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Properties of the Hadronic Shower produced by Neutrino and
Antineutrino Interactions in a Neon Target.

by

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in partial fulfilment of the
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ABSTRACT

Some results are presented from neutrino and antineutrino interactions in the Big European Bubble Chamber (BEBC) filled with a 70% molar neon / hydrogen mix and exposed to the CERN wide-band beam.

This analysis is based on $\sim 9,800$ neutrino and $\sim 17,000$ antineutrino charged current events identified as such using a large External Muon Identifier (EMI) of multiwire proportional chambers. Neutral current background to the charged current sample is estimated to be $< 1\%$ for events with muon momentum above $3 \text{ GeV}/c$ and is largely concentrated below $5 \text{ GeV}/c$.

The average charged multiplicity is seen to increase logarithmically with W^2 , in reasonable agreement with other experiments. Fragmentation functions are presented, which obey approximate isospin invariance. A simple analysis does not reveal any significant scaling violations or factorisation breaking for $W > 3 \text{ GeV}/c$ and $x_f > 0$. However, using π^- mesons of all x_f from events of all W shows some degree of Q^2 and x dependences in the fragmentation functions. A steady growth in $\langle P_t^2 \rangle$ with W^2 is found for fast forward fragments.

Evidence is presented for an azimuthal asymmetry about the current direction for pion production of $\langle \cos\phi \rangle = -0.050 \pm 0.012$ (-0.044 ± 0.012) for $\pi^+(\pi^-)$ in neutrino (antineutrino) interactions before correction for radiative effects and -0.039 ± 0.013 (-0.034 ± 0.014) after correction. This asymmetry is in qualitative agreement with a parton model calculation incorporating a quark primordial transverse momentum.

Final state bremsstrahlung is observed for muons and found consistent with QED expectations.

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TO

Mum, Dad and Wendy.

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INTRODUCTION

Deep inelastic scattering (DIS) experiments have been enormously successful in producing important physics results over the last two decades. In fact, over this time span, our understanding of the particulate nature of matter has moved forward in leaps and bounds, aided by such experiments. One particular DIS experiment, using neutrinos and antineutrinos was proposed in the late 1970's to do a high statistics survey of structure functions and the hadron final state and took data in the Spring of 1980. More than 10 institutes widely distributed geographically about Europe collaborated in this enterprise. In this thesis some results on the hadronic final state are presented, based on the data from this experiment. In particular, results on final state azimuthal asymmetries are given - the so-called " $\cos\phi$ " effect.

In chapter I a brief theoretical survey of neutrino interactions is presented, but, in such a small space hardly does justice to current knowledge in this field. Chapter II is devoted to apparatus details and data processing, whilst chapter III enlarges upon one particular aspect of the apparatus and presents results of work in assessing some background effects. Data selection and correction for some important systematic effects are described in chapter IV, which also covers some usual and not so usual features of neutrino interactions. In chapter V results are presented on hadronic multiplicities and some longitudinal properties of the hadron shower. Aspects of transverse and azimuthal degrees of freedom in the hadron shower are explored in chapter VI, which in particular discusses at length the presence of an azimuthal asymmetry in the production of final state hadrons. A brief summary of results presented, with particular reference to azimuthal asymmetries, is given in chapter VII.

Any experiment of this scale is naturally impossible for any one individual to perform and thus all participants are highly dependent on each other. However, the physics results presented and discussed in this thesis are the work of the author, unless stated otherwise.

In this thesis the word "neutrino" may refer to neutrinos and antineutrinos generically, as well as to the particle itself, depending on context.

Chapter I

THEORETICAL REVIEW

1.1 HISTORY

In 1896, Becquerel stumbled across the phenomenon of natural radioactivity and unknowingly heralded the dawn of the nuclear age. A decade and a half later, Rutherford put forward the idea that the atom has a substructure, as suggested by large angle scattering of α -particles by thin metallic foils. The nuclear atom was born.

Through the first half of this century, the idea that the atom was the fundamental building block of matter slowly crumbled. First the electron and proton were discovered, then the neutron, neutrino, muon and pion; some particles appearing in more than one electric charge state. The experimental discovery of such particles went hand in hand with the new theory of quantum mechanics (as developed by Bohr, Heisenberg, Schrodinger and fellow workers) which explained the particle dynamics. Dirac incorporated relativity into this framework, predicted the existence of the positron (later discovered by Anderson in 1932) and included the photon as the quantum of the electromagnetic interactions. As the years advanced, more particle states were added to this list, including the strange particles heralded by the so called "V" events. The lack of simplicity in the building blocks of matter and the various interactions between them led to a search for an underlying, simpler structure.

Broadly speaking, particles can be classified as hadrons (from Greek for the "strong" ones) or leptons (the "light" ones) depending on whether they interact through the strong force or not. The electron, its neutrino and their antiparticle sisters, the positron and its antineutrino are all leptons and are stable. In a similar

manner the muon and its neutrino can be grouped together. Hadrons, however, can be further classified as either baryons (such as the proton or neutron and all other particles which decay into them) and mesons (such as the pion). All mesons are unstable and ultimately decay into leptons. Both leptons and baryons carry intrinsic half integer angular momentum (or spin) and thus are fermions.

In the early 1960's, Gell-Mann proposed that hadrons could be grouped together in singlet, octet, decuplet etc. representations of the group SU(3) [1.1]. However, no grouping of hadrons could be found to fit the fundamental (triplet) in SU(3) representation. He thus proposed sub-constituent particles of hadrons, called quarks, to fill the triplet. Higher representations formed by taking combinations of triplets of quarks and antiquarks explained all the then known hadrons and even predicted the existence of some unknown particles which were later discovered. A great triumph for this theory was the prediction of the Ω^- and its subsequent discovery in 1964. A new quantum number, called flavour, was assigned to the three quarks to differentiate between them. In this theory, quarks carry fractional charge, fractional baryon number and like all leptons, carry spin 1/2 and are hence fermions.

TABLE 1.1 The Four Fundamental Forces of Nature			
Force	Relative Strength	Range	Boson
Strong	~ 1	10^{-15} m	gluon
Electromagnetic	$\sim 1/137$	"	photon
Weak	$\sim 10^{-5}$	10^{-17} m	W^+, W^-, Z^0
Gravity	$\sim 10^{-40}$	"	graviton

Interactions between fermions can occur through four forces (table 1.1). The strength of the gravitational force is exceedingly weak in comparison to the others and is very often neglected. Within the framework of Yukawa's exchange model of forces [1.2], the electromagnetic interaction is brought about by exchange of a massless, spin 1 boson (the photon) between electrically charged fermions and has infinite range. In contrast, the weak interaction is mediated by massive bosons (giving a finite range) and in addition may change both fermion flavour and charge. The exchange model picture has been tremendously successful and the strong interaction has been interpreted similarly with a group of eight vector bosons (called gluons) mediating interactions between fermions possessing an extra quantum number of colour charge.

As yet, free quarks have not been observed experimentally and it is believed that they are confined inside mesons and baryons by gluon exchange. To explore the quark dynamics it is thus necessary to give a bound quark a large 'kick'. This can be done very nicely using energy imparted by the exchange bosons of the weak interaction.

Neutrinos can be used to provide energy via the weak interaction to quarks bound in protons and neutrons. Some aspects of the strong interaction of quarks can hence be explored. However, since quarks are not observed directly, quark dynamics must be inferred from the hadrons observed after quark hadronization (fragmentation).

1.2 THE FUNDAMENTAL BUILDING BLOCKS

The present day picture of matter consists of quarks and leptons interacting via exchange of various quanta. The particles are itemized in table 1.2 together with some of their properties. For every particle listed there exists an antiparticle with opposite quantum numbers. The nature of the interactions naturally suggest placing quarks in doublets of contrasting electric charge and grouping

TABLE 1.2							
The Fundamental Fermion Doublets							
Generation (charge)	Leptons -1 0			Quarks +2/3 -1/3			
1	(e, ν)			(U, D)			
2	(μ, ν)			(C, S)			
3	(τ, ν*)			(T*, B)			
* Yet to be confirmed by experiment							
Q = charge							
I = isospin							
I3 = third component of I							
C = charm							
S = strangeness							
T = top							
B = bottom							
Quark Quantum Numbers							
Quark	Q	I	I3	C	S	T	B
Up	+2/3	1/2	+1/2	0	0	0	0
Down	-1/3	1/2	-1/2	0	0	0	0
Charm	+2/3	0	0	1	0	0	0
Strange	-1/3	0	0	0	-1	0	0
Top	+2/3	0	0	0	0	1	0
Bottom	-1/3	0	0	0	0	0	-1

them with one charged lepton and its neutrino into one 'generation'. So far three generations are believed to exist, each with a relationship between members comparable to the first generation. Members of each generation are typically more massive than the previous generation (neutrinos possibly excepted), but as to why this should be or indeed why there are generations at all is an open question.

Each quark is assigned a flavour (table 1.2) and it is the domain of weak interactions to bring about flavour transitions. Mixing can occur between the generations, most strongly between neighbouring generations, but almost negligibly between generations 1 and 3. The

extent of mixing is parameterized in terms of mixing angles which together constitute the 3 x 3 Kobayashi-Maskawa mixing matrix. The scene was originally set by D and S quark mixing with the Cabbibo mixing angle θ_c , but has been extended in the light of the discovery of the C and B quarks and the prediction of the T quark. The weak quark current, taking into account mixing, can be written

$$J = [u, c, t] \gamma_\mu (1 - \gamma_5) U \begin{bmatrix} d \\ s \\ b \end{bmatrix} \quad (1.1)$$

where $U = \begin{bmatrix} C_1 & S_1 C_3 & S_1 S_3 \\ -S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{i\delta} \\ -S_1 S_2 & C_1 S_2 C_3 - C_2 S_3 e^{i\delta} & C_1 S_2 S_3 - C_2 C_3 e^{i\delta} \end{bmatrix}$
and $C_i(S_i) = \cos\theta_i (\sin\theta_i)$, $\theta_1 = \theta_c$

This current, when contracted with a suitable lepton current, expresses all allowed weak charged current interactions.

The electromagnetic force couples via the charge of the fermions and is well described by a U(1) gauge symmetry, in which the exchanged boson (the photon) is uncharged. This theory is hence Abelian. In contrast, the weak force couples to the weak fermion charge and is described by a SU(2) gauge symmetry for the exchanged bosons. The latter exist in 3 charge states, W^+ , W^- and Z^0 . The first two account for charged current interactions whilst the latter is responsible for a weak neutral current similar to that mediated by the photon (although restricted to a finite range by the Z^0 mass).

The weak interaction has successfully been unified with the electromagnetic interaction (see next section) with an SU(2) x U(1) gauge symmetry. This theory has achieved enormous success, however, its non-Abelian nature raises complications for renormalization.

1.3 THE ELECTROWEAK THEORY

Up to this point it has been assumed that the weak interaction is a pointlike Fermi interaction. This is wrong as it leads to a violation of unitarity at high energy and does not exhibit a short range, characteristic of weak interactions. In analogy with quantum electrodynamics (QED), it is natural to use an exchange model where charged, massive, intermediate vector bosons, $W^\pm$, are exchanged (a charged current interaction). The culmination of this idea is in the electroweak model of Glashow, Weinberg and Salam in which QED and weak interactions are united. The theory is based on a weak isospin $SU(2)$ group of a massless triplet of vector bosons, W^1, W^2, W^3 and a weak hypercharge $U(1)$ group containing a massless singlet boson, B . Mixing occurs between the $I_3 = 0$ W boson and the B boson to produce the photon, A , and a neutral boson Z^0 whilst the $I_3 \neq 0$ components give rise to W^+ and W^- . By the process of "spontaneous symmetry breaking" the W^+ , W^- and Z^0 are given mass whilst the photon remains massless. The Lagrangian density for electroweak interactions is given by

$$L = gJ_\mu \cdot W_\mu + g'J_\mu^Y \cdot B_\mu \quad (1.2)$$

where J_μ and J_μ^Y are the weak isospin and hypercharge currents of the leptons and quarks and g and g' couplings to fields W_μ and B_μ respectively. In analogy with the definition of hypercharge $J_\mu^Y = J_\mu^{em} - J_\mu^{(3)}$ and hence

$$L = g(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-) + g(J_\mu^{(3)} - \sin^2 \theta_w J_\mu^{em}) Z_\mu + g \sin \theta_w J_\mu^{em} A_\mu \quad (1.3)$$

where $g'/g = \tan \theta_w$.

The first term gives rise to weak charged currents, the second to weak neutral currents and the third to electromagnetic neutral currents (i.e. QED).

The culmination of this theory was in the discovery at CERN of the Z^0 and $W^\pm$ [1.3, 1.4] by the UA1 collaboration. The theory is expounded in many texts (e.g. reference 1.5).

1.4 QUANTUM CHROMODYNAMICS

The enormous success of the electroweak theory has prompted a similar approach to the strong interaction. There are various sources of compelling evidence why quarks should appear in three states of a new quantum number called colour. This then takes into account a factor of 3 between experimental and theoretical values for the ratio of hadron to muon production in e^+e^- reactions. It also permits the formation of an antisymmetric wave function for the three quarks in the Ω^- . The strong interaction is explained by exchange of quanta called gluons, which couple to the colour charge of quarks. Leptons are colourless, which explains why they do not interact strongly. The three colours of quarks form a triplet in SU(3) whilst gluons form an octet. Since gluons also carry colour charge the theory is non-Abelian and hence permits gluon - gluon as well as quark - gluon couplings. The strong interaction coupling constant, α_s , decreases with increasingly energetic probes making the use of perturbation theory possible for high energy scattering processes. In fact the coupling is believed to be weak for small quark separations (so called asymptotic freedom) but approaches infinity as quarks are moved apart. This latter property leads naturally to quark confinement inside hadrons and explains the lack of evidence for free quarks. All observed groupings of quarks in hadrons ($Q\bar{Q}$ for mesons and QQQ for baryons) are colour singlets. Mesons are composed of colour - anticolour quark combinations whilst baryons are formed from three differently coloured quarks. Reference 1.6 covers this topic in much greater depth.

1.5 DEEP INELASTIC SCATTERING

1.5.1 Kinematics

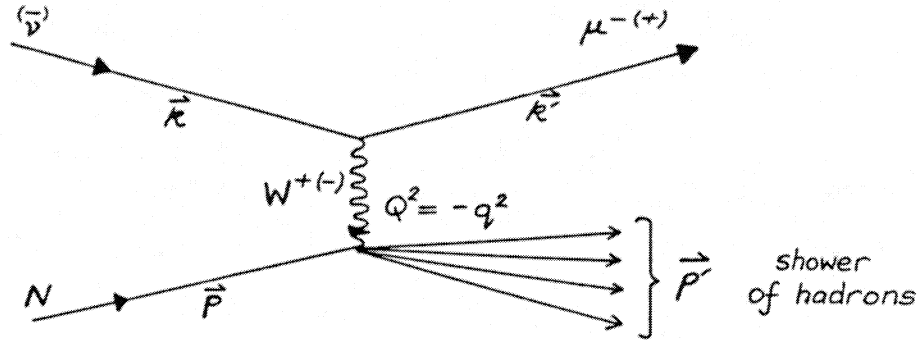


Figure 1.1: Deep inelastic scattering

Before embarking on a discussion of deep inelastic scattering it will be useful to introduce the variables commonly used. Deep inelastic scattering can be represented by a Feynman diagram (figure 1.1) in which k , p , k' and p' are the 4-momentum of incoming neutrino, target nucleon, outgoing muon and final state hadron system respectively. We define

$$\begin{aligned}
 s &= (k+p)^2 \\
 Q^2 &= -(k-k')^2 = -q^2 \quad (= -4EE' \sin^2(\theta/2)) \\
 \nu &= P \cdot q / M_p = E - E' \\
 x &= -q^2 / 2P \cdot q = Q^2 / 2M_p \nu \\
 y &= P \cdot q / P \cdot k = \nu / E \\
 W^2 &= (P+q)^2 = M_p^2 - Q^2 + 2M_p \nu
 \end{aligned} \tag{1.4}$$

where M_p is the proton mass, s is the total centre of mass energy squared and from $\delta P \cdot \delta x \geq \hbar$ Q^2 indicates the scale penetrated in the interaction. For high Q^2 smaller distances are probed. ν is the

energy transfer and W is the invariant mass of the final state hadronic system. x and y are the Bjorken scaling variables. Both x and y are bounded in the range $0 \leq x, y \leq 1$. If $x = 1$ then $y = 0$ and $W = M_p$ which is the case of elastic scattering, but for $x < 1$ the scattering is inelastic and for high Q^2 (i.e. $Q^2 \gg 1$) it is termed deep inelastic scattering. θ is the lepton scattering angle.

1.5.2 The Quark-Parton Model

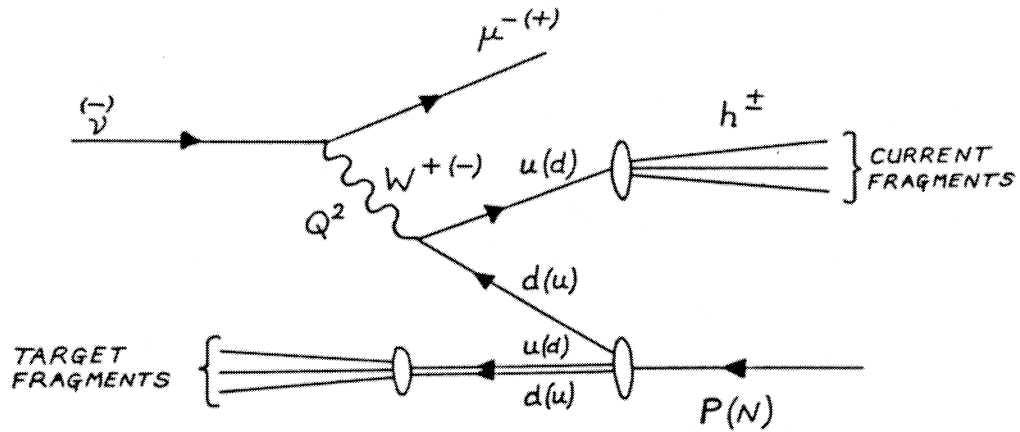


Figure 1.2: Deep inelastic scattering at the parton level

The quark parton model [1.7, 1.8] can be likened to the impulse approximation in nuclear physics. In a frame in which the target nucleon has a very large 3-momentum (the so-called infinite momentum frame) the nucleon can be viewed as a collection of partons travelling parallel (figure 1.2), each with negligible transverse momentum. The Heisenberg Uncertainty Principle gives $\delta E \delta T \geq \hbar$ and hence a high energy current interacts for only a very short time such that the struck parton can be taken to be quasi-free and the effects of

confinement within the nucleon can be neglected during this time. It can thus be assumed that :

1. during the time of the current absorption, interactions between partons can be neglected and
2. final state interactions can be ignored.

In fact, in such a frame partons appear not to interact with each other during the absorption of the current owing to time dilation. Deep inelastic scattering is interpreted as an incoherent sum of elastic scatters off these quasi-free partons and is more closely represented by figure 1.2.

The cross-section for this process is given by

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{G_F^2}{(2\pi)^2} \frac{E'}{E} \frac{(m_W^2)}{(m_W^2 + Q^2)} L_{\mu\nu} W^{\mu\nu} \quad (1.5)$$

where $L_{\mu\nu}$ and $W^{\mu\nu}$ are the leptonic and hadronic currents respectively and the term $m_W^2/(m_W^2 + Q^2)$ comes from the W boson propagator. $L_{\mu\nu}$ is well understood in the Standard Model but $W^{\mu\nu}$ can only be cast in terms of structure functions, namely $W_1(\nu, Q^2)$, $W_2(\nu, Q^2)$ and $W_3(\nu, Q^2)$, because of the composite nature of the nucleon and are determined by experiment. Other structure functions can be omitted as they are negligible. Equation (1.5) now becomes:-

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{G_F^2 E'^2}{2\pi^2} \frac{(m_W^2)}{(m_W^2 + Q^2)} (2W_1 \sin^2\theta/2 + W_2 \cos^2\theta/2 \pm W_3(E+E') \sin^2\theta/2) \quad (1.6)$$

where θ is the lepton scattering angle, '+' is for neutrino and '-' for antineutrino.

A consequence of the above assumptions is that since interactions are pointlike the cross-section should be independent of scale. Hence, the $W_i(\nu, Q^2)$ become independent of ν and Q^2 and only depend on the ratio x . In the limit of high Q^2 where individual partons can be

resolved the $W_i(v, Q^2)$ are replaced by $F_i(x)$. Quantum chromodynamics somewhat modifies these to be functions of Q^2 as well and leads to scaling violations at very high Q^2 . This effect can be interpreted as having resolved the cloud of gluons which surrounds individual quarks. However, neglecting such violations (which are very small at typical Q^2 's in this experiment) the cross-section becomes

$$\frac{d^2\sigma}{dx dy} = \frac{G_F^2 s}{2\pi} (xy^2 F_1(x) + (1-y)F_2(x) \pm y(1-y/2)x F_3(x)). \quad (1.7)$$

The nucleon structure functions $F_i(x)$ can be replaced by a sum of parton distribution functions $f_i(x)$ each giving the probability of hitting a quark of type i carrying a fraction x of the nucleon momentum thus,

$$\begin{aligned} F_2(x) &= 2x (\sum_q f_q(x) + \sum_{\bar{q}} f_{\bar{q}}(x)) \\ \text{and } xF_3(x) &= 2x (\sum_q f_q(x) - \sum_{\bar{q}} f_{\bar{q}}(x)) \\ \text{also for spin } \frac{1}{2} \text{ partons } 2xF_1 &= F_2. \end{aligned} \quad (1.8)$$

Equation (1.7) now becomes

$$\begin{aligned} \frac{d^2\sigma}{dx dy} &= \frac{G_F^2 s x}{\pi} (\sum_q f_q + (1-y)^2 \sum_{\bar{q}} f_{\bar{q}}) \text{ for neutrinos} \\ \text{and } \frac{d^2\sigma}{dx dy} &= \frac{G_F^2 s x}{\pi} ((1-y)^2 \sum_q f_q + \sum_{\bar{q}} f_{\bar{q}}) \text{ for antineutrinos.} \end{aligned} \quad (1.9)$$

It can be shown that $(1-y) = \frac{1}{2}(1+\cos\theta)$ and hence whilst νq and $\bar{\nu} \bar{q}$ scattering is isotropic $\nu \bar{q}$ and $\bar{\nu} q$ scattering has a strong dependence on the scattering angle θ . This is in fact a consequence of helicity conservation. Neglecting sea quark contributions, neutrino scattering is isotropic but antineutrino scattering has a $(1-y)^2$ dependence.

Another interesting feature arises as a consequence of isospin changes which can be brought about by the W^+ and W^- . Neutrino interactions proceed via a W^+ which can only be absorbed by D or $\bar{U}$

quarks (neglecting strange or charmed sea) but conversely antineutrino interactions proceed via a W^- and can only be absorbed by a U or $\bar{D}$ quark. This gives the possibility of selecting the flavour of the struck quark and hence (as will become apparent later) the leading meson after hadronization.

1.6 FRAGMENTATION

An event can be completely characterized by E_ν, Q^2 and x , but if single particle production is studied more variables are needed. Typical variables used are transverse momentum, P_t , of a hadron to the current direction, azimuthal angle, ϕ , about the current direction (with μ direction defining zero degrees) and z , the longitudinal momentum fraction of the struck quark momentum carried by the hadron.

Unfortunately, the momentum of the struck quark is unknown experimentally and only approximations can be made. A definition often used is

$$z = P^h \cdot P_n / q \cdot P_n \quad (1.10)$$

where P^h , P_n and q are the hadron, nucleon and current 4-momenta respectively. z is a Lorentz invariant, can be reduced to $z = E^h / (E^{\text{had}} - M_p)$ in the laboratory frame and is in fact the momentum fraction of the current carried by the hadron. A better estimate of the struck quark energy includes the nucleon mass (i.e. $z = E^h / E^{\text{had}}$). With this definition, z is bounded between 0 and 1 and this is the definition used in this thesis. (E^{had} = total hadronic energy).

Other useful variables (although not Lorentz invariant) are

$$\begin{aligned} x_f &= 2P_{\ell\ell}^* / W & \text{and} \\ y &= \frac{1}{2} \ln[(E^* + P_{\ell\ell}^*) / (E^* - P_{\ell\ell}^*)] \end{aligned} \quad (1.11)$$

where $P_{\ell\ell}^*$ is the hadron's momentum parallel to the current in the hadronic centre of mass (CMS) and E^* is the energy. x_f is bounded

between -1 and +1, the sign indicating whether the particle is forward (>0) or backward (<0) in the hadronic CMS. Transformation from one frame to another changes the rapidity (y) of all particles by a constant,

$$y' = y + \frac{1}{2} \ln[(1+\beta)/(1-\beta)] \quad (1.12)$$

where β is the velocity of the new frame with respect to the old.

1.6.1 Breit Frame

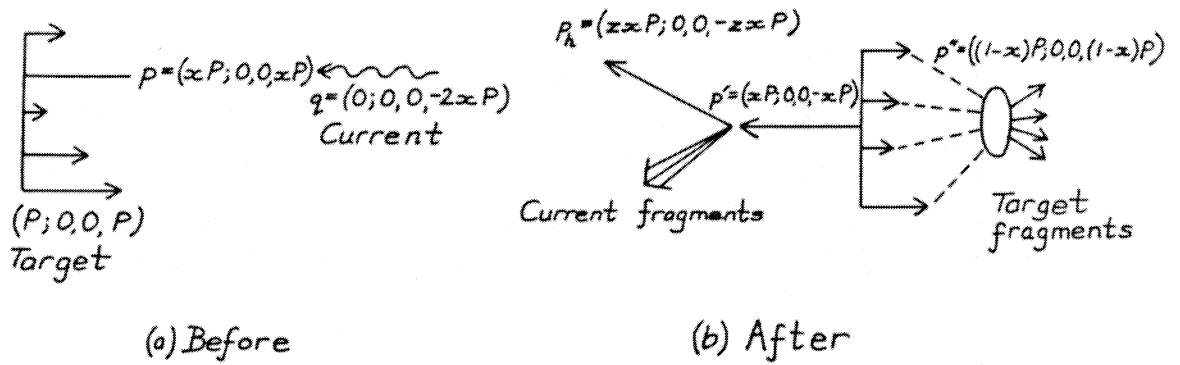


Figure 1.3: Scattering in the Breit frame

Neutrino-quark scattering and subsequent hadronization is most easily understood in the Breit frame in which the target-current system is boosted so that the current has zero energy. In this formalism the nucleon is taken as a collection of independent partons and has a momentum P (figure 1.3). The current, with momentum $q = (0; 0, 0, -2xP)$ is taken to interact incoherently with a parton with momentum $p = (xP; 0, 0, xP)$ head on. The struck parton thus has its momentum reversed, $p' = (xP; 0, 0, -xP)$ whilst the remaining partons have total

momentum $p'' = ((1-x)P; 0, 0, (1-x)P)$ and the two systems are separated by a large momentum. It is reasonable to assume that the subsequent hadronization is independent of the earlier current interaction but only depends on Z . The momentum of a hadron arising from the struck quark is hence $P_h = (zxP; 0, 0, -zxP)$.

1.6.2 Longitudinal Fragmentation

The hadronization phase of an interaction has unit probability of occurring since free quarks are not observed. However, a complete description of how quarks combine into observed hadrons is not yet possible. Whilst QCD is believed to be responsible for the processes involved in hadronization, the calculations necessary are non-perturbative and so cannot yet be completed. Hence, we have to be satisfied with parameterizations of hadronization in terms of models (see next section). Experimentally parton fragmentation functions are introduced into the single particle cross-section to describe hadronization and like structure functions are subjects for measurement. The single particle cross-section can be written

$$\frac{d^3\sigma^h}{dx dy dz} = \frac{G_F^2 s}{2\pi} \times (\text{Structure Function}) \times D_i^h(z) \quad (1.13)$$

where $D_i^h(z)$ is the fragmentation function and is the probability for producing hadron h from parton i with longitudinal momentum fraction z (thus neglecting P_t and ϕ). This function is naively expected to scale, but in practice it is found that fragmentation cannot be separated from the earlier current-parton interaction and is found to depend on the interaction scale. The fragmentation functions are hence expressed in the form $D_i^h(z, Q^2)$ which is scale violating.

By invoking isospin and charge conjugation invariance, relations between the various functions are found thus:-

$$\begin{aligned}
D_u^{\pi^+} &= D_d^{\pi^-} = D_{\bar{d}}^{\pi^+} = D_{\bar{u}}^{\pi^-} \\
D_d^{\pi^+} &= D_u^{\pi^-} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-} \\
D_s^{\pi^+} &= D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}
\end{aligned} \tag{1.14}$$

where heavy quark production has been ignored.

In neutrino scattering (putting $\theta_c = 0$ and ignoring charm) the single particle cross-section is

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz} (\nu P \rightarrow \mu^- h \dots) = \frac{d(x)D_u^h(z) + 1/3u(x)D_d^h(z)}{d(x) + 1/3u(x)} \tag{1.15}$$

but with the above relations simplifies to

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma}{dz} (\nu P \rightarrow \mu^- \pi^\pm \dots) = D_u^{\pi^\pm}(z) \tag{1.16}$$

and allows direct measurement.

1.7 HADRONIZATION MODELS

Our inability to calculate explicitly the hadronization process has led to a proliferation of models to explain it, each with varying degrees of success. Two such models will be discussed briefly.

1.7.1 The Cascade Model

Among the first models was that proposed by Feynman and Field [1.9] which follows a cascade picture (figure 1.4). The struck quark 'a' stretches a colour field between itself and the spectator diquark system, $q_1 q_2$, and from which quark - antiquark pairs are produced. The primary quark 'a' combines with a secondary antiquark ' $\bar{b}$ ' to produce a rank 1 meson ' $(a\bar{b})$ '. The remaining 'b' quark combines with a tertiary antiquark ' $\bar{c}$ ' to produce a rank 2 meson ' $(b\bar{c})$ ' and so on. The cascade finishes when a quark combines with the diquark system in the backward hemisphere. This occurs when little energy is left in the colour field and conserves baryon number. Clearly, there is a

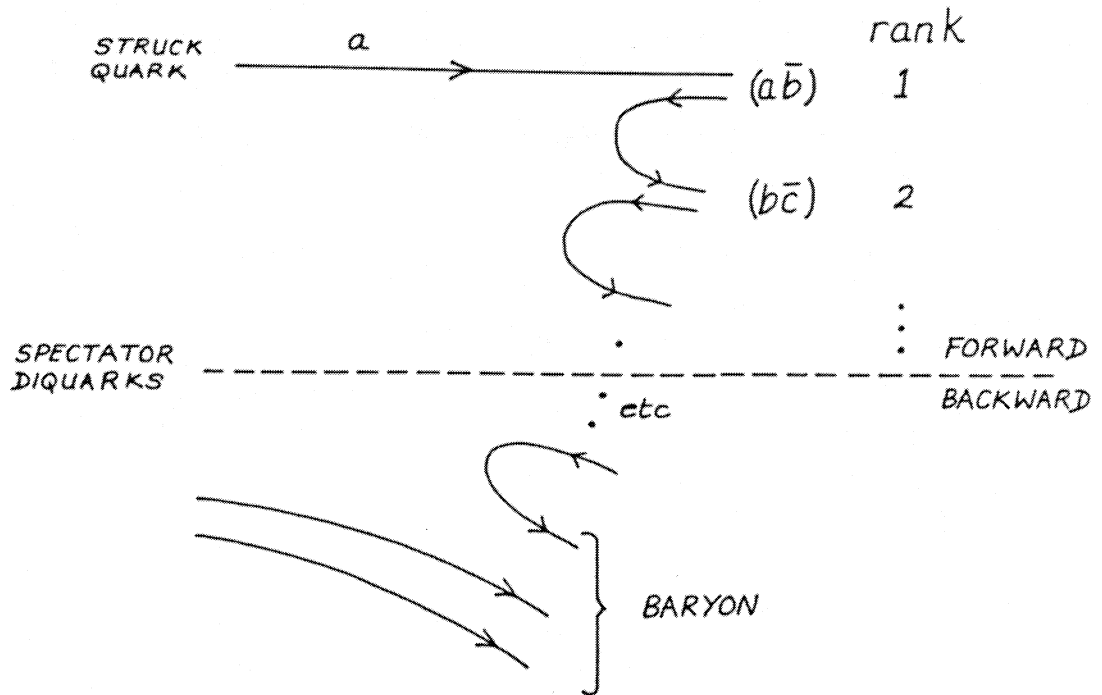


Figure 1.4: The cascade picture of fragmentation

link-up between forward and backward hemispheres and therefore a "leakage" of quantum numbers between the two. The probability of a meson to have a certain momentum is based on the ratio of meson and quark momenta with the consequence that the resulting z distributions are independent of the initial quark momenta. The probability of U,D,S,C etc. quark pair production in the colour field is fixed empirically as is the ratio of pseudoscalar and vector mesons produced. All resonances and π^0 's are decayed. Quark pairs are produced with equal and opposite transverse momenta according to a Gaussian distribution.

Whilst no direct comparisons are made in this thesis with this model, it is still helpful in understanding some of the processes involved.

1.7.2 The Lund Model

This model (owing to a group at the University of Lund, Sweden) has gained much popularity in recent years and is a logical extension of the cascade picture, putting it on a more formal footing. The model will only briefly be reviewed here, more thorough treatments are to be found in references 1.10, 1.11 and 1.12 .

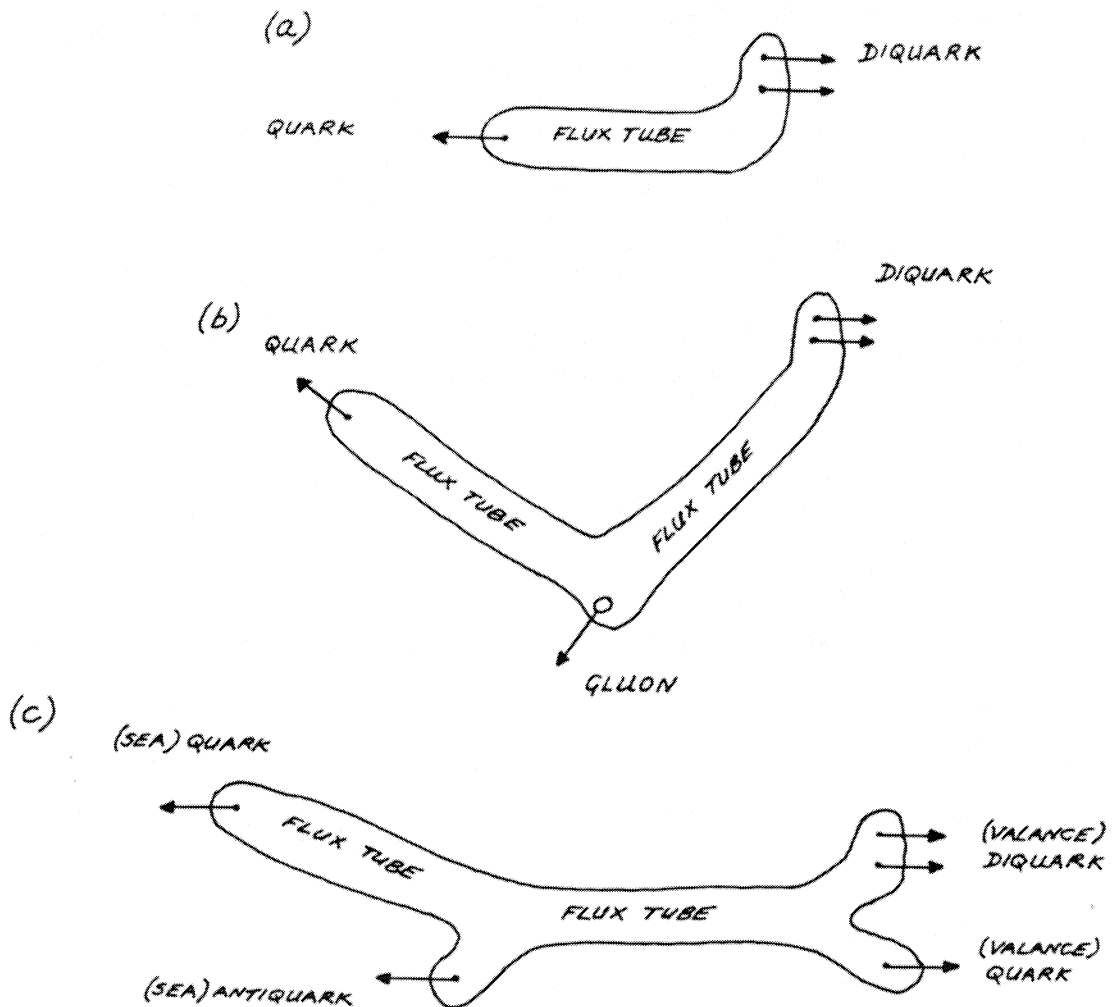


Figure 1.5: Quark and diquark systems connected by a QCD string

In the Lund model the QCD colour field between separating quark and diquark systems after scattering is confined to a flux tube (or string) stretched between them (figure 1.5a). As the two systems separate kinetic energy of the quarks is converted into potential energy in the extending string with a string tension (κ) of ~ 1 GeV/fm. A probability that the string will break yielding a $Q\bar{Q}$ pair is introduced and gives rise to 2 strings which may further divide. In this way all final state hadrons can be produced. Classically, however, the $Q\bar{Q}$ pair cannot be created at a point but instead at a finite separation along the string. The energy in the string between the pair provides their mass and transverse momentum. This can be treated as a tunnelling phenomenon with a probability proportional to

$$\exp(-\pi m_t^2/\kappa) = \exp(-\pi m^2/\kappa) \exp(-\pi P_t^2/\kappa) \quad (1.17)$$

where κ = string tension

m = quark mass

P_t = transverse momentum of quark to struck quark direction

$m_t = \sqrt{(m^2 + P_t^2)}$.

This leads to a Gaussian P_t spectrum which is flavour independent. The P_t is locally compensated between the two produced quarks as the string itself does not have any P_t . This formalism naturally suppresses heavy flavour production in the ratio $u:d:s:c = 1:1:0.3:10^{-11}$ and hence charm quarks can be neglected during fragmentation.

Gluon radiation by the struck quark is modelled by a kink in the string (i.e. a transverse excitation). A $q(qq)g$ event is simulated by stretching the string from q to g to qq (figure 1.5b) The only parameter in gluon generation is the strong coupling constant (α_s) which fixes the production rate. Fragmentation then proceeds with the strings independently in a similar manner as for one string.

Scattering from the sea is dealt with by assuming the struck sea quark and antiquark stretch out strings from the nucleon. The nucleon is taken to split into a quark - diquark system and all strings are then fragmented (figure 1.5c).

Diquark systems are dealt with by splitting them into primary and secondary quarks with a string stretched between them which also fragments. The secondary quark is connected to both the struck and primary quarks and so always picks up 2 quarks during fragmentation to become a baryon in the final state. Other baryon - antibaryon pairs may be created by producing diquark - antidiquark pairs when a string breaks (so conserving baryon number). The probability for this process against single quark pair production is fixed empirically to match available data.

A Monte Carlo program incorporating this fragmentation scheme is described in Appendix I and used in this thesis.

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Chapter II

EXPERIMENTAL DETAILS

2.1 INTRODUCTION

Neutrinos are weakly interacting particles and as such typically have a cross-section of the order of 10^{-38} cm² at 1 GeV/c which rises linearly with energy. This small cross-section dictates the need for a large mass target and a high neutrino flux at high energy. These conditions are largely met with the Big European Bubble Chamber (BEBC) filled with a heavy liquid and placed in the wide band neutrino beam at CERN. This filling gives higher gamma conversion and neutral particle detection efficiencies when compared with liquid hydrogen. Approximately 90% of event energy is observed. Energy correction methods were applied to estimate the neutrino energy and these will be discussed later.

2.2 THE CERN WIDE-BAND NEUTRINO BEAM

A wide-band neutrino beam does not use any momentum selection (unlike a narrow-band beam) and so gives the maximum possible neutrino flux over a large energy range. The CERN wide-band beamline is shown schematically in figure 2.1 and is discussed in reference 2.1.

For this experiment a 3 ms burst of some 10^{12} protons at an energy of 400 GeV/c was fired from the CERN Super Proton Synchrotron (SPS) every 10.8 seconds at a beryllium target of some 3 proton interaction lengths and a diameter comparable with the beam diameter (to minimize absorption loss of produced pions and kaons). The chief neutrino production mechanism was from the decays $\pi \rightarrow \mu + \nu$ and $K \rightarrow \mu + \nu$. Two magnetic horns [2.2], rotationally symmetric about the incident proton direction and having a large angular and momentum acceptance were used

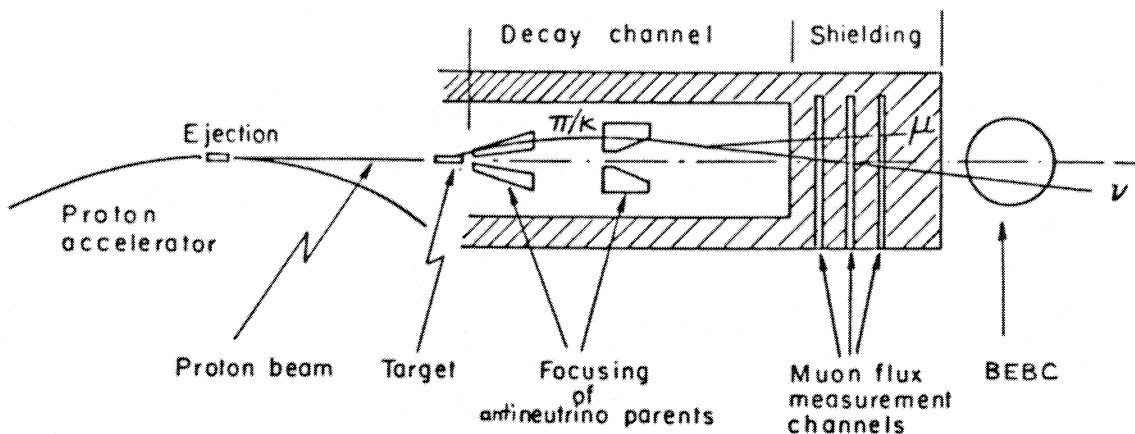


Figure 2.1: Schematic layout of the CERN wide-band neutrino beamline

to select and collimate the secondary π 's and K's, the neutrino parents. Switching from neutrino to antineutrino was accomplished by changing the sign of the horn magnetic field to select opposite sign parents. Particles of opposite charge were defocussed by the horn except for those travelling along the horn axis which met a beam stopper.

The focussed particles passed into a decay tunnel of some 290 metres and gave rise to a neutrino beam composed as shown in table 2.1. The tunnel was followed by 185 metres of iron shield to absorb remaining hadrons and charged leptons. The shield was broken in seven places by sets of concentric rings of solid state detectors which sampled the muon flux from pion and kaon decays [2.3]. Data from these detectors was recorded by a NORD computer. This information, when combined with data from the calibration detector arrays and nuclear emulsion exposures made it possible to estimate the neutrino flux through BEBC to an accuracy of about 10%. Following the iron shield was 173 metres of earth shield after which the beam emerged from the soil and traversed 50 metres of air before entering BEBC.

