

A measurement of Z production using tau final states with the LHCb detector



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Abstract

The LHCb experiment is one of the four main experiments located on the LHC ring. It collected 1 fb^{-1} of data in 2011 at a centre-of-mass energy of 7 TeV. In this thesis, the cross-section for $Z \rightarrow \tau\tau$ production is measured using the full 2011 dataset. A study of this process provides complementary measurements to LHCb's existing results in the $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ decay modes and tests lepton universality. Additionally it can be used to search for evidence of new physics processes producing high-mass tau pairs. This measurement is performed using four different di-tau final states, split into five analysis channels based on kinematic requirements. A selection scheme is developed for each and the background contributions and efficiencies for reconstructing and selecting signal events are determined using primarily data-driven methods. The results for the analysis channels are then combined, with correlations between the channels taken into account. The final measurement, $\sigma_{Z \rightarrow \tau\tau}$, represents the cross-section for the production of a tau pair through an intermediate Z boson where the mass of the Z is between 60 and 120 GeV, the transverse momenta of the tau leptons exceed 20 GeV, and the pseudorapidities of the tau leptons are between 2 and 4.5. The result is

$$\sigma_{Z \rightarrow \tau\tau} = 71.4 \pm 3.5 \pm 2.8 \pm 2.5 \text{ pb}$$

where the first uncertainty is statistical, the second is systematic and the third is due to the luminosity determination. The measurement is in agreement with previous results in the other leptonic decay modes of the Z at LHCb, and with theoretical predictions.

The VELO is a high precision tracking detector located around the interaction region. One of the key features of tau decays is their decay

length and this can be exploited to distinguish them from prompt backgrounds. A precise measurement of the decay length requires a high hit resolution in the VELO. In this thesis studies of the resolution of the VELO are performed. The single hit resolution of the VELO sensors is studied as a function of strip pitch, and a correction to improve the calculation of the impact positions of tracks on VELO sensors is presented. The best single hit resolution, at a strip pitch of $40\text{ }\mu\text{m}$ and a projected angle of between 10 and 14° is determined to be $3\text{ }\mu\text{m}$, while improvements in resolution of up to 30% for r -clusters at low projected angle are achieved by applying the correction to the calculation of the impact positions. The impact parameter resolution is also studied as a function of transverse momentum and found to be approximately $15\text{ }\mu\text{m}$ for the x and y components of the impact parameter for tracks with transverse momentum greater than 6 GeV .

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Declaration

This thesis is the result of my own work, except where explicit reference is made to the work of others, and has not been submitted for another qualification to this or any other university.

- Stephen Farry

I would like to dedicate this thesis to my family

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CHAPTER 1

INTRODUCTION

Particle physics is the study of the fundamental constituents of matter and the forces which govern their behaviour. The theoretical framework used to describe these interactions is known as quantum field theory. In quantum field theory, the theories of special relativity and quantum mechanics are combined, and particles are thought of, not as classical “billiard balls”, but as excitations of quantum fields, and their interactions described by the exchange of mediating particles. Currently, the best model of the fundamental particles and their interactions is known as the Standard Model, which has existed in its current form since the 1970s. The Standard Model describes the properties of the fundamental particles and three of the four fundamental forces in nature.

The framework of quantum field theory was first applied to the classical theory of electromagnetism, which unites electricity and magnetism, to produce the theory of quantum electrodynamics. It is considered the most successful of the current theories due its precise prediction of a large number of experimentally measured quantities, such as the anomalous magnetic moment of the electron. In the Standard Model, this prescription was also extended to apply to the weak and strong nuclear forces, which describe radioactivity and binding within the atomic nucleus. The Standard Model falls short of being a complete theory as it does not incorporate the theory of gravitation, described by general relativity. It also cannot account for the level of matter-antimatter asymmetry observed in nature. As a result, many searches are

underway to discover new physics beyond the Standard Model.

The predictions of the underlying theory of particle physics are tested experimentally using particle colliders. Particles such as electrons or protons are accelerated to high energies and collided at fixed points. Due to the mass-energy equivalence, ($E = mc^2$), these high energy collisions can produce particles with large masses such as W/Z bosons or top quarks which exist for short periods of time before decaying to more familiar, stable particles. Analysis of the final state particles produced by these collisions allows the intermediate particles to be reconstructed and studied. Today, the highest energy particle collider in the world is the Large Hadron Collider (LHC).

In this thesis, Z boson production is studied at the LHC through its decay to tau final states. The Z boson is considered a “standard candle” of particle physics, as its properties have been measured to high precision at colliders such as LEP, which preceded the LHC. The most precise measurement of Z boson production can be performed in its decays to electron and muon pairs, which present clean signatures and easily reconstructible final states. The measurement of Z production in its decay to the third lepton flavour, the tau, is more difficult as the tau itself subsequently decays to more stable particles. The Standard Model predicts that these three leptonic decay modes should occur at the same rate, a feature of the theory known as lepton universality, which can be tested experimentally by performing measurements of the rate, or cross-section, of each decay. Additionally, in many models of new physics beyond the Standard Model, tau pairs are produced by the decay of theorised new particles. A study of di-tau final states at the LHC has the potential to discover or rule out the existence of these particles.

This thesis is presented as follows. In Chapter 2 the theoretical background to the measurements presented is reviewed. The LHC and the LHCb detector are described in Chapter 3, with the reconstruction of the final state particles produced in the collisions described in Chapter 4. Studies of the resolution of the VELO sub-detector are described in Chapter 5, and finally a study of the $Z \rightarrow \tau\tau$ cross-section at LHCb is described in Chapter 6. A brief summary is given in Chapter 7.

CHAPTER 2

THEORETICAL REVIEW

2.1 The Standard Model

The Standard Model (SM) is a model of fundamental particles and how they interact. In this model, all matter is built from a small number of fundamental spin- $\frac{1}{2}$ particles known as fermions¹, where spin refers to their intrinsic angular momentum. These are split into two groups of six quarks and six leptons. The leptons consist of three charged particles, the electron, e , the muon, μ , the tauon, τ , and their corresponding neutral neutrino partners, the electron neutrino, ν_e , the muon neutrino, ν_μ , and the tau neutrino, ν_τ . These pairs are known as lepton ‘flavours’, with the masses of the charged leptons increasing with each flavour. The quarks carry fractional charges of $+2/3$ or $-1/3$ and are also grouped into pairs of quark ‘flavours’ of increasing mass. They are represented by the symbols u , d , s , c , b and t which stand for ‘up’, ‘down’, ‘strange’, ‘charm’, ‘beauty’ and ‘top’.

The Standard Model describes the interaction of the fundamental particles outlined above. There are four types of fundamental interactions: the strong, electromagnetic, weak and gravitational interactions. The SM attempts to describe three of these interactions by the exchange of integral spin particles, known as bosons, between the fermions. This is achieved by the introduction of four bosons; the gluon, g ,

¹The classification fermion is not limited to spin- $\frac{1}{2}$ particles but encompasses all half integer spin particles

<i>Generation</i>	1	2	3	Charge			
Quarks	u	c	t	$\frac{2}{3}$	Bosons	$W^\pm$	± 1
	d	s	b	$-\frac{1}{3}$		Z	0
Leptons	e	μ	τ	-1		γ	0
	ν_e	ν_μ	ν_τ	0		g	0

Table 2.1: The fundamental particles which make up the Standard Model of particle physics.

the photon, γ , and the W and Z bosons which mediate the strong, electromagnetic, and weak interactions.

The introduction of these bosons is achieved by promoting an existing global gauge symmetry of the theory to a local gauge symmetry. The invariance of the theory is preserved by the introduction of vector boson fields that interact with the existing fields in a gauge invariant manner. This transforms an originally free theory into an interacting theory. The number of boson fields which must be introduced depends on the symmetry group in question. In this context the SM is a Quantum Field Theory (QFT) based on the gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$. $SU(3)_C$ refers to the gauge field theory which describes the interaction of coloured quarks and gluons, Quantum Chromodynamics (QCD), while $SU(2)_L \otimes U(1)_Y$ is the symmetry group corresponding to the electroweak interaction, a unified description of the electromagnetic and weak forces.

2.1.1 Quantum Electrodynamics

Quantum Electrodynamics (QED) is the simplest gauge theory in particle physics. A free Dirac field, $\Psi(x)$, describes the behaviour of a particle with spin $s = \frac{1}{2}$, charge Q and mass m . Its dynamics are described by the Dirac equation [1].

$$(i\not{\partial} - m)\Psi(x) = 0 \tag{2.1}$$

which can be produced by applying the Euler-Lagrange equations [2] to its Lagrangian density, $\mathcal{L}$,

$$\mathcal{L} = \bar{\Psi}(x)(i\partial - m)\Psi(x) \quad (2.2)$$

This Lagrangian can be shown to be invariant under global $U(1)$ transformations such as $\Psi(x) \rightarrow e^{iQ\theta}\Psi(x)$ where θ is a continuous parameter. This global invariance implies the conservation of the electromagnetic current and the electromagnetic charge. The next step is to promote this global symmetry to a local symmetry, by allowing the continuous parameter θ to also depend on spatial position x so the transformation under consideration is $\Psi(x) \rightarrow e^{-iQ\theta(x)}\Psi(x)$. Under this transformation the Lagrangian above is not invariant. It requires the introduction of a gauge vector boson field, $A_\mu(x)$, which transforms as $A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu\theta(x)$, in order to restore invariance. It is convenient at this point to introduce the covariant derivative, D_μ , defined as

$$D_\mu\Psi(x) \equiv (\partial_\mu - ieQA_\mu)\Psi(x) \quad (2.3)$$

which has the same transformation properties as the field itself,

$$D_\mu\Psi(x) \rightarrow e^{-iQ\theta(x)}D_\mu\Psi(x) \quad (2.4)$$

A gauge invariant kinetic term, the electromagnetic field strength tensor is also introduced

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \quad (2.5)$$

Adding the photon field $A_\mu(x)$ and a term corresponding to $F_{\mu\nu}(x)$ to Equation (2.2) and substituting for D_μ gives the QED Lagrangian

$$\mathcal{L}_{QED} = \bar{\Psi}(x)(i\not{D} - m)\Psi(x) - \frac{1}{4}F_{\mu\nu}(x)F^{\mu\nu}(x) \quad (2.6)$$

It is interesting to note that the photon field introduced produces no mass term in the Lagrangian, and in fact that the addition of a mass term will break the symmetry of the Lagrangian. This is consistent with the massless photon observed in experiment.

2.1.2 Quantum Chromodynamics

QCD is a theory of the strong force, which describes the interaction of quarks and gluons (partons). It is a non-abelian¹ gauge theory based on the $SU(3)_C$ gauge group. The Lagrangian is produced in a similar fashion to the QED Lagrangian. A quark field, ψ , is now considered which is a triplet of Dirac fields and is given by

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} \quad (2.7)$$

Its Lagrangian is required to be invariant under the transformation $\psi \rightarrow e^{-iT^a \epsilon_a} \psi$ where a is an index running over from 1 to 8 and T^a represents the eight generators of the $SU(3)$ group. This produces a Lagrangian of the form

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G_{\mu\nu}^a G_a^{\mu\nu} \quad (2.8)$$

where $G_\mu^a(x)$ refers to the gluon fields with $G_{\mu\nu}^a \equiv \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c$ and the covariant derivative is represented by $D_\mu \equiv \partial_\mu - g_s G_\mu^a T^a$. The strong coupling constant is given by g_s and f^{abc} refer to the structure constants of the $SU(3)_C$ group. Extra terms arise from the fact that the $SU(3)$ group is non abelian and these extra terms represent the coupling of the gluon fields to each other. Gluons carry so called ‘colour charge’ and can interact with either themselves or quarks via the strong force.

The non-abelian nature of the QCD interaction means that it has interesting properties that distinguish its interactions from electroweak interactions. In QED, the virtual production of charged pairs results in vacuum polarisation around the particle. This causes the effective charge to vary with distance and results in the electromagnetic coupling constant, α_{em} , increasing with the energy scale of the interaction, Q^2 . While this phenomenon also occurs in QCD, with the virtual production of quark - anti-quark pairs, the quarks themselves carry colour charge and so can also additionally produce gluon pairs, and the net effect of the virtual pair production is

¹A non-abelian group is a group for which the result of applying the group operation on two or more group elements depends on the order in which they are applied.

a decrease in the strong coupling constant, α_s , with increasing Q^2 . This results in two unique phenomena at low and high energy scales known as ***confinement*** and ***asymptotic freedom*** respectively. As the force between two quarks increases with separation, it would in principle take an infinite amount of energy to separate them, and as such, quarks are never found in isolation, a property known as confinement. Also, in very high energy interactions, the strong coupling constant is small, and as a result quarks and gluons interact very weakly; this is known as asymptotic freedom.

2.1.3 Electroweak Theory

The Electroweak Theory is a unification of the Electromagnetic and Weak forces and is accomplished by requiring that the Lagrangian be invariant under gauge transformations corresponding to the gauge group $SU(2)_L \otimes U(1)_Y$. The Electroweak Theory describes the interaction of left and right handed lepton and quark fields, and purely left handed neutrino fields. The representations of the gauge group divide into left handed doublets

$$\begin{pmatrix} \nu_\ell \\ \ell \end{pmatrix}_L, \begin{pmatrix} u \\ d' \end{pmatrix}_L \quad (2.9)$$

and right handed singlets

$$\ell_R, u_R, d'_R \quad (2.10)$$

where $\ell = e, \mu, \tau$ and ν_ℓ is its corresponding neutrino, u is the ‘up’ type quark, and d' is the ‘down’ type quark weak eigenstate. If an electroweak doublet is denoted by χ , the interaction terms are revealed by requiring that the Lagrangian density, $\mathcal{L}$ is invariant under the transformation $\chi \rightarrow \chi + \delta\chi$ where

$$\delta\chi = [\vec{\epsilon} \cdot \vec{t} + \epsilon y] \chi \quad (2.11)$$

where $\vec{\epsilon}$ and ϵ represent a three-vector and vector respectively denoting an infinitesimal change and $\vec{t}$ is the three vector left handed generator of $SU(2)_L$ denoted by

$$t^i = \left(\frac{1 - \gamma^5}{2} \right) \frac{\sigma^i}{2} \quad (2.12)$$

The matrices σ^i are the Pauli-Matrices, the generators of the $SU(2)$ group, and $(1 - \gamma^5)/2$ is the left handed projection operator. The generator of the $SU(1)_Y$ group, y , is a linear combination of the left handed and right handed generators of $U(1)$, which are defined later. Thus four gauge fields, W_1^μ , W_2^μ , W_3^μ and B^μ , appear when local gauge invariance of the Lagrangian is imposed under this gauge group. The Lagrangian which describes the electroweak interactions contains a number of terms, and it can be represented as the sum of four distinct components.

$$\mathcal{L} = \mathcal{L}_{fermions} + \mathcal{L}_{gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{yukawa} \quad (2.13)$$

The dynamics of the fermion fields and their coupling to the gauge bosons is encoded in $\mathcal{L}_{fermions}$ and the kinetic terms of the gauge fields are given by $\mathcal{L}_{gauge}$. The remaining two terms, $\mathcal{L}_{Higgs}$ and $\mathcal{L}_{yukawa}$ describe the coupling of the gauge and fermion fields to a new boson field to be introduced, known as the Higgs field, as well as the dynamics of this field itself.

Analogously to QED, $\mathcal{L}_{fermions}$ describes the free fermion propagators as well as their coupling to the gauge bosons and is of the form

$$\mathcal{L}_{fermions} = \bar{\Psi} \gamma^\mu D_\mu \Psi \quad (2.14)$$

where D_μ represents the covariant derivative and for the electroweak theory is given by

$$D_\mu = \partial_\mu - ig\vec{t} \cdot \vec{W}_\mu - i\frac{g'}{2}yB_\mu \quad (2.15)$$

where g and g' represent the coupling strengths of the interactions. The gauge fields can be transformed into the physical fields observed in experiment, W_μ^+ , W_μ^- , Z_μ , and the familiar A_μ corresponding to the W and Z bosons and the photon.

$$W_\mu^+ = \frac{1}{\sqrt{2}} (W_\mu^1 + iW_\mu^2) \quad (2.16)$$

$$W_\mu^- = \frac{1}{\sqrt{2}} (W_\mu^1 - iW_\mu^2) \quad (2.17)$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W & -\sin \theta_W \\ \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad (2.18)$$

where θ_W is known as the weak mixing angle, which relates the couplings g and g' through $g \sin \theta_W = g' \cos \theta_W$.

From the QED Lagrangian (2.6), for a fermion f , the coupling to the electromagnetic field A_μ to the fermion field ψ^f is given by

$$ieQ^f\gamma^\mu \quad (2.19)$$

where Q^f is the charge of fermion f . Also, from Equations (2.14)-(2.17) it can be determined that the purely left handed coupling to the W_μ^+ and W_μ^- field is given by

$$ig \left(\frac{1 - \gamma^5}{2} \right) \gamma_\mu \quad (2.20)$$

However, the term describing the coupling to the Z_μ field is more complicated as it involves both left handed and right handed components arising from the generators of the $SU(2)_L$ and $U(1)_Y$ gauge groups. This is computed by considering the terms from (2.14) which contribute to the coupling of the neutral gauge bosons W^3 and B and substituting for our physical fields Z and A . This gives the expression

$$\begin{aligned} & -igt^3\gamma_\mu W^{\mu 3} - i\frac{g'}{2}y\gamma_\mu B^\mu \\ & = -i \left(g \sin \theta_W \left(\frac{1 - \gamma^5}{2} \right) I_3^f + \frac{g'}{2} \cos \theta_W Y^f \right) \gamma_\mu A^\mu + \\ & -i \left(g \cos \theta_W \left(\frac{1 - \gamma^5}{2} \right) I_3^f - \frac{g'}{2} \sin \theta_W Y^f \right) \gamma_\mu Z^\mu \end{aligned} \quad (2.21)$$

The first term corresponding to the photon field A^μ is the electromagnetic interaction and thus must have a coefficient equal to (2.19). This relation, along with $g \sin \theta_W = g' \cos \theta_W = e$, gives the relationship between the fermion charge, Q^f , the third component of weak isospin, I_3^f and weak hypercharge, Y^f which is known as the Gell-Mann Nishijima formula [3].

$$Q^f = I_3^f + \frac{Y^f}{2} \quad (2.22)$$

The Z interaction term can be re-expressed as

$$\begin{aligned} -i \frac{g}{\cos \theta_W} \left(\left(\frac{1 - \gamma^5}{2} \right) I_3^f - \sin^2 \theta_W Q^f \right) \gamma_\mu Z^\mu \\ = -i \frac{g}{\cos \theta_W} \gamma^5 \left(c_V^f - c_A^f \gamma^5 \right) \gamma_\mu Z^\mu \end{aligned} \quad (2.23)$$

where $c_V^f = I_3^f - \sin^2 \theta_W Q^f$ and $c_A^f = I_3^f$ are known as the vector and axial-vector couplings respectively. These couplings describe the coupling of the Z boson field to the fermions, known as the $V - A$ (vector - axial-vector) coupling.

The next term in the Lagrangian is $\mathcal{L}_{gauge}$, which contains the kinetic terms corresponding to the four gauge fields and is given by

$$\mathcal{L}_{gauge} = -\frac{1}{4} W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} \quad (2.24)$$

where the index i represents a sum over the three W fields. $W_{\mu\nu}^i$ and $B_{\mu\nu}$ are the field strength tensors and are defined as

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g \epsilon_{ijk} W_\mu^j W_\nu^k \quad (2.25)$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \quad (2.26)$$

A familiar consequence of gauge invariance is that the gauge bosons must be massless [4]. However, this is at odds with experimental data which shows three massive boson fields in addition to the massless photon. Theory and experiment must be reconciled by allowing for massive fields in the electroweak Lagrangian. This is accomplished by introducing a new field known as the Higgs field. The masses of both bosons and fermions can be explained by their interaction with this field by exploiting the principles of Spontaneous Symmetry Breaking and the Higgs mechanism.

Electroweak Higgs Mechanism

In the previous sections it was shown that an unbroken gauge theory requires massless gauge bosons. This is consistent with the massless photons and gluons seen in QED and QCD respectively. However, experimentally the gauge bosons involved in the weak interactions, the $W^\pm$ and Z bosons, have been observed to be massive.

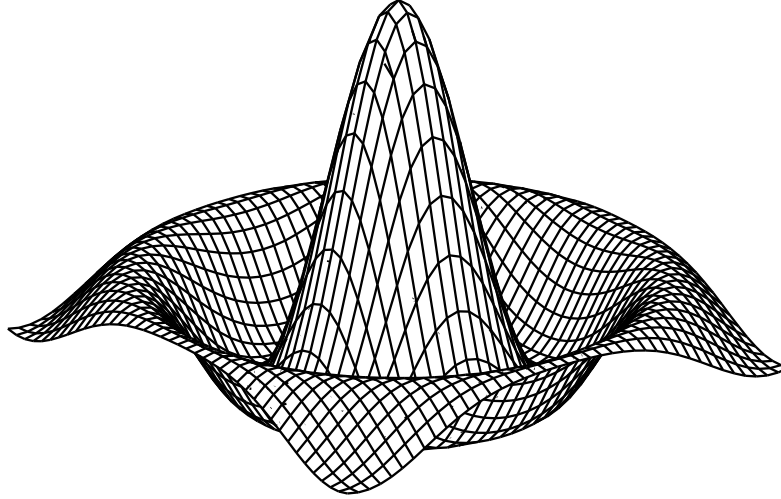


Figure 2.1: The Higgs Potential for $\mu^2 < 0$

Although mass terms for these bosons could simply be added by hand, this would spoil the gauge invariance of the Lagrangian. There exists however an elegant and physical method of adding the mass terms while preserving gauge invariance known as Spontaneous Symmetry Breaking. In its simplest manifestation, this relies on the introduction of an isospin $SU(2)$ doublet, Φ , of complex scalar fields to the Lagrangian.

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix} = \begin{pmatrix} \phi^1 + i\phi^2 \\ \phi^3 + i\phi^4 \end{pmatrix} \quad (2.27)$$

With an associated potential [5] of the form

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (2.28)$$

For $\mu^2 > 0$ this potential is symmetric and has a minimum at $\Phi = 0$. However, reversing the sign of μ^2 , as shown in Figure 2.1, means that the potential no longer has a single minimum at $\Phi = 0$ but instead an infinite number of minima at

$$|\Phi| = \sqrt{\frac{\mu^2}{2\lambda}} = \frac{\nu}{\sqrt{2}} \quad (2.29)$$

where $\nu = \mu/\sqrt{\lambda}$. The particular choice of vacuum corresponding to $\phi^1 = \phi^2 = \phi^4 = 0$ can be made which gives

$$\Phi_{min} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad (2.30)$$

This choice of a particular vacuum is referred to as the spontaneous symmetry breaking of the theory. The Lagrangian which describes the motion of the doublet Φ under the potential (2.28) is given by

$$\mathcal{L}_{Higgs} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - \mu^2 \Phi^\dagger \Phi - \lambda (\Phi^\dagger \Phi)^2 \quad (2.31)$$

Equation (2.27) can be re-parameterised as an expansion around Φ_{min}

$$\Phi = U(\xi)^{-1} \begin{pmatrix} 0 \\ \nu + H(x) \end{pmatrix} \quad (2.32)$$

where $U(\xi)$ is a unitary transformation given by $U(\xi) = \exp(-i\vec{t} \cdot \vec{\epsilon}/\nu)$ where $\vec{t}$ is the three-vector given in Equation (2.12) and $\vec{\epsilon}$ is a three-vector representing an infinitesimal change. A doublet of this form contains four degrees of freedom corresponding to the four real scalar fields, H and $\vec{\xi}$. However three of these fields are unphysical [6], producing a Lagrangian with massless particles known as Goldstone bosons. These fields can be removed by the gauge transformation $\Phi \rightarrow U(\xi)\Phi$, under which the Lagrangian is invariant. This leaves just the field H , and under this transformation the Lagrangian becomes

$$\begin{aligned} \mathcal{L}_{Higgs} = & \frac{1}{2} \partial_\mu H \partial^\mu H + \frac{1}{4} g^2 (\nu^2 + 2\nu H + H^2) W_\mu^+ W^{\mu-} + \\ & \frac{1}{8} (g^2 + g'^2) (\nu^2 + 2\nu H + H^2) Z_\mu Z^\mu + \\ & -\frac{\mu^2 \nu^2}{4} + \mu^2 H^2 - \lambda \nu H^3 - \frac{\lambda}{4} H^4 \end{aligned}$$

Identifying the mass terms corresponding to the W and Z fields gives us

$$m_W = \frac{1}{2} g \nu$$

$$m_Z = \frac{1}{2} \sqrt{g^2 + g'^2} \frac{1}{2} \nu = \frac{1}{2} \frac{g\nu}{\cos\theta_W} = \frac{m_W}{\cos\theta_W}$$

This gives an Electroweak theory with three massive bosons and one massless boson, consistent with experiment. In making the gauge transformation earlier three degrees of freedom corresponding to the three massless fields appear to have been lost. It can be seen however, that the $W^\pm$ and Z bosons have acquired mass, which now allows an additional polarization state, the longitudinal polarisation, previously forbidden for a massless particle. This requires an increment in the polarization states from two to three for each field. In this sense, the now massive bosons are said to have ‘eaten’ the Goldstone bosons, providing the longitudinal polarization of the massive fields. What is now called the Higgs field, $H(x)$, itself has a mass term determined by its vacuum expectation value ν .

Yukawa Coupling

A coupling between the fermions and the Higgs field can also be added to the Standard Model Lagrangian in order to give the fermions mass in a similar and consistent way. This is done by assuming a contribution to the Lagrangian, $\mathcal{L}_{yukawa}$, which describes the coupling between the fermions and the Higgs doublet, of the form

$$\mathcal{L}_{yukawa} = -g_f \bar{\Psi}_f \Psi_f \Phi^0 \quad (2.33)$$

The same expansion is performed around the vacuum expectation value of the Higgs doublet, ν , which will give

$$\mathcal{L}_{yukawa} = -g_f \nu \bar{\Psi}_f \Psi_f - g_f \bar{\Psi}_f \Psi_f H \quad (2.34)$$

The first term represents a mass term, where $m_f = g_f \nu / 2$. The coupling constant g_f can be chosen to match the fermion mass observed in experiment. The second term represents the coupling of the fermion field to the Higgs field, which is given by $2m_f / \nu$. In this way, the Higgs field couples to the fermions proportionately to their mass in the same way as the bosons. The Yukawa couplings, g_f , constitute free parameters in the Standard Model.

Neutrino Masses

It should be noted that in this treatment of the Standard Model, the neutrinos are assumed to be massless, with the theory consisting only of left handed neutrinos and right handed anti-neutrinos. However, the observed phenomenon of neutrino mixing, where neutrinos can oscillate between the different flavour states, implies that the neutrinos are in fact massive [7]. The Standard Model can be extended to accommodate neutrino masses by the introduction of a right handed neutrino field, acting as an $SU(2)$ singlet, which would give the neutrino mass in the same way as the other leptons, through the mixing of the left and right handed components of the field. Alternatively, a Majorana mass term can be added, whereby a mass term is produced by combining the left handed neutrino field with its complex conjugate [8]. The current best experimental upper bound on the combined mass of the three neutrino flavours is 0.28 eV [9].

2.1.4 Quark Mixing & $\mathcal{CP}$ Violation

In QFT, $\mathcal{CP}$ violation refers to the violation of the postulated $\mathcal{CP}$ symmetry, a combination of the $\mathcal{C}$ and $\mathcal{P}$ symmetries, which transform particles to their antiparticles, and perform spatial inversion of the Lagrangian respectively. It has been observed to be a perfect symmetry in both QED and QCD. However, in 1964, $\mathcal{CP}$ violation was discovered in the decay of neutral kaons [10]. This was due to transitions between different quark flavours which produced kaon mixing and led to the suggestion that the down type quark eigenstates mentioned above, d' , are not in fact the same as the d type quarks which participate in the strong interaction. The relation between them can be represented by a unitary matrix, V , known as the CKM matrix [11].

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.35)$$

The freedom to choose the quark phases, as well as the unitary matrix constraints, means there are just four free parameters in the matrix: three mixing angles, θ_{12} ,

θ_{23} and θ_{13} , and a complex phase δ . The matrix can then be represented as

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.36)$$

where $c_{ij} = \sin \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$. Non-zero mixing angles result in non-zero off diagonal elements and allow for transitions to different quark flavours in charged electroweak decays. The phase δ is known as the $\mathcal{CP}$ -violating phase, as a non-zero value will result in a $\mathcal{CP}$ violating term in the Lagrangian. The current experimental determination of the magnitude of the parameters of the CKM matrix is given below [12].

$$V = \begin{pmatrix} 0.97428 \pm 0.00015 & 0.2253 \pm 0.0007 & 0.00347^{+0.00016}_{-0.00012} \\ 0.2252 \pm 0.0007 & 0.97345^{+0.00015}_{-0.00016} & 0.0410^{+0.0011}_{-0.0007} \\ 0.00862^{+0.00026}_{-0.00020} & 0.0403^{+0.0011}_{-0.0007} & 0.999152^{+0.000030}_{-0.000045} \end{pmatrix} \quad (2.37)$$

Requiring that the matrix is unitary conserves a property of the Standard Model known as ***weak universality***, which is a consequence of the fact that the vector bosons couple equally to all SU(2) doublets. Any deviation from unitarity could be an indication of new physics and for this reason the unitarity nature of the matrix is subject to ongoing experimental tests [13].

