed in front of the hexagonal crystals. The idea of this arrangement was to combine information from them both in order to obtain a more accurate definition of the photon's impact point. To enable this the hodoscopes were displaced by half the width of an element, i.e. 0.5 cm, relative one another. Here the program looks for the bar in each hodoscope that contains the largest energy. Knowing the ξ -coordinate at the middle of the bars, one simply takes the average of the centre coordinates of the two bars with the maximum energies to represent the impact position; only element energies larger than 1 MeV are considered. The case when the bars are separated by more than half a unit is extremely unusual, but do nevertheless occur due to back-scattering from the hexagonal crystals. Something that happens more frequently on the other hand, is that only the inner hodoscope fires. Only 70% of the photons convert in the first hodoscope (cf. section 3.4.3.4). This leaves 70% of the remaining 30% to convert in the second hodoscope, which is 20% of the irradiating flux. The impact points of these latter photons are taken to be the centre ξ -coordinate of that bar of the inner hodoscope that contained the maximum energy. For all events the ξ -coordinate is later calculated with the centre-of-gravity algorithm described in section 3.4.3.3. During the run period A84 when two hodoscopes were mounted in front of BGO₂, the ξ -position, deduced as described above, was written onto the secondary generation of tapes, and the energy deposited in the hodoscope was added to the total energy of the hexagonal crystals. During the run period D83 only one hodoscope was used, and then the energies of the seven hodoscope bars were directly rewritten onto the secondary tapes.

Finally the pre-evaluation program also reads and accumulates scaler rates that count the signal rates at different levels and branches of the trigger electronics (cf. section 3.7). The most important rate is the telescope rate, i.e. the $\bar{p}$ -rate, as this is used to normalize the measured histograms. This number is crucial later when determining any yield of a particular reaction that might be observed. The **total** number of $\bar{p}$'s that arrived during the complete run period and were not vetoed by the "busy" signal from the online computer is of greatest importance. This number is the total number of $p\bar{p}$ annihilations that contributed to the studied event sample. Thus, it is the number a peak appearing in a spectrum should be normalized to in order to obtain the corresponding branching ratio. These rates are listed in table 4.1.

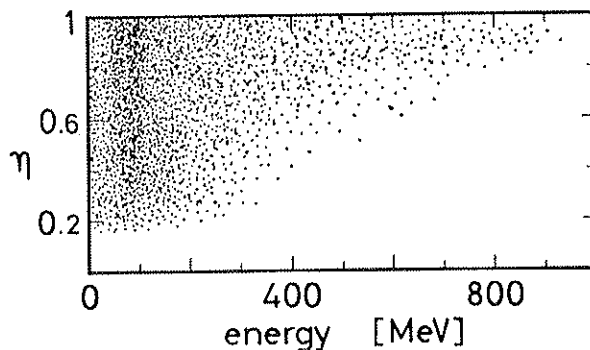


Fig. 4.2: Distribution of the ratio $\eta = E_c / E_{tot}$, i.e. the fraction of the energy contained in the central crystal, for accepted photons versus the photon energy.

Table 4.1: Total number of $\bar{p}$'s recorded in the different run periods.

D83	$2.953 \cdot 10^9$
M84	$1.491 \cdot 10^9$
A84	$2.026 \cdot 10^9$
M85	$5.769 \cdot 10^9$

4.1.2 Event Analysis

This second step of the data analysis is, like the first one, to a large extent performed by a computer program. Before playing through all data tapes and producing histograms, the absolute calibration has to be made. It has also first to be decided which kind of kinematic cuts should be made to reduce the background, and at which values to apply them. Thus, to investigate these topics, only a fraction of the data is analysed to start with. The absolute calibration is accomplished by adjusting the π^0 invariant mass peaks from the different detector combinations to the correct value, 134.96 MeV/ c^2 , and is further described in sub-subsection 4.1.2.2. Prior to the computation of the invariant mass m_0 of the photon pairs the opening angle ϕ between the two photons has to be known, which requires the reconstruction of impact positions. Sub-subsection 4.1.2.1 is devoted to the position reconstruction and discusses limitations and the impact it has on the measurements. Sub-subsection 4.1.2.3 explores possibilities to reduce background. For a sample of triple coincidence events some special kinematic checks were made which are discussed in sub-subsection 4.1.2.4. Finally there is a sub-subsection concerning the time analysis.

4.1.2.1 Reconstruction of the Impact Position

The principle for reconstructing the impact point is, as already extensively discussed in sections 3.4.3.3 and 3.5.3.2, to utilize the energy division between neighbouring modules, i.e. to enquire information about the shower topology. A simple and reasonably well working method of doing this is that of the centre of gravity, which works quite well for the BGO detectors. For the lead-glass array, however, the situation is somewhat different, and hence a variant approach was explored for its case.

4.1.2.1.1 BGO Algorithm

For reconstructing the impact point (ξ, ζ) we have used the weighted centre of gravity of the energy sharing among the seven crystals of a BGO assembly:

$$\xi = \frac{1}{N_E} \sum_{i=1}^7 \xi_i E_i^\alpha, \quad \zeta = \frac{1}{N_E} \sum_{i=1}^7 \zeta_i E_i^\alpha, \quad N_E = \sum_{i=1}^7 E_i^\alpha, \quad (3.41)$$

where (ξ_i, ζ_i) are the coordinates of the symmetry axes of the crystals. The $\xi\zeta$ -plane is the front face of the crystal. The exponent α is a free parameter optimized to 0.308. For more details about the inherent features of this method refer to section 3.4.3.3. The accuracy of the algorithm depends on the shower energy as the fluctuations in shower topology are more dramatic at lower energies than at higher. As shown in section 3.4.3.3 the resolution possible to achieve is about 1.3 cm full-width at half-maximum at a photon energy of 428 MeV. Unfortunately this algorithm systematically shuffles events towards the centre of the entrance surface. This tendency turns out to be impossible to cure by changing the only global parameter α . However, this effect is circularly symmetric around the central axis of the BGO assembly, and will therefore enter our measurement in a more random way and give an additional contribution to the position resolution. This contribution is in fact already implicit in the measured value of 1.3 cm at 428 MeV.

When the position (ξ, ζ) is calculated there is a low-energy cutoff at 2 MeV so that crystal energies E_i below that value are ignored. An exception occurs if the estimated impact position is compatible with $(\xi, \zeta) = (0, 0)$. The centre-of-gravity calculation is then conducted a second time with a yet lower limit of 0.5 MeV.

The ultimate motivation for making an accurate impact-position reconstruction is to be able to improve the energy measurement by performing a fit of the kinematics that will adapt the measured quantities to the invariant mass of the π^0 (or in occurrent case, of the η). To make the fitting optimally the experimental errors have to be provided. The kinematic fitting program works with the angles ϑ_1 and ϑ_2 between the γ -rays and the π^0 direction, respectively. Therefore, it is the angular resolution of one γ -ray that has to be delivered to the fitting program. It is computed from the transverse position resolution σ_e of the emission point and σ_s of the irradiation point on the detector front face together with the intermediate distance R :

$$\sigma_{\theta}(E_{\gamma}) = \frac{1}{R} \sqrt{\sigma_e^2 + \sigma_s^2(E_{\gamma})} = \frac{1}{R} \sqrt{\sigma_e^2 + E^{-0.7} \cdot 0.16 \text{ cm}^2}, (E \text{ in GeV}). \quad (4.9)$$

In all run periods except M85 the BGO sets were placed at a radial distance $R = 30$ cm from the target centre which results in an angular resolution

$$\sigma_{\theta} \approx E^{-0.35} \cdot 0.9^{\circ}, (E \text{ in GeV}). \quad (4.10)$$

In M85 the distance was $R = 50$ cm and subsequently a somewhat better value of $\sigma_{\theta} \approx E^{-0.35} \cdot 0.6^{\circ}$ (E in GeV) was achieved. Fig. 4.3 shows the dependence of σ_{θ} with shower energy E for the two radii of 30 and 50 cm, respectively. The figure also displays the angle resolution obtained from the hodoscope information. For comparison with the lead-glass situation (presented in the next paragraph) that curve is shown in the same figure.

4.1.2.1.2 Lead-glass Algorithm

The situation of the impact-point reconstruction of photons entering the lead-glass detector is somewhat different then of photons entering the BGO. Care has to be taken for the orientation of the block containing the maximum energy within the array. No constraints have been applied for lead-glass events when it comes to the energy sharing between neighbouring blocks analogous to the E_c/E_{tot} condition for BGO events. Thus, the event sample being subject to the position reconstruction consists of photons that could have entered anywhere in the array, including the edge blocks. The procedure considers a matrix of nine blocks covering the cluster, where the central block contains the maximum energy. Hence, it is possible that an outer row (or column) of three blocks does not "exist", so to say. Two mapping functions W_s and W_z that consider the location of the blocks in the array are introduced to overcome this problem. The impact position (s, z) is thus computed as

$$s = W_s(s', k, E), \quad z = W_z(z', k, E),$$

where $k = 1, \dots, 160$ is the middle-block number, and E the cluster energy and (s', z') is calculated according to eq. (3.64). The values of W_s and W_z are taken from a lookup table which corrects for the failure of the centre-of-gravity estimate (for more details refer to section 3.5.3.2). W_s and W_z are discrete functions of the ratio $(E_1 - E_2)/E_2$ and go in 50 steps. The steps are, on exit from the algorithm smeared effectively with a small random number.

As pointed out in the previous paragraph 4.1.2.1.1 it is of importance for the kinematic fitting to provide proper error definitions as they control the possible improvement of the energy measurement.

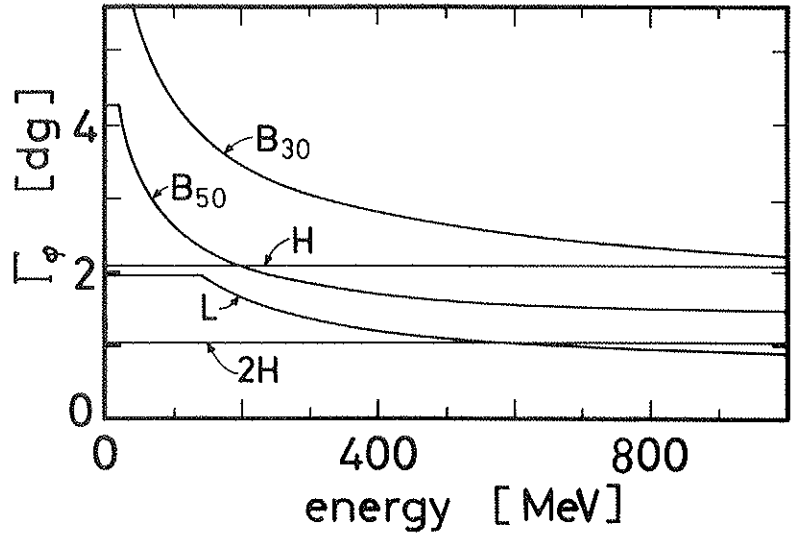


Fig. 4.3: Full-width at half-maximum $\Gamma\phi$ of the angular resolution for γ -rays detected by a BGO calorimeter at 30 cm distance from the target (B_{30}), and at 50 cm distance from the target (B_{50}), and for γ -rays detected by the lead-glass array (L) as a function of the photon energy. The resolution for photons detected by a BGO hodoscope (H, 2H) is constant.

It is the angular resolution of one γ -ray that has to be given, which is composed of an uncertain emission point and impact point ($R = 80$ cm):

$$\sigma_\phi(E_\gamma) = \frac{1}{R} \sqrt{\sigma_e^2 + \sigma_s^2} = \frac{1}{R} \sqrt{\sigma_e^2 + E^{-1} \cdot 0.19 \text{ cm}^2} \approx E^{-0.5} \cdot 0.4^\circ, (E \text{ in GeV}). \quad (4.12)$$

This error is relatively smaller than that in the energy measurement, wherefore a substantial improvement of the energy values can be expected. Fig. 4.3 shows the angle resolution for the leadglass together with the BGO curves. For a more extensive discussion of the inherent characteristics of this algorithm refer to section 3.5.3.2. This algorithm certainly exhibits some severe systematic shortcomings exactly as the BGO algorithm does. When measuring LL coincidences² this systematic is, however, completely randomized as it is only the relative angle between the γ -rays that is measured, and gives thus only an additional contribution to the resolution. The result should be compared to the position resolution obtained without performing any reconstruction. The omission of reconstruction yields a standard deviation $\sigma_b = 4.2 \text{ cm}/\sqrt{12} = 1.2 \text{ cm}$ at all energies, which is considerably worse at higher energies than the 0.5 to 0.7 cm achieved by the reconstruction procedure.

4.1.2.2 Absolute Calibration

The absolute calibration is accomplished by adjusting the π^0 invariant mass peaks to 134.96 MeV/c². The invariant mass m_0 of a photon pair is computed according to eq. (3.6) and entered a histogram. A separate histogram is generated for each detector combination from which the three different positions m_k , ($k = \text{'LL'}$, $\text{'B}_2\text{L'}$, and $\text{'B}_1\text{L'}$) of the π^0 -peaks are observed. Generally these values differ from $m_{\pi^0} = 134.96 \text{ MeV}/c^2$. The point is then to adjust them to the correct number by multiplying the energies E_i ($i = 1, 2$) with calibration factors c_k :

$$E_i' = c_k E_i. \quad (4.13)$$

It is then presumed that the angle is already properly calibrated (cf. section 3.6.2). If we start with LL coincidences the same calibration constant c_k is entering both energies E_1' and E_2' (provided that c_k is energy independent of course, i.e. assuming linearity³),

$$m_{\text{LL}}^2 = 4E_1 E_2 \sin^2(\phi/2), \quad m_{\pi^0}^2 = 4c_{\text{LL}}^2 E_1 E_2 \sin^2(\phi/2). \quad (4.14)$$

which gives

$$c_{\text{LL}} = \frac{m_{\pi^0}}{m_{\text{LL}}} = 1 - \frac{\Delta m}{m_{\text{LL}}}, \quad \Delta m = m_{\text{LL}} - m_{\pi^0}. \quad (4.15)$$

When c_{LL} is known the BGO energies can be calibrated in the same manner by using the π^0 invariant mass peaks produced by B_1L and B_2L coincidences:

$$m_{\pi^0}^2 = 4c_{\text{LL}} c_k E_1 E_2 \sin^2(\phi/2) \Rightarrow c_k = \frac{m_{\pi^0}^2}{m_k^2 c_{\text{LL}}} = \frac{m_{\text{LL}}^2}{m_k^2} c_{\text{LL}}, \quad (4.16)$$

where $k = \text{'B}_1\text{L'}$, $\text{'B}_2\text{L'}$.

² Caution: for BL coincidences one γ -ray is fixed in space by the BGO position relative to the lead-glass array; systematic effects could then become important.

³ The lead-glass system does in fact not respond exactly linearly, but this peculiarity has already been corrected for.

The precision of the calibration is given from the fitting of the peaks and amounts typically to about 0.5%. The calibration of the detectors was performed this way for the D83 data. For the later run periods the calibration was already good enough from the pre-analysis since the charged-pion peaks then could be adjusted to a, by then, known value (cf. section 3.4.3.1). The lead-glass had always to be calibrated this way.

When defining the calibration constants c_k any potential negative influence from the angle measurement should be minimized so that the constants are obtained as accurately as possible. The invariant mass is $m_0 \sim \sin(\phi/2)$ and hence is $dm_0 \sim \cos(\phi/2) d\phi$, so the proliferation of the angular error is mostly suppressed at 90° decay angle. Selection of only these decays is invoked by imposing the conditions that the π^0 -momentum $p_{\pi^0} = m_{\pi^0}$, and that the asymmetry $x \equiv (E_1 - E_2)/E_{\pi^0} \leq 10\%$ which guarantees that accepted events are restricted to the peak of the Jacobian distribution. These constraints can, of course, not be applied to B_1L coincidences, since this combination does not subtend 90° ; one has in this case to be satisfied with the closest angle covered by the configuration.

The success of the calibration is monitored by a variety of check-points. In the LL and B_2L π^0 -spectra there are signals from the process $p\bar{p} \rightarrow \pi^0(\rho^0, \omega)$ that peaks at 768 MeV/c. In the LL η -spectrum there is a signal from the reaction $p\bar{p} \rightarrow \eta(\rho^0, \omega)$ at 656 MeV/c. The calibration of the lead glass is revealed by both signals, whereas the BGO_2 calibration can only be seen from the π^0 -signal due to bad detection efficiency for η 's. The BGO_1 calibration cannot be revealed by any of these signals, since the combination B_1L is not sufficiently sensitive to those reactions. However, there is another signal appearing in this spectrum that allows inspection of the calibration.

It arises from monoenergetic π^0 's coming from the Panofski reaction $\pi^- p \rightarrow \pi^0 n$ which is induced by negative pions produced in $p\bar{p}$ annihilations. These π^0 's occur at a momentum of 28 MeV/c. The BGO_1 calibration is also perceptible in the photon energy spectrum obtained from B_1LL triple coincidences, of which the two photons detected by the lead glass are identified as a π^0 . The maximum photon energy is 933 MeV, which, together with knowledge about the resolution, settles the upper end-point of the spectrum. We can profit more from this triple combination by plotting the lead-glass π^0 -momenta under the boundary condition that the energy of the third photon is greater than 850 MeV/c which separates the two reactions $p\bar{p} \rightarrow 2\pi^0$ ($p_{\pi^0} = 929$ MeV/c) and $p\bar{p} \rightarrow \pi^0\gamma$ ($p_{\pi^0} = 933$ MeV/c). The retained events build a peak in the LL spectrum (fig. 4.4).

The D83 and M85 single photon measurements can apparently not be subject to any of the above signals, neither the invariant mass peaks nor the other signals. The calibration can in this case be founded on two other observables. On one hand the upper end-point of the photon-energy spectrum can be matched to the proper value (933 to 938 MeV), which, however, requires accurate knowledge about the resolution at this energy. On the other hand a signal from monochromatic photons can be observed. There is one such line, though of very poor statistics, originating from the other Panofski reaction $\pi^- p \rightarrow \gamma n$ ($E_\gamma = 129$ MeV). By matching these two pieces of information an absolute calibration of 1% could be achieved [130].

Finally, to summarize, the maximum absolute calibration errors can be observed a posteriori in units of the bin-width of respective histogram. These errors are of substantial importance later in the analysis, because they might introduce systematic errors when applying cuts in the spectra (see further next section).

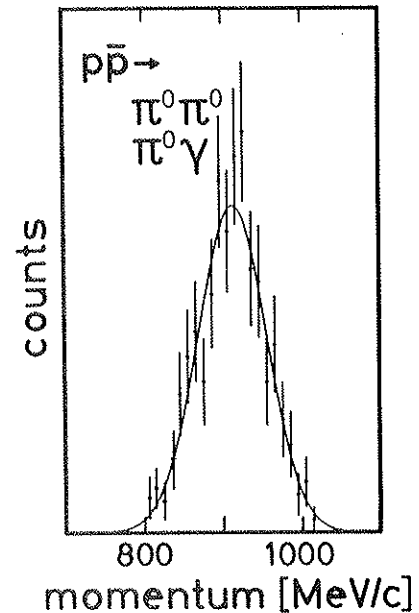


Fig. 4.4: Peak in the LL π^0 -momentum spectrum from the indicated reactions obtained when a high-energetic photon is detected by BGO_1 in the opposite direction.

4.1.2.3 Kinematical Investigations

The examination of the event kinematics commences by interrogating the invariant mass m_0 that has to belong to the $\pm 3\sigma$ interval around m_{π^0} . The standard deviation σ is given by a fit of the peak (see fig. 3.74). A $\pm 3\sigma$ cut corresponds to a confidence level⁴ of 99.8%. For the LL-spectrum the π^0 -window is [106, 162] (MeV/c²). By fitting a Gaussian plus a second-order polynomial to the $\pm 7\sigma$ region around the peak, the signal-to-background ratios are deduced.

Events that survive the invariant-mass cut are passed further to a one-constraint kinematic fitting program⁵ that forces the correct invariant mass to be reproduced by computing corrected quantities, and letting them supersede the measured ones. There are two virtues with such a kinematic fitting. Firstly, instead of merely deriving the π^0 -momenta from the two photon energies only, the measurement can be improved by letting the kinematic fitting "solve" the over-determined equation system that defines the event kinematics. Secondly, the χ^2 -value delivered by the fit is a sensitive measure of the goodness of an event under the hypothesis that it was a π^0 -decay, and provides consequently a potent event rejection. Before discussing the last issue we halt a moment at the former topic about the amelioration of the momentum resolution. To assure this point the level of precision in each individual measured quantity has to be identified and provided to the fitting program. If an error would be wrongly defined it could cause, not only a variable to be under-weighted, but even injury of the observations. These errors are used to weight the corrections of the measured quantities that the fitting does in order to fulfil the invariant-mass constraint. This means that the ratios between the relative corrections are proportional to the ratios between the relative accuracies. For numerical examples refer to section 3.6.3. The fitting is most relevant for LL coincidences and can then give a resolution three times smaller than that merely summing the photon energies leads to.

The other benefit of the fitting procedure was the resulting χ^2 -values. These are extremely useful in order to reject background from uncorrelated photon coincidences. The true π^0 -events will produce a proper χ^2 -distribution of one degree of freedom, and we employ a conventional cut at 95% confidence level, i.e. at $\chi^2 = 3.8$. Before trying to justify this cut it should just be demonstrated the excellent rejection power of background that this method possesses in comparison to simply making a cut in the invariant mass spectrum. For LL coincidences in the π^0 -window a χ^2 -cut at 3.8 reduced the data sample by 25%. The signal-to-background ratio in this window is $S/B = 1.04$ implying that 91% of the events thrown away are uncorrelated coincidences, or equivalently: 46% of the uncorrelated background is removed. Fig. 4.5 shows a result from a Monte Carlo simulation: the invariant mass distribution from uncorrelated coincidences retained by the χ^2 -cut. The dashed lines show the situation before the χ^2 -cut. If one simply makes a cut in the invariant mass spectrum at 95% confidence level it has to be applied approximately at $\pm 2\sigma$. This would only affect 10% of the data sample, i.e., hardly 17% of the uncorrelated background is removed in lieu of the 46% rejected by the χ^2 -cut.

To show that the $\chi^2 \leq 3.8$ is a reasonable requirement, the χ^2 -values from the kinematic fit are plotted (fig. 4.6). Before this distribution is compared to the theoretical function for one degree of freedom

⁴ See Appenix C.

⁵ The principles of such an iterative constraint fitting program are described in several articles; see for instance the one by B. Ronne in the CERN Report 64-13 (p. 87) from 1964. The basic idea is Lagrange's method of minimizing a multi-dimensional function for certain boundary conditions.

$$f(\chi^2) = \frac{1}{\sqrt{2\Gamma(1/2)}} \frac{1}{\sqrt{\chi^2}} e^{-\chi^2/2} = \frac{1}{\sqrt{2\pi\chi^2}} e^{-\chi^2/2}. \quad (4.17)$$

the contribution from the uncorrelated background has to be subtracted. The shape of the background distribution can be estimated in three ways:

- scrambling coincidences by using two photons from different events;
- comparing the χ^2 -plot for events belonging to a window below or above the π^0 -window;
- Monte Carlo simulate the background contribution.

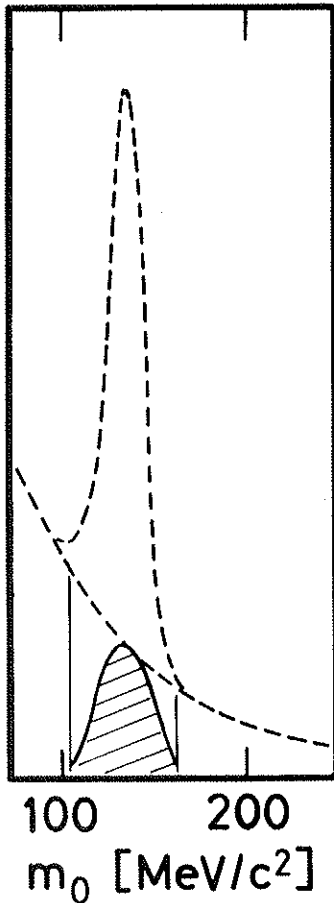


Fig. 4.5: Fraction of the uncorrelated background in the LL invariant mass spectrum that passes the χ^2 -cut as resulting from a Monte Carlo simulation.

Fig. 4.6: χ^2 -distribution from LL coincidences. The line is an estimation of the contribution from the uncorrelated background.

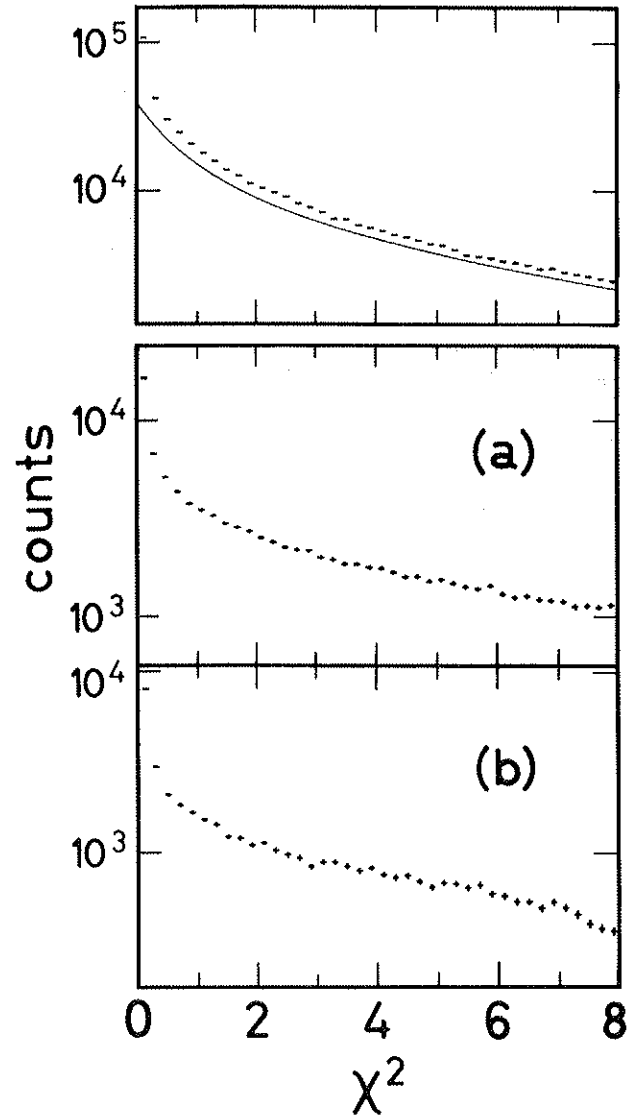


Fig. 4.7: χ^2 -distribution obtained from uncorrelated LL coincidences in a window (a) below and (b) above the π^0 -window in the invariant mass spectrum.

Figs. 4.7a–b show the χ^2 -plots for the window [50, 106] below and the window [162, 208] above the π^0 -window, respectively. A tentative form of the background in the π^0 -window is the binwise average of the two "side-bands" which yields the curve in fig. 4.6, scaled according to the signal-to-background ratio. This curve is in agreement with the result from a Monte Carlo simulation and the scrambling of disjunct events. Subtracting this contribution from the experimental data sample results in the distribution shown in fig. 4.8 which also contains the theoretical χ^2 -function for one degree of freedom [eq. (4.17)]. A reasonable choice of confidence level is evidently $\chi^2 = 3.8$.