TABLE 2.1					
Typical Wide-band Neutrino Beam Composition					
Parent	% ¹	Channel	Branch ²		
π^+	~80 (83)	$\mu^+ + \nu_\mu$	100.0	Focussed	Right Sign Neutrinos
K^+	~12 (4)	$\mu^+ + \nu_\mu$	63.6		
		$\pi^0 + \mu^+ + \nu_\mu$	3.2		
		$\pi^0 + e^+ + \nu_e$	4.8		
μ^+	~10 (10)	$e^+ + \bar{\nu}_\mu + \nu_e$	100.0		
K^0	~3 (7)	$\pi^- + e^+ + \nu_e$	39.0	Unfocussed	
		$\pi^- + \mu^+ + \nu_\mu$	27.1		
$\bar{K}^0$	~3 (7)	$\pi^+ + e^- + \bar{\nu}_e$		Unfocussed	
		$\pi^+ + \mu^- + \bar{\nu}_\mu$			
π^-		$\mu^- + \bar{\nu}_\mu$		Defocussed	Wrong Sign Neutrinos
K^-		$\mu^- + \bar{\nu}_\mu$			
		$\pi^0 + \mu^- + \bar{\nu}_\mu$			
		$\pi^0 + e^- + \bar{\nu}_e$			
μ^-		$e^- + \nu_\mu + \bar{\nu}_e$			
¹ percentage in parent beam for positive (negative) particles ² approximate branching ratio					

The relevant decay modes of the π and K are isotropic in their respective rest frames. Using relativistic kinematics and a Lorentz boost to the laboratory frame it is found that a neutrino can maximally take 43% of the original pion energy or 95% of the original

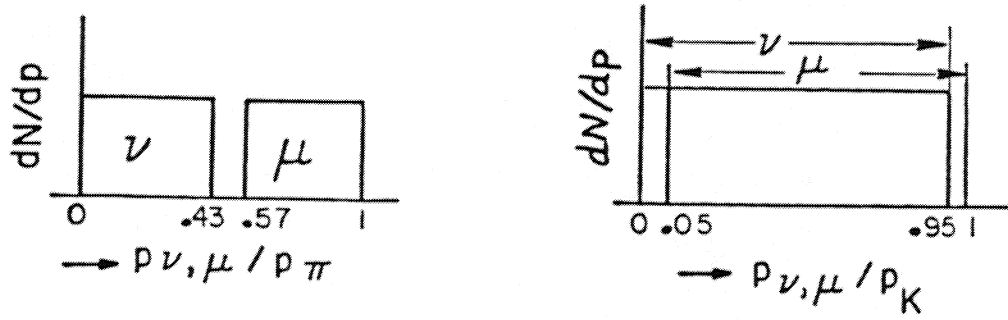


Figure 2.2: Decay probability for π or K into μ and ν

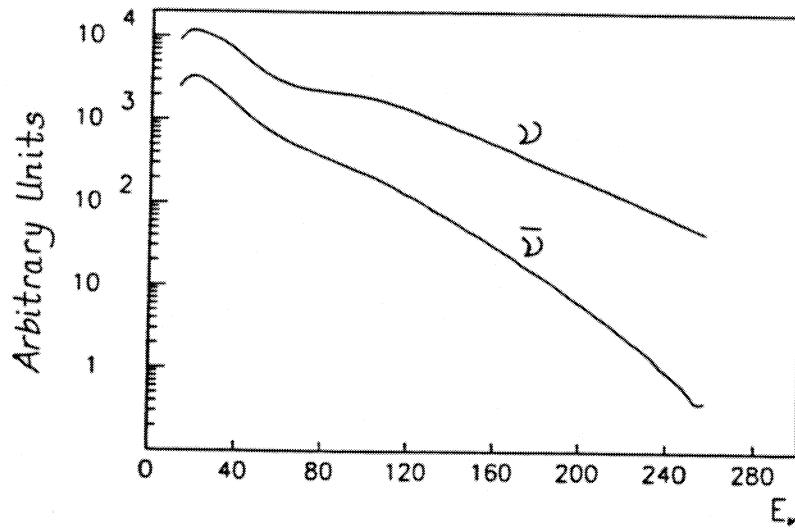


Figure 2.3: Typical neutrino energy spectrum from a wide-band beam

kaon energy, the probability distribution is shown in figure 2.2. Since the pions and kaons were produced with roughly the same energy spectrum the highest part of the neutrino energy spectrum arose from kaon decay. The spectrum is shown in figure 2.3

2.3 THE BIG EUROPEAN BUBBLE CHAMBER (BEBC)

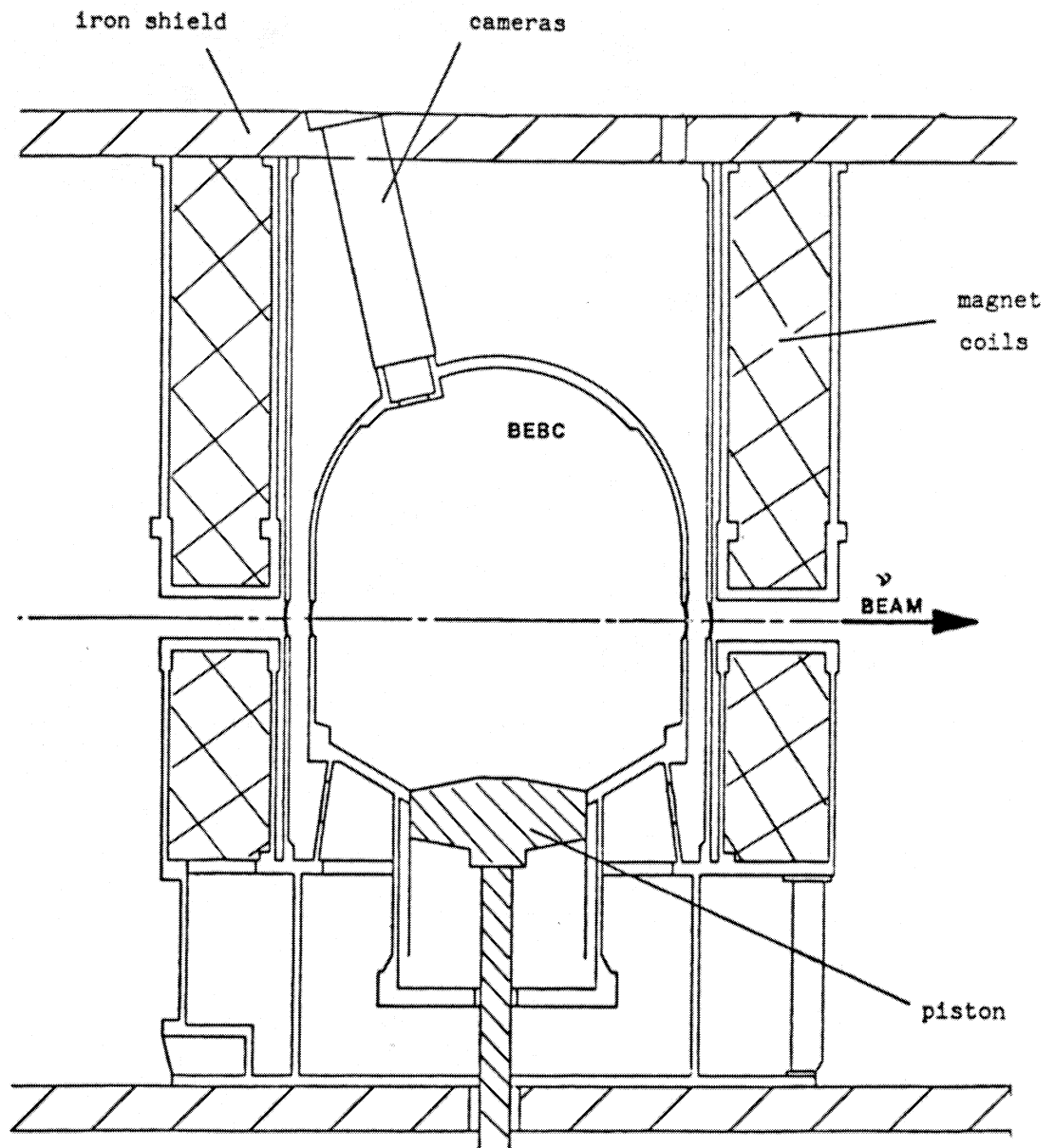
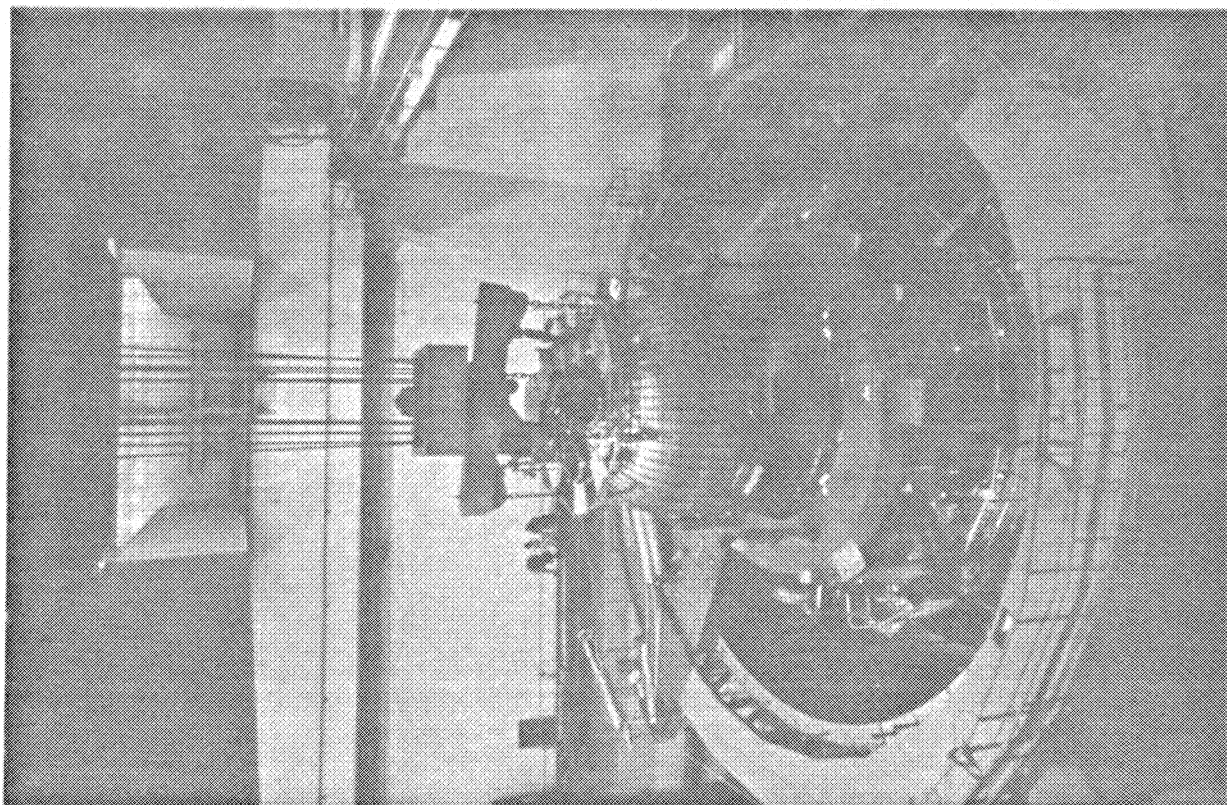
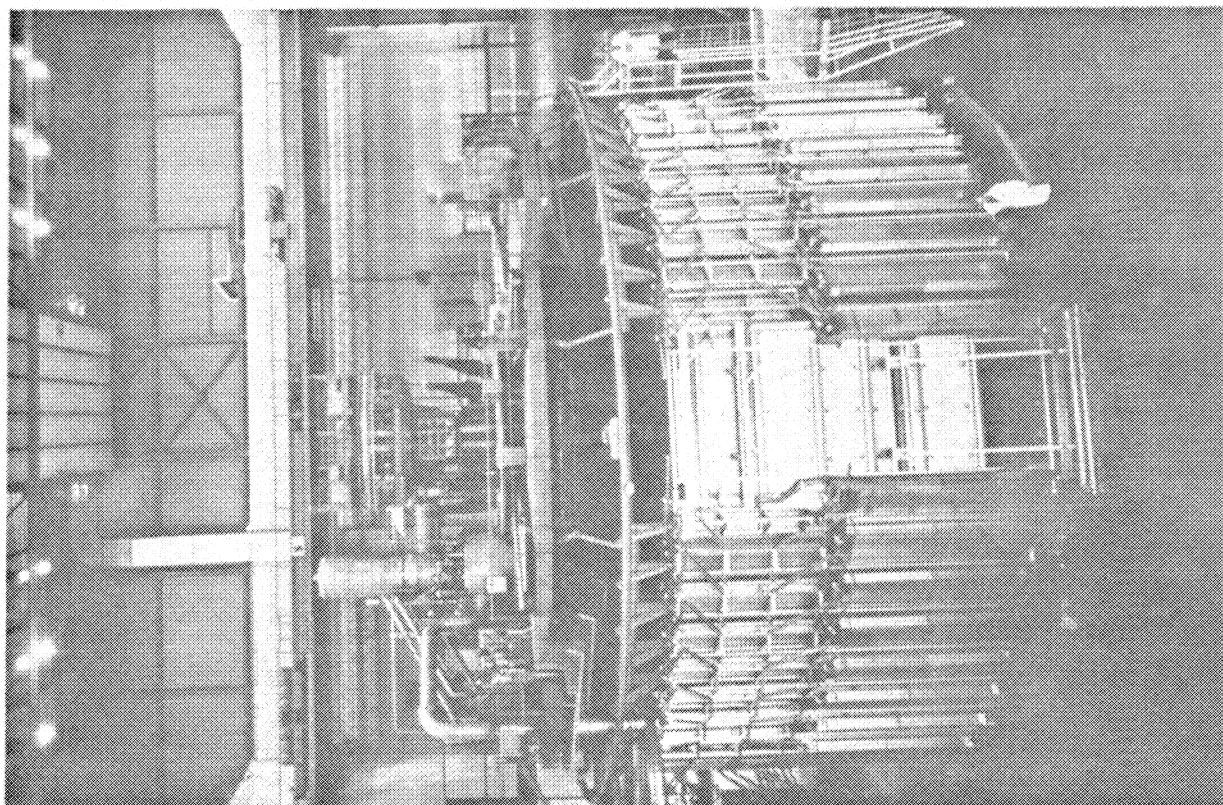
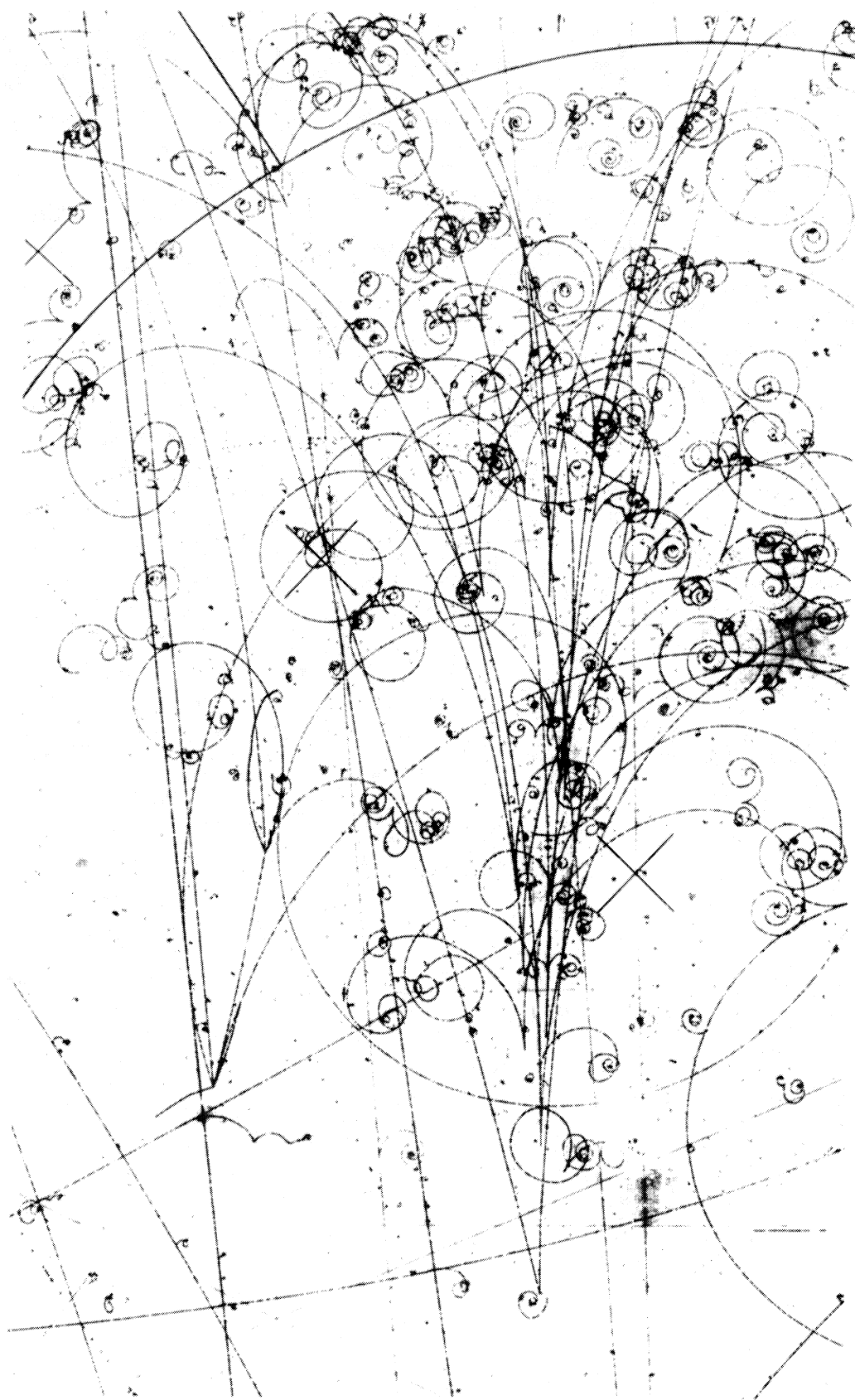


Figure 2.4: Side view of BEBC

BEBC was a cryogenic bubble chamber of approximate volume 38 m^3 [2.4]. The stainless steel pressure vessel was cylindrical in shape with a





diameter of 3.7 metres, a height of 2.0 metres and was capped by a dome of radius 1.85 metres (figure 2.4 & colour plate). Particle trajectories from interactions in the chamber were recorded on film using four cameras mounted on top of the chamber. Fish-eye optics were employed for the greatest coverage of the chamber volume. A fifth camera was also used with high resolution optics to photograph the central region of the chamber with a reduced depth of field. A typical antineutrino induced event is shown in the monochrome plate.

The chamber was filled with an approximately 75% molar liquid neon/hydrogen mixture of density 0.7 g/cm^3 which resulted in a hadron absorption length of approximately 1.5 m and a radiation length of 42 cm. This gave an improved energy resolution compared with a pure hydrogen filling, since a greater percentage of neutral hadrons and gamma rays were seen in the chamber. The chamber was operated at a temperature of 29.28 K and the camera flash occurred 8 ms after the beam spill giving a typical bubble size of 300 to 400 microns and density of 5 to 10 bubbles per cm. A picture was recorded by each camera for each beam spill. At the film scanning stage frames which did not conform to scan rules (see section 2.5) were rejected.

The bubble chamber was surrounded by a superconducting magnet operated at a nominal current of 5700 A which gave a uniform vertical field of some 3.5 Tesla throughout the chamber volume. To reduce stray field to a tolerable level and to provide a magnetic return yoke the whole assembly of chamber and magnet was enclosed by an iron shield. This high field enabled momentum measurements to be made for all tracks of reasonable length.

2.4 THE EXTERNAL MUON IDENTIFIER

Charged current (CC), neutral current (NC) and neutral hadron induced (from interactions prior to BEBC) events were separated by identifying all muons in events. This was done using an External Muon Identifier (EMI) consisting of two planes of multiwire proportional chambers.

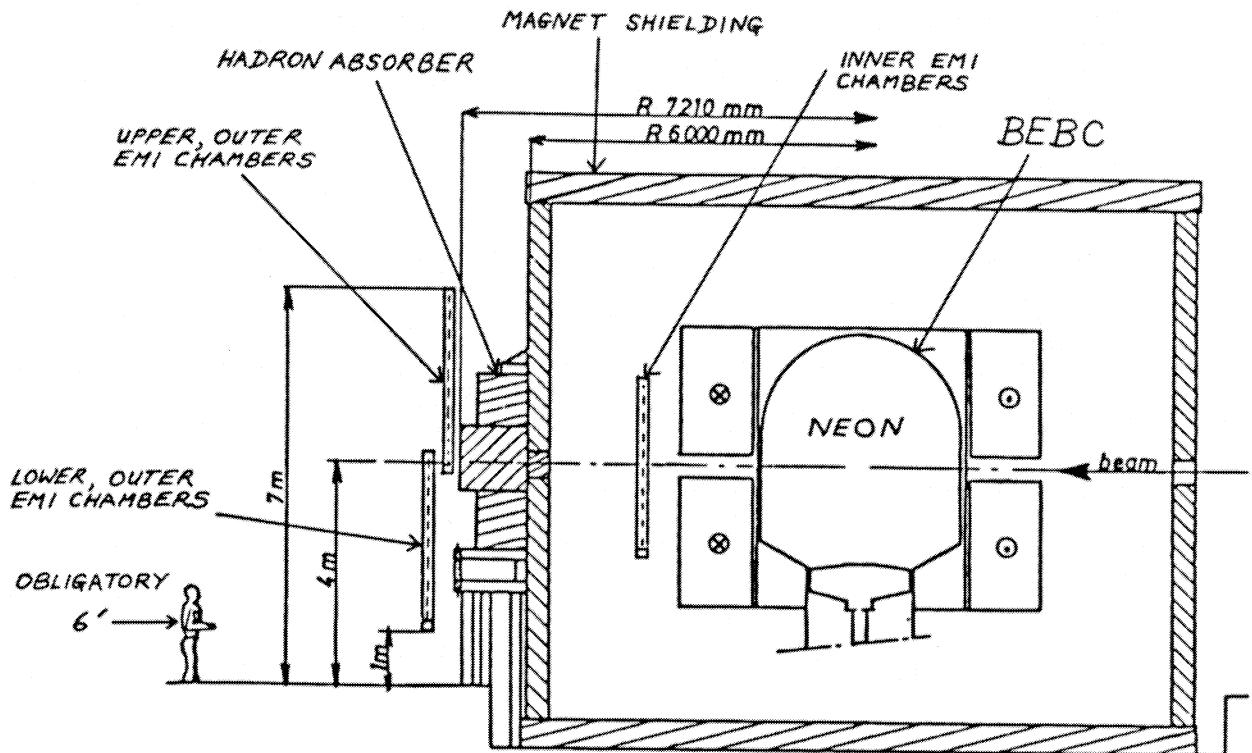


Figure 2.5: The External Muon Identifier and BEBC

The EMI was located just downstream of BEBC (figure 2.5), the whole assembly of BEBC and EMI is depicted in a colour plate. Muons were identified by demanding time coincidence and spatial correlation of signals from both planes. A much fuller discussion of the EMI, construction, operation and evaluation of acceptances and background is to be found in chapter III.

2.5 DATA REDUCTION

The data for this experiment (CERN code WA59) was taken in 1980 and totals approximately 36,300 neutrino and 61,000 antineutrino pictures. All frames of film were double scanned and all events found accepted as charged current candidates if they had:-

1. 2 or more prongs or 1 prong plus associated V^0 's or $\bar{\chi}$'s, and
2. 1 or more forward leaving tracks of positive or negative charge with curvature > 50 cm and length > 10 cm on two or more views.

Actual CC identification was only done after merging with EMI information. Such events were measured on all four views where possible and kept provided there was at least one right sign (i.e. negative for neutrino and positive for antineutrino) leaving track above 3 GeV/c, the vertex was within a fiducial volume defined by

$$\begin{aligned} R &< 155 \text{ cm} \\ |Z| &< 115 \text{ cm} \\ |Z| &< 235 - R \end{aligned} \tag{2.1}$$

(where $R^2 = (X+25)^2 + Y^2$ and X,Y,Z are the vertex co-ordinates, with origin at the centre of BEBC) or if the vertex was considered to be an N^* or V^0 possibly associated to another vertex on the frame. The high resolution view was not used.

2.6 GEOMETRICAL RECONSTRUCTION

To reconstruct the 4-vectors of all tracks of an event, each of the 4 views was projected in turn onto a calibrated measuring table and digitised. These measurements were fed into a geometrical reconstruction program (called GEOMETRY [2.6]) along with accurate constants describing the BEBC optics and magnetic field. The momentum of each track was determined from the curvature and the particle charge was deduced from the sign of the magnetic field. Various characteristics such as stopping tracks or decays in flight were used

to force particle identification. The measured rate of change of momentum was used to fit each mass hypothesis (pion or proton) in turn and the best fit was selected. Above about 1 GeV/c the fits were insensitive to mass hypothesis and by default a pion mass was assumed. 2-prong vertices on a frame were fitted for K^0 's, Λ^0 's and $\bar{\Lambda}^0$'s.

The results of this computation were saved in a format suitable for subsequent processing using the HYDRA memory management system [2.5]. This allowed the natural organization of data into banks of information linked into a tree structure in which (loosely speaking) nodes of the tree correspond to vertices and branches to tracks.

2.7 DATA PROCESSING

A program called EMI-PRODUCTION [2.7] was run after the geometry stage to identify all muon candidates. All tracks leaving BEBC, of both charges and with momentum greater than 3 GeV/c were extrapolated from BEBC to the EMI taking account of the magnetic field and intermediate material. For tracks which were predicted to have reached the EMI, information from the EMI chambers was scanned for acceptable hits. Muon candidates were accepted by another program (called EMI-DECIDE) if there was a time coincidence of hits which gave acceptable association and correlation χ^2 in both EMI planes. A more detailed description of the EMI and its use is given in chapter III.

A program (called SYNTHESIS) was used to connect neutron vertices to the primary vertex. All vertices on a frame were taken in reverse order to the beam direction. For each vertex, the best association to a previous vertex (which was not allowed to be that of a V^0 or $\bar{\Lambda}^0$) was found by taking the smallest P_t normalized by the number of secondary tracks. P_t was defined as the P_t of the secondary to the connecting line of flight. Each event was considered with and without the best neutral connection (if any) and checked for change in event P_t balance and lepton - hadron shower opening angle. The neutral vertex was

rejected if the change in $P_t < 200$ MeV/c or the change in lepton - hadron angle exceeded 10 degrees. Neutron vertices closer than 1cm were automatically connected. All tracks with momentum error exceeding 60% were set to have undefined charge. Some tracks seen to interact in BEBC had bad errors on momentum measurement (usually because of their short length) and in such cases it was often possible to use outgoing tracks from the secondary interaction to improve the momentum. Identified protons (e.g. stopping) of track length less than 4 cm were discarded from events on the basis of being low energy fragments from re-interactions in the neon nucleus.

Gamma tracks of length greater than 80 cm and momentum less than 1 GeV/c were also dropped. A large number of bremsstrahlung γ 's (brems) were measured which showed up as a large excess at low effective $\gamma\gamma$ mass. For each pair of γ 's (γ_1 being further from the interaction vertex than γ_2) a brem candidate was selected if :

1. $P_1 < P_2$ ($P = 3$ -momentum), and
2. $M_h < 0.01$, and
3. $M_v < 0.05$

where M_h (horizontal mass) and M_v (vertical mass) were defined in analogy to $M_{\gamma\gamma} = 2 P_1 P_2 (1 - \cos\Delta\theta)$ using angular separation in phi and dip (as defined in BEBC) respectively in place of θ (the $\gamma\gamma$ opening angle). The cut values were decided by Monte Carlo study. All brems thus identified were added to their parent if $P_1 > 0.3 + P_2$.

Two further programs (QUACK and PLAGUE) were employed to check event quality. The former program was used to check the integrity of the HYDRA data-structure whilst the latter was often employed to point out physically interesting or erroneous event features as well as to produce statistics for checking (see appendix B). Unacceptable events were corrected if possible and re-passed through all or part of the program chain as necessary.

The results of this data processing was 9,840 neutrino and 17,019 antineutrino CC events with EMI identified muons above 3 GeV/c. NC events were not measured intentionally. Events which turned out to be non-CC were rejected.

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Chapter III

THE EXTERNAL MUON IDENTIFIER (EMI)

3.1 INTRODUCTION

The external muon identifier (EMI) was constructed for identification of muons from interactions occurring within BEBC and consisted of two planes of multiwire proportional chambers. The requirements for this detector were such that an area of over 150 m^2 had to be covered and consequently the sheer scale of the system dictated that it should be self-triggering. A beam spill of 3 ms meant that several hundred hits had to be recorded without loss and the detector had to have a high geometrical acceptance. Measurements using three co-ordinates were necessary to resolve multi-hit ambiguities and a high counting efficiency had to be attained for reliable point reconstruction. The detector had to operate within the requirements of the BEBC hydrogen safety zone and had to be able to withstand the high stray magnetic field near BEBC.

In this chapter the construction and operation of the EMI and associated electronics are surveyed. The geometrical and electronic acceptances are discussed as is the reliability of muon identification using the EMI. Sources of background are evaluated and possible methods to account for them are presented.

3.2 THE EMI

3.2.1 Construction

A single plane EMI had unacceptably high backgrounds from muons produced in matter prior to BEBC in the high flux wide-band beam and so a two plane EMI consisting of 55 identical multiwire proportional chambers, each 3 m by 1 m, was used [3.1]. The outer plane of 47

chambers covered some 145 m^2 at $\sim 7.6 \text{ m}$ from the BEBC centre, whilst the inner plane of 8 chambers covered

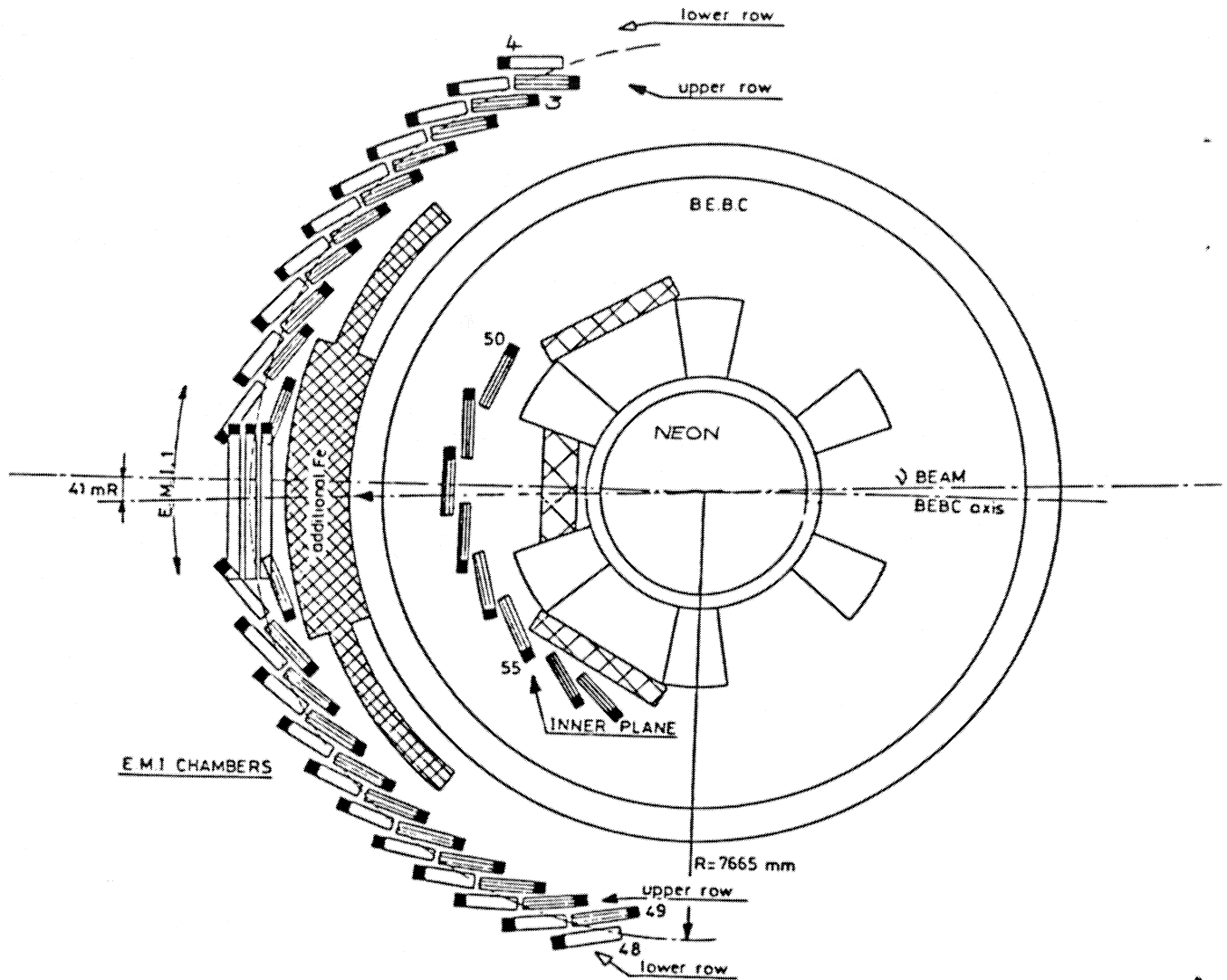


Figure 3.1: Layout of EMI, BEBC and hadron absorber

some 24 m^2 and was at $\sim 4 \text{ m}$ from the centre (figure 3.1). To reduce hadron punch-through greatly, iron shielding was placed between BEBC and the inner plane and again outside the BEBC outer casing in the

forward region where most fast hadrons were expected (figure 3.1). The solid-angle covered by the outer plane with respect to the BEBC centre was $\pm 80^\circ$ horizontally and $\pm 20^\circ$ vertically whilst the inner plane stretched from -43° to $+35^\circ$ horizontally and $\pm 18^\circ$ vertically. The BEBC magnet was operated such that right sign muons were swept to the side with the greater inner plane coverage.

All the chambers were identically constructed [3.2], each chamber consisting of two anode wire planes at $\pm 30^\circ$ to the shorter side and two cathode strip planes comprised of a vertical set split into two and a horizontal set not read out (figure 3.3). The structural framework supporting the anode planes is formed from two NOMEX honeycomb sandwich panels placed back to back and separated by a thin aluminium honeycomb sandwich (figure 3.4). The cathode strips were formed from silver painted strips on the inside of the NOMEX panels and were DC coupled to ground. The anode planes were capacitively read out. The chambers were run on a 70%/30% argon/carbon dioxide mixture which was ideal for use in the hydrogen safety zone around BEBC. This gave a sufficient operating plateau such that the chambers could be run satisfactorily at 3 KV.

The chambers were mounted in the outer plane as shown in figure 3.2 with effective channel spacing decreasing from 8 mm in the forward region through 16 mm to 32 mm in the extremes of the plane by grouping wires into 2 (A), 4 (B) and 8 (C) per channel respectively. This gave a spatial resolution varying from ± 0.5 cm to ± 3.0 cm. In the inner plane the lowest resolution chambers were used throughout.

For reconstruction purposes the whole EMI structure was accurately surveyed using standard techniques, the results being used subsequently in the EMI-PRODUCTION program.

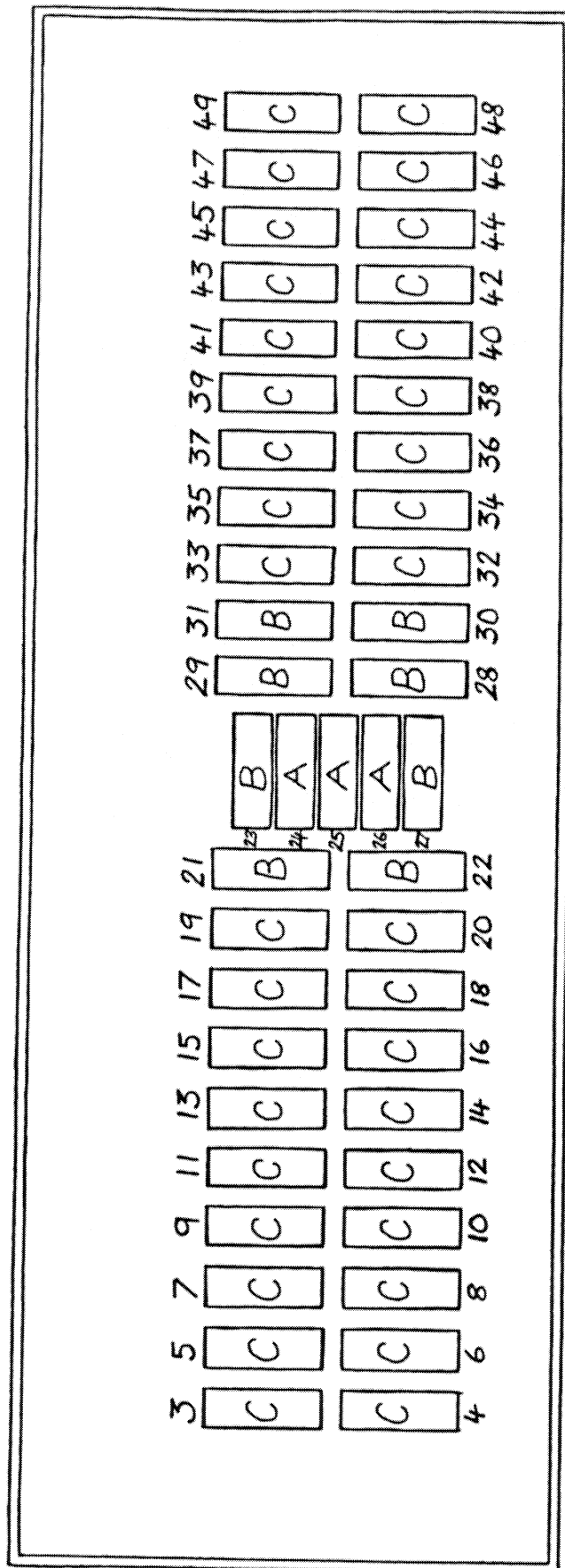


Figure 3.2: Outer plane chambers with differing spatial resolutions

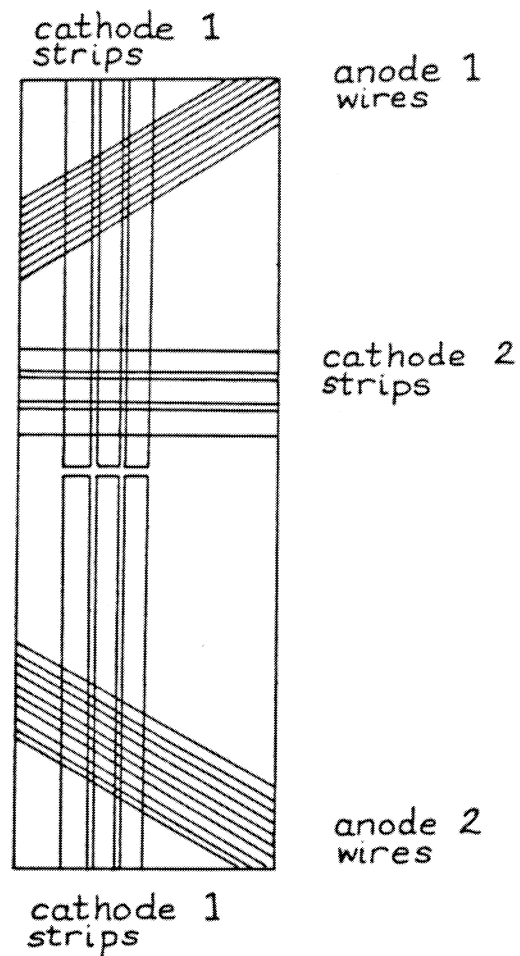


Figure 3.3: EMI chamber anodes and cathodes

3.2.2 Electronics and Read-out

The design requirements made it impossible for direct read-out of hits and so a system of buffers was employed to store data until the end of a beam spill when it was processed. During a beam spill, time slots (intervals) of about 250 ns were defined during which hits were recorded in a two level system of MOS shift register buffers. The first level was a buffer for each chamber with one input per channel and space for 39 hits in each channel (79 in regions of high hit

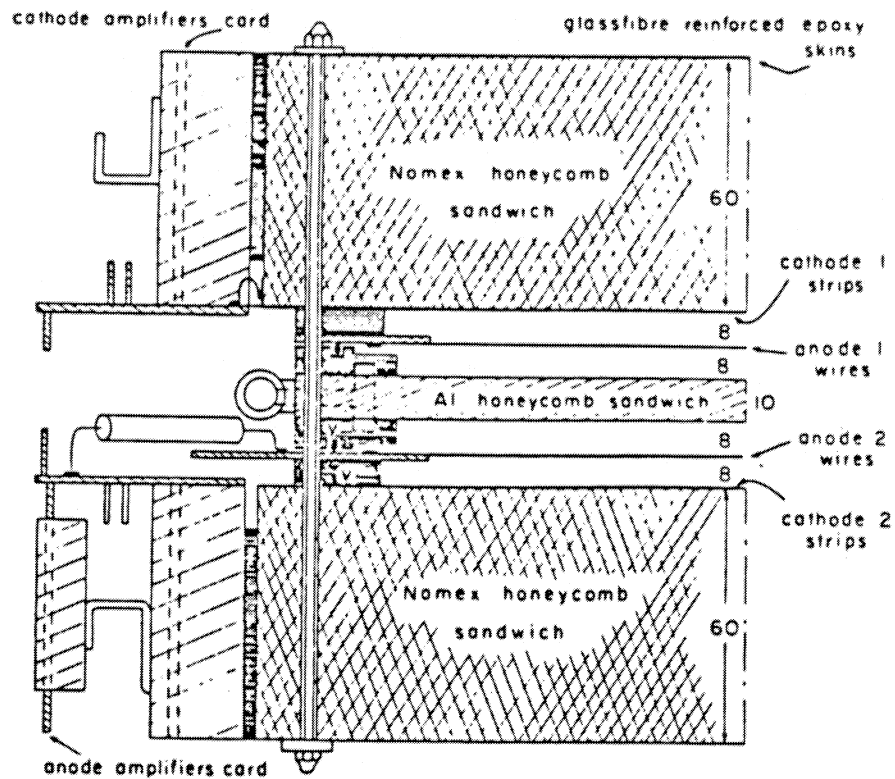


Figure 3.4: EMI chamber structural framework

rate). The second level was a main buffer with one input for each chamber, used to record chambers hit in each time slot and space for a maximum of 1023 time slots. This is shown schematically in figure 3.5.

In any given time slot, a coincidence between two anode wire planes of a chamber caused the whole EMI to record all channels hit in the first level buffer system, but only hit chambers occupied buffer space. In a similar manner, all hit chambers were recorded in the main buffer. In this way storage of null data was greatly reduced and the amount of buffer space needed kept to a minimum.

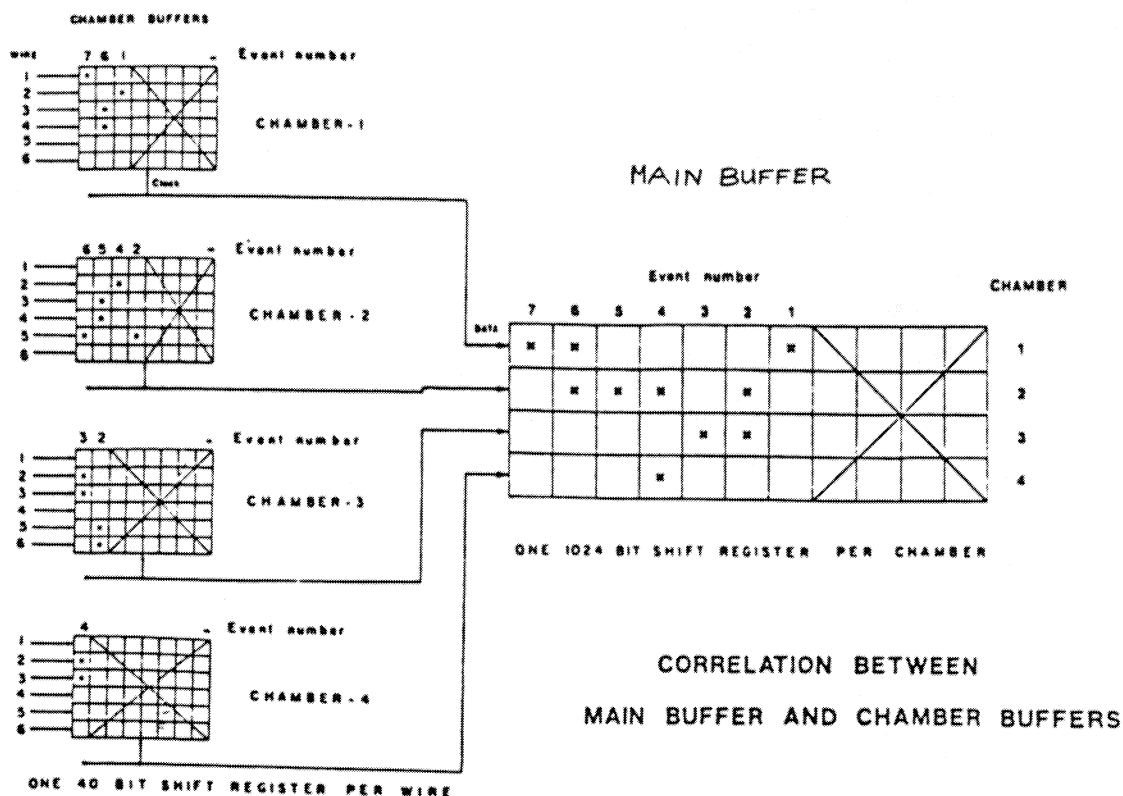


Figure 3.5: Usage of main and chamber buffers

Read-out was accomplished at the end of each beam spill via CAMAC interface to a NORD-10 computer. The main buffer was scanned in each recorded time slot for chambers showing hits. The hits were then read-out from the individual chamber buffers. This read-out method operated at 2 Mhz and gave a time resolution of some 250 ns to 300 ns, depending on event sequences. Finally, the data was formatted before being written onto magnetic tape.

3.2.3 Online Software and Operation

The EMI was controlled by software run on a NORD-10 computer in two different modes. Mode 1 (data taking mode) was made ready by an interrupt from the SPS at 600 ms before beam spill and started by a pre-pulse from the SPS at 50 μ s before spill. In the absence of other detectors to help in monitoring the EMI performance, a second mode (mode 2) of operation was automatically run in between beam spills to accumulate information from cosmic ray hits. This enabled the performance to be continually monitored and any faults reported to EMI operators, as well as recorded for future use during analysis. It was also possible to test EMI hardware electronically, under computer control.

3.3 EMI DATA

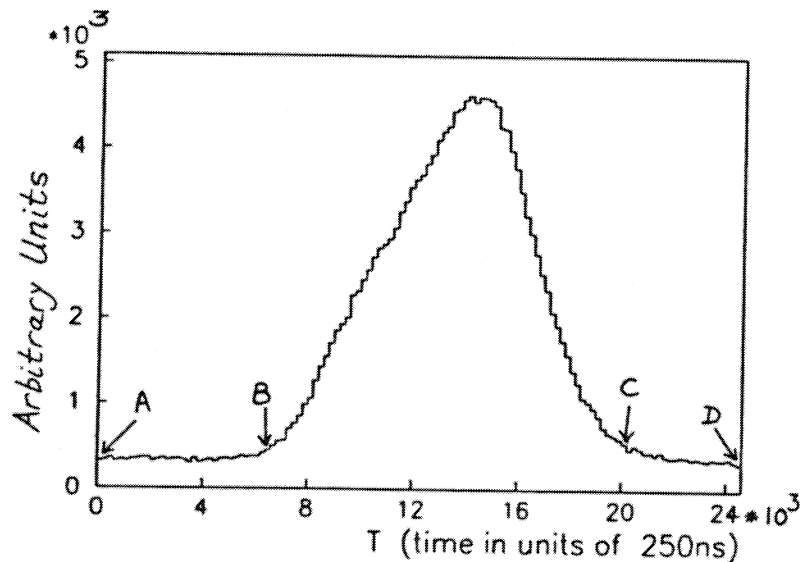


Figure 3.6: Density of hits before, during and after a beam spill

The data from the EMI was analysed separately to check performance. Figure 3.6 shows a typical distribution of the number of time slots

per spill containing hits averaged over some 500 frames. Between A and B or C and D cosmic rays are responsible for the hits, but at B the onset of the beam spill is clearly seen and lasts to C. Most of these hits come from beam associated muons from interactions in shielding prior to BEBC. The cosmic ray background component represents about 13.5% of hits during the beam spill.

Selecting only those hits during beam spill (B to C), figure 3.7a shows the distribution of hits amongst the outer plane chambers. The plot is normalized to the chamber with most hits. The number of hits in the inner plane is 8% of those in the outer plane and the muon flux in lower chambers of the outer plane is only 38% of that in the upper chambers. This is because beam associated muons were mostly swept (by the BEBC shielding magnet) over the top of BEBC and the inner plane, but caught the top of the outer plane. This sweeping was necessary to reduce through-going muon tracks in BEBC to a tolerable level. Figure 3.7b shows the distribution of hits in the outer plane after demanding a time coincidence between inner and outer planes and is normalized in the same way as before. It can be seen that a large proportion of the hits in chambers 31,33,35 and 37 in figure 3.7a are not in coincidence with the inner plane.

The time difference between successive recorded time slots (in units of 250 ns) (figure 3.8) shows an exponential shape. There are no entries in the first bin as hits arriving within 500 ns of each other are placed in the same time slot to avoid time slot boundary effects. Extrapolating back, the fraction of uncorrelated hits in any one time slot can be estimated to be ~10% of all hits.

It was vital that the EMI hardware was able to cope with this high flux of beam associated muons. The distribution of the numbers of time slots per beam spill (figure 3.9) has a mean of 385 and a maximum of 470, clearly showing the situation to be well within the design limit of 1023 imposed by the main buffer size.