2.1.5 Tau Leptons

The τ lepton is the heaviest of the three charged leptons, with a mass of 1.776 GeV [14] compared to masses of 0.105 GeV and 0.0005 GeV for the muon and electron respectively [12]. The lifetime of the tau and that of the muon are approximately related to their relative masses through the formula [15]

$$\frac{\tau_\tau}{\tau_\mu} = \frac{Br(\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e)}{Br(\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e)} \left(\frac{m_\mu}{m_\tau} \right)^5 \quad (2.38)$$

where $\tau_{\tau/\mu}$ refers to the tau/muon lifetime, m_τ/μ to their masses, and Br denotes the branching ratio for the indicated decay. Thus, due to its heavier mass, the tau

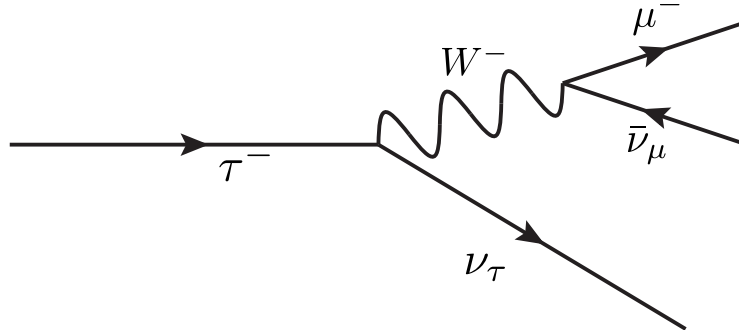


Figure 2.2: The decay of a τ lepton into a muon and two neutrinos via a W boson.

has a much smaller lifetime, which has been determined experimentally to be [12]

$$\frac{\tau_\tau}{\tau_\mu} = 1.3227 \pm 0.0005 \times 10^{-7} \quad (2.39)$$

The tau lepton is unique in that, due to its mass, it is the only lepton which can decay both leptonically and hadronically. This also allows the tau to act as a probe of both QCD and electroweak interactions. The branching ratios for its primary decay modes are tabulated in Table 2.2. Decays involving a single charged particle, such as an electron, muon or charged pion are referred to as single-pronged decays, while those involving three charged particles, such as three charged pions, are referred to as three-pronged decays. Three-pronged decays typically proceed through an intermediate meson resonance, such as a ρ , η or a_1 , for example in the decay $\tau^- \rightarrow a_1 \nu_\tau \rightarrow \rho \pi^- \nu_\tau \rightarrow \pi^- \pi^+ \pi^- \nu_\tau$ [16]. As a consequence of lepton family number conservation, tau decays are accompanied by one or more neutrinos, which typically escape detection in experiments. This makes the process of tau identification challenging in an experimental environment.

In addition, the polarization of the tau leptons must be taken into account in processes which produce a tau pair. For any inelastic process, the angular distribution of final state particles is dependent on their helicities, which can be represented probabilistically by their helicity density matrices [17]. Calculations of these matrices must take into account correlations between the initial and final state of a process in order to correctly predict the kinematics of the event. In the case of $Z \rightarrow \tau\tau$ events, due to the $V - A$ nature of the weak interaction, the helicities of

Final State	Branching Ratio
$e^- \bar{\nu}_e \nu_\tau$	0.1736 ± 0.0006
$\mu^- \bar{\nu}_\mu \nu_\tau$	0.1784 ± 0.0006
$\pi^- \nu_\tau$	0.1106 ± 0.0011
$\pi^- \pi^0 \nu_\tau$	0.2542 ± 0.0014
$\pi^- \pi^+ \pi^- \nu_\tau$	0.0916 ± 0.0010

Table 2.2: A summary of the branching ratios for different final states from τ^- decays [12].

the tau pair produced through the intermediate boson are correlated [18, 19]. This correlation must be taken into account in the subsequent decay of the taus in order to accurately predict the angular distribution of the final state particles.

2.1.6 Experimental Observables

Experimentally, the dynamics of elementary particles are probed using scattering experiments. Two beams of particles are accelerated to well defined momenta and collided. What is then measured is the the rate at which a particular final state is produced. These measurements can then be compared to theoretical predictions. To perform these predictions, the initial state, comprised of the two colliding particles, is given by Ψ_i and the final state of interest is given by Ψ_f . The probability of transition between Ψ_i and Ψ_f is encoded in the Scattering Matrix, S , known as the ***S-Matrix***.

$$S_{fi} = \langle \Psi_f | \Psi_i \rangle \quad (2.40)$$

It is useful to write this Scattering Matrix in terms of a delta-function free matrix element $\mathcal{M}_{fi}$ represented as

$$S_{fi} = \delta_{fi} - 2\pi i \delta^4(p_f - p_i) \mathcal{M}_{fi} \quad (2.41)$$

The delta function, δ_{fi} denotes the trivial case where no interaction occurs and the final state is the same as the initial state. The second delta function, $\delta^4(p_f - p_i)$ ensures energy-momentum conservation. In order to see how this matrix can be used to make predictions for experiments, the transition rate per unit volume, per

unit time, W_{fi} , is introduced where

$$W_{fi} = |S_{fi} - \delta_{fi}|^2 \quad (2.42)$$

The cross section, σ , is then given by this transition rate divided by the incident flux of the beam, F , and summed over the number of final states available

$$\sigma = \sum_f \frac{W_{fi}}{F} \quad (2.43)$$

In quantum theory, the number of final states per particle in unit volume with momentum in element d^3p is restricted to be $d^3p/(2\pi)^3$ [20]. This is then divided by $2E$, where E is the energy of the particle, as the wavefunctions can be normalised to have $2E$ particles per unit volume. The sum over the number of final states can then be expressed as an integral over the invariant phase space factor dQ . For a final state with n particles, this is given by

$$dQ = (2\pi^4)\delta^4(p_f - p_i) \prod_n \frac{d^3p_n}{2\pi^3 2E_n} \quad (2.44)$$

The incident flux is the number of colliding particles passing through unit area per unit time, and for two bunches of colliding particles with relativistic velocities v_A and v_B in the lab frame, and energies E_A and E_B , is given by [21]

$$F = |v_A - v_B| 2E_A 2E_B \quad (2.45)$$

The cross-section can then be expressed in a differential form

$$d\sigma = \frac{|\mathcal{M}|^2}{F} dQ \quad (2.46)$$

As F is determined by the experimental configuration, and dQ is simply a sum over the available phase space, the physics of the interaction is encoded in the matrix $\mathcal{M}$, also known as the invariant amplitude.

Perturbative Calculations

The scattering matrix provides the information necessary to predict the scattering of particles under an interaction potential. It is important to determine how to make these calculations. This can be done by following Reference [22] and considering degenerate energy eigenstates $\Psi(x)$ and $\Phi(x)$ of the interacting and free Hamiltonians respectively. The free Hamiltonian is represented by H_0 and the interacting Hamiltonian by $H(t)$ where $H(t) = H_0 + V(t)$, with $V(t)$ representing the interaction potential, which can be regarded as a perturbation to the free system. This gives the relations

$$H_0\Phi(x)_\alpha = E_\alpha\Phi(x) \quad (2.47)$$

$$H(t)\Psi(x)_\alpha = E_\alpha\Psi(x) \quad (2.48)$$

In the derivation of Equation (2.46) the initial and final states corresponded to eigenstates of H at times $-\infty$ and $+\infty$. The scattering matrix element representing the probability for scattering between these two states α and β is given by Equation (2.40). The calculation can be simplified by expressing the scattering matrix in terms of the free eigenstates. The interaction potential $V(t) \rightarrow 0$ as $t \rightarrow \pm\infty$, and so the eigenstates of both Hamiltonians are equivalent in the initial and final states. This allows the eigenstates to be related using the formula below

$$\Psi_\alpha^\pm = \Omega(\mp\infty)\Phi_\alpha \quad (2.49)$$

where $\Omega(\tau)$ is the time evolution operator and is given by

$$\Omega(\tau) = e^{iH\tau}e^{-iH_0\tau}. \quad (2.50)$$

$\Psi_\alpha^\pm$ represent the eigenstates of H at $t = \pm\infty$, often referred to as the “out” and “in” states. These relation then allows the scattering matrix to be expressed in terms of the free energy eigenstates $\Phi(x)$,

$$S_{\beta\alpha} = \langle \Phi_\beta | S | \Phi_\alpha \rangle \quad (2.51)$$

where S is the scattering matrix operator given by

$$S = \Omega(\infty)^\dagger \Omega(-\infty) = U(+\infty, -\infty). \quad (2.52)$$

$U(\tau, \tau_0)$ is now a unitary operator which transforms a state from time τ_0 to τ and is given by

$$U(\tau, \tau_0) = \exp(iH_0\tau) \exp(-iH(\tau - \tau_0)) \exp(-iH_0\tau_0). \quad (2.53)$$

Differentiating with respect to τ gives

$$i \frac{dU}{d\tau}(\tau, \tau_0) = V(\tau)U(\tau, \tau_0) \quad (2.54)$$

which has the solution

$$U(\tau, \tau_0) = 1 - i \int_{\tau_0}^{\tau} dt V(t) U(t, \tau_0). \quad (2.55)$$

This can be expanded to give a perturbative expression for the scattering matrix operator

$$\begin{aligned} S = U(+\infty, -\infty) = & 1 - i \int_{-\infty}^{\infty} dt_1 V(t_1) + \\ & (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 V(t_1) V(t_2) + \dots \end{aligned} \quad (2.56)$$

In this way the S-Matrix, and hence the matrix element, $\mathcal{M}$ can be calculated via a perturbative expansion in powers of the interaction field V using equations (2.51) and (2.56). For small values of V , the matrix is well defined by the early terms of its perturbative expansion. The terms of the matrix are referred to as leading order (LO), next to leading order (NLO), next to next to leading order (NNLO) in order of the perturbative expansion. Note that equation (2.56) is often more succinctly represented as

$$S = \mathcal{T} \left(\exp \left[-i \int_{-\infty}^{+\infty} dt V(t) \right] \right) \quad (2.57)$$

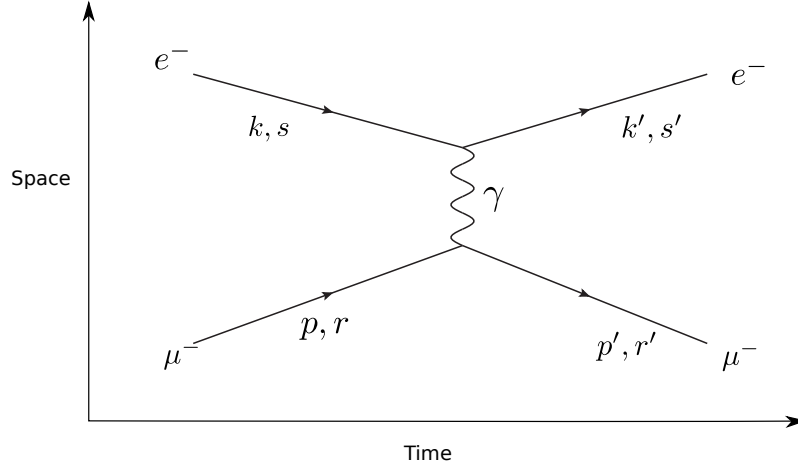


Figure 2.3: A simple Feynman diagram depicting the scattering of an electron and a muon with initial momenta k and p and spins s and r respectively to a final state with momenta k' and p' and spins s' and r' through the exchange of a virtual photon. The space and time axes are labelled explicitly. The muon and electron lines are external lines, the photon represents the propagator, and the points at which they intersect are known as vertices.

where $\mathcal{T}$ is the time ordering operator which orders operators in the Taylor expansion according to their time parameter and so preserves the time ordered structure of (2.56).

Feynman Diagrams

As can be seen in the above equation, the calculation of matrix elements requires the use of complex integrals. However, their regular structure can be exploited to represent them graphically as diagrams known as Feynman diagrams. These diagrams can be used to represent each term in the perturbative expansion. This is done by inserting (2.56) into (2.51) and expanding it using Wick's theorem [23]. Wick's theorem uses contraction and normal ordering to simplify the scattering matrix element into the sum of products of field commutators. A Feynman diagram then represents a term in the Wick's expansion. The diagrams consist of points called vertices and lines which connect the vertices either to the initial or final state (*external lines*) or to other vertices (*internal lines*). Charge, energy and momentum are conserved at each vertex. External lines represent real particles while

internal lines represent virtual or off shell particles. Each element of the diagram is assigned an algebraic factor, with the product of all the factors representing the matrix element $\mathcal{M}$. As the order of the calculation increases, the diagrams become more complex, involving self energy terms and loop contributions, as well as final states with additional external lines. Figure 2.3 shows a leading order diagram representing the scattering of a muon and an electron.

2.2 Hard Interactions at the Hadron Colliders

At hadron colliders, the composite nature of the colliding particles results in a rather complex event structure. However, the events of interest can often be broken down into two distinct processes: the hard scatter and the underlying event. The hard scatter refers to high energy interactions such as the production of high momentum lepton pairs through an intermediate Z boson. Processes such as this can be calculated to high precision using perturbative techniques as described in Section 2.1.6. However, the additional partons in the event which are not involved in this interaction can still partake in low energy interactions, collectively known as the underlying event. The behaviour of the underlying event is dominated by non-perturbative QCD effects, and consequently not as easily calculable. In order to compare the overall process to theoretical predictions, these low energy processes must be included in the calculations.

Certain phenomenological models can be employed to perform calculations involving these soft processes. The relationship between the cross section of the hard process, known as the partonic cross section, to the overall cross section, or hadronic cross section can be obtained by parameterising the internal structure of the proton through the use of Parton Distribution Functions (PDFs) and exploiting the factorisation theorem [24]. As the partonic cross-section can be calculated using perturbative QCD, a measurement of the hadronic cross section can also be performed in order to better understand the structure of the PDFs.

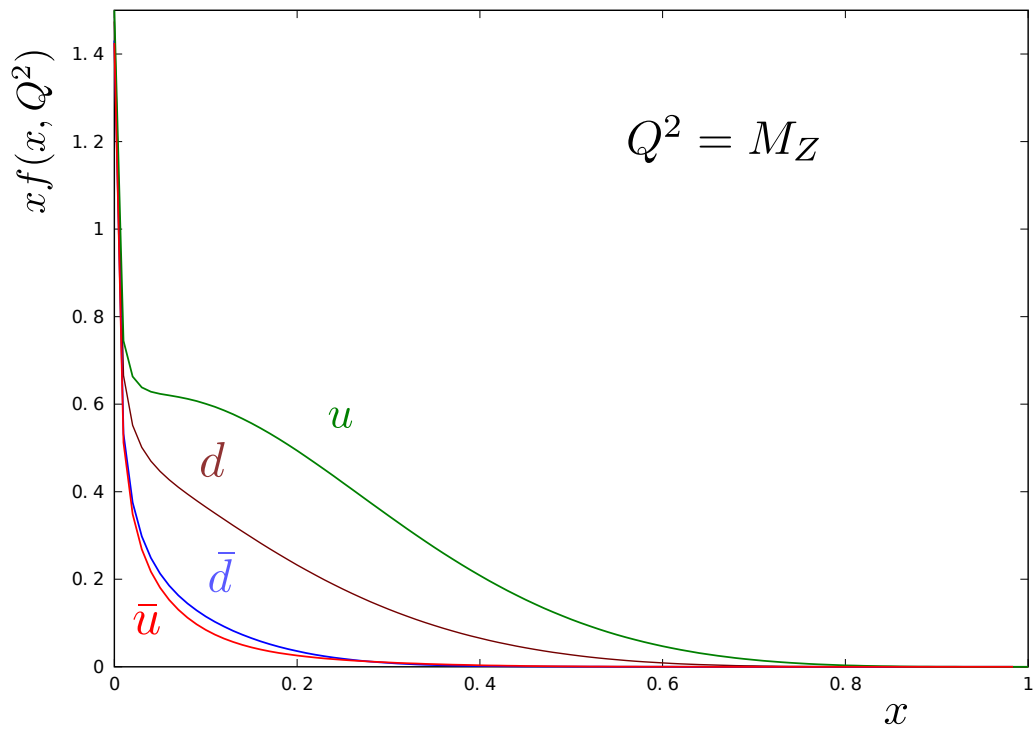


Figure 2.4: Parton Distribution Functions at $Q^2 = M_Z$ using the CTEQm PDF Set for the u (green), d (brown), $\bar{u}$ (red) and $\bar{d}$ (blue) quarks.

2.2.1 PDFs and the Factorisation Theorem

Parton Distribution Functions are process independent, non-perturbative functions which represent the probability density for finding a parton within a hadron with a certain longitudinal momentum fraction x at a given energy scale, Q^2 . Due to limitations in current QCD lattice calculations, these functions must be extracted at a given scale from experimental data. They can then be evolved to different energy scales using the DGLAP [25, 26, 27] equations and then used as input to theoretical calculations [28]. The PDFs can be extracted from measurements performed at the LHC by exploiting the factorisation theorem, which decomposes the hadronic cross section into the product of a partonic level cross-section and a parameterisation of the colliding hadrons, determined by the PDFs. The equation for the production of object X is given by

$$\sigma_{AB \rightarrow X} = \int dx_a dx_b f_{a/A}(x_a, Q^2) f_{b/B}(x_b, Q^2) \hat{\sigma}_{ab \rightarrow X} \quad (2.58)$$

where A and B refer to the colliding hadrons and a and b are the partons involved in the hard scattering process. The functions $f_{a/A}(x_a, Q^2)$ and $f_{b/B}(x_b, Q^2)$ are the PDFs for partons a and b as a function of the momentum fraction they carry, $x_{a/b}$, and the energy transfer of the hard scattering process, Q^2 . The partonic level cross-section, representing the cross-section for the production of X from partons a and b is given by $\hat{\sigma}_{ab \rightarrow X}$ and is perturbatively calculable.

2.2.2 Z Boson Production At The LHC

The leading order process for the production of a Z boson at the LHC is through the annihilation of a quark anti-quark pair. As the LHC is a proton proton collider, there are no valence anti-quarks present in the collision, and the anti-quarks instead come from the quark sea. The cross section for the production of a Z boson through the leading order Feynman diagram, illustrated in Figure 2.5, is easily calculable using the couplings that were derived in section 2.1.3. In the notation of Feynman diagrams, the vertex factor is given by

$$-i \frac{g}{\cos \theta_W} \gamma^\mu \frac{1}{2} (c_v^f - c_A^f \gamma^5) \quad (2.59)$$

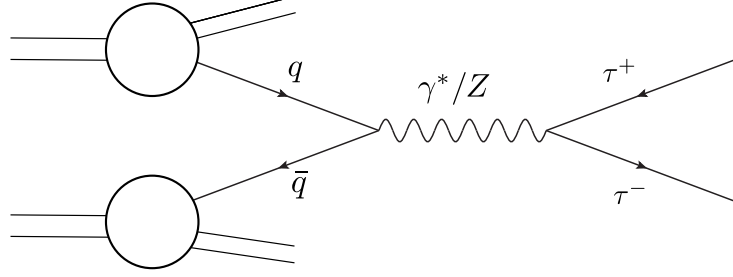


Figure 2.5: Di-muon production via the Drell-Yan process at a proton proton collider

In addition there are factors $u(p)_q^s$ and $\bar{v}(q)^{s'}$, which are Dirac spinors corresponding to internal lines representing a quark and anti-quark with spins s and s' respectively. There is also a factor of ϵ^λ , representing an outgoing vector boson of polarisation λ . The total cross-section is obtained by integrating the product of these particles over all the available phase space. At a particle collider such as the LHC, the initial partons involved in the interaction are not typically in a polarised state, and the initial spin states of the quarks are averaged when performing the calculation, as well as summing over the three polarisation states of the massive Z boson. The scattering amplitude is then given by

$$\hat{\sigma}_{q\bar{q}\rightarrow Z} = \frac{\pi}{3} \sqrt{2} G_F m_Z^2 (c_v^2 + c_A^2) \delta(\hat{s} - m_Z^2) \quad (2.60)$$

where the factor of $1/3$ arises from the fact that quarks have three colours and can only interact with a quark of the same colour. The delta function requires conservation of energy, where $\hat{s}$, the centre of mass energy of the parton interaction, is related to the centre of mass energy of the proton collision, s , by the momentum fractions, x_1 and x_2 , of the protons by $\hat{s} = x_1 x_2 s$. G_F is the Fermi coupling constant and is given by

$$G_F = \frac{\sqrt{2}}{8} \frac{g^2}{m_W^2} \quad (2.61)$$

Final State		Br
Neutrinos	ν_e, ν_μ, ν_τ	0.20
Leptons	e, μ, τ	3×0.0337
Hadrons		0.6991

Table 2.3: The branching ratios for the decay of a Z boson

In order to calculate the production cross section in proton-proton collisions, the partonic cross section must be convoluted with the (PDF) of the protons to give

$$\sigma_{pp \rightarrow Z} = \int dx_1 dx_2 \hat{\sigma}_{q\bar{q} \rightarrow Z} \sum_q [f_{q/p_1}(x_1, Q^2) f_{\bar{q}/p_2}(x_2, Q^2) + f_{\bar{q}/p_1}(x_1, Q^2) f_{q/p_2}(x_2, Q^2)] \quad (2.62)$$

where p_1 and p_2 represent the colliding protons and q and $\bar{q}$ represent the quarks involved in the hard interaction. The cross-section for the production of pairs of leptons via an intermediate Z is obtained by multiplying $\sigma_{pp \rightarrow Z}$ by the branching ratio, which represents the fraction of Z 's which decay through any particular lepton pair. The table of branching ratios is given in Figure 2.3. This calculation describes only the leading order process for Z boson production. Higher order processes must be considered for a full comparison with data, and include both virtual corrections, involving internal lines which will modify the cross section but produce the same final state, and real corrections, which produce additional partons in the final state, usually resulting in quark or gluon jets. Some of the higher order Feynman diagrams are represented in Figure 2.6.

2.2.3 Parton Showering

Even state of the art calculations, which have been performed up to NNLO, are missing higher order tree level contributions. These higher order contributions can contain enhancements in certain regions of phase space which means they are essential for a reliable description of the event shape in simulation. These effects can be estimated using the parton shower model [28] which models the higher order contributions by the radiation of quarks, gluons and photons from the partons involved in the hard matrix element. These emissions occur before and after the hard process,

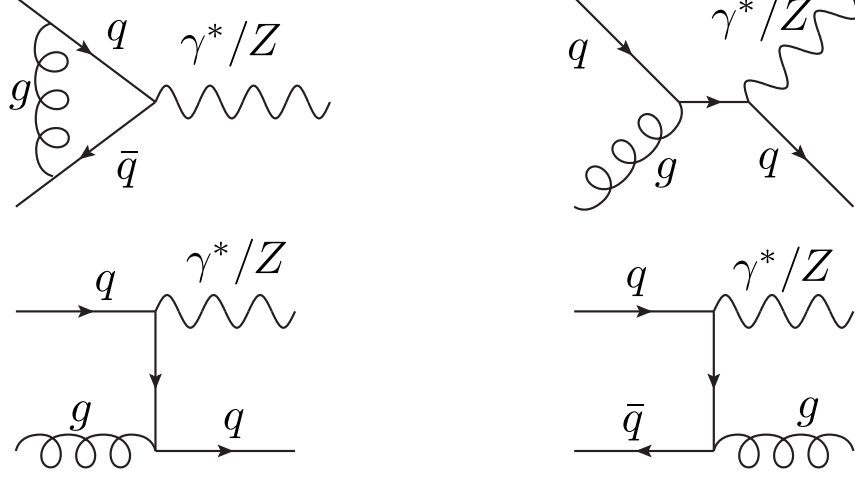


Figure 2.6: The Feynman diagrams for higher order processes leading to the production of a Z boson at a proton proton collider such as the LHC.

known as initial state (ISR) and final state (FSR) radiation respectively. ISR and FSR are both very similar in nature and are based on the processes $\ell \rightarrow \ell\gamma$, $q \rightarrow qg$, $g \rightarrow gg$ and $g \rightarrow g\bar{q}$ which can be generically represented as $a \rightarrow bc$. Each branching is characterised by a scale Q^2 , which evolves with each branching, and serves to order the branchings from higher to lower energies. This evolution is modelled using the Sudakov Form Factors [29] which describe the probability of evolving between different scales without branching. The kinematics of the branching for any of the above process is given by $P(z)_{a \rightarrow bc}$ which is known as the splitting function and describes the probability for a to branch where z is the fraction of the momentum of a obtained by b during the branching, with c obtaining a fraction of $1 - z$. The full parton shower is then obtained by choosing an energy scale for the hard process, and evolving backwards towards the initial parton to produce ISR, and evolving forward to produce FSR. This shower evolution is cut off at a minimum, Q_0 , when approaching the final state. The choice of which variable to use as the evolution variable, Q^2 , is not unique. Some examples are the energy transfer, $Q^2 = m^2$, the transverse momentum of the leading parton, $Q^2 = p_T^2$, and the angle of emission, $Q^2 \sim E^2\theta^2$.

2.2.4 Hadronisation

Hadronisation is the formation of hadrons from free quarks and gluons. After a collision, a number of quarks and gluons are produced but according to the principle of quark confinement, they cannot exist in a free state and so combine with $q\bar{q}$ pairs created from the vacuum to form hadrons. The underlying process that ultimately produces these hadrons is not perturbatively calculable and so is evaluated using a number of phenomenological models. Among these models are the String Fragmentation (SF) and Cluster Fragmentation (CF) models.

In the SF model [30], the potential of $q\bar{q}$ pairs is assumed to increase linearly with their separation analogously to a string. As they move sufficiently far from each other, the string can break, producing a new $q\bar{q}$ pair from the vacuum. When the $q\bar{q}$ form adjacent string breaks, these are combined to form mesons. The production of baryons is more complicated and produced using a quantum tunnelling mechanism. In the CF model [31] all outgoing gluons left over from the parton showering are split into light quark-antiquark or diquark-antidiquark pairs. Colour singlet clusters are formed among neighbours and are fragmented into hadrons based on the cluster mass.

2.2.5 Underlying Event and Pile-Up

In hadron hadron collisions, the composite nature of the colliding hadrons means that pairs of partons not involved in the primary process can interact. This interaction will be soft compared to the primary interaction but will give a non-negligible contribution to the overall event and its multiplicity. These multiple interactions can be evaluated by ordering the scattering in a falling scale of p_T , so that interactions are ordered from harder to softer scatterings. Correlations, such as those due to flavour and momentum conservation can be taken into account at each step.

It is also possible to have pile-up events, where multiple collisions occur per beam crossing. This also contributes to the event structure and can be evaluated if the luminosity per bunch crossing is known.

2.2.6 Monte Carlo Generators

The physics of particle colliders such as the LHC is predicted by Monte Carlo generators. These generators are software applications which simulate the processes that occur in high energy collisions. A number of different generators exist that provide predictions for a variety of physical observables which can be measured experimentally at the LHC. Calculation of the matrix element describing the hard process are typically performed at leading order for general purpose generators, and at higher orders for specialised generators. The non-perturbative component of the event is then estimated using the phenomenological models outlined in the previous section. Two of the most common generators are Pythia[32] and Herwig [33].

Pythia

Pythia is a general purpose Monte Carlo generator which calculates processes at LO. It has the ability to simulate a large number of events and allows different PDF sets and phase space cuts to be applied by the user. The original version was written in `FORTRAN77` and it has since being updated to `C++`. The latest `Fortran` version is Pythia 6, while Pythia 8 is the latest `C++` version. Both versions are maintained as they possess many different features. Pythia offers a parton shower model which can use either the transverse momentum or mass of the decaying parton as the evolution variable, Q^2 . It also uses the string fragmentation model to perform the hadronisation. Pythia 6 is the default generator in the LHCb simulation package. Pythia 8 is used in some of the analysis in this thesis as it has fully spin correlated tau decays implemented in the latest versions.

Herwig

Herwig is an alternative general purpose Monte Carlo generator with many similarities to Pythia. It is also available in both an older `Fortran` version and a new `C++` version, Herwig++. Herwig++ is available as an alternative generator in the LHCb software, and it can perform calculation to LO or NLO order. Herwig uses an angular ordered parton shower and the hadronisation step is performed with the cluster fragmentation model. Herwig also performs fully spin correlated tau decays [34].

CHAPTER 3

EXPERIMENTAL ENVIRONMENT

3.1 LHC

The LHC is the world's highest energy particle accelerator. It accelerates tightly packed bunches of protons in opposite directions around a circular accelerator, forcing them to collide at certain points where particle detectors are located. At optimal running conditions, these bunches will each have energies of 7 TeV, resulting in a centre of mass energy of 14 TeV.

3.1.1 Design Considerations

Particle Accelerators come in two main varieties, linear accelerators (linacs) and circular accelerators (synchrotrons). In a linear accelerator, bunches of particles are guided through straight waveguides with a periodic array of gaps, known as cavities. Radio-frequency (RF) oscillations are used to establish a moving electromagnetic wave in phase with the particles. Maintaining the phase between the two allows the particles to be accelerated. The maximum beam energy of a linear accelerator is limited by its length.

In a synchrotron, this limitation is avoided by accelerating the particles in a circular or near circular path by an array of magnets. The closed path allows the particles to be circulated around the ring repeatedly and to gain energy with every

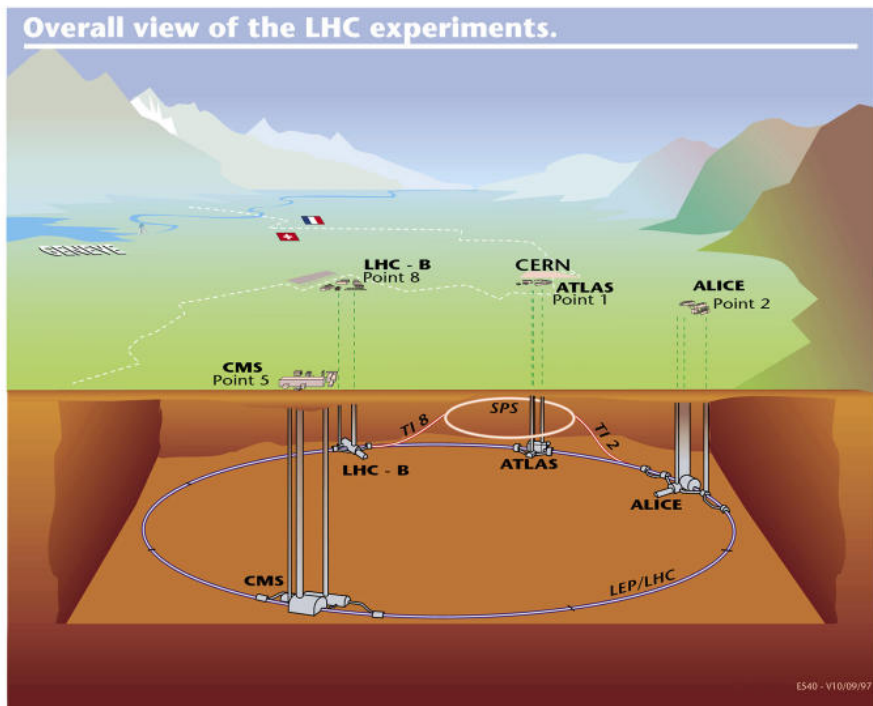


Figure 3.1: An overview of the LHC and the four main experiments [35]

revolution. A limitation of these accelerators arises in the form of synchrotron radiation, which is the electromagnetic radiation emitted when charged particles are accelerated radially. The energy lost by a particle of charge q per revolution of radius r is given by [15]

$$\Delta E = \frac{q^2 \beta^3 \gamma^4}{3\epsilon_0 r} \quad (3.1)$$

where ϵ_0 is the permittivity of free space and β and γ are the standard relativistic variables. For particles traveling close to the speed of light, $\beta \approx 1$ and $\gamma = E/m$ where m is the mass of the accelerating particle. In this case the synchrotron radiation is inversely proportionally to the fourth power of mass. For light particles such as the electron, these energy losses can thus be quite severe, restricting the centre-of-mass energy achievable by the accelerators. This loss can be minimised by using heavy particles such as protons, and accelerating them in a ring with a large radius. In this case, the energy achievable by the accelerator is limited by the magnet strength required to keep the colliding particles traveling in a circular path.