The momentum spectra are eventually produced from the events that were preserved by the χ^2 -condition whilst the momentum of the π^0 is computed from the fitted energies. The evaluation of the η -momentum spectrum is of course completely analogous to the π^0 -evaluation. The invariant mass cut is applied at $\pm 3\sigma$ around $m_\eta = 548.8$ MeV/ c^2 and the kinematic fitting is conducted according to this mass value instead of that of the π^0 .

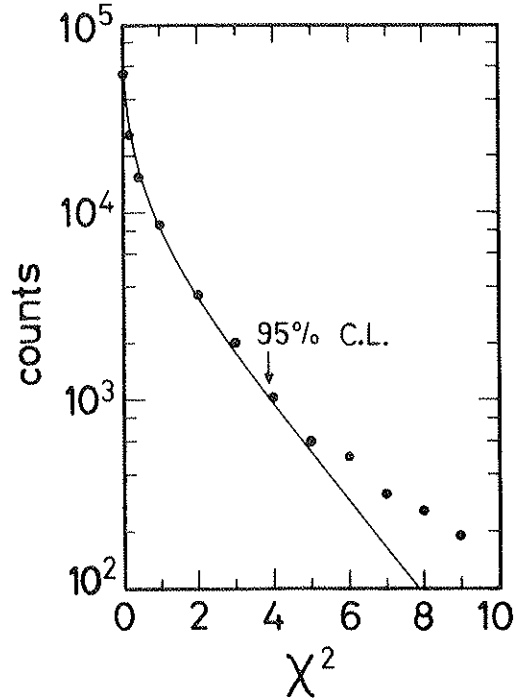


Fig. 4.8: Remaining χ^2 -distribution after the estimated contribution from the uncorrelated background has been subtracted. The line indicates the theoretical χ^2 -distribution for one degree of freedom.

4.1.2.4 Triple Photon Coincidence Events

This data sample, denoted B₁LL, is a subset of the LL coincidences recorded during the D83 run period. It consists of those events for which the photon pair detected by the lead-glass array is identified as a π^0 according to the cut conditions presented in the previous sub-subsection, and for which there was simultaneously a photon striking the BGO₁ detector in the opposite direction. Figs. 4.9a–b show the π^0 -momentum spectrum of the LL photon pair and the energy spectrum of the third photon as observed by BGO₁. Two additional quantities that can be derived from the three photons is the invariant mass M_0 and the missing mass MM :

$$M_0^2 = (E_{\pi^0} + E_\gamma)^2 - (p_{\pi^0} + p_\gamma)^2, \quad (4.18)$$

$$(MM)^2 = [2m_p - (E_{\pi^0} + E_\gamma)]^2 - (p_{\pi^0} + p_\gamma)^2$$

The corresponding spectra are displayed in figs. 4.9d–e. There is a prominent signal in the invariant mass spectrum stemming from ω -mesons that decay into $\pi^0\gamma$ (9% of all ω -decays). In the missing-mass spectrum there is a clear signal at zero which embraces $p\bar{p}$ annihilations into $2\pi^0$ and $\pi^0\gamma$ final states. The distribution of the angle Θ between the direction of the third photon and the π^0 -direction is shown in fig. 4.9c.

*We are no other than a moving row
Of visionary shapes that come and go
Round with this Sun-illumined Lantern held
In Midnight by the Master of the Show.*

(Omar Khayyam)

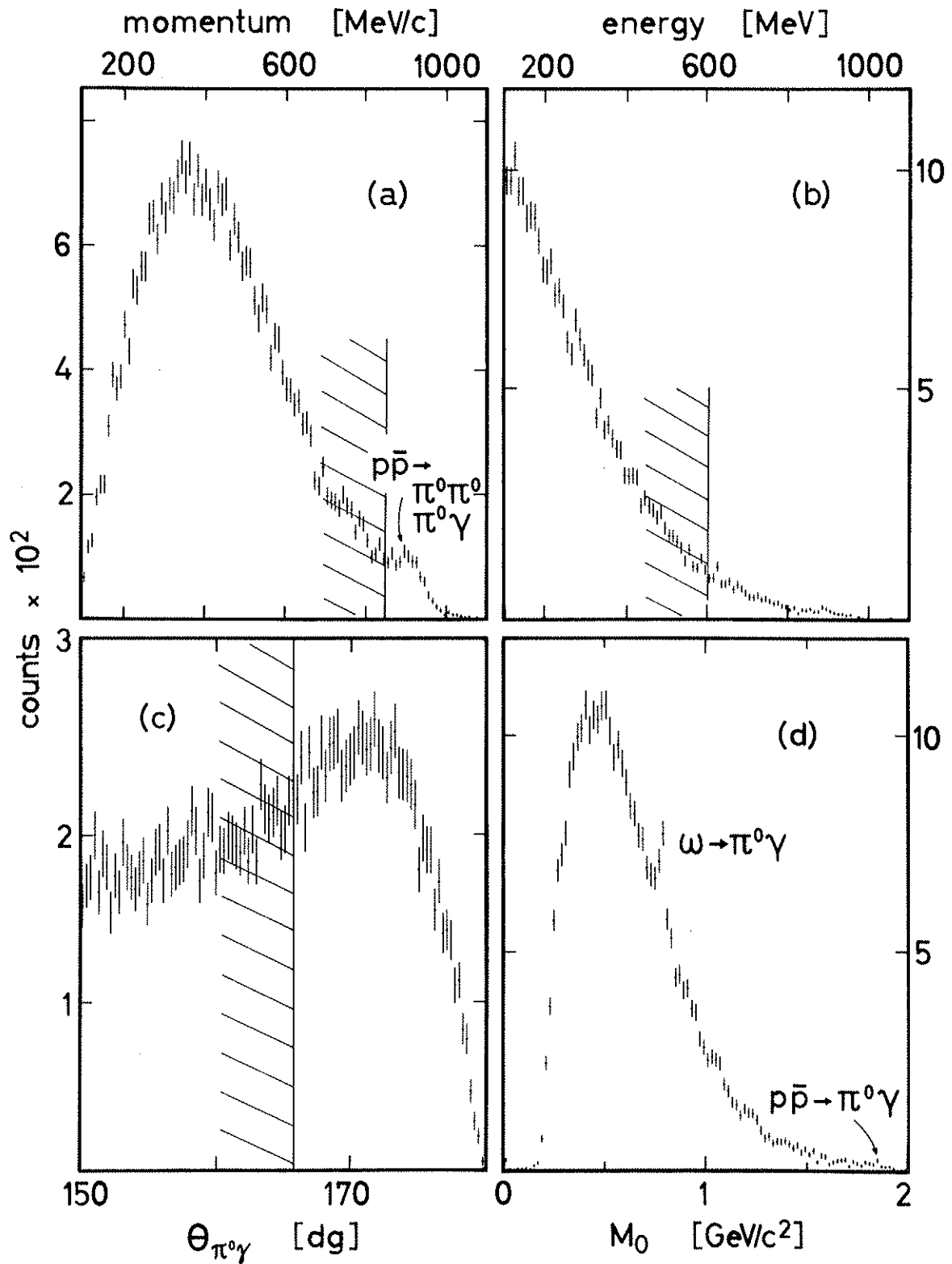


Fig. 4.9: Spectra of five different quantities derived from triple-photon coincidence events recorded during the D83 run period: (a) momentum of the π^0 detected by the lead glass, (b) energy of the photon detected by BGO₁, (c) angle $\theta_{\pi^0\gamma}$ between the photon and the π^0 directions, (d) the invariant mass M_0 , and (e) the missing mass MM . Striped regions indicate where the cut conditions are applied.

To discriminate contaminating background and separate the annihilation channels $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0\gamma$, constraints are imposed on the π^0 -momentum p_{π^0} , the photon energy E_γ , the angle Θ between their directions, and the missing mass MM , according to

- $p_{\pi^0} \geq 850 \text{ MeV}/c$
- $E_\gamma \geq 600 \text{ MeV}$ (4.19)
- $\Theta \geq 166^\circ$
- $(MM)^2 \leq 0.6(\text{GeV}/c^2)^2$

Even if the E_γ -cut and the Θ -cut are equivalent as they apply to the $2\pi^0$ -channel they might be complementary in suppressing background processes. The resulting p_{π^0} , E_γ , and Θ spectra are shown in figs. 4.10a–c. The detection efficiencies for different annihilation channels under these cut conditions are further presented in the next section.

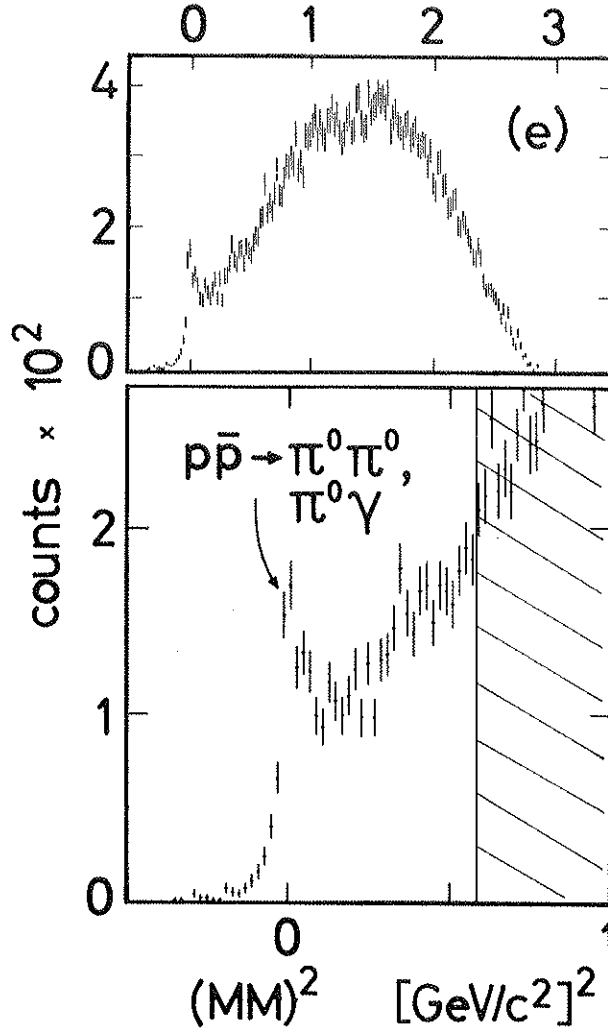


Fig. 4.9c: See figure caption on the former page.

4.1.2.5 Time Analysis

The purpose of implementing the TDCs in the trigger electronics is, in addition to reject very late events relative to the telescope signal, to offer the possibility of making a time-delayed analysis. The wish with such a time-delayed analysis is to distinguish secondary processes. These are for instance kaon decays $K \rightarrow \pi\pi^0$ (21%, 31%) that are delayed by the lifetime of the kaon ($\tau_{K^\pm} = 12.4 \text{ ns}$ and $\tau_{K^0} = 51.8 \text{ ns}$). Kaons are produced in $\sim 6.8\%$ of all $p\bar{p}$ annihilations [11]. To make the time resolution optimal two corrective measures are undertaken. One is to compensate for the different transmission times of the individual photomultipliers. The other is to amend the time-walk of the discriminators in the trigger electronics. The former correction is performed by histogramming TDC numbers in separate histograms depending on which BGO crystal contained the maximum energy. By requiring more than 50% of the shower energy to be contained in the module being plotted the trigger signal is dominated by the signal from the entrance crystal. This way the different transition times of the separate photomultipliers are probed, and relative time calibration factors are deduced:

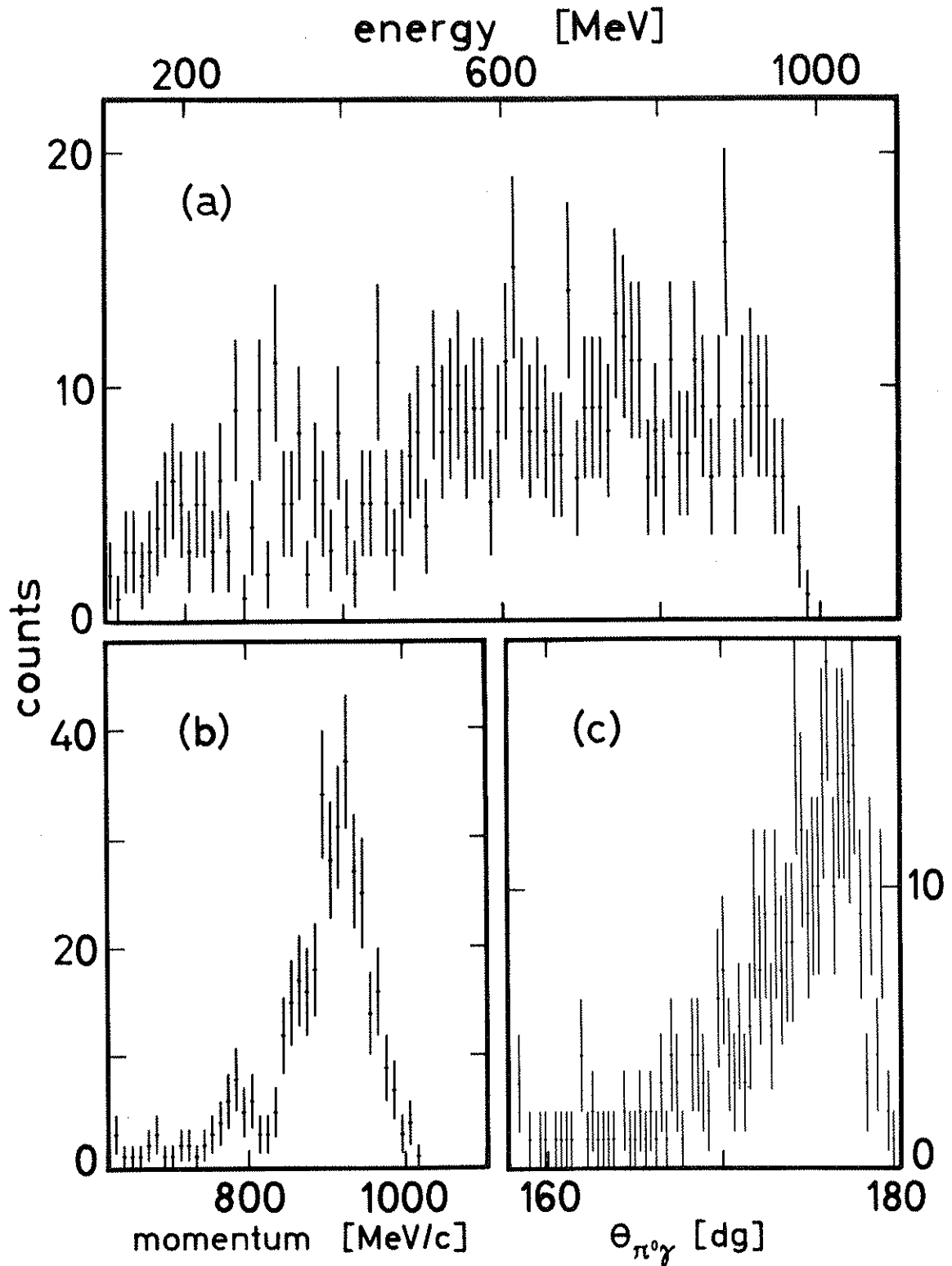


Fig. 4.10: The spectra of the same quantities as plotted in figs. 4.9a–c after the cut conditions have been applied.

A man should never be ashamed to own he has been in the wrong, which is but saying, in other words, that he is wiser today than he was yesterday.

(Alexander Pope, 1688–1744)

$$f_i = \frac{t_7 - t_0}{t_i - t_0}, \quad (4.20)$$

where $i=1,\dots,7$, and the corrected time is computed as

$$t' = \frac{t}{E_{tot}} \sum_{i=1}^7 f_i E_i. \quad (4.21)$$

The latter correction, of the time-walk, is carried out by monitoring the TDC values t as a function of shower energy E and deriving an energy dependent correction coefficient $c(E)$ according to

$$t'' = t'(E) + c(E) = \text{const.} \quad (4.22)$$

The final time spectrum of BGO₁ is shown in fig. 4.11a and shows the presence of a few late photons. The time resolution achieved after the corrections is about 320 ps.

To demonstrate secondary processes yielding π^0 's it requires that both photons of a coincidence event are late by the same time. In fig. 4.11b a scatter plot of B₁L coincidences from the event sample D83 is displayed. On the vertical axis is marked the lead-glass TDC value versus the BGO₁ TDC value on the horizontal axis, and one sees a clear diagonal band of late π^0 's, and perhaps a diffuse island in the upper right corner that are candidates for kaonic decays.

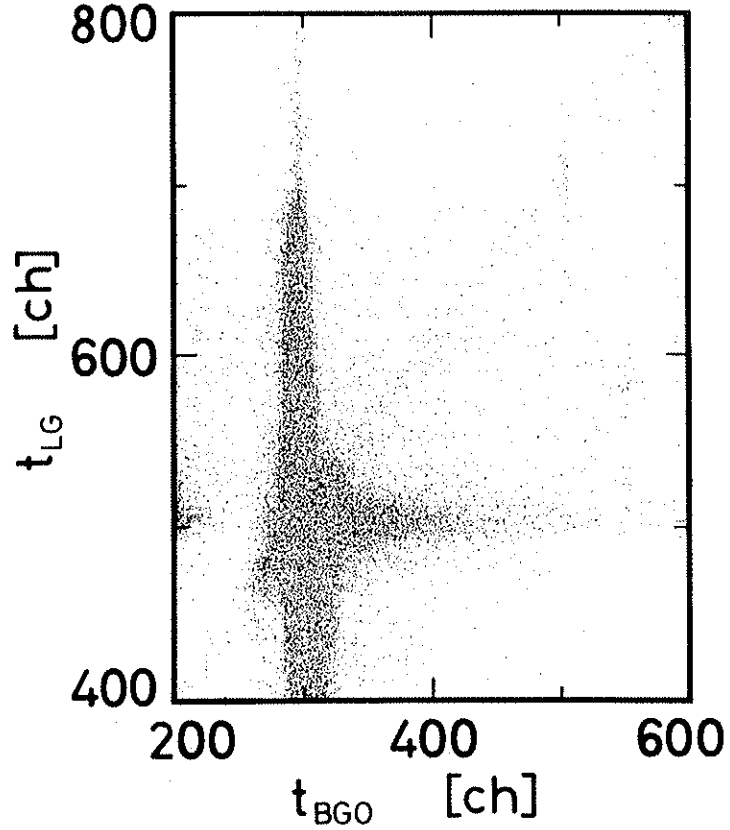


Fig. 4.11 b: Distribution of the lead-glass timing t_{LG} versus the BGO timing t_{BGO} for B₂L coincidences recorded during the D83 run period. Notice the weak diagonal band.

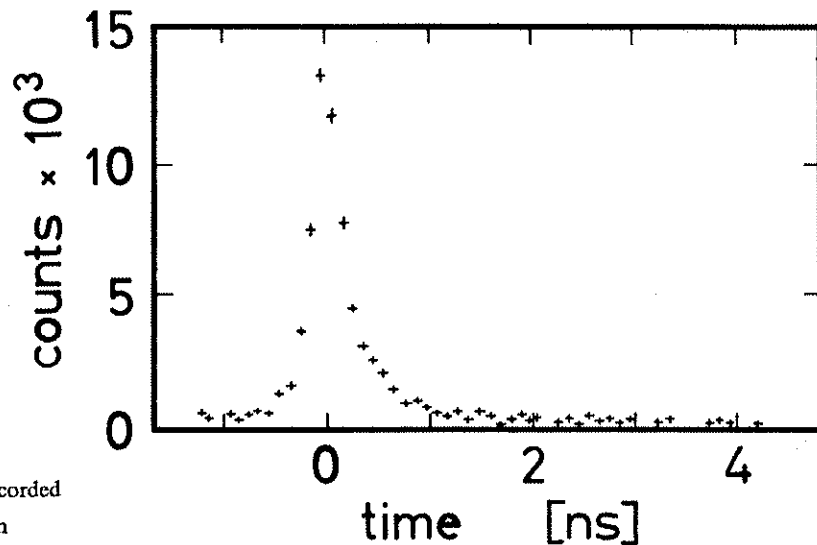


Fig. 4.11a: Time spectrum of BGO events recorded during the D83 run period. (Adapted from ref. [121]).

4.2. INTERPRETATION OF SPECTRA

In the previous section was described how the different spectra are extracted out of the raw data. It was presented what constraints were applied, both to the quality of the direct measurement of the detection phenomenon, i.e. the electromagnetic shower, and the event kinematics. The final stage of this offline analysis is a number of spectra. These are mainly π^0 -momentum spectra and the momentum spectrum of η -mesons emerging from the annihilation, and also the energy spectrum from single photons. In this section these spectra are interpreted including the deduction of yields from appearing signals, and comprehension of the general shape of the spectra. Thus, this section is confined to describing the methods, and the actual results from the interpretation are then collected in the succeeding section 4.3. The shapes of the π^0 -momentum and η -momentum spectra are dominated by the influence of the detector geometry. The overall shapes are understood from simulations of the many-body annihilation channels of the $p\bar{p}$ system by using Monte Carlo techniques. To deduce the size of a signal that appears above the continuous background the basic task is to parameterize the background in terms of a smooth (analytic) function. These procedures are described in subsections 4.2.1, 4.2.2, and 4.2.5 for the three types of inclusive spectra obtained from measuring an emitted π^0 , η , or γ , respectively. The sample of triple photon-coincidence events is subject to a more extensive interpretation that is discussed in subsection 4.2.3. Interesting information can also be extracted from the B_1L sum energy spectrum, which is presented in subsection 4.2.4.

4.2.1 Inclusive π^0 Momentum Spectra

Of the inclusive π^0 -momentum spectra the LL spectrum is perhaps the most comprehensive one. It contains high statistics and shows structures from resonances after the continuous background has been parameterized and subtracted. The high statistics of the spectrum offers high sensitivity for derivation of upper limits for any non-observed two-body disintegration modes of the $p\bar{p}$ system. The B_2L spectra from the run periods D83, M84, and A84 have their highest sensitivity at momenta around 200 MeV/c. They do therefore serve as complement to the LL spectrum when computing upper limits of two-body decays. The sensitivity of the LL combination is lower at low π^0 -momenta due to low detection efficiency. The B_1L combination is combined with LL to exclusively measure π^0 -coincidences in the lead-glass array that are accompanied by a third photon in the opposite direction. In addition B_1L is used to measure high-energetic back-to-background photon coincidences. Parenthetically it can be mentioned that the general shape of the B_1L spectrum offers a nice opportunity for a deeper understanding of the impact that the detector geometry has on π^0 -spectroscopy.

4.2.1.1 Double LL Coincidences

A first rough qualitative understanding of the general shape of the LL spectrum is apparent from comparing the spectrum of produced π^0 -mesons, i.e. as it would have been seen by a perfect 4π -detector (fig. 4.12). The LL spectrum has been simulated with Monte Carlo techniques. The annihilation channels included in this Monte Carlo simulation are those labelled "MC" in table 2.1, which are the dominant annihilation channels. These dominate the continuous background of the π^0 -spectrum. The resulting spectrum is shown in fig. 4.13a. Figs. 4.13b and 4.13c show the contributions from true π^0 -coincidences and from uncorrelated ones, respectively. The spectrum dropping to zero around 100 MeV/c is owing to the fact that the acceptance vanishes.

To subtract the continuous part of the data spectrum one needs a smooth (analytic) function that has the shape of the continuous background spectrum, i.e. that which has been simulated. We therefore try to parameterize the Monte Carlo spectrum (fig. 4.13a) in terms of an analytic function. The spectrum can be expressed as a set of numbers $c_i(p_i)$ being the bin contents where p_i are the momenta at the centre of the bins. Making a variable transformation from p_i to $x_i = p_0 - p_i$, where p_0 is the maximum occurring momentum, and plotting the spectrum $c_i(p_i)$ on a double logarithmic scale (fig. 4.14) shows that a large part of the spectrum becomes approximately linear. A straight line in this plot has the equation:

$$\ln(f) = \alpha + \beta \ln(x) \equiv \alpha + \beta \ln(p_0 - p) \quad , \quad (4.23)$$

which suggests that a large part of the spectrum can be fitted by the function⁶

$$f = e^{\alpha} (p_0 - p)^{\beta} = a(p_0 - p)^{\beta} \quad , \quad (4.24)$$

where a and β are free parameters. By subtracting a similar term with a "deeper knee", i.e. a larger exponent β , the curve can be bent down to follow the spectrum over a larger range. A function⁷ that fitted the whole Monte Carlo spectrum well was obtained with four such terms:

$$f(p) = A \sum_{l=1}^4 a_l (p_0 - p)^{\beta_l} \quad , \quad (4.25)$$

where the parameters were fitted (except a_1) to the following values:

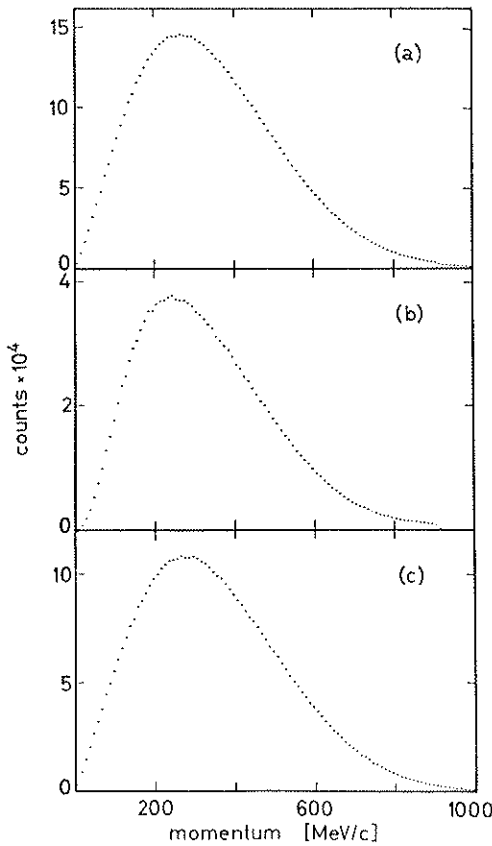


Fig. 4.12: Momentum spectra of the annihilation channels marked "MC" in table 2.1 as simulated with Monte Carlo techniques: (a) using all possible double photon coincidences, (b) of π^0 -mesons, and (c) of only uncorrelated photon pairs.

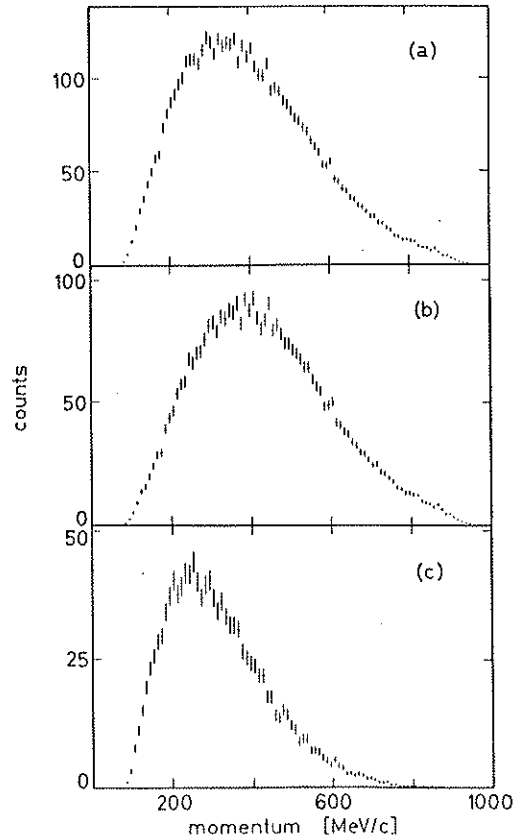


Fig. 4.13: LL momentum spectrum (a) as obtained from a Monte Carlo simulation. Spectra (b) and (c) are the contributions from true π^0 -coincidences and uncorrelated coincidences, respectively.