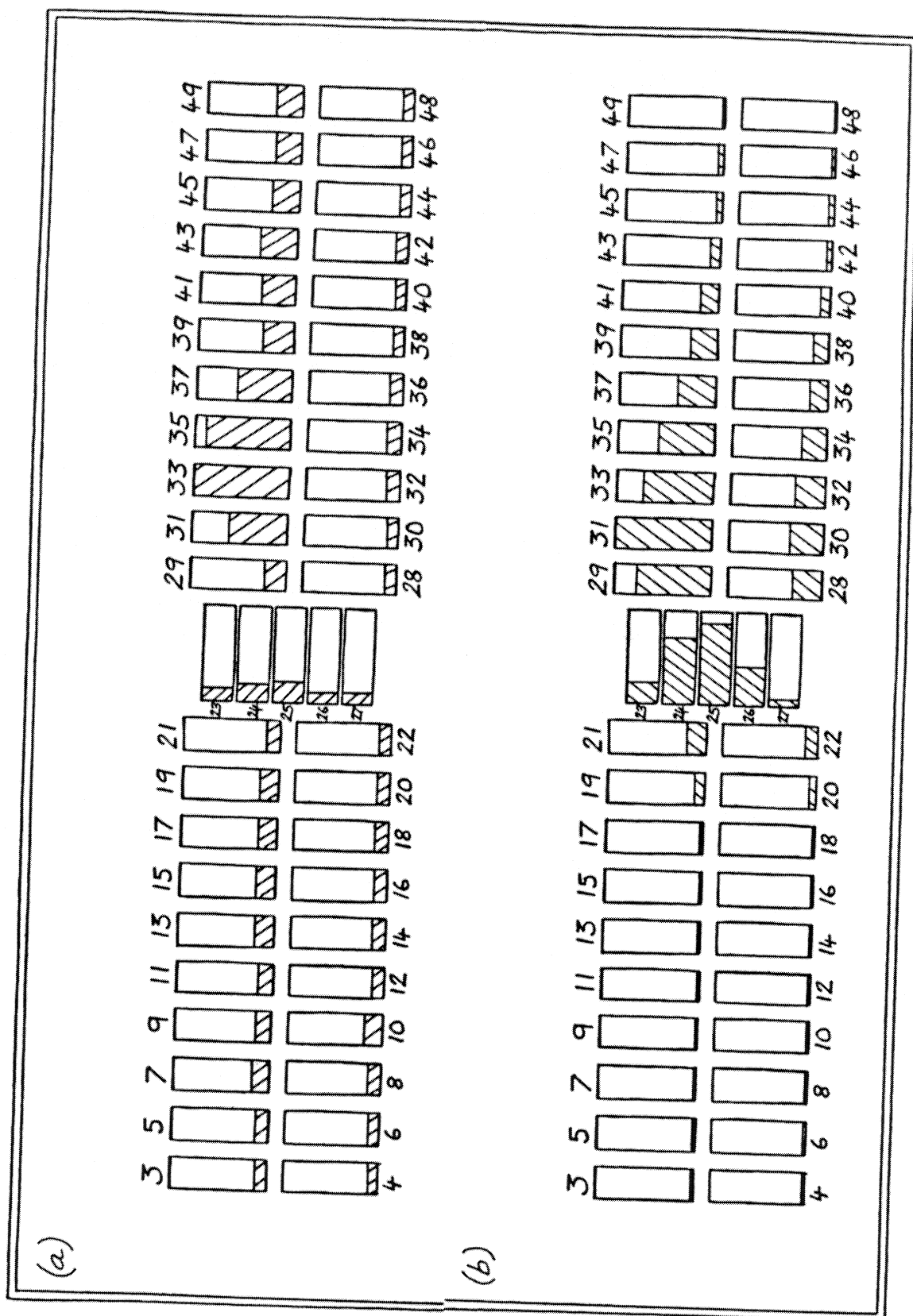


Figure 3.7: Distribution of hits amongst chambers

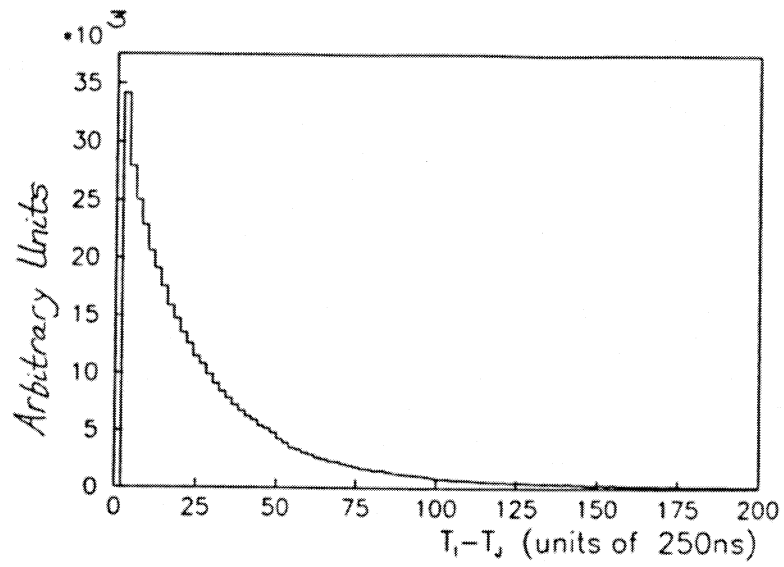


Figure 3.8: Time difference between successive time slots

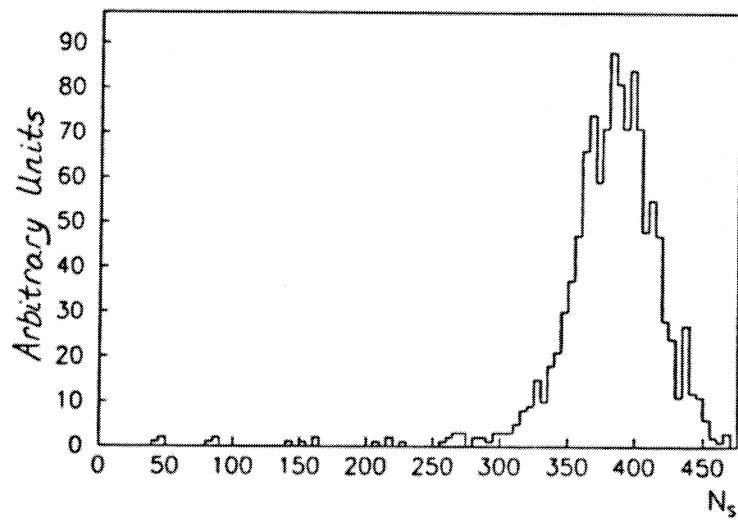


Figure 3.9: Distribution of number of time slots per beam spill

3.4 EMI ACCEPTANCES

TABLE 3.1							
EMI Geometrical Acceptances							
Neutrino (percentage)							
μ Mom. GeV/c	Sine of angle between μ and ν 0 - 0.1 - 0.2 - 0.3 - 0.4 - 0.5 - 1						ALL
3 - 4	90 $\pm$ 3	86 $\pm$ 3	74 $\pm$ 3	55 $\pm$ 3	43 $\pm$ 2	25 $\pm$ 1	60 $\pm$ 1
4 - 5	100 $\pm$ 3	97 $\pm$ 3	85 $\pm$ 3	64 $\pm$ 3	44 $\pm$ 2	32 $\pm$ 2	73 $\pm$ 1
5 - 6	100 $\pm$ 3	96 $\pm$ 3	88 $\pm$ 3	66 $\pm$ 3	47 $\pm$ 3	36 $\pm$ 5	79 $\pm$ 1
6 - 8	100 $\pm$ 2	98 $\pm$ 2	87 $\pm$ 2	72 $\pm$ 2	53 $\pm$ 3	60 $\pm$ 35	86 $\pm$ 1
8 - 10	100 $\pm$ 2	97 $\pm$ 2	89 $\pm$ 2	68 $\pm$ 3	56 $\pm$ 7	-	90 $\pm$ 1
10 - 15	100 $\pm$ 2	97 $\pm$ 2	88 $\pm$ 2	73 $\pm$ 3	-	-	94 $\pm$ 1
15 - 20	100 $\pm$ 2	97 $\pm$ 2	89 $\pm$ 3	71 $\pm$ 15	-	-	97 $\pm$ 1
20 - 25	100 $\pm$ 2	98 $\pm$ 2	90 $\pm$ 4	-	-	-	98 $\pm$ 1
25 - 30	100 $\pm$ 2	98 $\pm$ 3	90 $\pm$ 7	-	-	-	99 $\pm$ 2
30 - 40	100 $\pm$ 2	98 $\pm$ 2	89 $\pm$ 18	-	-	-	99 $\pm$ 1
40 - 50	100 $\pm$ 2	99 $\pm$ 3	-	-	-	-	100 $\pm$ 2
> 50	100 $\pm$ 1	99 $\pm$ 4	-	-	-	-	100 $\pm$ 1
ALL	100 $\pm$ 1	97 $\pm$ 1	87 $\pm$ 1	67 $\pm$ 1	47 $\pm$ 1	28 $\pm$ 1	90 $\pm$ 1
Antineutrino (percentage)							
μ Mom. GeV/c	Sine of angle between μ and ν 0 - 0.1 - 0.2 - 0.3 - 0.4 - 0.5 - 1						ALL
3 - 4	90 $\pm$ 3	88 $\pm$ 3	77 $\pm$ 3	58 $\pm$ 2	42 $\pm$ 2	25 $\pm$ 1	60 $\pm$ 1
4 - 5	100 $\pm$ 3	97 $\pm$ 3	84 $\pm$ 3	66 $\pm$ 3	42 $\pm$ 2	32 $\pm$ 2	74 $\pm$ 1
5 - 6	100 $\pm$ 3	95 $\pm$ 3	87 $\pm$ 3	70 $\pm$ 3	51 $\pm$ 3	34 $\pm$ 6	81 $\pm$ 1
6 - 8	100 $\pm$ 2	96 $\pm$ 2	87 $\pm$ 2	71 $\pm$ 2	55 $\pm$ 3	38 $\pm$ 22	86 $\pm$ 1
8 - 10	100 $\pm$ 2	97 $\pm$ 2	86 $\pm$ 2	71 $\pm$ 2	59 $\pm$ 9	-	91 $\pm$ 1
10 - 15	100 $\pm$ 1	97 $\pm$ 1	88 $\pm$ 2	75 $\pm$ 3	-	-	95 $\pm$ 1
15 - 20	100 $\pm$ 2	98 $\pm$ 2	89 $\pm$ 3	-	-	-	98 $\pm$ 1
20 - 25	100 $\pm$ 2	98 $\pm$ 2	91 $\pm$ 5	-	-	-	99 $\pm$ 1
25 - 30	100 $\pm$ 2	98 $\pm$ 3	84 $\pm$ 12	-	-	-	99 $\pm$ 2
30 - 40	100 $\pm$ 2	98 $\pm$ 3	-	-	-	-	99 $\pm$ 2
40 - 50	100 $\pm$ 3	99 $\pm$ 5	-	-	-	-	100 $\pm$ 2
> 50	100 $\pm$ 2	98 $\pm$ 7	-	-	-	-	100 $\pm$ 2
ALL	100 $\pm$ 1	97 $\pm$ 1	86 $\pm$ 1	69 $\pm$ 1	47 $\pm$ 1	27 $\pm$ 1	89 $\pm$ 1

During the run, wires which were found not to have any cosmics hits during mode 2 monitoring were noted by the NORD computer and flagged as "dead wires". Offline checking later revealed that whilst the odd

wire here and there was dead (sometimes for only a short while) a few small sections of the EMI were dead for the whole run. Fortunately, these sections were not in the most densely hit areas where losses could have been large. To correct all analyses for the lost CC events a set of "substitute" events was generated using events with muons passing through neighbouring (live) regions of the EMI of surface area equal to the dead regions. This ensured (to first order) the correct number of substitutes. These events were often used in addition to the data. I am grateful to A.Sansum for supplying these substitute events. An overall electronic EMI efficiency of 98% was estimated, excluding dead regions and accounts for occasional faulty channel electronics, gas pressure drops, etc.

Table 3.1 shows the geometrical acceptance as a function of muon momentum and angle between muon and neutrino. The acceptance was calculated using a Monte Carlo first written for the BEBC neutrino - hydrogen experiment by G.Corrigan and set up for this experiment by K.Varvell (and to whom I am grateful for supplying the table). The acceptance falls off rapidly for increasing angle between muon and neutrino and for higher muon momenta no events are produced at large angles (since energetic muons are very forward). The efficiency is relatively poor for $P_{\mu} < 10 \text{ GeV/c}$. The column 'ALL' is an average weighted according to the number of events in each bin. Efficiencies are comparable for neutrino and antineutrino. After folding in the muon momentum distributions the average correction factor was found to be 1.02, with very little tail to higher factors.

Correction for electronic inefficiency is a matter of overall normalization and not applicable in this thesis. Corrections for geometrical acceptance are small and have been neglected (except where explicitly stated). However, effects arising from specific dead wire regions have been corrected for.

3.5 IDENTIFICATION OF MUONS

A program (called EMI-PRODUCTION) using the HYDRA memory management system extrapolated all charged tracks of momentum > 3 GeV/c leaving BEBC, to both EMI planes in turn, using 4-vectors generated by the GEOMETRY program. Account was taken of intervening material in assigning an extrapolation error to all predicted hits (arising from multiple scattering). Actual hits were reconstructed from EMI data and matched against predictions for each time slot within extrapolation errors. Predictions and hits were measured in terms of co-ordinates (V = vertical, W = horizontal) local to each chamber and transformations to the global BEBC-EMI system were done at a latter stage. A χ^2 of association was defined by

$$\chi^2 = \frac{(\partial w^2 \cdot \Delta v^2 - 2\partial v \partial w \Delta v \Delta w + \partial v^2 \cdot \Delta w^2)}{\partial v^2 \partial w^2 - (\partial v \partial w)^2} \quad (3.1)$$

where $\Delta v = v(\text{hit}) - v(\text{predicted})$

$\Delta w = w(\text{hit}) - w(\text{predicted})$

$\partial v^2 = (\partial v)^2_{\text{hit}} + (\partial v)^2_{\text{pred}}$

$\partial v \partial w = \partial v_{\text{pred}} \times \partial w_{\text{pred}}$

and $\partial w^2 = (\partial w)^2_{\text{hit}} + (\partial w)^2_{\text{pred}}$

for each hit using extrapolation and reconstruction errors and had to be < 20 to be accepted. It was possible for more than one chamber to be hit in each plane as there were regions of chamber overlap, however, situations with > 2 hits only occurred in 18% of the muon sample. If the innermost hit was acceptable then extrapolation to the next chamber (usually in the outer plane) started from the hit rather than the prediction. In this case the resulting χ^2 at the next chamber hit was correlated to the first chamber and thus called the correlation χ^2 . For a sequence of hits to be acceptable to identify a muon, the correlation χ^2 had to be < 15 . In the case where there were more than 2 hits and the correlation χ^2 exceeded this threshold,

various combinations of hits were tried by neglecting one or more hits until the χ^2 was acceptable (if at all). In this way groups of hits in one or more time slots were formed for each candidate muon track.

In the EMI-DECIDE program further probability cuts were made before muon mass assignments were made. The best association χ^2 probability for a given muon candidate in a given time slot had to be $> 10^{-4}$ and within a factor of 10^3 of the best probability out of all time slots for the candidate. The values of these cuts were varied by an order of magnitude up and down without significant loss or gain of muons, indicating an optimum plateau removed from any thresholds. For a given time slot all association probabilities had to be $> 10^{-5}$ and within a factor of 10^4 of the best probability in the time slot. If these conditions were fulfilled along with a two plane correlation then the track was called a muon.

The mean association χ^2 per degree of freedom plotted for all muons in all acceptable time slots for the inner plane in various momentum bins is shown in figure 3.10. Assuming 2 degrees of freedom per hit, the association χ^2 probability is shown in figure 3.11. The corresponding plots for the outer plane are more or less identical. The mean correlation χ^2 per degree of freedom and probability is plotted as for the association χ^2 and is shown in figures 3.12 and 3.13.

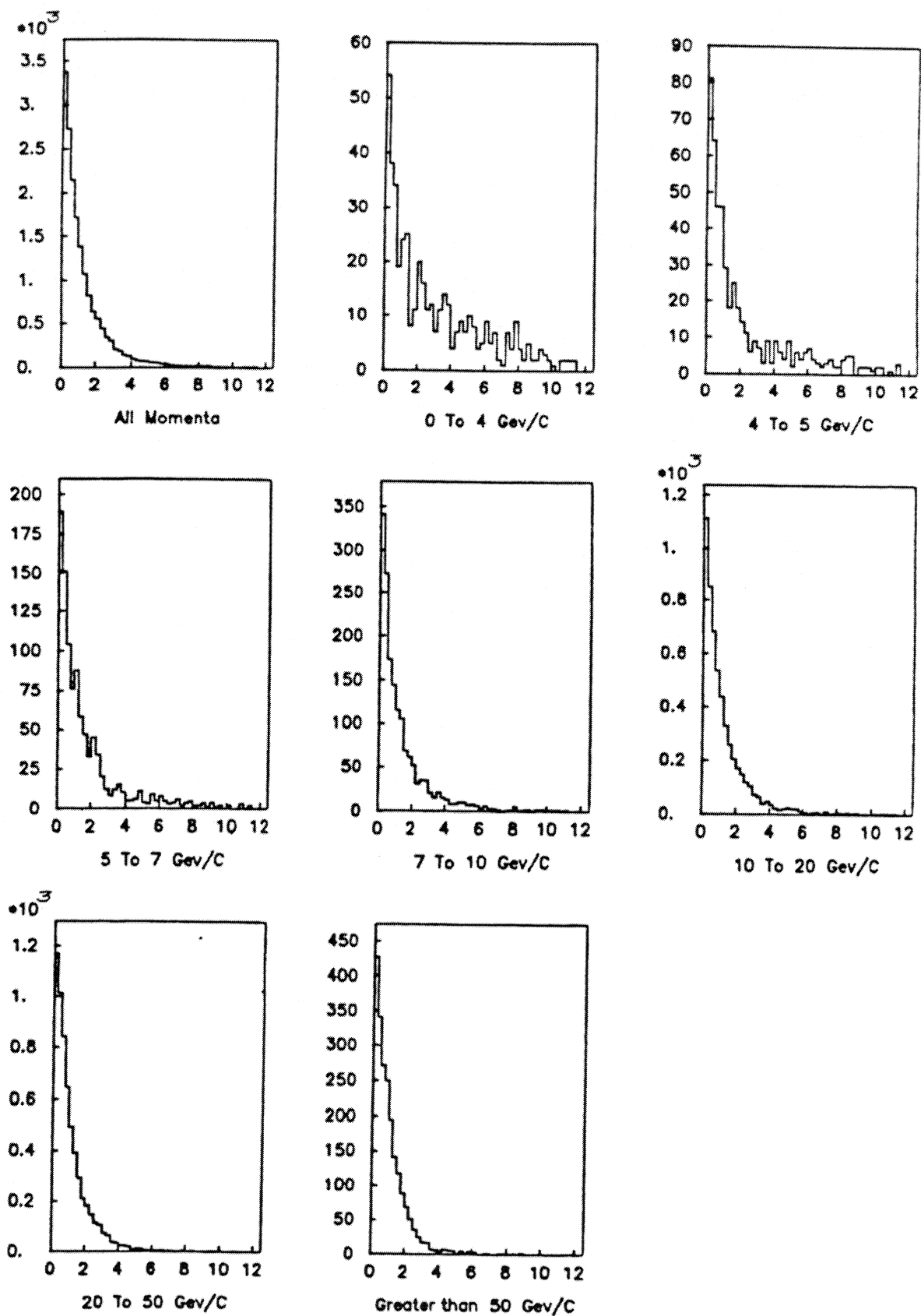


Figure 3.10: Mean inner plane association χ^2 per degree of freedom

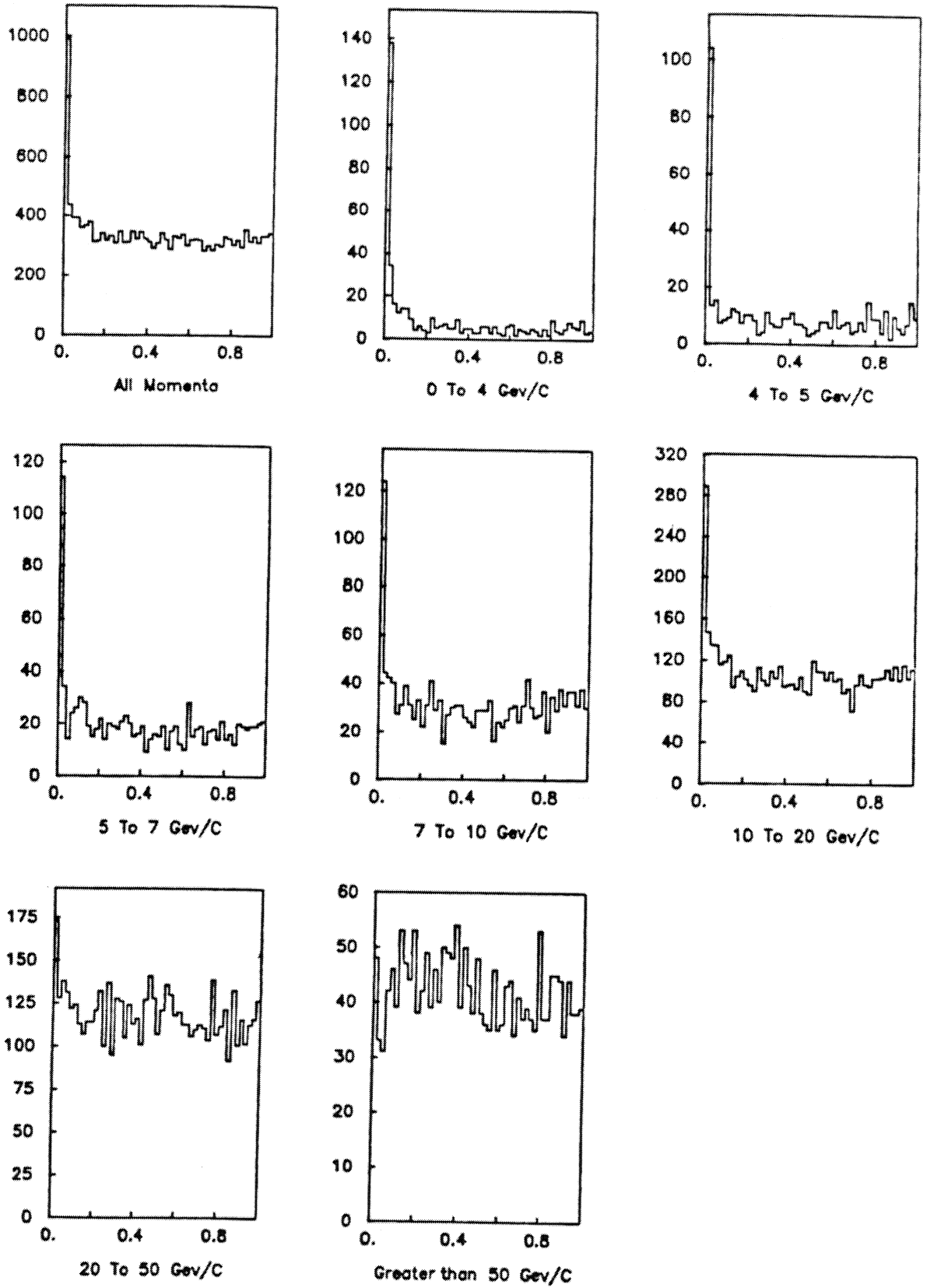


Figure 3.11: Mean inner plane association χ^2 probability

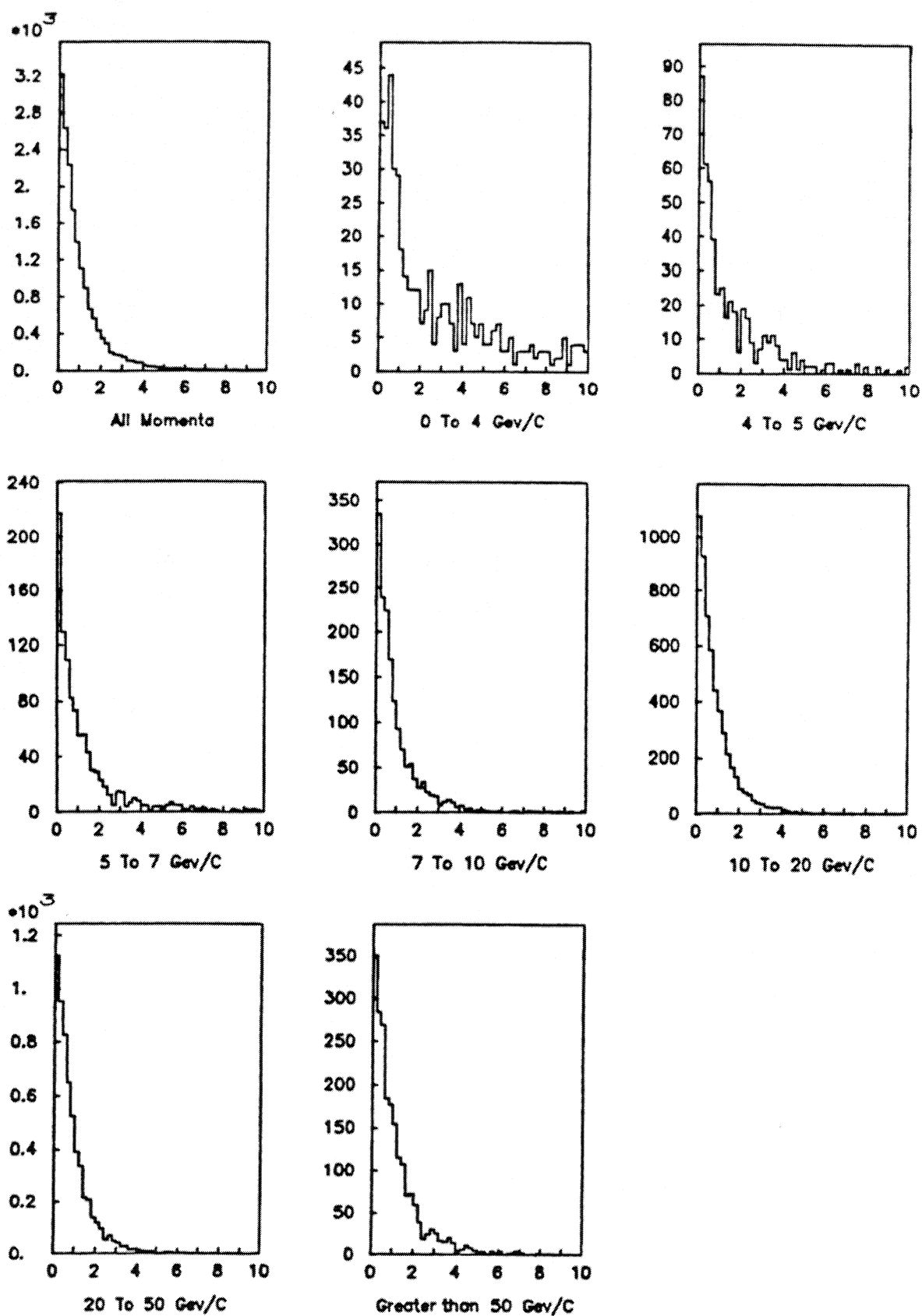


Figure 3.12: Mean correlation χ^2 per degree of freedom

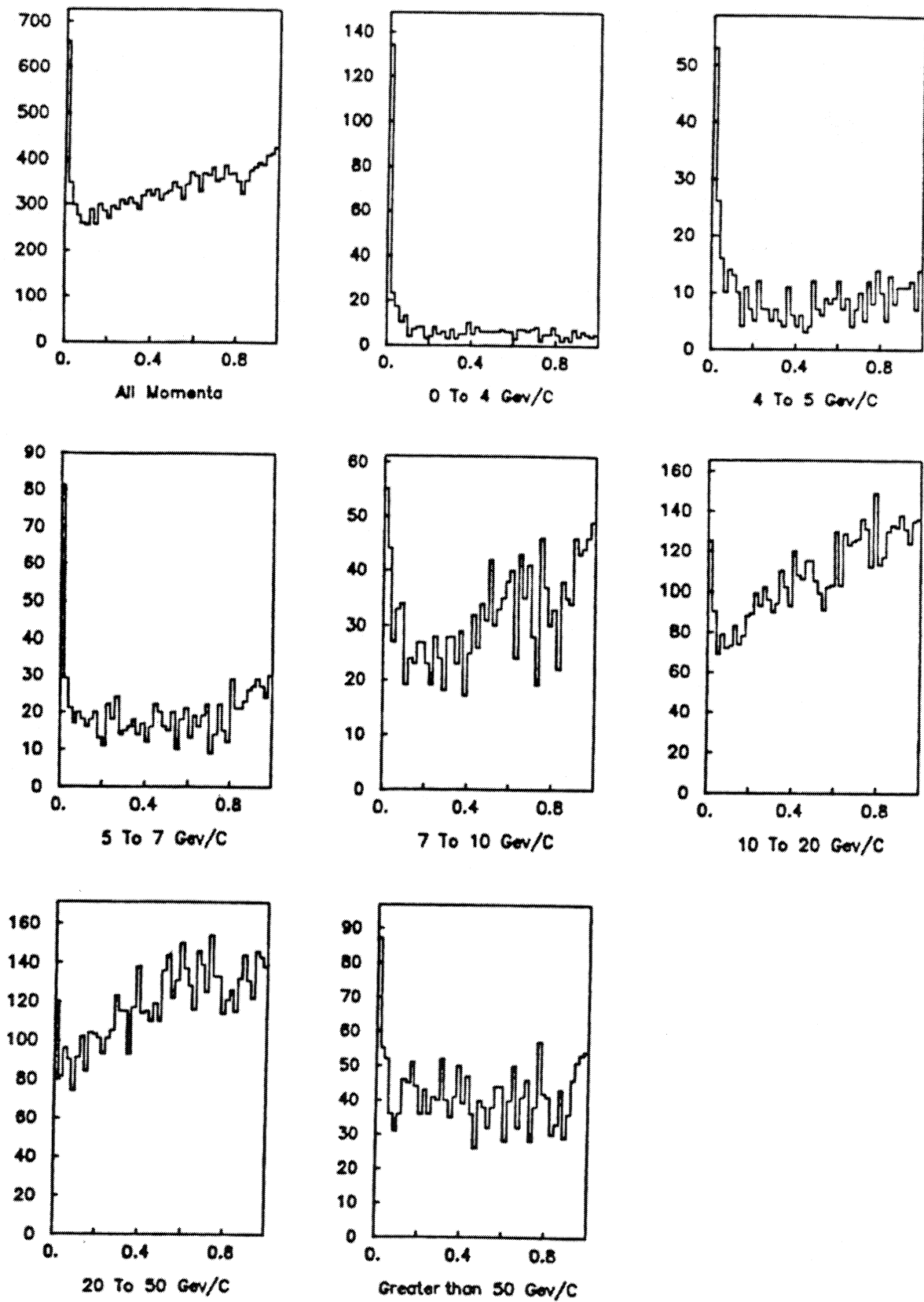


Figure 3.13: Mean correlation χ^2 probability per degree of freedom

3.6 BACKGROUND SOURCES

Hits in the EMI from sources other than event related muons can give rise to a background. The largest source of extraneous hits in the EMI was from beam associated muons arising from neutrino interactions in shielding just prior to BEBC, in the BEBC structure itself and in the hadron shield after BEBC, and far outweighs hits from cosmics during a beam spill. Iron shielding was used to reduce significantly the possibility of hadrons from BEBC events reaching the EMI. However, punch-through did take place at a much reduced level and some background came from hadron punch through to the inner plane combined with a beam associated muon hit in the outer plane.

The net result of these background hits was that some events were wrongly labelled charged current. Spurious hits contributed in one of two ways:-

1. a good muon had acceptable associations in more than one time slot of which only one could be correct (a multi-time slot muon).
2. a hadron track had acceptable associations in one or more time slots of which NONE were correct.

The former case did not change the number of muons in the event and so was unimportant save only to add confusion to the correct time slot for the muon. In the latter case, however, if the track belonged to a CC event then an extra muon was added to the event, thus simulating a multi-muon event. This second muon usually occurred in a different time slot to the good muon, at a much lower momentum and with significantly worse x^2 and so was filtered out. However, if the track belonged to a non CC event (i.e. NC, N^* or K^0 induced etc.) then the number of CC events was affected. It was these events (NC wrongly called CC) which were of concern.

3.7 ACCIDENTAL ASSOCIATIONS

A measure of the level of accidental muons was found (to first order) by using as input to the EMI program EMI data from time slots which could not have been associated with the events. This was achieved using EMI data from a frame number 4 (or 6) removed from the BEBC film frame number. A useful variable to describe muon associations combines the association χ^2 for the first hit with the correlation χ^2 of all subsequent hits to give a global χ^2 and a corresponding global χ^2 probability (with 2 degrees of freedom per hit), i.e.:-

$$\chi^2(\text{global}) = \chi^2(\text{assoc.}) + \sum \chi^2(\text{correl.}) \quad (3.2)$$

This takes account of the quality of all associations in a sequence of hits using linearly independent quantities. For the purpose of this analysis "wrong frame data" is taken to have a mis-match between EMI and BEBC frame numbers (in contrast to a match with "right frame data").

Figure 3.14 shows the global χ^2 probability for all muons in all time slots in various momentum bins for right frame data and figure 3.15 shows the corresponding information for accidental muons generated using the wrong frame method. The most significant features are the peaks at very low probability forming a large percentage of the wrong frame data and being mostly in the low momentum region. These peaks are also present in the right frame data and point to a contamination by accidental muons. At low momentum, errors on track extrapolations are largest owing to multiple scattering. This increases the probability of accidental hits giving association χ^2 's within the cuts.

The difference between figures 3.14 and 3.15 produces a statistical measure of the good muons and is shown in figure 3.16. The peaks at low probability are now noticeably reduced at low momenta, but at high momenta this is not the case. The remaining peaks may be interpreted in terms of EMI chamber mis-alignment.

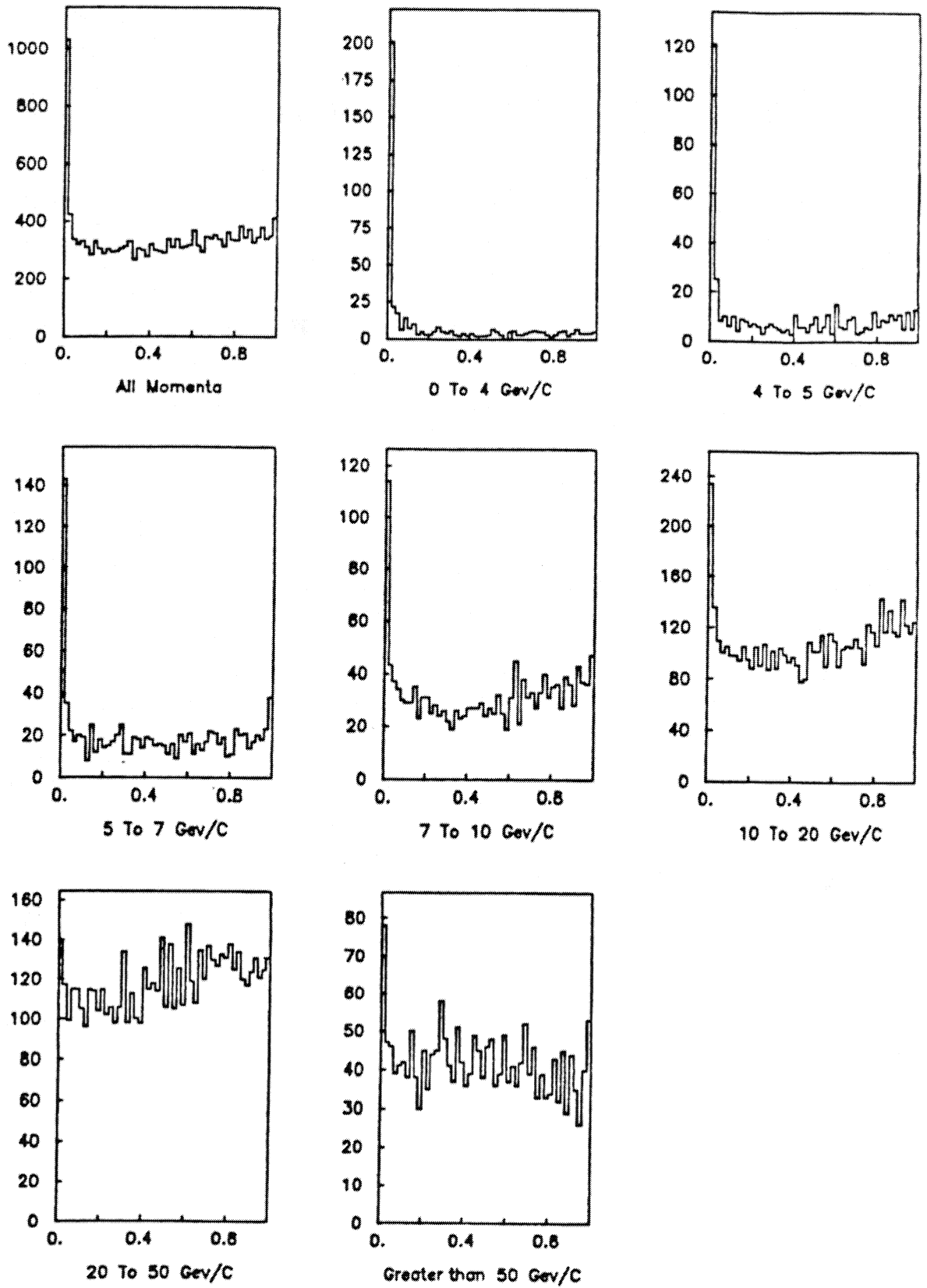


Figure 3.14: Global χ^2 probability (right frame data)

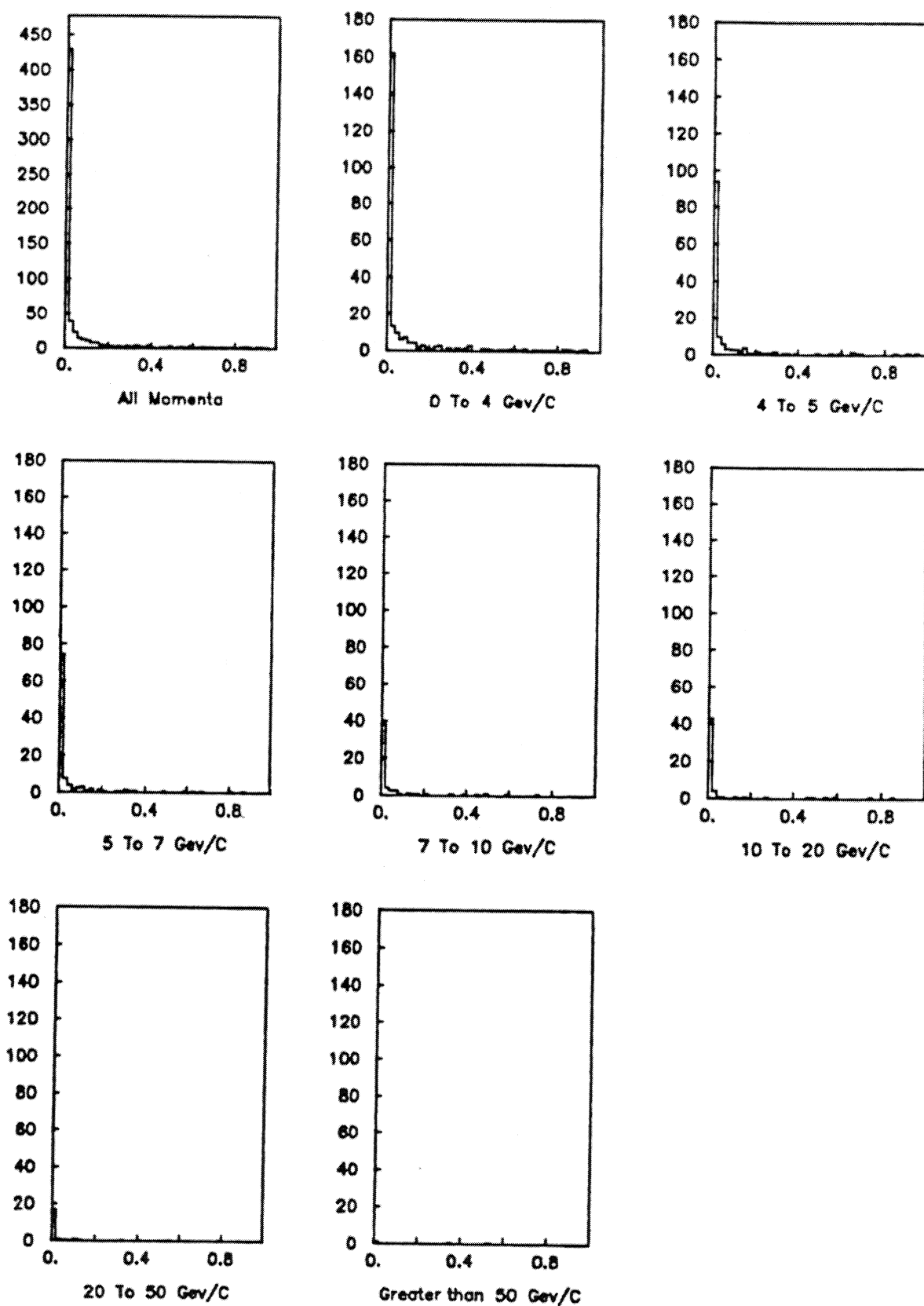


Figure 3.15: Global χ^2 probability (wrong frame data)

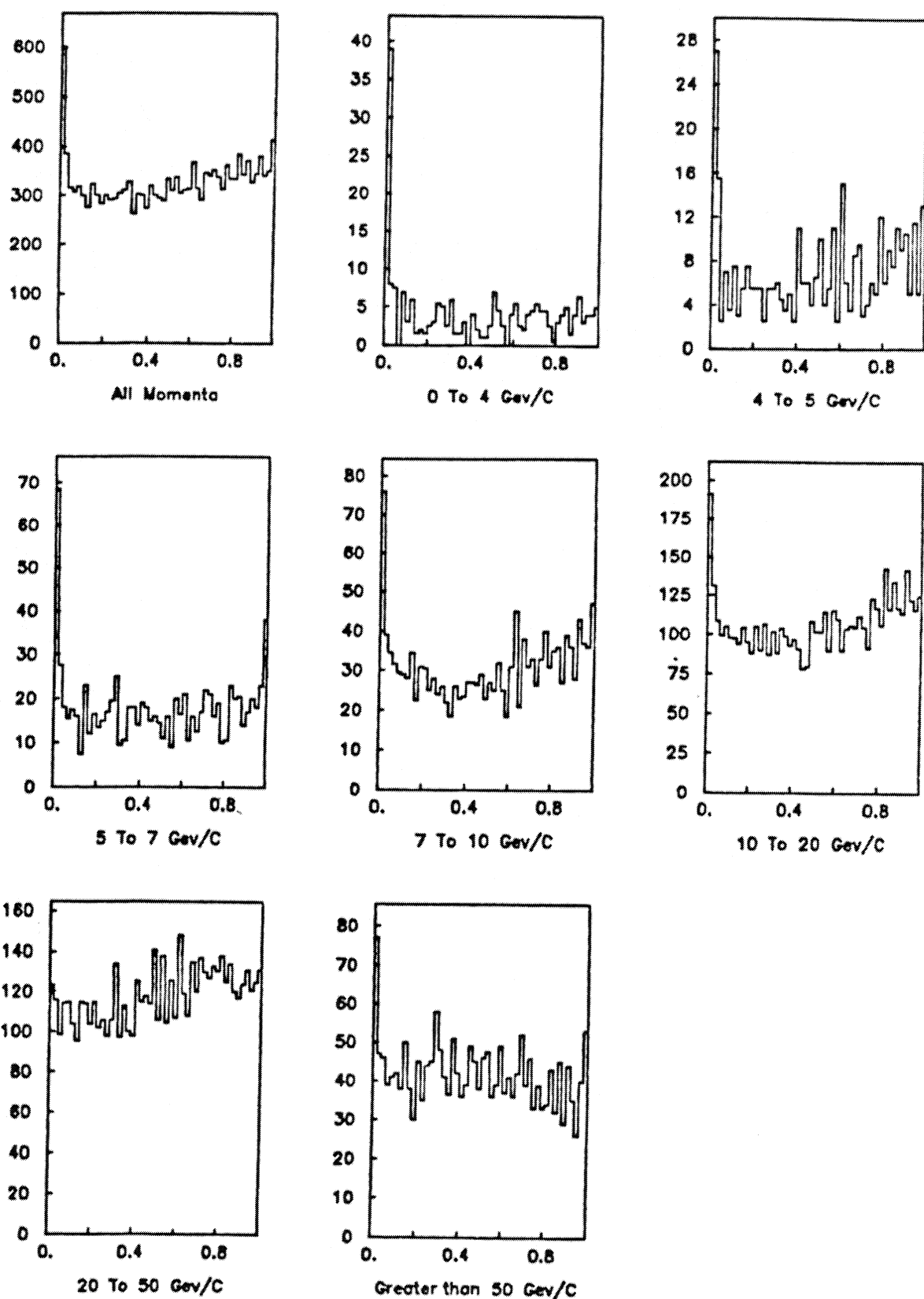


Figure 3.16: Global χ^2 probability (difference between wrong and right frame data)

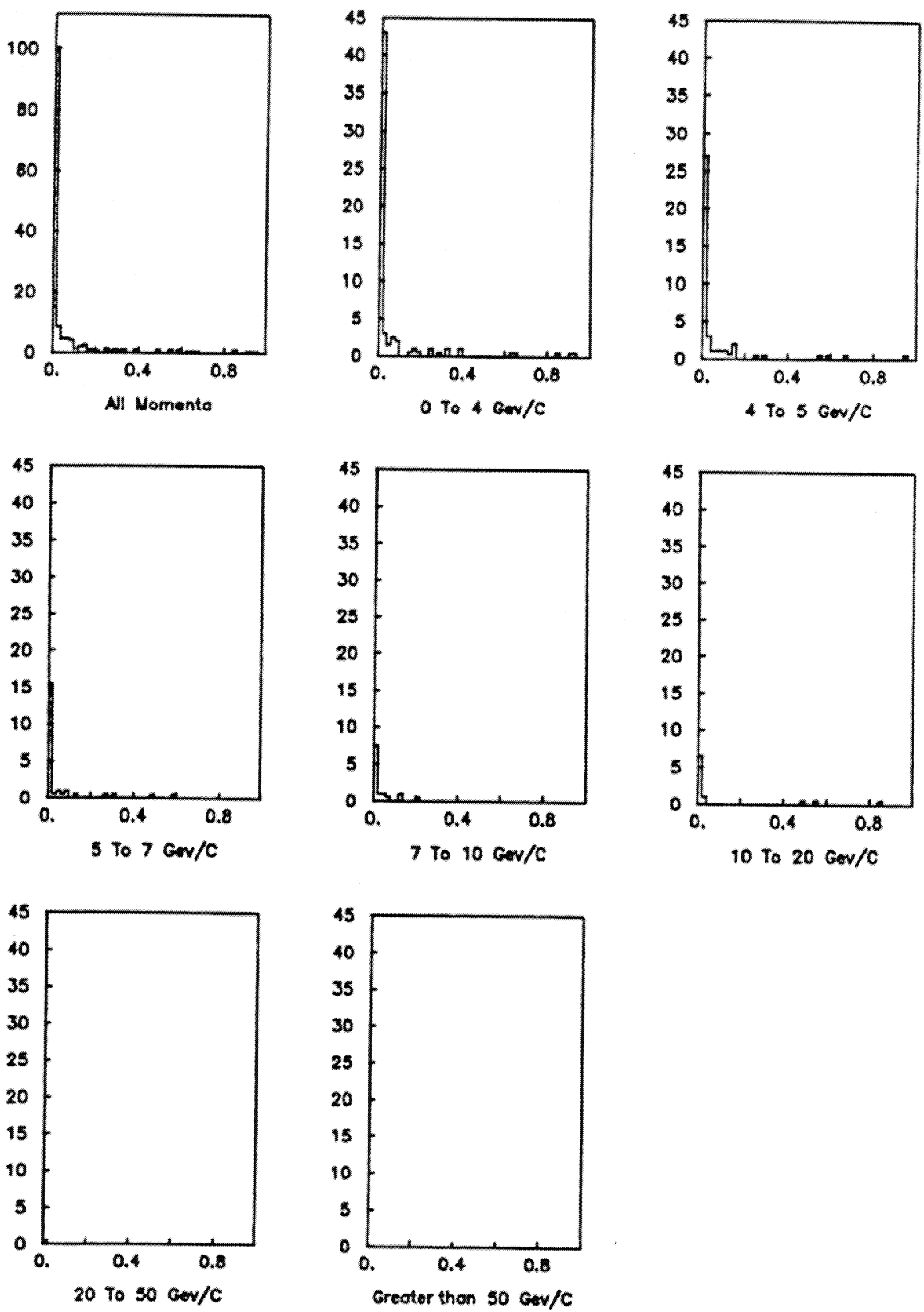


Figure 3.17: Global x^2 probability (NC-selected wrong frame data)

TABLE 3.2 Estimation of False Muons.					
Momentum (GeV/c)	CC (a)	CC (b)	CC (c)	CC (d)	NC (%) (e)
0 - 4	447	237.5	209.5	60	13.4
4 - 5	491	134	357	39.5	8
5 - 7	1010	104	906	21	2.1
7 - 10	1609	53.5	1555.5	11.5	0.7
10 - 20	5416	52.5	5363.5	9	0.2
20 - 50	5940	22	5918	0.5	0
> 50	2099	2	2097	0	0
All	17012	605.5	16406.5	141.5	0.8

Multi-time slot muons (1.8% of all muons) and accidental extra muons in a CC event (1.5% of all muons) do not affect the number of CC events, but the effect of non-CC events simulating CC events (NC background) can be studied by selecting known non-CC events for use with the wrong EMI data (the so called "NC selected wrong frame sample"). The low probability peaks remain (figure 3.17) and are still prominent at low momentum. Table 3.2 shows the number of muons in each momentum bin for (a) right frame, (b) wrong frame, (c) right frame - wrong frame, (d) NC-selected wrong frame and (e) contamination of CC events by NC events expressed as a percentage of right frame data. Whilst (c) gives a measure of the number of single muon, single time-slot CC events (e) gives the contamination of CC by NC events. The difference between (b) and (d) being that whilst the former is an estimate of the total number of muons in all time-slots, the latter ignores multi-time-slot muons and wrong multi-muon events. It can be seen that contamination is most significant below 5 GeV/c and in total forms 0.8% of all data.