The LHC is a synchrotron accelerator and consists of eight approximately circular arcs and eight straight line sections. An overview is shown in Figure 3.1. The main detectors, ATLAS, ALICE, CMS and LHCb are installed in four of these sections, with the remaining sections dedicated to machine operation. Two are reserved for beam cleaning, which controls the background particle rates from machine components, with RF and beam instrumentation, which controls beam losses, performed in another [36]. The final segment is reserved for the beam dump system which allows the beam to be absorbed safely in both planned and emergency dumps [37]. ATLAS and CMS are general purpose detectors, which will look for signs of new physics, such as the Higgs boson, extra dimensions, supersymmetry and others [38, 39]. In addition to proton-proton collisions, the LHC collides lead-ions at high energies to generate a quark-gluon plasma, a state of matter in which quarks and gluons are de-confined, and the ALICE detector studies the properties of this plasma [40]. The LHCb experiment, data from which is analysed in this thesis, will be described in detail in the next section.

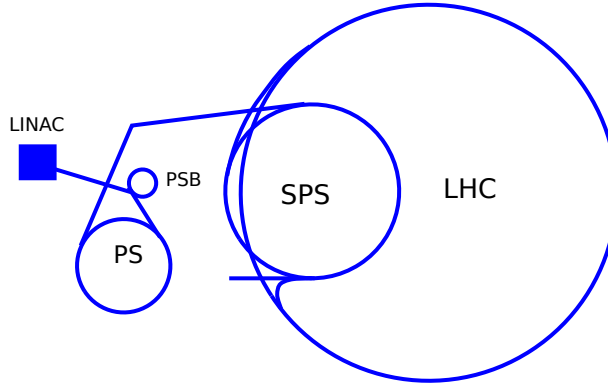


Figure 3.2: The LHC Accelerator System

3.1.2 Accelerator System

The LHC consists of a series of superconducting dipole magnets and RF cavities that, in 2010 and 2011, directed and accelerated two 3.5 TeV counter-rotating proton beams around the LHC ring. The beams are focused using a series of quadrupole magnets. At peak operation, 2808 proton bunches can circulate in the ring separated by a time interval of 25 ns [41]. In 2011, the collisions took place with a bunch spacing of 50 ns and up to a maximum of 1380 circulating bunches [42].

Before the protons are injected in to the main accelerator system of the LHC, they are prepared by a series of systems which successively boost their energy. The Linear Particle Accelerator (LINAC) accelerates their energy to 50 MeV, from there they are boosted into the Proton Synchrotron (PS) which accelerates them to 26 GeV before they are passed to the Super Proton Synchrotron (SPS) which boosts them to an energy of 450 GeV. From the SPS they are then injected into the main ring of the LHC. Here the proton bunches are accumulated and accelerated to their colliding energy [43]. The LHC accelerator system is illustrated in Figure 3.2.

3.1.3 Luminosity

The instantaneous luminosity, $\mathcal{L}$, can be expressed as the constant of proportionality relating the event rate, R_{EVT} and cross-section of a given process.

$$R_{EVT} = \sigma \cdot \mathcal{L} \quad (3.2)$$

The cross-section is related to the probability of the process to occur for specific initial conditions, such as the energy and nature of the colliding particles. What is of interest for the study of any process is the total number of events of that process produced at the LHC over a time period T , N_{EVT} . This is related to the cross-section by the time-integrated luminosity.

$$N_{EVT} = \sigma \int_0^T \mathcal{L} dt \quad (3.3)$$

The luminosity can also be expressed in terms of the beam parameters [44]

$$\mathcal{L} = f \frac{N_1 N_2}{4\pi\sigma_x\sigma_y} \quad (3.4)$$

where f is the beam crossing frequency, N_1 and N_2 are the number of particles in the bunches in each beam, and σ_x and σ_y describe the transverse beam profiles in the horizontal and vertical planes. Thus, once the luminosity is known and the number of events found for a particular process, its cross-section can be determined. In data taken during 2011 at the LHC, each bunch contained approximately 10^{11} protons and the beams had transverse dimensions of the order of 1 mm [45].

The main focus of the ATLAS and CMS experiments is the search for new physics. This requires high luminosities as the processes under study are expected to have low cross-sections [46]. High luminosities have the effect of increasing the average number of collisions, μ , per bunch-crossing. The presence of these multiple interactions, known as pile-up, makes reconstruction difficult and has specific problems for LHCb. As the main focus of LHCb is the study of the decays of B hadrons, it requires precise determination of primary and secondary vertices to isolate the signal from background processes. This is difficult in events with a large number of separate interactions. The ideal conditions for accurate vertex reconstruction at

LHCb is the presence of a single interaction in a bunch crossing. To this end, LHCb is designed to run at a lower luminosity to reduce pileup. This is achieved by offsetting the colliding beams in order to reduce the active collision area. This offset can be reduced as the proton beams decay in order to maintain a steady luminosity throughout the beam lifetime, a technique known as “luminosity levelling” [45].

3.1.4 Current Performance

The original commissioning road map of LHC was to run at centre-of-mass energy of 14 TeV after a period of commissioning. However after a serious incident in September 2008 the operation had to be suspended for a year to repair the damage [47]. The commissioning roadmap was then revised and it was decided to run the LHC at a centre-of-mass energy of 7 TeV for a period of two years before switching to an energy of 8 TeV in 2012 [48]. It will then be shut down for further commissioning before eventually running at the design energy of 14 TeV. The luminosity delivered by the LHC to the four main experiments in 2011 is shown in Figure 3.3.

3.2 LHCb

LHCb is an experiment dedicated to heavy flavour physics at the LHC. It searches for new physics in $\mathcal{CP}$ violation and the rare decays of hadrons containing beauty and charm quarks. Many models of new physics produce contributions that change the expectations of the $\mathcal{CP}$ violating phase, δ , as well as the branching ratios of rare decay modes [51, 52]. These models also provide for the existence of decay modes forbidden in the Standard Model which may be discovered at the LHC. Due to the large $b\bar{b}$ cross-section predicted at energies of 14 TeV, LHCb is in a position to study $\mathcal{CP}$ violation and rare decay modes with high statistics. To cope with the copious production of $b\bar{b}$ pairs, LHCb requires an efficient, robust and flexible trigger capable of coping with the harsh hadronic environment of a proton-proton collider. It also requires excellent vertex and momentum resolution.

The LHCb detector is a single-arm spectrometer with forward angular momentum coverage from 10 mrad to 300 mrad in the bending plane and 10 mrad to 250 mrad in the non-bending plane. This forward geometry is unique to LHCb and is

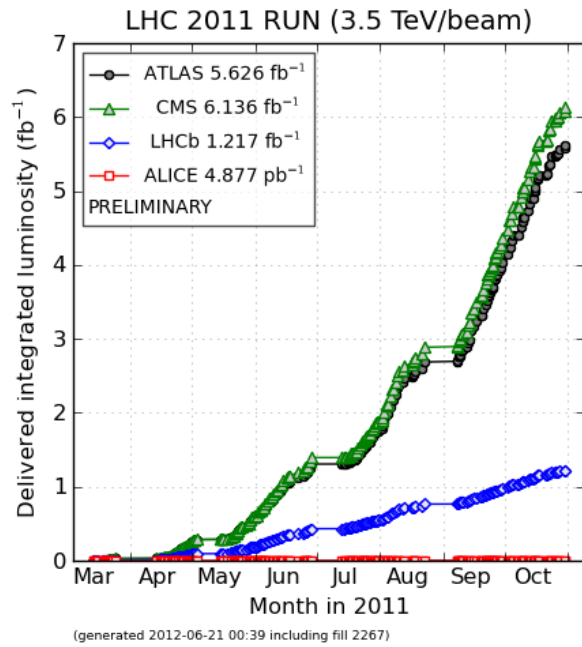


Figure 3.3: The integrated luminosity delivered by the LHC machine to the different experiments in 2011 [49]

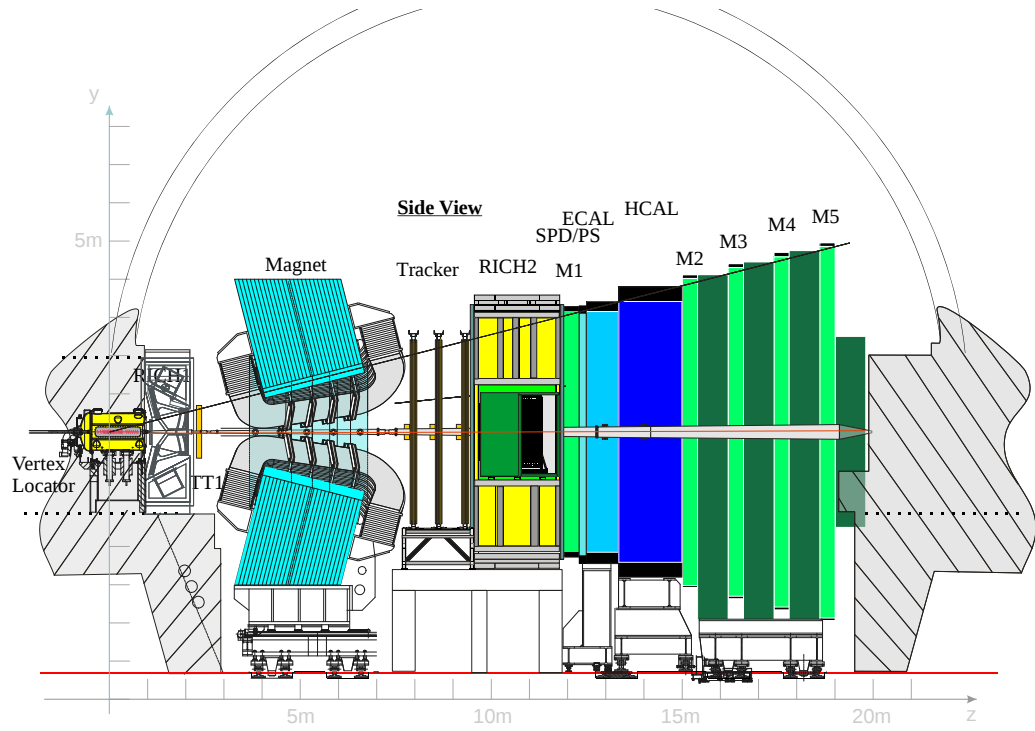


Figure 3.4: A schematic view of the LHCb detector [50]. The interaction point where the protons collide is on the left of the figure, and the individual sub-detectors are labeled.

chosen as B hadrons are predominantly produced in narrow angular cones in the forward and backward directions at high energies [53]. It is housed in the intersection point 8 of the LHC ring, previously used by the DELPHI [54] experiment at LEP. The interaction point of LHCb is displaced by 11.25m from the centre of the LHC optics to make optimal use of the cavern space.

A schematic of the LHCb detector is shown in Figure 3.4. It is designed to reconstruct the particles produced by a proton-proton collision at the LHC using a variety of detection techniques. It measures the momentum and direction of charged particles using a bending magnet and tracking system, and identifies them using specialised particle identification sub-detectors while energy deposits of both neutral and charged particles are also measured using a calorimetry system. The collisions take place at $z \sim 0$ on the left of the figure, and this interaction region is surrounded by the Vertex Locator which measures the direction of charged particles close to the interaction region. Further downstream are two Cherenkov detectors, RICH1 and RICH2, capable of distinguishing the mass of different particles, the Silicon Tracker and Tracker Turensis, which form part of the tracking system, and finally the calorimetry and muon systems. These components are now described in turn.

3.2.1 Vertex Locator

The VELO (**V**ertex **L**ocator) is used to measure particle trajectories close to the interaction region to high precision. These measurements are used to precisely locate and separate primary and secondary vertices. The VELO consists of 42 Modules positioned along the z axis each comprising of back to back silicon sensors with R and ϕ geometry respectively. Each module then consists of two halves centred on the beam axis which are closed during data-taking, with an area in the middle reserved for safe passage of the beams. In this nominal position, the active areas of the detector are located 8 mm from the beam, but, for safety reasons, both VELO halves are retracted to a distance of 29mm during beam injection, when the beam position is less stable. The VELO measures three dimensional space-points by combining a module's (r, z) and (ϕ, z) measurements. Each sensor contains strips of varying pitch which determine the resolution of a given hit. The VELO is described

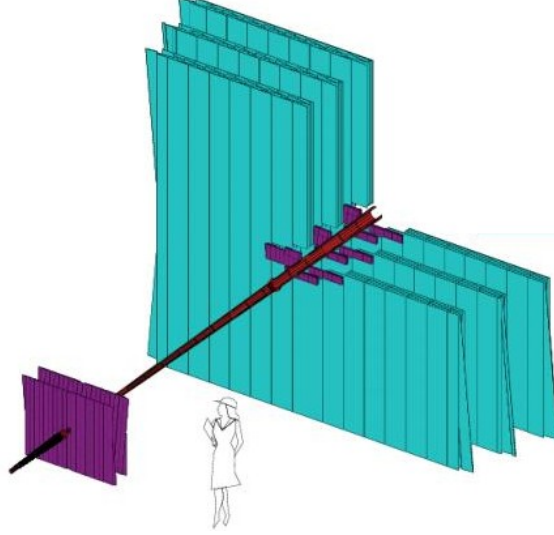


Figure 3.5: Overview of the LHCb tracking system, with the TT and IT indicated in purple, and the OT indicated in Green [50]

in detail in Chapter 5.

3.2.2 Tracking System

In addition to the VELO, a number of other subdetectors form part of the tracking system, and are placed either side of the magnet. These detectors reconstruct the trajectory of charged particles and determine their charge and momentum by the direction and magnitude of their deflection by the magnetic field.

The Silicon Tracker comprises two detectors, the Tracker Turicensis (TT) and the Inner Tracker (IT), where both use silicon microstrip sensors with a strip pitch of approximately $200\ \mu\text{m}$ to provide high resolution close to the interaction region. The Outer Tracker (OT), by contrast, is a drift-time detector, as it is placed further from the interaction region and covers a larger area. The arrangement of the tracking stations is illustrated in Figure 3.5. The TT is a planar track system located upstream of the dipole magnet, while the IT and OT both form part of three tracking stations located downstream of the magnet between 7 and 10 m from the interaction region. The IT occupies a cross-shaped region in the centre of each station, with the OT occupying the remaining outer regions.

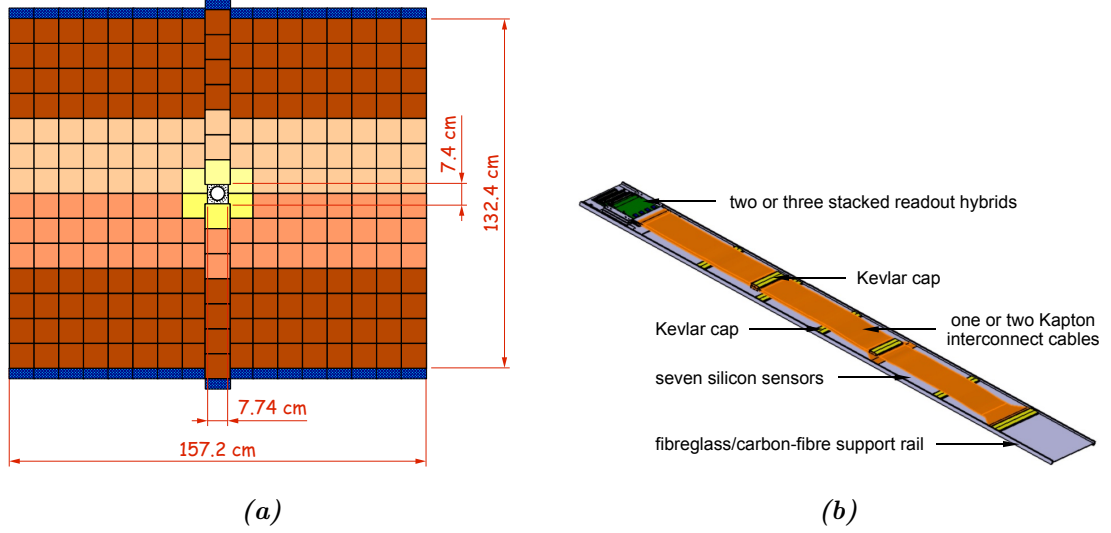


Figure 3.6: (a) Layout of a TT detection layer in the v configuration (b) A TT Detector Module [55]. The layer consists of 17 columns of full modules bound together.

Tracker Turensis

The TT comprises four detector layers, and is housed in a large light-tight, thermally and electrically insulated detector volume kept below a temperature of 5 °C. The four detector layers are arranged in two pairs (x, u) and (v, x) separated by approximately 27 cm along the LHC beam axis, where x denotes a horizontal alignment and u and v denote rotations of $\pm 5^\circ$ about the x -axis. The TT sensors are 500 μm thick, single sided p^+ on n silicon sensors¹. They are 9.64 cm wide and 9.49 cm long with 512 readout strips, each with a pitch of 183 μm .

The layers are assembled from half modules, consisting of rows of seven silicon sensors, which are placed above and below the LHC beampipe to make a full module. The first two detector layers contain seven columns of full modules, while the second two comprise rows of eight full modules. Adjacent modules are staggered by ~ 1 cm in z and have a small overlap to prevent acceptance gaps and also to facilitate alignment using the overlap region. The configuration of an x layer, as well as an individual detector module is illustrated in Figure 3.6.

¹Silicon sensors are described in detail in Chapter 5

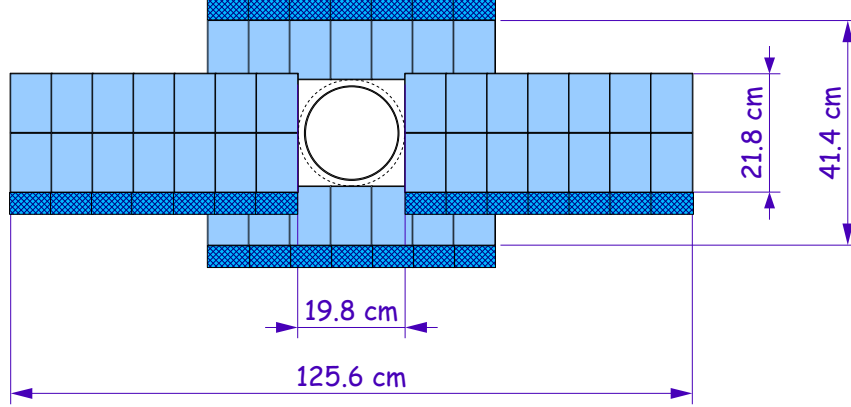


Figure 3.7: Layout of an IT detection layer [56]. Four individual directed boxes are arranged above, below and either side of the beam-pipe, which is located in the centre of the figure.

Inner Tracker

The IT is a 120cm wide, 40cm high cross shaped region in the centre of the three tracking station downstream of the magnet where the particle flux will be highest, and where the highest resolution is required for momentum reconstruction. Its dimensions are chosen in order to maintain the occupancy at a manageable rate for the read-out electronics. Each of the three IT stations consist of four individual detector boxes arranged around the beam-pipe as shown in Figure 3.7. As in the case of the TT, each detector box is light-tight, electrically and thermally insulated, and maintained at a temperature below 5°C. Adjacent modules are staggered by 4 mm in z and overlap by 3 mm in x to prevent acceptance gaps and allow for alignment using the overlap region. The IT sensors are also single-sided p^+ on n silicon sensors and are 7.6 cm wide and 11 cm long. They consist of 384 readout strips each with a strip pitch of 198 μm . The detector modules in boxes above and below the beam-pipe consist of single silicon sensors and corresponding readout hybrids. Those placed on the left and right consist of two silicon sensors and a readout hybrid.

Outer Tracker

The Outer Tracker covers the much larger remaining area of the tracking stations, and uses drift-time detector technology ideal for the tracking of charged particles over a large acceptance area. The OT is designed as an array of individual, gas-tight, straw-tube modules and consists of two staggered layers of drift-tubes with inner diameters of 4.9 mm [57]. Charged particles passing through one of the straw tubes will ionise the gas molecules and the liberated electrons will drift towards the anode wire. A sufficient field strength around the wire will initiate an electron avalanche which is collected by the wire and results in an electric current. The delay between the charged particle passing through the tube and its signal being read out is dominated by the drift-time of the ionised electrons. This delay can be used to determine the point at which the track traverses the detector. In the OT, a mixture of Argon (70%) and CO₂ (30%) is used to guarantee a fast drift-time (< 50 ns) while maintaining a sufficient drift co-ordinate resolution. The detector modules are arranged in the three tracking stations. Each station consists of four-layers, arranged in an x - u - v - x geometry. The dimensions of the inner-cross shaped boundary of the tracking stations, covered by the IT, ensure that the occupancies of the OT should not exceed 10% at nominal luminosity.

3.2.3 RICH

The RICH-1 and RICH-2 (Ring Imaging Cherenkov Detectors) are detectors used to distinguish particles of different mass at low and high-momentum respectively [58]. Particles which pass through these detectors at a speed greater than the speed of light in the medium of the detector, known as the radiator, emit electromagnetic radiation known as Cherenkov radiation [59]. This radiation is emitted in a cone whose polar angle, θ_C is known as the Cherenkov angle and is related to the particle velocity, v_p , by

$$\cos \theta_C = \frac{1}{n\beta} \quad (3.5)$$

where n is the refractive index of the material being traversed, and $\beta = v_p/c$. Combining this with the measurement of its momentum from the tracking stations provides an estimate of the mass and the particle can thus be identified. While the

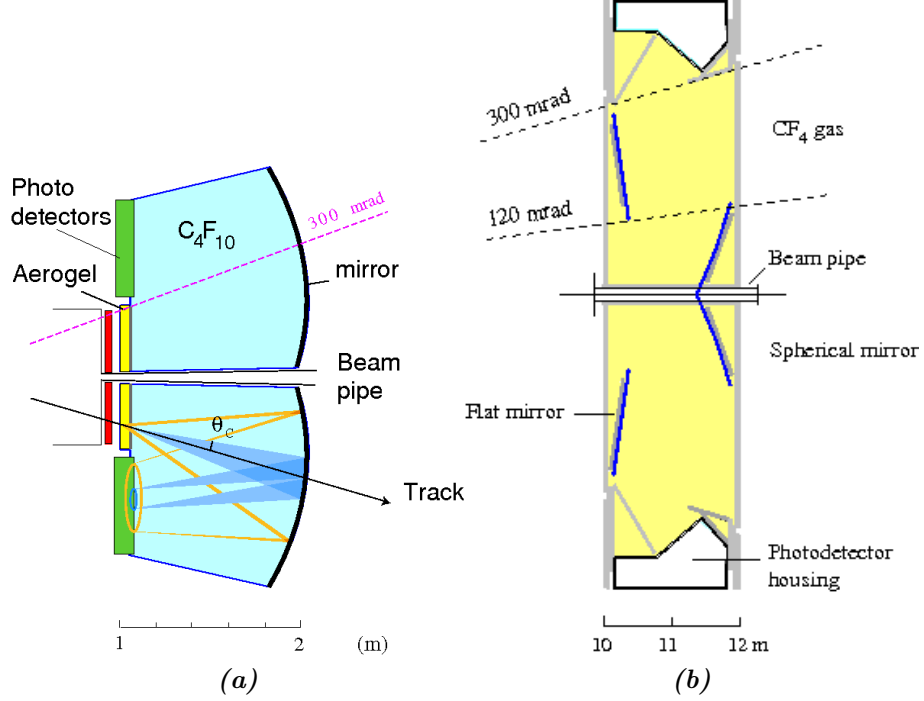


Figure 3.8: Schematics of (a) RICH-1 and (b) RICH-2 [58]

principal purpose of the RICH detectors is to distinguish between charged pions and kaons, particularly useful in the study of B hadron decays, it also complements the calorimeters and muon chambers in the identification of electrons and muons. One requirement of the RICH detectors is to provide $K - \pi$ separation over a momentum range of 1-150 GeV. This range is driven by the momentum spectrum of two body B decays, where it is required to distinguish between decays such as $B_s^0 \rightarrow K^+\pi^-$ and $B_s^0 \rightarrow \pi^+\pi^-$. To achieve this, the RICH system utilises three different types of radiators with different refractive indices, SA, gaseous C_4F_{10} and gaseous CF_4 . These radiators cover the low, intermediate and high momentum regions respectively. Spherical mirrors are used to focus the Cherenkov light emitted onto photodetectors. In order to reduce the material that particles in LHCb have to traverse, the photodetectors are situated outside of the LHCb acceptance in all cases [60]. This geometry is shown in Figure 3.8. RICH-1 covers the full LHCb angular acceptance and is situated upstream of the magnet. It provides $\pi - K$ separation

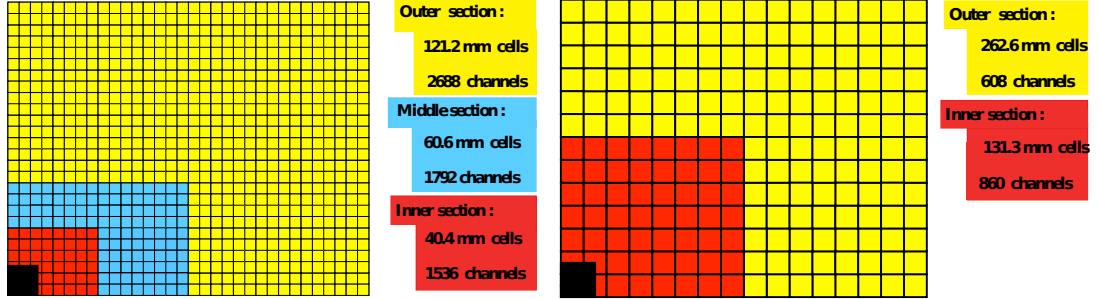


Figure 3.9: Illustration of the segmentation of the SPD/PRS and ECAL (left) and the HCAL (right) [62]

ration up to 50 GeV. RICH-2 is situated in the downstream region and distinguishes pions and kaons up to and beyond 100 GeV. It has a reduced angular acceptance compared to RICH-1. The RICH detectors have an angular resolution of $\sim 1 - 3$ mrad which allows them to distinguish between pions and kaons at the 3σ level [61].

3.2.4 Calorimeters

Calorimeters are designed to detect both charged and neutral particles by exploiting their interaction with material [63]. A shower is initiated when certain particles traverse material, producing a cascade of secondary particles whose energies are deposited, collected and measured by the calorimeter. Showers can be classified as either electromagnetic or hadronic in nature. An electromagnetic shower is produced by a particle which interacts primarily or exclusively through the electromagnetic force producing a shower of particles via Bremsstrahlung and pair-production, with the scale of the shower determined by the radiation length, X_0 , of the material. Although qualitatively similar, hadronic showers are initiated by the strong interaction. They are more complex because of the contribution of many different processes to the inelastic production of secondary hadrons. The scale of a hadronic shower is determined by the nuclear absorption length of the material, denoted λ_i . As this length is typically greater than the radiation length, hadronic calorimeters are in general larger and denser than electromagnetic calorimeters.

The calorimetry system at LHCb [64] measures the energy of electrons, photons

and both charged and neutral hadrons. It distinguishes between high transverse energy candidates for use in the earliest stage of the LHCb trigger, while offline, a matching of tracking and calorimetry information allows for the identification of electrons, photons and hadrons along with the measurement of their energies and positions. To perform these functions, the calorimetry system utilises Electromagnetic (ECAL) and Hadronic Calorimeters (HCAL). The difficult nature of the identification of electrons also necessitates the use of two further calorimeters. The Scintillating Pad Detector (SPD) and Pre-Shower (PRS) detectors are placed before the ECAL and are separated by a lead converter. As no shower is initiated before the SPD, it will only detect charged tracks, while an electromagnetic shower can be initiated in the lead converter and subsequently detected in the PRS. Thus the background from neutral pions can be suppressed by searching for hits in the SPD, while the PRS also suppresses the background of charged pions by requiring a shower profile at two segmented points along the beam axis. Calorimeters can be described as either homogeneous or sampling. In a homogeneous calorimeter, the same material is used both as an absorber and a detector while distinct materials are used for these purposes in a sampling calorimeter. LHCb is a sampling calorimeter which uses scintillating material interleaved with the absorbers to act as the detectors. The scintillating material absorbs energy from ionising energy and re-emits it in the form of light. This scintillation light is transmitted to a photo-multiplier tube by wavelength shifting (WLS) fibres. The segmentation of the calorimetry system is illustrated in Figure 3.9.

Scintillating Pad Detector / Pre-Shower Detector

The SPD/PRS system consists of two almost identical planes of rectangular scintillator pads with a total of 12032 detection channels with a 15mm lead converter, corresponding to a radiation length of $2.5 X_0$, sandwiched between. The distance between both scintillator planes along the beam axis is 56mm. To achieve a one-to-one projective correspondence with each other and with the ECAL, each scintillating pad is divided into inner, middle and outer sections with 4.04×4.04 , 6.06×6.06 , $12.12 \times 12.12 \text{ cm}^2$ dimensions and the dimensions of the SPD plane are smaller than the PRS by approximately 0.45%.

Electromagnetic Calorimeter

The LHCb electromagnetic calorimeter has a shashlik design consisting of a sampling structure, with scintillating tiles acting as the detectors and lead as the absorber. The ECAL is placed at a distance of 12.5 m from the interaction point and consists of a rectangular stack of sixty six modules arranged along the beam axis so that the scintillating tiles are perpendicular to the z axis. This corresponds to a total thickness of $25 X_0$. A module is built from alternating layers of 2 mm thick lead and 4 mm thick scintillator tiles. As the hit density increases rapidly as a function of distance from the beam-pipe, the calorimeter is divided into inner, middle and outer sections containing square cells of area 4.04×4.04 , 6.06×6.06 and $12.12 \times 12.12 \text{ cm}^2$ respectively. The energy resolution of the ECAL is found to be $\frac{8\%}{\sqrt{E}} \oplus 0.8\%$, where E is the measured energy [64].

Hadronic Calorimeter

The LHCb hadronic calorimeter is a sampling calorimeter with scintillating tiles acting as the detector and iron as the absorber. Contrary to the ECAL, the scintillating tiles run parallel to the beam axis, and they are interspersed with 1 cm of iron to form modules. This design is chosen due to its simplicity and cost effectiveness [65]. The HCAL assembly consists of two halves, each consisting of 26 modules. In the transverse direction, the length of the tiles and iron spacers corresponds to $5.6 \lambda_I$, where λ_I is the nuclear absorption length in iron[66]. The HCAL is segmented transversely into an inner and outer region containing square cells of area 13.13×13.13 and $26.26 \times 26.26 \text{ cm}^2$ respectively. The HCAL energy resolution is found to be $\frac{67\%}{\sqrt{E}} \oplus 9\%$ [64].