⁶ A function of this type has been used to successfully fit the inclusive photon spectrum; see section 4.2.5 and ref. [13].

⁷ Also the π^0 -spectrum might be possible to fit with a function of the same type as was later found to fit the γ -spectrum; see footnote on page 138.

$$\begin{array}{ll}
 a_1 = 1 & \beta_1 = 1.6 \\
 a_2 = e^{-13.5} & \beta_2 = 3.5 \\
 a_3 = e^{-63.6} & \beta_3 = 10.8 \\
 a_4 = e^{-83.9} & \beta_4 = 28.1
 \end{array}$$

A is also a free parameter but is not relevant for the shape of the spectrum, since it only determines the scale. The fit resulted in a χ^2 -value of 127 for 83 degrees of freedom. Fig. 4.15 shows the Monte Carlo spectrum with the fitted function. Fig. 4.16 shows the measured LL spectrum from M85 with the function $f_{back}(p) = Nf(p)$, where N is a normalization constant. As demonstrated by the figure the Monte Carlo simulation does not agree with the measured spectrum in detail. The reason for this is not understood, but seems to originate from annihilation channels with low π^0 -multiplicity but high multiplicity of charged pions, and could not thus possibly be connected to the rejection of showers due to firing veto counters.

The dashed curve is the contribution from the uncorrelated background as obtained from the Monte Carlo simulation, and is compared to the bulleted data points that are an estimation of the uncorrelated background from data. This estimation is computed as the average of the results from three different methods. One method consists of determining the signal-to-background ratio of 20 different invariant mass plots. Depending on which 50 MeV/c interval the π^0 -momentum belongs to a corresponding invariant mass plot is incremented. This method hinges on the assumption that the χ^2 -cut does not alter the shape of the uncorrelated background. To confirm or deny this assumption the following procedure was undertaken. The signal-to-background

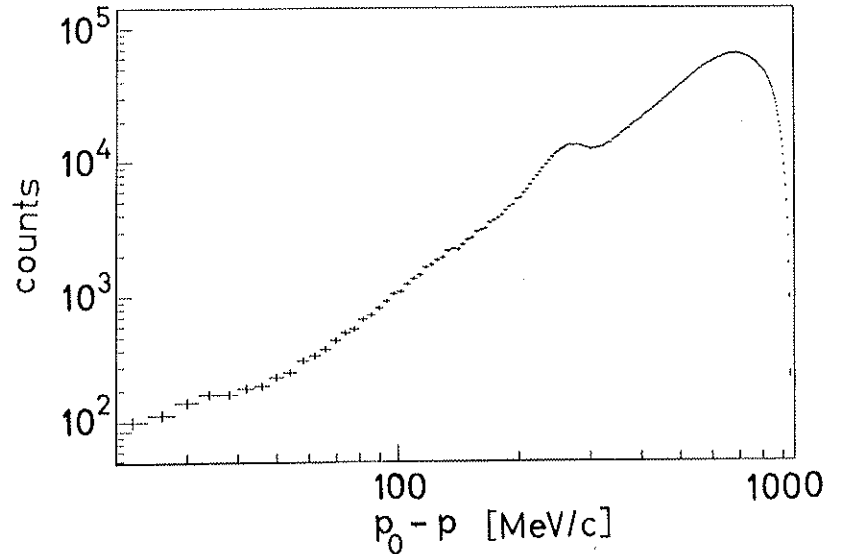


Fig. 4.14: LL π^0 -momentum spectrum recorded during A84 plotted on double logarithmic scale as a function of $p_0 - p$, where p_0 is the maximum occurring momentum and p is the π^0 -momentum.

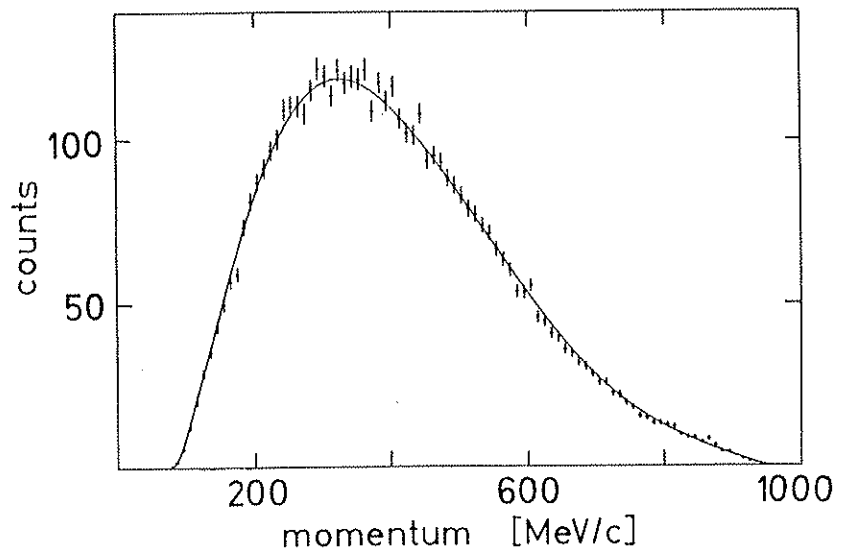


Fig. 4.15: A fit of the Monte Carlo simulated LL spectrum (fig. 4.13a).

ratios cannot be determined for invariant mass plots that are produced only from events that passed the χ^2 -cut, because the shape of the uncorrelated background has also a Gaussian shape after the χ^2 -cut (fig. 4.5). However, fruitful information is obtained by comparing the total number of entries of each invariant mass plot produced after the χ^2 -cut with the corresponding plot produced prior to the cut. Knowing that the χ^2 -cut rejects 5% of the true π^0 -events the rest of the removed events has to be uncorrelated background, i.e., if the area of the π^0 peak between the $\pm 3\sigma$ limits is N_{π^0} , and the total number of entries after the χ^2 -cut is n_{tot} , the number of uncorrelated coincidences n_{uc} after the χ^2 -cut is

$$n_{uc} = n_{tot} - n_{\pi^0} = n_{tot} - 0.95N_{\pi^0} \quad (4.26)$$

Thus, the signal-to-background ratio s/b after the χ^2 -cut is

$$\frac{s}{b} = \left[\frac{n_{tot}}{0.95N_{\pi^0}} \left(1 + \frac{1}{(S/B)} \right) - 1 \right]^{-1} \quad (4.27)$$

Comparing the 20 ratios $(S/B)_i$ obtained before the χ^2 -cut and the 20 ratios $(s/b)_i$ obtained after the cut indicates that higher momentum events are slightly more suppressed.

Another method to uncover the shape of the uncorrelated coincidences is to plot the content of each 50 MeV/c interval as a function of the χ^2 -cut value. When the cut is moved to larger values the events gained are practically only uncorrelated ones. The third method is to use the events in a window above and one below the π^0 -window, as was done to deduce the contribution to the χ^2 -distribution. Momentum spectra are subsequently produced by performing fits to m_0 -values at the centre of the windows. These are shown in fig. 4.17, and the shape in the π^0 -window is tentatively estimated by binwise taking the average of these two shapes. To estimate the contribution to the χ^2 -distribution one method employed was to synthesize coincidences from photons belonging to different events. This method can, on the contrary, not be used to estimate the momentum distribution of the uncorrelated background, because there is a substantially larger probability to pick a pair of photons where both have high energy than it is in a physical coincidence. In a physical event the total energy and momentum have always to be conserved, which does not hold for a scrambled coincidence.

In principle the goal is to subtract the continuous background in terms of an analytic function, and to fit the residual spectrum with a function $f_{peak}(p)$ that is a sum of peaks^a $f_r(p)$, where r labels a resonance:

$$f_{peak}(p) = \sum_r A_r f_r(p), \quad (4.28)$$

^a The peak function $f_r(p)$ is derived from a Breit-Wigner shape by weighting with the acceptance curve (fig. 3.81a) and folding with the resolution.

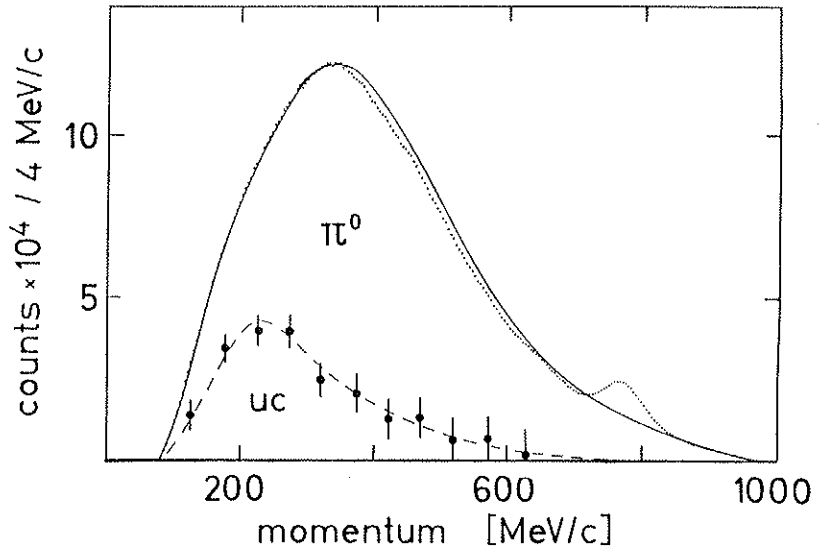


Fig. 4.16: The LL π^0 -momentum spectrum from M85 with the function fitted to the Monte Carlo spectrum. The dashed curve shows the contribution from uncorrelated coincidences obtained from the Monte Carlo simulation and is compared to an estimation based on data.

where A_i are the amplitudes of the signals from the respective resonances. However, due to the failure of the background function $f_{back}(p)$ to describe the shape accurately, this approach was altered. The π^0 -signals expected from observations of π^0 -associated two-body annihilations in earlier experiments are listed in table 4.2 together with the estimated yields. In fig. 4.18 these signals are exhibited as they would appear in the LL π^0 -spectrum if they all were produced with the same branching ratios. The figure also shows the expected signal from $p\bar{p}$ annihilations into $\pi^0\omega$, which has not been directly observed before. A structure function $f_{peak}(p)$ was composed being a sum of the four peaks $f_i(p)$ from table 4.1 with the amplitudes A_i as expected from the observed branching ratios. The function is shown in fig. 4.19 together with the data spectrum from A84. The spectrum is then fitted by the sum function

$$f_{spec}(p) = N_{peak} f_{peak}(p) + N_{back} f_{back}(p), \quad (4.29)$$

where the normalization factors N_{peak} and N_{back} are free parameters. In fact, also the parameters of f_{back} were refitted since f_{back} failed to describe the background exactly, but their values were altered by less than a standard deviation. Fig. 4.20a shows the total spectrum and the background function as resulting from the fit, while fig. 4.20b shows the residual spectrum after subtraction of this background, and with the structure function superimposed. The small cusp at 205 to 210 MeV/c is recognized as stemming from kaons produced in $p\bar{p}$ annihilations and stopping in the target, and decaying into $\pi\pi^0$. A small Gaussian was added to the fit function to represent this signal. Adding a peak for the process $p\bar{p} \rightarrow \pi^0\omega$ to the function f_{spec} resulted in a slightly better χ^2 -value of the fit and a value of A_ω that increased the area of the " $\rho\omega$ -peak" with roughly 25%. It should be emphasized, though, that the $\pi^0\omega$ -yield deduced in this way is con-

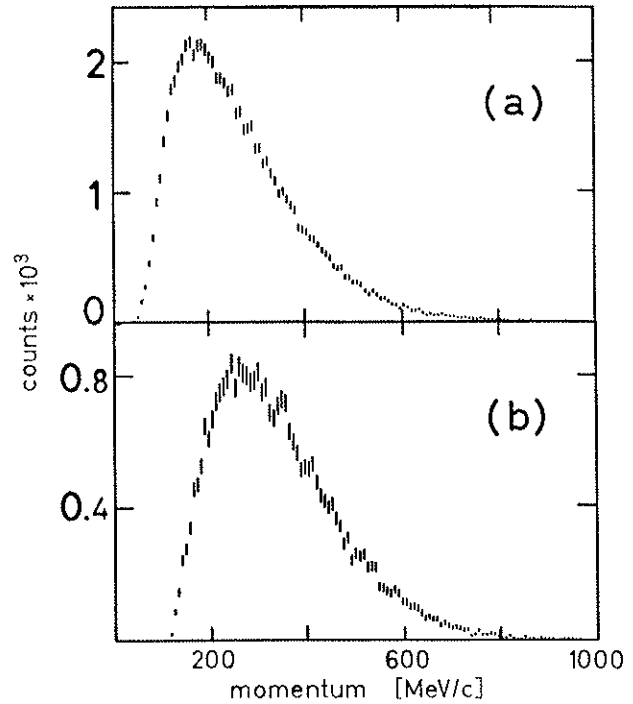


Fig. 4.17: Momentum spectra obtained from uncorrelated coincidences belonging to windows (a) below and (b) above the π^0 -window in the invariant mass spectrum.

Table 4.2: Expected π^0 -signals.

Reaction	Yield
$\pi^0\rho^0$	1.9%
$\pi^0\mathbf{B}^0$	0.4%
$\pi^0\mathbf{f}$	0.4%
$\pi^0\mathbf{A}_2^0$	2%

nected to the shape⁹ of the peak in the spectrum, and does thus hinge on the size of the signal in this spectrum from the channel $p\bar{p} \rightarrow \pi^0 \rho^0$.

The significance of the $\pi^0 \omega$ -signal can be enhanced by two means. On one hand, it can be discriminated from the $\pi^0 \rho^0$ -signal by interrogating the charged-pion distribution. On the other hand, there is the possibility of asking the presence of a third photon. The vast majority ($\sim 100\%$) of the ρ^0 -mesons decay into two charged pions (two neutral pions do not occur due to charge-parity conservation). Thus, the $\pi^0 \rho^0$ -events will be completely excluded by the requirement of a third photon, whereas a fraction of the $\pi^0 \omega$ -events will be retained due to the decay of the ω -meson into $\pi^+ \pi^- \pi^0$ or $\pi^0 \gamma$ yielding the third required photon. The third photon can be detected either by a BGO detector or by the lead-glass array. Due to the small solid angle of a BGO detector the already small $\pi^0 \omega$ -signal is further reduced so much by this condition that it is almost washed out. The π^0 -momentum spectrum in fig. 4.21 is obtained from M85 data, and is produced from those events that consisted of at least three showers in the lead-glass array, of which two were identified as a π^0 -decay. The signal at 768 MeV/c in this spectrum is exclusively coming from the decay $p\bar{p} \rightarrow \pi^0 \omega$. The area of this peak is accurately enough deduced by fitting a Gaussian plus a second order polynomial to this region of the spectrum, because the ω -resonance is narrow and thus the peak width is the detector resolution.

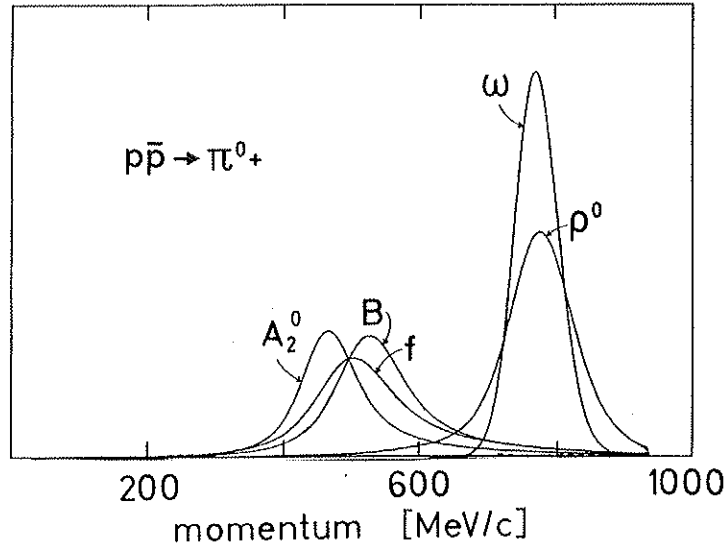


Fig. 4.18: Peaks expected in the LL π^0 -momentum spectrum from two-body annihilation channels.

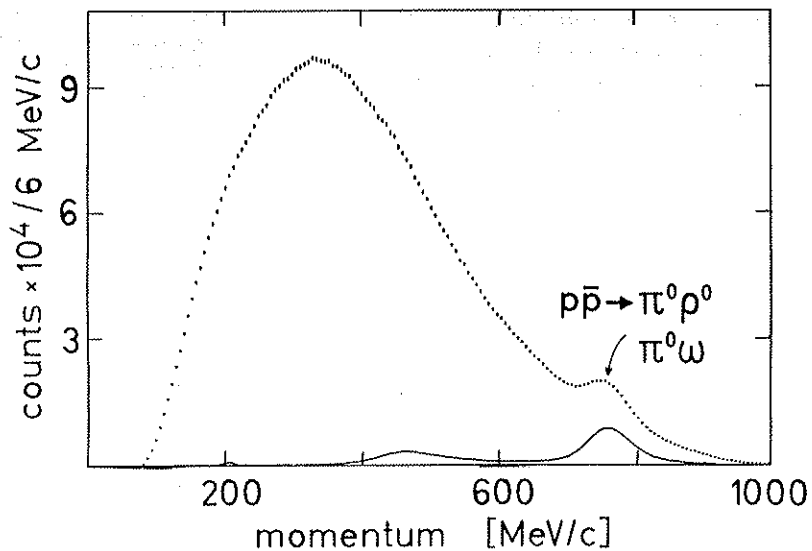


Fig. 4.19: The LL π^0 -momentum spectrum from A84 and the function representing the expected structures.

⁹ The peak shapes were derived from a classical Breit-Wigner shape, which does not give the completely correct shape, but does not alter the overall area significantly. However, when decomposing an observed structure into two peaks the chosen shape might influence the ratio.

The other method to separate the two signals $\pi^0\rho^0$ and $\pi^0\omega$ in the π^0 -momentum spectrum, is to utilize the information from the charged-pion hodoscope installed around the target during the M85 run period. This device consisted of 20 scintillator strips of dimensions $0.5 \times 2.0 \times 30$ cm³ assembled to a cylinder of 6 cm inner radius. Each scintillator strip subtends an azimuthal angle of 18°, and the complete cylinder covers 92% solid angle. The idea of this investigation is that of making use of the different spatial distributions of the $\pi^+\pi^-$ -pairs from the decays of the ρ^0 -mesons and the ω -mesons, respectively. The decays of these mesons are not isotropic in the centre-of-mass system but are correlated to the direction of motion as they are vector mesons, i.e. they carry spin. The momentum direction of the vector meson can be chosen as the quantization axis, since it is the opposite of that of the primary π^0 and thus known. The differential cross-sections [39] [147] for the two decays $\rho^0 \rightarrow \pi^+\pi^-$ and $\omega \rightarrow \pi^+\pi^-\pi^0$ are

$$\begin{aligned} d\sigma &\sim \sin^2\Phi \, d\Omega, \\ d\sigma &\sim \sin^2\Theta_N \sin^2\Theta \, d\Omega, \end{aligned} \quad (4.30)$$

respectively, where Φ is the helicity angle of the positive pion from the ρ^0 -decay, and Θ_N is the angle between the ω -momentum and the normal to its disintegration plane, and Θ is the helicity angle of the positive pion in the centre-of-mass system of the $\pi^+\pi^-$ from the ω -decay.

While this thesis is being written these investigation have just started and not yet yielded final results. A Monte Carlo simulation [100] of the expected charged-pion distributions from

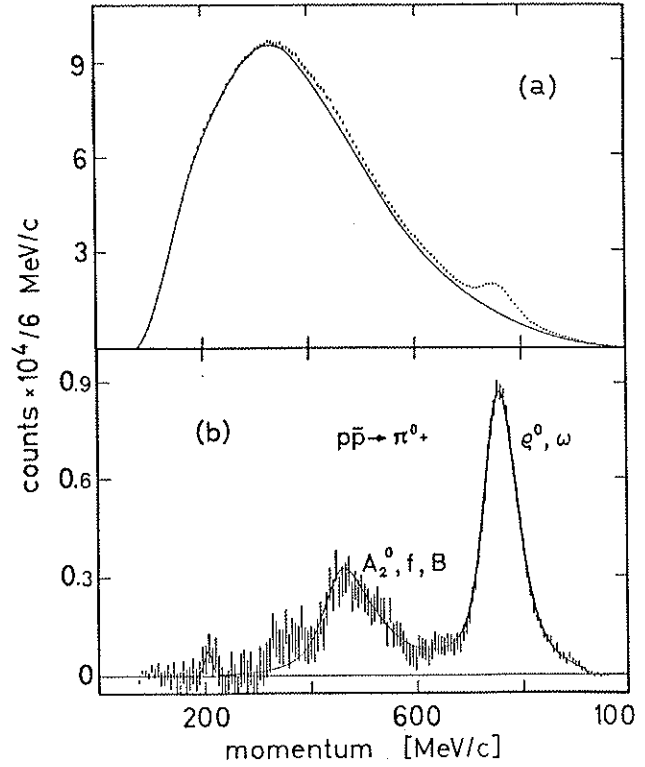


Fig. 4.20: The LL π^0 -momentum spectrum from A84 with the function representing the continuous background superimposed (a). The residual spectrum after subtraction of the continuous background with the structure function superimposed is shown in (b).

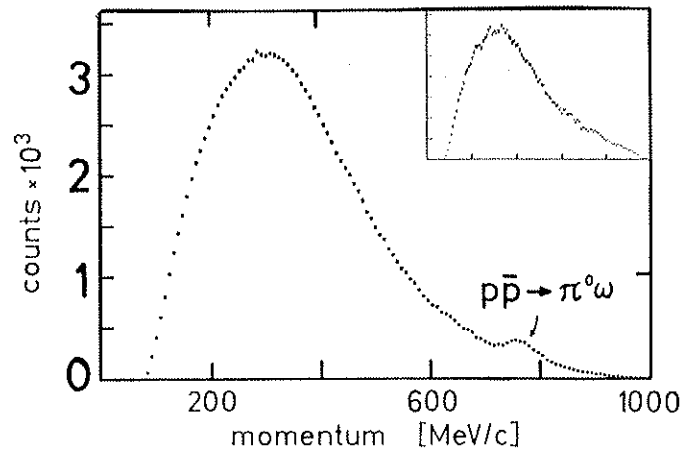


Fig. 4.21: LL π^0 -momentum spectrum from M85 obtained from events yielding three photon showers in the lead-glass array. Inset shows the spectrum when requiring no charged pions.

these two annihilation channels was done using a computer program by Amsler [10] that incorporates the spin correlation¹⁰ of the decays. The difference in the distributions of the charged pions from the two decays $\rho^0 \rightarrow \pi^+ \pi^-$ and $\omega \rightarrow \pi^+ \pi^- \pi^0$, respectively, is perhaps best visualized in a two-dimensional plot with one axis being the number $i=1, \dots, 20$ of that scintillator strip fired by one of the charged pions, and the other axis being the number $j=1, \dots, 20$ of that scintillator strip fired by the other charged pion. Scintillator strip "1" is then defined as the one the primary π^0 passed through, which direction is calculated from the kinematics of the two decay photons detected by the lead-glass array. In this plot the central cross $i=j=10$ represents the opposite direction of the primary π^0 . Apparently, all $\pi^0 \rho^0$ -events will occur in quadrants 2 and 4 symmetric around the diagonal, because the two charged pions in these decays have to emit symmetrically in the projection plane perpendicular to the $\pi^0 \rho^0$ -decay line (fig. 4.22). This circumstance does not hold for the $\pi^0 \omega$ -events, since the ω -decay is a three-body decay which will allow the charged pions to appear on the "same side" of the $\pi^0 \omega$ -direction, so to say. The resultant distributions from the Monte Carlo investigation are shown in

fig. 4.23.

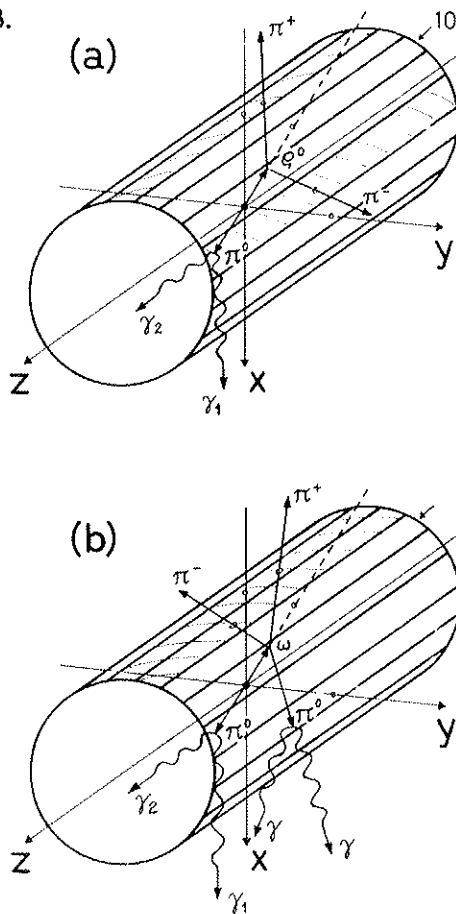


Fig. 4.22: The two charged pions from a ρ^0 -decay (a) do always hit the multiplicity counter on opposite sides of the $\pi^0 \rho^0$ -line, which is in contrast to the two charged pions from an ω -decay (b).

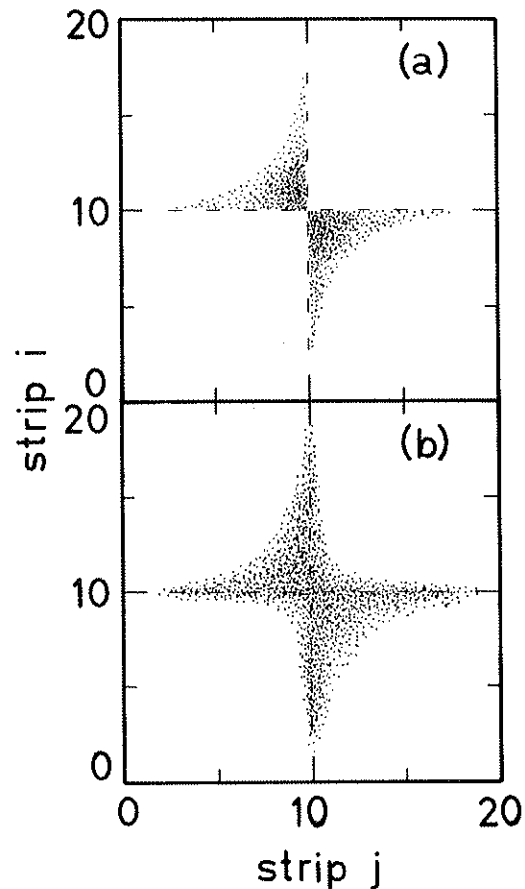


Fig. 4.23: Distributions of pairs of fired scintillator strips i, j of the multiplicity counter as resulting from a Monte Carlo simulation of (a) ρ^0 -decays, and (b) ω -decays.

¹⁰ In practice, Monte Carlo programs of this type work in the way that, firstly, an event is generated randomly so that energy and momentum is conserved according to the mass specifications given as input. After that, a weight factor w_p is calculated which informs about the probability that this particular momentum configuration occurs, owing to phase space considerations. Similarly, a weight factor w_s can be calculated from considering spin-correlation effects so that the overall probability for a particular decay to occur is given by the product $w_p w_s$.