A global probability cut (even momentum dependent) removes more good CC events than background and so is not very useful. A more

attractive way to deal with the background is to subtract the NC-selected wrong frame events from the (right frame) data in any analysis. This second approach has been adopted for all results presented in this thesis.

3.8 COMPARISON OF GOOD CC AND NC BACKGROUND EVENTS

TABLE 3.3			
Average Kinematical Values for NC-background Events			
		$\bar{\nu}$	ν
E (visible)	GeV	30.61 ± 1.95	40.93 ± 4.35
E (corrected)	GeV	39.51 ± 2.55	51.44 ± 5.46
X (Bjorken)		0.079 ± 0.009	0.058 ± 0.008
Y (Bjorken)		0.77 ± 0.014	0.802 ± 0.015
W (visible)	GeV/c ²	5.04 ± 0.27	6.16 ± 0.39
W (corrected)	GeV/c ²	7.02 ± 0.25	7.55 ± 0.30
P (muon)	GeV/c	5.27 ± 0.21	5.83 ± 0.34
Q ²	GeV ² /c ⁴	2.77 ± 0.28	3.50 ± 0.48
ν	GeV	31.42 ± 1.98	37.37 ± 2.86
Q _h		-0.02 ± 0.07	1.77 ± 0.12
P _t (in, R.M.S.)	GeV/c	0.77	0.75
P _t (out, R.M.S.)	GeV/c	0.71	0.84
P _t (event)	GeV/c	1.03 ± 0.05	1.19 ± 0.08
P _t (muon)	GeV/c	0.56 ± 0.03	0.54 ± 0.03
P _t (hadronic)	GeV/c	0.98 ± 0.05	1.11 ± 0.08

The effect of background events is more prominent in particular kinematic regions, although only a comparatively small number of events contribute. Table 3.3 shows the average values of some kinematic quantities which can be compared with table 4.4. The average energy of NC background events is lower than for good CC events. However, the biggest distortions in NC background events are at very low x , very high y , low muon momentum, low Q^2 and extraordinarily high ν . P_t distortions are also present (e.g. low muon P_t).

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Chapter IV
EVENT SELECTION, CORRECTION AND KINEMATICS

4.1 INTRODUCTION

TABLE 4.1 Event Categories		
	ν	$\bar{\nu}$
non-CC	6145	10512
wrong charge CC	216	3620
multi-muon CC	95	137
1-prong CC	45	968
all other single μ CC	9840	17019

The data reduction and event reconstruction processes have been described in chapter II. The data was then split into categories as outlined in table 4.1. Although the film was not scanned for NC events, a number did result from rejections from the CC sample and have been labelled non-CC as they include K^0 and neutron induced interactions. They form an incomplete NC sample. Only single muon CC events are used in subsequent analyses presented. The non-CC events were originally measured because they had leaving tracks > 3 GeV/c in momentum, but subsequently did not prove to have a muon. Wrong charge muon events arose from contamination of the antineutrino beam by neutrinos and vice versa [2.1] and form 16.6% of the antineutrino CC

data but only 2.1% of the neutrino CC data. Multi- μ and 1-prong events have not been used.

Although the EMI may have flagged 2 muons in an event only those muons appearing in the same time slot were taken to be dimuon candidates. In all other cases the muon was selected as

1. the fastest candidate if it also had the smallest χ^2 , or else
2. the fastest of the 2 tracks if $P_{\mu}^a \geq 2P_{\mu}^b$, or else
3. the highest P_t track.

and the other track was taken to be a pion. The multi- μ candidates represent 0.9% of the total neutrino CC sample and 0.6% of the antineutrino CC sample.

A major problem with events in heavy liquids (such as neon) is the fair proportion of close, secondary interactions of tracks from the primary vertex. The momentum for these tracks was often badly determined and sometimes the charge could not be found as the track was too short and straight. The SYNTHESIS program improved measurements by using the momenta of secondary tracks to set a lower limit. For tracks with $\delta P/P \geq 60\%$, the charge from the GEOMETRY stage (if known) was set undefined.

Events containing short stubs had usually undergone nuclear re-interactions in which a primary track interacted with nucleons of the same neon nucleus. Most of these fragments (or stubs) were low energy ($\ll 1$ GeV/c) and so were not measured since they did not arise from the primary interaction.

The numbers of events resulting from this experiment are among the highest from a bubble chamber neutrino experiment to date.

4.2 PARTICLE MOMENTA AND MEASUREMENT ERRORS

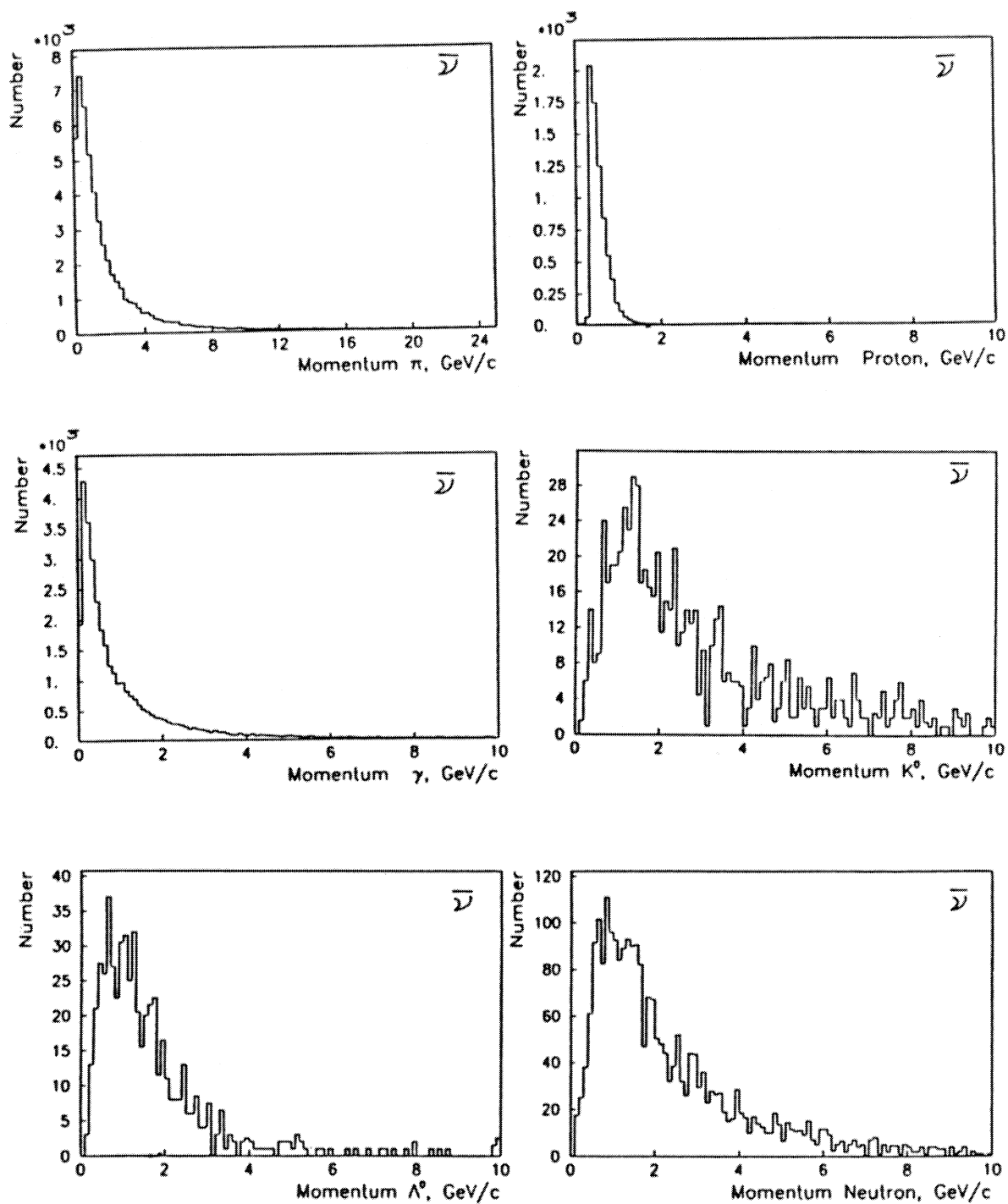


Figure 4.1: Momentum distributions, π , P , γ etc.

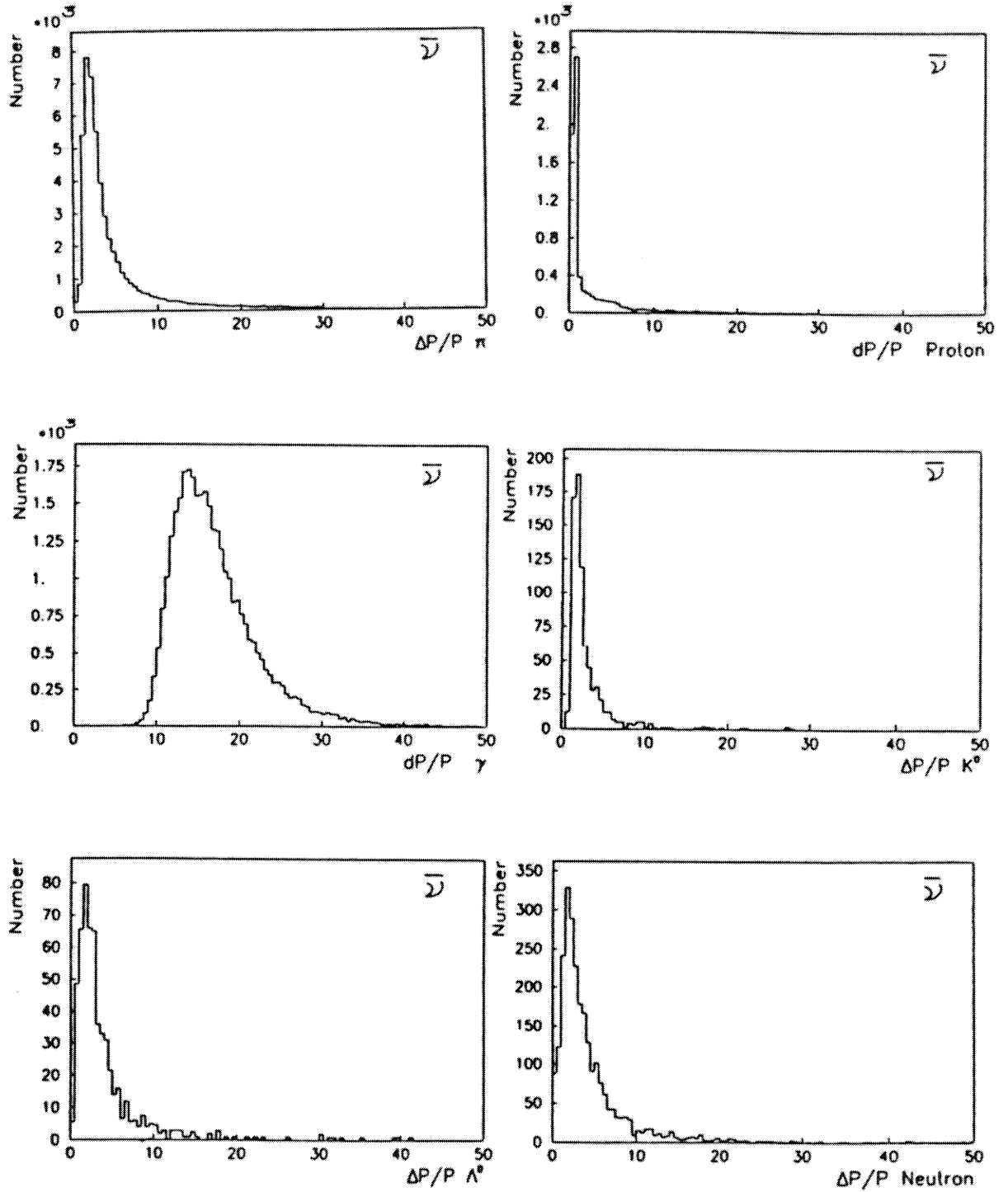


Figure 4.2: Momentum measurement errors, π, P, γ etc.

The momentum distributions (figure 4.1) for pions, protons, gammas, etc. all show evidence of losses at low momenta, caused either by very short tracks being confused with stubs, or by a low scanning

efficiency for these tracks. However, the 300 MeV/c cut on protons and the maximum momentum at which a proton can be identified are clearly seen. The short tail of protons from 1 to 2 GeV/c is due to distinct identification from range etc. Figure 4.2 shows the corresponding momentum measurement error distributions. The long tail for π 's is due to short, interacting tracks and events close to the downstream wall of BEBC. The errors on γ 's are typically much larger because the e^+e^- pair usually undergo many small angle changes. This is due to radiation of many bremsstrahlung γ 's and makes fitting difficult. On the other hand, muons are particularly well measured, even though they are fast and often straight, as the EMI hits were used to improve the momentum determination.

4.3 SYSTEMATIC INEFFICIENCIES

At low neutrino energy, event multiplicity is typically low and scanning efficiency is poorest. Such events also have low momentum muons which have the lowest EMI geometrical detection efficiency.

TABLE 4.2 $\bar{\nu}$ Film Scanning Efficiency.						
# Prongs	1	2	3	4	5	>5
% efficiency	82.2	94.3	95.7	96.6	99.4	98.7

The antineutrino film was subjected to 2 normal and 2 special scans. The scanning efficiency has been estimated using methods outlined in reference 4.1 and is shown in table 4.2. (I am grateful to A.Sansum of the WA59 collaboration for supplying the details.) The efficiency is clearly topology (and hence energy) dependent. Events

with a low number of prongs are not so easily spotted since they are not so obvious amongst other "activity" on the film. Scan and EMI weights have not been used (except where explicitly stated) as the corrections are negligible.

4.4 MISSING NEUTRAL ENERGY

4.4.1 Event Transverse Momentum Imbalance

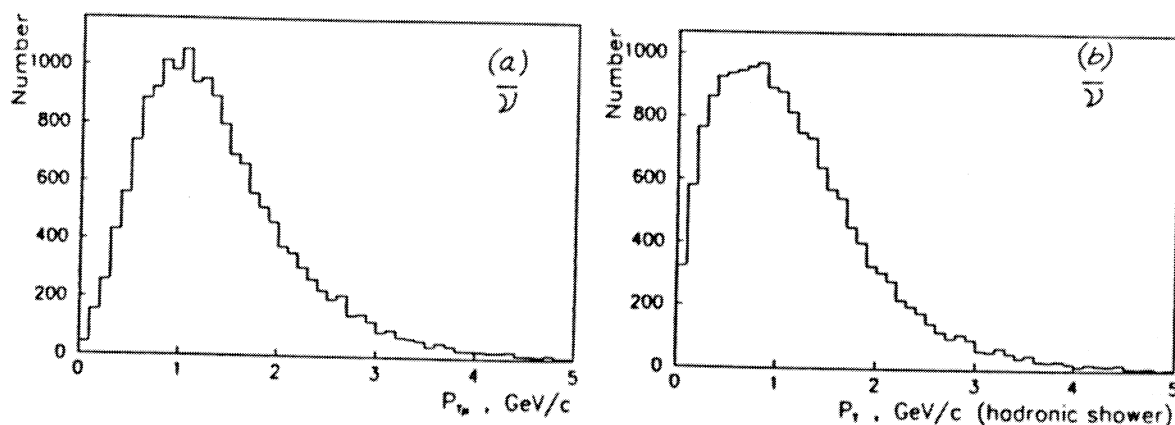


Figure 4.3: Antineutrino data, (a) muon P_t and (b) hadronic P_t

In an ideal detector all particles would be seen. Sadly, BEBC was not ideal in that a proportion of neutral particles (mostly γ 's from π^0 decay and some neutrons and K_L^0 's) escaped unseen. This manifests itself in momentum imbalance both parallel and perpendicular to the beam direction. Along the beam axis no imbalance can be detected experimentally as the initial neutrino energy can only be inferred from the energy of the final state products. However, in the transverse direction, the imbalance can be observed in the final state particles alone. In an ideal world, the muon P_t (figure 4.3a) should totally balance the hadronic P_t (figure 4.3b) but as can be seen, the hadronic P_t is typically smaller. This is best illustrated by

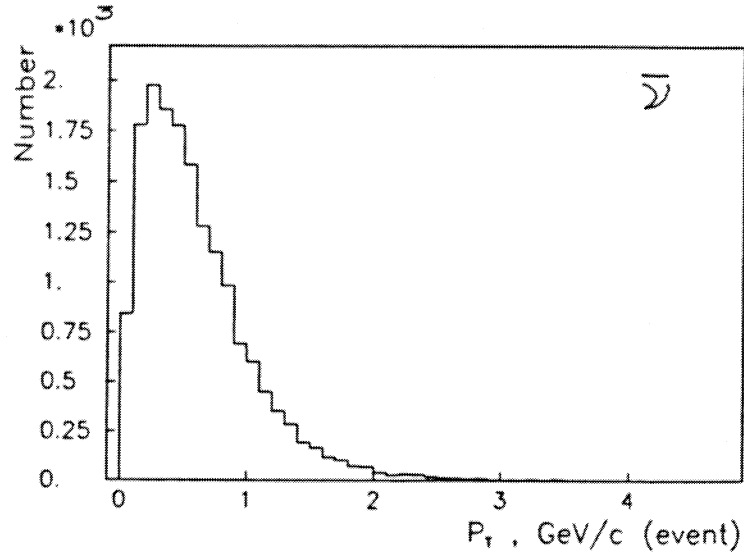


Figure 4.4: Total event P_t , antineutrino

considering the total event P_t (figure 4.4) which ideally should be zero. The issue is further complicated when nuclear Fermi motion (which is a few hundred MeV/c) is considered. This, however, can not account for the larger values shown in figure 4.4.

In practice, a lower limit for the neutrino energy can be set from the visible energy and an upper limit from the primary proton beam energy. In 1-prong events the minimum energy can be constrained to be at least the neutron mass, since although not detected, a baryon has to be present to obey conservation laws. Estimates can be made of the missing energy and such methods are usually based on transverse momentum balance of an event about the neutrino beam direction.

4.4.2 Energy Correction Methods

The underlying assumption of the method owing to Myatt [4.2], is that the seen and missing hadron systems are both approximately co-linear with the 3-momentum transfer of the current. The neutrino energy is given by $E^\nu = P_\ell^\mu + P_\ell^S + P_\ell^u$ where $P_\ell^{S(u)}$ is the seen (unseen)

hadronic longitudinal momentum and P_ℓ^μ is the longitudinal muon momentum. P_ℓ^u , although unknown, can be estimated by scaling P_ℓ^s by the event P_t imbalance, thus

$$E^v = P_\ell^\mu + V \cdot P_\ell^s, \quad \text{where } V = P_t^\mu / P_t^s \quad (4.1)$$

and P_t^s is positive when on the opposite side of the beam direction to the muon. As P_t^s becomes small, V becomes unreasonably large and for P_t^s on the same side of the beam as the muon (i.e. $P_t^s < 0$), V becomes negative and actually reduces the seen energy. In the situation where $|P_t^s| > |P_t^u|$ the seen energy is scaled down.

The method owing to Heilmann [4.3] attempts to overcome some of these problems and is of the form

$$E^v = E^\mu + E^s (1 + |P_t^{ev}| / \sum_h |P_t^h|) - M_p \quad (4.2)$$

where E^s is the seen hadronic energy, E^μ is the muon energy, M_p is the target nucleon mass, P_t^{ev} is the summed P_t of the seen hadrons and muon (i.e. the missing P_t) and P_t^h is the transverse momentum of hadron 'h' to the beam direction. This method has the advantage that $\sum_h |P_t^h|$ can not approach zero and so avoids an unreasonably large correction. However, since typically $\sum_h |P_t^h| > |P_t^{ev}|$ the method systematically produces lower results than the first method above (except for one missing particle when the effect is the same). This is not a great drawback in this experiment as under-correction is compensated for by nuclear Fermi motion (by accident).

An analytic approach to the problem of energy correction [4.4] shows that the 5 unknowns E^v , P_x^u , P_y^u , P_z^u and m^u (neutrino energy, 3-momentum and mass of the missing neutral system respectively) can be reduced by energy and momentum conservation to one unknown (m^u for example). This enables a lower limit on E^v to be set by defining a minimum mass m^u for the missing neutral system, thus

$$E^v = \frac{m'^2 - m^u{}^2 - 2.M_p \epsilon' + M_p^2}{2.(\epsilon' - M_p - x')} \quad (4.3)$$

where x' , m' and ϵ' are the longitudinal momentum, mass and energy of the muon and charged hadrons. A value $m^u = 0$ allows for missing γ 's. The maximum value of E^v is constrained by the maximum primary proton energy of 400 GeV.

4.4.3 Evaluation of Correction Methods

TABLE 4.3 Monte Carlo Comparison of Energy Correction Methods			
Method	$\langle E_c/E_t \rangle$ (%)	width (%)	$E_c/E_t > 1$ (%)
Myatt	96.7	12.7	23.3
Grant	97.1	12.1	22.4
Heilmann	96.6	9.0	20.3

The three correction methods described were originally designed for use in neutrino - hydrogen experiments and the applicability to a complex nuclear target is not obvious. The chief complication is nuclear Fermi motion. In addition much more neutral energy is picked up compared to hydrogen. Figure 4.5 shows the ratio of corrected to true event energy (E_c/E_t) plotted against the ratio of visible to true energy (E_v/E_t) from an LPS Monte Carlo simulation of the Heilmann correction in antineutrino events. The cases where E_v/E_t exceed 1 arise from picking up particles from re-interactions. The average, $\langle E_v/E_t \rangle$, is 91.9%, which becomes 96.6% for $\langle E_c/E_t \rangle$ (figures 4.6a & b respectively). Table 4.3 compares the different methods. Also of importance is the width of E_c/E_t and the proportion of events which are over-corrected (i.e. $E_c/E_t > 1$). All $\langle E_c/E_t \rangle$'s are very similar.

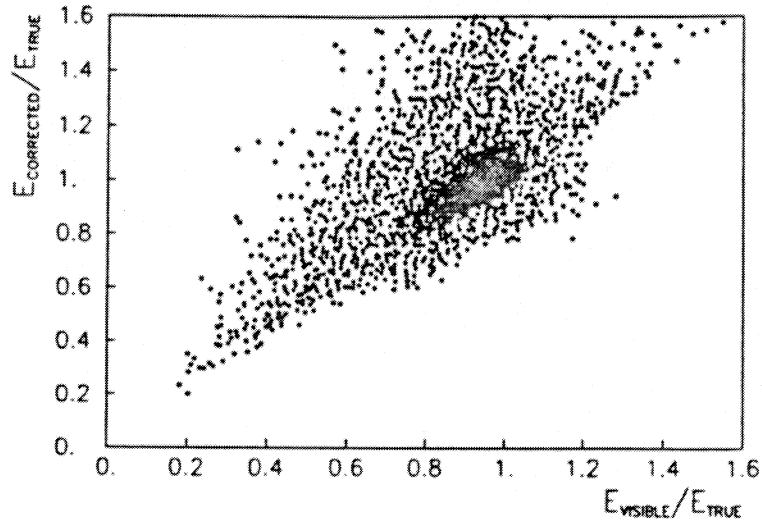


Figure 4.5: E_c/E_t .vs. E_v/E_t

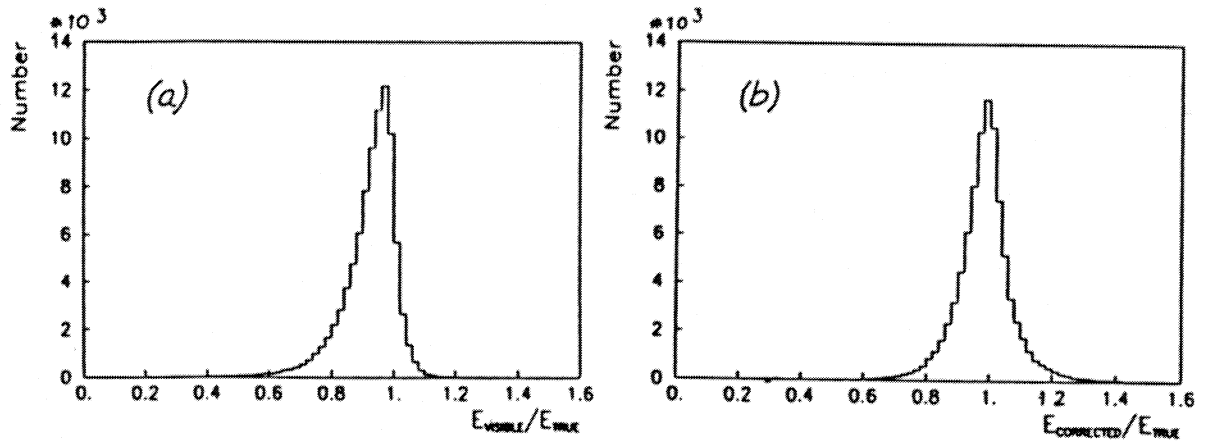


Figure 4.6: Energy ratios (a) E_v/E_t and (b) E_c/E_t

The Heilmann method has been used where energy corrected quantities are required throughout this thesis. The average E_c/E_t is more or less constant at unity throughout the whole of the E_t range (figure 4.7) with standard deviation constant at just over 0.1.

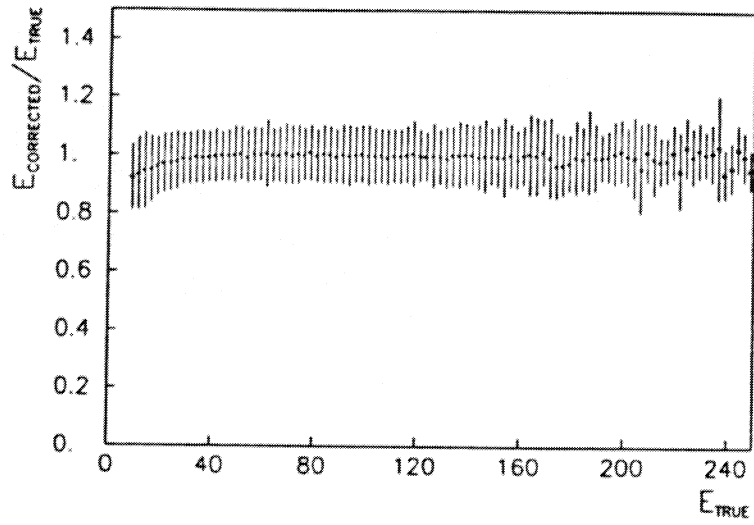


Figure 4.7: Energy ratio E_c/E_t .vs. E_t

4.5 KINEMATIC DISTRIBUTIONS

TABLE 4.4			
Average Kinematical Values			
		$\bar{v}$	v
E (visible)	GeV	37.18 ± 0.22	50.76 ± 0.41
E (corrected)	GeV	39.59 ± 0.23	54.59 ± 0.44
x (Bjorken)		0.216 ± 0.001	0.243 ± 0.002
y (Bjorken)		0.336 ± 0.002	0.450 ± 0.003
W (visible)	GeV/c ²	3.26 ± 0.02	5.11 ± 0.11
W (corrected)	GeV/c ²	4.07 ± 0.02	5.34 ± 0.03
P (muon)	GeV/c	26.15 ± 0.18	29.54 ± 0.31
Q ²	GeV ² /c ⁴	4.56 ± 0.05	10.29 ± 0.14
v	GeV	13.26 ± 0.12	24.12 ± 0.25
Qh		0.24 ± 0.01	1.75 ± 0.01
Pt (in R.M.S)	GeV/c	0.40	0.44
Pt (out R.M.S)	GeV/c	0.39	0.43
Pt (event)	GeV/c	0.61 ± 0.004	0.65 ± 0.005
Pt (muon)	GeV/c	1.37 ± 0.01	1.73 ± 0.01
Pt (hadronic)	GeV/c	1.16 ± 0.01	1.53 ± 0.01
Effective number of events		16990	10120

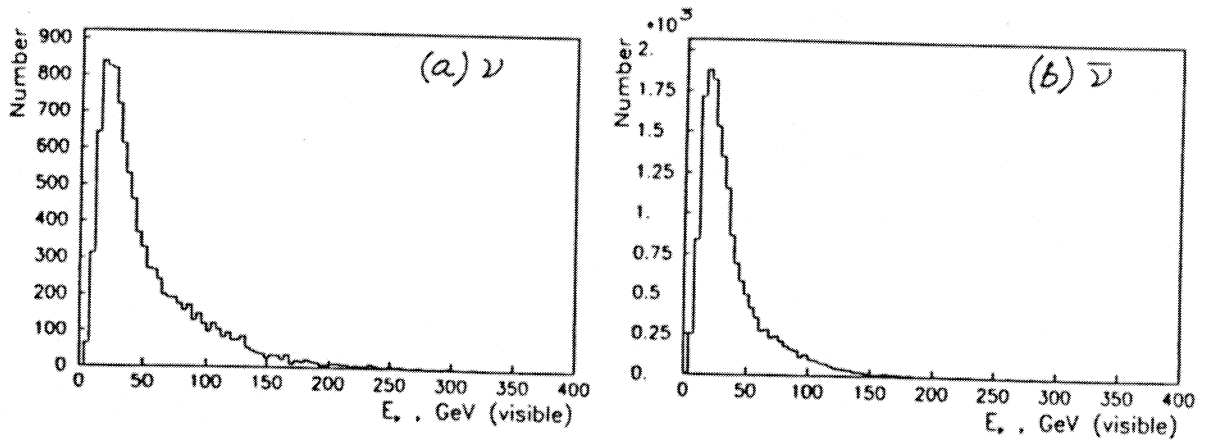


Figure 4.8: Total visible event energy (a) ν , (b) $\bar{\nu}$

Table 4.4 gives the average values of various kinematical quantities for neutrino and antineutrino events. Figure 4.8 shows the total visible event energy for both data samples. In general, the mean energy for neutrino is higher than for antineutrino. After correction of the hadron shower for missing neutral energy, the spectrum shape remains very similar but the average increases slightly (see table 4.4). Thus, as expected from the neon/hydrogen mix, a large fraction of neutral energy is picked up. Scanning losses show up as a loss of events at low energy. In these figures and in the rest of this thesis all events with just one visible track (the muon) have been excluded because (a) such events were scanned for differently in neutrino and antineutrino film and (b) these events are not used in any later analysis described anyway. This excludes events of the type $\bar{\nu}P \rightarrow \mu^+P\pi^-$ in which the proton is slow and hence missed at the scanning stage or cut out by the 300 MeV/c cut and the π^- disappears via the reaction $\pi^-P \rightarrow \pi^0N$ with subsequent loss of neutrals. Reactions of the type $\bar{\nu}P \rightarrow \mu^+N\pi^0$ are also excluded.

The muon momentum distributions for neutrino and antineutrino are comparable (figure 4.9). Figure 4.10 shows y , in which neutrino data

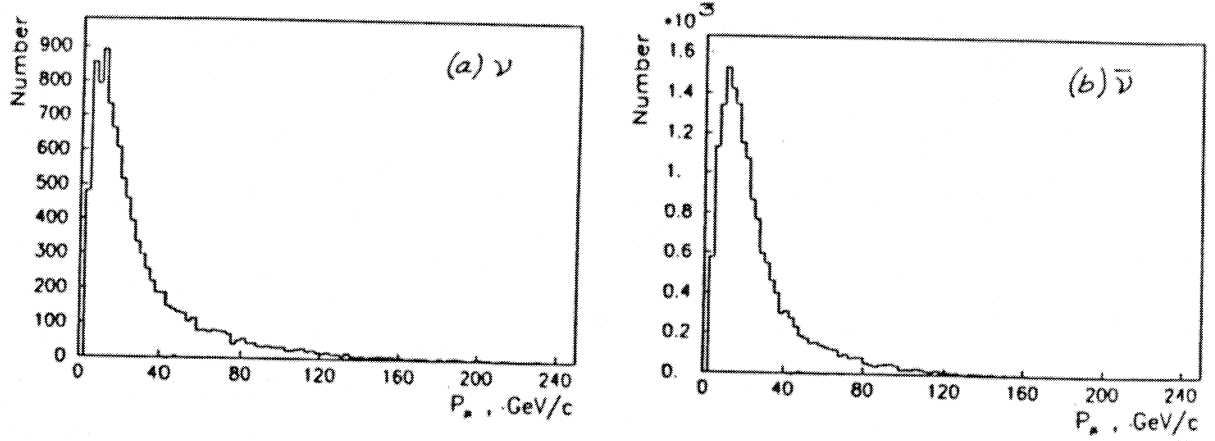


Figure 4.9: Muon momentum (a) ν , (b) $\bar{\nu}$

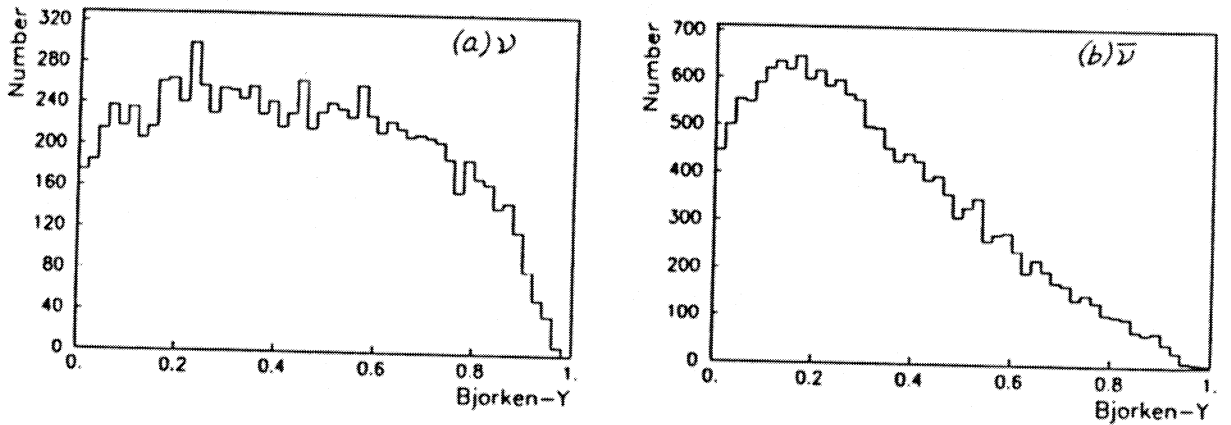


Figure 4.10: Bjorken y , (a) ν , (b) $\bar{\nu}$

is seemingly flat and antineutrino data is approximately of the form $(1-y)^2$ except for losses. Exclusion of 1-prong events has caused a deviation from the expected form with a loss showing up at low y . The loss of high y events (in both cases) is due to the 3 GeV/c cut on muon momentum and the EMI geometrical acceptance.

The x dependence is shown in figure 4.11 and is similar for neutrino and antineutrino. Both show scattering from the sea, where

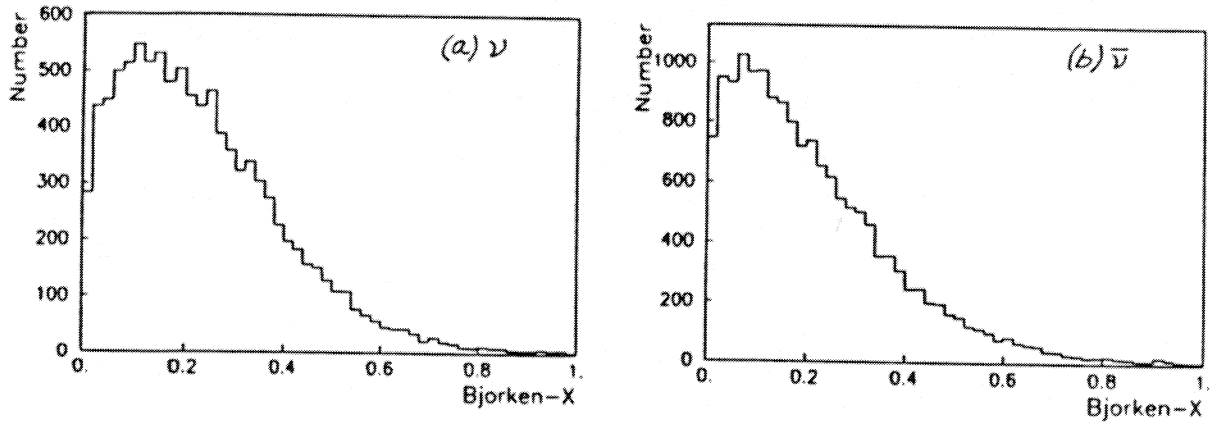


Figure 4.11: Bjorken x, (a) ν , (b) $\bar{\nu}$

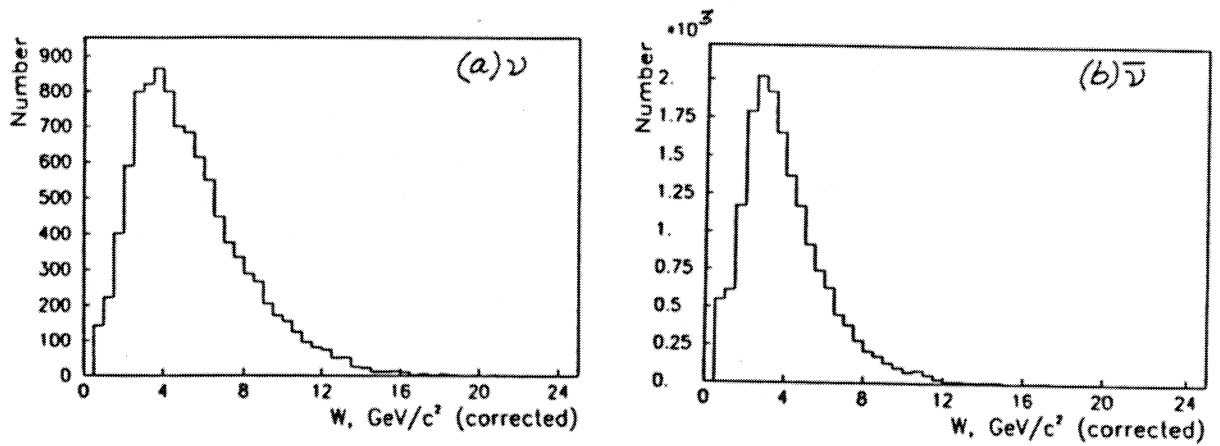


Figure 4.12: Effective mass of hadronic shower, W, (a) ν , (b) $\bar{\nu}$

$x < 0.1$. Monte Carlo studies have shown that the x distribution can be reasonably well described by the functional form $x^\alpha(1-x)^\beta$ ($\alpha=0.7$, $\beta=4$) for valance quarks and $(1-x)^\gamma$ ($\gamma=5.5$) for sea quarks. The effect of suppression of high y events can also be seen in the hadronic shower energy W (corrected for missing neutrals) (figure 4.12), where the mean W is higher for neutrino than antineutrino (table 4.4). Very little structure can be seen, not even the Δ^{++} , since the quasi-

elastic channel has been removed. The mean W is slightly less before energy correction (table 4.4)

4.6 THE $Q^2 - \nu$ PLANE

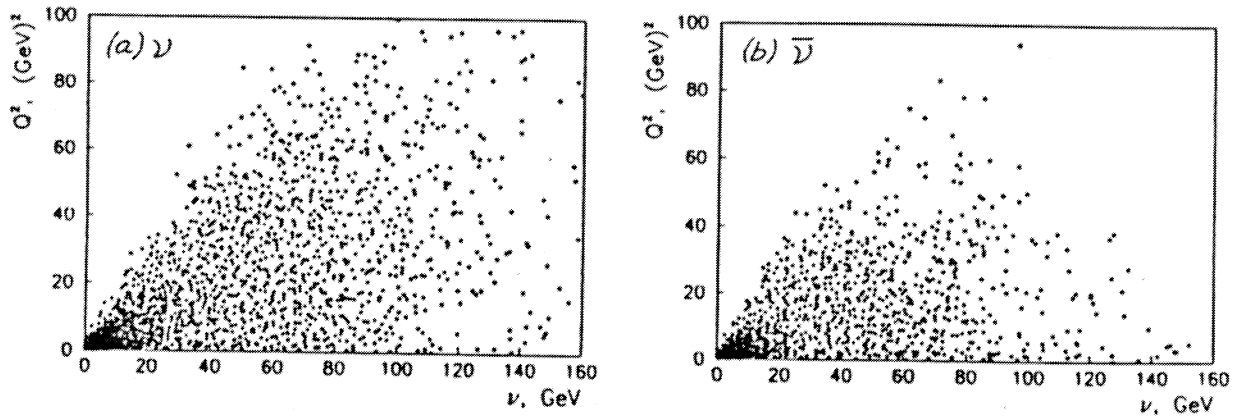


Figure 4.13: The $Q^2 - \nu$ plane, (a) ν , (b) $\bar{\nu}$

A fair proportion of information is contained in a $Q^2 - \nu$ plot (figure 4.13). Both Q^2 and ν have been corrected for the effect of missing neutrals. Different regions of the plot are inhabited by events from different corners of phase space.

The slope of any line drawn from the origin is x , and is bounded by $0 \leq x \leq 1$. When $x = 1$ elastic scattering occurs and $W = M_p$. Lines drawn parallel to this are in areas of inelastic events, have increasing W and hence can in principle contain resonances (e.g. $\Delta^{++}(1236)$). This is more easily seen by considering $W^2 = M_p^2 + 2M_p\nu - Q^2$. The upper bound on the ν axis is just the neutrino energy, E_ν . Lines drawn from the point of intersection of the $\nu = E_\nu$ and $Q^2 = 0$ lines are at constant scattering angle θ . Hence, events are expected to populate a triangle and this is clearly seen in figure 4.13. The deep inelastic region is defined to be at large Q^2 and ν (i.e. $Q^2 \gg M_p^2$, $\nu \gg M_p$).

Although in principle any resonance behaviour should show up as a band parallel to the $x = 1$ contour, in practice both Q^2 and ν are smeared by the poor knowledge of E_ν . In addition, smearing causes some events to lie outside the physical region described (e.g. beyond the $x = 1$ contour). For this experiment most events lie in the bottom left hand corner and are at modest Q^2 and ν compared to the regions accessible to muon or narrow-band neutrino beams. However, the majority of events are still deep inelastic. Neutrino events spread further into the plane than antineutrino events because of the $(1-y)^2$ factor in the antineutrino cross-section.

4.7 RE-INTERACTIONS AND EVENT CHARGE

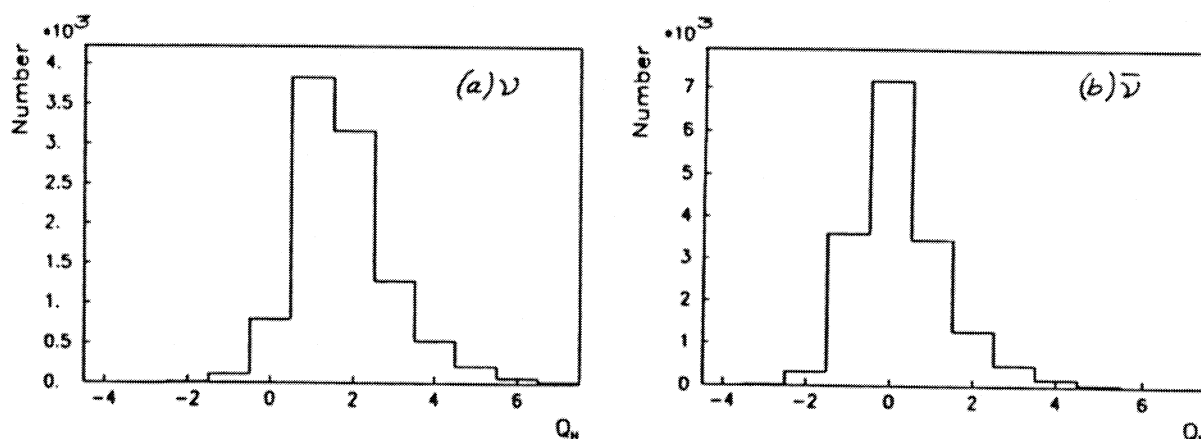


Figure 4.14: Hadronic final state charge, Q_h , (a) ν , (b) $\bar{\nu}$

A consequence of a complex nuclear target such as neon is that re-interactions of final state particles in the struck nucleus often occur. Interaction fragments may elastically scatter giving rise to low momentum "boil-off" protons and neutrons, the former appearing as short, unmeasureable stubs on bubble chamber film. However, the presence of stubs was noted on the antineutrino film so that the

TABLE 4.5 Hadronic Final State Charge	
Process	Q_h
$\bar{\nu} P \rightarrow \mu^+ H^0$	0
$\bar{\nu} N \rightarrow \mu^+ H^-$	-1
$\nu P \rightarrow \mu^- H^{++}$	+2
$\nu N \rightarrow \mu^- H^+$	+1
where H is the final state hadron system	

absence of stubs could be used to enrich a sample of events not having undergone re-interactions. Other inelastic processes can also occur, such as $\pi^- P \rightarrow \pi^0 N$, $\pi^+ N \rightarrow \pi^0 P$ etc. Both processes give rise to neutral particles which may not be detected. It may be concluded that re-interactions lead to unwanted smearing of final state particles.