3.2.5 Muon System

The muon system is designed to detect and reconstruct muons, which are present in the final states of many interesting processes, by exploiting their penetrative power. It uses five stations, M1-M5, with the last 4 stations interleaved with thick iron absorbers to absorb hadronic backgrounds. M1 is placed in front of the the PRS. Each station is divided into 4 different regions, R1-R4 to give a total of twenty

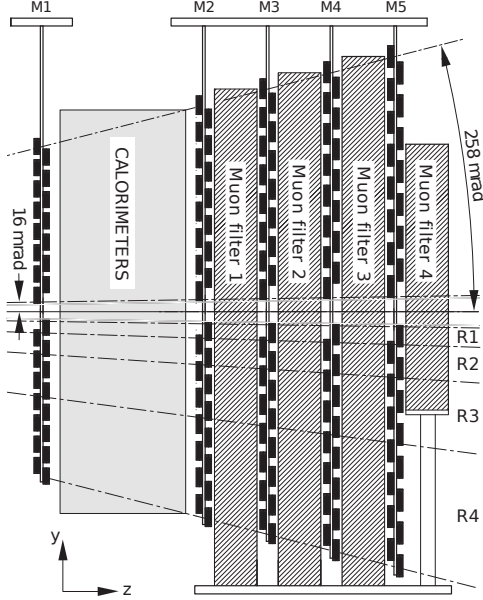


Figure 3.10: A side view of the muon system, with the five stations indicated [50]

regions. These regions are designed to have approximately equal occupancies, and as such the spatial resolution decreases moving away from the interaction region, with regions R4 having a factor of 8 worse resolution than R1 in each station. A good spatial resolution is required to provide a measurement at the earliest stage of the trigger. The better the spatial resolution, the better the resolution on the momentum of tracks reconstructed using the muon system alone, known as muon stand-alone tracks. The first three muon stations are primarily used for these measurements, while the later stations have a factor of two worse spatial resolution, and are primarily used for identification of penetrating particles. As a result, the muon system provides a p_T resolution of approximately 20% on candidate muon tracks [67].

The calorimeters and iron absorbers, each of which is 80 cm thick act as a muon shield, which absorbs hadrons and electrons and allows those particles that pass through the shield to be identified as muons. The total material thickness between the interaction point and the fifth muon station is 20 nuclear interaction lengths. A momentum of greater than 5 GeV is required for a muon to pass through all five

stations, and so momentum dependent identification requirements are necessary in order to positively identify muons over a full momentum range.

Multi-Wire Proportional Chambers (MWPCs) are used in all but one of the twenty regions [68]. In the innermost region of M1 (M1R1), triple-GEM (Gas-Electron-Multiplier) technology is used. This is because of its increased radiation hardness which allows it to cope better with the high particle flux in this region [69].

CHAPTER 4

EVENT RECONSTRUCTION

Event reconstruction is the process of interpreting electrical signals from the detector and using them to reconstruct the particles produced in a proton-proton collisions. This is achieved using a series of algorithms, performed both in real time while the collisions are occurring, known as online, and after the data has been written to disk, known as offline. Due to time constraints, the online event reconstruction is partial, and only the minimum reconstruction necessary to monitor the detector performance and decide which events should be written to disk is performed. The offline event reconstruction involves a full reconstruction using all available detector information in order to enable as precise a measurement of the final state particles as possible for physics analyses. Here the offline algorithms, for track and vertex reconstruction, and particle identification, are described first, followed by the online trigger system.

4.1 LHCb Track Reconstruction

As a charged particle passes through the tracking stations of a detector, it leaves a series of hits tracing out its path. This path can be reconstructed, and the momentum of the particle measured, by fitting these individual hits to a trajectory using a series of algorithms, where the reconstructed trajectory is known as a track. The tracking system at LHCb consists of the VELO, TT, IT and OT Detectors. A

track is defined by a set of states which are tangential to the particle's momentum at given z positions. It is convenient to define the states at the z positions where the tracking system makes its measurements, known as the measurement or sensor planes. A state, $\vec{x}$, is fully defined as

$$\vec{x} = \begin{pmatrix} x \\ y \\ t_x \\ t_y \\ q/p \end{pmatrix} \quad (4.1)$$

where x, y, z are the usual cartesian coordinates with the origin at the interaction point, the z -axis oriented along the beam axis and the y -axis in the direction of the magnetic field. The track slopes are given by $t_x = \frac{dx}{dz}$, $t_y = \frac{dy}{dz}$ and q/p is the charge (± 1) divided by the magnitude of the momentum.

4.1.1 Pattern Recognition

The first step in the track reconstruction procedure is the pattern recognition. This involves a series of algorithms which identify subsets of measurements belonging to a single track from the ensemble of measurements present in any single event. In doing this, the number of ghost tracks must be minimised, where ghost tracks refer to fake tracks which consist of a random selection of hits, either from real particle hits belonging to different tracks or from noise in the detector. The pattern recognition can be performed in a number of different ways depending on the type of track. There are five distinct types of track in LHCb corresponding to the detector elements traversed by that track [70]. They are illustrated in Figure 4.1 and listed here.

- **Long Tracks:** will traverse the full system. They are the most useful tracks for physics as they have the most accurate momentum information.
- **Velo Tracks:** only have measurements in the VELO, and as such have no momentum information but can be used in primary vertex reconstruction. They can also be used to identify tracks traveling in the backward direction,

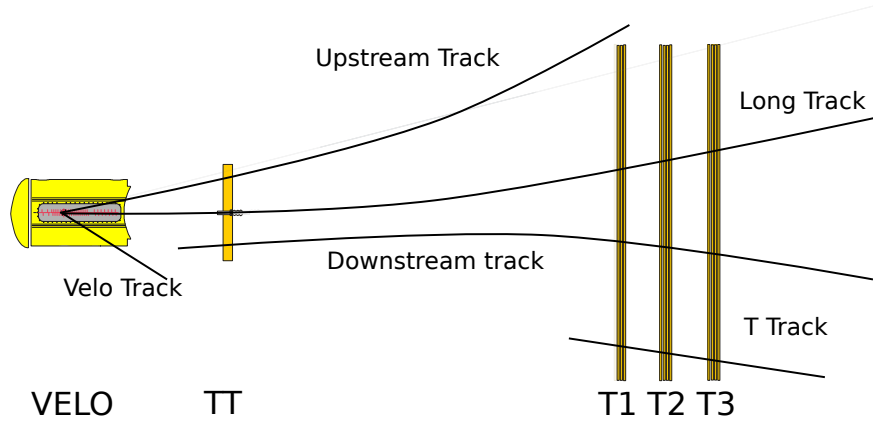


Figure 4.1: The different type of tracks reconstructed by the LHCb tracking software.

which is useful in the study of processes such as exclusively produced di-muon pairs [71].

- **Upstream Tracks:** only traverse the VELO and TT stations but do not reach the T Stations. They can be used in the RICH 1 Reconstruction.
- **Downstream Tracks:** only traverse the TT and T Stations. They can be used to reconstruct particles decaying outside the VELO acceptance.
- **T Tracks:** only traverse the T Stations. They can be used in RICH 2 Reconstruction.

The pattern recognition procedure uses six different algorithms to identify as many tracks of each variety as possible.

- **VELO Seeding:** This algorithm searches for straight lines in the VELO. It makes triplets of space points from the r and ϕ VELO measurements and combines those formed close to the interaction region with those formed in the most downstream stations. The track is then extrapolated from the downstream stations backwards towards the interaction point to search for more measurements close to the trajectory which can be added. The resulting track segments, known as VELO seeds, act as seeds for the other pattern recognition algorithms.

-
- **T-Seeding:** Here seeds are formed using the T-Stations. Although the magnetic field is present in the T-Stations, the bending takes place in the y direction, and so tracks can be identified from hits forming a straight-line in the $(x-z)$ plane. The resulting track segments are known as T-Seeds.
 - **Forward Tracking:** This algorithm uses the VELO seeds and combines them with single hits in the tracking stations, defining a trajectory. Other hits are identified as belonging to the track if they are sufficiently close to this trajectory.
 - **Track Matching:** In this algorithm, unused VELO seeds are matched with T-Seeds by extrapolating both to a central plane of the magnet. Where two seeds are found to be compatible, a search is also performed for hits in the TT which can be added.
 - **Upstream Tracking:** This algorithm searches for Upstream Tracks by using VELO seeds remaining after the forward tracking and track matching algorithms have been run. The VELO seeds are extrapolated to the $(y-z)$ plane of the TT stations. The momentum is calculated separately for the VELO seed and each compatible TT Hit. When a minimum of three momentum calculations are compatible, the VELO seed and the compatible TT hits are identified as an upstream track.
 - **Downstream Tracking:** Downstream tracks are found by starting with a T-Seed and trying to add TT Hits to make a trajectory. Like forward tracking, hits are then found by comparison to this trajectory.

In all cases, the output of the pattern recognition algorithms is a set of hits which are then passed to the fitting algorithms to fully reconstruct their trajectory.

4.1.2 Track Fitting

After a set of measurements have been identified by the pattern recognition, they are fitted to a trajectory using track fitting algorithms. In LHCb the Kalman Filter algorithm is used to reconstruct the track states at each measurement position [72].

The Kalman Filter proceeds iteratively from one measurement to the next, improving the knowledge of the trajectory with every step. It is preferable to a simple linear least squares fit [73] for track reconstruction as it does not require the time-expensive inversion of large matrices, and can also discard hits in the course of fitting [74]. It also allows the removal of individual measurements from a fitted track without the need to perform a full track refit. Its final result is equivalent to a least squares fit as it is based on minimising the χ^2 of the track [75]. At LHCb, the state vector, $\vec{x}$, is defined as a function of the z -coordinate along the beam line, $\vec{x} = \vec{x}(z)$. For a discrete number, k , of z positions corresponding to the intersection points of the track with the measurement planes, $\vec{x}_k = \vec{x}(z_k)$. Treating the track trajectory as a discrete dynamical system, its evolution from $k - 1$ to k is described by

$$\vec{x}(z_k) = \vec{x}_k = f_{k-1}(\vec{x}_{k-1}) + w_{k-1} \quad (4.2)$$

where f_{k-1} is the track propagator function which propagates the track state from $\vec{x}_{k-1}$ to $\vec{x}_k$ and w_{k-1} is a constant representing the process noise, in this case the random track disturbance caused by multiple scattering and energy loss between z_{k-1} and z_k . The full Kalman Fit consists of three steps to calculate any given state, $\vec{x}_k$.

- **Prediction** - Prediction is the prediction of $\vec{x}_k^{k-1}$ from $\vec{x}_{k-1}$ using (4.2).
- **Filter** - The prediction $\vec{x}_k^{k-1}$ is updated with the measurement m_k to produce $\vec{x}_k^k$, by weighting both the prediction and the measurement with their respective covariance matrices and the response matrix¹ of the detector.
- **Smoothing** - Smoothing is the the reversal of the prediction procedure where the fitting is performed in the other direction. The estimate, $\vec{x}_k^k$, is now updated with the reverse prediction based on the future measurements, $\vec{x}_k^{k+1}$ to produce $\vec{x}_k$.

This procedure is performed for all measurements to produce a set of states which collectively define the track and map out its trajectory. Studies of the tracking

¹For a distribution of a physical observable with N bins, the response matrix, R , is the $N \times N$ matrix whose R_{ij} element represents the probability of reconstructing the observable with a value in bin i when its true value is in bin j .

performance of the LHCb detector are presented in Chapter 5.

4.2 Vertexing

Another important set of observables at particle colliders are the production points of particles, known as vertices. These vertices can be reconstructed as the intersection point of tracks known to have been produced at the same point. In a proton-proton collider, the primary vertex (PV) refers to the point at which the protons collide, and the majority of the particles in the event are created. A precise knowledge of the primary vertex in each event is necessary to identify interesting events.

4.2.1 Seeding

The reconstruction of primary vertices first requires the identification of seed positions for the fitting algorithm. At LHCb, the seed positions are determined using the method of analytical clusterisation [76]. A cluster in this case is simply a one dimensional coordinate, z^{clu} , as well as an associated uncertainty, σ_z^{clu} . The initial clusters are simply the intersections of the tracks with the z -axis obtained by extrapolation. The clusters are then iteratively merged together. Pairs with a separation, D , of less than 5 are merged into one cluster where D is defined as

$$D = \frac{|z^{clus1} - z^{clus2}|}{\sqrt{(\sigma_z^{clus1})^2 + (\sigma_z^{clus2})^2}} \quad (4.3)$$

The combined z^{clu} of the pair is weighted according to the uncertainties of the individual values as

$$z^{clu} = \frac{w_1 z^{clus1} + w_2 z^{clus2}}{w_1 + w_2} \quad (4.4)$$

where the weights, w_1 and w_2 , are given by $w_{1,2} = 1/\sigma_z^{clus1,2}$. These iterations continue until there is no longer any pairs of clusters to be merged. The remaining clusters act as the seeds for the vertex fitting algorithm.

4.2.2 Fitting

The clusters in Section 4.2.1 act as the seeds for the vertex fitting algorithms, which proceed in order of decreasing multiplicity of the seed clusters. A number of track types, long, velo and downstream, are considered in the fitting, with the tracks required to have a distance of closest approach of less than 30 mm to the seed position. The primary vertex is determined using a least squares method minimising the primary vertex χ_{PV}^2 , defined as

$$\chi_{PV}^2 = \sum_j^{tracks} \frac{ip_j^2}{\sigma_{ip_j}^2} \quad (4.5)$$

where ip refers to the track impact parameter, defined as the distance of the track with respect to the vertex, and σ_{ip} is its associated uncertainty. After each iteration the track with the highest impact parameter significance, $IPS = ip/\sigma_{ip}$, is removed and the vertex position recalculated with the smaller subset of tracks. The next iteration removes tracks based on their IPS with respect to the refitted vertex. This process is continued until all tracks have $IPS < 4$. The PV candidate is discarded if the final multiplicity is less than six. This procedure is then repeated for the next seed with the tracks already used excluded. The primary vertex resolution for a vertex composed of 25 tracks is measured to be (13.0, 12.5, 68.5) μm in the (x , y , z) directions [77].

4.3 Particle Identification

The LHCb particle identification system consists of the two RICH detectors, the calorimeter system and the muon detector. The purpose of the system is to identify stable particles from their tracks by exploiting their unique properties. The RICH system differentiates particles, particularly pions and kaons, using their respective masses, while the calorimeter and muon system exploit the differing interaction properties of electrons, muons and hadrons to identify them. The information from the various methods can be combined by constructing likelihood variables from each system.

4.3.1 Rich Identification

Particle identification in the RICH detector is performed by observing the pattern of hit pixels detected by the RICH photodetectors as a reconstructed track passes. This pattern is compared to the pattern expected for a set of different mass hypotheses and a likelihood variable for each hypothesis constructed. As the momentum coverage of the RICH does not extend beyond 150 GeV, it cannot in general be used to distinguish between high momentum particles produced by the decays of W and Z bosons.

4.3.2 Photon Identification

At LHCb, photons are identified as neutral clusters in the ECAL, with the clusters tested for matching with all well reconstructed tracks. All long tracks in an event are extrapolated to the ECAL, and a cluster-track position matching estimator, χ_γ^2 , is built for each cluster. Clusters due to charged particles are characterised by large peaks at small values, while those with $\chi_\gamma^2 > 4$ are identified as photons.

4.3.3 Electron Identification

Electron identification also relies heavily on the identification of clusters in the electromagnetic calorimeter. The electron estimator χ_e^2 is built which uses the ratio of the energy of a charged cluster in the ECAL to the momentum of the closest track as a distinguishing variable in addition to the cluster-track position matching used in the photon estimator, χ_γ^2 . Also included in the estimator is the Bremsstrahlung photons emitted by the electrons before the magnet. Bremsstrahlung occurs when the electrons are bent by the magnet, and as the material in the magnet region and hence the multiple scattering is minimal, the position of ECAL clusters corresponding to Bremsstrahlung photons can be accurately predicted and the clusters searched for. The photons identified in Section 4.3.2 are used as the candidates for this Bremsstrahlung recovery process. Finally the track's energy depositions in the PRS and the HCAL are added to construct a likelihood variable from all the available information. For the identification of high-momentum electrons with momenta exceeding the coverage of the RICH detector, an alternative identification method

Muons	
Required Hits	Momentum Range
M2, M3, M4	$p < 6 \text{ GeV}$
M2, M3, M4 or M5	$6 < p < 10 \text{ GeV}$
M2, M3, M4, M5	$p > 10 \text{ GeV}$

LooseMuons	
Required Hits	Momentum Range
Two of M2, M3	$p < 10 \text{ GeV}$
Three of M2, M3, M4, M5	$p > 10 \text{ GeV}$

Table 4.1: The conditions for positive identification for the two classes of muons, Muons and LooseMuons.

is required, and is described in Section 6.4.3.

4.3.4 Muon Identification

Muon identification is performed by extrapolating tracks within the acceptance of the M2 and M3 stations with $p > 3 \text{ GeV}$ into the muon stations. A search for hits is performed around a Field of Interest (FOI) of the extrapolated position at the z position of each station. This FOI is parameterised as a function of momenta for each station and region. The parameters are chosen to maximise efficiency while maintaining an acceptable background rate due to misidentification. There are two classes of muons, Muons and LooseMuons defined in LHCb corresponding to the hits observed and the momentum of the track [78]. The conditions are given in Table 4.1.

Further discrimination can be provided by a comparison of the slopes of the tracks in the muon system to those in the main tracker, and also the average distance from the extrapolated position to the muon hit. These are used to create a muon likelihood function which, if required, can be used to select muons with a higher purity in offline selection algorithms. Studies of the efficiency of the muon identification procedure are performed in Chapter 6.

4.4 LHCb Trigger

A trigger system is a system that rapidly decides whether an event should be written to disk for offline analysis. This is necessary as only a certain portion of the total collisions can be recorded due to limitations in data storage capacity and disk I/O rate. The rate of collisions at the LHC producing particles detectable by the LHCb detector is estimated to be 10 MHz [79] which must be significantly reduced, with the LHCb data acquisition system designed to record events at a rate of approximately 2 kHz [80]. The reduction is achieved by a sequence of triggers which impose requirements on the event at each trigger stage. Only events satisfying these requirements, known as “triggered” events, are then recorded. The trigger requirements are optimised to achieve a high efficiency for retaining events which will later be selected by offline algorithms, known as offline selected events, and rejecting background events. The LHCb trigger comprises two trigger steps; the Level-0 (L0) and High Level Trigger (HLT) [55].

4.4.1 L0

The L0 (Level-0) trigger is a hardware level trigger which reduces the rate to 1 MHz. It uses four components to decide whether to accept or reject an event in 4 μ s: the L0 calorimeter trigger, the L0 muon trigger, the pile up system, and the L0 decision unit. The decision unit combines information from the other components to make the trigger decision.

Pile-Up Trigger

The Pile-Up system [81] consists of two VELO R sensors (A and B) perpendicular to the beam axis and located upstream of the VELO¹. These sensors measure radial positions, r_a and r_b at z positions, z_a and z_b of the A and B planes respectively. An estimate can be made of the z position of the vertex at which the track originated,

¹Upstream of the VELO refers to the region along the beam-axis between the VELO and the collision point

z_v , by assuming the vertex is located at the beam axis using the following formula

$$z_v = \frac{kz_a - z_b}{k - 1} \quad (4.6)$$

where $k = \frac{r_a}{r_b}$. Any peaks in the z_v distribution are indicative of interaction vertices, and so events containing multiple interactions can be identified as those with more than one peak. The pile-up trigger is capable of rejecting these events and selecting only single interaction events by identifying secondary peaks in the z_v variable and requiring that the number of tracks in this peak be below a certain value.

Calorimeter Trigger

The Calorimeter trigger uses zones of 2 x 2 cells which are large enough to contain most of the energy deposition of a single particle while small enough to prevent the overlap of two separate depositions. Candidates for different particle types: hadrons, electron, photons and neutral pions are selected, with the highest transverse energy, E_T , candidate selected for each type, and entered into the trigger decision. The trigger comprises three steps.

- High E_T deposits in the ECAL and HCAL are selected. The HCAL deposits provide the hadron candidates.
- The ECAL deposits are merged with the PRS and SPD information. These electromagnetic candidates are then identified as electrons, photons or neutral pions and the highest transverse energy candidate per type is selected.
- The total E_T deposited in the HCAL and the SPD multiplicity of the event are also stored. Large values are indicative of events with a large number of final state particles, which require a large processing time, and can be rejected at the trigger stage by placing requirements on these event variables.

Muon Trigger

At the L0 trigger stage, muon tracks are reconstructed by performing a search for hits satisfying a straight line hypothesis pointing towards the muon stations from the primary vertex. For each hit in M3, a search is performed in M2, M4 and

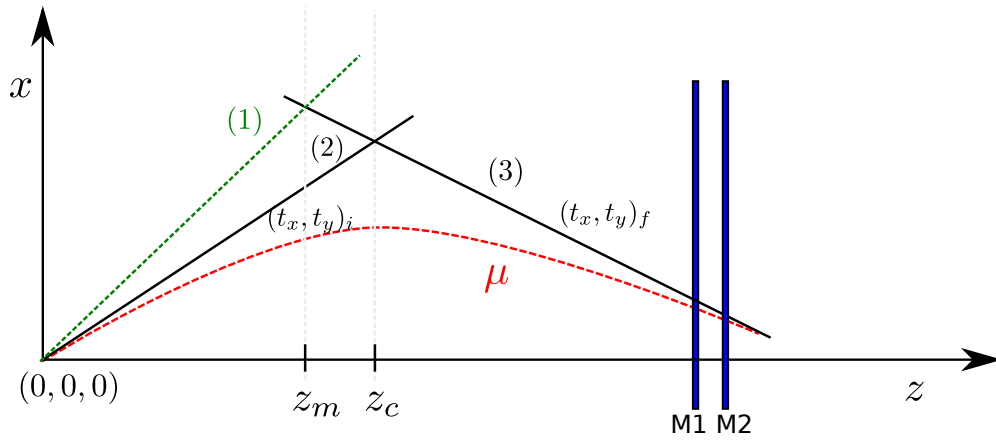


Figure 4.2: Illustration of the ‘p-kick’ method used in the estimate of track momentum in the muon trigger. The particle is assumed to come from the primary vertex and the effect of the magnet is approximated as a single kick at the magnet centre. The centre of the magnet is estimated in a two-stage process, first using the spatial centre, z_m , and then by integrating the magnetic field along the path joining lines (1) and (3) to calculate a more accurate centre, z_c . The momentum is then estimated using the difference in the slopes of lines (2) and (3) given by $(t_x, t_y)_i$ and $(t_x, t_y)_f$ using Equation (4.7). The true path of the muon is shown by the dashed red line.

M5 by extrapolating along a straight line joining the primary vertex to the M3 hit to the other stations and searching for hits in a field of interest centred on the extrapolated position. An extrapolation is then performed to the M1 plane, and the closest M1 hit to the extrapolation point is selected. M1 and M2 are utilised to perform a p_T measurement by assuming the particle came from the interaction point and approximating the effect of the magnet as a single ‘p-kick’ at the magnet centre. This method is illustrated in Figure 4.2. The variable which provides the highest sensitivity is the change of momentum in the bending plane, Δp_x , with the relation between it and the momentum given by [72]

$$\Delta p_x = p \left(\frac{t_{x,f}}{\sqrt{1 + t_{x,f}^2 + t_{y,f}^2}} - \frac{t_{x,i}}{\sqrt{1 + t_{x,i}^2 + t_{y,i}^2}} \right) = q \int \left| \vec{B} \times d\vec{l} \right|_x \quad (4.7)$$

where t_x and t_y are the track slopes defined in Section 4.1 and the subscripts i and f indicate whether the slopes refer to the path before or after the ‘kick’ at the magnet centre. The magnetic field is given by $\vec{B}$, with the charge and path of the track given by q and $d\vec{l}$ respectively. A determination of the track slopes and the integral of the magnetic field along the path allows the track momentum to be estimated. The L0 muon processor looks for up to two high p_T muon tracks in an event, where the highest p_T tracks are chosen if more than two are present. The p_T of these tracks then enter the trigger decision, where different triggers select events with one or two high p_T muons.

Decision Unit

The Decision Unit gathers the information collected by the Pile-Up, Calorimeter and Muon Triggers. Events are selected if they satisfy any one of a number of different criteria, with different thresholds in place for selecting events containing one or two high transverse momentum muons, or events with high transverse energy electrons, hadrons, or neutral photon or pion candidates. It computes the decision and sends it to the Readout Supervisor which makes the final decision on whether or not an event is to be accepted. Events are selected if they satisfy one of five specific criteria; if they contain a single high p_T muon, two high p_T muons (with lower thresholds for the individual muons than in the single muon case), a high E_T hadron candidate, a

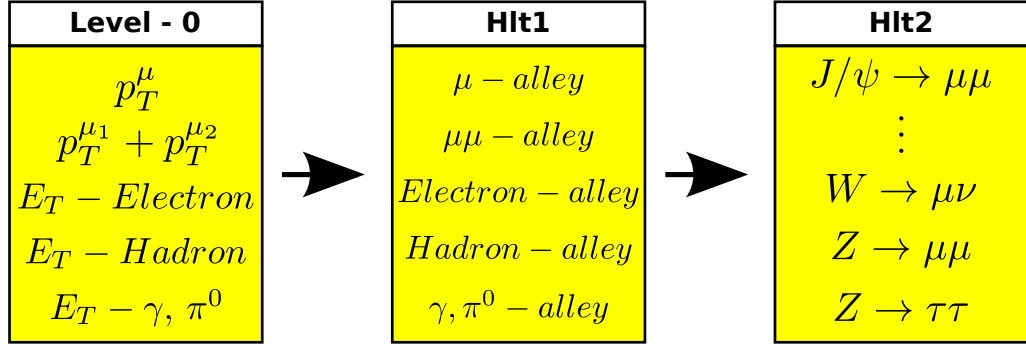


Figure 4.3: The LHCb trigger flow. Hlt1 uses alleys to confirm the decisions made by L0, while Hlt2 contains a large number of different trigger lines to select events for offline analysis.

high E_T electron candidate, or a high E_T γ or π^0 candidate.

4.4.2 HLT

The HLT is the second and final level of trigger. It is a software level trigger running on events which pass the L0 trigger. It aims to reduce the rate to a few kHz through a sequence of C++ applications run on the Event Filter Farm, a computing farm containing up to 2000 computing nodes. Each application has access to all the data in an event and could in principle execute all the offline selection algorithms. However, given the 1 MHz input rate from L0, the HLT aims to reject uninteresting events efficiently by using the minimum amount of data possible. It consists of two steps, HLT1 and HLT2.

HLT1

HLT1 starts with a sequence of alleys which correspond to the trigger types of the L0 trigger. As HLT1 has less time constraints and access to more information, it can use simple algorithms to perform more precise measurements and confirm or reject the L0 decisions. It reconstructs VELO seeds in two stages. First it reconstructs two dimensional ($r - z$) tracks and matches them to the L0 object, calculating a matching χ^2 . If this χ^2 is sufficiently low, the VELO ϕ -sensor information is added to construct a full three dimensional track. T-seeds are also reconstructed in the

T-stations, decoding hits in a window around the trajectory obtained by assuming the L0 object comes from the interaction region. These seeds can be combined to produce tracks traversing the whole detector which can be used to confirm the L0 decisions. The L0 Muon object is not used in HLT1, and instead a full muon reconstruction is performed to confirm the L0 Muon decision. The HLT1 output rate is ~ 30 kHz.

HLT2

HLT2 performs the full off-line track reconstruction. The only difference is that due to CPU constraints, the HLT-tracks are not reconstructed with the full Kalman filter and the full covariance matrix is not obtained. It is thus important that loose track quality cuts are applied before use in the offline analysis. A series of different selections are written for different physics processes and these aim to reduce the rate to 2 kHz which will then be written to storage for use in offline analysis. The full trigger flow is illustrated in Figure 4.3.

4.5 Software

The LHCb software [82] is built on the Gaudi [83] framework. It contains a number of different software applications which are used to produce the data summary tables (dst's) used in offline analysis. These applications can be broken down into the three distinct functions that they perform.

4.5.1 Simulation

Simulation of both physics processes and the response of the LHCb detector is essential to the understanding of the performance of the entire data processing chain. It is also vital in order to perform comparisons between data and theoretical predictions.

In LHCb this simulation is performed in various steps using the Gauss and Boole applications. Gauss employs and configures an external Monte Carlo generator, such as Pythia, to generate any number of physics processes. It then simulates the interaction between these generated particles and the detector using the GEANT4 [84]

application. This simulation includes effects such as multiple scattering, photon conversions and the decay of unstable particles in the detector volume.

Boole emulates the electronic readout of the detector using the data produced by Gauss, and is tuned using test beam data. It produces a digitised output indistinguishable from real data read out from the LHCb detector.

4.5.2 Reconstruction

The LHCb reconstruction software is known as Brunel and uses either digital output from the detector (in the case of real data) or the Boole application (in the case of simulation) to perform full event reconstruction. It creates tracks and vertices, as well as particle objects known as proto-particles. These proto-particles are basic unidentified particles with associated RICH, calorimeter and muon identification information which can be used in offline analysis.

4.5.3 Analysis

Analysis at LHCb is typically performed using the DaVinci physics analysis software package. Final states of interest can be selected by making appropriate particle combination and applying suitable selection criteria using what are known as offline selection algorithms.

CHAPTER 5

TRACKING PERFORMANCE OF THE VERTEX LOCATER

5.1 The Vertex Locater

LHCb is an experiment optimised for the study of decays of hadrons containing b quarks. One of the most distinctive features of decays of this type is the relatively long life-time of the hadrons under study ($\tau \sim 10^{-12}$ s) [85]. This results in a secondary vertex displaced from the interaction region by ~ 1 cm [53] which can be used to separate these decays from background processes. The VELO provides a precise measurement of track co-ordinates close to the interaction region, which can be used to identify these secondary vertices. This allows LHCb to provide an accurate measurement of the lifetime of particles, and also to use the lifetime information to tag the flavour of the decaying particle.