A similar data plot of the events between 680 MeV/c and 850 MeV/c in the π^0 -momentum spectrum from M85, and for which exactly two strips of the charged-pion hodoscope fired, does, however, not show any conspicuous clue how to separate the two channels $p\bar{p} \rightarrow \pi^0 \rho^0$ and $p\bar{p} \rightarrow \pi^0 \omega$. The hope is that the continuous background under the peak in the π^0 -momentum spectrum is flat and smooth in the charged-pion plot. Whether this is true has, however, to be further examined before any conclusions can be drawn from this plot.

Finally, information can be extracted from this π^0 -momentum spectrum about any other two-body decay of the $p\bar{p}$ system that result in a π^0 by deducing a "sensitivity curve". This curve gives an upper limit for how frequently any non-observed two-body decay $p\bar{p} \rightarrow \pi^0 X$ can occur. The limit is deduced such that, if a process $p\bar{p} \rightarrow \pi^0 X$ occurs with a branching ratio given by this sensitivity curve, it would result in a " 3σ " peak in the LL π^0 -momentum spectrum. The unit " σ " for denoting significance is indeed a confused concept in the literature. I have adopted the definition given in Appendix C. The size of a 3σ -peak is obtained by adding an artificial peak to the spectrum and fitting the function Cf_{spec} with only the scale C as a free parameter, and calculating the increase of the χ^2 -value (cf. Appendix C). The branching ratio corresponding to such a peak is then derived by dividing the total number of counts in the peak with the π^0 -acceptance for its mean¹¹ π^0 -momentum. Subsequently the branching ratio for the associated hypothetical two-body decay $p\bar{p} \rightarrow \pi^0 X$ is obtained by normalizing the yield to the total number of $p\bar{p}$ interactions that were contributing to the spectrum.

The sensitivity curve deduced from the LL spectrum from M85 is shown in fig. 4.24. On the vertical axis the branching ratio $R(p\bar{p} \rightarrow \pi^0 X)$ is read directly as a function of the momentum of the recoiling π^0 .

It should be emphasized that the sensitivity curve in fig. 4.24 is derived under the assumption that the unknown state X is narrow enough not to cause the momentum distribution of the recoiling π^0 to be broader than $\sim 10\%$ of the resolution at the mean π^0 -momentum, which allows the mass distribution of the X -state to be about 20% broader. For a larger lifetime of the X -state the branching ratio has to be larger to yield a 3σ -effect in the spectrum.

It should finally also be pointed out that the entire discussion above is only strictly valid if the X -state does not disintegrate into some fatal charged-particle distribution that will impair the π^0 -measurement. This would reduce the effective π^0 -acceptance and thus saturating the sensitivity by requiring a larger branching ratio.

Fig. 4.25: B₂L π^0 -momentum spectrum from A84 with a spline function representing the continuum superimposed. The dashed line is the contribution from the uncorrelated background.

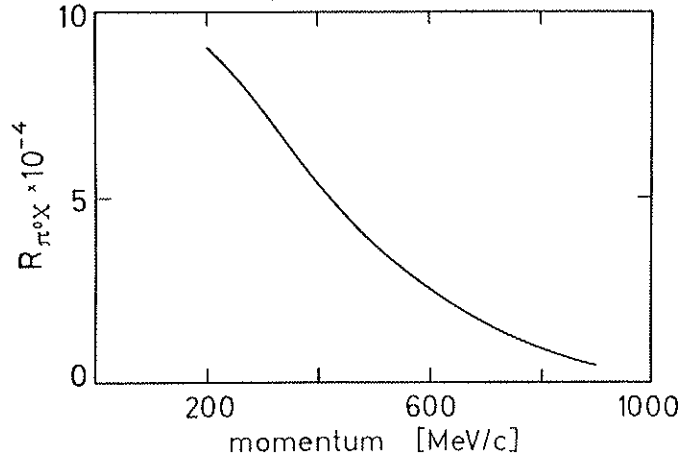
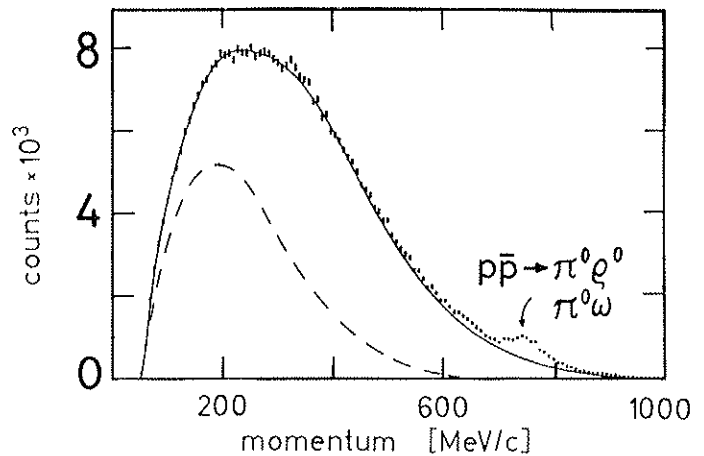


Fig. 4.24: Upper limit for the branching ratio $R_{\pi^0 X}$ (see text) with 95% confidence level as a function of the π^0 -momentum.



¹¹ The acceptance should be taken at the mean value of the peak, because the momentum is resolved after the π^0 has been accepted by the geometry of the detector.

4.2.1.2 Double B_2L Coincidences

There are three B_2L π^0 -momentum spectra recorded during three different run periods. The resolution differs between the spectra due to the different number of BGO hodoscopes mounted in the BGO system, and in addition the D83 arrangement subtended another angular interval. Thus, the acceptance of the D83 spectrum is somewhat different, and care has to be taken if summing the three spectra together. Fig. 4.25 shows the A84 spectrum together with two curves, the dashed one resulting from a Monte Carlo simulation and representing the contribution from the uncorrelated coincidences. The contribution from the uncorrelated background was in this case also estimated from data as was done for the LL spectrum.

The continuous background of this spectrum could not be parameterized in terms of an analytic function of the kind used for the LL spectrum, but is well parameterized by a spline function of fourth order with seven knots. The smoothness of this function was examined by computing the derivative of it, which was found to be monotonically decreasing to zero and then remaining negative, making no rapid variations.

A sensitivity curve (fig. 4.26) was also derived for this spectrum by using this spline function plus a peak function for the $\rho^0\omega$ -signal.

4.2.1.3 Double B_1L Coincidences

There are also three different B_1L π^0 -momentum spectra existing from the same three different run periods, viz. D83, M84, and A84. The spectra from D83 and A84, are more similar in the sense that they both cover about the same π^0 -momentum range, which differs slightly from the M84 spectrum. The B_1L spectrum measured during the A84 period is shown in fig. 4.27. The spectrum contains mostly of uncorrelated coincidences which is demonstrated by the invariant mass plot (fig. 3.74c). The monochromatic π^0 -signal at 28 MeV/c springs from the Panofski reaction $\pi^-p \rightarrow \pi^0n$ which is induced by π^- -mesons produced in $p\bar{p}$ annihilations and stopped in the target. This signal has been used for calibration.

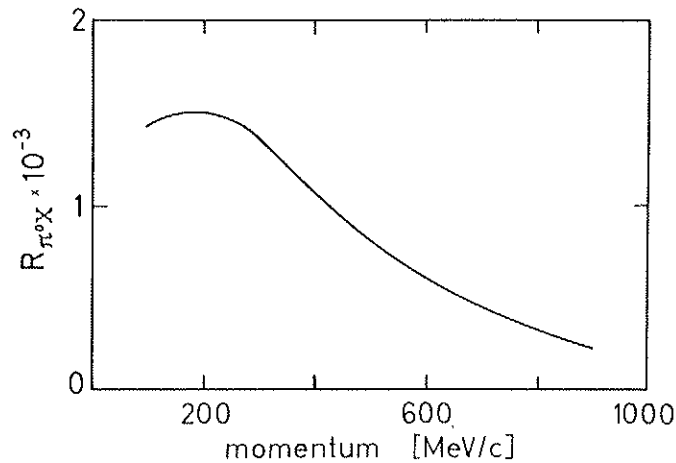


Fig. 4.26: Upper limit for the branching ratio $R_{\pi^0 X}$ (see text) with 95% confidence level as a function of the π^0 -momentum.

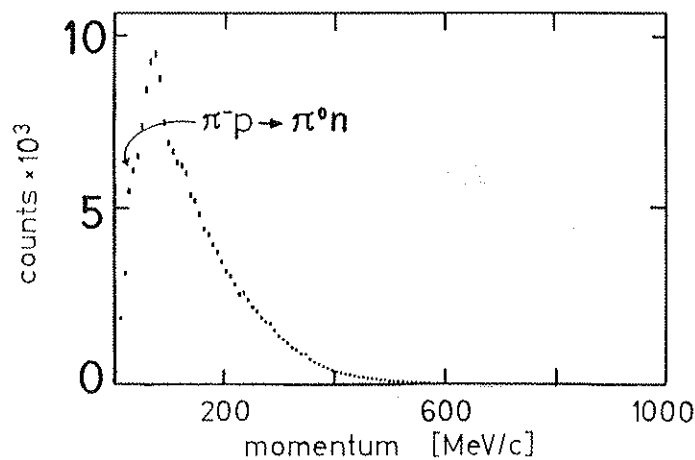


Fig. 4.27: B_1L π^0 -momentum spectrum from A84.

The somewhat queer general shape of this spectrum offers a relishing opportunity to further understand the impact the detector geometry has on the spectral shape. The bump around 110 MeV/c on the upper falling slope of the B_1L spectrum shows also up in a Monte Carlo simulation of the spectrum, and has therefore most likely to be due to the detector geometry. A tentative explanation could be the following. Let us consider a uniform π^0 -energy distribution between 135 and 1000 MeV (fig. 4.28a). The momentum distribution is then as indicated in the figure. If the detector now only accepts coincidences in a narrow aperture around, let us say, 120° the accepted spectrum will have a shape according to the Jacobian-shaped distribution under the uniform one, where the peak is obtained for the momentum that corresponds to a minimum angle of 120° . In a diagram where one decay-photon energy E_1 is plotted versus the other photon energy E_2 , these accepted events will lie on the hyperbola (fig. 4.28b)

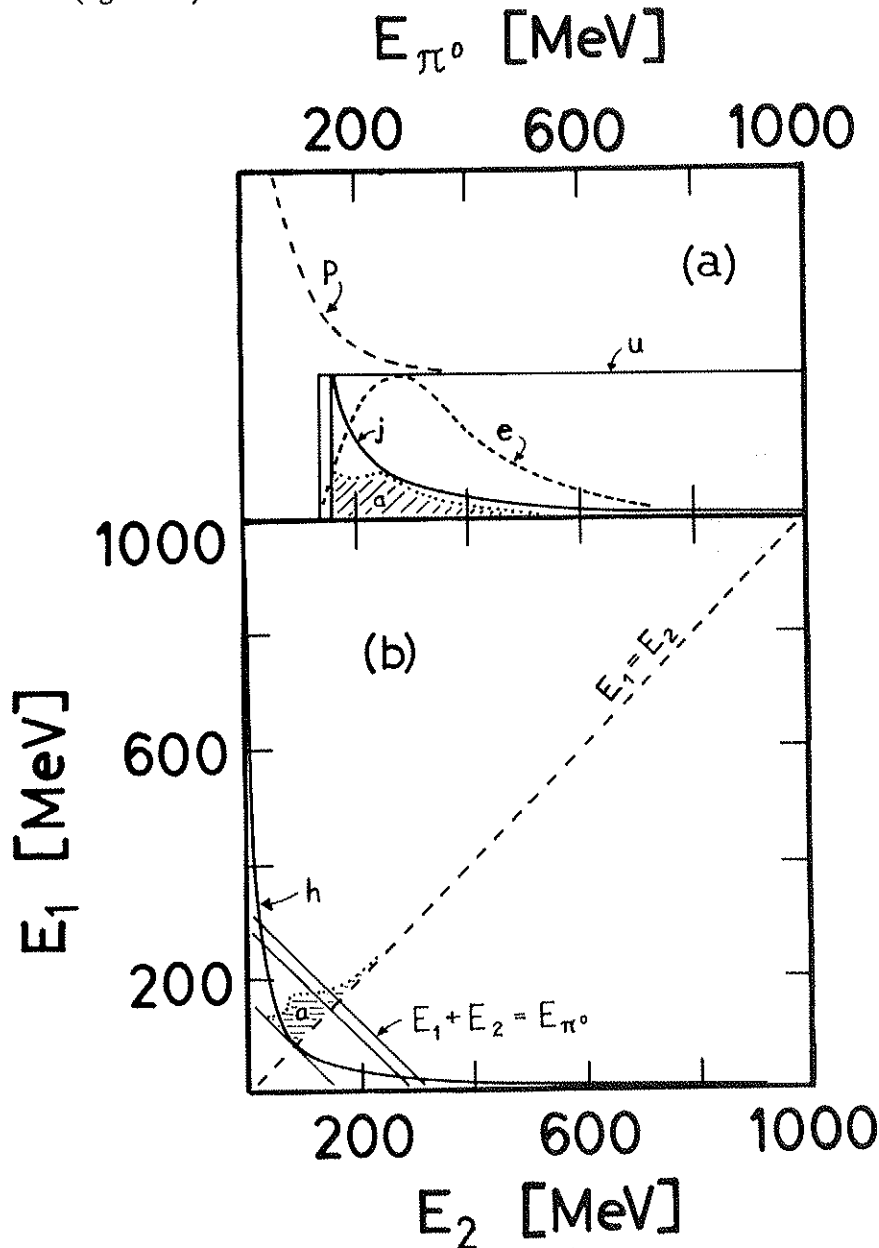


Fig. 4.28: The accepted part j of a uniform π^0 -energy distribution u in a small angular interval around 120° . Thus, a is the accepted part of the π^0 -spectrum e from pp annihilations. In a plot (b) of one photon energy E_1 versus the energy E_2 of the other photon, π^0 's occur on the hyperbola h , and the π^0 -energy spectrum is the projection onto the diagonal $E_1 = E_2$.

$$E_1 = \frac{m_0^2}{4E_2^2 \sin^2(\phi/2)} = \frac{(135)^2}{4 \sin^2 60^\circ} = \frac{6075}{E_2} \quad (4.31)$$

In this two-dimensional plot the π^0 -energy distribution is the projection onto the diagonal $E_1 = E_2$. The π^0 -energy distribution after acceptance is obtained by sampling the different Jacobian-peak distributions at 120° . In the E_1 -versus- E_2 plot the events in one and the same bin in the π^0 -energy spectrum are lying within a diagonal strip $E_1 - E_2 = E_{\pi^0}$ as illustrated in the figure. If we now go back and remind that the produced π^0 -energy spectrum is not uniform, the accepted spectrum has to be scaled binwise so that the striped shape in fig. 4.28 is obtained. To get the measured spectral shape we have to sum the contribution from several hyperbolas down to $p_{\pi^0} = 0$. The rising edge of the spectrum is thus given by the shape of the produced spectrum. This rise is abruptly cut off to a sharp cusp as the range vanishes and the bump on the falling slope is a remainder of the maximum of the produced π^0 -spectrum. I hope this discussion has made it conceivable that the bump around 110 MeV/c the spectrum is marred with is **not** a signal, and has shown the importance of examining structures observed with a γ -pair spectrometer with delimited solid angle.

It could also be illustrative here to mention the difference between the two ways the π^0 -momentum can be obtained from the two measured photon energies, either by performing a fitting of the decay kinematics or not. The measured π^0 -events will not lie exactly on the hyperbola (4.31) in fig. 4.28 but scatter around it due to detector resolutions. If no kinematic fitting is conducted the π^0 -energy will directly be projected onto the diagonal $E_1 = E_2$ as if the event would have been lying on the hyperbola "behind" (seen from the diagonal $E_1 = E_2$). If a fitting of the kinematics is performed, on the other hand, the event will firstly be projected onto the closest point on the hyperbola, and then projected onto the diagonal $E_1 = E_2$.

A final remark concerning the B_1L spectrum is why the spectrum of uncorrelated events vanishes in the vicinity of the origin. Uncorrelated coincidences would be expected to occur isotropically and completely random at arbitrary coincidence angles, as for example 180° and 120° , i.e. as likely at 0 momentum as at 78 MeV/c. The reason is simply that the solid-angle element vanishes at the origin. The number of coincidences detected in a certain angular interval $[\phi, \phi + d\phi]$ is

$$dN = 2\pi R \sin(\pi - \phi) d\phi, \quad (4.32)$$

where $R = 80$ cm is the distance to the lead-glass array, and ϕ is the opening angle between the two γ -rays. Thus, the angle spectrum should grow from zero at 180° as $\sin(\pi - \phi)$ until the circular solid-angle element infringes the edge of the lead-glass array, which happens when $R \sin(\pi - \phi) = 12.8$ cm, i.e. when $\phi = \pi - \arcsin(12.8/R) = 11^\circ$. From then on it decreases with the fraction of the circular solid-angle element that is covered by the array:

$$dN = 2\pi R \sin(\pi - \phi) d\phi \cdot \frac{\delta}{2\pi}, \quad \delta = 4 \arcsin \left(\frac{12.8}{R \sin(\pi - \phi)} \right). \quad (4.33)$$

In fig. 4.29 the B_1L angle spectrum is shown together with the curve of the solid-angle element $d\Omega$ as a function of the opening angle ϕ .

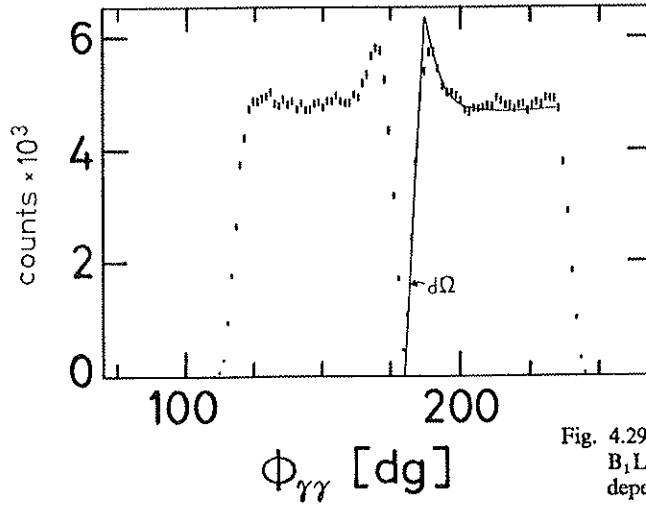


Fig. 4.29: Distribution of opening angles $\phi_{\gamma\gamma}$ as observed with the B₁L detector combination in A84. The line shows the expected dependence.

The reason why the spectra from the other detector combinations do not have a similar sharp top as the B₁L π^0 -momentum spectrum is simply that they are convolutions of a much larger number of hyperbolas in the E_1 -versus- E_2 plot, since those detector combinations covers larger energy ranges.

4.2.2 Inclusive η Momentum Spectrum

The η -momentum spectrum was measured during two run periods, viz. A84 and M85 (fig. 4.30). Only the LL combination is sufficiently sensitive to η -decays in order to obtain a useful spectrum. The general shape of the spectrum can be understood as being dominated by uncorrelated background from pionic annihilation channels, which is demonstrated by a Monte Carlo simulation (fig. 4.31). The curve superimposed on the Monte Carlo spectrum is a fitted spline function $f_{back}(p)$. Fig. 4.32a shows the spectrum from A84 with the background normalized according to the signal-to-background ratio for that spectrum. The residual spectrum after subtracting the background is shown in fig. 4.32b. The prominent signal at 656 MeV/c is caused by monoenergetic η 's recoiling from ω -mesons: $p\bar{p} \rightarrow \eta\omega$. This peak can possibly also conceal a fraction of $\eta\rho^0$ -events, but only a relatively smaller fraction, since the peak has the width of the resolution. The peak is too narrow to house any comparable amount of $\eta\rho^0$ -decays as is apparent from fig. 4.33. As demonstrated by this plot the annihilation channel $p\bar{p} \rightarrow \eta\rho^0$ would result in a substantially broader peak than the one observed.

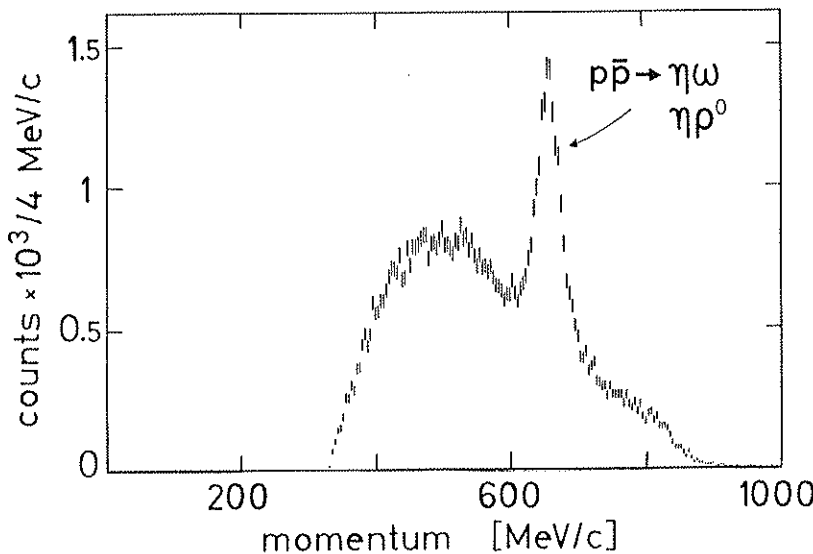


Fig. 4.30: The η -momentum spectrum from M85.

The next item is the broad structure around 800 MeV/c. A fair assumption of the origin of this structure could be that it arises from the reactions $p\bar{p} \rightarrow 2\eta$ and $p\bar{p} \rightarrow \pi^0\eta$. However, it ought then to be possible to discern two distinct peaks. In addition there seem to be a tail towards lower momenta. These two remarks certainly cast some doubt on the hypothesis. To trace the origin of this particular structure we interrogate the charged-particle multiplicity counter around the target. The hypothesis can be examined by demanding the absence of any charged pion, which would be anticipated not to affect the $\pi^0\eta$ -yield at all and the 2η -yield

by maximum 25%, and possibly only suppress background and thus enhance the structure. What in reality happens is that the count rate of the structure is almost exhausted. This hypothesis was consequently abandoned. After consulting table 2.1 for possible candidate processes for the "wedge" shaped η -distribution, the idea rose that it can spring from the secondary decay of A_2 -mesons produced in $p\bar{p}$ annihilations:



The branching ratio for $A_2 \rightarrow \eta\pi$ is 14.5%. The upper end of this distribution¹² is given by the η -momentum obtained in the lab-frame when it is forward boosted. The mean A_2 -energy is

$$\begin{aligned} E &= \frac{1}{4m_p} (4m_p^2 + m_{A_2}^2 - m_\pi^2) = \\ &= 1.4 \text{ GeV} \Rightarrow \beta = 0.33, \end{aligned} \quad (4.35)$$

which gives the Lorentz boost of the η -momentum

$$\begin{aligned} p_\eta^{\text{lab}} &= \gamma p_\eta^{\text{CM}} + \beta \gamma E_\eta^{\text{CM}} = \\ &= 835 \text{ MeV/c}. \end{aligned} \quad (4.36)$$

To enhance the signal from this process we request those events of which exactly one strip of the charged-pion multiplicity counter has fired. We then select events of the decay chain (4.34) when the primary pion and the secondary pion both passed through one and the same strip of the multiplicity counter. The geometry of such an event is illustrated in fig. 4.34a. This demand means that the monitored events had the secondary pion running in the backward direction relative to the direction of the A_2 -meson. This implies in its turn that the η -meson was emitted in the forward direction and will consequently acquire its maximum momentum in the lab frame. Thus, only the events belonging to the upper end of the η -distribution will be

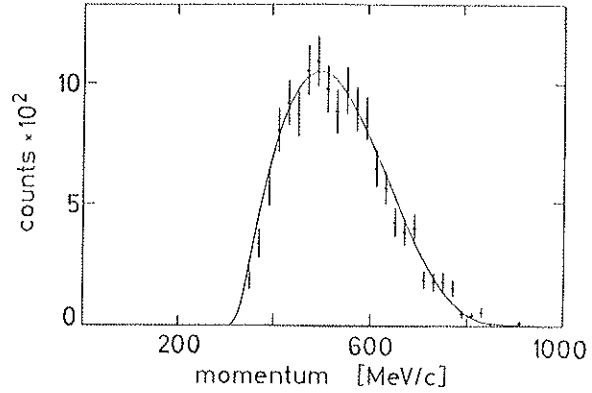


Fig. 4.31: Contribution to the η -momentum spectrum from uncorrelated pionic background as resulting from a Monte Carlo simulation.

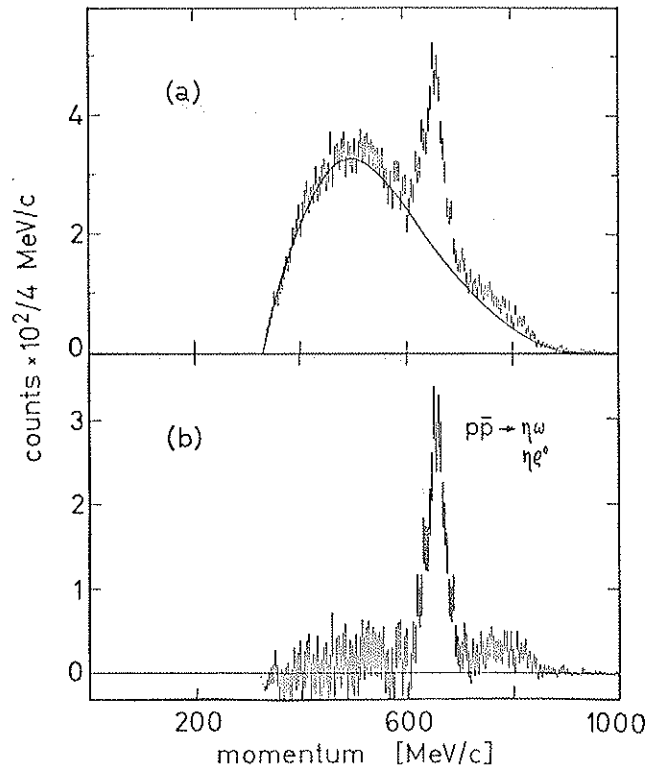


Fig. 4.32: The η -momentum spectrum from A84 (a) with a function representing the background superimposed, and (b) the remaining spectrum after subtraction of this background.

¹² A two-body decay in flight results in a square energy distribution if the decay products are massless (cf. Appendix A). When the decay particles are massive the distribution attains a concave top with an average slope. The η -distribution in this case is further distorted by the broad mass distribution of the A_2 -meson and the acceptance curve of the detector combination, and also smeared out by the resolution. The result is a triangular-shaped distribution.

maintained, which is also confirmed from the spectrum in fig. 4.35. To get the shape of this η -distribution a Monte Carlo simulation was performed. The result shape can be parameterized by the combined function (fig. 4.34b)

$$f_A(p) = \begin{cases} Ae^{-(p-\mu)^2/2\sigma^2} & , p > \mu \\ A \frac{p-p_0}{\mu-p_0} \frac{p}{\mu} \sqrt{\frac{\mu^2+c^2}{p^2+c^2}} & , p < \mu \end{cases} \quad (4.37)$$

where $p_0 = 380$ MeV/c, $c = 549$ MeV/c, $\mu = 820$ MeV/c, and $\sigma = 20$ MeV/c. The η -spectrum was fitted by the sum function of f_{back} and a function f_{struc} composed from $f_A(p)$ and two peak functions (fig. 4.36):

$$f_{struc}(p) = \alpha f_A(p) + \beta f_\omega(p) + \gamma f_\rho(p). \quad (4.38)$$

The $\eta\omega$ -decays can be discriminated from the $\eta\rho^0$ -decays in the same way as in the $\pi^0\omega$ -case, by requiring a third photon to be detected in the lead-glass array (fig. 4.37).