As a consequence of re-interactions, the final state event charge often alters. The summed hadronic charges expected in events without re-interactions are outlined in table 4.5. In practice these charges are smeared and are shown in figures 4.14a & b for neutrino and antineutrino respectively. Most noticeable is the tail out to positive summed hadronic charge, Q_h , which results from re-interactions of the kind $\pi^+ P \rightarrow \Delta^{++} \rightarrow \pi^+ P$, where the proton is energetic and passes the 300 MeV/c stub cut. By this mechanism events can pick up one or more units of charge. The reaction $\pi^- P \rightarrow \pi^0 N$ has a similar effect. The short tail to negative Q_h is due to the proton stub cut removing valid protons. Scanning and measuring losses can contribute to both tails.

Clearly, it is not possible to identify events as having occurred from an interaction on a proton or neutron. For example, an

antineutrino induced event with $Q_h = 0$ may be the result of an interaction with a neutron and subsequent pick-up of a nuclear proton.

4.7.1 Coherent Diffractive Production

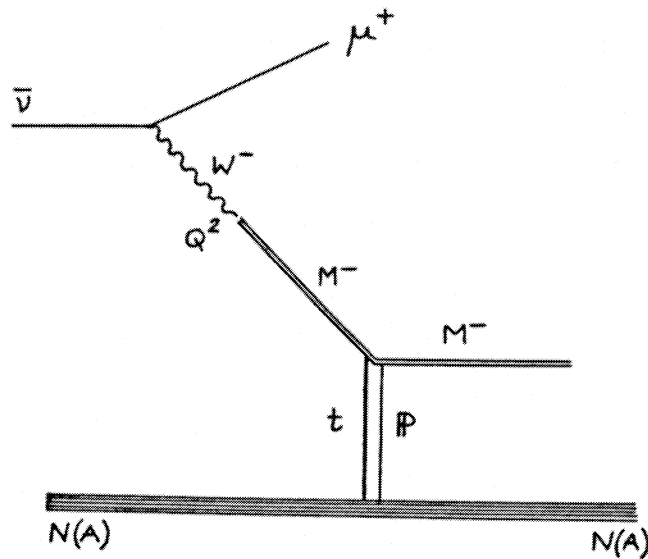


Figure 4.15: Mechanism for coherent diffractive production

Evidence for coherent diffractive production by antineutrinos has been observed in this experiment. In this process the W^- couples directly to a vector or axial-vector meson or to a pseudoscalar meson via a longitudinally polarized W^- (figure 4.15). This meson, M^- , then scatters coherently via pomeron exchange with the nucleus. Such events characteristically occur at low t , low multiplicity and have no stubs ($t = 4$ -momentum transfer squared from meson to nucleus). The results of this analysis are presented in reference 4.5.

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Chapter V

MULTIPLICITIES AND LONGITUDINAL PROPERTIES OF THE HADRON SHOWER

5.1 INTRODUCTION

A complete and exhaustive study of the hadronic final state covers many areas, all probing the properties of the struck quark and how it fragments. Unfortunately, all such studies fall into the regime of non-perturbative QCD and cannot be calculated. However, measurements on the struck quark can be made to some extent by inference from the final state fragments. One such analysis, namely a search for an asymmetry in struck quark production, will be studied in detail in the next chapter.

In this chapter, some longitudinal properties of the hadron shower will be reviewed and compared with other experiments. Some multiplicity distributions will also be presented.

5.2 MULTIPLICITIES

TABLE 5.1 Average Particle Multiplicities		
	$\bar{v}$	v
Overall	6.00 ± 0.03	8.02 ± 0.04
+ve's	2.24 ± 0.01	3.24 ± 0.02
-ve's	1.89 ± 0.01	1.97 ± 0.01
π^+	1.90 ± 0.01	2.71 ± 0.01
π^-	1.85 ± 0.01	1.91 ± 0.01
proton	1.38 ± 0.01	1.47 ± 0.01
$\bar{x}$	2.91 ± 0.02	3.53 ± 0.03
Charged	3.93 ± 0.02	5.05 ± 0.03

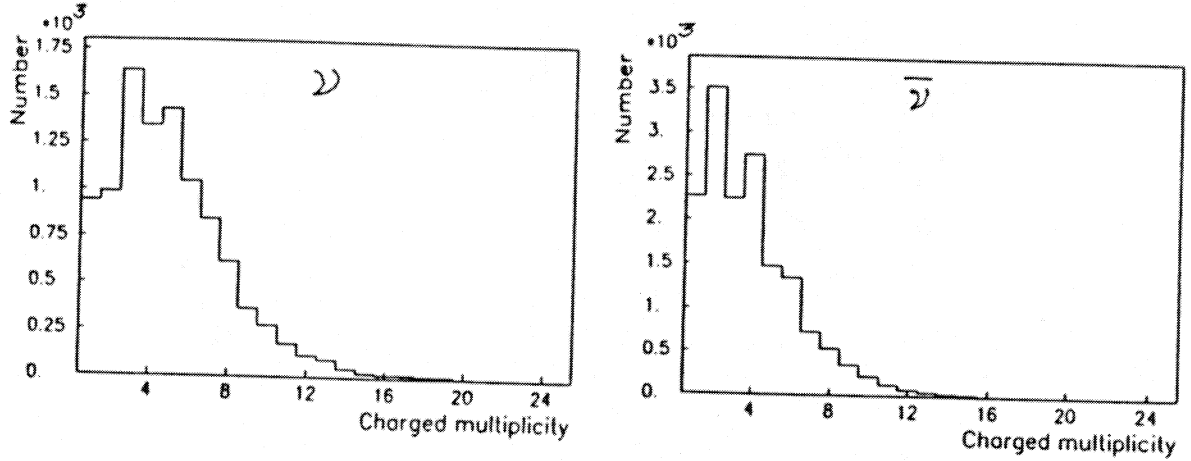


Figure 5.1: Charged multiplicity

Table 5.1 shows the average multiplicity for various particles and categories. Clearly, there is a tendency for higher multiplicities in neutrino interactions. This is a result of the greater energy available for hadron creation. The ratio of average multiplicities, $\langle N_{\pi^+} \rangle / \langle N_{\pi^-} \rangle$, is seen to be greater than 1 for neutrino events, but less than 1 for antineutrino events and is consistent with expectations from the quark-parton model. There is a fair contamination of the π^+ sample by protons. This leads to an over-estimate of this ratio for neutrino events, but an under-estimate for antineutrino. The overall charged multiplicity (figure 5.1) has a tail up to 15 (19) for antineutrino (neutrino) events.

Figure 5.2 shows the topological cross sections $P(N_{ch}, W^2) = \sigma(N_{ch}, W^2) / \sigma_{tot}(W^2)$ for the charged multiplicities $N_{ch} = 1$ to 13 in neutrino events (even prong events have been excluded for clarity, but qualitatively show the same features, the curves are merely to guide the eye). It can be seen that $P(1, W^2)$ decreases rapidly with W^2 . As W^2 increases the thresholds for production of higher prong topologies are passed. Each topology rises in turn to a maximum before giving way to a higher prong topology. Thus, an increase in hadronic energy

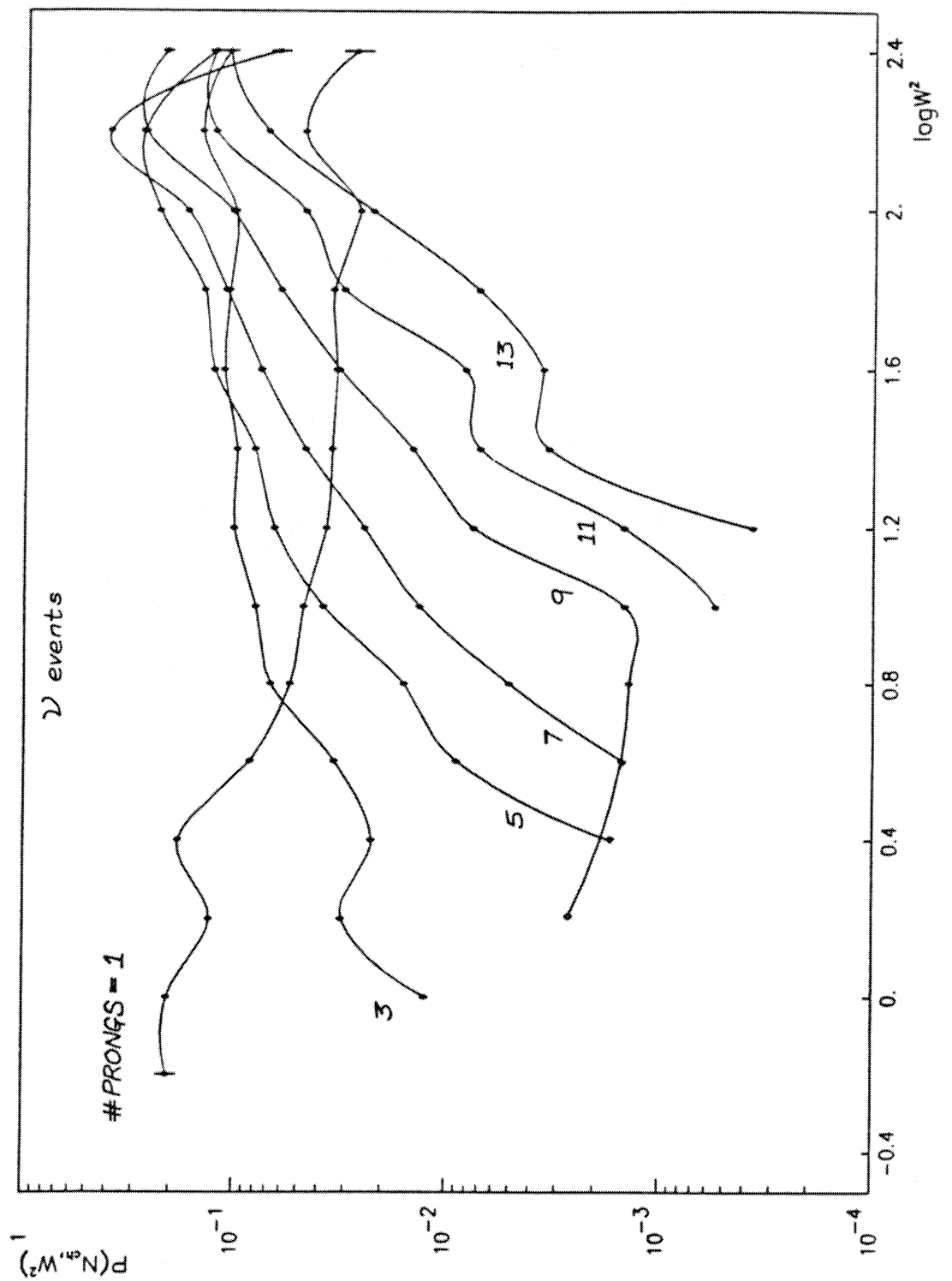


Figure 5.2: Topological cross-sections as a function of W

leads to the creation of more particles rather than a constant number of particles, each with more energy.

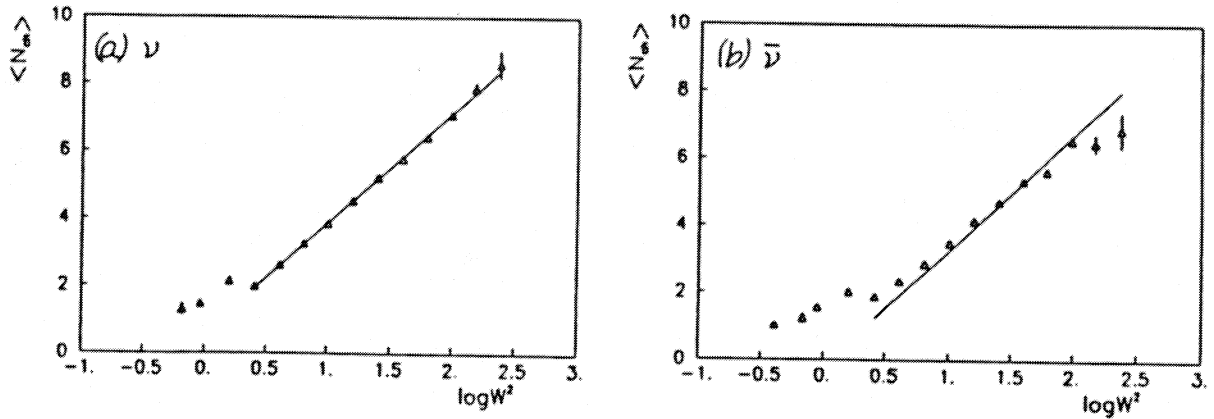


Figure 5.3: Dependence of $\langle N_{ch} \rangle$ on $\ln W^2$

TABLE 5.2				
Results of Fits to $\langle N_{ch} \rangle = a + b \cdot \ln W^2$				
Reaction	W^2 range	a	b	Ref.
νN	3-180	0.58 ± 0.21	1.42 ± 0.07	this thesis
$\bar{\nu} N$	3-180	-0.21 ± 0.17	1.50 ± 0.06	this thesis
νP	4-200	0.37 ± 0.02	1.33 ± 0.02	[5.1]
	4-100	0.80 ± 0.10	1.25 ± 0.04	[5.2]
$\bar{\nu} P$	4-100	-0.44 ± 0.13	1.48 ± 0.06	[5.3]
	6-140	0.18 ± 0.20	1.23 ± 0.07	[5.4]
νn	4-100	0.21 ± 0.10	1.21 ± 0.04	[5.2]
$\bar{\nu} n$	4-140	0.79 ± 0.09	0.93 ± 0.04	[5.4]

Figure 5.3a & b shows $\langle N_{ch} \rangle$ vs. $\ln W^2$, which above the resonance region (2 GeV/c² or so) appears to be linear. The result of a weighted least-squares fit above the resonance region of the form $\langle N_{ch} \rangle = a + b \cdot \ln W^2$ is given in table 5.2 and is compared with other

experiments. The measurements from this experiment are high and can probably be attributed to increased multiplicity from re-interactions. The situation is further complicated in that separation of proton and neutron targets is not possible and only an average of them can be obtained.

5.3 SEPARATION OF CURRENT AND TARGET FRAGMENTS

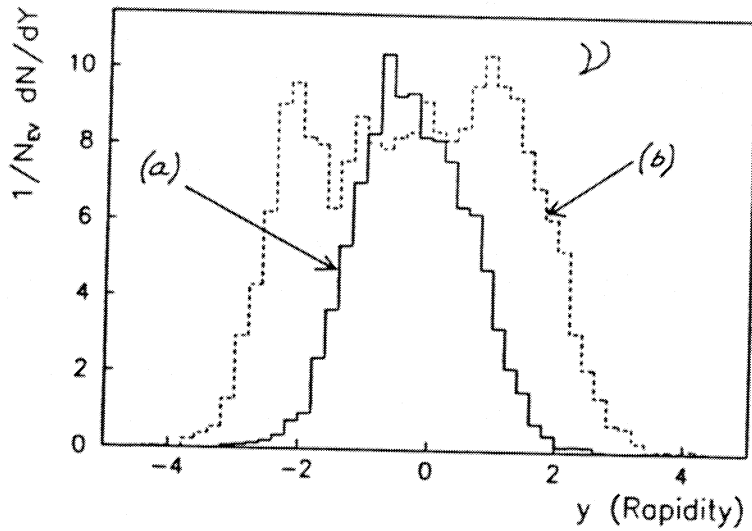


Figure 5.4: Rapidity for positive hadrons (a) $W < 3$, (b) $W > 8$

Distinction between fragments from the struck quark (current fragments) and the spectator diquark (target fragments) becomes easier as W increases. This is probably best seen by considering the centre of mass rapidity for tracks at fixed W . (As an example, the neutrino data is used.) At low W ($< 3 \text{ GeV}/c^2$) the rapidity distribution for positive hadrons is peaked around $y = 0$ (figure 5.4.a). But at high W ($> 8 \text{ GeV}/c^2$) there is a quite pronounced plateau which extends over more than 3 units of rapidity (figure 5.4.b) and indicates some separation of fragments. A similar effect can be seen for negative

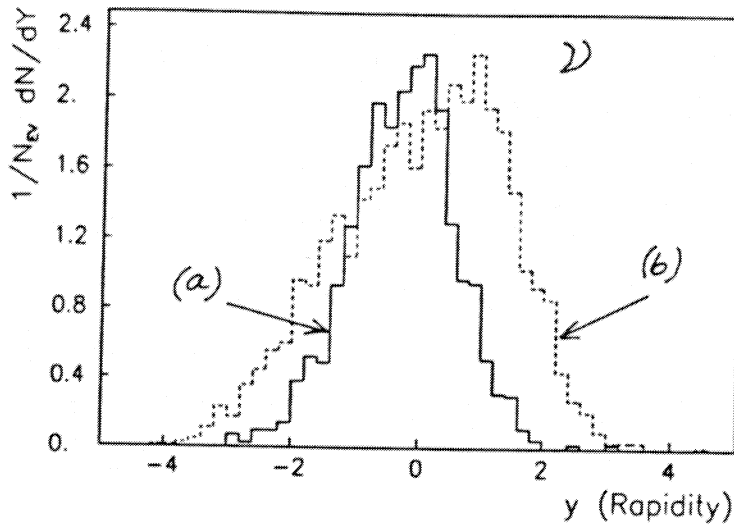


Figure 5.5: Rapidity for negative hadrons (a) $W < 3$, (b) $W > 8$

hadrons (figure 5.5), though not so striking since the negative multiplicity in neutrino events is typically lower than the positive multiplicity. In this chapter, $x_f > 0$ has been used to select current fragments for events with $W > 3 \text{ GeV}/c^2$.

A roughly equivalent separation can be brought about by using fragments with $z > 0.2$. This cut is also improved by demanding $W > 3 \text{ GeV}/c^2$. In such events target fragments are mostly concentrated below $z = 0.2$. This cut will be used frequently in the next chapter.

5.4 FRAGMENTATION FUNCTIONS

The basic definition of a fragmentation function is given in chapter I. Experimentally they are measured as

$$D_q^h(z) = \frac{1}{N_{\text{evts}}} \frac{dN_q^h}{dz} \quad (5.1)$$

where dN_q^h is the number of hadrons of type 'h' arising from the fragmentation of a quark (or quark system) of type 'q' in an interval

dz. In neutrino scattering, in general, the struck quark is of known flavour (neglecting sea contributions) and hence explicit measurement of $D_q^h(z)$ is possible for U and D quarks. Whilst the current fragments are of reasonably well defined origin in this experiment, the target fragments are not so well understood. Since separation of interactions on protons and neutrons is not possible, it is impossible to say whether the target diquark system was composed of UU or UD (UD or DD) quarks in neutrino (antineutrino) interactions. Fragmentation results of the diquark system will not be discussed for this reason. Figure 5.6 shows the fragmentation functions for U (neutrino events) and D (antineutrino events) quarks into π^+ and π^- . Only tracks with $x_f > 0$ in events with $W > 3$ have been used. Correction for smearing in z have not been applied. Such corrections tend to have most effect at high z and can exert a great influence on final conclusions. Correction for particle mis-identification has also not been applied. The depletions at low z are a result of the W and x_f cuts, which have caused some soft, current fragment tracks to be excluded.

Isospin invariance during hadronization requires $D_u^{\pi^+} = D_d^{\pi^-}$ and $D_d^{\pi^+} = D_u^{\pi^-}$. A small excess can be seen in the π^+ fragmentation functions when compared with the same distributions for π^- . This may be explained by proton contamination. Figure 5.7 shows that within experimental errors these relations are approximately verified.

The integral of equation (5.1) gives the average multiplicity of hadrons of type 'h', i.e.

$$\langle N^h \rangle = \int_0^1 D^h(z) dz \quad (5.2)$$

The values for $\langle N^h \rangle$ are summarized in table 5.3 and can be compared with explicit measurements in table 5.1.

In a similar manner, the average momentum fraction carried by hadrons of type 'h' in an event can be estimated from

$$\text{momentum fraction} = \int_0^1 z D^h(z) dz \quad (5.3)$$

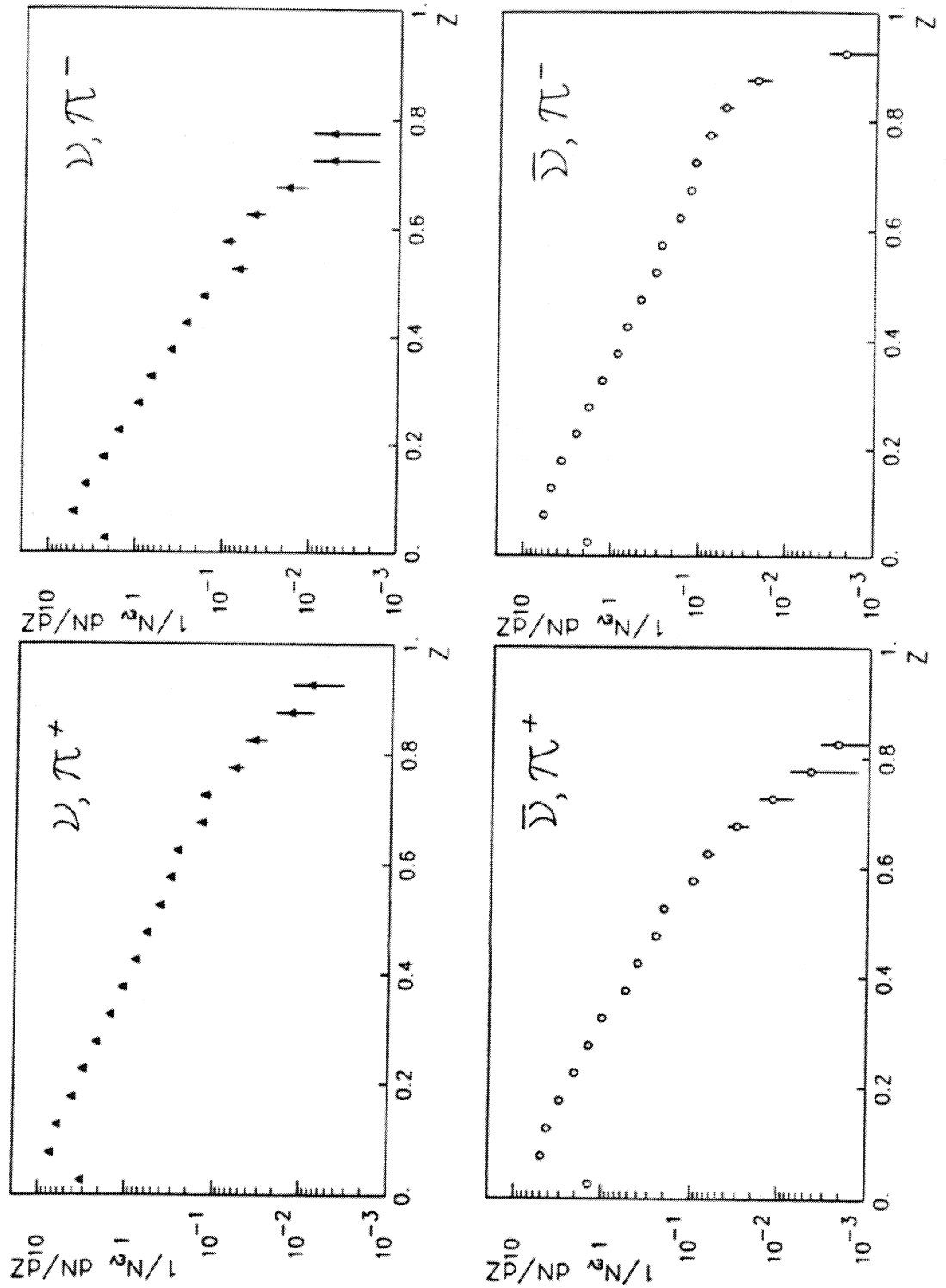


Figure 5.6: ν and $\bar{\nu}$ fragmentation functions for π^+ and π^-

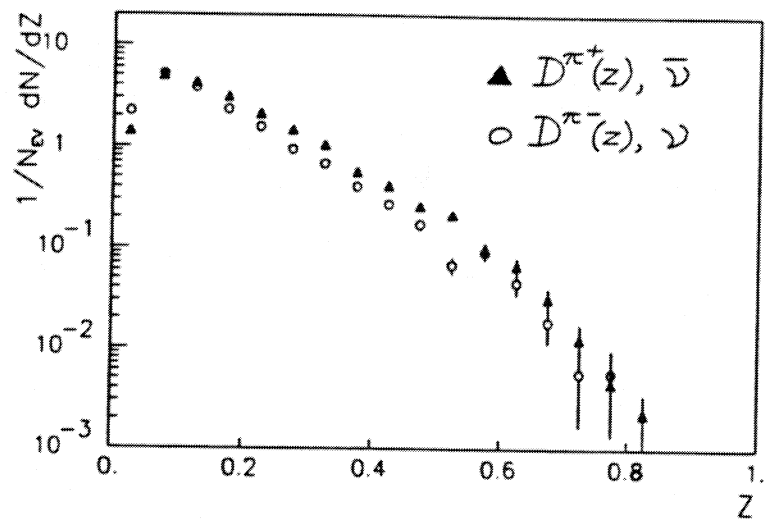
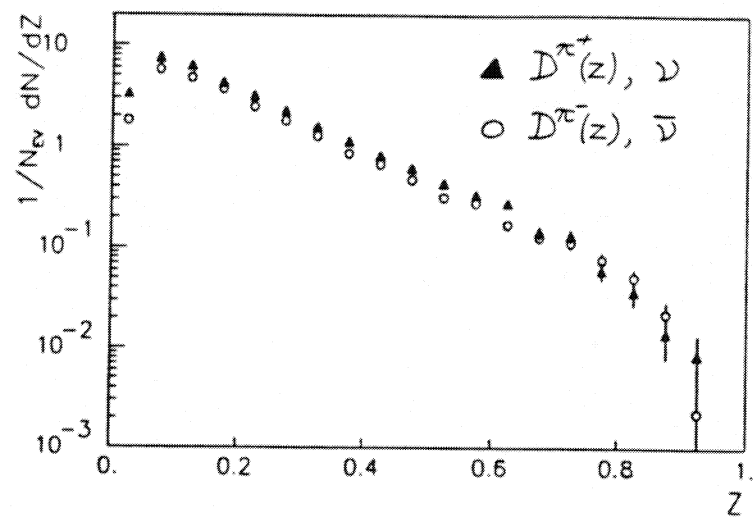


Figure 5.7: Isospin test of fragmentation functions

Table 5.4 shows the values obtained in this analysis.

TABLE 5.3				
Integral of Fragmentation Functions				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\int$	1.45	2.65	1.71	1.52

TABLE 5.4				
Average Momentum Fraction Carried by Hadrons of Type h				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
<i>FRACT.</i>	0.14	0.34	0.24	0.19

5.5 LEADING PARTICLE EFFECTS

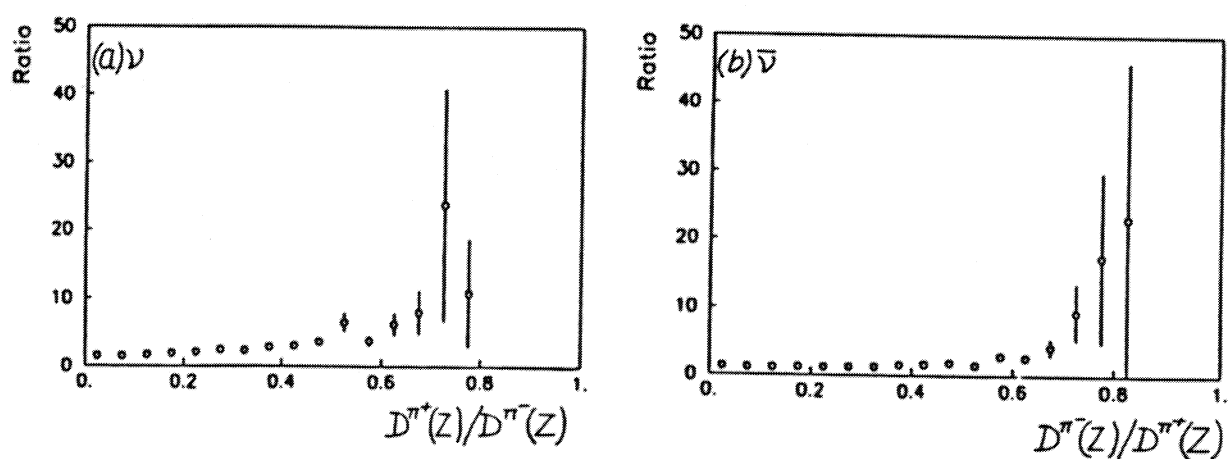


Figure 5.8: Ratio of fragmentation functions for (a) ν and (b) $\bar{\nu}$

In the cascade picture of fragmentation (see chapter I) the primary meson (also expected to be the most energetic) is limited in flavour and charge possible. Neglecting strange sea contributions in the cascade, a U (D) quark can pick up a $\bar{D}$ ($\bar{U}$) quark to form a π^+ (π^0) or π^0 (π^-) meson respectively. Hence, in neutrino (antineutrino) interactions the leading charged meson is expected to be a π^+ (π^-). This feature of hadronization is found to have relevance in chapter VI.

In neutrino interactions the ratio $D^{\pi^+}(z) / D^{\pi^-}(z)$ is expected to approach infinity as z approaches unity. The inverse of this quantity is expected to behave in the same way for antineutrino interactions. This is upheld experimentally as shown in figures 5.8a & b. The ratio at low z (in either plot) is unity, but above $z \approx 0.5$ it increases rapidly. A similar result has been found in antineutrino interactions in the Fermilab 15' bubble chamber [5.5].

5.6 ORIGIN OF THE SEMI-INCLUSIVE HADRON CROSS-SECTION

As suggested in chapter I, the semi-inclusive cross-section for hadron production of type 'h' results from combining the parameterizations of the parton density distributions and fragmentation functions with the calculable parton-current interaction cross-sections [5.6], thus

$$\frac{d\sigma^h}{dx dy dz dP_t' d\phi} = \sum_{ab} \int dx_p dz_p dP_t d\xi d\xi' \delta(x - \xi x_p) \delta(z - \xi' z_p) \quad (5.4)$$

$$\times \delta(P_t' - \xi' P_t) F_a(\xi, Q^2) \frac{d\sigma_{ab}}{dx_p dy dz_p dP_t d\phi} D_b^h(\xi', Q^2)$$

where $a(b)$ is the initial (final) parton of momentum P_a (P_b), $x_p = Q^2/2P_a \cdot q$ and $z_p = P_a \cdot P_b / P_a \cdot q$. The function $F_a(\xi, Q^2)$ is the probability density function for finding a parton of type 'a' with momentum fraction ξ of the nucleon momentum and $D_b^h(\xi', Q^2)$ is the probability for parton 'b' to fragment into hadron 'h' with momentum fraction ξ' . The parton-current cross-section is given by

$$\frac{d\sigma_{ab}}{dx_p dy dz_p dP_t d\phi} = \frac{\alpha_s(Q^2) G_f^2 Q^2}{(2\pi)^3 y} \frac{K \delta[P_t - (Q^2 z_p (1-z_p)(1-x_p))^{\frac{1}{2}}]}{x_p} \quad (5.5)$$

$$\times (A + B \cos\phi + C \cos 2\phi)$$

This contains contributions from zeroth and first order QCD diagrams and suffers from singularities in the limit of soft gluons. These can be absorbed into the parton distribution and fragmentation functions in the form of a Q^2 dependence, the evolution of which can be predicted by the Altarelli-Parisi equations [5.7]. Thus scaling violations in fragmentation functions may be expected.

In addition to the Q^2 scale breaking in D_b^h an x dependence is also predicted, thereby breaking factorization of the interaction and fragmentation processes.

5.7 SCALING VIOLATION AND FACTORIZATION BREAKING

To search for scaling violation effects the data was divided into two Q^2 bins, namely $1 < Q^2 < 4 \text{ (GeV/c}^2\text{)}^2$ and $Q^2 > 6 \text{ (GeV/c}^2\text{)}^2$. Figure 5.9 shows $D^{\pi^\pm}(z)$ for neutrino and antineutrino interactions in these bins with $x_f > 0$ and $W > 3 \text{ GeV/c}^2$. Again a dip at low z is evident. The apparent difference between the two curves in this region is false, as the cut in W removes more low z tracks for low Q^2 than for high Q^2 . This is a consequence of the correlation between W and Q^2 (equation 1.4). There is no evidence for scaling violation in any of the channels shown. Figure 5.10, however, shows the same distributions as above, but this time including all W and x_f (and hence the backward fragments). Only the results for π^- 's have been shown since π^+ 's are contaminated by protons up to quite high z . A significant excess can be seen at low z for high Q^2 events compared with low Q^2 in both neutrino and antineutrino. This may be taken as evidence of the occurrence of scaling violations. However, although not expected to have a Q^2 dependence, the target fragments are substantial in this region and may exert some influence.

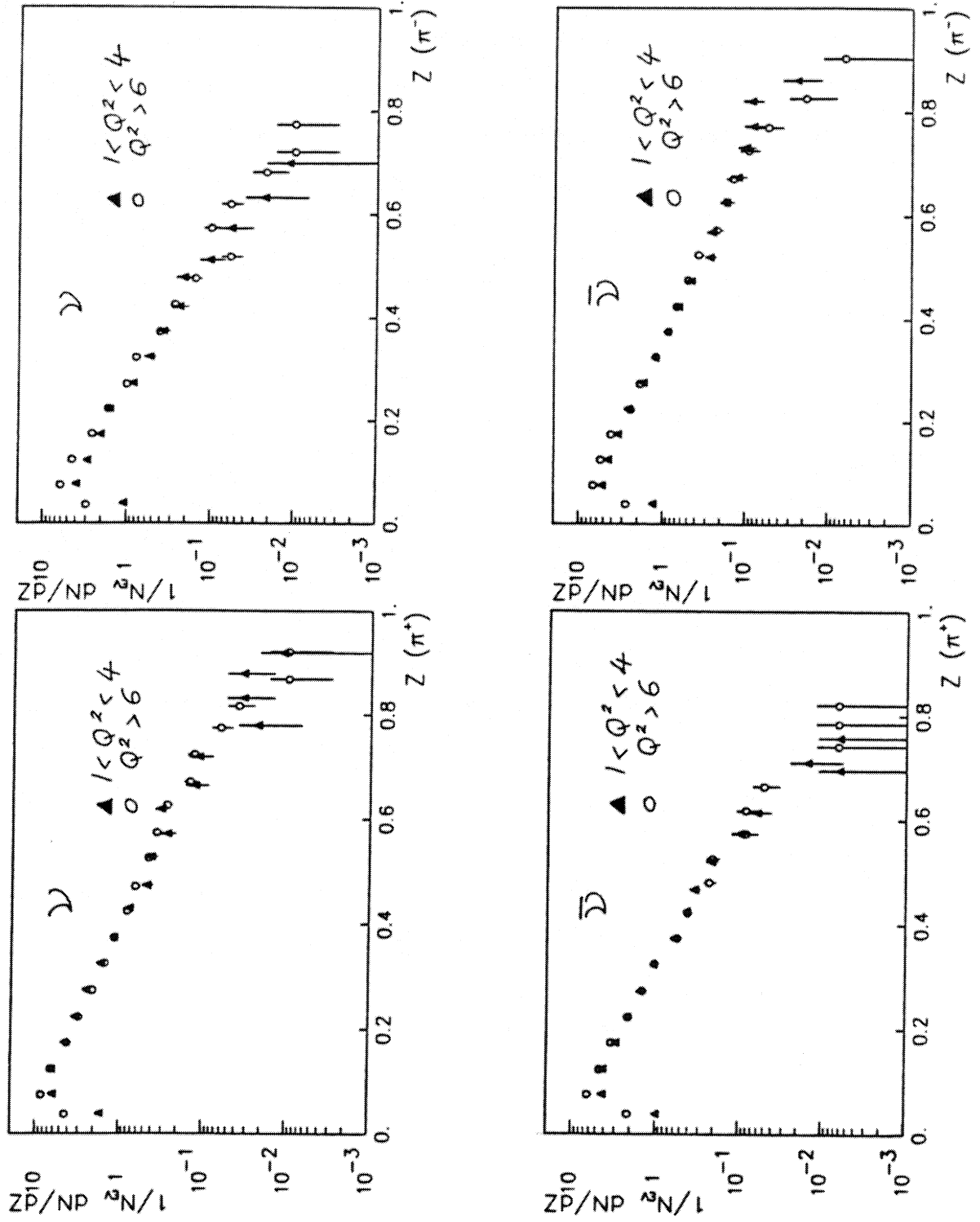


Figure 5.9: $D^{\pi^{\pm}}(z)$ for two ranges of Q^2

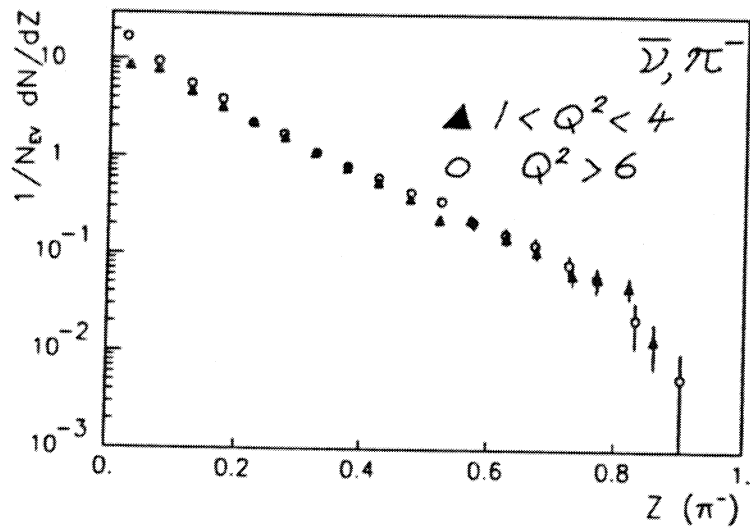
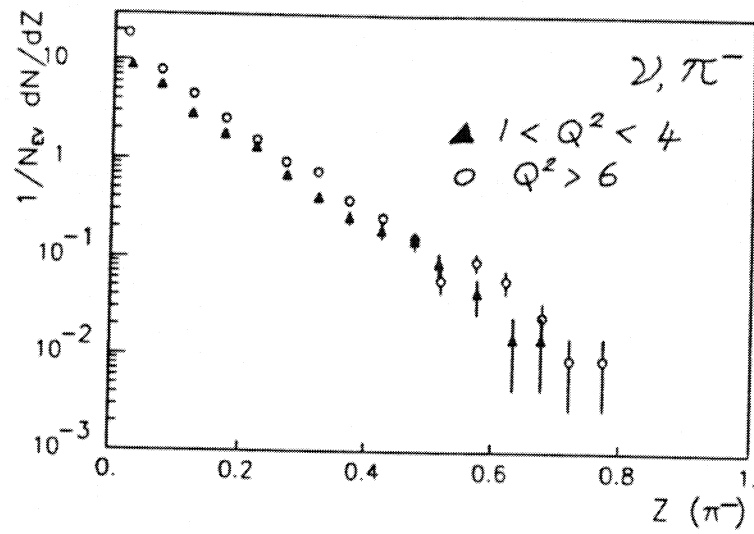


Figure 5.10: $D^{\pi^-}(z)$ for two ranges of Q^2 (no W or x_f cuts)