5.1.1 Silicon Detectors

The trajectory of charged particles can be measured to high precision at particle colliders using silicon detectors. In a crystalline such as silicon, the energy levels available to the constituent electrons of the atoms lie in distinct regions known as bands. If an electron acquires sufficient energy it is promoted from the valence band to the conduction band and can move freely between atoms. This leaves a missing

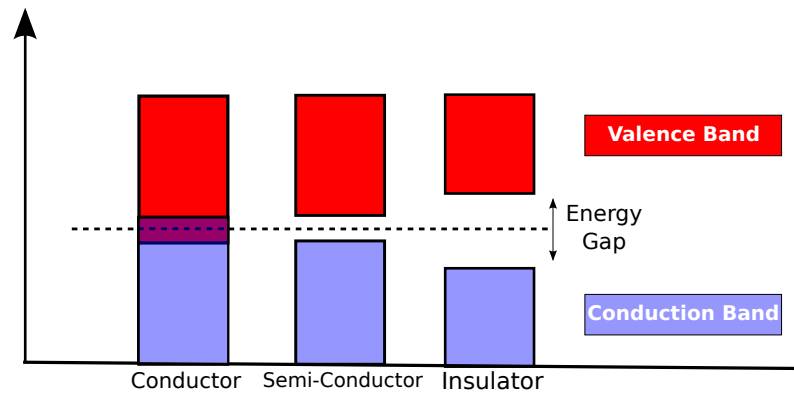


Figure 5.1: Illustration of the band structure of conductors, semi-conductors and insulators.

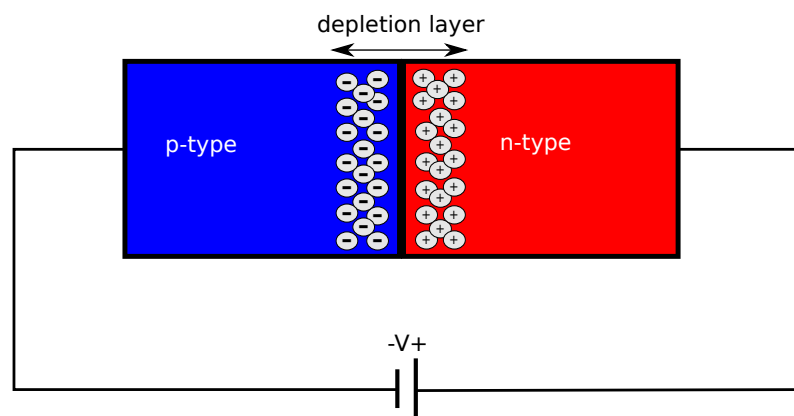


Figure 5.2: A reverse biased p-n junction. The width of the depletion layer is dependent on the material and will increase with increasing reverse bias

electron, or hole, in the valence band which acts as a positive charge carrier. The band gap is an energy range between the valence and conduction bands where no electron states can exist, and represents the energy required to free a bound electron from its shell so that it can become a charge carrier. Materials with large band gaps are known as insulators, those with small band gaps are known as semi-conductors, and those which have small or non-existent band gaps are known as conductors. This band structure is illustrated in Figure 5.1. Silicon is a semi-conductor with four electrons in its valence band and a band gap of 1.11 eV at a temperature of 300 K [86].

Doping is the introduction of impurities to an otherwise pure semi-conductor in order to modify its electrical properties. This doping is achieved by introducing certain atoms, known as dopant atoms, to the material in order to increase the number of free charge carriers. In p-type (positive) doping, the doping material takes outer electrons from semi-conductor atoms leaving holes and producing an excess of positive charge carriers. In n-type (negative) doping, the dopant atoms provide extra conduction electrons to produce an excess of negative charge carriers.

P-n junctions [87] are junctions formed at the boundary of a p-type and n-type semiconductor created in two separate regions of a single crystal by doping. Electrons near the p-n junction diffuse into the p region leaving donor atoms while holes near the p-n junction diffuse into the n region leaving acceptor atoms. This forms what is known as a depletion layer in the boundary region. The formation of this layer produces an electric field which opposes the diffusion, and thus an equilibrium state can be reached. A reverse-bias voltage can be applied where the voltage at the cathode is higher than the anode to increase the width of the depletion layer as is shown in Figure 5.2. The bias voltage at which the depletion layer occupies the entire p-n junction is known as the depletion voltage. A fully depleted p-n junction can be used as a particle detector, with electron-hole pairs generated as a charged particle traverses the crystal and ionises the atoms, with the electron-hole pairs collected on the n or p side. A silicon microstrip sensor consists of a large array of silicon strips with each strip capable of detecting the passage of charged particles. Its design is illustrated in Figure 5.3. It is comprised of a silicon bulk and a backplane. Strips are situated in the bulk and are delimited by narrow silicon implants. In the VELO, the bulk is lightly doped n-type, while the backplane

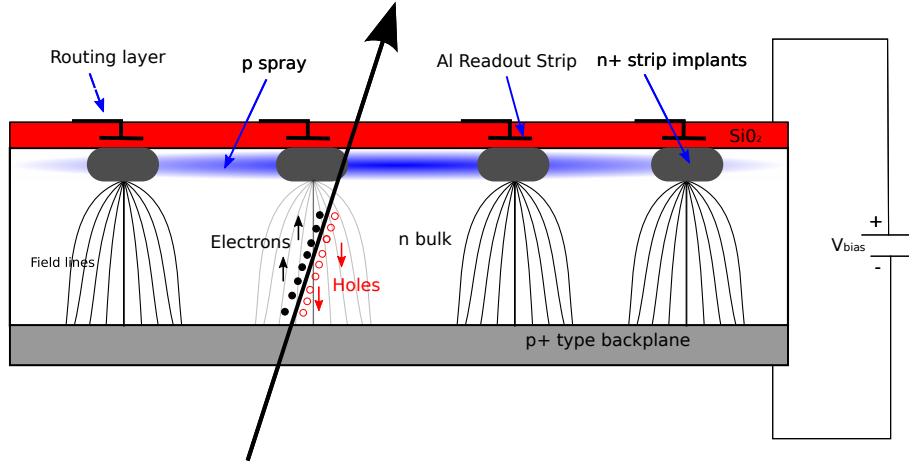


Figure 5.3: Diagram of a silicon microstrip sensor. The p-n junction is formed between the p+ type silicon backplane and the n type silicon bulk.

is heavily doped p-type (p+). Heavily doped n-type (n+) is used for the implants. A p-n junction is formed between the backplane and the bulk. Electrons generated in the bulk drift towards the implants. The signal induced by the particle's passage is read out by aluminium readout strips. The readout strips are isolated via a silicon dioxide layer and the silicon strips via a thin layer of p-type silicon known as a p-spray. A routing layer enables signal readout to the front end electronics.

5.1.2 Detector Overview

The VELO consists of 84 silicon microstrip sensors split into two detector halves. Each sensor consists of 2048 microstrips and they are arranged in a series of modules placed along the beam axis as shown in Figure 5.4, where the sensors close to the interaction point are equidistant from each other, and sensors further downstream, in what is known as the “gap” region, are staggered in z position. Each module consists of an r -measuring sensor and a ϕ -measuring sensor, which are arranged back to back to provide measurements of both the radial distance and azimuthal angle with respect to the primary vertex and provides an orthogonal (r, ϕ) pair. The sensors are illustrated in Figure 5.5. This co-ordinate measuring system is preferred due to its ability to form a fast two dimensional reconstruction of (r, z) tracks for use in the LHCb trigger [89]. In order to achieve high precision, the VELO

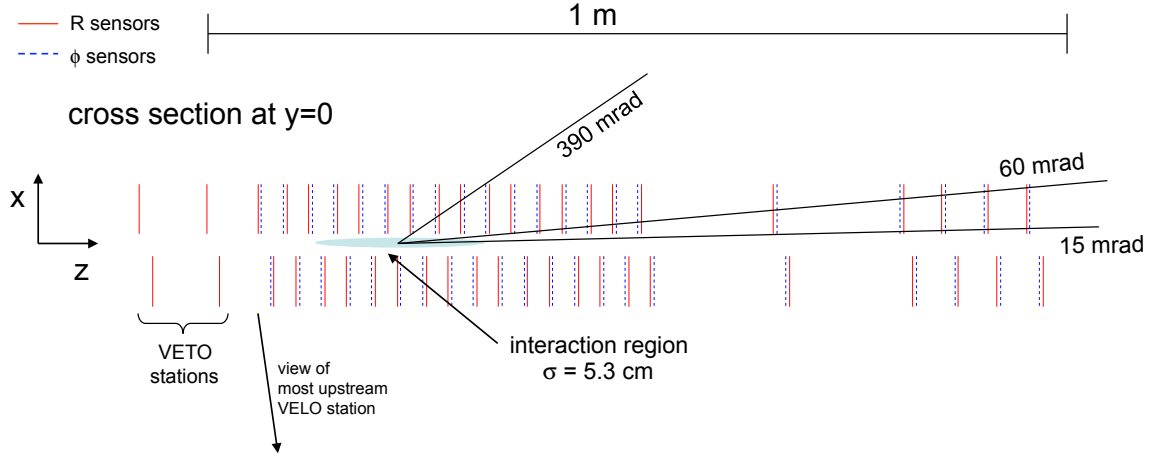


Figure 5.4: Overview of the VELO detector. Shown above is the positioning of the r and ϕ sensors along the beamline. [50]

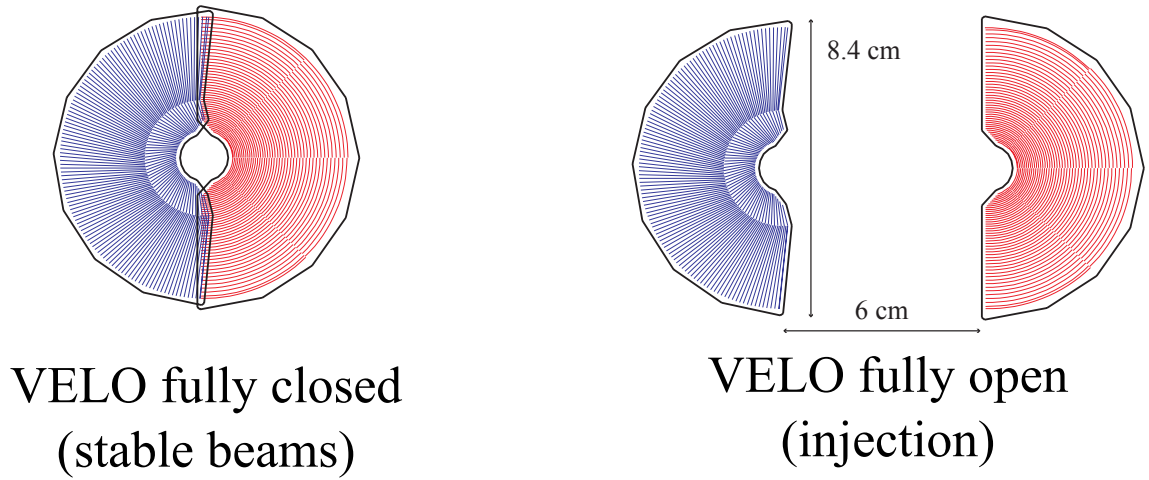


Figure 5.5: An illustration of the sensors in the fully closed and fully open positions. [88]

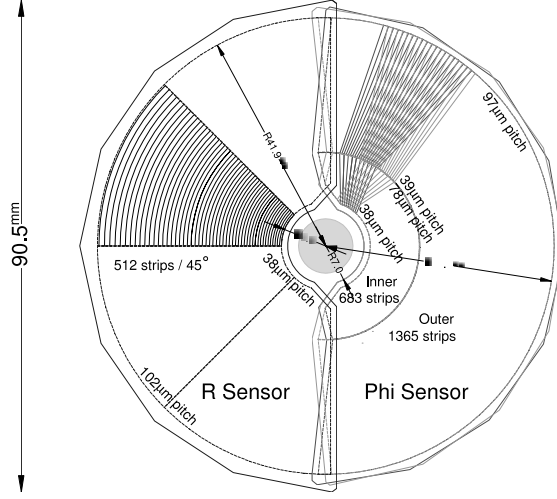


Figure 5.6: Shown are the strip geometries of the r and ϕ type sensors. Only a fraction of strips are illustrated. For the ϕ sensors, the strips of two adjacent sensors are overlaid to highlight the alternating skews of the ϕ sensors[50].

sensors are placed as close as possible to the interaction region to reduce the track extrapolation error. The active area of the VELO sensors are placed at a radial distance of 8 mm from the beam in the fully closed position. This is closer than the minimal aperture required by the LHC during injection, and so requires that the VELO be retractable to a distance of 29 mm during injection. In r -sensors, the $n+$ strip implants are concentric semi-circles centred at the nominal LHC beam position. Each strip is divided into four 45° regions to minimise occupancy. The minimum pitch, located at the innermost region is $35\ \mu\text{m}$, increasing linearly to $101.6\ \mu\text{m}$ at the outer radius of 41.9 mm. The ϕ sensors read out the azimuthal angle, orthogonal to the radial distance. These sensors are divided into two regions, inner and outer, to moderate strip occupancy. A skew is also introduced to improve the pattern recognition and reject hits due to detector noise. The slope of the inner region makes an angle of approximately 20° with respect to the radial component at 8 mm, and the outer strips make an angle of 10° at 17 mm. This arrangement is shown in Figure 5.6.

Both sensors are $300\ \mu\text{m}$ thick and are connected via pitch adapters to Beetle chips where Beetle chips are front end readout chips designed for LHCb [90]. Each sensor has 16 Beetle chips, with charge from 128 strips processed by each Beetle

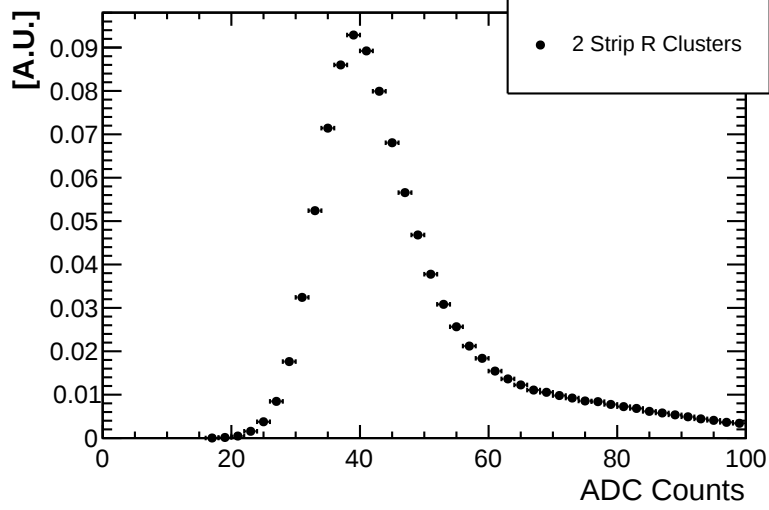


Figure 5.7: The total charge deposited in a two strip cluster in a VELO r-sensor expressed in ADC counts

chip. The back to back r and ϕ sensor pairs, as well as the Beetle chips, are glued and electrically bound on printed circuit boards known as hybrids. The hybrids are rigid structures which cool the silicon and are in turn mounted on a pedestal. This assembly constitutes a module. Twenty one VELO modules are arranged along the beam axis.

The charge on the silicon strips is read out from the Beetle chip in analogue form, and sent to the repeater board. There it is amplified and then sent to the TELL1 board for digitisation and suppression. The TELL1 is a common readout board designed for LHCb [91]. The TELL1 boards host four 10-bit ADCs (Analogue to Digital Converters) which convert the analogue information from the Beetle chips into digital values known as ADC counts.

Appropriate threshold cuts are put into place to separate signal and noise readings and create VELO clusters which are then used in the track reconstruction process. Two thresholds are defined for the VELO, a seed and an inclusion threshold; the seed threshold on each strip is taken to be six times the mean noise, typically of the order of 12 - 18 ADC counts, and the inclusion threshold is set at 40% of the seed threshold, typically of the order of 5-7 ADC Counts. To be defined as signal, the ADC value on any strip must be above the seed threshold. The adjacent strips are

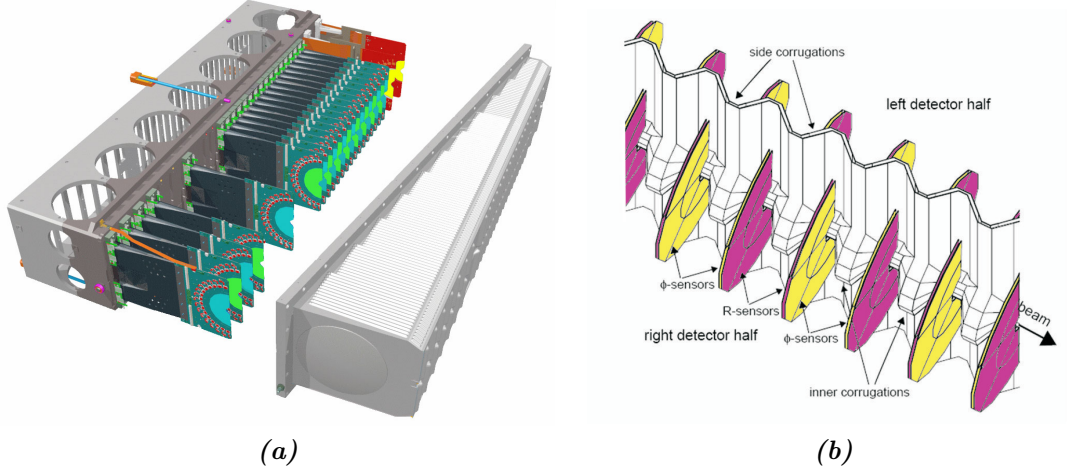


Figure 5.8: (a) Exploded view of the module support and modules (left) and the RF box (right), (b) Zoom view of the inside of an RF-Foil with detector halves in fully closed position. [50]

then searched to see if any exceed the inclusion threshold, and if so they are added to the cluster containing the seed strip. A cluster can contain a maximum of four strips. The distribution of summed ADC counts for two-strip clusters in r-sensors is shown in Figure 5.7, and is well described by a landau shape representing the signal convoluted with a Gaussian representing detector noise. Each detector half is enclosed in a $300\text{ }\mu\text{m}$ thick corrugated aluminium foil known as the RF-Foil. This foil insulates the modules from the vacuum of the LHC beam and also shields the detector from any RF pickup possible due to the proximity of the sensors to the LHC beam. The foil allows for an overlap of the sensor halves as well as allowing space for safe passage of the beams. An overview of the modules and RF-Foil is shown in Figure 5.8. To avoid deformation of the foil structure the pressure between the LHC vacuum (10^{-9} mbar) and the RF-foil is minimised by maintaining a vacuum pressure of 10^{-6} mbar within the detector volume. This detector vacuum is maintained within the VELO vacuum vessel in which the detector halves, readout electronics, RF-foils, kapton readout cabling and cooling system are enclosed. It is shown in Figure 5.9.

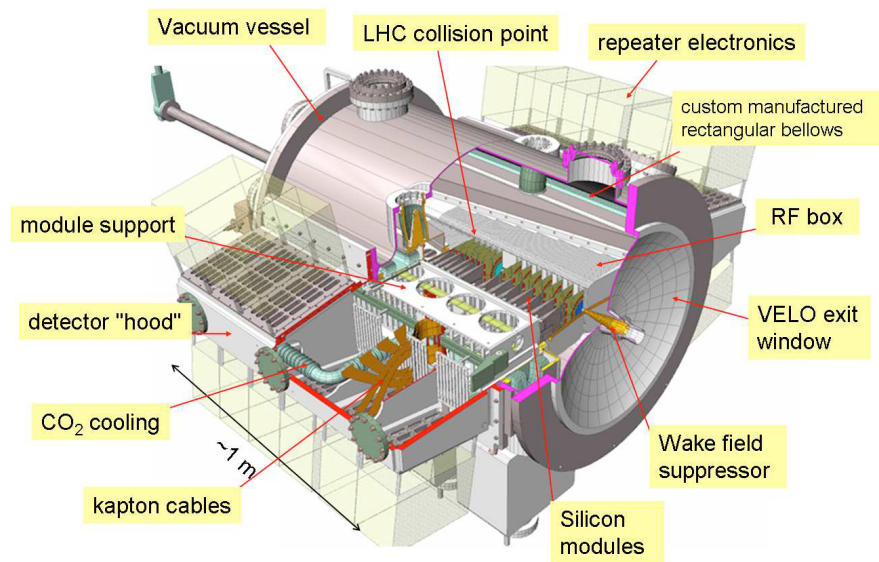


Figure 5.9: Overview of the VELO vacuum vessel with the constituent parts labelled. [50]

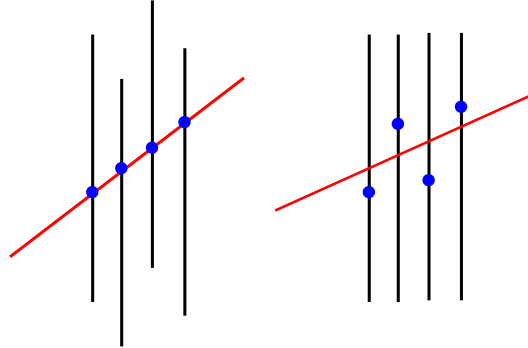


Figure 5.10: Illustration of the effects of not correcting for misalignment. On the left is the real situation, while on the right is the track reconstructed if misalignments are not taken into account.

5.1.3 Alignment

Alignment refers to the precise knowledge of the position of detector parts, which is vital in order to correctly reconstruct objects such as tracks, as illustrated in Figure 5.10. In the case of the VELO, after precise construction and assembly, a survey of the individual modules and assembled halves was performed to a precision of better than $10\ \mu\text{m}$ [92]. This provides the initial positions for a software alignment using reconstructed tracks and also provides the final measurement for some positions which cannot be determined using the software alignment, such as the z positions of the sensors.

The software alignment for the VELO consists of three stages; firstly, the r and ϕ sensors within each module are aligned with respect to each other, then the relative positions of the modules within each VELO detector half are determined, and finally the two detector halves are aligned with respect to each other. Each method is performed using track residuals, which are defined as the shortest distance from the fitted track trajectory to the track's measurements and are described in more detail in Section 5.2.

For the relative alignment of the r and ϕ sensor pairs, magnetic field and multiple scattering effects are ignored and the tracks in the VELO are estimated to be straight lines. The relationships between the residuals and misalignments in each sensor

plane are given by [93]

$$\epsilon_r = -\Delta x \cos \phi_{track} + \Delta y \sin \phi_{track} \quad (5.1)$$

$$\epsilon_\phi = \Delta x \sin \phi_{track} + \Delta y \cos \phi_{track} + \Delta_\gamma r_{track} \quad (5.2)$$

where ϕ_{track} is the azimuthal angle of the reconstructed track, $\epsilon_{r/\phi}$ refer to the r/ϕ residuals, Δx is the x misalignment, Δy is the y misalignment, and Δ_γ refers to the rotation of the sensor around the z axis. The misalignment parameters can be obtained by fitting the residual distributions as a function of ϕ_{track} for each sensor to the above functions.

The two remaining steps of the alignment are calculated via a non-iterative method involving matrix inversion. This is based on a χ^2 function produced from the residuals with a dependence on the track and alignment parameters. The parameters are then obtained through minimisation of this function. Partial matrix inversion is used to solve the simultaneous equations obtained from the derivatives of the χ^2 with respect to the parameters. This matrix inversion is performed using the Millipede [94] program. For the alignment of the two detector halves, only overlap tracks are used, that is, tracks which pass through the overlap region of the detector halves and have hits in both.

5.2 Detector Resolution

Detector resolution is a measure of how accurately a detector measures quantities such as the positions of the hits along a track trajectory. The resolution of any measurable quantity is given by the square root of the variance of the distribution of residuals, which are defined as the difference between the measured and true values. The ***single hit residual*** is the distance between the measured hit position and the true hit position in a sensor plane and can be used to determine the single hit resolution. Knowledge of the single hit resolution is important in track reconstruction as it determines if hits should be included in or discarded from a track trajectory, and also determines the χ^2 of the fitted track. As the true hit position is not known, the single hit residuals and consequently the single hit resolution must be determined from the track residuals.

Where a hit is fitted to a track trajectory, its **track residual** is defined as the distance between the hit position and position of the track when extrapolated to the sensor plane. A hit residual is said to be **biased** when it is calculated with respect to the track fitted including the hit, and **unbiased** when it is calculated with respect to the track re-fitted with the hit removed. For a perfectly aligned detector, the track residual distribution for the measurements of reconstructed tracks will be represented by Gaussian shapes centred at zero, with a width determined by the intrinsic resolution of the detector. Deviations from a zero mean indicate simple translational misalignments while non-Gaussian shapes point to more complicated misalignments, such as rotations. Thus, the study of residuals provides a good analysis of detector alignment and intrinsic resolution.

In the VELO, the single hit resolution is dependent on the pitch and number of strips in which the track has deposited charge, known as the cluster size. For a single strip cluster, where the track has deposited charge in just one strip, the residual distribution is represented by a uniform distribution ranging from $-p/2$ to $p/2$, where p is the pitch of the strip. The resolution for a hit in a single strip cluster is then given by the square root of the variance of this distribution as

$$\sqrt{\frac{1}{p} \int_{-p/2}^{p/2} x^2 dx} = \frac{p}{\sqrt{12}} \quad (5.3)$$

This is known as the binary resolution. For an analogue detector such as the VELO, which measures the charge deposited on each strip in ADC counts, improved resolution can be obtained in the case of multi-strip clusters by assuming a linear charge distribution with the hit position then given as the weighted mean of the strip ADC counts, which, for an n -strip cluster, is given by

$$x_{hit} = \frac{\sum_i^n x_i \cdot ADC_i}{\sum_i^n ADC_i} \quad (5.4)$$

where i is an index running over the strips in the cluster, with x_i and ADC_i representing the corresponding position and ADC count value. For multi-strip clusters, the distribution of the single hit residuals is represented by a Gaussian shape, with the resolution given by its width.

In this chapter three different studies of detector resolution using data collected by the LHCb detector in 2010 are presented. Firstly, in Section 5.2.1, a study of the effect of the calculation of the *inter-strip fraction* using Equation (5.4) on the VELO resolution is performed, where the inter-strip fraction refers to the fractional position between two strips. This calculation assumes a linear charge sharing of the track's charge deposition amongst neighbouring strips, as illustrated in Figure 5.11. A possible correction to the linear charge sharing assumption is evaluated and its effects on the detector resolution studied. In Section 5.2.2 the *single hit resolution* of the VELO sensors is studied as a function of both strip pitch and projected angle, where projected angle refers to the angle of incidence of the track on the sensor. The standard method of calculating single hit resolutions at LHCb is to utilise variables calculated iteratively in the Kalman fit described in Section 4.1.2. In the fit, the biased track residual and the uncertainty on both the single hit residual and the biased track residual are calculated at each step. These are then related to the single hit residual by

$$\underbrace{(x_{hit} - x_{track})}_{\text{biased hit residual}} \frac{\delta x_{hit}}{\delta(x_{hit} - x_{track})} \approx \underbrace{x_{hit} - x_{true}}_{\text{single hit residual}} \quad (5.5)$$

where x_{hit} is the reconstructed hit position, x_{track} is the position of the track reconstructed to the sensor plane, x_{true} is the true hit position and δx_{hit} and $\delta(x_{hit} - x_{track})$ represent the uncertainty on the hit position and the biased residual respectively. The ratio of the two uncertainties reflects the ratio of the resolution of the single hit residual to that of the biased track residual, and so the single hit residuals, and consequently the single hit resolution can be extracted from the biased track residuals using this approach. Results using this method are presented in Reference [95]. However, an alternative approach is presented in this thesis which explicitly calculates the unbiased track residuals and derives the single hit resolution. A third study is also performed in Section 5.2.3 of the *impact parameter resolution* of tracks reconstructed by the tracking system, whose resolution is largely dominated by the VELO.

All three studies are performed using a sample of events triggered using a minimum bias trigger in 2010. The minimum bias trigger selects events using the mini-

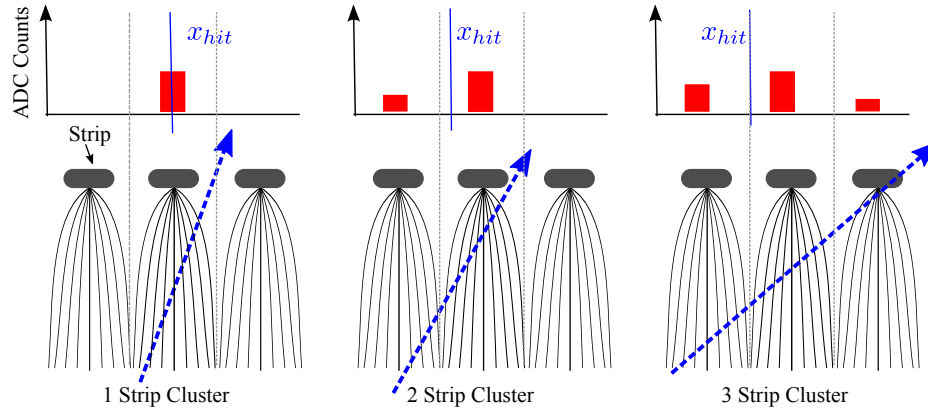


Figure 5.11: An illustration of the different size of clusters that can be produced at LHCb. The curved lines represent the electric field lines of each silicon strip and the blue dashed line represents the trajectory of a track traversing the detector. A graphical illustration of the calculation of the hit position, x_{hit} , is also shown, where the charge collected by each strip is represented by the red bands, and the calculated positions by the blue lines.

imum requirements necessary to ensure it can be subsequently reconstructed by the LHCb detector. In this way, a relatively unbiased data sample can be selected for detector studies.

5.2.1 Inter-strip Fraction

The inter-strip fraction, as determined from Equation (5.4), is calculated using a weighted mean method, based on the assumption that the charge deposited by a passing track is distributed linearly onto the strips it traverses. This is not expected to be true for low incidence angles, where diffusion is expected to be the main cause of charge spread [96]. In these cases, the weighted mean method will incorrectly calculate the hit position and result in a degradation of the single hit resolution. The weighted mean method can be tested by studying the unbiased hit residual distributions as a function of the inter-strip position. In particular, the mean of the distribution in different bins of inter-strip position should be consistent with zero for perfectly calculated inter-strip fractions.

The track residuals of all r -clusters used in the reconstruction of long tracks in a minimum bias sample are considered. For the determination of the unbiased track

residual, the residual is unbiased by the removal of a module, that is, an r and ϕ -sensor pair, from the track fit. The residual is given by the distance between the radial position of the hit in the sensor and the radial position of the track when extrapolated to the sensor z coordinate, where the radial distance is taken with respect to the centre of the sensor. In this fashion, the unbiased track residual is calculated for each VELO measurement on the track.