4.2.3 Triple B_1LL Coincidences

This detector combination allows to exclusively measure decay channels of the $p\bar{p}$ system that result in at least one additional photon emitted in the opposite direction from the π^0 . When the kinematical cuts are applied (cf. section 4.1.2.4) only two-body and three-body decays are preserved. Because the two photons detected by the lead-glass array are identified as a π^0 , the presence of a third photon then signifies that the final state merely comprises neutral particles. Contributions can thus be expected from the reactions: $p\bar{p} \rightarrow$

- | | |
|--|---------------------------------|
| a) $\pi^0\gamma$ | f) $\eta\omega$ |
| b) $2\pi^0$ | g) $3\pi^0$ |
| c) $\pi^0\eta, \eta \rightarrow 2\gamma$ | h) $\pi^0 f \rightarrow 3\pi^0$ |
| d) $\pi^0\omega, \omega \rightarrow \pi^0\gamma$ | i) $2\pi^0\eta$ |
| e) 2η | |

(4.39)

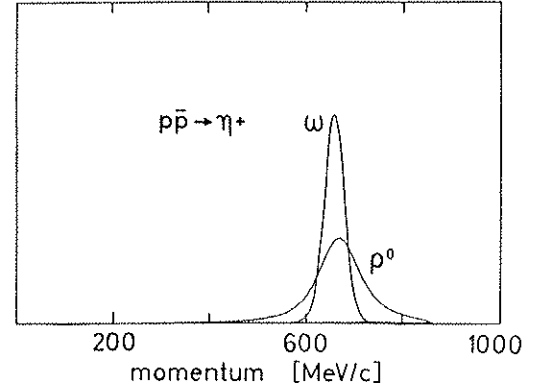


Fig. 4.33: Comparison between the shapes of the peaks from η 's recoiling from ρ^0 -mesons and ω -mesons, respectively.

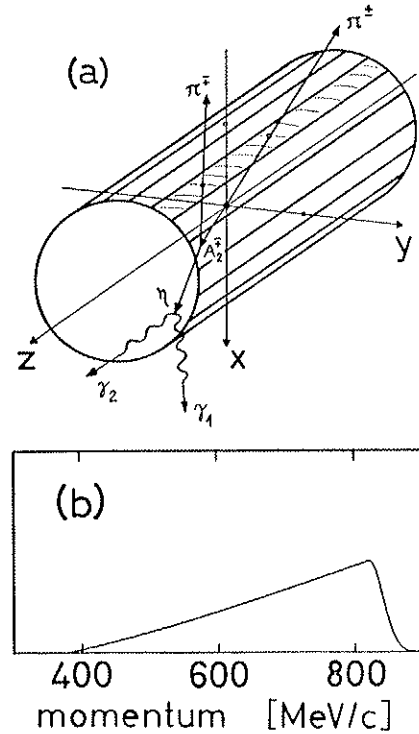


Fig. 4.34: Momentum distribution (b) of η -mesons from decays (a) of A_2 -mesons.

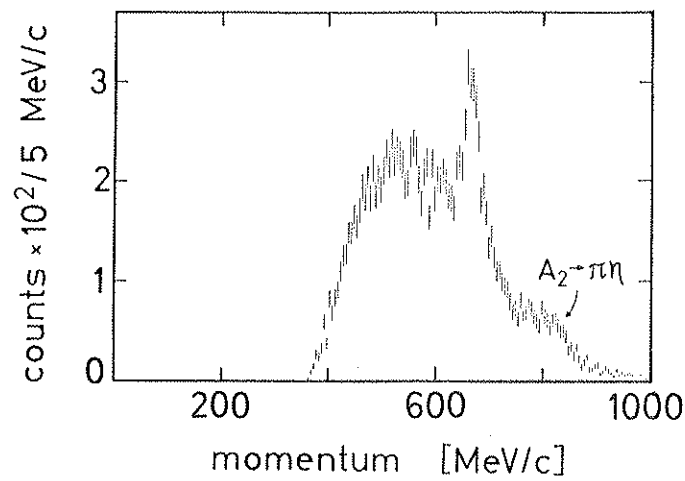


Fig. 4.35: The η -momentum spectrum obtained under the condition that exactly one scintillator strip of the multiplicity counter fired.

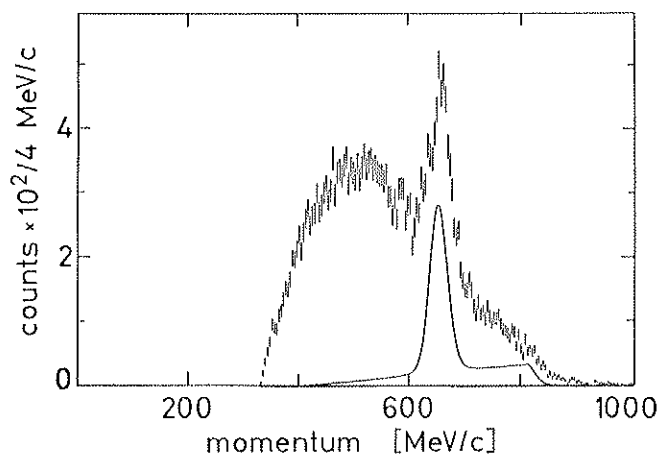


Fig. 4.36: The η -momentum spectrum from A84 with the structure function superimposed.

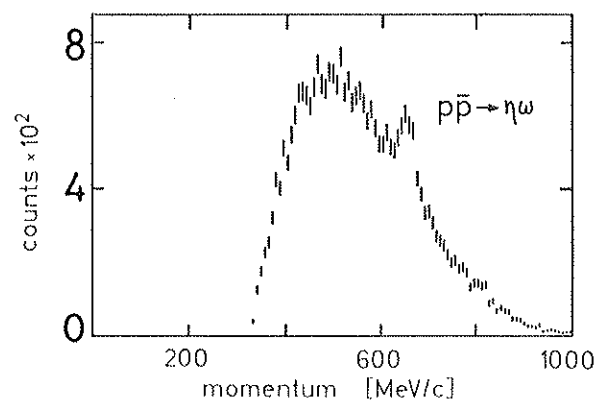


Fig. 4.37: The η -momentum spectrum from M85 produced from events yielding three photon showers in the lead-glass array.

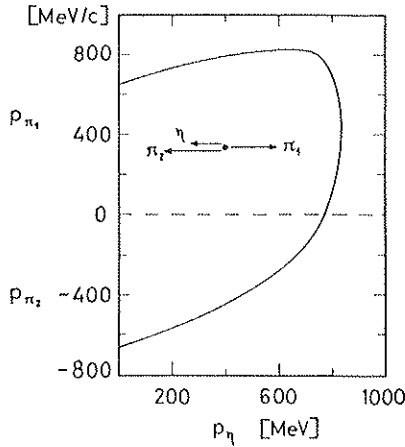


Fig. 4.38: Momentum dependence between the particles of the reaction $p\bar{p} \rightarrow 2\pi^0\eta$.

Several of these channels are rejected by one or the other cut condition. A π^0 from the reaction (i) will never reach a momentum above 850 MeV/c (fig. 4.38). In practice also channel (h) can be excluded, since it is only a few events in the tail of the mass distribution of the f -meson that produce a π^0 -momentum above threshold. The $\eta\omega$ -channel is also rejection by the π^0 -momentum threshold. The remaining reactions that could contribute under the current constraints are thus reactions (a), (b), (d), and (g). Fig. 4.39 shows the contributions from these channels to the BGO_1 energy spectrum, which is the relevant spectrum to study as it expose the quite different shapes of the contributions from the channels (4.39). In this figure the fraction of events occurring in each 10 MeV interval is plotted as compared to the total number of events occurring in the target. From the inclusive η -spectrum an upper limit can be determined for the channels $p\bar{p} \rightarrow \pi^0\eta$ and $p\bar{p} \rightarrow 2\eta$. The 1σ -limits are $R_{\pi^0\eta} < 7 \cdot 10^{-5}$ and $R_{2\eta} < 10^{-4}$, respectively. From the inclusive π^0 -spectrum was found

that the branching ratio of $p\bar{p} \rightarrow \pi^0\omega$ is less than 1%. Thus, only the four annihilation channels mentioned above remains.

The distributions of the retained four reactions have been fitted [62] to the E_{BGO} spectrum by using the fit program MINUIT [101]. Fig. 4.40 shows the result for $\chi^2/f \approx 0.86$ from this fit of the function

$$f(E) = \sum_{i=1}^3 N_i f_i(E) \quad (4.40)$$

folded with the $i=1$ detector resolution, where N_i are the number of entries in the spectrum belonging to the distribution $f_i(E)$. These distributions are, in turn, the ones originating from the reactions $p\bar{p} \rightarrow \pi^0\gamma$ ($i=1$), $p\bar{p} \rightarrow 2\pi^0$ ($i=2$), and $p\bar{p} \rightarrow (3\pi^0, \pi^0\omega)$ ($i=3$). The two last channels are taken together because they yield (small) distributions of very similar shape, the branching ratios being $R_{3\pi^0} \sim 5\%$ [48] and $R_{\pi^0\omega} \sim 5\%$ (from the inclusive π^0 -spectrum), respectively. The fitting was performed under the boundary condition that the function $f(E)$ should be normalized to 299, the observed number of events in the energy interval from 600 to 1000 MeV.

I will end this subsection by shortly mentioning two methods that were explored [63] in order to separate the two decay channels $p\bar{p} \rightarrow \pi^0\gamma$ and $2\pi^0$. First method: the triple event sample was processed by two different kinematic fitting procedures, one five-constraints fit to the hypothesis that the decay was $p\bar{p} \rightarrow \pi^0\gamma$, and one four-constraints fit to the hypothesis that it was $p\bar{p} \rightarrow 2\pi^0$. However, the resulting χ^2 -distributions have four degrees of freedom and 5 degrees of freedom, respectively, and have therefore too similar shapes to offer a powerful separation of the two channels. Moreover, too many $2\pi^0$ -decays percolate the five-constraints fit yielding a good χ^2 -value. Second method: based on the idea of more efficiently utilizing the information concealed in the angle distributions. By parameterizing the distributions of measured quantities for the two hypothesis, respectively, a maximum-likelihood analysis was performed. For this procedure the three quantities $\mathbf{x} = (p_{\pi^0}, E_{\gamma}, \theta_{\pi^0\gamma})$ were chosen. The likelihood function $P(\mathbf{x}, N_1, N_2)$, where N_1 is the number of $\pi^0\gamma$ -events and N_2 is the number of $2\pi^0$ -events, was defined as

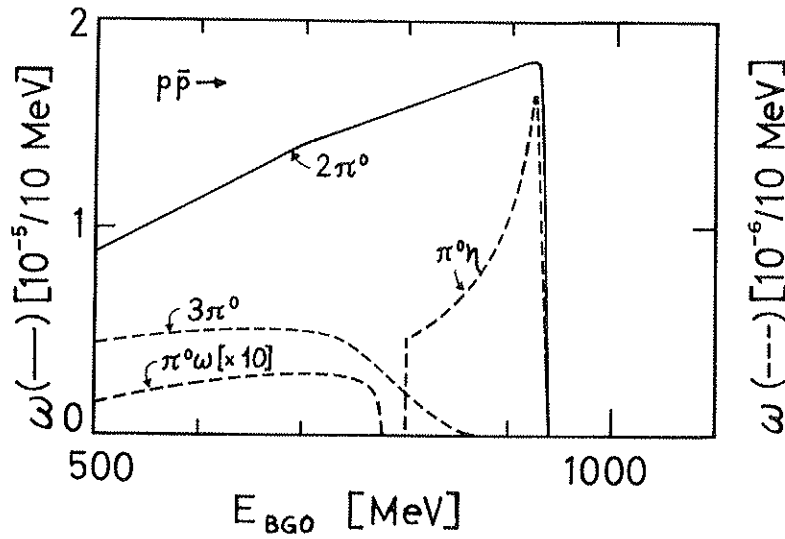


Fig. 4.39: Efficiency ω for detecting a triple B_1LL photon coincidence under kinematical constraints (see text). (Adapted from ref. [62]).

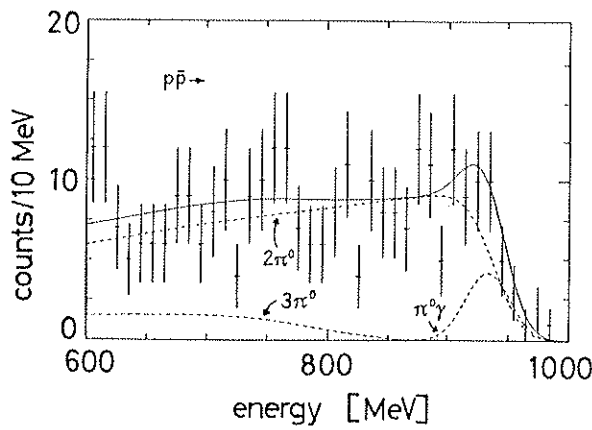


Fig. 4.40: BGO₁ energy spectrum obtained from the event sample of triple photon coincidences from D83 with the fitted contributions from the indicated annihilation channels.

$$P(\mathbf{x}, N_1, N_2) = \frac{1}{N} [N_1 f(\mathbf{x}) + N_2 g(\mathbf{x}) + (N - N_1 - N_2)h(\mathbf{x})] , \quad (4.41)$$

where $N=331$ was the total number of events in the sample. The probability-density functions f , g , and h were obtained from Monte Carlo simulations of the individual annihilation channels. They pertain to the $\pi^0\gamma$ -events, the $2\pi^0$ -events, and the two background channels ($3\pi^0$, $\pi^0\omega$), respectively. By evaluating $P(\mathbf{x})$ for each event, the likelihood function was obtained

$$\ell(N_1, N_2) = \sum_{i=1}^N \ln [P(\mathbf{x}_i, N_1, N_2)] . \quad (4.42)$$

Fig. 4.41 shows the likelihood function $\ell(N_1, N_2)$. The 1σ -level is defined by $\ell_{1\sigma} = \ell_{\max} - 2.3$ [56], and the two numbers N_1 and N_2 of $\pi^0\gamma$ -events and $2\pi^0$ -events, respectively, can be read with their errors from the projections. The result from this method was, however, not significantly better than from the fit procedure described above.

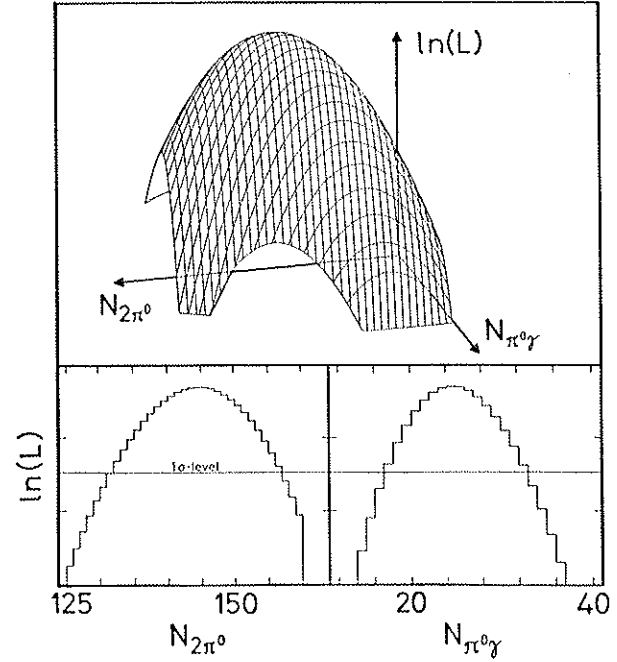


Fig. 4.41: Likelihood function L obtained from an analysis of the triple B_1LL coincidences from D83.

4.2.4 Back-to-back B_1L Coincidences

From the B_1L data from D83 is also extracted the sum-energy spectrum of back-to-back photon coincidences. The hypothesis is then **not** that the photon pair originates from a zero-momentum π^0 . The invariant mass cut and the kinematic fitting along with χ^2 -cut are thus omitted. The purpose of this event sample is instead to be sensitive to the annihilation channels; $p\bar{p} \rightarrow 2\gamma$, $p\bar{p} \rightarrow \pi^0\gamma$, $p\bar{p} \rightarrow 2\pi^0$, and $p\bar{p} \rightarrow 3\pi^0$. The spectrum of the sum energy $E_1 + E_2$ of the two photon energies is produced under the conditions

$$\begin{aligned} E_1, E_2 &\geq 450 \text{ MeV} , \\ \theta_{12} &\geq 170^\circ , \end{aligned} \tag{4.43}$$

where θ_{12} is the angle between the two γ -rays. The contributions to this spectrum from different annihilation channels are shown in fig. 4.42. In the figure is plotted the fraction of events occurring in each 20 MeV interval relative the total number of events occurring in the target. Some of the channels in fig. 4.42 can be neglected, because of the small detection efficiency for them and their small branching ratios. From the inclusive η -spectrum an upper limit can be determined for the channels $p\bar{p} \rightarrow \pi^0\eta$ and $p\bar{p} \rightarrow 2\eta$. The 1σ -limits are $R_{\pi^0\eta} < 7 \cdot 10^{-5}$ and $R_{2\eta} < 10^{-4}$, respectively. From the inclusive π^0 -spectrum

was found that the branching ratio of $p\bar{p} \rightarrow \pi^0\omega$ is less than 1%.

Thus, only the four annihilation channels mentioned above remains. The distributions from these channels have been fitted [62] by using the fitting program MINUIT [101]. The result is displayed in fig. 4.43, and was obtained for $\chi^2/f \approx 0.6$ and the yield of the channel $p\bar{p} \rightarrow \pi^0 \gamma$ being fixed to the value $1.9 \cdot 10^{-5}$ which was deduced from the data sample of triple B_1LL coincidences (see next subsection 4.2.3).

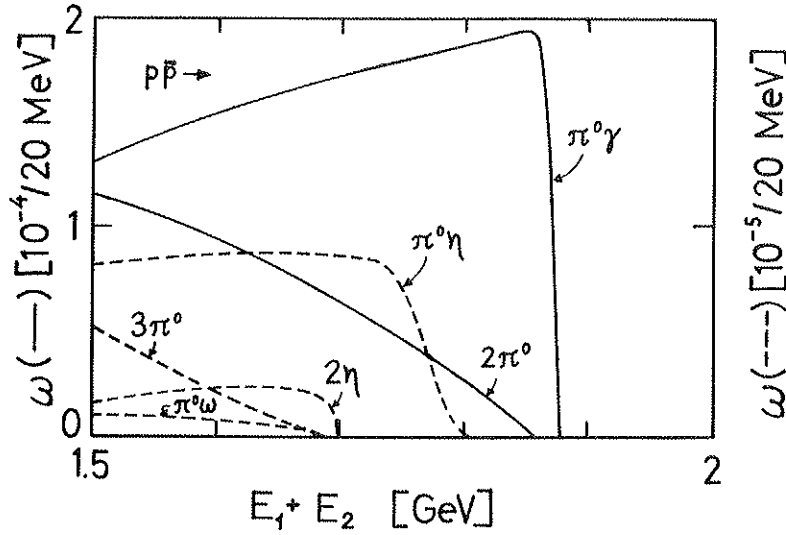


Fig. 4.42: Efficiency ω for detecting a high-energetic back-to-back photon coincidence from the indicated annihilation channels as a function of the sum $E_1 + E_2$ of the two photon energies. (Adapted from ref. [62]).

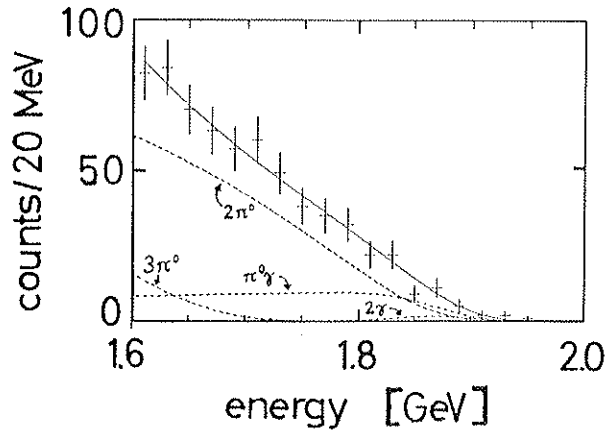


Fig. 4.43: B_1L spectrum from D83 of the sum energy of high-energetic back-to-back photon coincidences with the fitted contributions from the indicated annihilation channels.

4.2.5 Photon Energy Spectrum

The inclusive photon energy spectrum produced in $p\bar{p}$ annihilations at rest was measured during the two run periods D83 and M85. The M85 spectrum is shown in fig. 4.44. The high-energy part is also shown on logarithmic scale to visualize the "shoulder" of the square distribution from the reaction $p\bar{p} \rightarrow \pi^0(\rho^0, \omega), \pi^0 \rightarrow 2\gamma$. The D83 spectrum was fitted by a polynomial, and the subtracted spectrum (fig. 4.45) shows two clear signals, viz. from the two Panofski reaction $\pi^- p \rightarrow \gamma n$ and $\pi^- p \rightarrow \pi^0 n$, $\pi^0 \rightarrow 2\gamma$, the former reaction corresponds to a yield of about 0.7%. The M85 spectrum was simulated with Monte Carlo technique using the decay channels listed in table 2.1 and marked "MC" (fig. 4.46a).

This Monte Carlo spectrum was fitted [130] by the function¹³ for $\chi^2/f=1.19$.

$$f_{back}(E) = A [a_1(E_0 - E)^{\beta_1} + a_2(E_0 - E)^{\beta_2} - a_3 b_3^E - a_4 b_4^E] , \quad (4.44)$$

The parameter values delivered by the fit were:

$$\begin{array}{lll} a_1 = e^{-33.8} & a_4 = e^{-6.63} & b_3 = 0.972 \\ a_2 = e^{-19.3} & \beta_1 = 4.22 & b_4 = 0.997 \\ a_3 = e^{-5.00} & \beta_2 = 1.56 & E_0 = 917 \text{ MeV} \end{array}$$

The remainder of the Monte Carlo spectrum after subtraction of $f_{back}(E)$ is plotted in fig. 4.46b. The M85 data spectrum could afterwards be fitted by the function $f(E) = N f_{back}(E)$, with merely the overall normalization being a free parameter. The residual spectrum after subtraction of the background function is shown in fig. 4.47. The square-shaped part of the spectrum is predominantly arising from monochromatic π^0 's recoiling from ρ^0 -mesons and ω -mesons. This part of the spectrum embraces around 30 000 counts, which corresponds to a value of approximately 2.6% for the combined yield of $p\bar{p} \rightarrow \pi^0(\rho^0, \omega)$ in agreement with the result obtained from the inclusive π^0 -measurement.

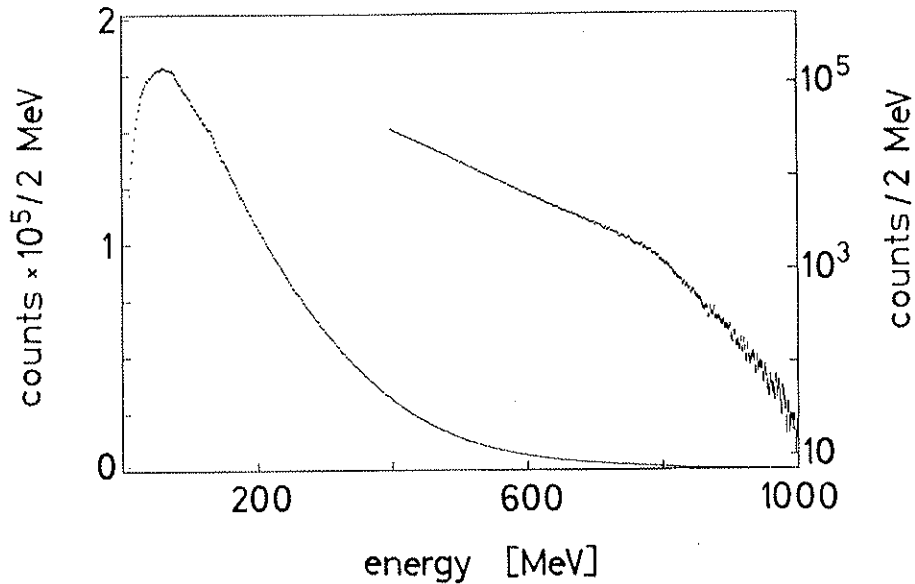


Fig. 4.44: Inclusive photon energy spectrum from M85. The high energy part is shown also in logarithmic scale.

¹³ It was later found that the γ -spectrum is well fitted by an expression of the type $\sin \epsilon [1 - \alpha \cos \epsilon]^\beta$, $\epsilon = \pi E/E_0$, having only two free parameters.

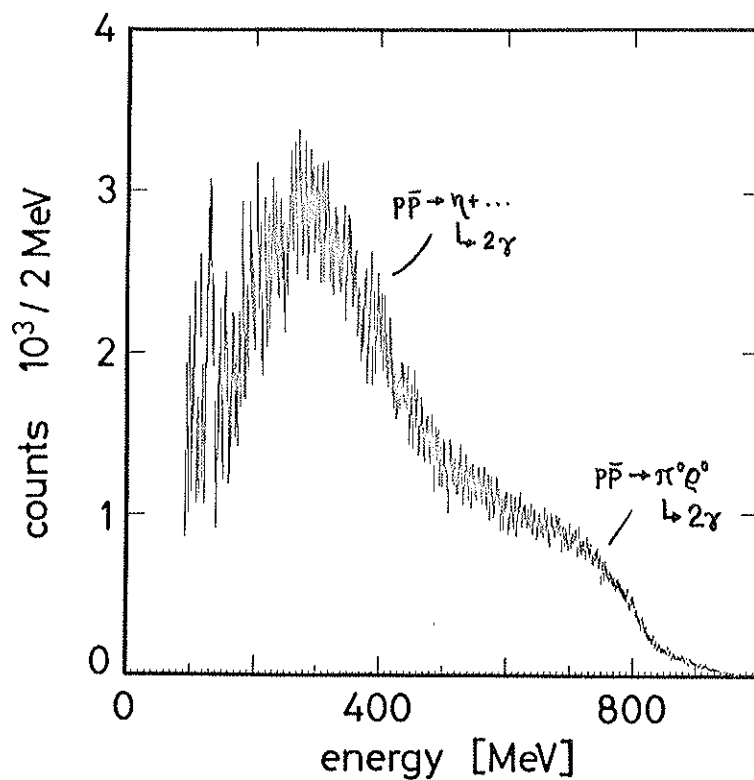


Fig. 4.47: Photon energy spectrum from M85 (50% of full statistics) after subtracting the background function obtained from the Monte Carlo simulation.