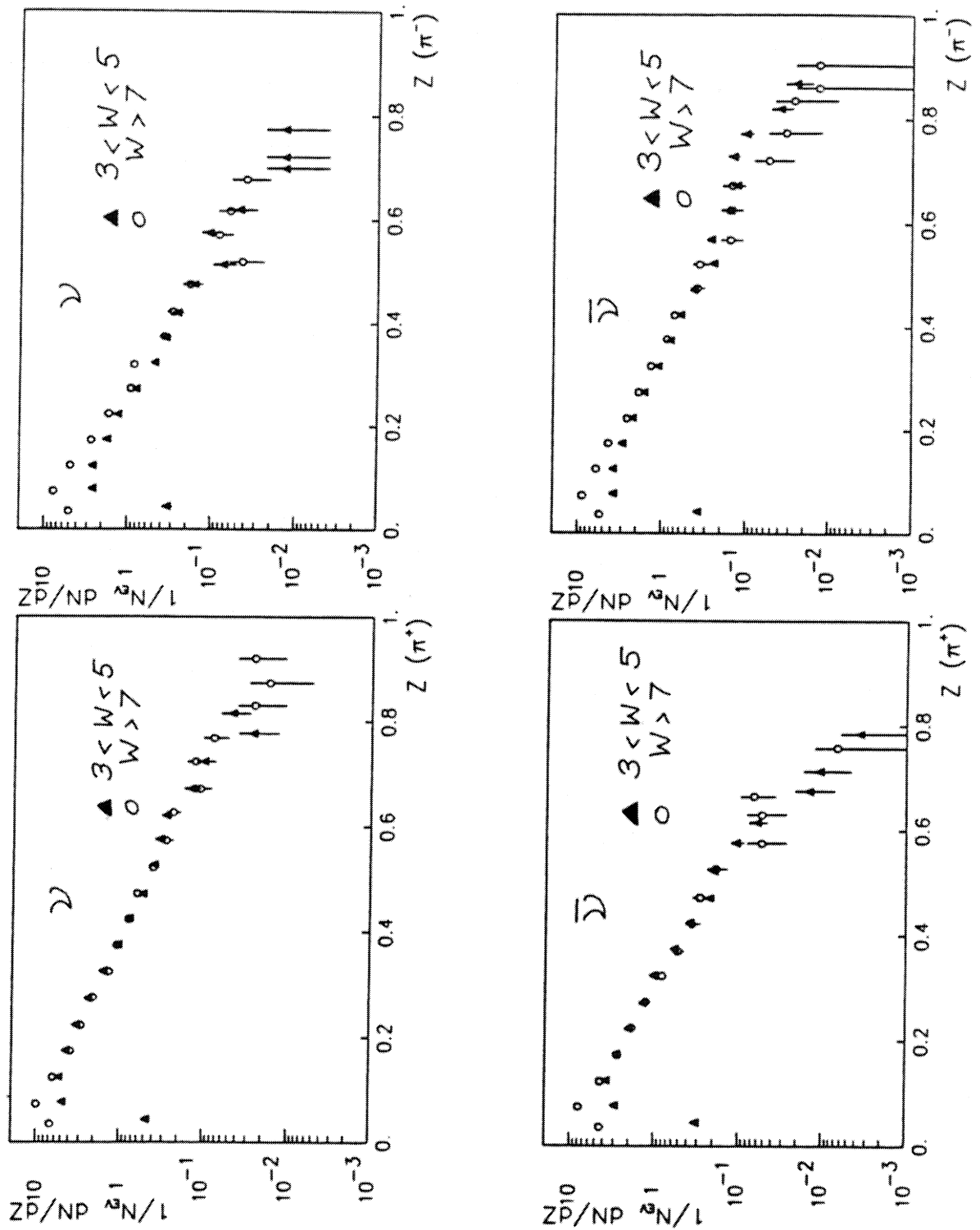


Figure 5.11: $D^{\pi^{\pm}}(z)$ for two ranges of W

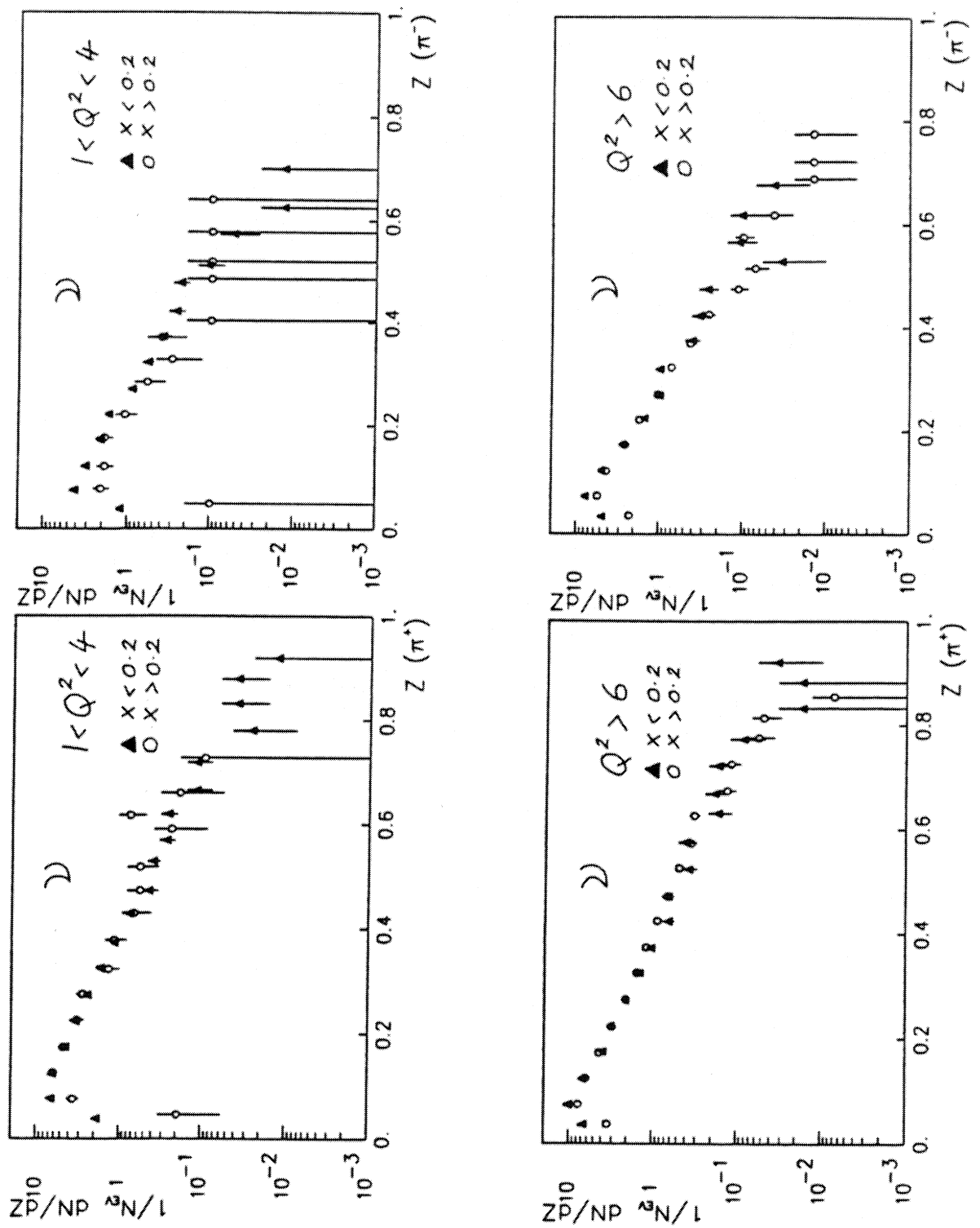


Figure 5.12: $D^{\pi^\pm}(z, Q^2)$ for two ranges of x , neutrino

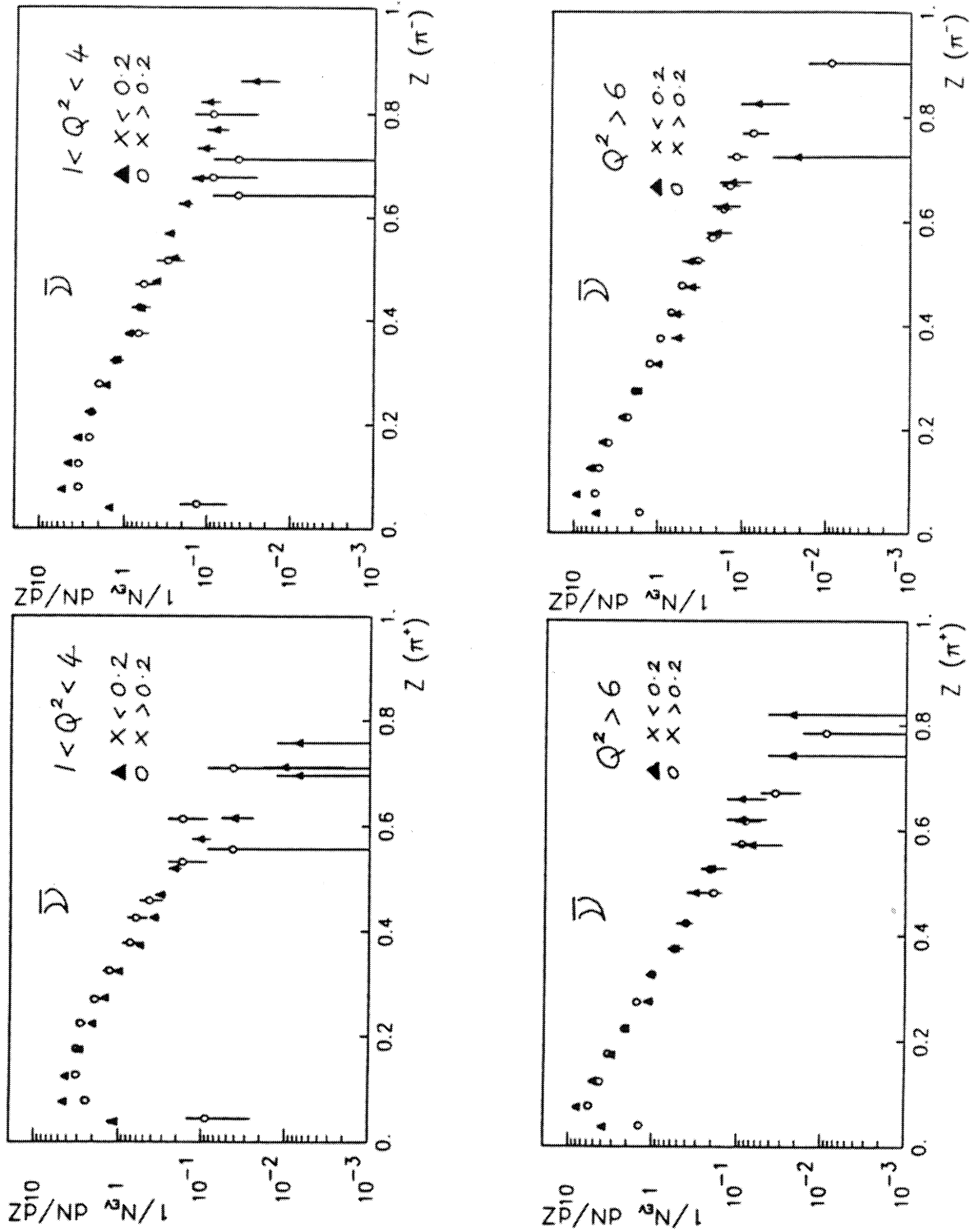


Figure 5.13: $D^{\pi^\pm}(z, Q^2)$ for two ranges of x , antineutrino

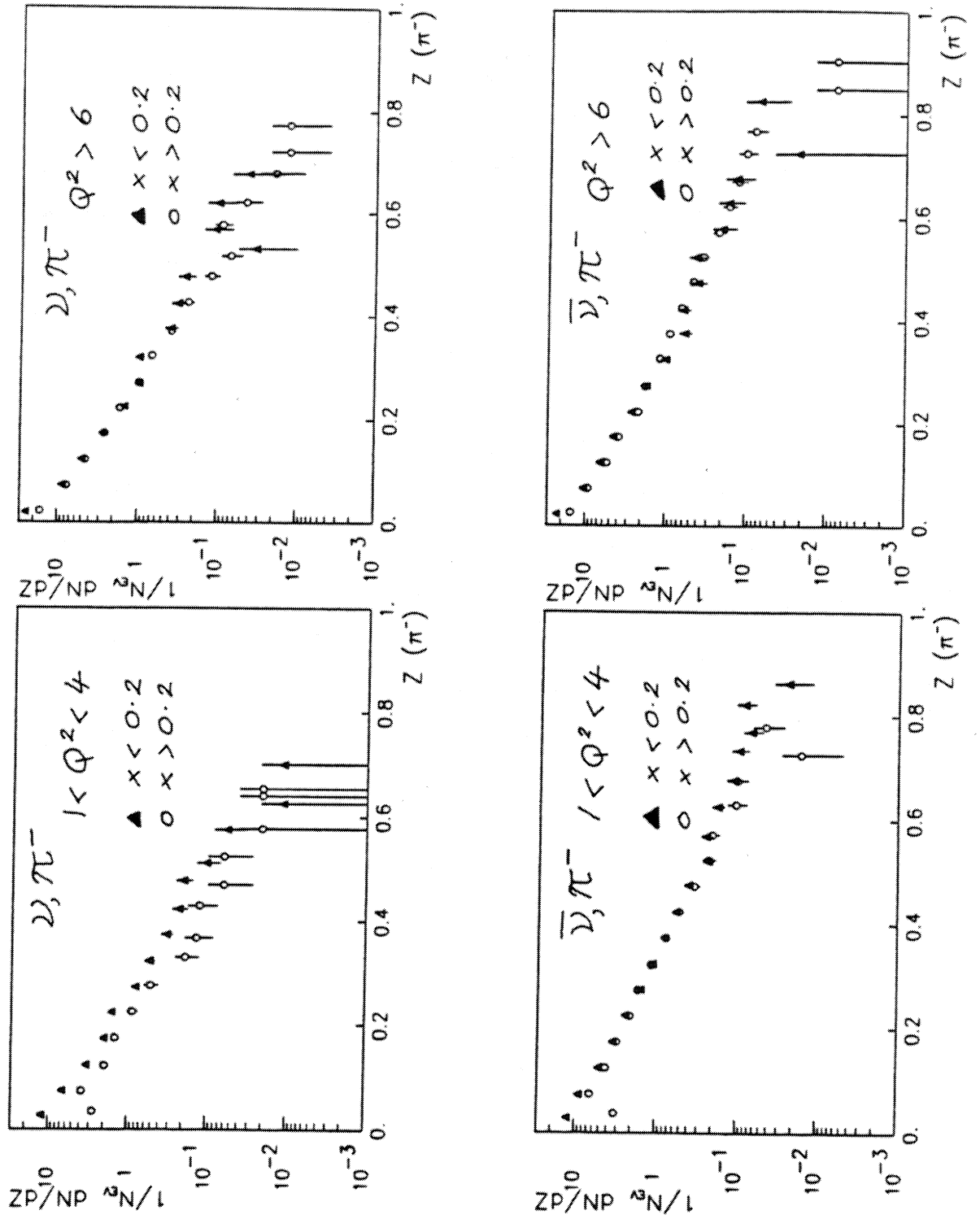


Figure 5.14: $D^{\pi^-}(z, Q^2)$ for two ranges of x (no W or x_f cuts)

Ignoring the low z region, no variation of $D^{\pi^{\pm}}(z)$ is found with W (figure 5.11) for the ranges $3 < W < 5 \text{ GeV}/c^2$ and $W > 7 \text{ GeV}/c^2$ for pions with $x_f > 0$.

Within the Q^2 ranges defined above, the data was further divided between low (< 0.2) and high (> 0.2) x . Figure 5.12 show $D^{\pi^{\pm}}(z, Q^2)$ for these regions for events with $W > 3 \text{ GeV}/c^2$ and $x_f > 0$. The depletion at low z persists (for the reasons given above). Elsewhere in z no difference between curves is detectable at low or high Q^2 in any of the channels, within errors. Figure 5.14 shows the same as the above, but without W and x_f cuts for π^- mesons only. Within the Q^2 ranges defined, a variation of the fragmentation functions with x is observed at low z , which may indicate a breaking of factorization. Once again the comments made on target fragments and scale breaking may apply.

On the whole, measurements of scale and factorization breaking are beset with problems, since observed effects are in regions where current and target fragments are most difficult to separate. Scaling deviations have been reported by a number of experiments (e.g. references 5.8, 5.9 and 5.10) based on moments analyses. Non-factorization effects have also been reported (e.g. reference 5.11). On the other hand another experiment, using antineutrinos [5.12], has reported no scaling violation or factorization breaking. Mis-identification of a proton as a π^+ can cause spurious scale breaking at low Q^2 and low W (e.g. a fast ($> 1 \text{ GeV}/c$) target fragment proton can move into the forward region when assigned the pion mass). Smearing can cause the leakage of slow pions between forward and backward regions and give rise to anomalies at low z . To some extent this is avoided by taking z moments, thereby weighting to higher z , where the particle origin is more certain. However, this too is problematic in that the high z particles are also badly smeared and the effects seem to be at low z anyway.

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Chapter VI

TRANSVERSE AND AZIMUTHAL PROPERTIES OF THE HADRON SHOWER

6.1 INTRODUCTION

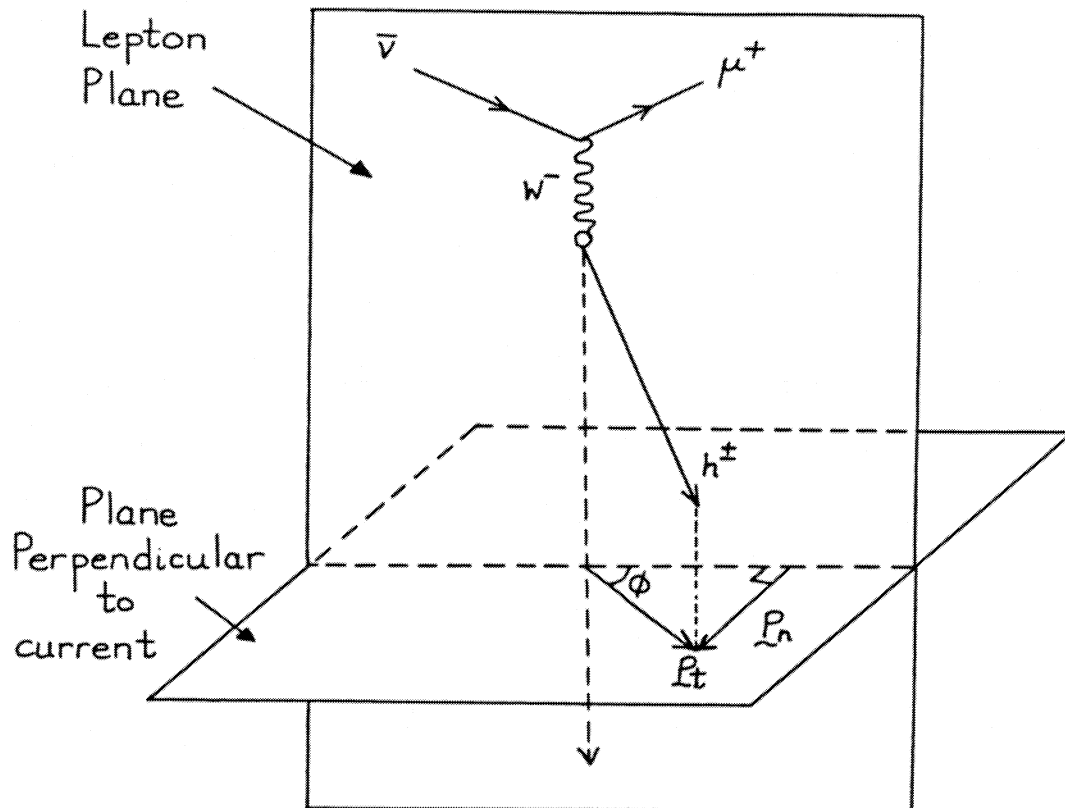


Figure 6.1: Definition of P_t and ϕ in neutrino interactions

Within the framework of the simple parton model there is no scope for explaining the presence of hadronic final state transverse momenta

(P_t), and azimuthal angle (ϕ). A more detailed analysis incorporating the primordial transverse momentum of confinement, k_t , and gluon radiation processes in perturbative QCD is more able to explain experimental results. Figure 6.1 defines P_t and ϕ in neutrino interactions. ϕ is measured from the projection of the outgoing muon direction into a plane perpendicular to the current direction as origin. The "leptonic plane" is defined by the incoming and outgoing leptons and by momentum conservation also contains the current. It is often useful to discuss P_n , the component of P_t perpendicular to the lepton plane (see later).

In this chapter, two possible sources of asymmetry in ϕ for final state pions will be discussed and evidence for such an effect presented. Some results on transverse momenta will also be mentioned.

6.2 TRANSVERSE MOMENTA

6.2.1 Sources

It has been mentioned already that in the naive parton model there is no scope for final state transverse momenta, P_t , with respect to the current direction. However, a little more thought shows that observed P_t 's can come from a number of contributions, namely

$$\langle P_t^2 \rangle = (\langle k_t^2 \rangle + \langle P_{t \text{ QCD}}^2 \rangle) \langle z^2 \rangle + \langle P_{t \text{ FRAG}}^2 \rangle \quad (6.1)$$

where k_t = non-perturbative transverse momentum
(see section 6.3.3)

$P_{t \text{ QCD}}$ = contribution from gluon bremsstrahlung

and $P_{t \text{ FRAG}}$ = contribution from hadronization.

The factor $\langle z^2 \rangle$ multiplies both $\langle k_t^2 \rangle$ and $\langle P_{t \text{ QCD}}^2 \rangle$ as these two quantities relate to a parton before fragmentation. A fuller discussion of P_t sources is to be found in reference 6.1. It is expected that distributions in k_t and $P_{t \text{ FRAG}}$ should be Gaussian, with average values of a few hundred MeV/c and independent of event energy.

However, the P_t resulting from gluon bremsstrahlung is believed to be energy dependent.

6.2.2 Dependence on W

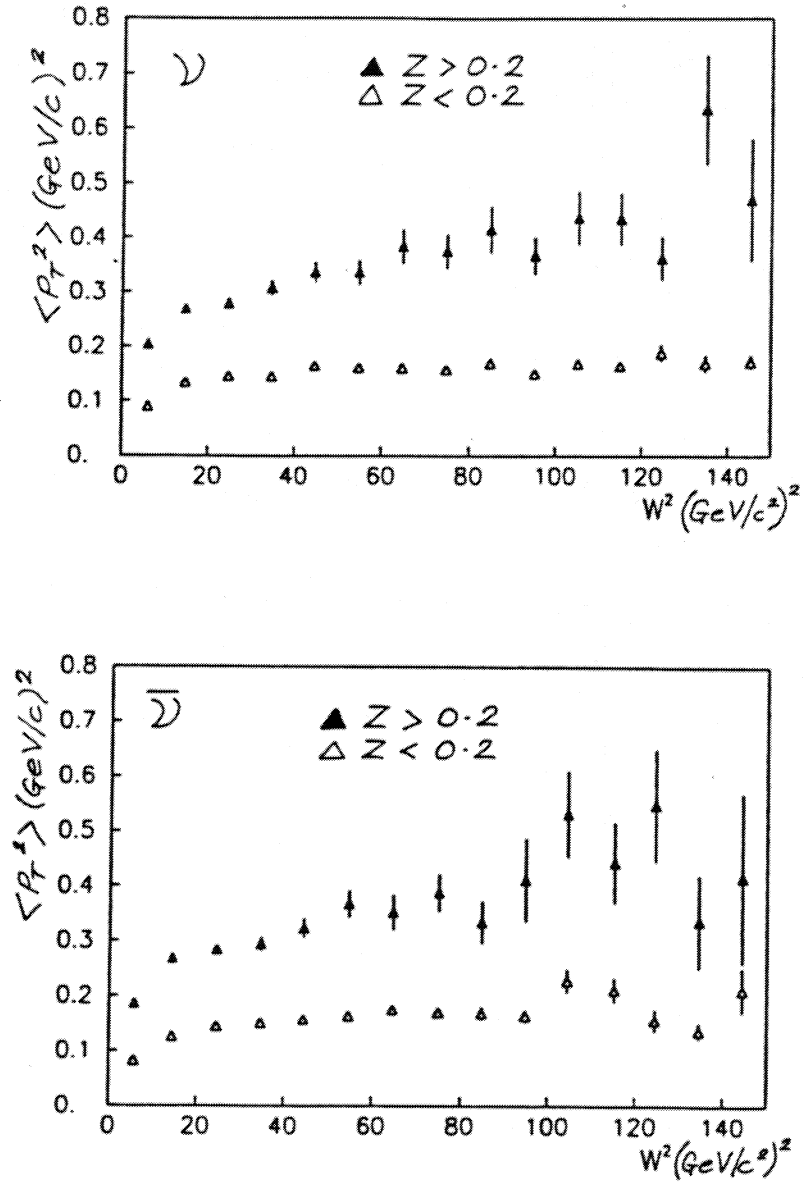


Figure 6.2: P_t as a function of W^2 , (a) $z < 0.2$, (b) $z > 0.2$

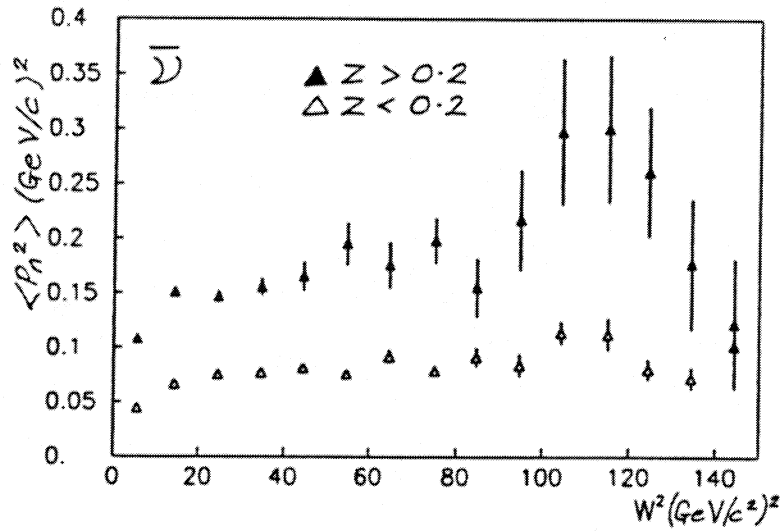
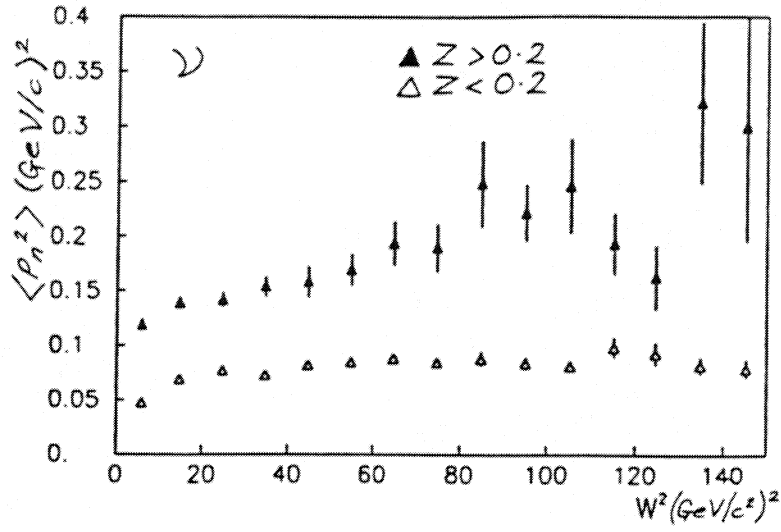


Figure 6.3: P_n as a function of W^2 , (a) $z < 0.2$, (b) $z > 0.2$

$\langle P_t^2 \rangle$ is shown as a function of W^2 in figure 6.2 for charged pions with (a) $z < 0.2$ and (b) $z > 0.2$. These two categories are used to separate current and target fragments. After an initial rise, the $z < 0.2$ fragments show $\langle P_t^2 \rangle$ more or less constant with W^2 , but for $z > 0.2$ there is a steady rise with W^2 . It is not clear whether this is due to more phase space at high W in the forward region or to gluon

bremsstrahlung from the struck quark. Other neutrino experiments [6.2, 6.3] have observed similar effects.

A more reliable variable to use is P_n^2 (figure 6.1) as this avoids the effects of smearing in the lepton plane. It is defined by $P_n^2 = P_t^2 \sin^2 \phi$. Assuming symmetry in ϕ , $\langle P_n^2 \rangle = \frac{1}{2} \langle P_t^2 \rangle$. Figure 6.3 shows the same as figure 6.2 but using $\langle P_n^2 \rangle$. The same dependence on W is observed and the above relation is approximately verified.

6.2.3 Dependence on Q^2

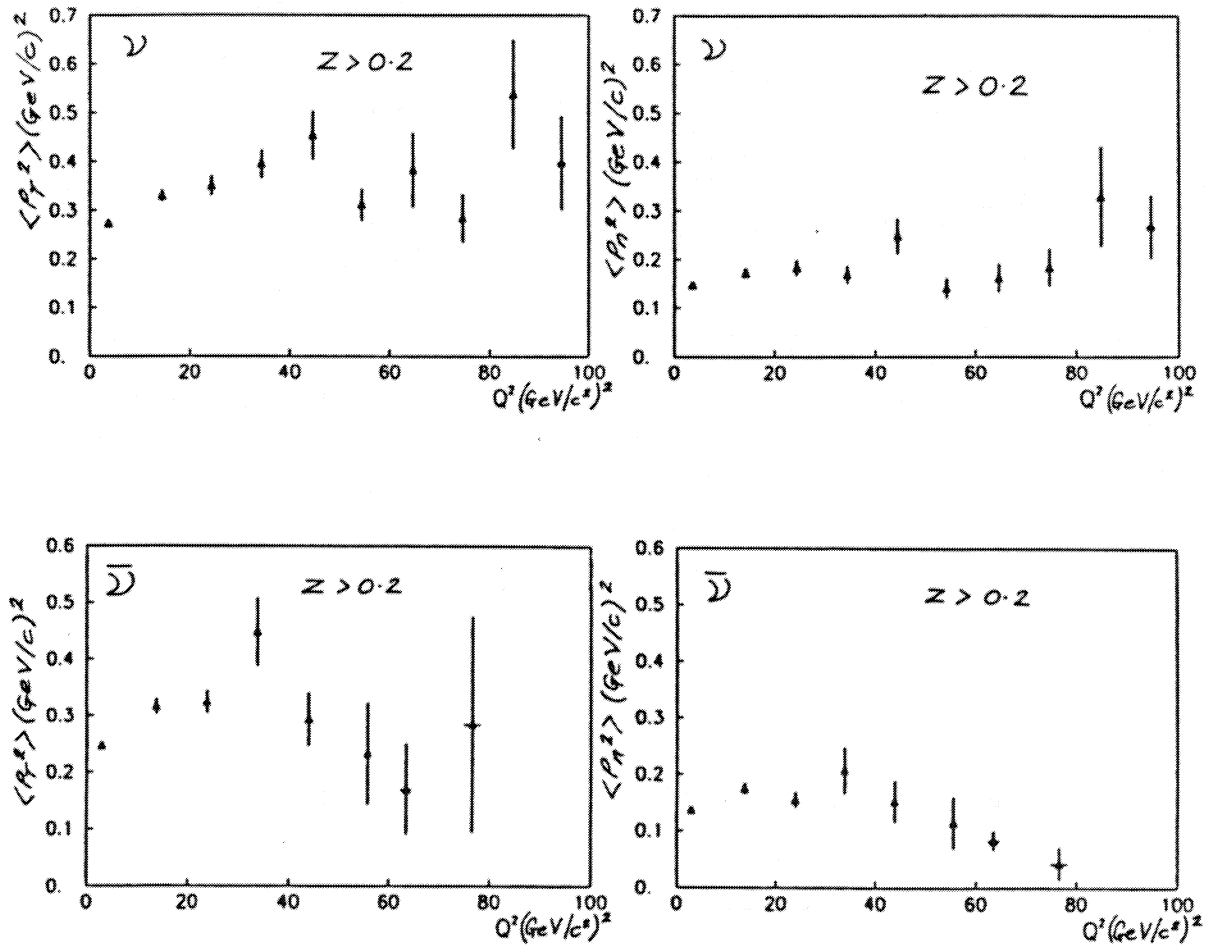


Figure 6.4: (a) P_t & (b) P_n as a function of Q^2 , $z > 0.2$

Figure 6.4 shows the dependence on Q^2 of $\langle P_t^2 \rangle$ and $\langle P_n^2 \rangle$ for current fragments ($z > 0.2$). Both $\langle P_t^2 \rangle$ and $\langle P_n^2 \rangle$ tend to fall off for high Q^2 in antineutrino events but are consistent with being flat for neutrino events. Some predictions are to be found in reference 6.4 with which the data is qualitatively consistent. The turn-over of the antineutrino distributions, after an initial rise, is expected from phase space limitations.

6.3 SOURCES OF AZIMUTHAL ASYMMETRY

Ravndal [6.5] has pointed out that an asymmetry in the azimuthal angle, ϕ , can come about from parton model considerations. Georgi and Politzer [6.6] have suggested that similar effects may be observed as a result of QCD processes. In what follows, a brief argument for an asymmetry is followed by an elaboration on the above two ideas.

6.3.1 Intuitive Picture

A rigorous derivation of the origin of an azimuthal asymmetry is beyond the scope of this thesis, however, it is relatively easy to see how it may come about.

The lepto-production cross-section is proportional to

$$d\sigma \propto \frac{2E'}{E} (j_\mu J^\mu)^2 \quad (6.2)$$

where j_μ and J^μ are the leptonic and hadronic currents respectively and E (E') is the incoming (outgoing) lepton energy. It is easiest to work in an helicity basis defined by

$$\begin{aligned} q &= (q^0:0,0,|q|) && \text{current vector} \\ \epsilon_{\ell,r} &= \sqrt{\frac{1}{2}}(0:1,\pm i,0) && \text{left,right polarization vectors} \\ \epsilon_s &= 1/\sqrt{Q^2} (|q|:0,0,q^0) && \text{longitudinal polarization vector.} \end{aligned} \quad (6.3)$$

In this basis $j_\mu = A_{\ell(r)}\epsilon_{\mu\ell} + A_{r(\ell)}\epsilon_{\mu r} + A_s\epsilon_{\mu s}$ for a neutrino (antineutrino), with $A_\ell = \sqrt{(E/2E')}$, $A_r = \sqrt{(E'/2E)}$ and $A_s = 1$. The incoming neutrino and outgoing muon form the lepton plane, whilst the incoming and outgoing quark define a hadronic plane inclined at an angle ϕ to the lepton plane. The latter plane can only be defined if the incoming quark (for one reason or another) has a momentum component transverse to the current direction. Conserving helicity in the hadronic frame the lepton current is rotated thus

$$j_\mu = A_{\ell(r)}\epsilon_{\mu\ell}e^{-i\phi} + A_{r(\ell)}\epsilon_{\mu r}e^{i\phi} + A_s\epsilon_{\mu s} \quad (6.4)$$

and when contracted with J^μ gives a cross-section proportional to

$$\begin{aligned} d\sigma_\nu &= [1+(1-y)^2]d\sigma_{\ell\ell} + 2(1-y)d\sigma_{ss} \\ &+ 2(1-y)[\text{Re } d\sigma_{\ell r}\cos 2\phi - \text{Im } d\sigma_{\ell r}\sin 2\phi] \\ &+ 2\sqrt{2}[(1-y)^{\frac{1}{2}} + (1-y)^{3/2}][\text{Re } d\sigma_{\ell s}\cos \phi - \text{Im } d\sigma_{\ell s}\sin \phi] \end{aligned} \quad (6.5)$$

for neutrinos and

$$\begin{aligned} d\sigma_\nu &= [1+(1-y)^2]d\sigma_{rr} + 2(1-y)d\sigma_{ss} \\ &+ 2(1-y)[\text{Re } d\sigma_{\ell r}\cos 2\phi - \text{Im } d\sigma_{\ell r}\sin 2\phi] \\ &+ 2\sqrt{2}[(1-y)^{\frac{1}{2}} + (1-y)^{3/2}][\text{Re } d\sigma_{rs}\cos \phi + \text{Im } d\sigma_{rs}\sin \phi] \end{aligned} \quad (6.6)$$

for antineutrinos, where $d\sigma_{\alpha\beta} = (\epsilon_{\mu\alpha}J^\mu)^*(\epsilon_{\nu\beta}J^\nu)$, $\alpha, \beta = \ell, r, s$ and gives the hadronic component of the cross-section for different polarization states of the current. It can be seen that angular asymmetries can arise from interference terms. $\sin\phi$ and $\sin 2\phi$ terms change sign under time reversal and if the latter is assumed to be an exact symmetry it can be neglected in the approximation that final state interactions are ignored [5.6] (see section 6.8). The term in $\cos\phi$ comes about owing to interference between longitudinal and transverse polarizations whilst the $\cos 2\phi$ term can be attributed to interference between left and right transverse polarizations. This is more easily seen if the current's wave function is cast in terms of spherical harmonics

$$\begin{aligned}
 \ell &\rightarrow Y_1^1 && \propto -\sin\theta e^{i\phi} \\
 s &\rightarrow Y_1^0 && \propto \cos\theta \\
 r &\rightarrow Y_1^{-1} && \propto \sin\theta e^{-i\phi}
 \end{aligned}
 \tag{6.7}$$

Interference terms between ℓ and s or r and s spherical harmonics gives rise to $\cos\phi$ dependences whilst interference between ℓ and r gives rise to $\cos 2\phi$ effects.

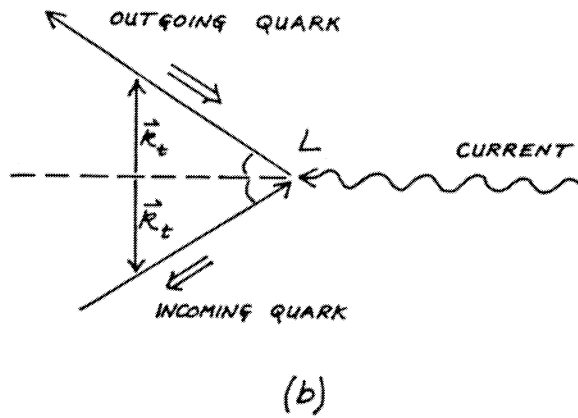
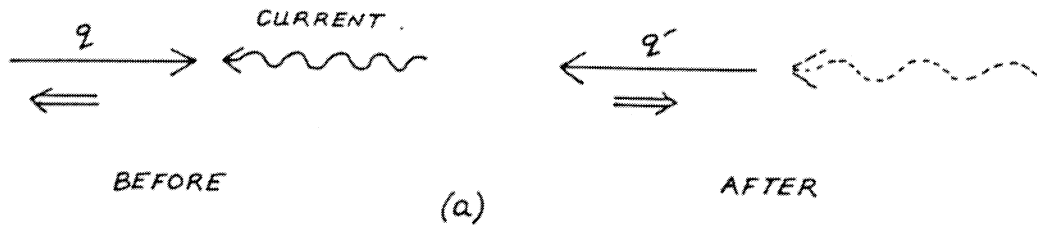


Figure 6.5: Absorption of (a) transverse & (b) longitudinally polarized currents

A crucial point in the above argument is that the incoming and outgoing quarks should define a plane. As will be seen later, this can come about in one of two ways. First, however, consider the

situation in which the current and quark (q) collide co-linearly (figure 6.5a). In the limit of zero mass, quarks are purely left handed and to conserve helicity the quark spin must flip by 1 unit. This can only happen if the current is transversely polarized ($Y_1^{\pm 1}$). On the other hand, in the situation where the incoming quark has a transverse momentum to the current direction, helicity can be conserved if the quark spin flip is provided by the quark angular momentum relative to the current. This permits absorption of a longitudinally polarized current (figure 6.5b). Hence, any process where a quark P_t is present can also potentially give a non-zero coefficient to $\cos\phi$ terms in equations 6.5 and 6.6. Such a P_t can come about either from a primordial transverse momentum within the nucleon, or as a result of QCD processes. These two possibilities will be discussed separately in the following sections.

6.3.2 QCD Sources of Azimuthal Asymmetry

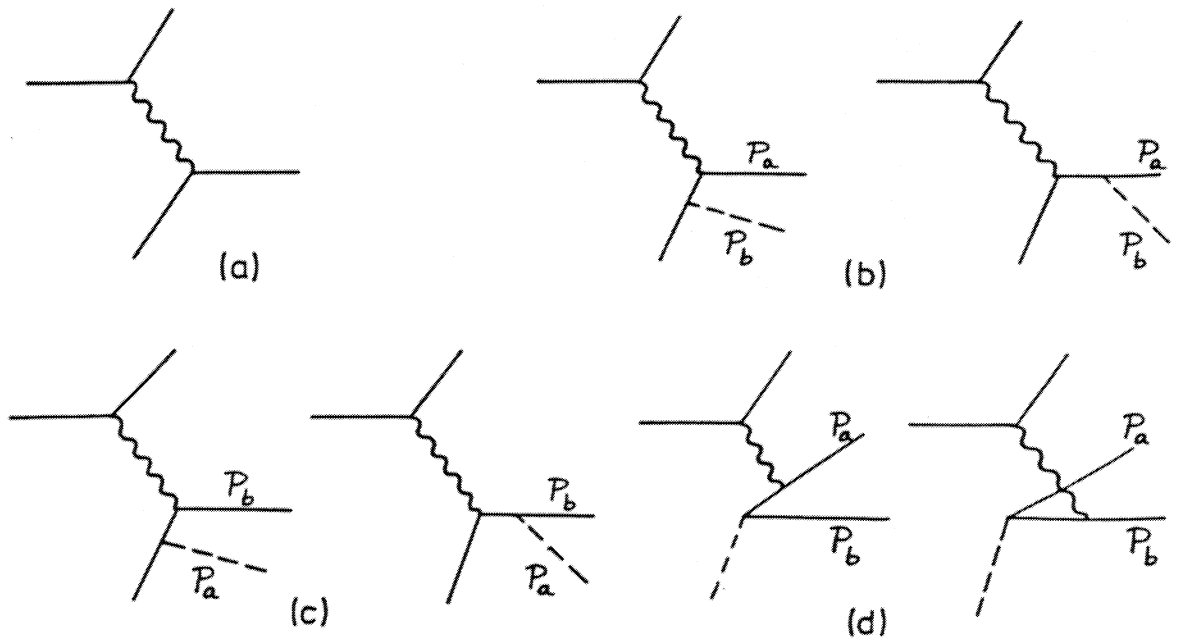


Figure 6.6: Feynman diagrams for zeroth and first order processes

Equations (5.4) and (5.5) give the full semi-inclusive pion production cross-section incorporating QCD effects. The relevant Feynman diagrams are shown in figure 6.6 and give rise to the semi-inclusive cross-section of equation (5.4). Substitution of equation (5.5) in equation (5.4) yields

$$\frac{d\sigma^h}{dx dy dz dP_t' d\phi} = (A_0 + A_i) + B_i \cos\phi + C_i \cos 2\phi \quad (6.8)$$

for the angular dependence of the semi-inclusive cross-section after integration over ξ, ξ' and P_t . Explicit forms for the coefficients A_0, A_i, B_i and C_i are given in reference 5.6. A_0 arises from the zeroth order diagram and carries no angular dependence. A_i, B_i and C_i arise from first order diagrams where, $i=1,2$ and 3 corresponds to b,c and d respectively of figure 6.6. A number of firm predictions can be extracted. Firstly $C_i > 0$, $i=1,2,3$ and hence there is a bias towards pion production in the regions $-45^\circ \leq \phi \leq 45^\circ$ and $135^\circ \leq \phi \leq -135^\circ$ (i.e. close to the lepton plane). Secondly, $B_1 < 0$ indicates greater pion production in the backward ($90^\circ \leq \phi \leq -90^\circ$) than in the forward ($-90^\circ \leq \phi \leq 90^\circ$) hemisphere for pions originating from quark decay in hard gluon bremsstrahlung events (figure 6.6b). Conversely, $B_2 > 0$ indicates higher pion production in the forward hemisphere for pions originating from gluon decay in similar events (figure 6.6c). Thirdly, B_3 can be positive or negative and does not therefore give a preference for either hemisphere (figure 6.6d). The target fragment contributions, A_i , are expected to be ϕ independent.

The relative importance of these contributions is highly dependent on the fragmentation functions assumed for quarks and gluons and any prediction for $\langle \cos\phi \rangle$ is immediately model dependent. When all values of z are included, the sign of $\langle \cos\phi \rangle$ is a result of the relative average multiplicities of quark and gluon fragments. If quark and gluon fragmentation functions are identical then $\langle \cos\phi \rangle = 0$ as the ratio of multiplicities is then unity and full cancellation follows.

It is more usual to make a cut at $z > 0.2$ to remove the effect of target fragments. In this instance terms B_1 and B_2 are both suppressed, but B_2 more so. B_3 also becomes negative and a net negative $\langle \cos\phi \rangle$ should be observed. Softer fragmentation for gluons than for quarks would make $\langle \cos\phi \rangle$ more negative by greater suppression of B_2 . Predictions for $\langle \cos 2\phi \rangle$ are not discussed in detail (see later).

6.3.3 Non-Perturbative Sources of Azimuthal Asymmetry

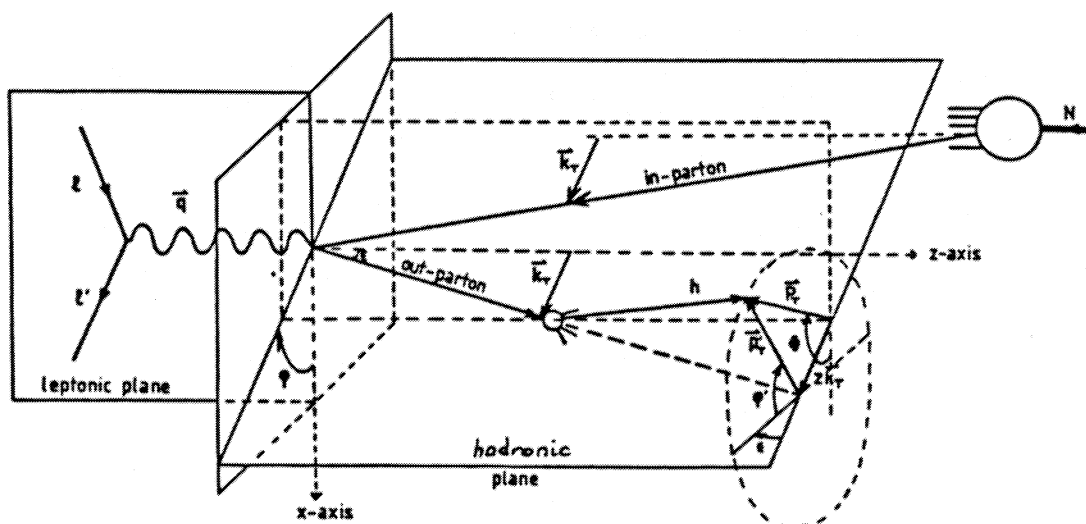


Figure 6.7: Parton scattering with primordial k_t

Cahn [6.7] has pointed out that a negative $\langle \cos\phi \rangle$ can also be expected from a parton model calculation incorporating primordial transverse momentum, k_t . A similar discussion is to be found in reference 6.4. Figure 6.7 portrays the process in detail.

The long range forces of parton confinement inside the nucleus are at present incalculable. Even so, such forces can be described

phenomenologically by a primordial or "intrinsic" transverse momentum distribution, k_t , in the parton distribution function. By momentum conservation k_t and ϕ are preserved through the interaction. A hadron fragment, 'h', takes a fraction, z , of the outgoing parton momentum and picks up an additional transverse momentum P_t' and angle ϕ' (around the decaying parton direction) owing to hadronization. The observable P_t and Φ of 'h' are given by

$$P_t^2 = z^2 k_t^2 + P_t'^2 + 2z k_t P_t' \cos \phi' \quad (6.9)$$

$$\text{and } \Phi = \phi + \tan^{-1}\{(P_t' \sin \phi') / (z k_t + P_t' \cos \phi')\}$$

Assuming k_t and P_t' have Gaussian forms, $\exp\{-ak_t^2\}$ and $\exp\{-bP_t'^2\}$ respectively, it can be shown [6.4] that the semi-inclusive cross-section for this process is

$$\frac{d\sigma}{dx dQ^2 dz d\Phi} = \frac{G_f^2 \int dk_t^2 dP_t'^2 a \cdot \exp\{-ak_t^2\} b \cdot \exp\{-bP_t'^2\} D^\pi(z, Q^2)}{(2\pi)^2} \times \left\{ A + \frac{BI_1 k_t \cos \Phi}{\sqrt{Q^2}} + \frac{CI_2 k_t^2 \cos 2\Phi}{Q^2} \right\} \quad (6.10)$$

where A, B and C are functions of parton density functions and include y dependences. I_1 and I_2 are the results of integrating over ϕ' and are both functions of $z k_t / P_t'$.

The resulting asymmetry is entirely due to intrinsic k_t and modified by the fragmentation P_t' . For $k_t \ll P_t'$ I_1 and I_2 approach 0 and for $P_t' \ll k_t$ approach 1, with I_2 always less than I_1 . Since $\langle k_t \rangle$ is similar to $\langle P_t' \rangle$ (i.e. a few hundred MeV/c or so) neither extreme is reached and a negative $\langle \cos \Phi \rangle$ can be expected (as for QCD).

6.3.4 Specific Predictions

A negative $\langle \cos \Phi \rangle$ is expected, whether arising from QCD or k_t effects. The QCD inspired asymmetry should show a $1/\ln Q^2$ dependence, but a $1/\sqrt{Q^2}$ dependence is expected for a k_t induced asymmetry (which will

thus fall off faster with increasing Q^2). Since both sources of asymmetry may be present, a clear distinction between the two models is not possible, except at high Q^2 , where QCD should dominate. At low z the complications of target fragments dominate, leading to no significant asymmetry.

A negative $\langle \cos\phi \rangle$ implies a tendency for hadrons to be produced on the opposite side of the current direction from the muon, whilst a positive $\langle \cos 2\phi \rangle$ implies a tendency for production close to the lepton plane. A non-zero value of $\langle \sin\phi \rangle$ implies asymmetric production about the lepton plane.

6.4 ASYMMETRY MEASUREMENTS

6.4.1 Experimental Problems

TABLE 6.1				
LPS Monte Carlo Estimate of Biases, $\times 10^2$				
Energy corrected current direction				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\langle \cos\phi \rangle$	-7.03 ± 0.59	-9.62 ± 0.40	-9.75 ± 0.53	-11.64 ± 0.58
$\langle \cos 2\phi \rangle$	-0.56 ± 0.59	0.27 ± 0.41	-1.36 ± 0.55	0.84 ± 0.59
$\langle \sin\phi \rangle$	0.17 ± 0.59	-0.10 ± 0.41	0.86 ± 0.55	0.02 ± 0.58
$\langle \sin 2\phi \rangle$	0.41 ± 0.59	-0.07 ± 0.41	0.08 ± 0.54	0.96 ± 0.58
Projected current direction				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\langle \cos\phi \rangle$	0.95 ± 0.59	0.26 ± 0.40	-0.56 ± 0.53	0.25 ± 0.59
$\langle \cos 2\phi \rangle$	-1.06 ± 0.59	-1.02 ± 0.41	-3.26 ± 0.55	-0.32 ± 0.59
$\langle \sin\phi \rangle$	0.24 ± 0.60	-0.04 ± 0.41	0.89 ± 0.55	-0.17 ± 0.59
$\langle \sin 2\phi \rangle$	-0.16 ± 0.59	0.10 ± 0.40	0.40 ± 0.54	1.07 ± 0.59

Most critical to measurement of $\langle \cos\phi \rangle$ (and to P_t) is a reliable definition of the current direction. LPS Monte Carlo studies have revealed that calculation of this direction using the observed muon and an energy corrected neutrino lead to a systematic negative $\langle \cos\phi \rangle$. However, similar simulation has shown that a suitable definition (which overcomes the loss of missing neutrals) is obtained by projecting the total observed hadron shower momentum (charged and neutral) into the lepton plane. Results of the simulations are shown in table 6.1 for the four orthogonal functions $\cos\phi$, $\cos 2\phi$, $\sin\phi$, $\sin 2\phi$. All values shown are multiplied by 100 for clarity. It can be seen that $\langle \cos 2\phi \rangle$ remains biased whichever method is used and this is the reason why it is not discussed much in what follows. The averages of all four functions ($\cos\phi$, $\cos 2\phi$, $\sin\phi$, $\sin 2\phi$) will be presented, although only $\langle \cos\phi \rangle$ will be discussed in detail. A projected current direction has been used for all subsequent results.

Any asymmetry, however generated, is greatly smeared by the hadronization process. In addition many hadrons come from resonance decays, which, like fragmentation, dilute any asymmetry by adding in further P_t 's and angular distributions. Furthermore, in a nuclear target, dilution also results from re-interactions. Putting all this together, the observation of any asymmetry of significance indicates a greater signal present at the parton level (assuming all dilutions to be random).

6.4.2 Kinematical Cuts

A number of constraints were applied to the data to avoid certain kinematical regions. A minimum of 3 hadrons (including neutrals and $\bar{\nu}$'s) were required in order that a reasonable definition of the current direction could be made. The lowest muon momentum accepted was 5 GeV/c. This cut excluded a fair proportion of NC background events. The minimum W and Q^2 allowed was 3 GeV/c² and 1 (GeV/c²)²

respectively. This removed the resonance region and selected events from the deep inelastic region. Clearly, there is a correlation between these two cuts.

Only charged pions were selected (which included a contamination from charged kaons). In principle π^0 's can also show an asymmetry but matters are complicated by γ detection efficiencies etc. and so have been neglected. By demanding $z > 0.2$ a sample of largely current fragments were selected. Tracks of undefined charge were not used. This was not found to introduce any biases. Unless stated otherwise, these cuts have been applied globally for the remaining results presented in this chapter.