The track residual for a given hit on the track will have contributions both from the single hit resolution of the individual hit, and also from the single hit resolution of the other hits on the track, known as the extrapolation precision. To compare single hit residuals for different hits, the contribution from the extrapolation precision is minimised by ensuring that the track is well defined in the region of the hit. This is achieved by requiring that there are adjacent hits on the sensors either side of the hit in question, and that these hits have a well defined position. The adjacent hits are required to correspond to two-strip clusters, as two-strip clusters have a better resolution than the binary resolution of one-strip clusters and three-strip clusters suffer from a large rate of ‘fake’¹ clusters which degrade the hit resolution. Additionally, the clusters are required to have a total deposited charge between 30 and 70 ADC counts where the charge deposited in two strip r clusters in ADC counts was shown in Figure 5.7. Requiring that the hits should have a charge in the region of the peak of the distribution reduces the rate of fake clusters produced by detector noise. As the resolution is expected to scale approximately linearly with strip pitch, in all cases the residuals are divided by the strip pitch to facilitate a comparison between different strips.

The residuals are split into twenty bins of inter-strip fraction between 0 and 1, and the mean of the distribution in each bin shown in Figure 5.12a. A large deviation from the expected value of 0 is observed, with a clear structure. The residuals can further be split into different projected angle bins. Two of these distributions are shown in Figure 5.12b. The distribution is almost flat for high projected angles, with greater deviations seen at low projected angles, as expected with diffusion effects being more prominent at lower projected angles. A correction is defined by splitting the distribution into 20 projected angle bins and fitting a 7th order polynomial,

¹An n -strip cluster is referred to as ‘genuine’ when the track trajectory it is fitted to has traversed all n strips, and ‘fake’ when one or more of the strips is the result of detector noise.

$f_i(x_{hit})$, to each, where i is an index running over all the projected angle bins. Subtracting this polynomial from the calculated position flattens the distribution of means and provides a method of modelling the diffusion effects more correctly. The correction is then applied to x_{hit} as

$$x_{hit}^{corr} = x_{hit} - f_i(x_{hit}) \quad (5.6)$$

where x_{hit}^{corr} represents the corrected position, and the polynomial $f_i(x_{hit})$ corresponding to the projected angle of the track is used. The effect of the corrections can be seen by comparing the hit resolutions before and after the correction is applied, where the resolution is given by the width of distributions fitted to a Gaussian. The hit resolution as a function of projected angle before and after these corrections are applied is shown in Figure 5.13. A clear improvement can be seen, especially at low projected angles where, as mentioned previously, the diffusion effect is greatest. Improvements of 20-30% in hit resolution are seen below projected angles of 0.1 radians.

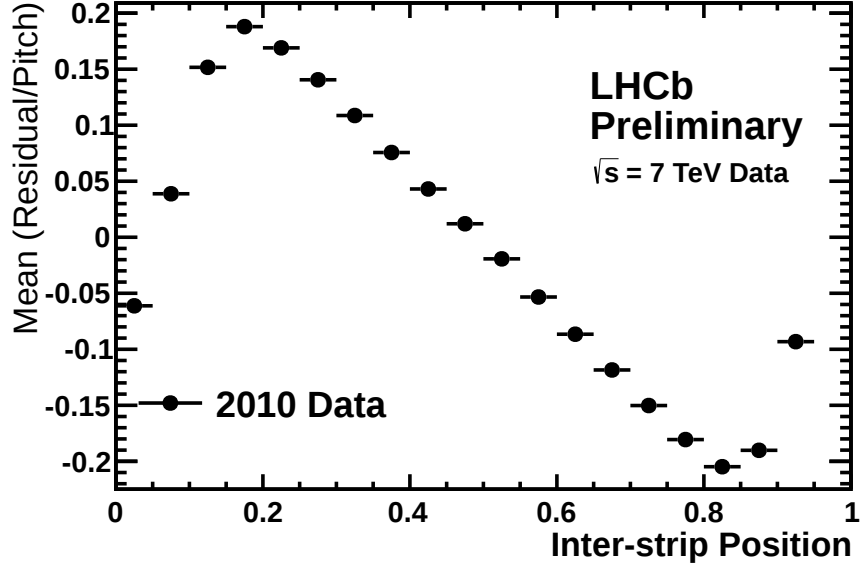
5.2.2 Single Hit Resolution

As described in the previous section, the hit resolution for any hit along a track is the combination of two effects, the single hit resolution of the hit, and the extrapolation precision of the track. The contribution from the extrapolation precision is again minimised by requiring hits corresponding to two-strip clusters with a charge of between 30 and 70 ADC counts either side of the relevant hit. However, in order to precisely measure the single hit resolution, the two effects must be separated. This is achieved by first considering the relationship between the single hit resolutions of strips of different pitch.

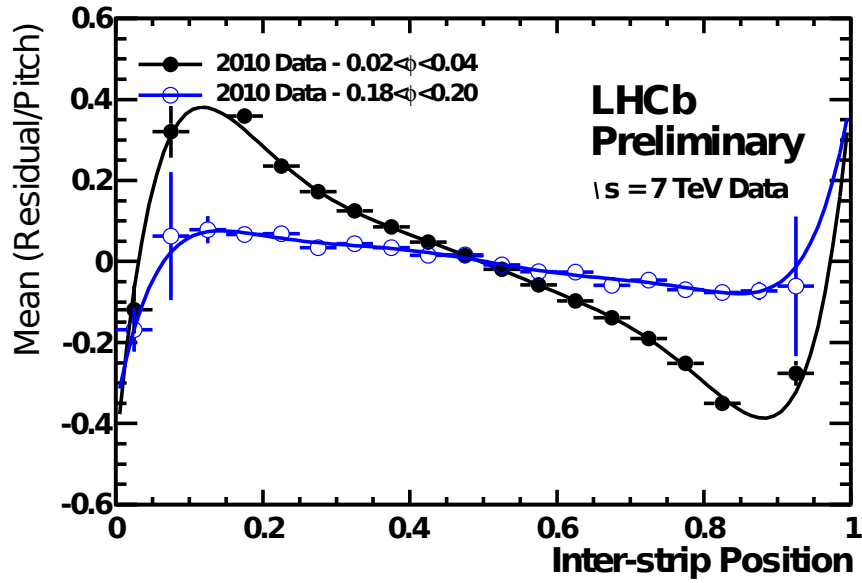
For two hits, $x_{hit}^{(1)}$ and $x_{hit}^{(2)}$ in strips with pitches p_1 and p_2 , the single hit resolutions, $\sigma_x^{(1)}$ and $\sigma_x^{(2)}$, are assumed to scale only with pitch, so that

$$\sigma_x^{(1)} = \frac{p_1}{p_2} \sigma_x^{(2)} \quad (5.7)$$

With this assumption, for a track travelling at an angle θ , as illustrated in Figure 5.14, the ratio of the pitches of the strips it traverses on two sensors separated



(a)



(b)

Figure 5.12: (a) Distribution of mean against fractional position integrated over all projected angles, (b) Distribution of mean against fractional position in two different projected angle bins. The polynomial fit used to correct the distribution is also plotted. A greater deviation from zero is observed in low projected angles.

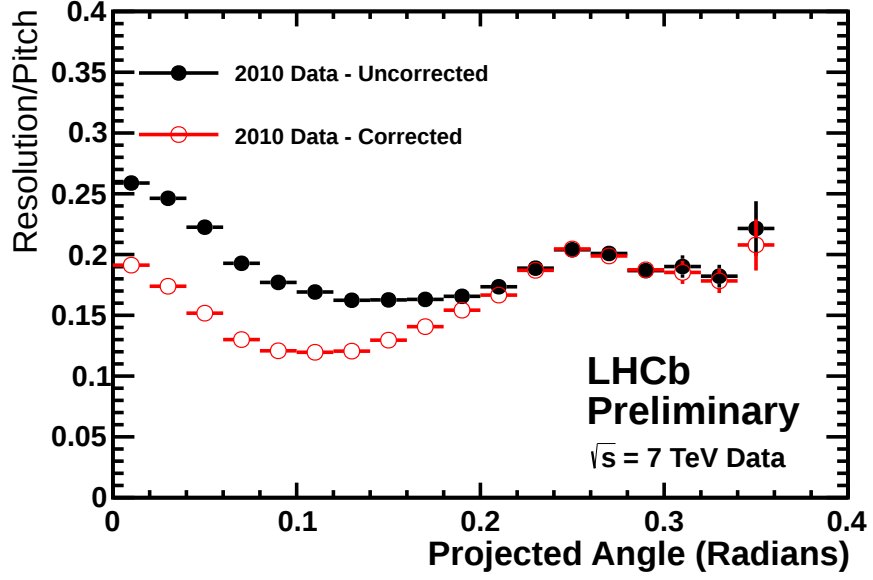


Figure 5.13: A comparison of the resolution/pitch as a function of the projected angle before (black) and after (red) the corrections are applied. A clear improvement of up to 30% can be seen in the low projected angle region.

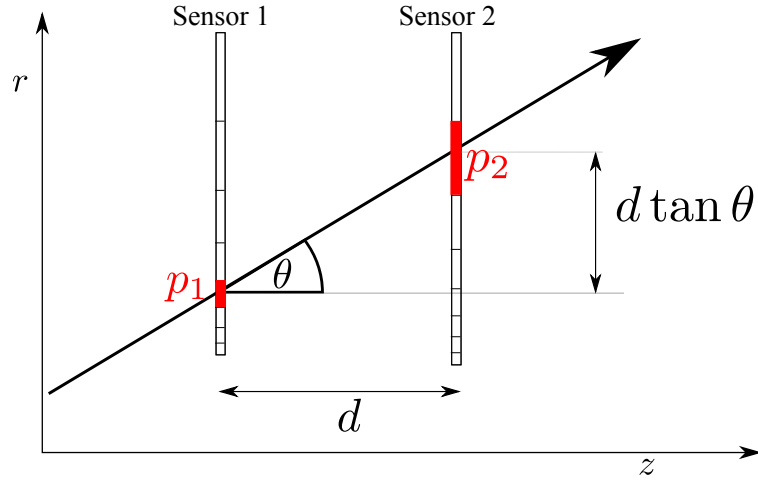


Figure 5.14: A track travelling at an angle of θ through two sensors placed a distance of d apart and depositing charge in strips of pitch p_1 and p_2 .

by a distance, d , is given by

$$a = \frac{p_1}{p_2} = d \tan(\theta) (1.887 \times 10^{-3}) \quad (5.8)$$

where 1.887×10^{-3} is the rate of change of pitch as a function of radial distance for the r -sensors. Consequently the single hit resolution of two neighbouring hits along a track trajectory are related by a multiplicative factor a . In the case of the VELO, it is convenient to choose the sensors before the “gap” region, as was shown in Figure 5.4 where all sensors are placed a uniform distance of 30 mm apart, and d can be taken to be constant. The single hit resolution of the central hit to be determined is denoted by σ , and the single hit resolution on the hits on the sensors left and right of the central hit are then given by σ/a and $a\sigma$ respectively. The single hit resolutions of the three hits are assumed to give the only contribution to the total hit resolution, and as the track is unbiased, with the central hit not included in the track fit, all three are assumed to have an equal contribution to the resolution, and so are added in quadrature to give a total hit resolution of $\sqrt{1 + \frac{1+a^4}{a^2}}\sigma$ on the central strip. This relates the single hit resolution to the total hit resolution, and so the single hit residuals can be determined from the total hit residual using the formula

$$\underbrace{(x_{hit} - x_{track}^{ubs})}_{\text{unbiased residual}} \cdot 1 / \sqrt{1 + \frac{1+a^4}{a^2}} \approx \underbrace{x_{hit} - x_{true}}_{\text{single hit residual}} \quad (5.9)$$

analogously to Equation (5.5), where x_{track}^{ubs} refers to the position of the unbiased track extrapolated to the sensor plane. The single hit resolution is then determined to be the Gaussian width of the distribution of the single hit residuals.

As the resolution is heavily dependent on projected angle, and pitch, the resolution is determined as a function of pitch for all r -strip clusters used to reconstruct long tracks in two separate projected angle bins, namely 0-4° and 7-11°. The results are shown in Figure 5.15 and contrasted with the binary resolution of $p/\sqrt{12}$.

From the plot the best resolution, obtained by extrapolating to a pitch of 40 μm and a projected angle of 7-11°, is $\sim 3 \mu\text{m}$, significantly better than the binary resolution of 11.5 μm . This also compares very favourably to other detectors, such as CMS and CDF which have best resolutions of 14 μm [97] and 9 μm [98] respectively.

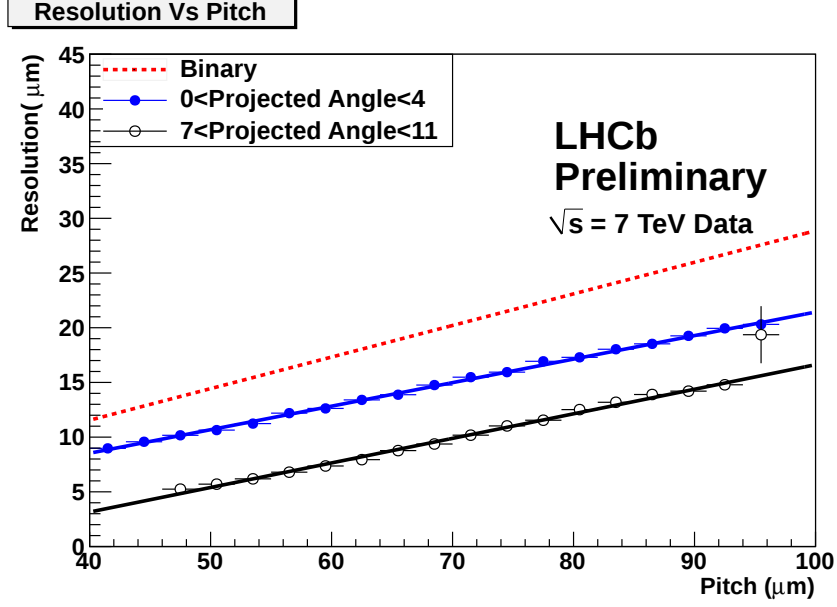


Figure 5.15: VELO single hit resolution as a function of strip pitch in two projected angle bins and compared to the binary resolution

This precise single hit resolution in the VELO allows LHCb to reconstruct track trajectories close to the interaction region with high precision. This in turn allows it to measure and distinguish primary and secondary vertices to high precision to help with the identification of decays coming from short lived particles such as B mesons or τ leptons.

5.2.3 Impact Parameter Resolution

The impact parameter of a track with respect to any given vertex is the distance of closest approach of the track's trajectory to that vertex. The calculation of the impact parameter is illustrated in Figure 5.16 and is now described. The distance between the primary vertex and a track trajectory, $\vec{r}(t)$, is given by

$$\vec{r}(t) = \vec{x} - \vec{v} + \hat{k}t \quad (5.10)$$

where $\vec{v}$ is the primary vertex position, $\vec{x}$ is the position of the first state, $\hat{k}$ is the unit momentum vector and t is a parameter that determines the position along the

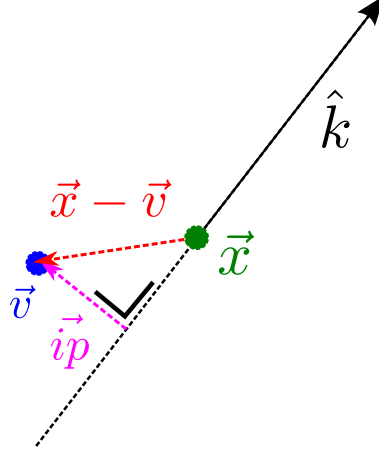


Figure 5.16: An illustration of the calculation of the impact parameter vector $\vec{ip}$ using Equation (5.12).

track's trajectory. The impact parameter vector, $\vec{ip}$, is given by $\vec{ip} = \vec{r}(t_{min})$, where t_{min} is the value of parameter t obtained by minimising $|\vec{r}(t)|$ with respect to t , given by

$$t_{min} = -(\vec{x} - \vec{v}) \cdot \hat{k} \quad (5.11)$$

with $\vec{ip}$ then given by

$$\begin{aligned} \vec{ip} = \vec{r}(t_{min}) &= \vec{x} - \vec{v} - \hat{k} \left((\vec{x} - \vec{v}) \cdot \hat{k} \right) \\ &= \hat{k} \times (\vec{x} - \vec{v}) \times \hat{k} \end{aligned} \quad (5.12)$$

The impact parameter is then given by the magnitude of the impact parameter vector. The impact parameter of a track with respect to the vertex at which it was produced is by definition identically zero, but in reality is determined by the resolution of the detector. Tracks coming from secondary interactions can be identified as those having large impact parameters with respect to the primary vertex. In this way, impact parameters serve as vital variables in order to identify certain particles and events in LHCb. However, in order to understand what is meant by “large” it is important first to understand the resolution with which the impact parameter can be determined, and a study of this resolution is now described.

Parameterisation

To determine the impact parameter resolution, the impact parameter in one dimension, ip , is considered to be the sum of two principal contributions: a contribution from the single hit resolution of the hits making up the track, ip^A ; and the multiple scattering of the track as it passes through detector material, ip^B . The impact parameter for an individual track is then given as $ip = ip^A + ip^B$, while the impact parameter resolution, σ_{ip} , is given by

$$\sigma_{ip} = \sigma_{ip}^A \oplus \sigma_{ip}^B \quad (5.13)$$

where σ_{ip}^A and σ_{ip}^B are the resolution of the impact parameter due to the single hit resolution and multiple scattering effects respectively. These two resolution contributions are now described.

The impact parameter resolution due to the single hit resolution of the track measurements, σ_{ip}^A , can be determined by first considering the calculation of the impact parameter using measurements in just two sensor planes as shown in Figure 5.17. For two measurements, (d_1, z_1) and (d_2, z_2) , the impact parameter is given by

$$ip^A = \frac{d_1 z_2 - d_2 z_1}{d_1 - d_2} \quad (5.14)$$

The resolution σ_{ip}^A can be determined using the propagation of errors formula on Equation (5.14), where the uncertainties on the positions d_1 and d_2 are given by the single hit resolution, σ_{HR} ,

$$\sigma_{ip}^A = \left(\frac{d_1 z_2 - d_1 z_1}{(d_1 - d_2)^2} \oplus \frac{d_2 z_1 - d_2 z_2}{(d_1 - d_2)^2} \right) \sigma_{HR} \quad (5.15)$$

As a track is constructed from more than two hits in reality, the approach is generalised to a system with n measurements, d_i , each with a hit resolution of σ_{HR} , and the impact parameter resolution contribution is given by [99]

$$\sigma_{ip}^A = \frac{\sqrt{\sum_i^n \left(\sum_j^n d_i^2 - d_i \sum_k^n d_k \right)}}{n \sum_i^n d_i^2 - \sum_i^n d_i \sum_k^n d_k} \sigma_{HR} \quad (5.16)$$

The contribution, σ_{ip}^B , arises from the fact that a particle traversing detector material is deflected through a small angle, θ_{MS} , by its interaction with nuclei. The distribution of the scattering angles is given by a Gaussian with a width, $\sigma_{\theta_{MS}}$, determined by the equation for multiple scattering [12].

$$\sigma_{\theta_{MS}} = \frac{13.6 \text{ MeV}/c}{\beta c p} \cdot \sqrt{\frac{x}{X_0}} \cdot \left[1 + 0.038 \ln \left(\frac{x}{X_0} \right) \right] \quad (5.17)$$

where x/X_0 is the thickness of the material in radiation lengths, p is the particle's momentum and β is the usual relativistic variable. After a track has traveled a distance d , the impact parameter displacement, ip^B , due to multiple scattering by an angle θ_{MS} is given by $ip^B = r \tan \theta_{MS}$, as can be seen in Figure 5.18. In the small angle approximation it is given by $ip^B \sim r \theta_{MS}$. If d is the distance from the primary vertex along the beam axis and X is the thickness of the material parallel to the beam axis, for a track traveling at an angle θ , the distance it travels is given by $d/\sin \theta$ and the thickness of material it traverses given by $X/\sin \theta$. The resolution due to multiple scattering, σ_{ip}^B is the impact parameter displacement resulting from an angular deflection of the mean scattering angle $\sigma_{\theta_{MS}}$. Using the relation $p_T = p \sin \theta$, σ_{ip}^B is given as

$$\sigma_{ip}^B = \frac{r}{p_T \sin^{1/2} \theta} 13.6 \text{ MeV}/c \cdot \sqrt{\frac{X}{X_0}} \cdot \left[1 + 0.038 \ln \left(\frac{X}{\sin \theta X_0} \right) \right] \quad (5.18)$$

From Equation (5.13), the impact parameter resolution is then given by

$$\sigma_{ip} = a \oplus \frac{b}{p_T} \quad (5.19)$$

where a and b can be determined by comparing Equation (5.19) with Equations (5.13), (5.16) and (5.18) and are approximated to be constant. From this equation the impact parameter resolution as a function of $1/p_T$ will be close to a straight line. The intercept of the slope gives an estimate of the contribution from the hit resolution, while the slope will reflect the multiple scattering effects.

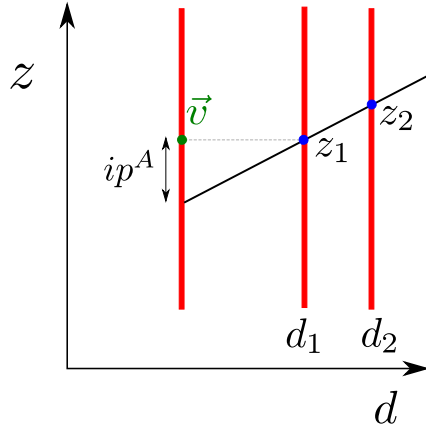


Figure 5.17: An illustration of the calculation of the impact parameter using two track measurements (d_1, z_1) and (d_2, z_2)

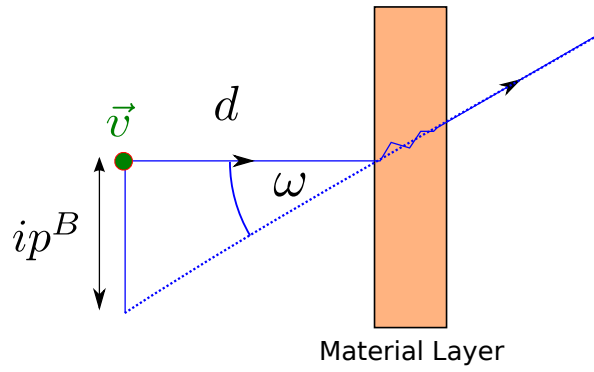


Figure 5.18: An illustration of the impact parameter displacement due to multiple scattering in a material layer.

Method

The method used to calculate the impact parameter resolution uses an assumption that all tracks in a minimum bias event originate from the primary vertex. This assumption implies that the true value of the impact parameters are zero, and so the measured values give the impact parameter residuals, which can be used to determine the impact parameter resolution. In order to clean up the event and track sample used, certain selection criteria are applied.

- Only events with one primary vertex reconstructed from more than 25 tracks are considered. This removes the complications arising from having events with multiple interactions as well as requiring a well resolved vertex.
- Only long tracks with more than 5 VELO r -sensor hits are considered. This ensures a track with momentum information well defined by the VELO.
- Low quality tracks are rejected by requiring that the track χ^2 probability is greater than 1%. The χ^2 probability of a reconstructed track represents the probability that the χ^2 for a track should be greater than or equal to the χ^2 of the reconstructed track for the number of degrees of freedom of the track.
- All tracks are required to have associated hits in the TT sub-detector. As the TT is not required in the pattern recognition stage for identifying long tracks, requiring TT hits provides a further track constraint to reduce the number of ghost tracks in data.

For each track passing the selection, the primary vertex is refitted without that track in order to calculate an unbiased impact parameter. The impact parameter is then determined from Equation (5.12).

The impact parameter vector, $\vec{ip}$, is decomposed into its x and y components, IPx and IPy . The track sample is split into 20 bins of p_T^{-1} between 0 and 3 GeV^{-1} , and each distribution is fitted to a Gaussian. Two of the fitted distributions are given by way of example in Figure 5.19. The resolution in each bin is given by the width of the fitted Gaussian shape. The distributions obtained in both data and simulation are shown in Figure 5.20. The linear dependence of the impact parameter on the track's transverse momentum can be clearly seen, and the resolutions at different

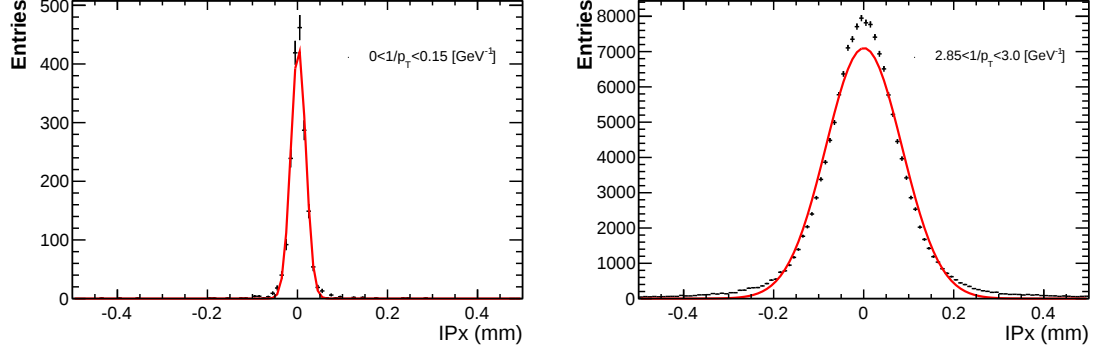
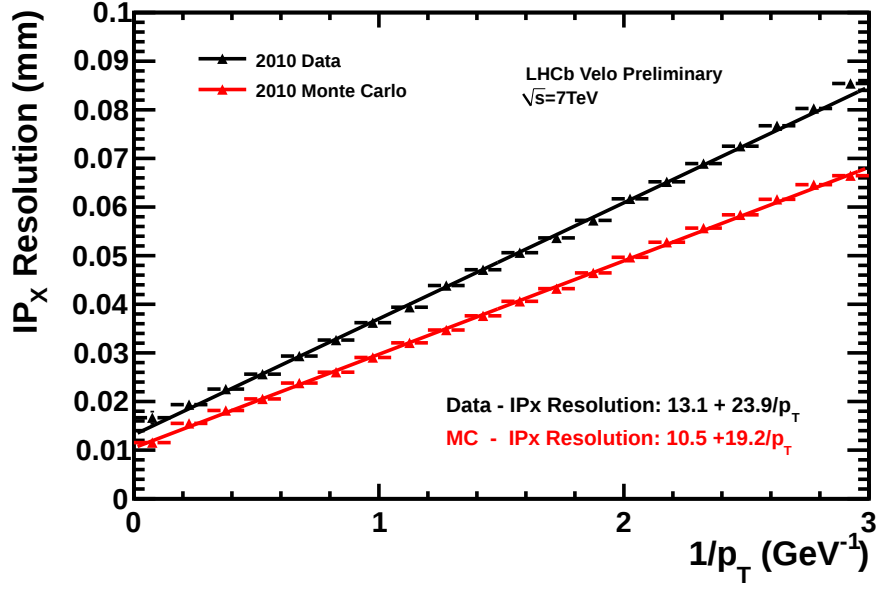
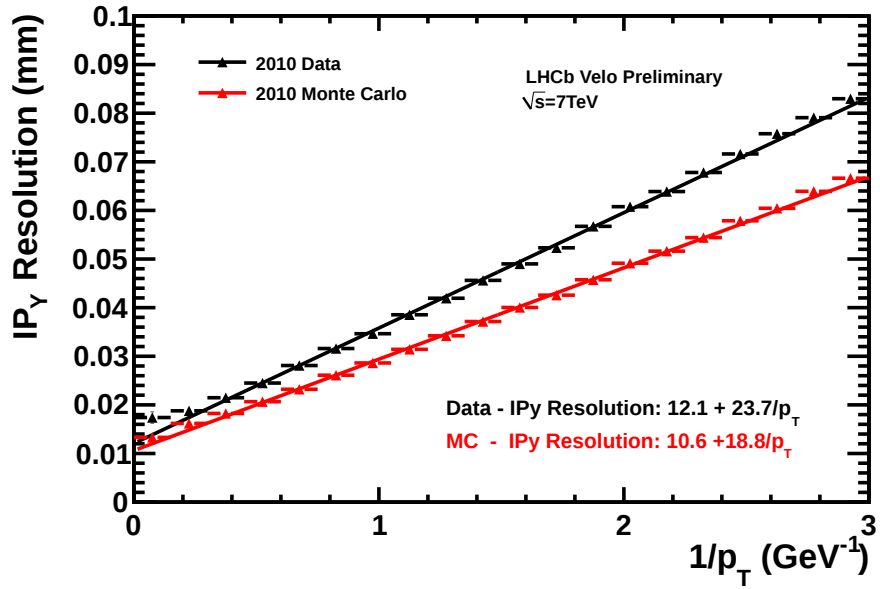


Figure 5.19: The IPx distribution in two bins of p_T^{-1} fitted to Gaussian shapes.

transverse momenta can be obtained from the graph. The distributions in data and simulation do not agree, with the discrepancy in the slope of the distribution suggesting that the multiple scattering effects are underestimated in simulation. Knowledge of the impact parameter resolution is essential as it is one of the key parameters used to determine if a particle is associated to a primary vertex or has come from some secondary interaction.



(a)



(b)

Figure 5.20: The impact parameter resolutions as a function of p_T^{-1} split into their (a) x and (b) y components.

CHAPTER 6

A MEASUREMENT OF THE $Z \rightarrow \tau\tau$ CROSS-SECTION AT LHCb

This chapter presents the measurement of the cross section of Z production via the decay of tau leptons. First the motivation for making such a measurement at LHCb will be discussed, followed by the characteristics of the signal and background events, and the selection scheme developed. Finally an overall measurement will be presented and compared to theoretical predictions and other measurements made at LHCb.

6.1 Motivation

The cross-section times branching ratio for $Z \rightarrow \tau\tau$ production has been measured to high precision in electron-positron collisions at LEP [100] while more recent measurements have also been made at ATLAS [101] and CMS [102] in proton-proton collisions. This section will discuss the motivation for measuring the cross-section for the production of $Z \rightarrow \tau\tau$ events at the LHCb experiment.

6.1.1 Lepton Universality

In the Standard Model, a universal coupling is assumed between the three generations of leptons and the gauge bosons. This property of the theory is known as

lepton universality, and is a feature of the underlying symmetry of the theory. It in turn predicts that, in the approximation that the masses of the leptons are identical, the branching ratio for the decay of the gauge bosons into leptons is flavour independent. This can be tested experimentally by measuring the branching ratios for the decay of charged and neutral gauge bosons to different leptonic final states. At LEP, lepton universality has been confirmed to a precision of better than 1% in the three leptonic decay modes of the Z boson [103], but a 2.8σ excess was observed in the branching ratio for the decay of W bosons to a tau and a tau neutrino compared to the other lepton flavours [104]. A number of models have been proposed to account for the excess [105, 106, 107] and it motivates continuing tests of lepton universality at the LHC.