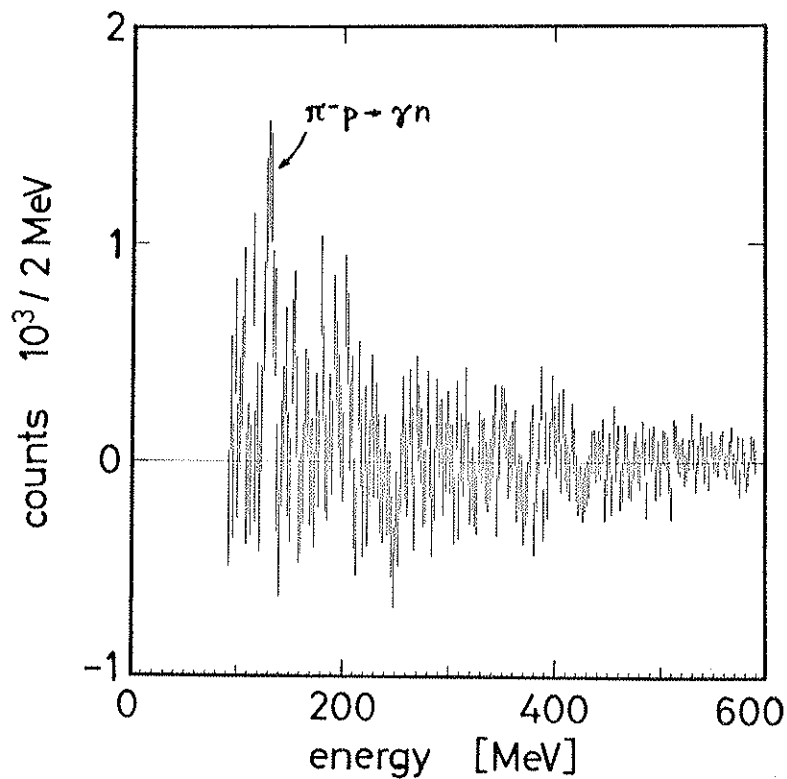


Fig. 4.48: Photon energy spectrum from M85 after subtraction of the expected contributions from η -associated annihilation channels and π^0 -associated two-body channels.

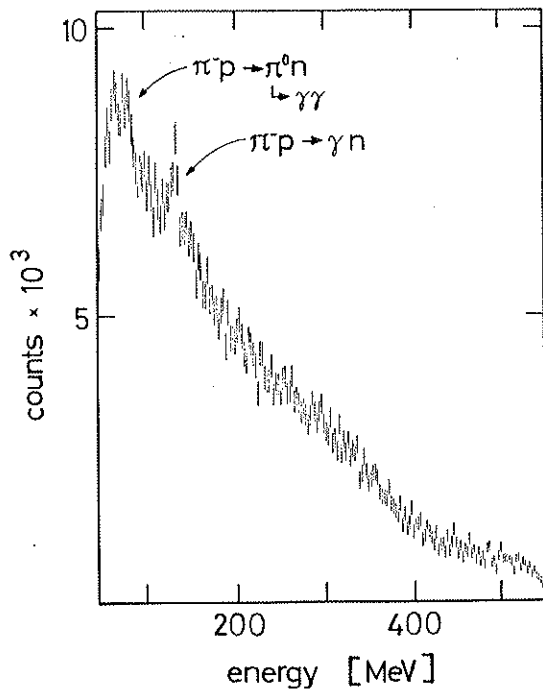


Fig. 4.45: Inclusive photon energy spectrum from D83 (50% of full statistics), the background being partially subtracted.

The tail extending to the highest energies comprises events from the decays $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0\gamma$, but also events from $p\bar{p} \rightarrow 3\pi^0$ and $p\bar{p} \rightarrow \pi^0\omega$, $\omega \rightarrow \pi^0\gamma$. The rest of the spectrum above the square-shaped part is attributed to η -mesons decaying into 2γ . These yield square-distributions between $E_\gamma = (1 \pm \beta_\eta)E_\eta/2$. All these square-distributions, for any η -energy contain the energy $E_\gamma = m_\eta = 275$ MeV, which thus will be the maximum of the total distribution from η 's of different energies. This part of the spectrum is roughly 200,000 counts large and corresponds to a total η -yield of about 4.3%. After subtraction [139] of the two contributions discussed above the spectrum seems to be exhausted of structures (fig. 4.48) except of the peak from the Panofski reaction $\pi^-p \rightarrow \gamma n$, which corresponds to a yield of 2.4%; notice the larger yield of this process in M85 as compared to that obtained in the run D83 when the target was only half as long as it was in M85, and thus the liquid-hydrogen volume was only half as large.

A sensitivity curve for the reaction $p\bar{p} \rightarrow \gamma X$ is derived from the statistics recorded in M85, and is presented in fig. 4.49, where it is assumed that the unknown state X is narrower than the limit obtained from the resolution.

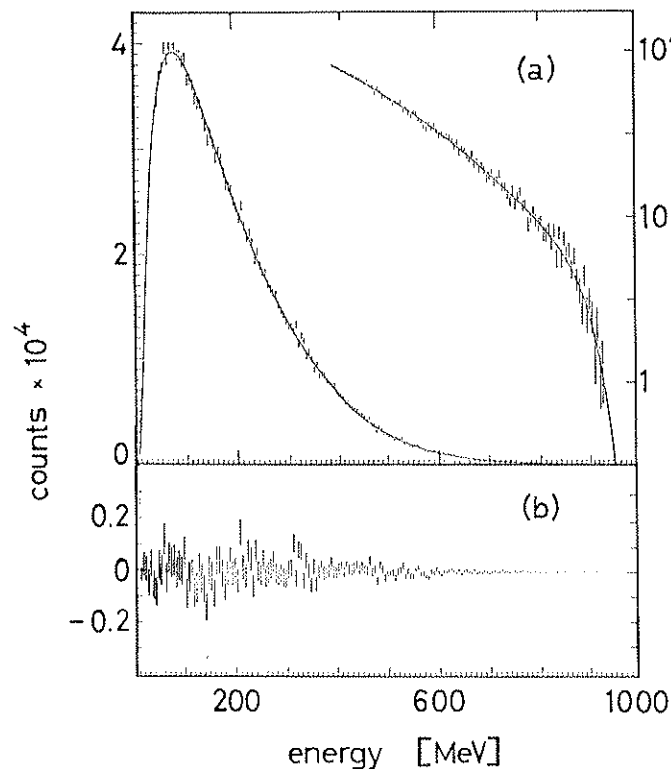


Fig. 4.46: Monte Carlo simulation of the photon energy spectrum (a) with a fitted function superimposed, and (b) the residual after subtracting this function.

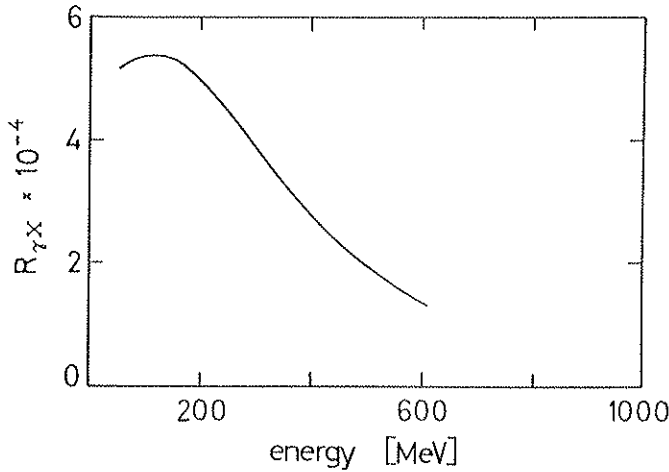


Fig. 4.49: Upper limit for the branching ratios $R_{\gamma X}$ (see text) with 95% confidence level as a function of the photon energy.

4.3. RESULTS

The previous sections of this chapter discussed the treatment of the different data samples to extract information. In this section the results from those procedures consisting of branching ratios for neutral decay channels of the $p\bar{p}$ system, plus upper limits for the occurrence of other non-observed channels are summarized. The results are structured according to which particle is emitted and available for spectroscopy, i.e. whether the $p\bar{p}$ annihilation is associated with photon radiation (subsection 4.3.2), or associated with emission of π^0 -mesons (subsection 4.3.3), or associated with emission of η -mesons (subsection 4.3.4). The limits obtained for the population of exotic or quasi-nuclear states are collected in a separate subsection 4.3.1, for the two cases of photon emission and π^0 -emission. All branching ratios derived in this section are corrected for lost events. The "standard corrections" are the following two, and are implicit if nothing else is mentioned. Firstly, 5% of the detected π^0 's or η 's were rejected by the cut in the χ^2 -distribution from the fitting of their decay kinematics. Secondly, there is a finite probability of 1.8% that a photon converts before it reaches the detector, and will thus be rejected as if it was a charged particle. Of course the branching ratios $R(\pi^0 \rightarrow 2\gamma) = 98.8\%$ and $R(\eta \rightarrow 2\gamma) = 39.1\%$ are also taken into account.

4.3.1 Population of Exotic States

The highest sensitivity for determination of upper limits of the branching ratios of any unknown states is obtained when the statistics from the different run periods are added together. For the case when the annihilation is associated with emission of a π^0 -meson the LL spectra from M85 and A84, and the B_2L spectra from A84, M84 and D83, are used to derive a sensitivity curve. The information from the two different detector combinations is merged such that the highest sensitivity is chosen out of the two at each π^0 -momentum. Because the resolutions of the LL spectra do not differ significantly, these two spectra are summed. Similarly, the three B_2L spectra are summed. The B_2L spectra from D83 have somewhat different acceptance, since the range was not exactly the same during that run period. Thus, the acceptance of the summed spectrum is the weighted average of the two acceptances. The resultant sensitivity curve is displayed in fig. 4.50, and gives the upper limit with 95% confidence level (cf. Appendix C) of the branching ratio $R_{\pi^0 X}$ for the reaction

$$p\bar{p} \rightarrow \pi^0 + X,$$

where X is a resonance with **unknown** mass m_X and an intrinsic width of less than 80 MeV/c². If the X-resonance is broader, i.e. has a lifetime less than about 10^{-21} s, the upper limit of $R_{\pi^0 X}$ is larger. The limit grows faster than the lifetime, so that a twice as large intrinsic width as given by the figure corresponds to an upper limit that is more than twice the number given in fig. 4.50. Alternatively, one can take a twice as large limit, but then suffice with a smaller confidence level.

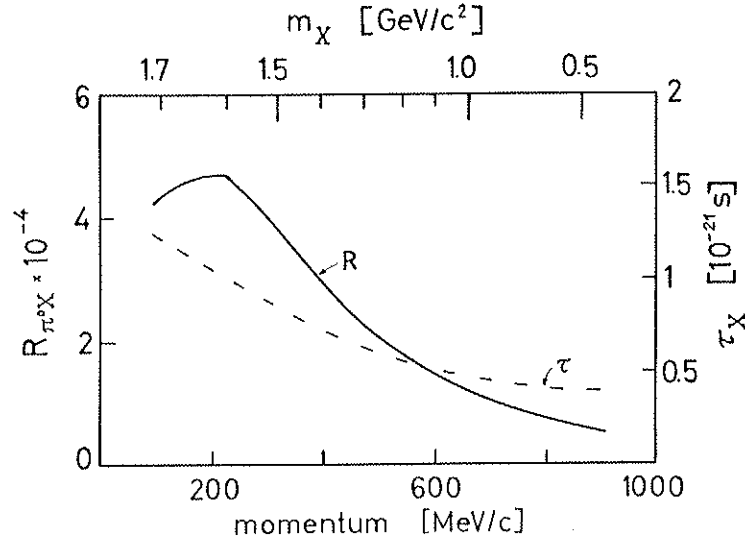


Fig. 4.50: Upper limit for the branching ratios R_{π^0} (see text) with 95% confidence level as a function π^0 -momentum and the mass m_X of the recoiling object.

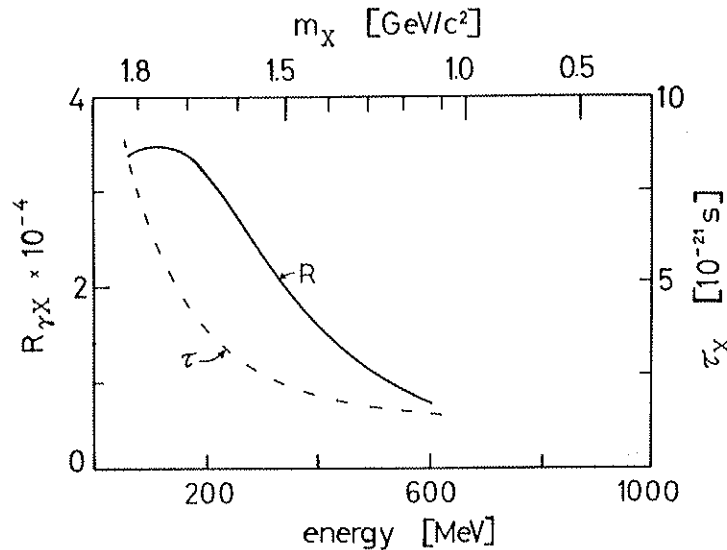


Fig. 4.51: Upper limit for the branching ratios $R_{\gamma X}$ (see text) with 95% confidence level as a function of photon energy and the mass m_X of the recoiling object.

"Who invented the human heart, I wonder", said Justine.
 "Tell me, and then show me the place where he was hanged". (L. Durrell)

An analogous sensitivity curve is derived from the single photon energy spectra from M85 and D83 for the case the annihilation is associated with photon emission (fig. 4.51). The curve gives the upper limit with 95% confidence level of the branching ratio $R_{\gamma X}$ for the reaction

$$p\bar{p} \rightarrow \gamma + X,$$

where X again is a resonance with **unknown** mass m_X , but has now to have an intrinsic width of less than 40 MeV/c². The limit of $R_{\gamma X}$ given by this curve is only strictly valid if the subsequent decay of the X-resonance is completely charged:

$$p\bar{p} \rightarrow \gamma + X^0, \quad \begin{matrix} \searrow \\ \rightarrow n\pi^+ + n\pi^- \end{matrix} \quad (4.46)$$

which has been considered as a likely decay mode. In case the X-decay would generate photons a strict limit has to be taken from the curve in fig. 4.49, which is based on only the M85 data.

4.3.2 Channels Associated with Photon Emission

By observing three photons in coincidence it has been possible to measure the rare annihilation channel

$$p\bar{p} \rightarrow \pi^0\gamma. \quad (4.47)$$

By observing two high-energetic colinear photons an upper limit for the decay rate of

$$p\bar{p} \rightarrow 2\gamma \quad (4.48)$$

is obtained. The corresponding branching ratios $R_{\pi^0\gamma}$ and $R_{2\gamma}$ of these two annihilation channels, respectively, are somewhat sensitive to the absolute energy calibration of the detector system, which might introduce systematic errors. From the observed positions of identifiable signals the absolute energy calibration is seen to be better than $\pm 1\%$. This systematic uncertainty implies in the worst case that all 2γ -events can be $\pi^0\gamma$ -decays or $2\pi^0$ -decays instead. Thus, only an upper limit is achieved for the 2γ -channel.

The numbers of $\pi^0\gamma$ -decays and of 2γ -decays observed were deduced from spectral fits described in sections 4.2.3, 4.2.4, respectively. To obtain the corresponding branching ratios these numbers have to be scaled according to the detection efficiency for each decay type, respectively, and then normalized to the total number of $p\bar{p}$ -interactions (table 4.1) that contributed to the measurement. In addition to the standard corrections, for χ^2 -rejected π^0 's and finite conversion probability of photons, the number of $\pi^0\gamma$ -decays has to be corrected for lost due to the π^0 -momentum cut at 850 MeV/c, and for 5% lost due to a requirement of having at least four columns of lead-glass blocks between two adjacent shower maxima (cf. section 4.1.1.3). In all run periods except D83 only three columns were required. The two branching ratios for the annihilation channels $p\bar{p} \rightarrow \pi^0\gamma$ and $p\bar{p} \rightarrow 2\gamma$ are then, respectively,

$$R_{\pi^0\gamma} = (1.9 \pm 0.9) \cdot 10^{-5} \quad (4.49)$$

$$R_{2\gamma} < 1.2 \cdot 10^{-6} \quad (4.50)$$

4.3.3 Channels Associated with Emission of π^0 -mesons

The inclusive π^0 -spectrum from $p\bar{p}$ annihilations at rest shows structures from several ordinary mesonic resonances, as for instance the A_2 and the ρ . Our measurement is consistent with expectations (see table 4.2). The branching ratio $R_{\pi^0\omega}$ for $p\bar{p} \rightarrow \pi^0\omega$ can be estimated in several ways¹⁴. One approach is to study the shape of the (ρ^0, ω) -peak in the π^0 -momentum spectrum. This gives a small statistical error, but, on the other hand, a substantial systematic error is introduced due to uncertain definition of the background and possibly wrong peak shape used in the fitting. Another approach is to use the spectrum produced under the condition that a third photon was detected, which excludes all $\pi^0\rho^0$ -decays, but this of course reduces the statistics. The combined yield for $p\bar{p} \rightarrow \pi^0(\rho^0, \omega)$ is derived by taking the weighted average of the two numbers obtained from the LL spectra from M85 and A84:

$$R_{\pi^0(\rho^0, \omega)} = (2.6 \pm 0.08 \pm 0.4)\% \quad (4.51)$$

The last error is a systematic error coming from uncertainty in defining the background, which was given by the fit described in subsection 4.2.1.1. To minimize this error the number (4.51) was based on the peak area inside the full-width at half-maximum, which contains the $\pi^0\omega$ -yield and 50% of the $\pi^0\rho^0$ -yield. When deriving the combined branching ratio $R_{\pi^0(\rho^0, \omega)}$ a small correction was made, except the standard corrections, of the contribution from A84 owing to a larger π^0 -detection efficiency. This higher detection efficiency was due to the fact that no cut was imposed, in the offline analysis, on the z -coordinates of the impact positions of photons entering the lead-glass array. However, most of the π^0 's of which one of the decay photons entered close to the edge of the detector are rejected by the cut in the χ^2 -distribution obtained from fitting the π^0 -decay kinematics. By subtracting the branching ratio for $p\bar{p} \rightarrow \pi^0\rho^0$, which is $R_{\pi^0\rho^0} = (1.9 \pm 0.2)\%$ according to ref. [65], from the combined yield (4.51), the branching ratio for $p\bar{p} \rightarrow \pi^0\omega$ is obtained to be

$$R_{\pi^0\omega} = (0.7 \pm 0.2 \pm 0.4)\% \quad (4.52)$$

From the exclusive π^0 -spectrum produced from π^0 -events associated with a third photon the branching ratio for $p\bar{p} \rightarrow \pi^0\omega$ is directly observed to be:

$$R_{\pi^0\omega} = (0.48 \pm 0.05 \pm 0.33)\% \quad (4.53)$$

Here, the systematic error mainly is due to uncertainty in determining the detection efficiency for observing three photons from $p\bar{p} \rightarrow \pi^0\omega$.

By measuring three photons in coincidence of which the two detected by the lead-glass array are identified as a π^0 -decay, the branching ratio for the neutral annihilation channel

$$p\bar{p} \rightarrow 2\pi^0 \quad (4.54)$$

is obtained. The number $R_{2\pi^0}$ is derived from a flat spectral region in the energy spectrum measured with BGO₁. Thus, the end-point has a negligible influence on the yield and makes the number insensitive to the absolute energy calibration. The value deduced from the fit described in subsection 4.2.3 is

¹⁴ An illustrative method to reveal any potential $\pi^0\omega$ -events is to make a (discrete) Fourier transformation of the spectrum. Because the ω -resonance is substantially narrower than the ρ , the ω -content of the peak appears as "higher frequency" components in the transformed spectrum.

$$R_{2\pi^0} = (2.2 \pm 0.3) \cdot 10^{-4} \quad (4.55)$$

The neutral annihilation channel $p\bar{p} \rightarrow 3\pi^0$ is also observable in this data sample of triple photon coincidences. However, only an upper limit is obtained, because an uncertain fraction of the events might originate from $\pi^0\omega$ -decays:

$$R_{3\pi^0} < 3.2 \cdot 10^{-3} \quad (4.56)$$

Both the branching ratios $R_{2\pi^0}$ and $R_{3\pi^0}$ were deduced using the same corrections as applied to $R_{\pi^0\gamma}$.

4.3.4 Channels Associated with Emission of η -mesons

The inclusive η -momentum spectrum generated from $p\bar{p}$ annihilations at rest uncovers one sharp resonance, viz. the ω . A fraction of the peak can come from $\eta\rho^0$ -events. Thus the combined yield $R_{\eta(\rho^0, \omega)}$ for $p\bar{p} \rightarrow \eta(\rho^0, \omega)$ is derived from the weighted average of the two numbers obtained from the data samples recorded during the run periods M85 and A84:

$$R_{\eta(\rho^0, \omega)} = (7.9 \pm 0.1 \pm 0.5) \cdot 10^{-3} \quad (4.57)$$

The last error is again the systematic error coming from uncertainty in defining the background under the peak. The number (4.57) is based on the area of the peak that is limited of the full-width at half-maximum of the ρ^0 -peak (cf. fig. 4.33). Also in the η -case the yield expected from earlier measurements [65] of $p\bar{p} \rightarrow \eta\rho^0$ can be subtracted, but it is just 50% of the yield $R_{\eta\rho^0} = 0.2\%$ that appears in the considered interval. Thus, the branching ratio for $p\bar{p} \rightarrow \eta\omega$ is

$$R_{\eta\omega}' = (7 \pm 1 \pm 0.5) \cdot 10^{-3} \quad (4.58)$$

The branching ratio $R_{\eta\omega}$ can be directly deduced from the exclusive η -momentum spectrum produced under the condition that a third photon is present in the lead-glass array, which excludes the $\eta\rho^0$ -events but reduces also the detection efficiency for $p\bar{p} \rightarrow \eta\omega$. The obtained value is

$$R_{\eta\omega} = (6.0 \pm 0.3 \pm 2.8) \cdot 10^{-3} \quad (4.59)$$

Here, the last systematic error is mainly caused by uncertainty in estimating the detection efficiency for observing three photons from $p\bar{p} \rightarrow \eta\omega$.

The η -distribution from the secondary decays of A_2 -mesons produced in $p\bar{p}$ annihilations is used to estimate the total A_2 -production in $p\bar{p}$ annihilations in liquid hydrogen. The branching ratio for $A_2 \rightarrow \pi\eta$ is measured [46] to be $(14.5 \pm 1.2)\%$. The total production rate of A_2 -mesons is thus estimated to be

*Take her up tenderly,
Lift her with care,
Fashion'd so slenderly
Young, and so fair!*

(Thomas Hood)

$$R_{A2} = (5.0 \pm 1.7)\%.$$

(4.60)

The number is taken as the weighted average of the two values obtained from M85 and A84 data, respectively, and the error is mainly due to uncertain background definition.

The signals from the following annihilation channels are **absent** in the inclusive η -spectrum:

$$p\bar{p} \rightarrow \pi^0\eta \quad (4.61)$$

$$p\bar{p} \rightarrow 2\eta \quad (4.62)$$

$$p\bar{p} \rightarrow \eta\eta' \quad (4.63)$$

By summing the statistics from M85 and A84 upper limits (95% confidence level) for the corresponding branching ratios are derived:

$$R_{\pi^0\eta} < 7 \cdot 10^{-5}, \quad (4.64)$$

$$R_{2\eta} < 1 \cdot 10^{-4}, \quad (4.65)$$

$$R_{\eta\eta'} < 5 \cdot 10^{-4}. \quad (4.66)$$

*If a lion could speak we wouldn't
understand him.* (L. Wittgenstein)

*And you want to travel with him,
you want to travel blind
and you think maybe you'll trust him
because he touched your perfect body
with his mind.
And you want to travel with her
and you want to travel blind
and you're sure that she can find you
because she's touched her perfect body
with her mind.*

(Leonard Cohen)

CHAPTER 5

DISCUSSION AND CONCLUSIONS

The results to date from the experiment PS182 are compared in this chapter to theoretical predictions and results reported by earlier experiments. The primary results from PS182 are the upper limits obtained for the population of exotic states (section 5.1). In addition, it has been possible to measure rare neutral annihilation channels of the $p\bar{p}$ -system. These results are compared to predictions by the quark "Rearrangement Model" (section 5.2), and by the "Vector Dominance Model" (section 5.3). The observation of the annihilation channel $p\bar{p} \rightarrow 2\pi^0$ is used to estimate the P -wave contribution to $p\bar{p}$ annihilations in liquid hydrogen (section 5.4).

5.1. EXOTICS REMAIN ELUSIVE

The upper limits obtained in this experiment for the population of (narrow) exotic states X through the reaction $p\bar{p} \rightarrow \gamma X$ are not compatible with the yields of the signals reported by Richter et al. [123], and neither with the theoretical predictions by Dover [53], but are nevertheless consistent with other theoretical predictions by J.P. Ader [3]. The limits obtained in this experiment are compared to the theoretical predictions, and to numbers reported by a pre-LEAR experiment in table 5.1. The significance (cf. Appendix C) of the signals reported by that experiment are for comparison "translated" according to table C.1 and listed together with the corresponding confidence levels in the fifth column of table 5.1. Our result is consistent with the upper limit of $4 \cdot 10^{-4}$ for the yield of the reaction $p\bar{p} \rightarrow \gamma X$, $X \rightarrow 2\pi^+ 2\pi^-$, reported by Ahmad et al. [7].

The upper limits obtained in this experiment for the population of (narrow) exotic states through the reaction $p\bar{p} \rightarrow \pi^0 X$ are given by fig. 4.50 and are around $5 \cdot 10^{-4}$ with 95% confidence level. This result can be viewed in the context of limits reported by other experiments [93] [7] on the occurrence of the reaction $p\bar{p} \rightarrow \pi^\pm X^\mp$, which are of the same order of magnitude.

Table 5.1: Comparison between the yields of the signals reported by Richter et al. [123] and the upper limits obtained in this experiment. Two different theoretical predictions are also compared.

Photon energy [MeV]	PS182 (95% CL)	Experiment Richter et al.	$\Sigma[CL]$	Theory Ader	Dover
108	$3.5 \cdot 10^{-4}$				$5 \cdot 10^{-3}$
180	$3.3 \cdot 10^{-4}$	$(16 \pm 5) \cdot 10^{-4}$	3.1σ	88%	$5 \cdot 10^{-3}$
200				$2.0 \cdot 10^{-4}$	$5 \cdot 10^{-3}$
220		$(30 \pm 11) \cdot 10^{-4}$	2.7σ	85%	$5 \cdot 10^{-3}$
290	$2.4 \cdot 10^{-4}$	$(12 \pm 5) \cdot 10^{-4}$	2.7σ	85%	$5 \cdot 10^{-3}$
420		$(1 \pm \frac{1}{2}) \cdot 10^{-4}$	0.3σ	17%	$5 \cdot 10^{-3}$
550	$1.1 \cdot 10^{-4}$	$(9 \pm 3) \cdot 10^{-4}$	3.0σ	89%	$5 \cdot 10^{-3}$

As evident from table 5.1 the results obtained in this experiment deny the signals reported by Richter et al., which seem difficult to understand from a physics point of view as the measurements are of the same kind. One could of course argue that the reported yields and significances of those signals were over-estimated. A signal appearing in both experiments, however, is the Panofski peak arising from the secondary reaction $\pi^- p \rightarrow \gamma n$, and should serve as a rather fair comparison. The signals reported by Richter et al. are of the same strength as the Panofski signal [123], and the yield of the Panofski reaction scales (to first order) with the target volume whereas the population of exotic states should not. Thus, one would expect those signals found by Richter et al. to appear stronger than the Panofski signal in this experiment PS182, because our target volume is smaller. But the fact is the contrary, the Panofski peak is present and distinct while the surrounding spectrum scatters. These results do, however, not imply that exotic states do not exist. As pointed out in chapter 1, care should be taken before making any conclusions from spectroscopic results about the existence of exotic states. These can have a substantial intrinsic width and may also overlap with one another which would render them difficult to identify in a spectroscopic measurement like this. The conclusion seem to be that one has to perform scattering experiments with polarized beams and targets, and make a subsequent partial-wave analysis in order to give more definite answers to the still opened question whether exotic kinds of resonances exist or not.