6.4.3 Results

Figure 6.8 shows distributions for $\cos\phi$ and ϕ for π^- and π^+ in antineutrino events. An asymmetry in the $\cos\phi$ distributions is not immediately obvious. Perhaps more obvious is the higher proportion of tracks which lie at ± 180 degrees in the ϕ distribution for π^- compared with $\phi = 0$. The distributions for neutrino events are very similar. However, the average values for all four orthogonal functions under the cuts of section 6.4.2 are given in table 6.2 for neutrino and antineutrino events without correction for any remaining systematic effects. Of most significance are the deviations from zero of $\langle \cos\phi \rangle$ for π^+ in neutrino events and π^- in antineutrino events (the "signal" channels) which are 4.3 and 3.6 standard deviation effects respectively (marked by a '*' in table 6.2). All other entries in the table are within two standard deviations of zero. Errors quoted are statistical. The channels showing an asymmetry are also the ones in which a leading particle effect has been observed (see section 5.6). This seems to indicate that a memory of the struck parton is preserved through to the final state. All values for $\langle \cos 2\phi \rangle$ and $\langle \sin 2\phi \rangle$ are consistent with zero and more will be said on $\langle \sin\phi \rangle$ in section 6.8.

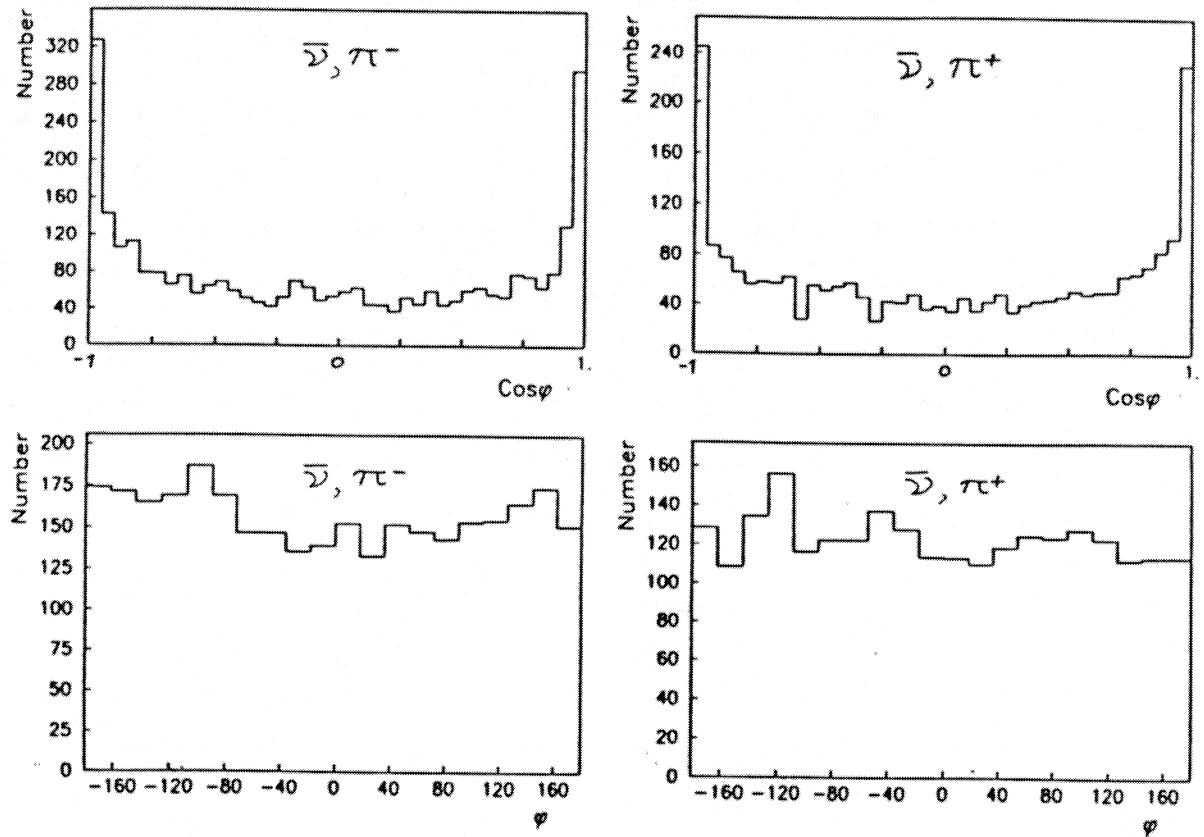


Figure 6.8: $\text{Cos}\phi$ and ϕ distributions for π^- & π^+ in $\bar{\nu}$ events

TABLE 6.2				
Azimuthal Asymmetries, $\times 10^2$				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\langle \cos\phi \rangle$	-0.70 ± 1.83	* -5.00 ± 1.17	* -4.42 ± 1.24	-0.02 ± 1.40
$\langle \cos 2\phi \rangle$	-1.73 ± 1.84	-0.85 ± 1.19	-1.24 ± 1.26	-2.77 ± 1.41
$\langle \sin\phi \rangle$	-2.60 ± 1.86	-2.19 ± 1.18	-1.83 ± 1.26	-2.03 ± 1.43
$\langle \sin 2\phi \rangle$	1.22 ± 1.85	-0.54 ± 1.17	0.78 ± 1.25	0.70 ± 1.42

The stability of the $\langle \cos\phi \rangle$ asymmetry may be judged from table 6.3

TABLE 6.3				
$\langle \cos\phi \rangle \times 10^2$ for various cuts and conditions				
ν			$\bar{\nu}$	
Cut	π^-	π^+	π^-	π^+
(a)	0.54 ± 0.64	-0.77 ± 0.49	-1.54 ± 0.54	0.53 ± 0.55
(b)	-1.75 ± 1.97	-5.89 ± 1.26	-5.57 ± 1.32	1.14 ± 1.51
(c)	-0.92 ± 1.99	-5.27 ± 1.30	-4.00 ± 1.48	-0.28 ± 1.70
(d)	-0.56 ± 1.91	-4.71 ± 1.25	-4.89 ± 1.39	-0.26 ± 1.55
(e)	-0.57 ± 1.83	-4.90 ± 1.17	-4.37 ± 1.24	-0.07 ± 1.40
(f)	-0.70 ± 1.83	-5.03 ± 1.17	-4.43 ± 1.24	-0.02 ± 1.40
(g)	-	-	-2.91 ± 2.04	-0.76 ± 2.28
(h)	-	-	-4.77 ± 1.52	0.48 ± 1.55
Key (a) no cut in Z (b) $P_t > 0.2$ (c) $Q^2 > 3$ (d) minimum number of hadron tracks = 5 (e) corrected for EMI geometrical acceptance (f) corrected for scanning inefficiencies (g) events with stubs only (h) events with no stubs only				

in which results are presented for various cuts and conditions. The cuts were each applied separately. Including tracks of all z (a) substantially reduces $\langle \cos\phi \rangle$ in magnitude in signal channels, although a 3σ effect remains for π^- in antineutrino events. Excluding low P_t tracks (b) (< 0.2 GeV/c) enhances $\langle \cos\phi \rangle$ a little. More will be said on z and P_t dependences in the next section. Little change is made by demanding $Q^2 > 3$ (GeV/c²)² (c). On average, a more reliable current direction is used by selecting events with 5 or more hadrons. However, little change in $\langle \cos\phi \rangle$ is noticed in any channels (d).

The effect of correcting for EMI geometrical acceptance (e) or for scanning efficiencies (f) has virtually no effect. The antineutrino film, however, was scanned for the presence of short proton stubs, which are indicative of nuclear re-interactions. Events with evidence

of re-interactions (g) show a reduction in a $\langle \cos\phi \rangle$ signal compared to events without such evidence (h), but are consistent within errors.

6.5 KINEMATIC DEPENDENCES OF $\langle \cos\phi \rangle$

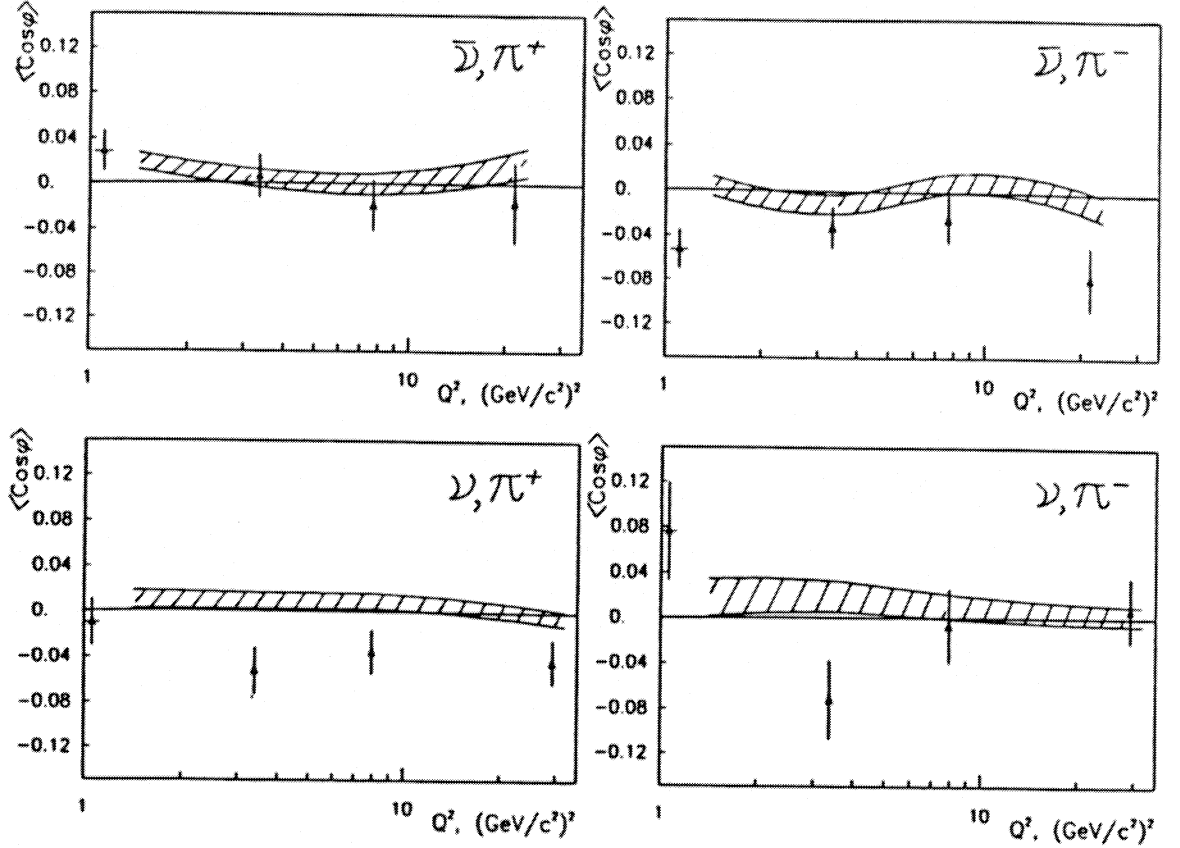


Figure 6.9: $\langle \cos\phi \rangle$.vs. Q^2

The dependence of $\langle \cos\phi \rangle$ on the kinematic variables Q^2 , W , x , y , z and P_t is explored in figures 6.9 to 6.14 respectively. The shaded bands are results from LPS Monte Carlo simulations (phase space) under the same cuts with $\pm 1\sigma$ width. The bands are generated by joining points and hence values between points are found by interpolation. For each figure, data is presented for π^+ and π^- mesons for neutrino and antineutrino events.

All phase space predictions are consistent with being flat and zero. In general, the channels showing an overall asymmetry (π^- in

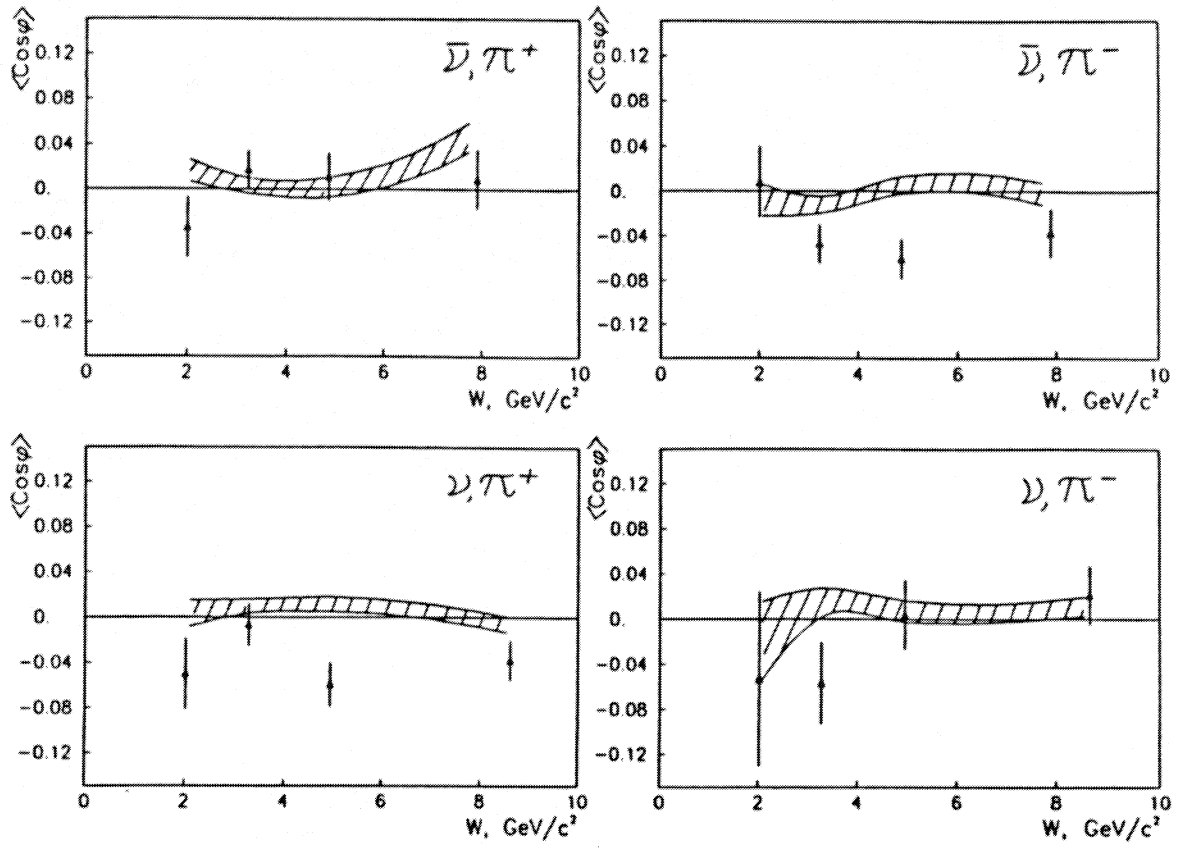


Figure 6.10: $\langle \cos\phi \rangle$.vs. W

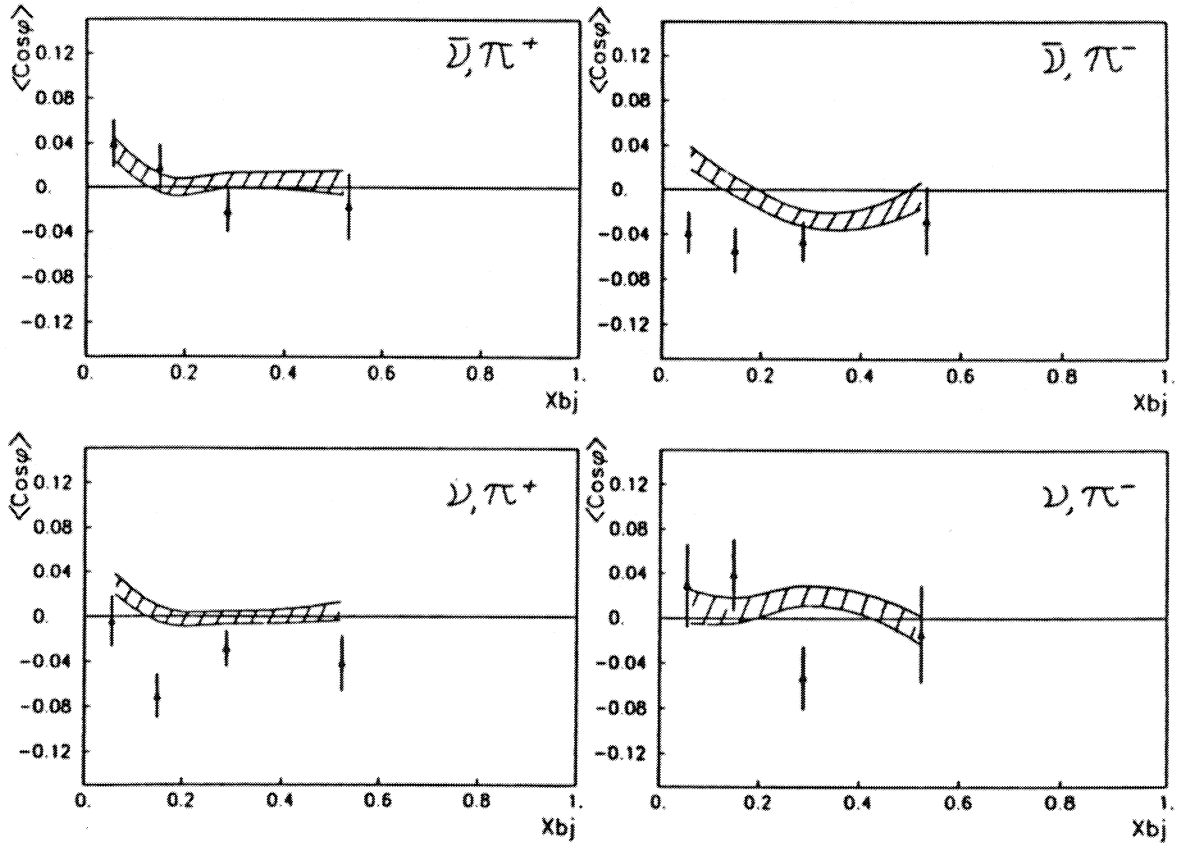


Figure 6.11: $\langle \cos\phi \rangle$.vs. x

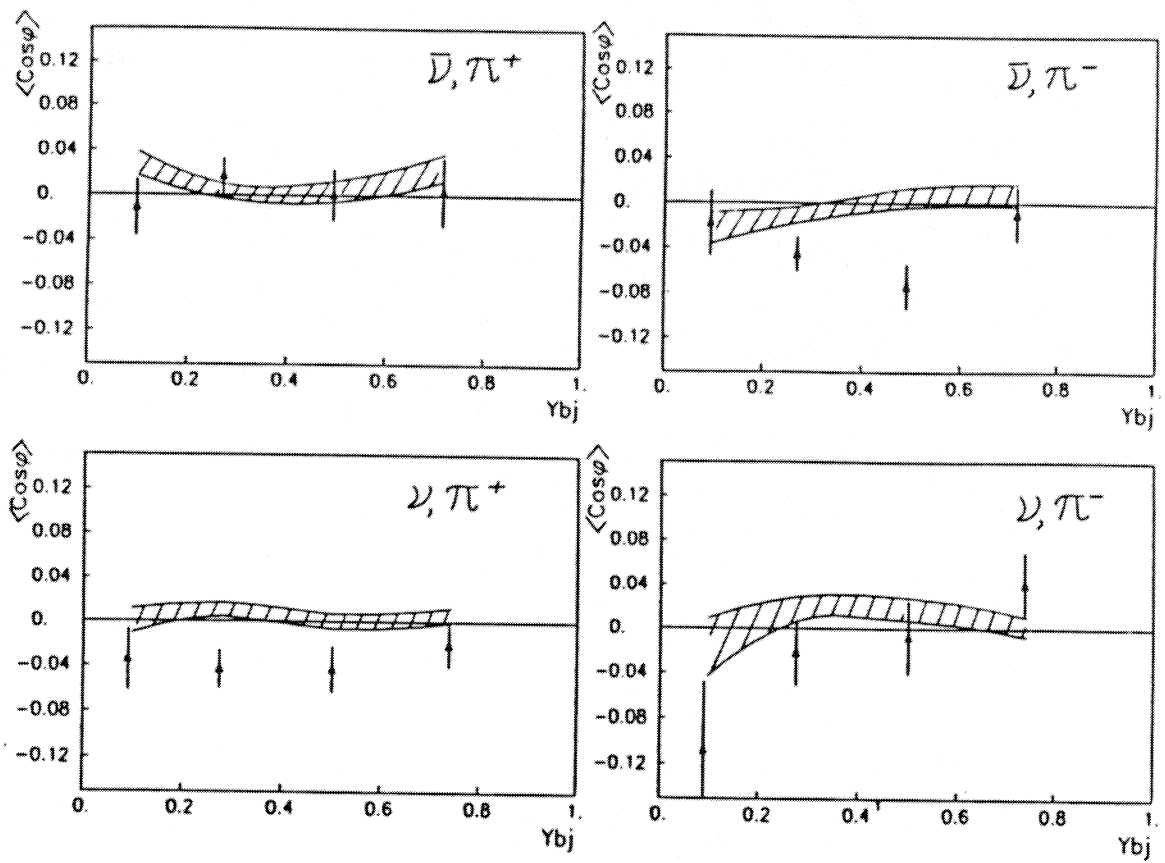


Figure 6.12: $\langle \cos\phi \rangle$.vs.y

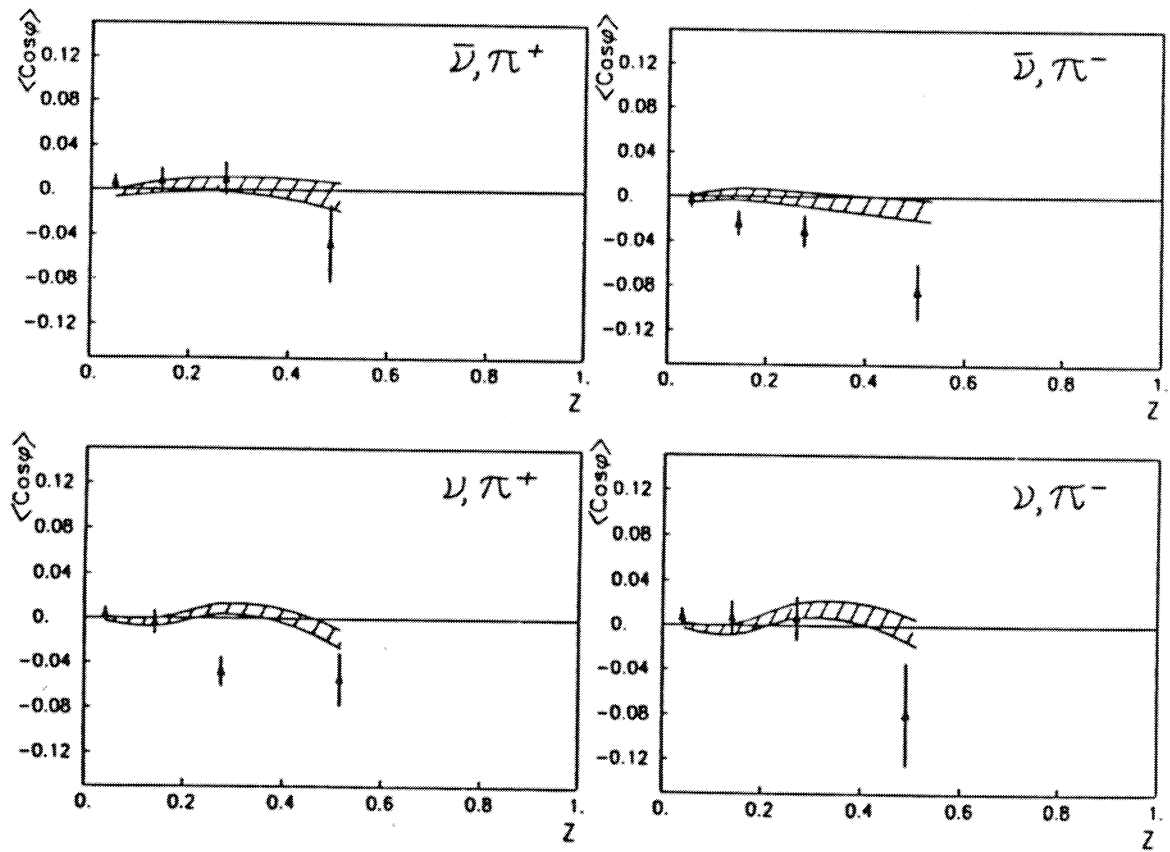


Figure 6.13: $\langle \cos\phi \rangle$.vs.z

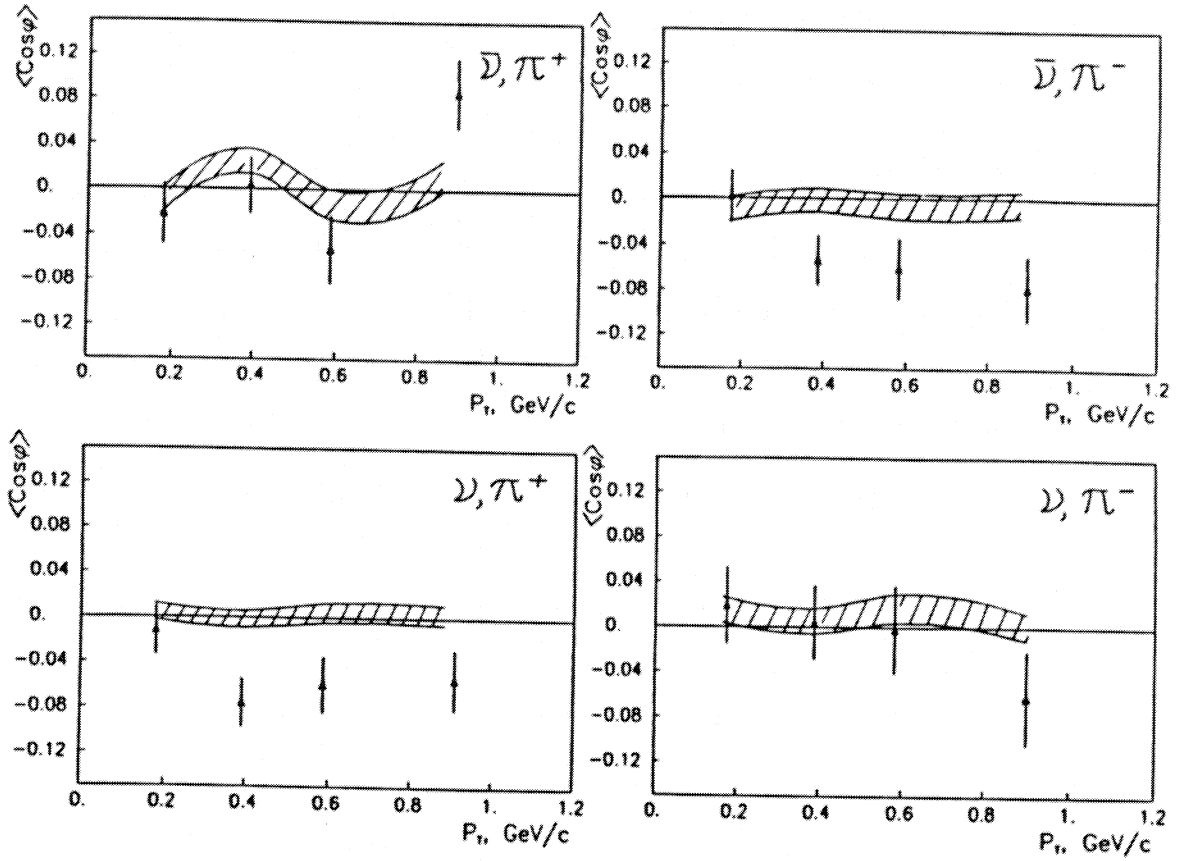


Figure 6.14: $\langle \cos\phi \rangle$.vs. P_t

antineutrino events and π^+ in neutrino events) differ from the phase space predictions and are generally negative. This is most prominent when $\langle \cos\phi \rangle$ is plotted versus z (figure 6.13) and P_t (figure 6.14).

At low z , where target fragments dominate, $\langle \cos\phi \rangle$ is consistent with zero in all channels. As z increases, struck quark fragments are selected and $\langle \cos\phi \rangle$ becomes more negative. This is most prominent in the signal channels. In the non-signal channels a negative $\langle \cos\phi \rangle$ is observed for the highest z data point and could be explained, for example, by events where the leading quark has formed a spin 1 meson. Subsequent decay then leads to fast fragments of both charges.

At low P_t (figure 6.14) one can expect the introduction of random transverse momenta due to fragmentation, resonance decays, re-interactions and smearing to move randomly any hadron to an arbitrary ϕ about the current direction. At higher P_t these effects no longer dominate and a net negative $\langle \cos\phi \rangle$ persists in signal channels [6.8].

Figure 6.9 depicts the Q^2 dependence of $\langle \cos\phi \rangle$ which, whilst showing a net negative $\langle \cos\phi \rangle$ in signal channels, does not hint at either a $1/\ln Q^2$ or $1/\sqrt{Q^2}$ form within the statistics and Q^2 range available. Similarly, no conclusions may be drawn about any W,x or y dependences (figures 6.10, 6.11 and 6.12 respectively).

6.6 MODEL COMPARISONS

The discussion above indicates that the $\langle \cos\phi \rangle$ results are inconsistent with phase space predictions in signal channels. Comparisons with the two models presented at the beginning of this chapter have been made using the Lund Monte Carlo program with varying parameter settings.

TABLE 6.4				
QCD Expectations for $\langle \cos\phi \rangle \times 10^2$ from the Lund Monte Carlo, Antineutrino Events				
(a)	% composition	$\langle \cos\phi \rangle_q$	$\langle \cos\phi \rangle_{\pi^-}$	$\langle \cos\phi \rangle_{\pi^+}$
Q	98.3	-0.30 ± 0.24	-0.09 ± 0.51	-0.05 ± 0.51
QG	0.1	-	-	-
Q $\bar{Q}$	1.6	1.38 ± 1.95	4.08 ± 3.41	2.26 ± 3.67
overall	100.0	-0.29 ± 0.24	0.02 ± 0.50	0.01 ± 0.50
(b)	% composition	$\langle \cos\phi \rangle_q$	$\langle \cos\phi \rangle_{\pi^-}$	$\langle \cos\phi \rangle_{\pi^+}$
Q	83.0	-0.52 ± 0.30	1.28 ± 0.57	0.83 ± 0.81
QG	5.7	-19.70 ± 1.00	-4.48 ± 1.69	-3.05 ± 2.33
Q $\bar{Q}$	11.3	-1.52 ± 0.70	0.34 ± 1.28	-0.12 ± 2.03
overall	100.0	-1.71 ± 0.20	0.65 ± 0.50	0.36 ± 0.71
(c)	% composition	$\langle \cos\phi \rangle_q$	$\langle \cos\phi \rangle_{\pi^-}$	$\langle \cos\phi \rangle_{\pi^+}$
Q	68.6	-1.11 ± 0.29	1.57 ± 0.66	1.05 ± 0.94
QG	18.2	-20.44 ± 0.55	-3.38 ± 0.96	-4.36 ± 1.34
Q $\bar{Q}$	13.2	1.32 ± 0.69	0.22 ± 1.20	-0.26 ± 1.99
overall	100.0	-4.40 ± 0.24	0.00 ± 0.50	-0.68 ± 0.72

The possible QCD sources of asymmetry were explored with (a) essentially no, (b) 5.7% and (c) 18.2% of gluon bremsstrahlung in antineutrino events. These proportions were the result of applying specific cuts at the event generation stage. The various proportions of events with (i) quark (Q), (ii) quark - gluon (QG) and (iii) quark - antiquark (Q $\bar{Q}$) in the final state (corresponding to Feynman diagrams in figures 6.6b, c and d respectively) is given in table 6.4 along with the estimated $\langle \cos\phi \rangle$ for the struck quark and observed π^+ and π^- after accounting for fragmentation, resonances, smearing, re-interactions, etc.

The cuts of section 6.4.2 were applied. For this comparison the parton transverse momentum, k_t was set to zero and the P_t of fragmentation to 0.4 GeV/c.

The most important conclusion is that, whilst asymmetries can be expected at the quark level, they have completely disappeared when pions are considered and all events included. This gives an impression of the impact of the processes which lie between quarks and observed hadrons. At the quark level very large asymmetries exist in quark-gluon events which persist through to the observed pions ((b) and (c) in table 6.4). Quark and quark - antiquark events do not show an asymmetry at parton or pion levels. It would appear that to reproduce the data nearly all events would have to be of the quark-gluon type. At present it is not possible to run the Lund program allowing vast numbers of events with soft gluons as unitarity would be violated at high W.

The Lund Monte Carlo also seems incapable of reproducing the difference between signal and non-signal channels. This is most noticeable in the results in the remainder of this section.

The Lund Monte Carlo program has also been used in a mode where, as for (a) above, virtually no gluon bremsstrahlung is generated but an angular dependence on ϕ for the struck quark along the lines of the

TABLE 6.5				
$\langle \cos\phi \rangle \times 10^2$ from the Lund Monte Carlo Including k_t Effects				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\langle \cos\phi \rangle$	-2.01 ± 0.69	-2.52 ± 0.39	-5.11 ± 0.51	-3.83 ± 0.51

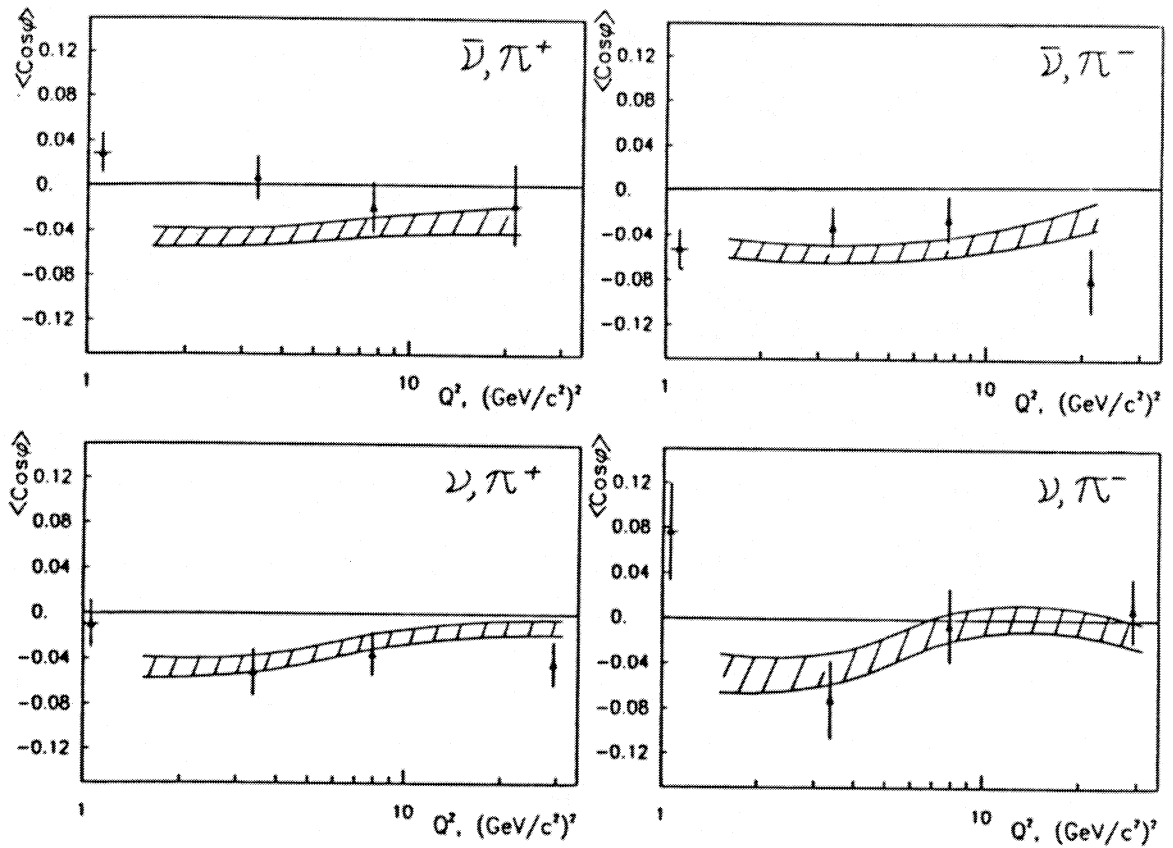


Figure 6.15: $\langle \cos\phi \rangle$.vs. Q^2 , comparison with k_t induced effects

formalism developed by Cahn [6.7]. Table 6.5 shows the results of this. The kinematic dependences of $\langle \cos\phi \rangle$ are reproduced in figures 6.15 to 6.20 with a comparison with the non-perturbative model. A value of $\langle k_t \rangle = 0.35$ has been used and is seen to reproduce the data reasonably well, at least in the signal channels. In non-signal

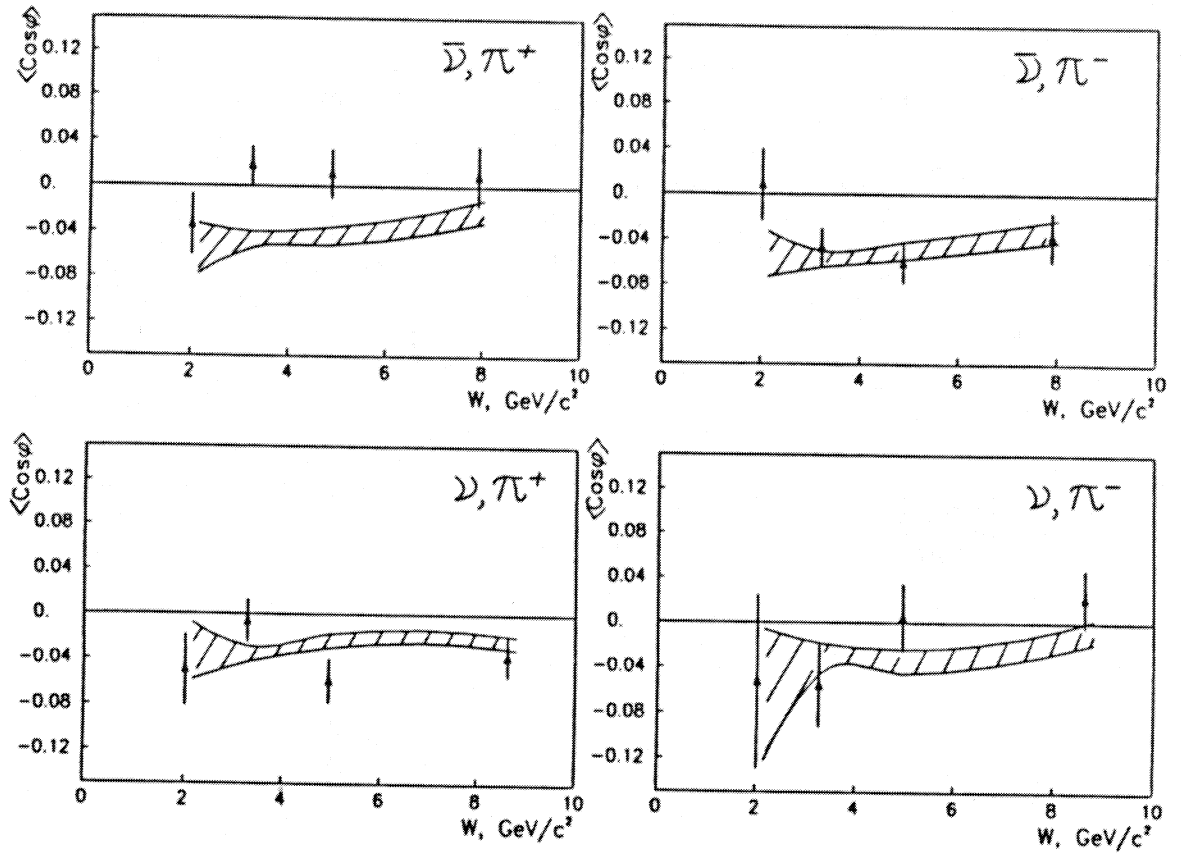


Figure 6.16: $\langle \cos\phi \rangle$.vs. W , comparison with k_t induced effects

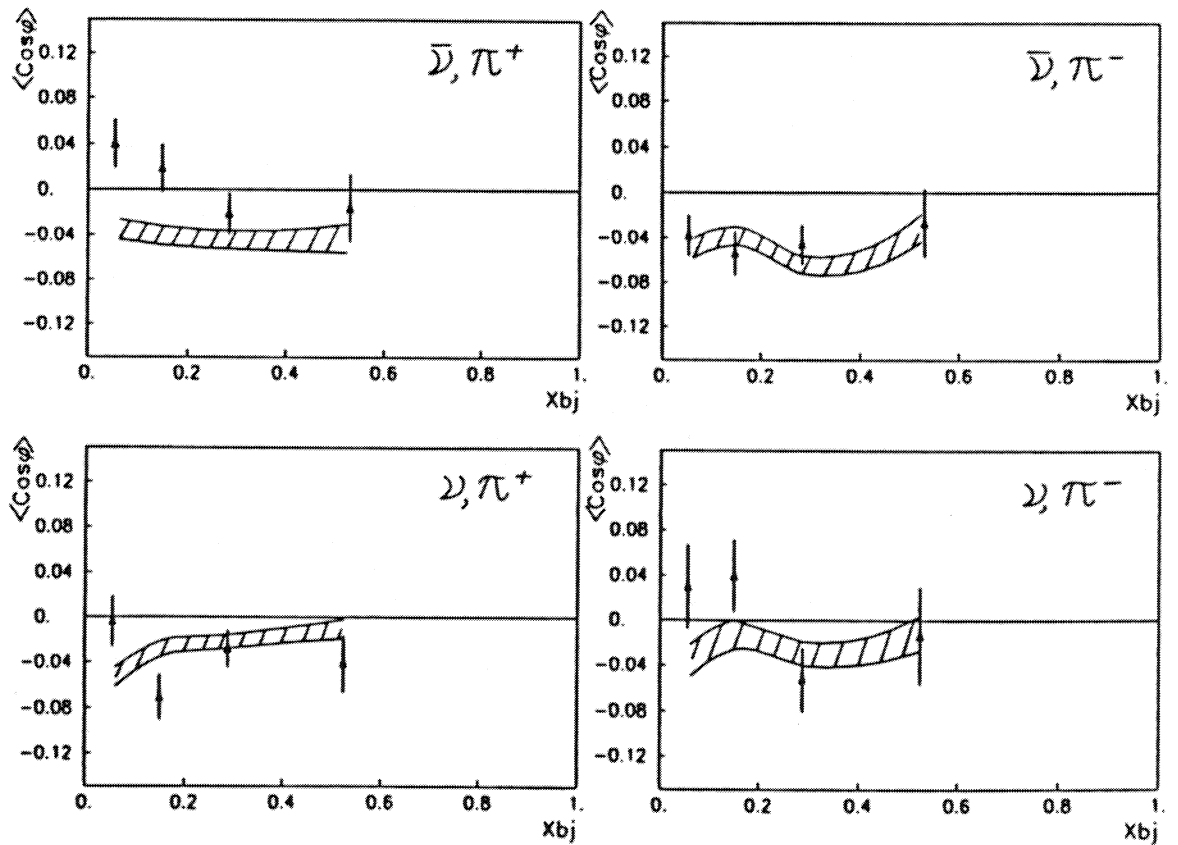


Figure 6.17: $\langle \cos\phi \rangle$.vs. x , comparison with k_t induced effects

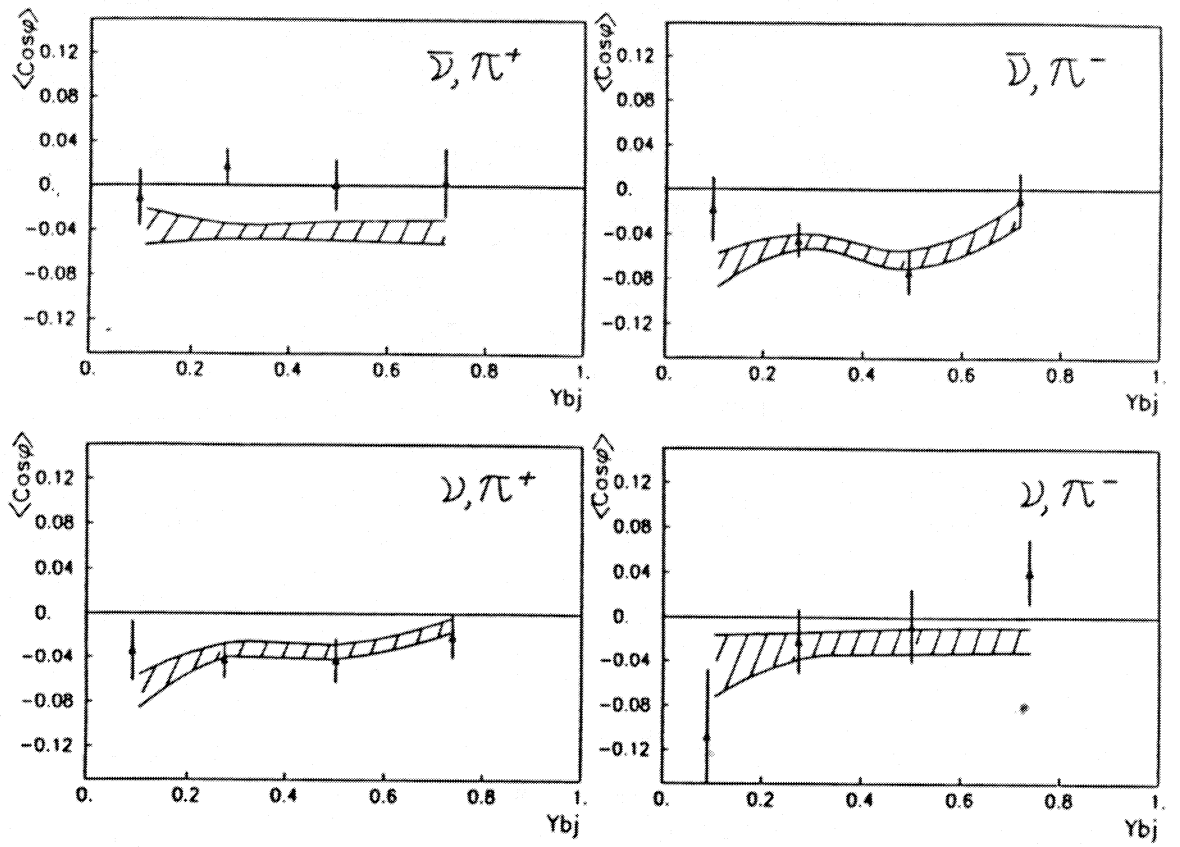


Figure 6.18: $\langle \cos\phi \rangle$.vs. y , comparison with k_t induced effects

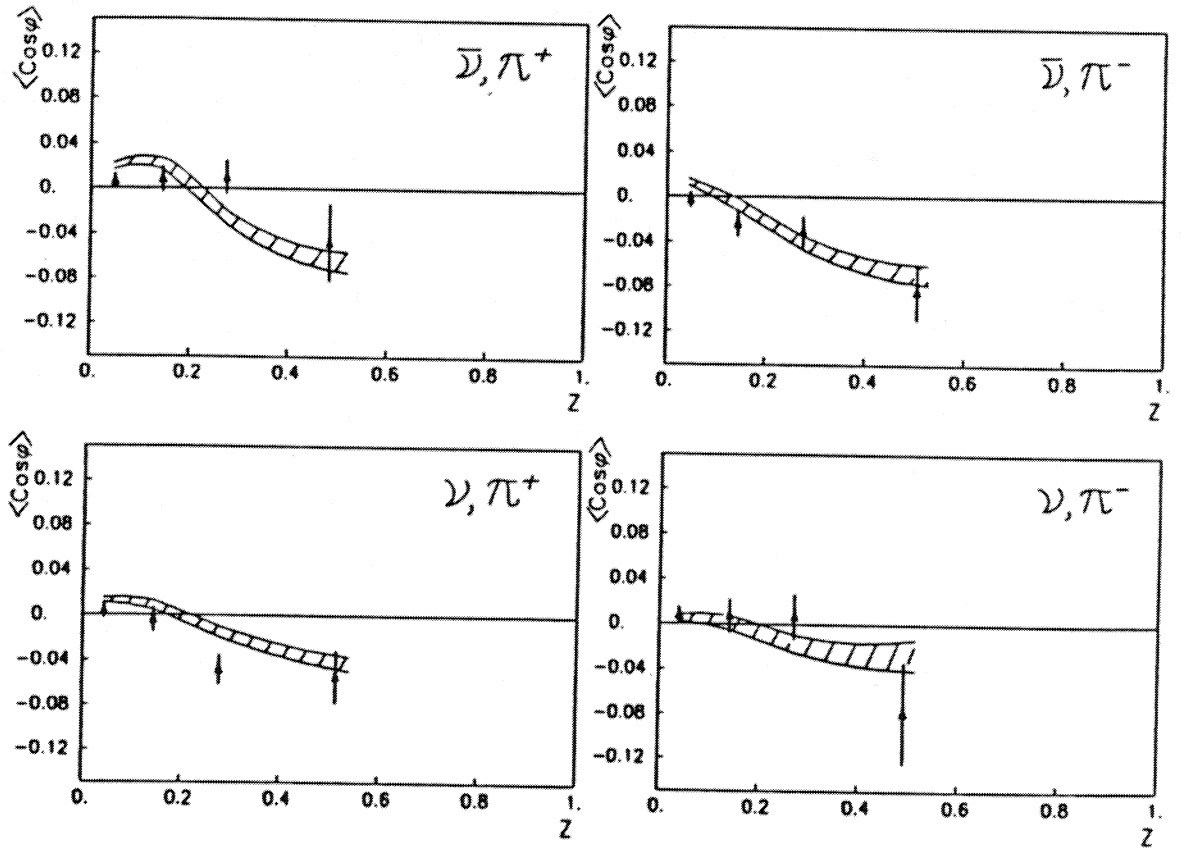


Figure 6.19: $\langle \cos\phi \rangle$.vs. z , comparison with k_t induced effects

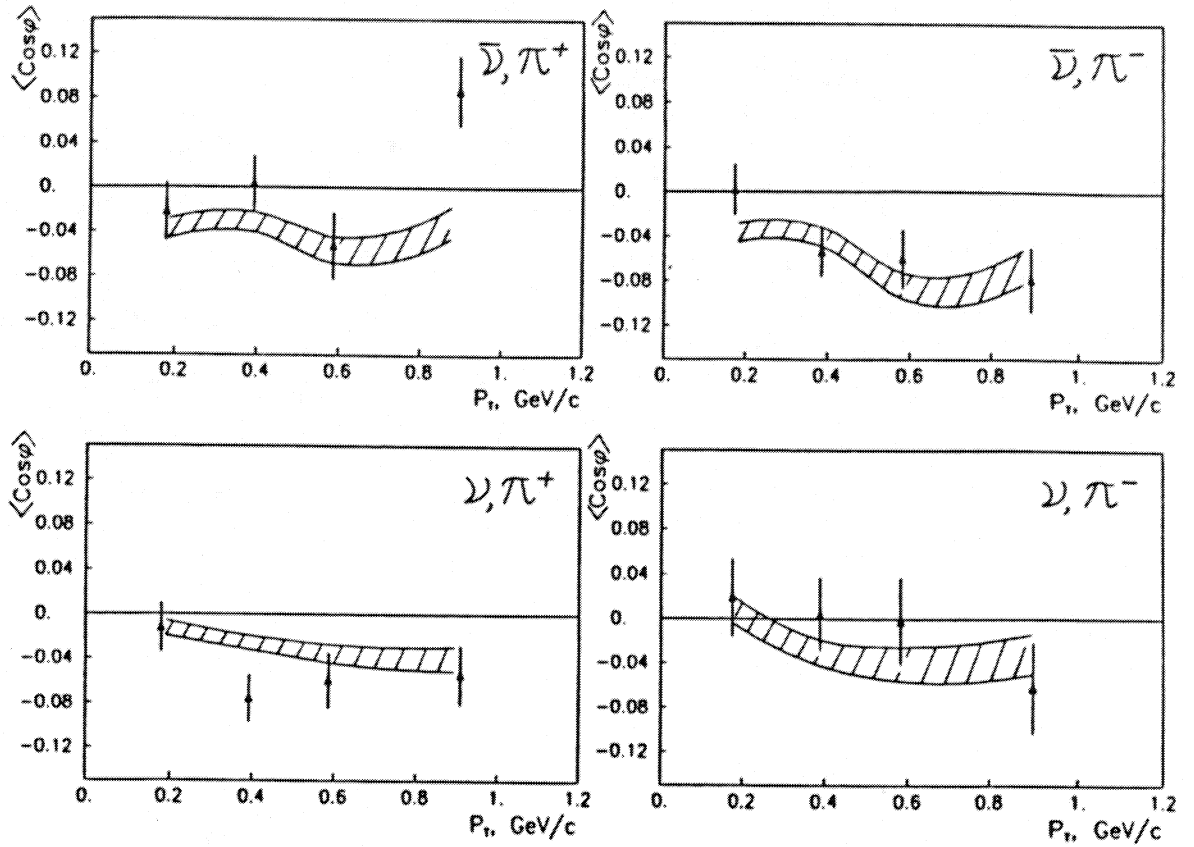


Figure 6.20: $\langle \cos\phi \rangle$.vs. P_t , comparison with k_t induced effects

channels the Monte Carlo shows a tendency to lie below the data. As yet, this is not understood. A higher value of $\langle k_t \rangle$ (e.g. 0.44) produced a more negative effect in all channels and the difference between signal and non-signal channels diminished. A lower value of $\langle k_t \rangle$ (e.g. 0.3) produced a better fit in non-signal channels but tended to be systematically too close to zero in signal channels. The Monte Carlo prediction does not extend to very low Q^2 (see Appendix A) because of cuts in event generation. The same reason explains the large width of the Monte Carlo band at low W . The data is reasonably well reproduced in the signal channels for y and z (and in the latter qualitatively agrees in non-signal channels). Not much can be learnt from comparisons as a function of x , but as a function of P_t the data is in reasonable agreement with the simulation.