As measurements of the cross-sections for the $Z \rightarrow ee$ [108] and $Z \rightarrow \mu\mu$ [109] processes have already been performed at LHCb, lepton universality can be tested by performing a measurement in the $Z \rightarrow \tau\tau$ channel.

6.1.2 New Physics

Tau leptons play a significant role in searches for new physics phenomena at the LHC. New processes producing tau pairs with an invariant mass in the region of the on-shell Z mass will manifest themselves as an excess in the $Z \rightarrow \tau\tau$ measurement. As the properties of the Z boson have been studied to high precision at LEP, with its branching fractions measured to a precision of greater than 0.25% [103], the underlying partonic processes producing a Z boson at the LHC can be accurately calculated, and convoluted with the Parton Distribution Functions to predict the $Z \rightarrow \tau\tau$ cross-section at the LHC. The observed cross-section can then be compared with this prediction to identify any excess.

Within the framework of the Standard Model, ongoing searches for the Higgs boson have excluded masses below 115.5 GeV and above 127 GeV [110], with a Higgs like particle recently observed at a mass of approximately 125 GeV [111, 112]. At this mass, the Higgs decays to tau pairs approximately 8% of the time, as shown in Figure 6.1. As the cross-section for the production of a Standard Model Higgs is up to two orders of magnitude lower than that of a Z boson [113, 114], a high precision of measurement is required to identify or exclude a Standard Model Higgs

boson.

High mass tau lepton pairs are also produced in a number of theories beyond the Standard Model. Supersymmetry refers to a symmetry between fermions and bosons which, if it exists in a broken form, predicts that supersymmetric fermion or boson partners exist for each particle in the Standard Model. Supersymmetry models also require a minimum of four Higgs bosons, three of which are neutral and decay to tau pairs [115, 116]. In a number of supersymmetric models, heavy supersymmetric scalar neutrinos also have tau decay modes [117]. Possible extensions of the electroweak symmetry of the Standard Model also predict the existence of a fourth generation of fermions [118] which would produce final states with taus, as well as final states with muons and electrons similar to those searched for in this analysis.

Where the tau pairs produced by any of the above processes have an invariant mass in the range of the Z pole, they can contribute to an excess in a measurement of the cross-section for the $Z \rightarrow \tau\tau$ process. Where no significant excess is observed, limits can be placed on a number of parameters in the theories.

6.1.3 Parton Distribution Functions

As described in Section 2.2.1, the parton distribution functions, which are dependent on both the energy scale of the interaction, Q , and the longitudinal momentum fraction of the partons, x , are extracted from measurements made at particle colliders. The PDFs can be extrapolated to different energy scales using the DGLAP equations introduced in Section 2.2.1. However, a theoretical error inherent to this evolution procedure is present for the PDF distributions in the (x, Q^2) space not directly measured by experiment. The uncertainty can be reduced, and the PDFs constrained by performing measurements in a wide range of phase space.

For the Drell-Yan process, where a di-lepton pair is produced through an intermediate virtual photon or Z boson, the Q^2 is given by the momentum transfer of the interaction, the mass of the photon or Z boson, M , with the longitudinal fractions of the struck partons, $x_{1,2}$, determined by the rapidity, y of the intermediate boson as

$$x_{1,2} = \sqrt{\frac{M^2}{s}} \exp(\pm y) \quad (6.1)$$

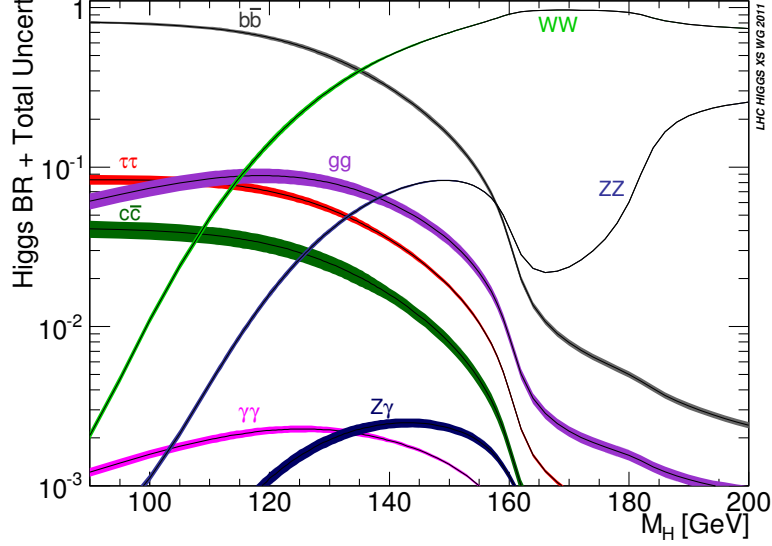


Figure 6.1: Branching ratios for the decay of a standard model Higgs boson in the mass range 100-200 GeV. The band width represents the uncertainty on that ratio [119]. The mass ranges outside of 115.5 to 127 GeV have been excluded by previous measurements [110].

where y is defined as

$$y = \ln \left(\frac{E - p_z}{E + p_z} \right) \quad (6.2)$$

with E representing the energy of the particle, and p_z its momentum along the beam axis. The unique forward rapidity region of LHCb allows it to explore two distinct regions of (x, Q^2) space as is shown in Figure 6.3. Note that a related quantity which will be used later in the chapter, is the pseudorapidity, η , in which the E in Equation 6.2 is replaced by the magnitude of the momentum, $|p|$. The uncertainty contribution to the Z production cross-section from the PDFs as a function of y is shown in Figure 6.2. The large uncertainties of up to 3% at large rapidities can be reduced by measurements in this region by LHCb. However, as studies in the $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ channels present cleaner signals and provide more accurate measurements than the $Z \rightarrow \tau\tau$ channel, constraining the PDFs does not constitute the primary motivation for the measurement presented in this thesis.

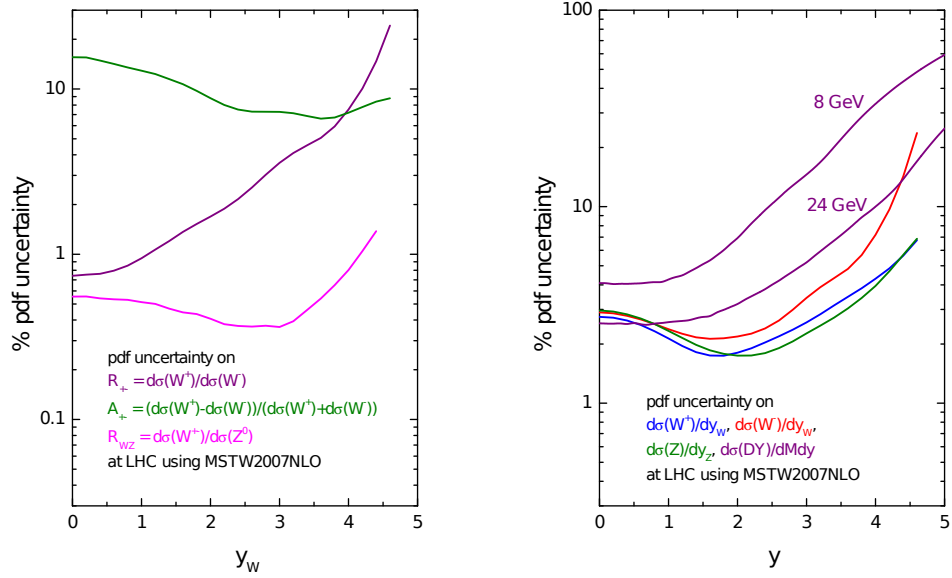


Figure 6.2: The uncertainties on the cross-section for a number of different physical observables at the LHC due to the PDFs. The uncertainty on the Z production cross-section is shown in green on the right-hand side plot [120].

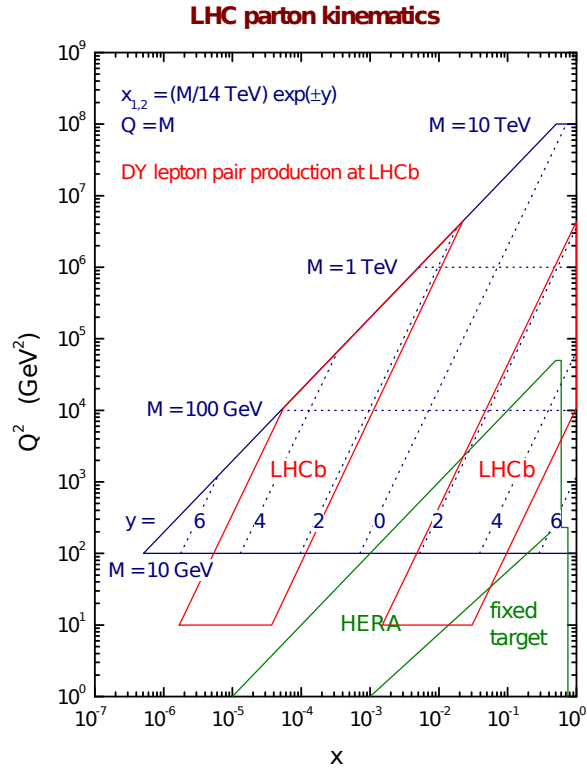


Figure 6.3: The (x, Q^2) measurement phase space where the regions which have been explored by previous experiments are indicated [120]. The red bands indicate the regions which will be covered by LHCb at a centre-of-mass energy of 14 TeV. The low- x , high Q^2 region is currently un-explored by other experiments.

6.2 Signal and Background Processes

In contrast to the $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ channels, which present clear di-muon and di-electron final states respectively, a study of the $Z \rightarrow \tau\tau$ channel requires the analysis of a number of different final states due to the subsequent decay of the tau leptons. In this analysis, the decays of the tau to single charged particles, specifically muons, electrons and hadrons with zero or more neutral hadrons are considered. Five analysis channels corresponding to four distinct di-tau final states are analysed: $\tau_\mu\tau_\mu$, $\tau_\mu\tau_e$, $\tau_e\tau_\mu$, $\tau_\mu\tau_h$ and $\tau_e\tau_h$, where the subscripts represent the visible decay product of the tau, with h representing a charged hadron. The final state containing a muon and an electron is split into two analysis channels, $\tau_\mu\tau_e$ and $\tau_e\tau_\mu$, based on kinematic requirements which will be described in Section 6.5.1. The decay product of the first tau is referred to as the first decay product, with the decay product of the second tau referred to as the second decay product. In the $\tau_\mu\tau_\mu$ final state, the first decay product refers to the muon with the highest transverse momentum, with the lower momentum muon representing the second decay product.

The branching ratios for the primary decay modes of the τ lepton are given in Table 2.2 and the final states analysed account for 44% of all $Z \rightarrow \tau\tau$ events. Each $Z \rightarrow \tau\tau$ final state is characterised by a pair of high transverse momentum oppositely signed particles produced in isolation from other event activity. The lifetime of the decaying τ lepton also results in its decay products having a large impact parameter¹ with respect to the primary vertex. These properties can be exploited to separate these events from background processes which produce similar final states. The background contributions which contribute a significant number of events are now described.

QCD Events are events produced through the strong interaction which can produce high momentum leptons in the final state in a number of different ways. Genuine muons and electrons are produced through the semi-leptonic decay of B and D mesons, or through the decay-in-flight of pions and kaons. Additionally, pions and kaons can be misidentified as electrons, when they leave large deposits in the ECAL, or as muons when they punch-through to the muon chambers. The final state particles in these events are typically contained within hadronic jets of

¹The impact parameter of a track was defined in Section 5.2.3

particles.

Electroweak Events contain a single high momentum muon or electron produced from a hard electroweak process which acts as the first decay product, with the second decay product coming from an associated jet or the underlying event. This background consists mostly of $W \rightarrow \ell \nu_\ell$ events, but also includes $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ events where one of the leptons is not reconstructed in the fiducial region.

The production of **pairs of gauge bosons** or **top quarks** also contribute background events, with the hard process contributing both final state particles. In the $\tau_\mu\tau_\mu$ channel a large background also exists from di-muon production through the **Drell-Yan** process. The same process also contributes a small number of events to the $\tau_\mu\tau_h$ and $\tau_e\tau_h$ channels when one of the Drell-Yan produced final state leptons is mis-identified as a hadron.

6.3 Dataset and Trigger Configuration

The dataset used in this analysis corresponds to the complete dataset collected by LHCb in 2011 at a centre of mass energy of 7 TeV. The events selected are triggered using independent muon and electron trigger paths which are now described.

- The **muon trigger path** comprises the L0Muon, Hlt1SingleMuonHighPT and Hlt2SingleMuonHighPT trigger lines. The L0Muon line is described in Section 4.4.1 and the two latter lines represent algorithms put in place at the HLT1 and HLT2 trigger stages to select events containing muons with a transverse momentum greater than 10 GeV. Muons are said to have been triggered when they have passed all three lines.
- The **electron trigger path** comprises the L0Electron, Hlt1SingleElectronNoIP, Hlt2SingleElectronTFHighPt and Hlt2SingleElectronTFVHighPt lines. The Hlt1SingleElectronNoIP line, which confirms the L0Electron decision described in Section 4.4.1 at the HLT1 stage, was absent in the early part of the 2011 data taking and only data collected after its introduction is considered in the analysis. The remaining two lines represent the algorithms put in place at the HLT2 level to select high transverse momentum electrons and have p_T thresholds of 10 and 15 GeV. The rate on the first line was judged to be too

high midway through the year, and so it was pre-scaled¹ and the line with the higher threshold subsequently introduced. Triggered electrons are required to have passed the L0 and Hlt1 Lines, and either of the Hlt2 lines.

The $\tau_\mu\tau_h$ and $\tau_\mu\tau_e$ final states are required to have triggered on the muon representing the leading decay product, and the $\tau_\mu\tau_\mu$ final state on either of the final state muons. The $\tau_e\tau_h$ final state is required to have triggered on the final state electron, while the $\tau_e\tau_\mu$ final state is required to have triggered on either final state lepton.

In all cases, the simulation samples were produced using Pythia 6.4 and the Gauss framework with the running conditions present in the 2011 data-taking. In most cases, these samples were produced centrally by the LHCb collaboration, with the exception being the $t\bar{t}$ and WW samples, which were produced locally using the same packages and configuration.

Throughout the analysis $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ data samples act as control samples which provide pure samples of muons and electrons for efficiency and selection studies. These samples are obtained by applying the event selection requirements described in References [109] and [108] to the 2011 dataset. Muons and electron are identified using the same procedure as described in Section 6.4; the transverse momentum of both leptons is required to be greater than 20 GeV, and the invariant mass of the lepton pair to be between 60 and 120 GeV for the $Z \rightarrow \mu\mu$ channel, and between 40 and 120 GeV for the $Z \rightarrow ee$ channel, unless otherwise stated. A sample of minimum bias data is also used for some selection and efficiency studies. This sample corresponds to events collected at LHCb throughout the 2011 data-taking where minimum requirements are applied at the trigger stage in order to select events which can be subsequently reconstructed by the detector.

6.4 Particle Identification

Analysis of the $Z \rightarrow \tau\tau$ final states requires the identification of three distinct particles: muons, electrons and hadrons. In this section, the track quality requirements for all candidate particles is first described, followed by the identification requirements used for each particle type.

¹A trigger line is said to be pre-scaled when only a certain percentage of the events it selects are written to disk

6.4.1 Track Selection

The procedure for reconstructing tracks at LHCb was described in Section 4.1. A track pre-selection is performed and only long tracks with a chi-squared probability greater than 0.1% within the pseudorapidity region $2 < \eta < 4.5$ are selected in order to reject poorly reconstructed tracks and ghost tracks, where long tracks and ghost tracks were described in Section 4.1. Additionally, due to the reduced acceptance of the hadronic calorimeter, tracks which are identified as hadrons are required to be in the region $2.25 < \eta < 3.75$. This is described in Section 6.6.5.

6.4.2 Muon Identification

Muons are selected using the standard muon identification procedure outlined in Section 4.3.4. For the momentum range of the muons in this analysis, this is equivalent to requiring that the long track muon candidates should be matched to hits in the four outermost muon stations. This ensures a high identification efficiency as well as a good rejection of pions and kaons by requiring that the candidate muons have traversed over 20 hadronic interaction lengths of detector material.

6.4.3 Electron Identification

High momentum electron identification at LHCb is challenging due to the saturation of the ECAL, which limits the energy readout of an individual cell. Figure 6.4a shows the maximum energy deposited in an individual ECAL cell by electrons in $Z \rightarrow ee$ events and the peak at ~ 0.28 GeV represents the energy saturation of the cell. This saturation prevents the energy of the photons radiated by the electron through the Bremsstrahlung process being fully recovered, and results in the momentum distribution of electrons being smeared to lower values. The invariant mass of electron pairs in $Z \rightarrow ee$ events is shown in Figure 6.4b [108], where the peak is spread to values below the on shell Z mass as a result of the smearing. The electron identification procedure outlined in Section 4.3.3 is insufficient for identifying high momentum electrons and so in its place a cut based selection is performed, which exploits the energies deposited in the PRS, ECAL and HCAL calorimeters associated to long tracks to identify the electron. Electrons are expected to have large

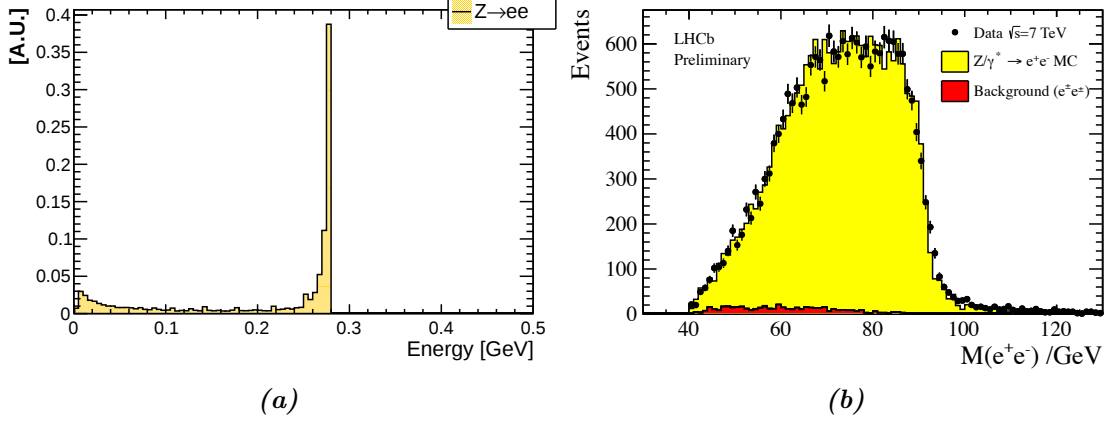


Figure 6.4: (a) Normalised distribution of the maximum energy deposited in an individual ECAL cell for $Z \rightarrow ee$ events. (b) The Z mass peak obtained from electron pairs at LHCb [108].

deposits in the PRS and ECAL, and small deposits in the HCAL. These distributions are shown for $Z \rightarrow ee$ events in data and simulation in Figure 6.5 while the corresponding distributions produced by tracks with a transverse momentum greater than 5 GeV in a minimum bias sample, largely consisting of pions, and muons from $Z \rightarrow \mu\mu$ events are shown in Figure 6.6. By comparing data and simulated $Z \rightarrow ee$ events, good agreement is observed in the E_{ECAL}/P and E_{HCAL}/P distributions, but the E_{PRS} distribution is poorly described in simulation. Electrons are selected by requiring that the associated calorimeter energies should satisfy the requirements

$$\begin{aligned}
 E_{\text{ECAL}}/P &> 0.1 \\
 E_{\text{HCAL}}/P &< 0.05 \\
 E_{\text{PRS}} &> 0.05 \text{ GeV}
 \end{aligned}
 \tag{6.3}$$

where E_{ECAL} , E_{HCAL} and E_{PRS} are the ECAL, HCAL and PRS energies associated to the track, and P is the magnitude of the track's momentum. In addition the candidate tracks are required to fail the LooseMuon requirement, ensuring that they are matched to less than three hits in the outermost muon stations.

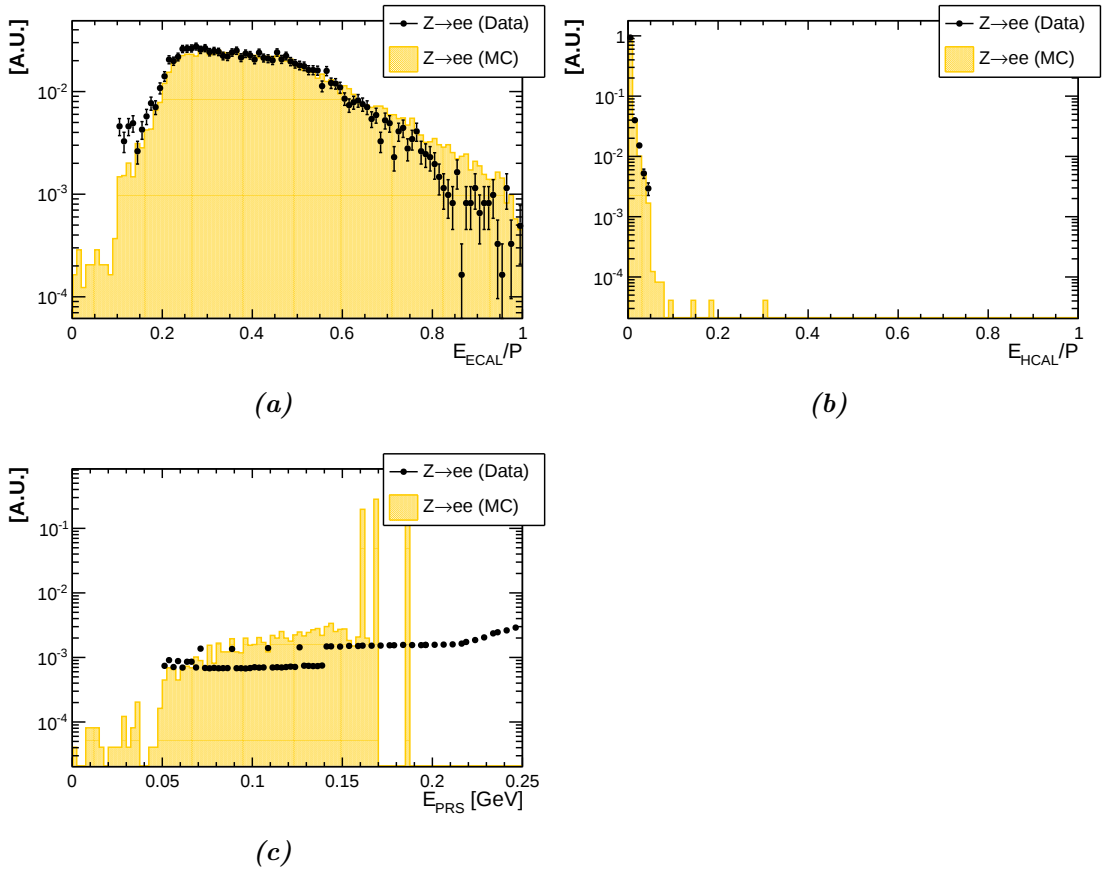


Figure 6.5: The calorimeter response due to electrons produced in $Z \rightarrow ee$ events. Shown are the normalised (a) E_{ECAL}/P , (b) E_{HCAL}/P and (c) E_{PRS} distributions.

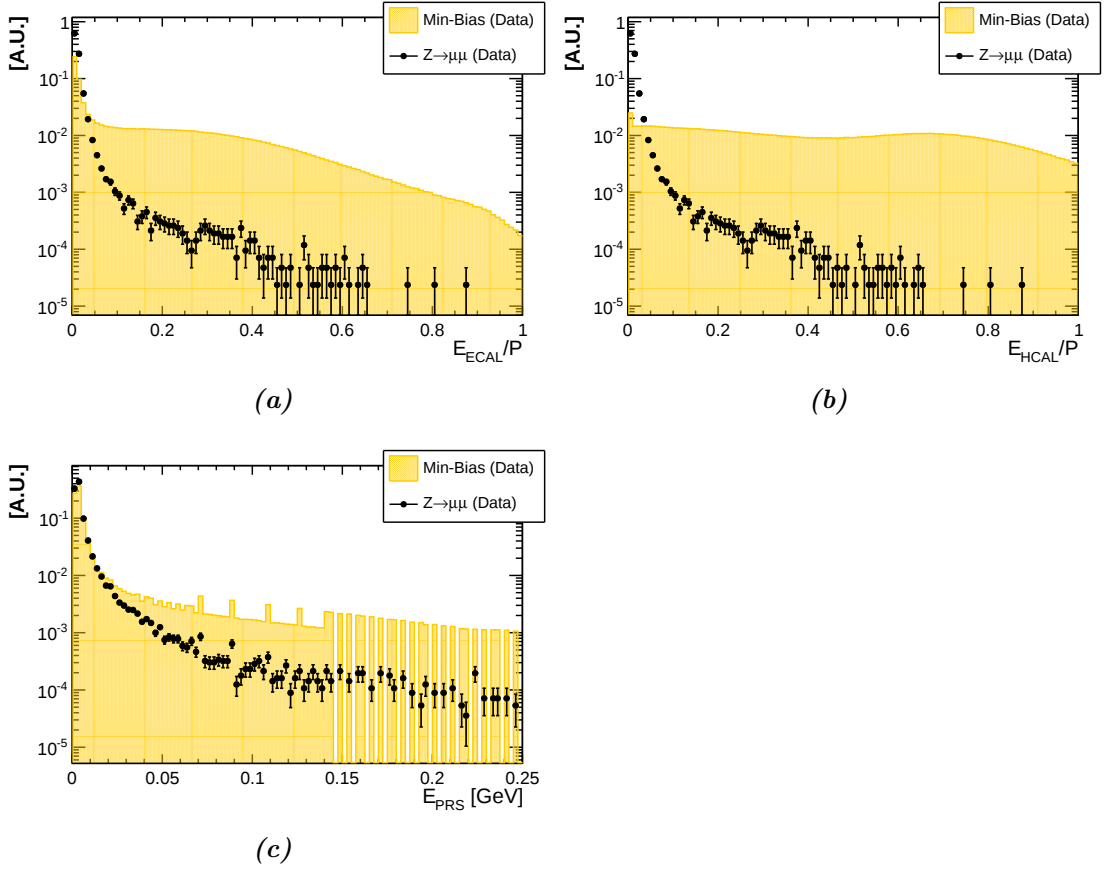


Figure 6.6: Normalised distributions for (a) E_{ECAL}/P , (b) E_{HCAL}/P and (c) E_{PRS} in samples of high p_t tracks in minimum bias events and identified muons in $Z \rightarrow \mu\mu$ events.

6.4.4 Hadron Identification

Hadrons are identified by exploiting the fact that, unlike electrons and muons, they shower in the hadronic calorimeter. Information from the RICH system is not used as its momentum coverage does not span the full momentum spectrum of the hadronic tau decays from $Z \rightarrow \tau\tau$ events. The HCAL energy deposits for particles in minimum bias events, muons and electrons are given in Figures 6.6b and 6.5b. Hadrons are selected as those tracks satisfying the requirement

$$E_{\text{HCAL}}/P > 0.05 \quad (6.4)$$

Similarly to electrons, hadron candidates are required to fail the LooseMuon requirement.

6.5 Signal Selection

Tau identification in a hadronic environment such as the LHC is challenging as a result of the large background encountered from QCD processes. To ensure a sufficiently pure sample of $Z \rightarrow \tau\tau$ events, a number of observables which distinguish the signal from the background are considered, with requirements on these variables applied where necessary. The observables are now described in turn.

6.5.1 Transverse Momentum

Lepton pairs produced through an intermediate Z boson have a large transverse momentum due to the mass of the decaying particle. Where the leptons are taus, the observable products have a softer momentum spectrum compared to the other leptonic Z decays due to the subsequent decay of the taus, but their momenta can still be used to distinguish them from low-momentum background processes. The transverse momentum distribution of the visible decay product of the tau in $Z \rightarrow \tau\tau$ events is shown in Figure 6.7a, where the decay product is a muon, electron or single charged hadron. For each analysis channel, events are selected which contain two oppositely signed tracks identified as the first and second decay products, where the first decay product is required to have a transverse momentum above 20 GeV,

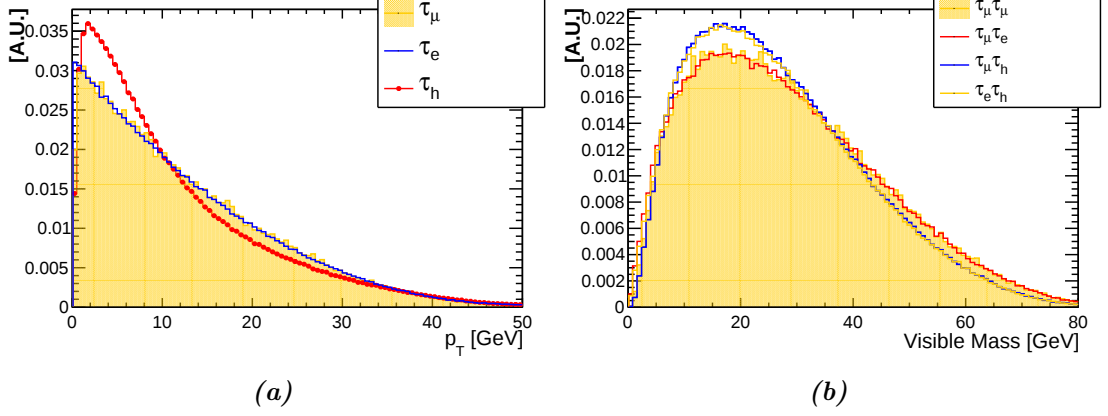


Figure 6.7: (a) The transverse momentum distribution of muons, electrons, and single charged hadrons produced in the decay of the tau leptons in $Z \rightarrow \tau\tau$ events, and (b) the visible mass spectrums for the four $Z \rightarrow \tau\tau$ final states considered in the analysis. The distributions are taken from simulation, calculated using the Pythia 8 program and are normalised to unity.

and the second decay product to have a transverse momentum above 5 GeV. In addition, the transverse momentum of the muon in the $\tau_e\tau_\mu$ channel is required to be less than 20 GeV. These transverse momentum requirements split the final state containing a muon and an electron into two distinct analysis channels with no overlapping events.

6.5.2 Visible Mass

The visible mass is defined as the mass of the combined four-vectors of the visible decay products of the final state. It is given by

$$M_{\text{vis}} = \sqrt{\left(\sum_i E_i\right)^2 - \left(\sum_i |\vec{p}_i|\right)^2} \quad (6.5)$$

where i is an index running over the visible decay products, with E_i and $\vec{p}_i$ representing their associated energy and momentum. In fully reconstructible $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ events, the visible mass represents the mass of the intermediate Z boson¹.