5.2. QUARK REARRANGEMENT

The upper limit of $3.2 \cdot 10^{-3}$ for the branching ratio of $p\bar{p} \rightarrow 3\pi^0$ found by this experiment is not directly compatible with the prediction by Maryama and Ueda using the Rearrangement Model. The strength of the $3\pi^0$ -channel as predicted by Maryama and Ueda [95] and listed in table 2.2 is around 1%, and this discrepancy with our experimental limit cannot be explained as being caused by form-factor effects (which are expected to be substantial) since these are implicitly contained in the fitting of parameters of their model. However, their prediction is not necessarily in disagreement with our result due to the following reason [110]. When deriving the number $\sim 1\%$ for the " $3\pi^0$ -yield" they have always counted a photon pair as a π^0 , which implies that their published number, in fact, also embraces the 6γ -branching ratio of the channels $p\bar{p} \rightarrow 2\pi^0\eta$, $p\bar{p} \rightarrow 2\eta\pi^0$, and $p\bar{p} \rightarrow 3\eta$, and this brings the predicted branching ratio for pure $3\pi^0$ -yield down to $\sim 0.3\%$. Another theoretical prediction of the $3\pi^0$ -channel is given by Niskanen et al. [107] also based on the Rearrangement Model. They give the yield for $\pi^+\pi^-\pi^0$ that should be multiplied [110] by a factor $3/2$, the ratio of the squared $SU(3)$ recoupling coefficients, which gives a predicted $3\pi^0$ -yield of the order of $0.1 - 0.2\%$.

Our upper limit of $3.2 \cdot 10^{-3}$ for the branching ratio of $p\bar{p} \rightarrow 3\pi^0$ is not compatible with the number reported by Devons et al. [48] of $(7.6 \pm 2.3) \cdot 10^{-3}$. As described in section 4.2.3, our result relies on a Monte Carlo simulation of the detection efficiency for this annihilation channel which assumed the decay to be isotropic. The Dalitz plot shown in ref. [48] is, however, non-uniform, and thus our result might be misleading, since we only scan a limited region of the Dalitz plot. When comparing the two results one should maybe also consider the fact that Devons et al. fitted the events to the hypothesis $p\bar{p} \rightarrow 2\pi^0 + 2\gamma \rightarrow 6\gamma$, in which the two photon pairs with invariant masses "nearest" (it is not said how near) the π^0 -mass were "constrained" to have the π^0 -mass. This π^0 -recognition might be based on necessary but not sufficient tests, so to say.

Other results from PS182 that can be compared to theory are the ratios $R_{\pi^0(\rho^0/\omega)}$ and $R_{\eta(\omega/\rho^0)}$ between the channels $p\bar{p} \rightarrow \pi^0\rho^0$ and $p\bar{p} \rightarrow \pi^0\omega$, and the channels $p\bar{p} \rightarrow \eta\omega$ and $p\bar{p} \rightarrow \eta\rho^0$, respectively. The numbers obtained from our experiment are

$$R_{\pi^0(\rho^0/\omega)} = \frac{R_{\pi^0\rho^0}}{R_{\pi^0\omega}} = \frac{1.9 \pm 0.2}{0.5 \pm 0.3} = 3.8 \pm_{17}^{67}, \quad 2.1 < R_{\pi^0(\rho^0/\omega)} < 11 \quad (5.1)$$

$$R_{\eta(\omega/\rho^0)} = \frac{R_{\eta\omega}}{R_{\eta\rho^0}} = \frac{7 \pm 1}{2 \pm 1} = 3.5 \pm_{15}^{80}, \quad 2 < R_{\eta(\omega/\rho^0)} < 8. \quad (5.2)$$

The final states $\pi^0\rho^0$ and $\eta\omega$ are reached via S -wave annihilation (dominant in liquid hydrogen) from an initial $p\bar{p}$ -state with isospin $I=0$ and spin $S=1$. The Rearrangement Model [107] predicts that the initial $p\bar{p}$ -state with isospin $I=0$ and spin $S=1$ should be a factor of 7 to 8 more favoured as compared to $I=1, S=1$, which is in agreement with our results. Discrepancy between theory and experimental data could be explained by the argument that the Stark effect (cf. chapter 2) should cause the different initial $p\bar{p}$ -states to occur with a statistical ratio [110].

In an extended version of the, so called, " $^{13}\text{P}_0$ -Model" [108] the rearrangement matrix elements squared gives the values $R_{\pi^0(\rho^0/\omega)} \approx 9$ and $R_{\eta(\omega/\rho^0)} = 9$, which also is in reasonable agreement with our experimental results.

The upper limits we obtain for $p\bar{p}$ annihilations into final states 2η and $\eta\eta'$ combined with the measured branching ratio $R_{2\pi^0}$ for $p\bar{p} \rightarrow 2\pi^0$, allow us to compute limits of the ratios $R_{2\eta/2\pi^0}$ and $R_{\eta\eta'/2\pi^0}$ between the branching ratios for $p\bar{p} \rightarrow 2\eta$ and $p\bar{p} \rightarrow 2\pi^0$, and the branching ratios for $p\bar{p} \rightarrow \eta\eta'$ and $p\bar{p} \rightarrow 2\pi^0$, respectively:

$$R_{2\eta/2\pi^0} = \frac{R_{2\eta}}{R_{2\pi^0}} < \frac{1}{2.2 \pm 0.3} = 0.53, \quad (5.3)$$

$$R_{\eta\eta'/2\pi^0} = \frac{R_{\eta\eta'}}{R_{2\pi^0}} < \frac{5}{2.2 \pm 0.3} = 2.6. \quad (5.4)$$

This is in agreement with the theoretical predictions stated in chapter 2.

5.3. VECTOR DOMINANCE

Our measured branching ratio $R_{\pi^0\gamma} = (1.9 \pm 0.9) \cdot 10^{-5}$ for $p\bar{p} \rightarrow \pi^0\gamma$ can be compared to the number predicted by the Vector Dominance Model (see chapter 2). The predicted number is $R_{\pi^0\gamma} \approx 1.4 \cdot 10^{-4}$, which evidently is in contradiction to our measurement. However, there are effects that are not taken into account in the formula (2.33), which relates the $\pi^0\gamma$ -channel to the real $\pi^0\rho^0$ -channel (main contribution). Such an effect is the influence of form factors. Because the dynamics of the two reactions $p\bar{p} \rightarrow \pi^0\rho^0$ and $p\bar{p} \rightarrow \pi^0\gamma$ are quite different, $\pi^0\rho^0$ involving rather low momentum transfers whereas $\pi^0\gamma$ is a hard process, form-factor effects are expected to enter importantly. Further-more, in the real $\pi^0\rho^0$ -case the substantial intrinsic width of the ρ -meson increases the phase space integral [106] [109] in contrast to the $\pi(\rho^0) \rightarrow \pi^0\gamma$ -case where this effect should not appear at all. For these two reasons $\pi^0\gamma$ is, apart from scaling with the fine structure constant, further suppressed as compared to real $\pi^0\rho^0$. Thus, the number obtained from eq. (2.33) should be multiplied by another factor $\sim 1/20$ [110]. Consequently the measured small branching ratio for $p\bar{p} \rightarrow \pi^0\gamma$ seems to be understandable from a theoretical point of view.

Similarly, the predicted number for the channel $p\bar{p} \rightarrow 2\gamma$ [eq. (2.40)] should be multiplied by a factor $\sim 1/400$ resulting in a number of $R_{2\gamma} \sim 10^{-8}$, which is two orders of magnitude below our measured limit. An earlier experiment [71] has measured the cross-section for $p\bar{p} \rightarrow 2\gamma$ at a total centre-of-mass energy of $\sqrt{s} = 2.24$ GeV to be $d\sigma/d\Omega(90^\circ) = (160 \pm_{80}^{190})$ nb/sr. By only considering the different phase space for atomic annihilations as compared to annihilations at $\sqrt{s} = 2.24$ GeV, this cross-section can be translated to a corresponding yield of $R_{2\gamma} = (32 \pm_{16}^{37}) \cdot 10^{-6}$ for annihilations at rest. However, there is no argument for the matrix element to be the same for atomic¹ annihilations as it is at $\sqrt{s} = 2.64$ GeV. At low momentum transfers one has to consider [19] corrections to the first-order Feynman diagram from the propagator being different from $1/(\not{p} - m_p)$ as well as the fact that the vertices get modified (they

¹ A way to calculate atomic annihilations [19], that works well for positronium, is to consider the initial state as a superposition of $p\bar{p}$ -states with a weight factor $\phi(\mathbf{p})$, being the Fourier transform of the wave-function $\psi(\mathbf{r})$ of protonium (which is a hydrogen-like atom). However, in the $p\bar{p}$ -case, the wave-function $\psi(\mathbf{r})$ is not as well known as in the positronium case since the $p\bar{p}$ -potential is not only the electromagnetic one.

are, by the way, not given by the normal form factors since one photon is off-shell).

5.4. S-WAVE OR P-WAVE ANNIHILATION

By comparing our measured branching ratio $R_{2\pi^0} = (2.2 \pm 0.3) \cdot 10^{-4}$ for $p\bar{p} \rightarrow 2\pi^0$ with the branching ratio $R_{\pi^+\pi^-} = (3.2 \pm 0.3) \cdot 10^{-3}$ [14] for $p\bar{p} \rightarrow \pi^+\pi^-$ the P -wave contribution to $p\bar{p}$ annihilations in liquid hydrogen can be estimated. The $2\pi^0$ -channel can only proceed via P -wave while both S -wave and P -wave are allowed for the $\pi^+\pi^-$ -channel. If we assume spin statistics for the initial state and if ν is the fraction of P -wave annihilations, we can write

$$\nu = \frac{P\text{-wave}}{\text{all}} = \frac{2 \cdot R_{2\pi^0}}{R_{\pi^+\pi^-}} = \frac{2 \cdot (2.2 \pm 0.3) \cdot 10^{-4}}{(3.2 \pm 0.3) \cdot 10^{-3}} = 0.14 \pm 0.03, \quad 0.11 < \nu < 0.17. \quad (5.5)$$

This is consistent with the result obtained by studying $p\bar{p}$ annihilations into neutral kaons [23], which indicated that the P -wave contribution has to be less than 20%. It is also consistent with a theoretical prediction [24] which gives the contribution to be 10% – 40%.

When computing the ratio ν for the P -wave contribution to $p\bar{p}$ annihilations in liquid hydrogen, we have assumed that all annihilations take place from an initial atomic state. This is, however, not completely true, since there is a finite probability that an antiproton hits a proton before it is slowed down. These in-flight annihilations do not necessarily have the same relative P -wave contribution as the atomic annihilations. During the run period D83 (when the $2\pi^0$ yield was measured) the range of the $\bar{p}$ -beam was 5 cm of liquid hydrogen, and the $\bar{p}$ -momentum was then around 200 MeV/c over roughly 80% of the range. The fraction of the annihilations that take place in flight can then be estimated from the total annihilation cross-section [eq. (2.30)] to be less than 1%, which does not alter the derived value of the P -wave contribution significantly. However, if the P -wave contribution to the in-flight annihilations is as important [106] as the S -wave contribution down to 10 to 20 MeV, one could suspect that the fraction of P -wave annihilations from atomic states are less, since the annihilation cross-section plausibly increases towards lower momenta.

*Ni kan inte dela ut recept på hur man gör en mozartsk
melodi på grundval av ta en fjärdedelsnot, sedan en åtton-
delsnot, sedan en sextondelsnot etc...
Ni ber inte en konstlärare om ett recept på hur man gör en
Leonardo-teckning.
Härav den oerhörda tråkigheten, orsakad av den gängse
professorliga dokumentationen eller högsträvande avhand-
lingen i prosodi.*

(Ezra Pound)

APPENDIX A

KINEMATICS OF TWO-PHOTON DECAYS

The so called Jacobian peak distribution is the angular correlation function $J(\phi)$ of the two photons from the decay of a π^0 , or η , or any other particle that can decay into two photons. It gives the distribution of opening angles $\phi_{\gamma\gamma}$ between the directions in the lab frame of the photons, and is derived under the assumption that the decays occur isotropically in the centre-of-mass system. The derivation outlined here follows the one given by Källén [87], and the reader is referred to this book for further details.

The phase space integral I of the decay can be written (notations from fig. A.1)

$$I = \int d^4k_1 d^4k_2 \delta(k_1^2) \delta(k_2^2) \theta(k_1) \theta(k_2) \delta(P - k_1 - k_2), \quad (\text{A.1})$$

and the probability for the opening angle $\phi_{\gamma\gamma}$ to be within the interval from ϕ to $\phi + d\phi$ is then

$$J(\phi) d(\phi) = \frac{2}{\pi} \int d^4k_1 d^4k_2 \delta(k_1^2) \delta(k_2^2) \delta(P - k_1 - k_2) \times \quad (\text{A.2})$$

$$\times \theta(k_1) \theta(k_2) \delta\left(\cos\phi - \frac{k_1 \cdot k_2}{E_1 E_2}\right) \sin\phi d\phi.$$

We can solve this expression by firstly eliminating k_2 with the help of the third δ -function, and then integrate over E_1 by using the first δ -function and the first step-function:

$$J(\phi) = \frac{2}{\pi} \int dk_1 \delta(k_1^2) \delta(-m^2 - 2Pk_1) \theta(k_1) \theta(P - k_1) \delta\left(\cos\phi - \frac{k_1 \cdot p - k_1^2}{E_1 E_2}\right) \sin\phi = \quad (\text{A.3})$$

$$= \frac{2}{\pi} \int \frac{d^3k_1}{2E_1} \theta(E - E_1) \delta(2EE_1 - m^2 - 2p \cdot k_1) \delta\left(\cos\phi - \frac{2EE_1 - m^2 - 2E_1^2}{2E_1(E - E_1)}\right) \sin\phi =$$

$$= \frac{1}{|p|} \int_0^E dE_1 \delta\left(\cos\phi - \frac{2EE_1 - m^2 - 2E_1^2}{2E_1(E - E_1)}\right) \theta[4p^2 E_1^2 - (2EE_1 - m^2)^2] \sin\phi,$$

where the last step makes use of the last δ -function and introduces a step-function to assure that the magnitude of the cosinus of the angle between p and k_1 does not exceed unity. Integrating over E_1 then finally yields:

$$J(\phi) = \frac{2}{|p|} \int_0^E dE_1 \cdot E_1 (E - E_1) \delta[2(1 - \cos\phi)(E_1^2 - E_1 E) + m^2] \times \quad (\text{A.4})$$

$$\times \theta[4m^2 E_1 (E - E_1) - m^4] \sin\phi =$$

$$= \frac{m^2}{2|p|} \frac{\sin\phi}{(1 - \cos\phi)^2} \theta\left(\frac{2}{1 - \cos\phi} - 1\right) \int_0^E dE_1 \delta\left[\left(E_1 - \frac{E}{2}\right)^2 - \left(\frac{E^2}{4} - \frac{m^2}{2(1 - \cos\phi)}\right)\right] =$$

$$= \frac{m^2}{|p|} \cdot \frac{\sin\phi}{(1 - \cos\phi)^2} \cdot \frac{\theta[E^2(1 - \cos\phi) - 2m^2]}{\sqrt{E^2 - 2m^2/(1 - \cos\phi)}} =$$

$$= \frac{1-\beta^2}{2\beta} \frac{\cos(\phi/2)}{\sin^2(\phi/2)} \frac{\theta[\beta^2 - \cos^2(\phi/2)]}{\sqrt{\beta^2 - \cos^2(\phi/2)}}.$$

The step-function defines the minimum opening angle that can occur:

$$\phi_{min} = 2\arccos(\beta) = 2\arcsin\left(-\frac{m}{E}\right) = 2\arctan\left(-\frac{m}{p}\right). \quad (\text{A.5})$$

Fig. A.2 shows $J(\phi)$ for different values of $\beta = p/E$. The top axis gives the momentum p if $m = m_{\pi^0}$.

The energy distribution of the decay photons turns out to be uniform between the maximum and the minimum energies

$$E_{max} = \frac{E_{\pi^0}}{2}(1+\beta), \quad E_{min} = \frac{E_{\pi^0}}{2}(1-\beta), \quad \beta = \frac{p_{\pi^0}}{E_{\pi^0}}. \quad (\text{A.6})$$

This can be shown in the following way. The number dN of states per solid-angle element $d\Omega_{CM}$ in the centre-of-mass system is constant, since the π^0 is a scalar meson and decays isotropically. The density of states as a function of the observed photon energy E in the lab system is then

$$\frac{dN}{dE} = \frac{dN}{d\Omega_{CM}} \cdot \frac{d\Omega_{CM}}{d\Omega} \cdot \frac{d\Omega}{dE} = \text{const.} \cdot \frac{\sin\theta_{CM}}{\sin\theta} \frac{d\theta_{CM}}{d\theta} \frac{\sin\theta}{dE/d\theta}, \quad (\text{A.7})$$

where θ_{CM} and θ are adapted from fig. A.1. The Lorentz contraction of space $d\Omega_{CM}/d\Omega$ can be evaluated from the aberration formula in special relativity:

$$\cos\vartheta = \frac{\cos\theta - \beta}{1 - \beta\cos\theta} \quad (\text{A.8})$$

$$\Rightarrow \frac{\sin\theta_{CM} d\theta_{CM}}{\sin\theta d\theta} = \frac{1 - \beta^2}{(1 - \beta\cos\theta)^2}. \quad (\text{A.9})$$

The photon energy E in the lab-frame is obtained as a function of the angle θ to the π^0 -direction in the lab-frame by evaluating the Lorentz invariant $P_{\pi^0}^\mu P_{\gamma\mu}$ in the centre-of-mass system and in the lab-system:

$$P_{\pi^0}^\mu P_{\gamma\mu} \big|_{CM} = (m_{\pi^0}, 0)(E_{CM}, k_{CM}) = m_{\pi^0} E_{CM} = m_{\pi^0}^2/2 \quad (\text{A.10})$$

$$P_{\pi^0}^\mu P_{\gamma\mu} \big|_{lab} = (E_{\pi^0}, p_{\pi^0})(E, k) = E_{\pi^0} E - p_{\pi^0} \cdot k = E(E_{\pi^0} - p_{\pi^0} \cos\theta) = EE_{\pi^0} (1 - \beta\cos\theta)$$

$$\Rightarrow E = \frac{m_{\pi^0}^2}{2E_{\pi^0} (1 - \beta\cos\theta)}$$

$$\frac{dE}{d\theta} = \frac{m_{\pi^0}^2 \beta \sin\theta}{2E_{\pi^0} (1 - \beta\cos\theta)^2}. \quad (\text{A.11})$$

We then eventually find that the angle dependence is exactly cancelled by the Lorentz contraction of space:

$$\begin{aligned}
\frac{dN}{dE} &= \text{const.} \cdot \frac{1-\beta^2}{(1-\beta\cos\theta)^2} \cdot \sin\theta \cdot \frac{2E_{\pi^0}(1-\beta\cos\theta)^2}{m_{\pi^0}\beta\sin\theta} = \\
&= \text{const.} \cdot \frac{2E_{\pi^0}(1-\beta^2)}{m_{\pi^0}\beta} = \text{const.} \cdot \frac{1}{m_{\pi^0}} \cdot \frac{2}{\beta} \sqrt{1-\beta^2},
\end{aligned} \tag{A.12}$$

which obviously is constant for fixed β , i.e. monoenergetic π^0 's.

The distribution of decay angles θ between a photon direction and the original π^0 -direction is found in exactly the same way

$$\frac{dN}{d\theta} = \frac{dN}{d\theta_{\text{CM}}} \frac{d\theta_{\text{CM}}}{d\theta} = \text{const.} \cdot \frac{\beta\sin\theta}{(1-\beta\cos\theta)^2}. \tag{A.13}$$

This distribution is displayed in fig. A.3 for different values of β .

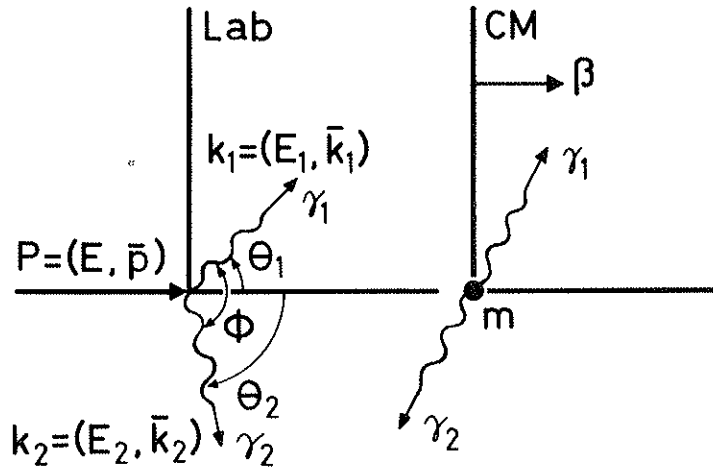


Fig. A.1: Kinematics of a π^0 -decay.

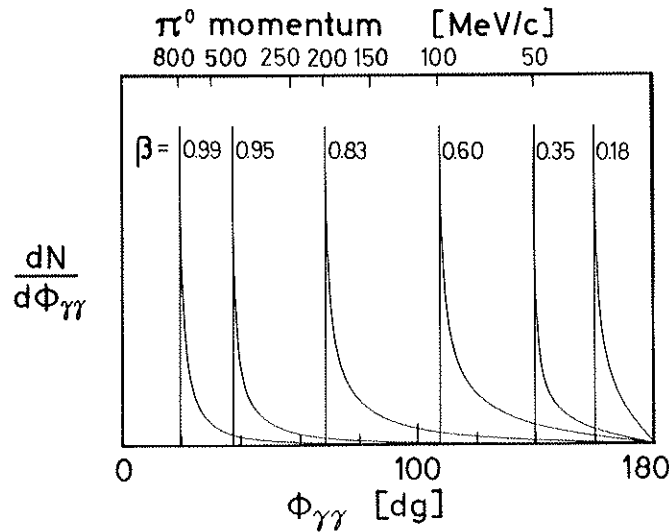


Fig. A.2: Jacobian peak distributions for different values of β . Top axis shows π^0 -momentum for corresponding minimum angles.

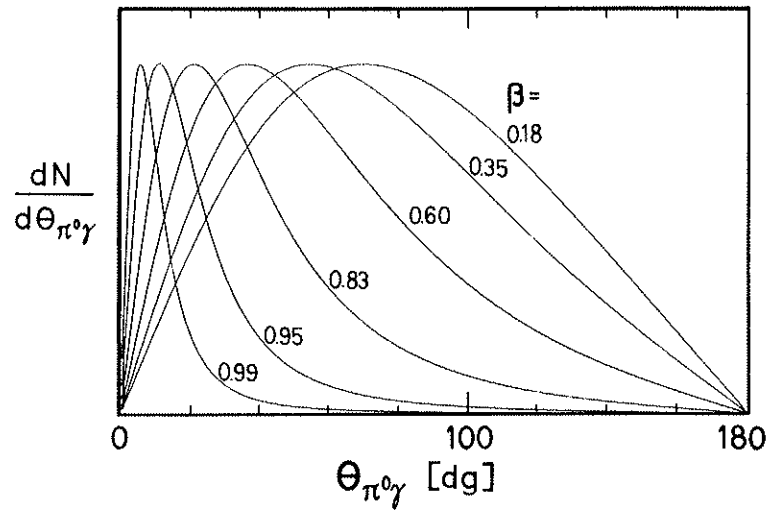


Fig. A.3: Distributions of decay angles $\theta_{\pi^0\gamma}$ between the π^0 and the photon directions for different values of β .

APPENDIX B

ELECTROMAGNETIC SHOWERS

Whenever an electron or a photon of high energy propagates through matter, there will occur a multiplicative process of bremsstrahlung alternated with pair production. This phenomena² is called an "electromagnetic shower" (fig. B.1). When an electron passes near a nucleus there is a finite probability that it emits a photon through bremsstrahlung. The other major interaction, that of pair production, is when a photon materializes into an electron-positron pair when passing through the field from a nucleus. In both these processes there is a recoil of the nucleus to conserve energy and momentum, but because of its large mass the energy absorbed is completely negligible. There are also other processes present in an electromagnetic shower, but the two interactions mentioned above are the ones playing the dominant role for the development of a shower, and determine the basic character of the shower. Of somewhat less importance are the inelastic collisions between high-energy electrons and atomic electrons. Since these collisions are extremely numerous but with very little energy transfer in each collision, the phenomenon can be considered as a continuous energy loss — "ionization loss" (cf. section 3.3.2). The different cross-sections of the main processes occurring in an electromagnetic shower are compared in fig. B.2.

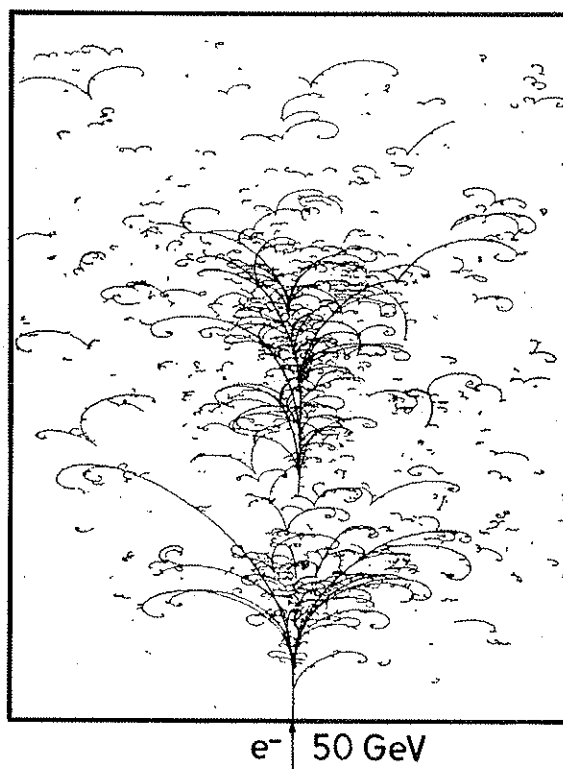


Fig. B.1: Bubble chamber picture of an electromagnetic shower.

² For a survey see refs. [9] [81].

The cross-sections³ of the two predominant processes, bremsstrahlung and pair production, that profoundly determine the development of the electromagnetic shower, were derived by Heitler [72]. In the extreme relativistic limit they can be written, respectively,

$$\sigma_B = 4\sigma_0 \left[\ln \left(\frac{2E_0}{m_e c^2} \right) - \frac{1}{3} \right] , \quad (\text{B.1})$$

$$\sigma_P = \sigma_0 \left[\frac{28}{9} \ln \left(\frac{2E_0}{m_e c^2} \right) - \frac{218}{27} \right] , \quad (\text{B.2})$$

where E_0 is the energy of the primary electron or photon upon entry, respectively, and $\sigma_0 = Z^2 r_0^2 / 137$, $r_0 = 2.8$ fm being the Bohr radius of the electron. These formulae are, however, derived under the basic assumptions that the Born approximation is valid, and that the screening of the nucleus is negligible. For heavy elements ($Z > 50$) the screening of the nucleus becomes substantial; the cross-sections saturate, and the expressions (B.1) and (B.2) modify to the asymptotic formulae.

$$\sigma_B^{sc} = 4\sigma_0 \left[\ln(183Z^{-1/3}) + \frac{2}{9} \right] , \quad (\text{B.3})$$

$$\sigma_P^{sc} = \sigma_0 \left[\frac{28}{9} \ln(183Z^{-1/3}) + \frac{2}{27} \right] . \quad (\text{B.4})$$

Screening effects become important if

$$\frac{E_0}{2m_e c^2} > 137Z^{-1/3} . \quad (\text{B.5})$$

For larger Z the Born approximation rarely gives very correct results, particular not for small input energies E_0 when it is hardly legitimate to compute the matrix element by inserting plane waves either. E.g., for lead and photon impact energies around 150 MeV the Born approximation gives a factor of 2 too small value for the pair-production cross-section σ_P . The validity of the Born approximation is given by the condition

$$\frac{2\pi Z}{137\beta} \ll 1 . \quad (\text{B.6})$$

The Born approximation is therefore generally valid for GeV-energies and Z 's of typical crystal calorimeters. The screening, on the other hand, is seldom negligible but not complete either for crystal-calorimeter materials. The cross-sections can be corrected for the failure of the Born approximation by adding [72]

³ The Feynman amplitude for the bremsstrahlung process is derived in the book by Björken & Drell and the amplitude for pair production is readily deduced from this one with the well-known substitution rules for "crossed" diagrams.