6.7 SYSTEMATIC ERRORS IN MEASURING $\langle \cos\phi \rangle$

Up to this point, little has been mentioned of systematic errors, save in connection with the choice of definition of current direction. Corrections for smearing in ϕ were explored using the LPS Monte Carlo and found to be negligible. Of much greater importance were radiative corrections. These will be discussed below from the point of view of a systematic effect in $\cos\phi$ asymmetry measurements. However, their observation is of interest in its own right and so will be discussed in some detail.

6.7.1 Inner Bremsstrahlung

It is possible for the outgoing muon to radiate a photon in the field of neighbouring particles. In a similar manner the struck quark can also radiate and tentative evidence for this has been found in this experiment but is the subject of much further analysis. Such photons from the muon (often referred to as inner bremsstrahlung or 'IB' for short) will be mis-assigned to the hadron shower and lead to a miscalculation of both the lepton scattering plane and the direction of the total hadron shower. In the discussion which follows γ 's from antineutrino events have been used, although very similar results were obtained when neutrino events were analysed.

The majority of γ 's come from π^0 and η decays, the remainder coming from single γ decay modes and inner bremsstrahlung. However, since some γ 's from π^0 's and η 's escape detection the single γ sample is greatly enhanced. After treatment of normal bremsstrahlung γ 's as described in section 2.7 the effective $\gamma\gamma$ mass distribution from all combinations in an event (figure 6.21) clearly shows a peak at the π^0 mass, although virtually no η signal can be seen. The continuous curves in this figure are from a simulation using the LPS Monte Carlo. If events with only 2 γ 's are used the π^0 shows up very well. The width of the peak at its base was used to define a window $0.08 <$

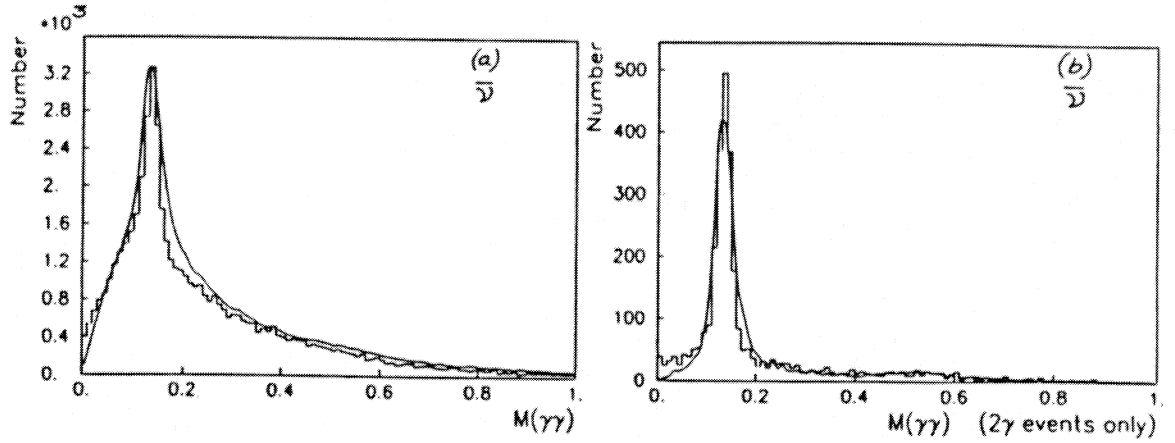


Figure 6.21: $\gamma\gamma$ mass distribution (a) all γ 's, (b) 2 γ events only

$\text{Mass}(\gamma\gamma) < 0.19 \text{ GeV}/c^2$ in which γ 's were flagged as belonging to a π^0 if, when combined with another γ in the event, gave a mass within the window.

The distribution of γ 's in azimuth, ϕ_γ , about the beam axis (figure 6.22a) shows a small enhancement at zero degrees, corresponding to the muon direction. Once again the curves are from LPS Monte Carlo simulations. The dotted curve does not include muon bremsstrahlung effects, whereas the solid curve does (the latter overlapping the former for much of the plots). γ 's not attributable to a π^0 (within the above the window) show a larger enhancement (figure 6.22b) here when compared with those believed to originate from a π^0 (figure 6.22c), although a small "leakage" signal is also present in the latter.

Looking in azimuth about the current direction, ϕ_c , (figure 6.23a) an excess at low angles is observed, which corresponds to a correlation with the muon direction. This effect is more marked for γ 's not belonging to π^0 's (figure 6.23b) as opposed to those which are (figure 6.23c), although there is also a small presence of IB γ 's here too. In the centre of mass system of the neutrino and struck nucleon

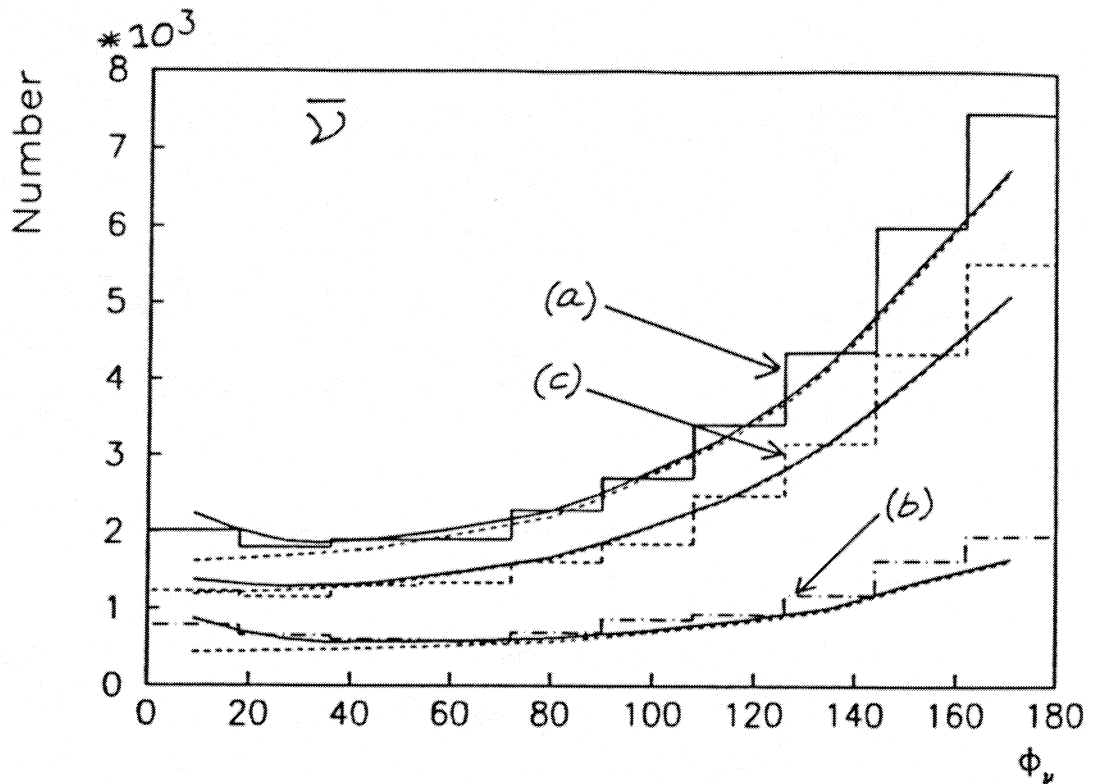


Figure 6.22: Angle, ϕ_ν , between μ and γ about beam axis

(assumed to be at rest) the hadrons and muon go off back to back and γ 's originating from the hadron shower would be expected mostly towards 180 degrees in angle $\theta^*(\mu\gamma)$, with the muon (0 degrees). This is borne out experimentally (figure 6.24), although a small spike is visible at $\cos\theta^*(\mu\gamma) = 1$. Selecting γ 's with $\phi_\nu > 90$ degrees about the beam axis a large proportion are seen to be at large angles $\theta^*(\mu\gamma)$ from the muon in this frame (figure 6.25a). In contrast γ 's with $\phi_\nu < 90$ degrees about the beam show a marked excess at small $\theta^*(\mu\gamma)$ (figure 6.25b).

The above is evidence of muon bremsstrahlung. Very similar results have been reported in another neutrino experiment [6.9].

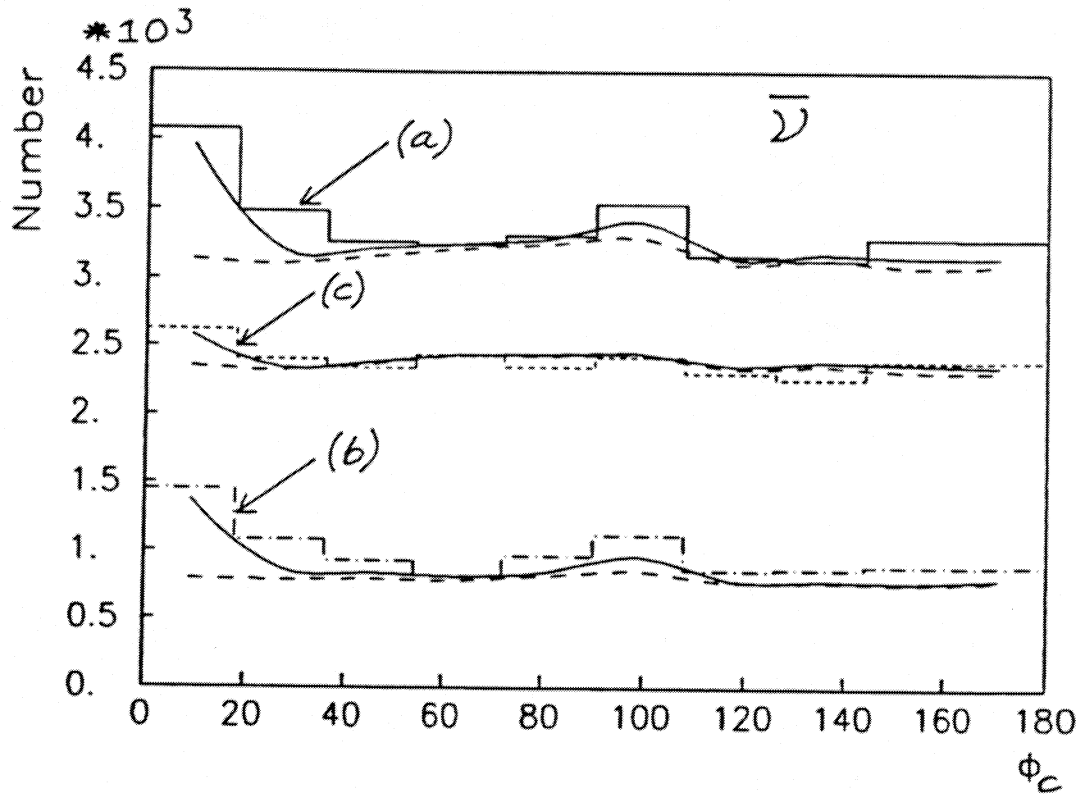


Figure 6.23: Angle, ϕ_c , between μ and $\bar{\nu}$ about current direction

6.7.2 Simulation of Inner Bremsstrahlung

Radiative corrections in neutrino interactions have been considered by a number of authors [6.10, 6.11]. Appropriate Feynman diagrams which should be taken into account in calculating radiative corrections are shown in figure 6.26. The diagrams relevant for calculation of bremsstrahlung are (a), (b) and (f). Not only should bremsstrahlung be observed from the muon but also from the struck quark. Unfortunately, the latter is masked by the complexities of the hadron shower and hence observation is made difficult. A leading log formula [6.10] incorporating these diagrams transforms the bare cross-section (σ_b) into the observed cross-section (σ_{obs}) as follows :-

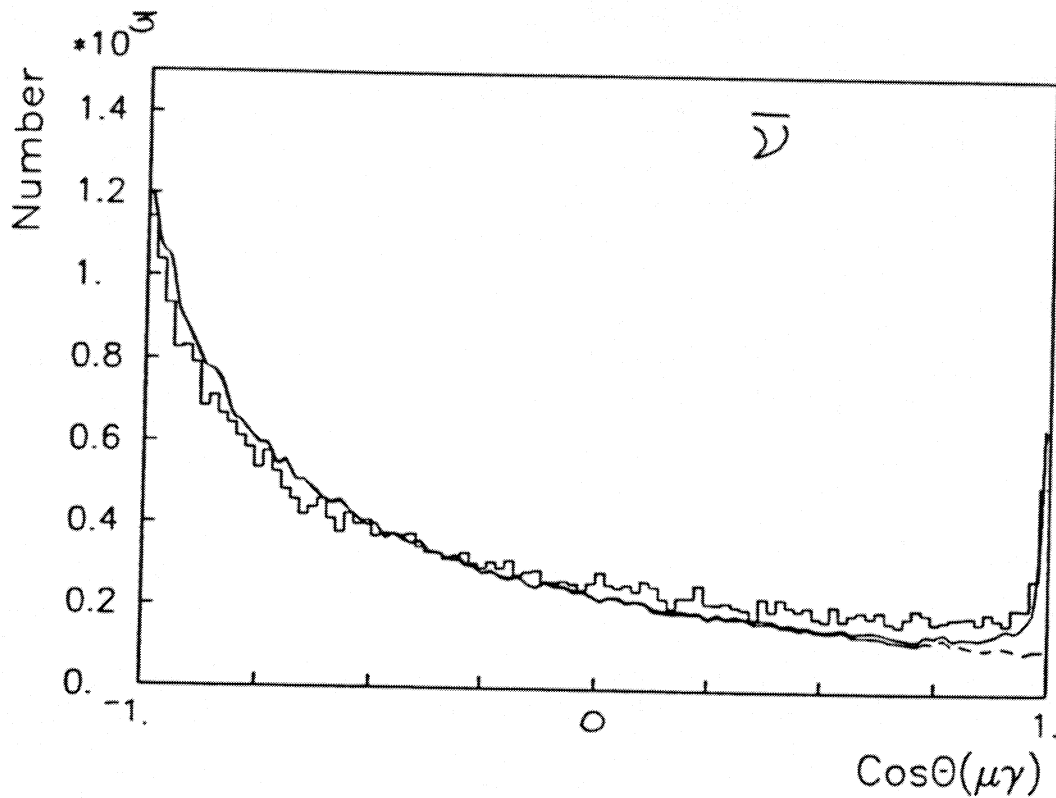


Figure 6.24: Angular separation of μ and $\bar{\nu}$, $\cos\theta^*(\mu\bar{\nu})$ in neutrino - nucleon centre of mass

$$\sigma_{\text{obs}} = \sigma_b + \alpha/2\pi\{\ln(Q^2/\mu^2)\}F(\sigma_b) + \alpha/2\pi\{\ln(Q^2/M^2)H + \alpha G/2\pi \quad (6.11)$$

where μ = muon mass

M = typical hadronic mass

$F(\sigma_b)$ a function of muon variables

H a function of quark variables

and G is a constant term.

The term with coefficient $F(\sigma_b)$ contains the μ (leptonic) contributions, the term in H is a result of quark radiation and the term in G contains remaining $O(\alpha)$ corrections, which are small. At low Q^2 where $\ln(Q/\mu)^2 \sim O(1)$ corrections are of order α/π but for $\ln(Q/\mu)^2 \gg 1$ corrections are of order $(\alpha/\pi)\ln Q^2$.

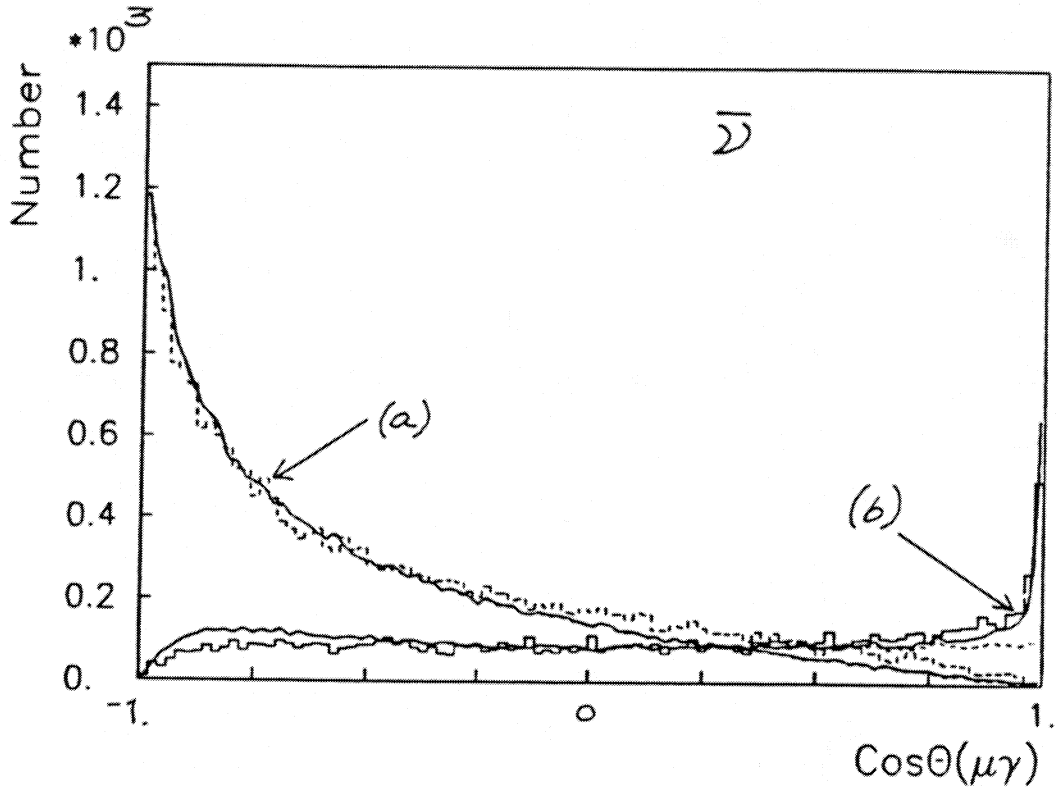


Figure 6.25: As figure 6.24 but with (a) $\phi_\nu > 90^\circ$ and (b) $\phi_\nu < 90^\circ$

In the analysis presented here the transformation of the cross-section is not so important as the simulation of muon bremsstrahlung. There is no reason to suppose that the bremsstrahlung should be anything but symmetric in azimuth about the muon direction in the neutrino - nucleon centre of mass, but in angular separation ($\theta^*(\mu\gamma)$) and energy one may expect specific distributions. The angular dependence is predicted to be proportional to $1/(1-\beta\cos\theta^*)$, where β is the muon velocity in this frame and the energy spectrum is expected to be proportional to $(1+z^2)/(1-z)$, where $(1-z)$ is the fractional energy of the muon given to the photon. The soft photon singularity ($z \rightarrow 1$) cancels when virtual and real photons are taken into consideration. These features were added into the LPS Monte Carlo and

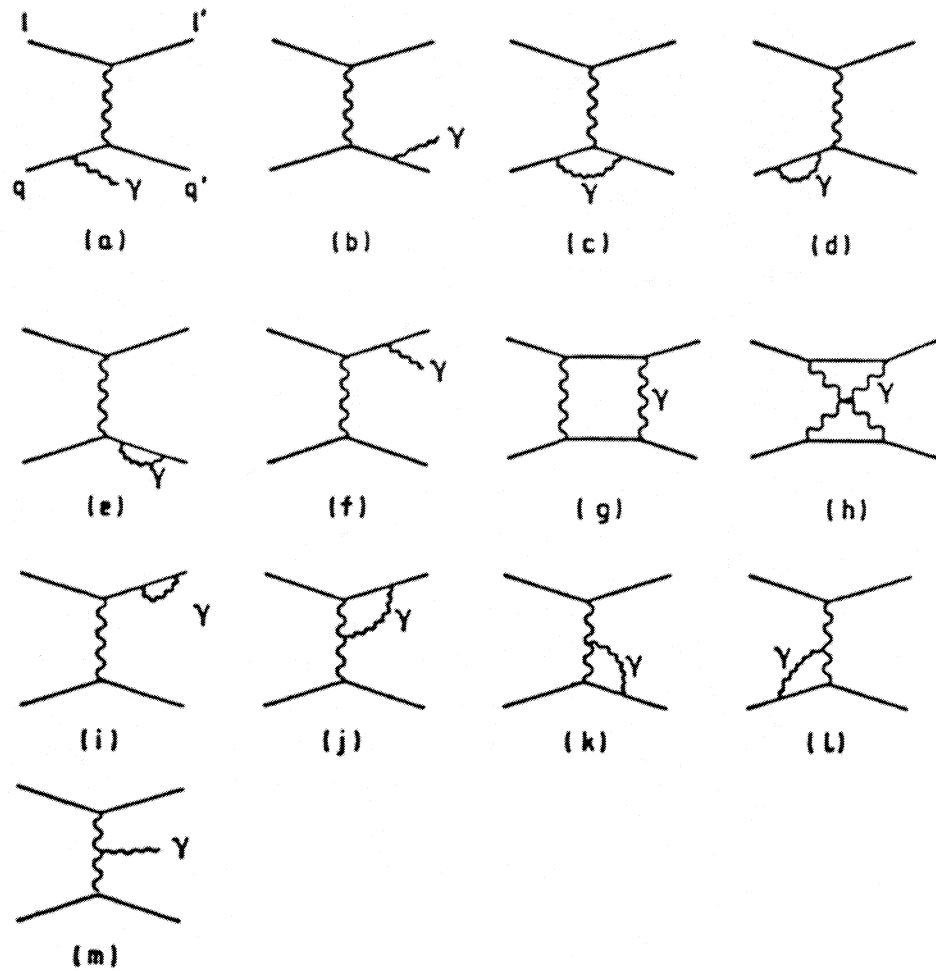


Figure 6.26: Feynman diagrams contributing to radiative corrections

a minimum value of z was set to avoid the singularity. The value was chosen so that even for the highest energy muons the lowest possible energy photons were below the scan limit (of ~ 75 MeV/c) for photons in general. This scan energy cut was also applied. The production rate was normalized for a reasonable fit to the data.

The solid curves in figure 6.22 to 6.25 show the results of the simulation. The dashed curves show the situation with no generation of muon bremsstrahlung.

6.7.3 Effect of Muon Bremsstrahlung on $\langle \cos\phi \rangle$ Measurements

TABLE 6.6				
$\langle \cos\phi \rangle \times 10^2$ for LPS Monte Carlo with μ Brems.				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
$\langle \cos\phi \rangle$	0.01 ± 0.62	-1.19 ± 0.43	-1.76 ± 0.54	-1.31 ± 0.59

TABLE 6.7				
$\langle \cos\phi \rangle \times 10^2$ Corrected for Effect of μ Brems.				
	ν		$\bar{\nu}$	
	π^-	π^+	π^-	π^+
(a)	-0.71 ± 1.93	-3.81 ± 1.25	-2.66 ± 1.35	1.29 ± 1.52
(b)	0.36 ± 1.91	-3.94 ± 1.29	-3.36 ± 1.36	1.04 ± 1.50

The effect of muon bremsstrahlung on $\langle \cos\phi \rangle$ measurements is shown in table 6.6. On average, the size of the systematic correction is of the order of one standard deviation in the uncorrected result. (a) in table 6.7 shows the effect on the data results of a straight subtraction of the above; errors have been combined in quadrature. There is no reason to suppose, however, that the correction should differ between neutrino and antineutrino or between π^+ and π^- and hence the spread of values in table 6.6 may be statistical. With this in mind, perhaps a better approach is to take the average of the biases of table 6.6 and subtract from each channel in turn to produce (b) in table 6.7. The effect on the distribution of $\langle \cos\phi \rangle$ with Q^2

etc. (figures 6.9 to 6.14 and 6.15 to 6.20) is to move the Monte Carlo prediction bands more negative by about 1σ (i.e. about half the width of the bands in the figures).

6.8 <SINφ> EFFECTS

TABLE 6.8 <sinφ>×10 ² With & Without Stubs		
$\bar{\nu}$		
	π^-	π^+
(a) with stubs	-4.83 ±2.1	-2.10 ±2.40
(b) without stubs	-0.23 ±1.54	-1.52 ±1.74

Recent results from another neutrino experiment [6.12] have reported an asymmetry in hadron production about the lepton plane, which is dominant in events known to have been affected by nuclear re-interactions. In this experiment the antineutrino film was scanned for very short proton stubs, which were indicative of the presence of nuclear re-interactions. Table 6.8 gives <sinφ> (under the same cuts as for <cosφ>) for events with and without stubs. All numbers are consistent with zero except perhaps for π^- tracks in events with stubs, which may be significant. A direct comparison of these numbers with the other experiment is difficult as the cuts applied were different (although in principle a similar analysis is possible).

The ratio $R = (N^+ - N^-) / (N^+ + N^-)$, where $N^+(N^-)$ is the number of hadrons with $0 < \phi < 180$ ($-180 < \phi < 0$) degrees, can be related to <sinφ> by $R = 4 \langle \sin\phi \rangle / \pi$. This gives a value of $R = -0.061 \pm 0.027$ for π^- tracks in events with stubs and is roughly half the asymmetry observed in the other experiment (which used all hadron tracks). An

observation of a negative $\langle \sin\phi \rangle$ of a few percent may be expected from consideration of one-loop QCD diagrams [6.13], but this cannot be confirmed (within errors) by this experiment. ($\langle \sin 2\phi \rangle$ effects are expected to be positive and just a fraction of a percent and are thus beyond the reach of data discussed here.)

6.9 ASYMMETRIES IN RECONSTRUCTED JETS

TABLE 6.9 $\langle \cos\phi \rangle_{\text{jet}} \times 10^2$				
ν			$\bar{\nu}$	
	Forward	Backward	Forward	Backward
(a)	-2.29 ± 0.75	2.56 ± 0.69	0.14 ± 0.61	0.70 ± 0.57
(b)	-3.94 ± 0.29	3.78 ± 0.27	-5.71 ± 0.27	5.24 ± 0.25
(c)	0.13 ± 0.27	-0.15 ± 0.24	0.05 ± 0.28	-0.14 ± 0.26
Key (a) data (b) Lund model incorporating primordial transverse momentum effects (c) LPS Monte Carlo				

An attempt was made at reconstructing the struck quark direction by summing the momenta of all charged and neutral fragments with $z > 0.2$ ($z > 0.1$ for $\bar{\nu}$'s). Naturally there is a loss of current fragments and a gain of target fragments at low z . In a similar manner, a target "jet" (diquark fragments) was defined using the tracks which failed the above cuts. The azimuthal distribution of the resultant jets about the projected current direction was then calculated. The results for $\langle \cos\phi \rangle_{\text{jet}}$ are shown in table 6.9.

Results for neutrino events (a) show an asymmetry for both current and target jets of about equal magnitude but opposite sign. However,

the same can not be said for the antineutrino data where no significant asymmetry is observed for either jet. A simulation using the Lund Monte Carlo incorporating k_t effects (b) tends to support the effect seen in the neutrino, but not the antineutrino events. The effects observed do not lead to any firm conclusions as the results for neutrino and antineutrino are not consistent and together do not match the Monte Carlo simulation. It is possible that the lower average multiplicity of antineutrino events implies that the quark and diquark directions are poorly reconstructed. Results from the LPS Monte Carlo (c) no systematic biases are present.

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Chapter VII

SUMMARY

The kinematic distributions of chapter III are as expected from theory. Correction for missing neutral energy appears to be adequate on a statistical basis, but event by event has a spread about the ideal.

The average charged multiplicity is found to increase linearly with $\ln W^2$ and with a slope slightly higher than previous experiments. This can be explained by the complications introduced by a nuclear target.

The fragmentation functions of U and D quarks (scattering from the sea has been neglected) into charged pions obeys approximate isospin invariance. Establishing whether Q^2 and x dependences of the fragmentation functions exist is not so easy. It would appear that a simple analysis may hint at the presence of these effects in the low W region but a more detailed analysis (e.g taking moments) is necessary to reach firmer conclusions.

It is fairly clear that an azimuthal asymmetry is present in $\langle \cos \phi \rangle$ for both neutrino and antineutrino data and is dominant in π^+ and π^- (leading) channels respectively, as expected. The values obtained for $\langle \cos \phi \rangle$ in this analysis (before any corrections) are -0.050 ± 0.012 (-0.044 ± 0.012) for π^+ (π^-) in neutrino (antineutrino) interactions. The choice of the definition of an unbiased current direction for $\langle \cos \phi \rangle$ measurements has precluded a study of $\langle \cos 2\phi \rangle$ effects. However, such effects are expected to be smaller than $\langle \cos \phi \rangle$ and hence more difficult to measure. The fact that the $\langle \cos \phi \rangle$ asymmetries appear in opposite channels when neutrino and antineutrino are compared is a good indication of some dynamics at work. An antineutrino experiment in the Fermilab 15' bubble chamber [7.1] has

also observed an asymmetry of similar proportions to that presented here (and also a difference between channels). However, they showed, using a Monte Carlo, that the effect of nuclear re-interactions and smearing can reduce an effect by about a factor of two. Also, no radiative corrections were made. Correction for nuclear re-interactions has not been made in the analysis presented here.

Asymmetries can also be expected in muon scattering experiments and have indeed been observed [7.3]. The EMC collaboration, working at much higher Q^2 and W , have reported a negative $\langle \cos\phi \rangle$ [7.2], but a direct comparison with their results is made difficult in that they have removed y dependences. However, their results agree qualitatively with this experiment, particularly with respect to trends in $\langle \cos\phi \rangle$ versus z and in a lack of structure as functions of Q^2 and W .

The comparisons made with the Lund Monte Carlo suggest that a k_t induced effect with $\langle k_t \rangle = 0.35$ GeV/c explains the data reasonably well and that a QCD induced asymmetry is not necessary, but cannot be excluded. The value of $\langle k_t \rangle$ is lower than used in many other experiments. The mis-match between Monte Carlo and data in non-signal channels suggests that the dilution of the original quark asymmetry is not fully understood.

Muon bremsstrahlung is observed and found to reduce $\langle \cos\phi \rangle$ measurements by about 10. With this correction $\langle \cos\phi \rangle = -0.039 \pm 0.013$ (-0.034 ± 0.014) for π^+ (π^-) in neutrino (antineutrino) interactions.

No great advantage is gained in trying to reconstruct the original quark direction by summing fragments.

$\langle \cos 2\phi \rangle$ has not been covered in detail because of systematic biases. $\langle \sin\phi \rangle$ and $\langle \sin 2\phi \rangle$ results have been reported, but little exists in the literature on them.

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Appendix A

MONTE CARLOS

Two Monte Carlos have been used in this thesis. The first is based on Longitudinal Phase Space and was chiefly used for correction of systematics. The second was based on the Lund model of fragmentation and was used mostly for model comparisons.

A.1 THE LPS MONTE CARLO

The Longitudinal Phase Space Monte Carlo generates individual events. Antineutrino energy and interaction vertex position in BEBC are based on the measured, radially dependent flux for this experiment. The vertex is confined within the fiducial volume (equation 2.1) and the antineutrino origin is generated at random from within the beam decay tunnel. A proportion (~5%) of quasi-elastic events are generated, the rest being based on the appropriate Bjorken-x and Bjorken-y distributions [A.1] for a proton or neutron, account being taken of the sea. If the event lies outside the EMI geometrical acceptance it is dropped at this stage. Fermi motion and a virtual mass assignment of the neon nucleus is parameterized according to the results of Bodek and Ritchie [A.2] and is applied to both the primary interaction and any subsequent re-interaction.

Charged and neutral multiplicities for all strong interaction vertices (including the primary) are generated from a Poisson distribution with a maximum multiplicity of 20 and a dispersion that is a linear function of the mean. The average charged multiplicity is a linear function of $\ln(s)$, but is constrained at low multiplicities to conserve charge and to produce two or more hadrons. The neutral multiplicity is generated in a similar manner but with a mean which

produces on average half as many final state neutral as charged non-strange mesons after accounting for η decays.

Hadrons from the primary interaction, with energy less than ten times their mass, are allowed a finite probability of undergoing nuclear re-interaction. Neutral hadrons are tracked until they interact, decay or leave BEBC (except if below energy thresholds for neutrons and γ 's, set to avoid generation of many elastic scatters and small γ 's respectively). Charged hadrons from primary and other vertices within 0.4 cm of the primary are tracked for a short distance to allow for close interactions which would be indistinguishable from their production vertex on BEBC film. Nuclear boil-off protons with momentum exceeding 0.3 GeV/c are generated forward in BEBC in accordance with the measured backward (in the neutrino - nucleon centre of mass) protons seen in the data.

To account for detection and measuring inefficiencies events are smeared according to the following rules:-

1. Tracks with momentum < 10 MeV/c were dropped.
2. Tracks with a stop length < 2 cm in BEBC were dropped unless able to decay or inelastically scatter.
3. Tracks were smeared in $1/P$, λ (dip) and ϕ (azimuth) as prescribed by Gluckstern [A.3] .
4. For tracks with $\delta P/P > 30\%$ then
 - a) if a muon, the error is reduced according to the track length to the EMI,
 - b) if an electron or track leaving BEBC, the limit $\delta P/P < 0.5 P$ is imposed,
 - c) if a proton which stops in BEBC, then a proportion are misidentified as pions,
 - d) if an interacting track then the momentum is reduced by about 20%,

- e) if a short track with large error and small curvature, then the charge charge was set undefined.
5. Protons with momentum $< 0.3 \text{ GeV/c}$ are rejected and others are mis-identified as pions depending on momentum.
 6. All kaons are called pions except for 10% of those with momentum below 1 GeV/c .
 7. Momentum errors of V^0 's and γ 's are based on measurements.
 8. All strongly interacting neutrals and K_L^0 's are called neutrons.
 9. Unseen neutrals, all neutrons, 30% of γ 's and charged tracks with momentum below 0.3 times their mass are dropped from all N^* vertices, the remains then being treated according to the SYNTHESIS program (see section 2.7).
 10. Scan efficiencies are imposed for V^0 's, γ 's and N^* vertices.

A.2 THE LUND MONTE CARLO

The popular Monte Carlo attributable to the group at the University of Lund, is more sophisticated than the above in that it takes into account QCD corrections (and hence scaling violations) to the parton model by including QCD matrix elements. These matrix elements are integrated numerically over a grid of x and W values accessible in this experiment. At each point in the grid the probabilities for the final state partons to be (i) Q , (ii) $Q\bar{G}$ or (iii) $Q\bar{Q}$ are computed. Events between grid points are generated by interpolation. Hard gluon bremsstrahlung ((ii) above) was governed using a value of 0.3 for the QCD- Λ parameter. The hadronization scheme has been described in chapter I.

As already mentioned, there is no transverse momentum intrinsic in a string and yet final state hadrons do possess P_t . There are a number of ways this can come about :-

1. hard or soft gluon bremsstrahlung, or

2. P_t of $Q\bar{Q}$ pairs created during fragmentation, or
3. resonance decays, or
4. primordial transverse momentum (k_t) of the struck quark within the nucleon.

Process (4) comes about because partons confined inside a nucleon are expected to possess the sub-nuclear equivalent of Fermi motion. A struck parton is very likely to have a momentum component perpendicular to the current which is preserved through the interaction and is manifested in the final state as a P_t . An average value of $k_t = 0.35$ was used, perhaps a little lower than choices made by other experiments.

Using spin counting the proportion of pseudoscalar and vector mesons is expected to be 1:3, but empirically it is found to be nearer 1:1. This ratio was fixed as 1:1. A value of 0.3 was used for the ratio of S quark to U or D quark production during fragmentation and a value of $\langle P_t \rangle$ of hadronization was set to 0.4 GeV/c.

The following cuts were enforced at event generation to prevent the Monte Carlo program looping.

$$Q^2 > 0.6 \text{ (GeV/c)}^2$$

$$W^2 > 5 \text{ (GeV/c)}^2$$

$$\nu > 1 \text{ GeV}$$

$$E_\nu > 10 \text{ GeV}$$

The energy at which string fragmentation was stopped was set to 1.1 GeV and gave a reasonable fit to data multiplicity distributions.

Smearing and experimental losses were imposed in a similar manner to the previous Monte Carlo.

Appendix B
THE PLAGUE SUITE OF SUBROUTINES

PLAGUE is a set of Fortran subroutines for checking the physical sense of events. Checking of events at the scanning stage is very time consuming and so it is one of the aims of PLAGUE to reduce this effort greatly by producing printout of salient quantities (for all or selected classes of events) highlighting any irregular features. Statistical information can be accumulated and is often very useful for comparing laboratories.

For each event data is organised according to a format used by the HYDRA system [2.5] based on blocks (or banks) of information. These banks are arranged sequentially in a linear vector of computer memory and are linked together to form a tree structure using pointers from one bank to another. The data is organized into banks for geometrical reconstruction (information on vertices, tracks, particle identification etc) and for EMI data (hits, timing, etc). Using the information from these two groups of banks another group of banks are produced during processing containing data important for physics analysis (the PHYS bank or event summary structure). Broadly speaking, the subroutines are divided into three categories.

1. event printing
2. statistical information accumulated over a set of events
3. service routines for the above 2 categories (which will not be discussed further here).

Figure B.1 shows PLAGUE output for a typical CC event. The first part consists of information on event, roll, frame, vertex position etc. and status flags (e.g. on which scans found), followed by the summed energy of particle categories (e.g. charged hadrons, N^* , etc.)

STATISTICS	SCAN		EVENT SCANNING SUMMARY.						
	STEP 1	2	OVERFLOW			0.	UNDEFINED		0.
			3	4	5	6	7	8	9
1	ALL 34859	SGPR 34859	SCN1 24926	SCN2 24703	BOTH 21759				
2		6PR 2337	SCN1 1911	SCN2 1887	BOTH 1784				
3		9PR 856	SCN1 705	SCN2 707	BOTH 669				
4		4PR 3537	SCN1 2782	SCN2 2756	BOTH 2537				
5		1PR 6831	SCN1 3491	SCN2 3634	BOTH 2536				
6		>=10 1420	SCN1 1179	SCN2 1179	BOTH 1127				
7		3PR 5695	SCN1 4340	SCN2 4234	BOTH 3799				
8		7PR 2031	SCN1 1700	SCN2 1704	BOTH 1622				
9		5PR 3928	SCN1 3228	SCN2 3221	BOTH 3031				
10		2PR 6969	SCN1 4532	SCN2 4331	BOTH 3652				
11		8PR 1237	SCN1 1050	SCN2 1039	BOTH 994				
12		OPR 18	SCN1 8	SCN2 11	BOTH 8				

Figure B.2: Scanning Statistics as a Function of Topology

and the number of particles in the category. The second part provides information on individual tracks, in particular, mass, charge, energy, momentum, momentum error, track length and labels, flags indicating various conditions and the track fit residuals (in μm) for various mass hypotheses. A "U" at the beginning of a track description indicates that a unique track fit was found. In the case of an EMI identified muon, information on time slot, chambers hit, predicted and actual hit co-ordinates, χ^2 and amount of material between BEBC and EMI chambers (in grammes) are given. Information on secondary

STATISTICS	ESCN		SCAN SUMMARY IN ENERGY BINS.						
			OVERFLOW			0.	UNDEFINED		0.
	STEP 1	2	3	4	5	6	7	8	9
1	ALL 34859	SCN1 24926	SCN2 24703	BOTH 21759					
2		> 10 27194	SCN1 20594	SCN2 20436	BOTH 18461				
3		5-10 2867	SCN1 1866	SCN2 1837	BOTH 1539				
4		< 3 3269	SCN1 1575	SCN2 1594	BOTH 1051				
5		3-5 1529	SCN1 891	SCN2 836	BOTH 708				

Figure B.3: Scanning Statistics as a Function of Energy

vertices is given below the track. The third and final part provides comments and messages resulting from a number of tests carried out on an event. The tests can range from charge conservation checks to examination of the residuals of other possible mass fits to see if the best mass has been selected.

Routines in the second category above can be used to give a statistical summary of kinematic fits (in experiments using a hydrogen target), track labels as a function of particle mass, number of events found on each scan (figures B.2 & B.3) and the number of leptons in each CC event (and their ordering in decreasing momentum).

Appendix C
WA59 PUBLICATIONS

Study of μ -e Events Produced in Antineutrino Interactions.

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An investigation of the EMC effect using antineutrino interactions in deuterium and neon.

A.M.Cooper et.al., Phys.Lett. 141B No1,2 (1984), 133

Observation of coherent diffractive charged current interactions of antineutrinos on neon nuclei.

P.Marage et.al., Phys.Lett. 140B No1,2 (1984) 137

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