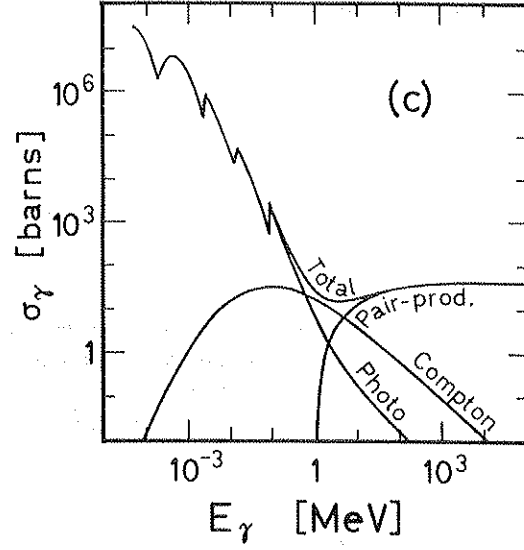
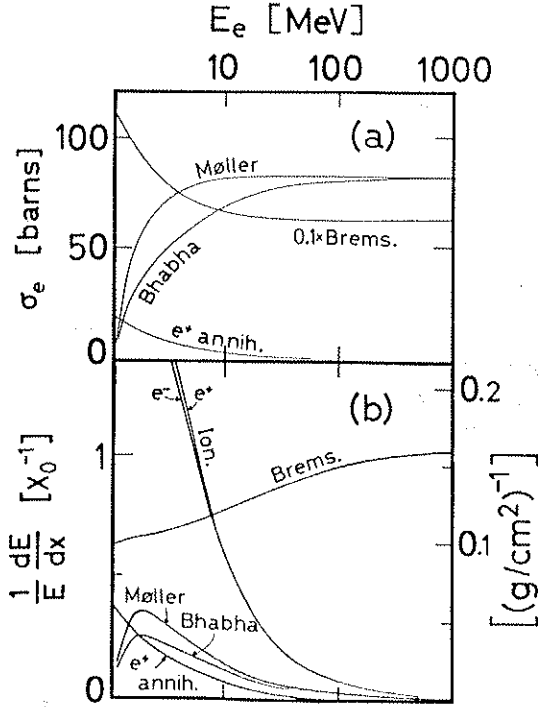


Fig. B.2: Cross-sections for lead of different processes occurring in an electromagnetic shower; (a) for electrons, (c) for photons, and (b) is the differential energy loss of electrons due to different processes. (Adapted from ref. [98]).

$$\sigma' = 2\sigma_0 Q(Z), \quad Q(Z) = 2.414 \cdot (Z/137)^2, \quad Z \ll 137. \quad (\text{B.7})$$

The bremsstrahlung cross-section increases rapidly for small energies of the emitted photon — the so called "infrared catastrophe". It is, however, reasonable not to consider particles below a certain cutoff energy E_c , i.e. the minimum kinetic energy that can be detected in the calorimeter. If we only consider particles above the cutoff energy E_c we see a rapid build-up in number of particles to a maximum followed by a slower fall-off as the shower develops (fig. B.3). From one primary high-energy electron the average energy of the secondary electrons decreases uniformly until it is comparable to the thermal electron energy. The longitudinal development of the electromagnetic shower in a particular medium is determined by the *radiation length* X_0 . The radiation length is defined as the distance, in which the energy E_0 of a high-energy electron will drop, on average, to $e^{-1}E_0$, i.e.

$$E(t) = E_0 e^{-t/X_0}. \quad (\text{B.8})$$

by bremsstrahlung *only*. Because the pair-production cross-section σ_p is approximately $7/9 \cdot \sigma_B$ a high-energy photon will convert within a distance X_0 into an electron-positron pair with the probability $7/9$. The distance $9/7 \cdot X_0$ is thus sometimes referred to as the "conversion length".

The lateral spread of the shower is mainly due to multiple Coulomb scattering of the electrons that do not radiate but have a large enough energy to travel far from the axis. The energy of an electron that loses as much energy due to excitation and ionization as by radiation is called the *critical energy* ϵ .

The formulae for X_0 and ε are rather complicated [126], but for rapid estimates one can use the approximate expressions:

$$X_0 \simeq 180 \frac{A}{Z^2} [\text{g/cm}^2] , \quad \frac{\Delta X_0}{X_0} < \pm 20\% \text{ for } 13 \leq Z \leq 92, \quad (\text{B.9})$$

$$\varepsilon \simeq \frac{550}{Z} [\text{MeV}] , \quad \frac{\Delta \varepsilon}{\varepsilon} < \pm 10\% \text{ for } 13 \leq Z \leq 92. \quad (\text{B.10})$$

A quantity usually introduced to represent the spatial development of the shower is the differential distribution of the *track length* T , defined as the sum of the tracks of all the charged particles. T depends of course on the cutoff energy E_c . For $E_c = 0$ all particles are detected and we have $T/X_0 = E_0/\varepsilon$. For increasing values of E_c , the fraction F of useful track length decreases as

$$T = F(z) \frac{E_0}{\varepsilon} X_0 , \quad (\text{B.11})$$

where

$$F(z) \simeq e^z \left[1 + z \ln \left(\frac{z}{1.526} \right) \right] . \quad (\text{B.12})$$

The formula (B.12) is valid within 10% for $z \leq 0.3$; universal for all materials with [104]

$$z = 4.58 \cdot \frac{Z}{A} \frac{E_c}{\varepsilon} . \quad (\text{B.13})$$

Note that because $T \sim E_0$ calorimetry is possible at all, since ionization and excitation are continuous and uniform along the track.

The distribution of the longitudinal track length, which is proportional to the longitudinal energy deposition is given by Longo [92]:

$$\frac{dE}{dy} = E_0 \frac{b^{\alpha+1}}{\Gamma(\alpha+1)} y^\alpha e^{-by} , \quad y = \frac{t}{X_0} , \quad (\text{B.14})$$

where α and b are two constants related to the shower maximum [1]:

$$y_{\max} = \alpha/b \simeq y_{\text{med}} - 1.5 , \quad (\text{B.15})$$

where y_{med} is the median depth, i.e. where half of the incident energy E_0 is deposited (fig. B.3):

$$y_{\text{med}} = a + \ln \frac{E}{\varepsilon} , \quad a = \begin{cases} 0.4 & \text{for electrons} \\ 1.2 & \text{for photons} \end{cases} \quad (\text{B.16})$$

The knowledge of the longitudinal distribution of the shower allows to compute the thickness t of the calorimeter needed to contain a fixed fraction of the incident energy:

$$t(98\%) \cong 3y_{med}. \quad (B.17)$$

This discussion is true on the average. Typically fluctuations multiply or divide the average deposited energy by a factor 1.5 [84]. However, the energies deposited at different depths are strongly correlated [36].

The lateral spread of the shower is usually measured, adequately enough, in Molière units R_M , which is the transversal spread of an electron beam of energy E_0 after having penetrated a thickness of X_0 :

$$R_M = \frac{E_s}{\epsilon} X_0, \quad (B.18)$$

where $E_s = 21$ MeV is the constant appearing in multiple scattering theory [126]. There are, however, two different mechanisms contributing to the transverse distribution as shown by fig. B.4 [146]. The central part scales as R_M , while the peripheral part is mainly due to the propagation of photons that are less attenuated in matter (10 to 20 MeV in lead) (fig. B.5). The spatial distribution of this component is determined by the minimum value of the photon attenuation coefficient which has no simple dependence upon A and Z [1] [105] [146]. Still 95% of the total energy is contained in a cylinder of radius

$$R_{95} \simeq 2R_M \simeq 14 \frac{A}{Z} [\text{g/cm}^2]. \quad (B.19)$$

Owing to statistics in the elementary processes there will be a limit on the optimal energy resolution [92], that is possible to achieve with a calorimeter:

$$\frac{\sigma(E)}{E} = \frac{\sigma(T)}{T} \simeq \frac{0.7\%}{\sqrt{E [\text{GeV}]}} \cdot \frac{E_c}{\epsilon} = \frac{0.5}{11.8} = 0.04. \quad (B.20)$$

Any lack of containment spoils the resolution and gives the lineshape a tail towards zero. In this context longitudinal losses are much more dangerous than lateral ones.

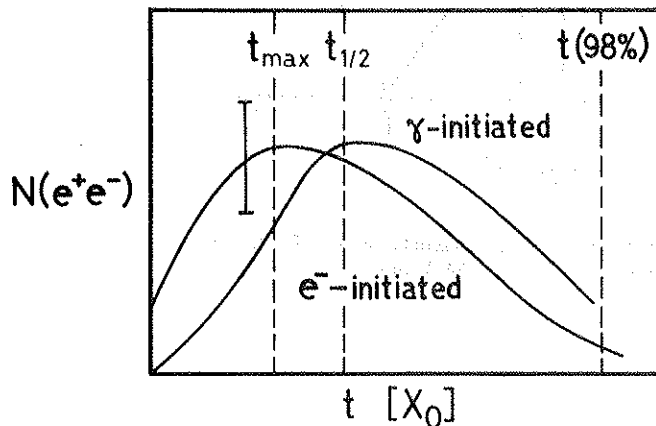


Fig. B.3: Number of tracks $N(e^+e^-)$ present in an electromagnetic shower after penetration of the thickness t . (Adapted from ref. [74]).

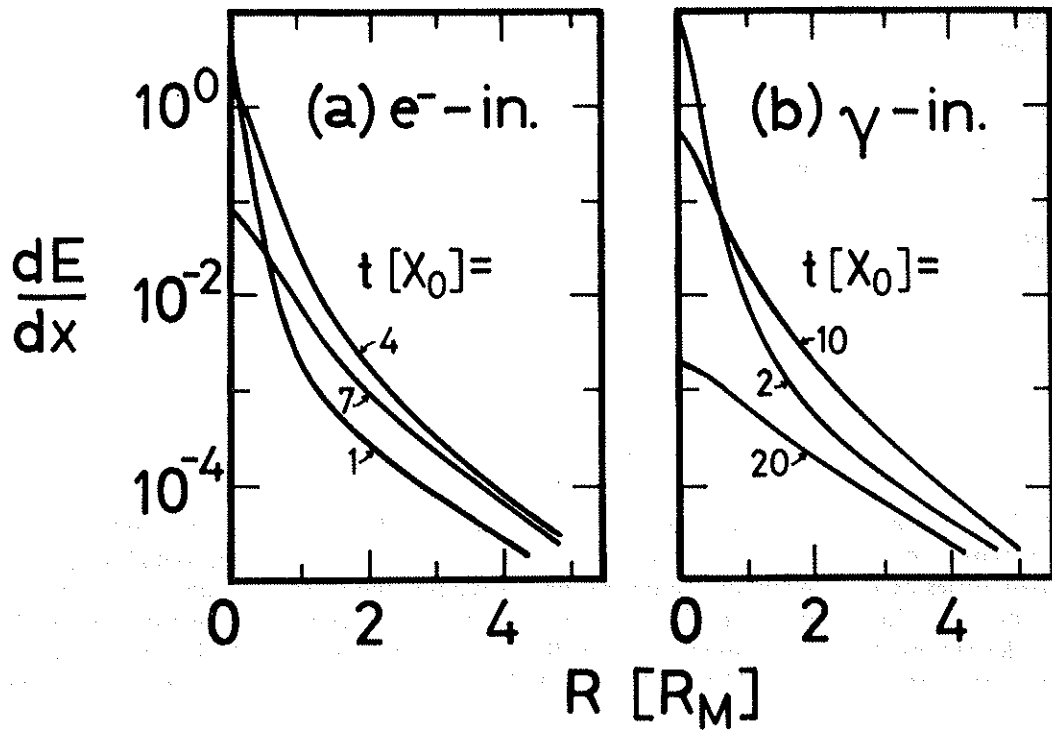


Fig. B.4: Radial energy distribution of an electromagnetic shower at different depths t . (Adapted from ref. [9]).

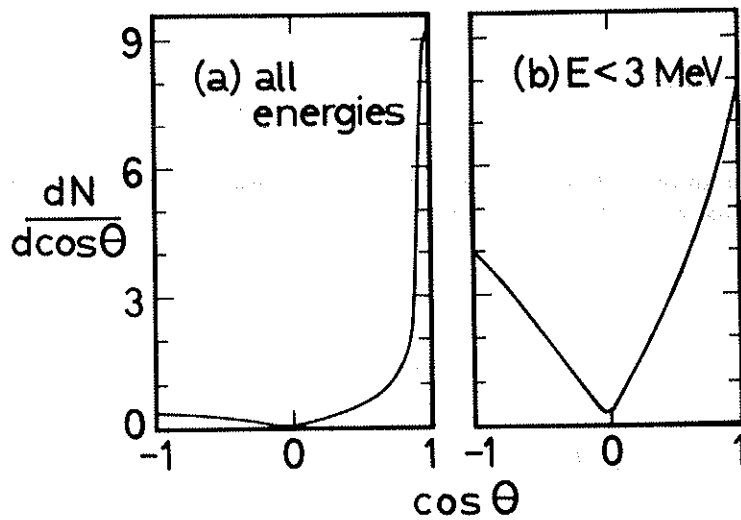


Fig. B.5: Angular distribution of tracks in an electromagnetic shower for (a) any particle energy, and (b) particle energies below 3 MeV only. (Taken from ref. [9]).

APPENDIX C

SIGNIFICANCE OF SIGNALS

The method of least squares is built on the hypothesis that the optimum description of a set of data is one which minimizes the weighted sum of squares of deviations of the data points y_i from the fitting function $y(x_i)$. This sum is characterized by the variance s^2 of the fit, which is an estimate of the variance σ^2 of the data. For a function with n parameters, the variance of the fit to N data points is

$$s^2 = \frac{\frac{1}{N-n} \sum_{i=1}^N \left\{ \frac{1}{\sigma_i^2} [y_i - y(x_i)]^2 \right\}}{\frac{1}{N} \sum_{i=1}^N \frac{1}{\sigma_i^2}}, \quad (C.1)$$

where the factor

$$f = N - n \quad (C.2)$$

is the number of degrees of freedom left after fitting N data points to n parameters. The weighting factor for each data point is the inverse of the variance σ_i^2 which describes the uncertainty of that particular data point i , normalized to the average of all the weighting factors.

The variance s^2 of the fit is also characterized by the statistic χ^2 :

$$\chi_f^2 = \frac{\chi^2}{f} = \frac{s^2}{\bar{\sigma}^2}, \quad (C.3)$$

where χ_f^2 is the so called "reduced chi-square", and $\bar{\sigma}^2$ is the weighted average of the individual variances. This definition of χ^2 , as the ratio of the estimated variance s^2 to the parent variance $\bar{\sigma}^2$ (times the number f of degrees of freedom), makes it a convenient measure of the *goodness* of the fit, and yields the maximum likelihood that the fitting function $y(x)$ represents the parent distribution [21]. If the fitting function is a good approximation to the parent dependence, then the estimated variance s^2 should agree well with the parent variance $\bar{\sigma}^2$, and the value of the reduced chi-square should be approximately unity. A value less than 1 does not indicate an improvement of the fit, but is simply a consequence of the uncertainty in the determination of s^2 .

A stochastic variable x is χ^2 -distributed if it has a probability density function

$$P(x) = \frac{1}{2^{f/2} \Gamma(f/2)} x^{f/2-1} e^{-x/2}. \quad (C.4)$$

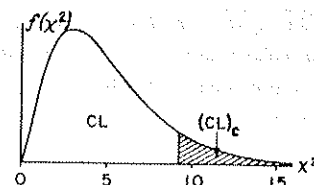
The mean and variance are f and $2f$, respectively. The *confidence level* (CL) is defined as

$$CL = \int_0^y P(x) dx, \quad (C.5)$$

and can be read from the diagram in fig. C.1. When evaluating $P(x)$ Stirling's approximation for the Γ -function may be useful:

$$\Gamma(x) = (x-1)! \cong 2.507 e^{-x} x^{x-1/2} (1 + 0.0833/x), \quad (C.6)$$

which is accurate to $\pm 0.1\%$ for $x \geq 1/2$.



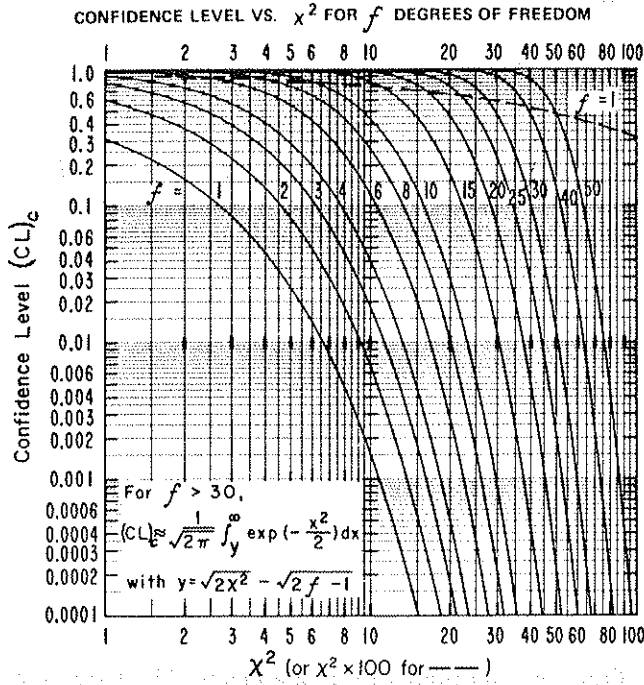


Fig. C.1: (Taken from ref. [122]).

defined region in the spectrum around the peak, and its significance is most expediently expressed in units of the statistical uncertainty of the background. On the other hand, if the peak position is completely unknown, the significance has in addition to depend on the total number of data points in the entire spectrum, since the probability of having a statistical fluctuation just anywhere in the spectrum has to increase with the number of data points.

Suppose $y(x_i)$, $i=1, \dots, N$, is a function that emulates the "background" of the spectrum of data points y_i at the ordinates x_i . Any other shapes, local or global, but the structure under current concern are comprehended as "background". Further assume that $y(x_i)$ fits the rest of the spectrum, without the considered peak, with a goodness $\chi^2/f \approx 1$. If a fit of $y(x_i)$ to the complete spectrum, including the peak, yields a goodness $\chi^2/f = \sqrt{2f}$, the *significance* of the peak is said to be "1 σ ". The unit "1 σ " is then the *standard deviation* $\sqrt{2f}$ of the relevant χ^2 -distribution for $f = N - n$ degrees of freedom, where N is the total number of data points, and n is the number of free parameters in $y(x)$.

Suppose there is a peak present that covers a number k of bins j . Let us divide the contents c_j of the bins that cover the peak into two (unknown) numbers b_j and p_j , being the number of counts belonging to the background and the peak, respectively. We can then write the contribution to the χ^2 -value from each one of these bins as

$$\begin{aligned} \sigma_j^{-2} [c_j - y(x_j)]^2 &= \sigma_j^{-2} [b_j + p_j - y(x_j)]^2 = \\ &= \sigma_j^{-2} [b_j - y(x_j)]^2 + p_j \sigma_j^{-2} [p_j + 2b_j - 2y(x_j)] \end{aligned} \quad (\text{C.8})$$

The significance Σ of the peak is thus approximately given by

* According to R.A. Fisher (Statistical Methods for Research Workers, Oliver and Boyd, Edinburgh and London, 1958) a better approximation is that χ , not χ^2 , becomes normally distributed, so that

$$x' = \sqrt{2x} - \sqrt{2f} - 1 \in G(0, 1), \quad x \in \chi^2(f). \quad (\text{C.7})$$

This is particular true for confidence levels close to 1. Thus, for $f=50$ and $\chi^2=80$, the true value is $1 - CL = 0.45\%$, but the Gaussian approximation yields $(\chi^2 - f)/\sqrt{2f} = 3.0$ corresponding to $1 - CL = 0.13\%$, while Fisher's approximation gives $x' = 2.7$ corresponding to $1 - CL = 0.35\%$.

For large numbers of degrees of freedom ($f > 30$) the χ^2 -distribution is well approximated⁴ by a Gaussian $G(f, \sqrt{2f})$ with mean value f and standard deviation $\sqrt{2f}$.

The significance of a structure observed in a spectrum of data points y_i is indeed a confused concept. Numbers quoted in the literature are usually in units of "1 σ ", which seem to denote an arbitrary unit without conventional definition. My personal intuition is that the concept *significance* of a structure should carry information about the probability that the structure **not** just is a statistical fluctuation, and thus I have adopted the definition given below.

There are two distinct cases: either the ordinate position at which the peak should occur is known in advance, or it is not. If the peak position is known the significance only depends on the statistics of that con-

$$\Sigma \simeq \sum_{j=1}^k \left\{ \frac{p_j}{b_j} [p_j + 2b_j - 2y(x_j)] \right\}, \quad (\text{C.9})$$

if the peak spans over k bins. If we assume the background under the peak to vary slowly, so that all $b \simeq \bar{b}$, it can be meaningful to estimate the height above the background, in units of $\sigma \equiv \sqrt{\bar{b}}$, of a peak having the significance $\Sigma = m\sigma = m\sqrt{2f}$, $m = 1, 2, \dots$. The average height $\bar{p}$ above the background of the dominant peak bins is then approximately

$$\bar{p} \simeq \left[\sqrt{1 + \sqrt{2f} \cdot n/k} - 1 \right] \sqrt{\bar{b}}. \quad (\text{C.10})$$

Vice versa the significance Σ of a peak with dominant bins of average height $\bar{p}$ above the background with average height $\bar{b}$, is approximately given by (units of σ):

$$\Sigma \simeq \frac{k}{\sqrt{2f}} \left[\left(\frac{\bar{p}}{\sqrt{\bar{b}}} + 1 \right)^2 - 1 \right], \quad f \gg k. \quad (\text{C.11})$$

It should be emphasized that the formulae (C.9) and (C.10) are only meaningful if the number of data points are much larger than the number of points spanned by the peak structure, i.e. $f \gg k$. In table C.1 the peak height $\bar{p}$ is listed for different significances $\Sigma = m\sigma$ and different values of the ratio k/f , i.e. the fractional region of the spectrum that is spanned by the peak. If $f < 50k$ eq. (C.11) is not meaningful any more and the significance has to be exactly derived from the fit. The significance approaches the limit given by the statistical uncertainty $\sigma_i = \sqrt{\bar{b}_i}$ of the background. However, if f becomes too small compared to k one can wonder if the systematic error introduced by inaccurate definition of the background $y(x)$ will not be significantly larger than the statistical error, and thus turn the result of the significance test completely corrupt. One has, so to say, tested another hypothesis than that intended.

It appears to me as if it was very common to always denote significance in terms of the statistics **under the peak only**, even when the peak position was unknown prior to the measurement. How large a region that is considered as being under the peak is then very unclear. There are arguments, that one should take ± 2 standard deviations of the peak width, or convert the peak shape to a square with the same area and height. In any case, if the peak position was unknown before the measurement, this method bears little resemblance with the probability that the peak is merely a statistical fluctuation, and yields a rapidly increasing fault of the significance for the peak to have a physics reason, if the number of data points is very large compared to the peak.

Table C.1: Relation $\Sigma/\sqrt{\bar{b}}$ between the significance Σ of a signal that appears at an unknown position and the uncertainty of the background $\sqrt{\bar{b}}$. k/f is the fractional region of the spectrum spanned by the peak.

		$k/f[\%]$				
		0.4	0.8	1.2	1.6	2
Σ	1σ	3	2	1.2	1.1	1
	2σ	6	4	2.4	2.2	2
	3σ	9	6	3.5	3.3	3

$$m_{\text{eff}} = \frac{1}{2} \left(\frac{1}{m_1} + \frac{1}{m_2} \right)$$

The effective mass m_{eff} is defined as the mass of a single particle that would have the same kinetic energy as the two particles in the center of mass frame. It is given by:

$$\frac{1}{m_{\text{eff}}} = \frac{1}{m_1} + \frac{1}{m_2}$$

where m_1 and m_2 are the masses of the two particles. The effective mass is used to simplify the analysis of two-body collisions.

$$\frac{1}{m_{\text{eff}}} = \frac{1}{m_1} + \frac{1}{m_2} \Rightarrow m_{\text{eff}} = \frac{m_1 m_2}{m_1 + m_2}$$

For a two-body collision, the effective mass m_{eff} is the mass of a single particle that would have the same kinetic energy as the two particles in the center of mass frame. It is given by:

$$\frac{1}{m_{\text{eff}}} = \frac{1}{m_1} + \frac{1}{m_2}$$

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π^0 , γ , AND η SPECTROSCOPY OF $p\bar{p}$ ANNIHILATIONS AT REST

Staffan Carius

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π^0 , γ , AND η SPECTROSCOPY OF $p\bar{p}$ ANNIHILATIONS AT REST

STAFFAN CARIUS

Thesis for the degree of Doctor of Technology

Abstract

The experiment PS182 performed at the Low Energy Antiproton Ring at CERN is presented. It was designed to search for exotic states (narrow) produced in proton-antiproton annihilations at rest. A spectrometer consisting of two sets of bismuth germanium oxide (BGO) crystals and a lead-glass array has been used to measure the π^0 and η inclusive momentum spectra and the inclusive photon energy spectrum produced in $p\bar{p}$ annihilations at rest. The tests of the BGO sets in an electron beam of energies from 50 to 450 MeV is described, as well as procedures to extract the energy and position resolutions for detected photons. Also discussed are the methods of calibration and reconstruction of impact position of photons detected by the lead-glass array

For the production rate of narrow exotic states, an upper limit is given as a function of the momentum of the emitted π^0 -meson or as a function of the energy of the emitted photon, respectively. The measured inclusive spectra show structures of ordinary mesonic resonances, which allows deduction of the branching ratios for the two neutral annihilation channels $p\bar{p} \rightarrow \pi^0 \omega$ and $p\bar{p} \rightarrow \eta \omega$. From the absence of signals upper limits are extracted for the neutral two-mesonic annihilation channels $p\bar{p} \rightarrow \pi^0 \eta$, 2η , $\eta\eta'$. By measuring coincidence events of three photons, the branching ratios of the rare annihilation channels $p\bar{p} \rightarrow 2\pi^0$ and $p\bar{p} \rightarrow \pi^0 \gamma$ are measured with precision, as well as an upper limit for the channel $p\bar{p} \rightarrow 3\pi^0$. Discriminating events of two high-energetic back-to-back photons resulted in an upper limit for $p\bar{p} \rightarrow 2\gamma$.

To be presented, with the permission of the Faculty of Applied Physics and Mathematics of the Royal Institute of Technology, for public criticism at the Research Institute of Physics (main auditorium) on May 30 1986 at 10.00.