¹In reality, the mass spectrum will be spread to lower values due to final state radiation of the leptons. This is often referred to as the radiative tail of the distribution.

However, in the case of $Z \rightarrow \tau\tau$, the loss of information from neutrinos will result in a mass spectrum spread to a wider range at lower values, as shown in Figure 6.7b. In this analysis, the visible mass is required to be greater than 20 GeV for each final state. In the case of the $\tau_\mu\tau_\mu$ channel, the mass region $80 < M_{\mu\mu} < 100$ GeV is also excised to remove the events located around the Z pole occurring directly through the $Z \rightarrow \mu\mu$ decay channel.

6.5.3 Isolation

The leptonic products of Z decays are in general produced in isolation. This is in contrast to QCD events, where the tracks are typically contained within hadronic jets, and surrounded by other charged tracks. This is illustrated in Figure 6.8 and is an important property that can be exploited to separate signal events from the background. A radial distance in $\eta - \phi$ space, R , between two tracks is defined as

$$R = \sqrt{\Delta\phi^2 + \Delta\eta^2} \quad (6.6)$$

where $\Delta\phi$ and $\Delta\eta$ are the differences in azimuthal angle and pseudorapidity of the tracks. The isolation variable, I_{pT} , is defined as the transverse component of the vectorial sum of the momentum of all long tracks in a cone of radius, $R = 0.5$, around the candidate track, where the candidate track itself is excluded. For N tracks in the cone, I_{pT} is given by

$$I_{pT} = \left(\sum_i^N p_i \right)_T \quad (6.7)$$

where p_i represents the three-momentum of each charged track in the cone. The distribution of the maximum I_{pT} of the first and second decay candidates is shown for simulated $Z \rightarrow \tau\tau$ events, QCD background events obtained from data and $Z \rightarrow \mu\mu$ events in data and simulation in Figure 6.9. $Z \rightarrow \tau\tau$ and $Z \rightarrow \mu\mu$ events are peaked towards zero, while QCD events are spread to larger values. The simulation correctly describes the trend in the data with Z decays to leptons having low values for I_{pT} . However the agreement is not perfect, with the simulation underestimating the tail of the $Z \rightarrow \mu\mu$ distribution. This is due to the description of the underlying event

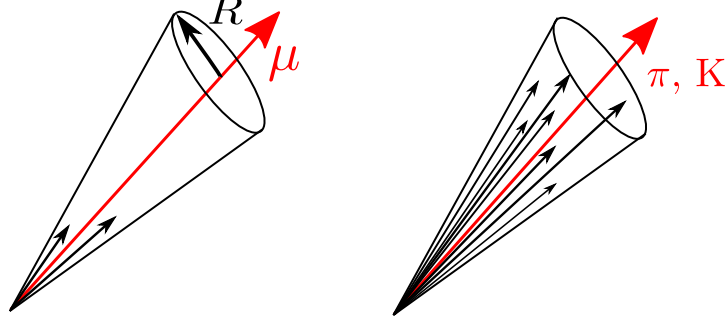


Figure 6.8: On the left a muon produced through a Z decay is illustrated, with a small number of tracks in the surrounding cone. On the right, a background QCD event is illustrated, where a pion or kaon is produced surrounded by a large number of charged tracks and mis-identified as a muon.

in simulation. Isolated events can be selected by requiring low values of I_{pT} , and so both decay products are required to satisfy $I_{pT} < 2$ GeV in the $\tau_\mu\tau_\mu$, $\tau_\mu\tau_e$ and $\tau_e\tau_\mu$ analysis channels and $I_{pT} < 1$ GeV in the $\tau_\mu\tau_h$ and $\tau_e\tau_h$ channels due to higher backgrounds.

6.5.4 Impact Parameter

An important property of $Z \rightarrow \tau\tau$ events which distinguishes them from background events is the lifetime of the tau lepton itself. The tau has a lifetime of 2.9×10^{-13} s [12] in its rest frame and travels a distance of ~ 1 cm in a $Z \rightarrow \tau\tau$ event at the LHC before decaying. This results in its decay products having a large impact parameter with respect to the primary vertex compared to prompt background events. As two signal tracks are present, the sensitivity can be further improved using an impact parameter sum method [121] where each of the impact parameters is signed and the significance of the sum used as a distinguishing variable, IPS . The variables used in the calculation of IPS are illustrated in Figure 6.10. By convention, the sign of the impact parameter is chosen to be the sign of the z -component of the particle's angular momentum with respect to the primary vertex. The signed impact

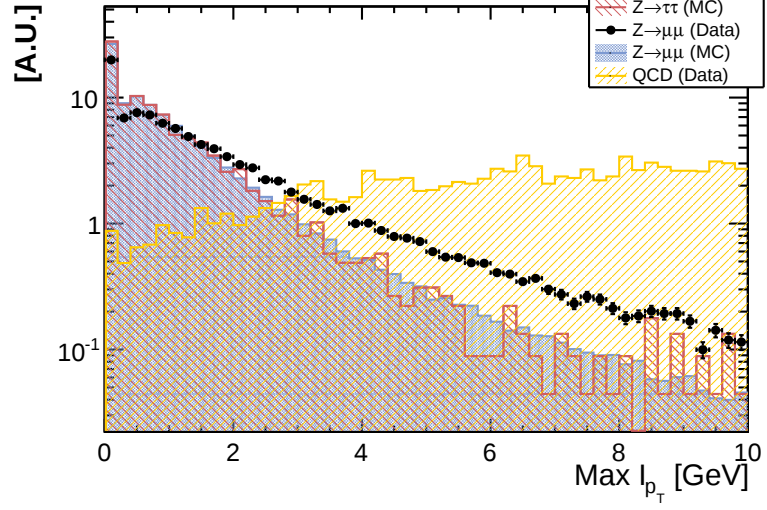


Figure 6.9: The normalised distributions of the maximum I_{p_T} for both decay products in an event for simulated $Z \rightarrow \tau\tau$ (simulation), $Z \rightarrow \mu\mu$ (data and simulation) and QCD (data) events.

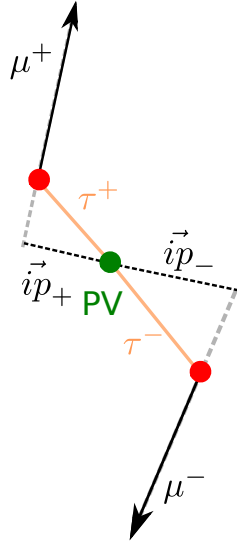


Figure 6.10: Illustration of the the impact parameter vectors $\vec{ip}_{\pm}$ for the particles in a $\tau_{\mu}\tau_{\mu}$ event. The magnitude of each vector is signed and the significance of the signed sum used as the discriminating variable IPS .

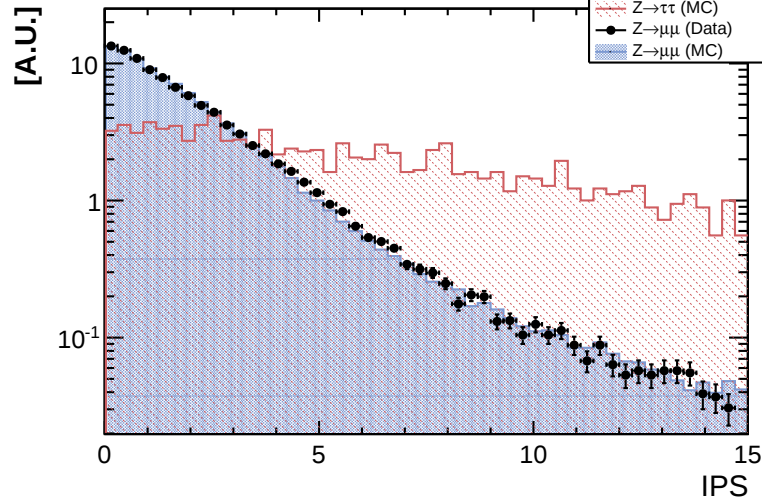


Figure 6.11: The normalised IPS distributions for $Z \rightarrow \tau\tau$ in simulation and $Z \rightarrow \mu\mu$ events in data and simulation

parameter, d , is then given by

$$d = |\vec{i}\vec{p}| \cdot \frac{(\vec{i}\vec{p} \times \vec{p}) \cdot \hat{z}}{\left| (\vec{i}\vec{p} \times \vec{p}) \cdot \hat{z} \right|} \quad (6.8)$$

where $\vec{i}\vec{p}$ is the impact parameter vector, which points from the point of closest approach of the track to the primary vertex, $\vec{p}$ is the particle's three-momentum, and $\hat{z}$ is a unit vector along the z -axis. The uncertainty on the impact parameter is taken by extrapolating the track to the point of closest approach and using the track covariance matrix. The impact parameter sum, IPS , is then given by

$$IPS = \frac{d_+ + d_-}{\sqrt{\delta d_+^2 + \delta d_-^2}} \quad (6.9)$$

where d_+ and d_- represent the signed impact parameters of the positive and negative tracks respectively, with δd_+ and δd_- representing their associated uncertainties. By comparing the means of the IPS distributions for $Z \rightarrow \mu\mu$ events in data and simulation, the simulation is found to be related to the data by a constant

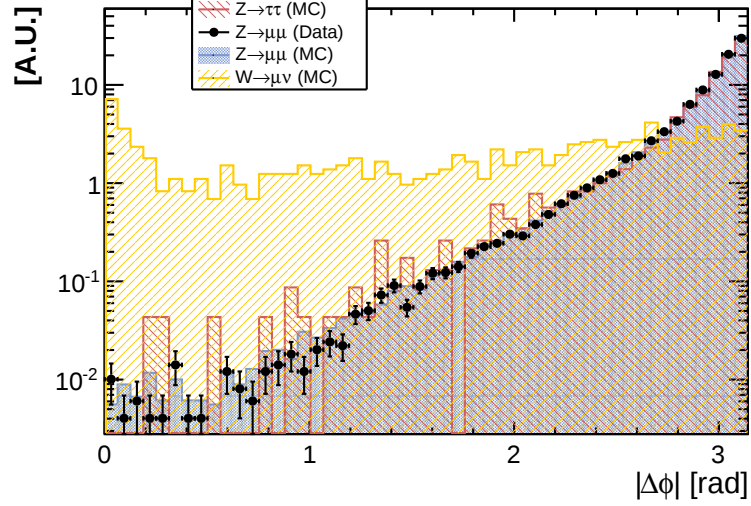


Figure 6.12: The normalised $|\Delta\phi|$ distributions for $Z \rightarrow \mu\mu$ events in data and simulation, and $Z \rightarrow \tau\tau$ and W events in simulation.

scaling factor of 1.12 ± 0.01 . The IPS distributions for $Z \rightarrow \mu\mu$ events in data and simulation and simulated $Z \rightarrow \tau\tau$ events are shown in Figure 6.11, where the simulated distributions are multiplied by the scaling factor. The $Z \rightarrow \mu\mu$ distributions are peaked towards zero while the $Z \rightarrow \tau\tau$ distribution is spread to larger values as expected. A requirement of $IPS > 9$ is applied in the $\tau_\mu\tau_\mu$, $\tau_\mu\tau_h$ and $\tau_e\tau_h$ analysis channels, which have large backgrounds from Drell-Yan produced muon pairs and electroweak events.

6.5.5 Azimuthal Angle Separation

The azimuthal angular separation of two tracks, $|\Delta\phi|$, is given by

$$|\Delta\phi| \equiv \begin{cases} |\phi_1 - \phi_2| & \text{if } |\phi_1 - \phi_2| \leq \pi \\ 2\pi - |\phi_1 - \phi_2| & \text{if } |\phi_1 - \phi_2| > \pi \end{cases} \quad (6.10)$$

The leptonic decay products of Z bosons tend to be back to back in the transverse plane, as the Z is typically produced with low transverse momentum. This is not true of many of the backgrounds, where there is only a weak correlation between

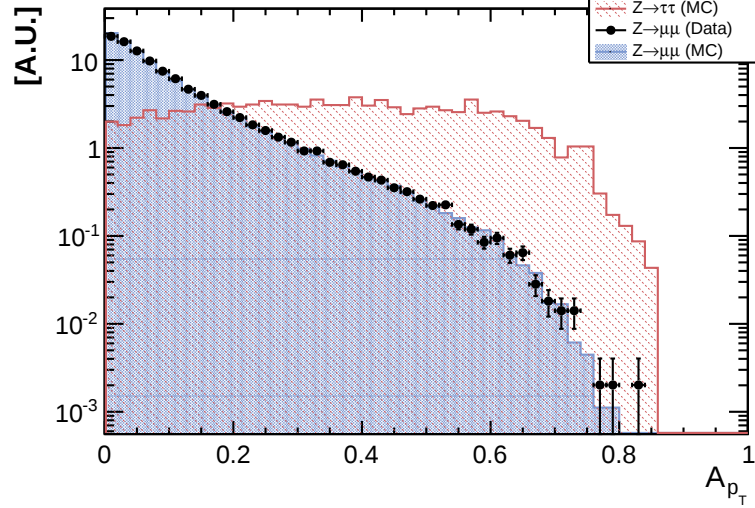


Figure 6.13: The normalised A_{p_T} distributions for $Z \rightarrow \mu\mu$ (data and simulation) and $Z \rightarrow \tau\tau$ events (simulation) are shown.

the direction of the two candidate tracks. The $|\Delta\phi|$ distributions are shown for simulated $Z \rightarrow \tau\tau$ and $W \rightarrow \mu\nu_\mu$ events, and for $Z \rightarrow \mu\mu$ events in data and simulation in Figure 6.12. The $Z \rightarrow \tau\tau$ and $Z \rightarrow \mu\mu$ distributions are peaked towards π radians, while the W events have a flatter distribution. A good agreement is observed between data and simulation in the $Z \rightarrow \mu\mu$ channel, and so the $Z \rightarrow \tau\tau$ simulation is expected to model the $Z \rightarrow \tau\tau$ data distribution well. In all analysis channels, $|\Delta\phi| > 2.7$ radians is required to enhance the signal purity.

6.5.6 Transverse Momentum Asymmetry

The asymmetry variable, A_{p_T} , is defined as

$$A_{p_T} = \frac{p_T^{(1)} - p_T^{(2)}}{p_T^{(1)} + p_T^{(2)}} \quad (6.11)$$

where $p_T^{(1)}$ and $p_T^{(2)}$ represent the transverse momentum of the first and second decay products respectively. Although the background events produced by a virtual Z boson decaying directly to a di-muon pair in the $\tau_\mu\tau_\mu$ final state are largely removed

Channel	N
$\tau_\mu\tau_\mu$	124
$\tau_\mu\tau_e$	421
$\tau_e\tau_\mu$	155
$\tau_\mu\tau_h$	189
$\tau_e\tau_h$	101

Table 6.1: The number of selected events for each analysis channel

by excising the region of the Z mass peak, events at lower masses produced through the photon propagator still contribute to the sample. This background contribution is reduced by exploiting the fact that the intermediate boson is typically produced with low transverse momentum. Momentum conservation then requires that the p_T of the two leptonic products should be balanced and the A_{p_T} distribution peaked towards zero. In $Z \rightarrow \tau\tau$ events, the A_{p_T} distribution is flatter as a portion of the the tau momenta is lost to the neutrinos and neutral hadrons. A comparison of this variable in data and simulation for the $Z \rightarrow \mu\mu$ channel and simulation for the $Z \rightarrow \tau\tau$ channel is shown in Figure 6.13. The distribution is peaked towards zero in the $Z \rightarrow \mu\mu$ channel, as expected, so in the $\tau_\mu\tau_\mu$ channel, the muons are further required to satisfy $|A_{p_T}| > 0.3$. As good agreement is observed in data and simulation for $Z \rightarrow \mu\mu$ events, the $Z \rightarrow \tau\tau$ distribution obtained from simulation is expected to accurately reflect the data distribution.

6.6 Cross-Section Determination

After the selection requirements have been applied to the dataset, the number of events, N , selected for each analysis channel is given in Table 6.1. The cross-section for the $Z \rightarrow \tau\tau$ process, $\sigma_{Z \rightarrow \tau\tau}$, is then calculated using the formula

$$\sigma_{Z \rightarrow \tau\tau} = \frac{\sum_i^N 1/\varepsilon_{rec}^i - \sum_j^{N_{bkg}} 1/\varepsilon_{rec}^j}{\varepsilon_{sel} \cdot \int \mathcal{L} \cdot \mathcal{A} \cdot BR} \quad (6.12)$$

where N_{bkg} is the number of background events, $\int \mathcal{L}$ is the integrated luminosity of the sample, $\mathcal{A}$ is an acceptance factor used to correct the measurement to the quoted

fiducial region, BR is the branching ratio for the particular final state and ε_{sel} is the efficiency for selecting the signal events. The reconstruction efficiency, ε_{rec} , is calculated for each event and is dependent on the momentum and pseudorapidity of the first and second decay products. These components and the methods for determining them will now be described in turn.

6.6.1 Luminosity

At LHCb the luminosity is determined using two different methods. A Van der Meer Scan [122, 123] is performed, where one of the beams is swept sideways across the other and the collision rate measured as a function of beam displacement to determine the beam profile. Additionally, a beam gas method [124] is used, where vertices arising from interactions with the material are reconstructed in order to trace the beam profile. A knowledge of the beam profile allows the luminosity to be calculated using Equation (3.4) and both methods are combined to obtain a precision of 3.5% [125]. For the $\tau_\mu\tau_\mu$, $\tau_\mu\tau_e$ and $\tau_\mu\tau_h$ analysis channels, the luminosity is determined to be $1.024 \pm 0.034 \text{ fb}^{-1}$, while for the $\tau_e\tau_\mu$ and $\tau_e\tau_h$ analysis channels, only events after the introduction of the electron trigger at Hlt1 level are considered, and the sample corresponds to $0.955 \pm 0.033 \text{ fb}^{-1}$.

6.6.2 Branching Ratio

The branching ratio, BR , represents the fraction of $Z \rightarrow \tau\tau$ events which produce any given final state, and is calculated from the PDG [12] values. For degenerate states such as $\tau_\mu\tau_\mu$, it is given by the square of the branching ratio for the individual tau decay mode, while for non degenerate states it is given by twice the product of the branching ratio for each individual decay mode. The branching ratio for the hadronic decay mode represents the sum of the branching ratios for all tau decays producing a single charged hadron in the final state. The branching ratios for each analysis channel are tabulated in Table 6.2.

BR (%)	
$\tau_\mu\tau_\mu$	3.031 ± 0.014
$\tau_\mu\tau_e$	6.208 ± 0.020
$\tau_e\tau_\mu$	6.208 ± 0.020
$\tau_\mu\tau_h$	16.933 ± 0.056
$\tau_e\tau_h$	17.341 ± 0.057

Table 6.2: The branching ratio for each $Z \rightarrow \tau\tau$ final state analysed [12].

Channel	N_{ss}	r^{QCD}	r^{EWK}	f^{QCD}	f^{EWK}	N_{QCD}	N_{EWK}
$\tau_\mu\tau_\mu$	9	1.30	2.00	1.00	0.00	11.69 ± 3.57	00.00 ± 3.47
$\tau_\mu\tau_e$	100	1.11	1.16	0.65	0.35	72.42 ± 2.34	40.32 ± 4.49
$\tau_e\tau_\mu$	52	1.03	1.56	1.00	0.00	53.97 ± 3.03	00.00 ± 1.50
$\tau_\mu\tau_h$	53	1.00	0.98	0.79	0.21	41.83 ± 0.54	10.83 ± 0.53
$\tau_e\tau_h$	34	0.98	1.02	0.73	0.27	24.52 ± 0.59	9.29 ± 0.46

Table 6.3: The background parameters used to determine the number of QCD and electroweak background events.

6.6.3 Background Estimation

The QCD and electroweak background contributions are estimated by applying the full selection for each analysis channel to the data sample, but requiring that the final state particles have the same charge. When one or both decay product candidates are contained in jets, their charges will have only a weak correlation, and so both processes are expected to contribute similar numbers of events to a same-sign sample as an opposite-sign sample. The fractional contributions of the QCD and electroweak backgrounds, f^{QCD} and f^{EWK} , to the same-sign sample for each analysis channel are obtained by fitting template shapes using the method in Reference [126] to the p_T difference distribution, $p_T^{(1)} - p_T^{(2)}$, where $p_T^{(1)}$ and $p_T^{(2)}$ refer to the transverse momentum of the first and second decay products respectively. A QCD template is obtained from data, where the full selection is applied but the isolation requirement is reversed, with I_{p_T} required to be greater than 10 GeV for both final state particles, enriching the sample in QCD events. The electroweak template is obtained from simulation by applying the selection to $W \rightarrow \mu\nu$ and $W \rightarrow e\nu$ simulated events,

which are expected to be characteristic of the electroweak events. The electroweak template is shifted to larger values due to the hard momentum spectrum of the leading decay product. The results are shown in Figure 6.14.

Additionally, ratios, r^{QCD} and r^{EWK} which relate the number of opposite-sign events to same-sign events for each process are determined by comparing the number of events in the templates obtained from same-sign and opposite-sign data samples. These templates are given for the $\tau_\mu\tau_e$ channel in Figure 6.15 where the opposite-sign templates have a small excess of events over the same-sign templates in both QCD and electroweak events. The number of QCD and electroweak background events for each analysis channel are then given by

$$N_{QCD} = r^{QCD} \cdot f^{QCD} \cdot N_{ss} \quad (6.13)$$

$$N_{EWK} = r^{EWK} \cdot f^{EWK} \cdot N_{ss} \quad (6.14)$$

where N_{ss} is the number of same sign events obtained and N_{QCD} and N_{EWK} represent the estimated number of QCD and electroweak background events respectively. The fit uncertainty is added in quadrature with the statistical uncertainty of the templates to obtain an overall systematic uncertainty on the two background contributions. The parameters used to determine N_{QCD} and N_{EWK} are tabulated in Table 6.3. The contributions from WW and $t\bar{t}$ events are obtained by applying the full selection to simulated WW and $t\bar{t}$ samples, which are normalised to NLO cross-sections to produce an expected number of events, N_{WW} and $N_{t\bar{t}}$ for each analysis channel. As only a small number of events survive the selection for each final state, the statistical uncertainty on the number of remaining events is deemed sufficient as a systematic error.

The Drell-Yan background contribution to the $\tau_\mu\tau_\mu$ analysis channel is estimated by normalising a $Z/\gamma^* \rightarrow \mu\mu$ template shape to the data in the range $80 < M_{\mu\mu} < 100$ GeV around the residual $Z \rightarrow \mu\mu$ mass peak. The integral of the normalised template below the peak then represents the number of $Z/\gamma^* \rightarrow \mu\mu$ background events in the selected sample. This template is obtained by applying the full $\tau_\mu\tau_\mu$ selection, but reversing the impact parameter cut and requiring $IPS < 1$, to enrich the sample in $Z/\gamma^* \rightarrow \mu\mu$ events. A similar procedure is employed to determine the contribution to the $\tau_\mu\tau_h$ channel. A $Z/\gamma^* \rightarrow \mu\mu$ template is again obtained

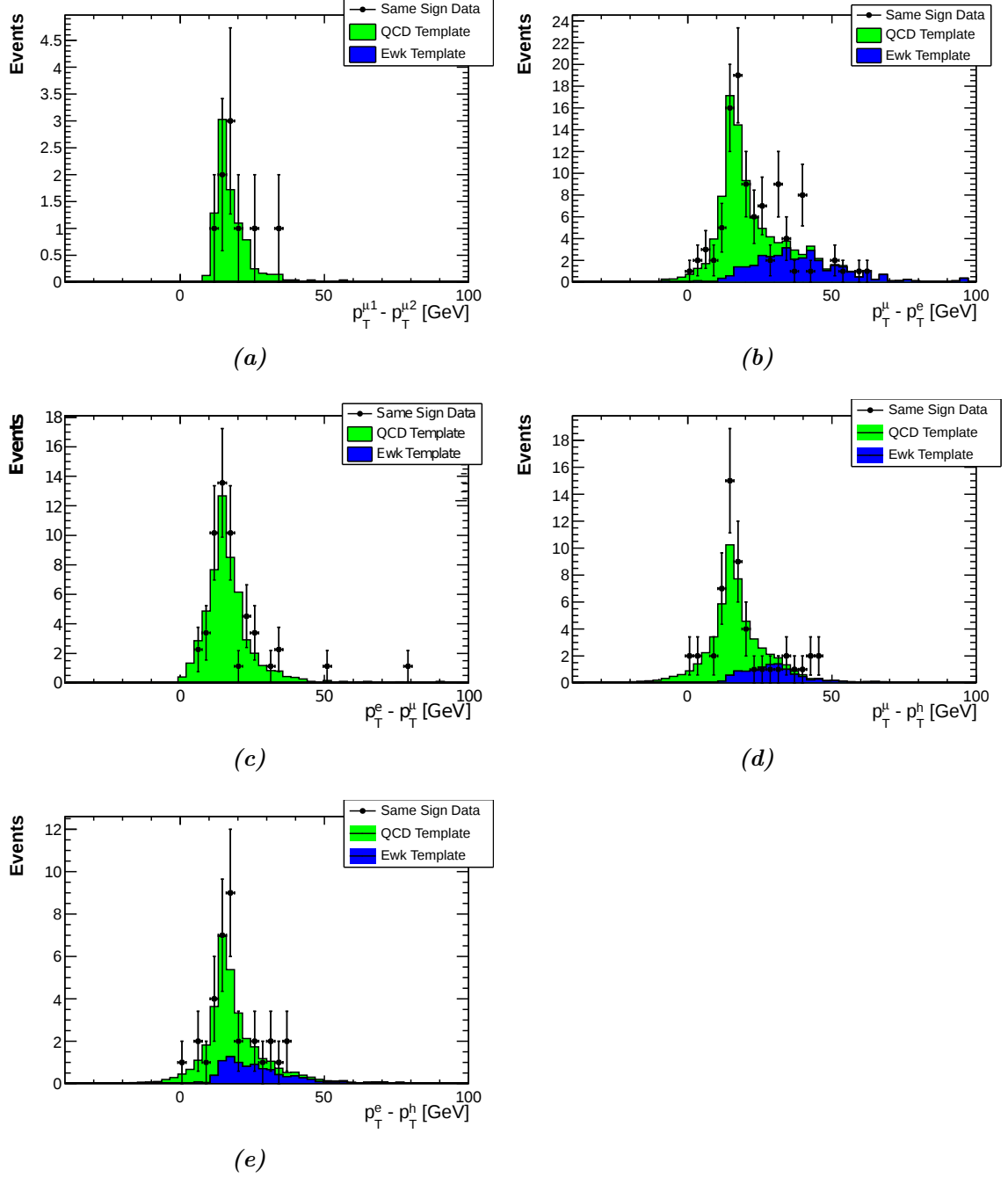


Figure 6.14: The background fits used to determine the QCD and electroweak background contributions in the (a) $\tau_\mu\tau_\mu$, (b) $\tau_\mu\tau_e$, (c) $\tau_e\tau_\mu$, (d) $\tau_\mu\tau_h$, and (e) $\tau_e\tau_h$ final states.

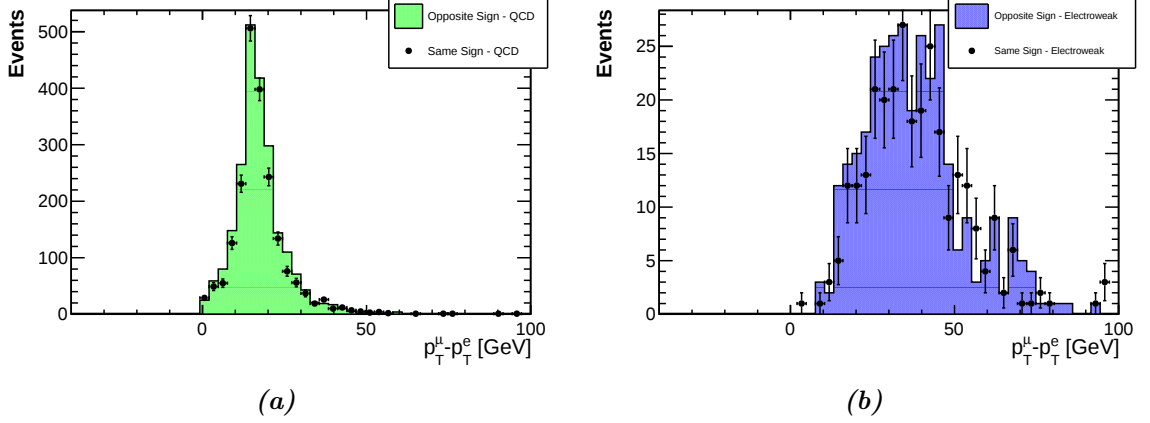


Figure 6.15: The templates obtained for (a) QCD and (b) electroweak events in opposite and same-sign samples in the $\tau_\mu\tau_e$ channel. The ratios r_{QCD} and r_{EWK} are determined from the ratio of the number of events in the opposite sign template to those in the same sign template

from data, and is normalised to the data in the peak region where the $\tau_\mu\tau_h$ selection requirements are applied but both tracks are required to be identified as muons. The template is then scaled by the probability of a muon to be misidentified as a hadron. This mis-identification probability is calculated by selecting events with two oppositely signed tracks in the region of the Z mass peak where one track is identified as a muon; the percentage of events where the second track satisfies the hadron identification requirements represents the misidentification probability, and is determined to be $0.06 \pm 0.01\%$. For the $\tau_e\tau_h$ channel, the same method is employed but both the template shape and the mis-identification efficiency are obtained from simulation. The $\tau_e\tau_h$ selection requirements are applied to the data where both tracks are required to be identified as electrons. The template shape is then normalised to the data in the region of the Z mass peak and scaled by the mis-identification probability of $0.63 \pm 0.02\%$. The background uncertainties are given by the statistical error on the number of selected events in the normalisation range added in quadrature with the statistical error on the template shape, with the uncertainty of the mis-identification probability also added in quadrature for the $\tau_\mu\tau_h$ and $\tau_e\tau_h$ channels.

The total number of selected and background events and their associated un-

	$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$
N	124	421	155	189	101
N_{QCD}	11.69 ± 3.57	72.42 ± 2.34	53.97 ± 3.03	41.83 ± 0.54	24.52 ± 0.59
N_{EWK}	0.00 ± 3.47	40.32 ± 4.49	0.00 ± 1.50	10.83 ± 0.53	9.29 ± 0.46
N_{WW}	0.03 ± 0.01	3.65 ± 0.24	1.02 ± 0.01	-	0.66 ± 0.37
$N_{t\bar{t}}$	0.03 ± 0.01	13.35 ± 0.42	1.57 ± 0.01	0.02 ± 0.01	0.05 ± 0.05
N_{DY}	29.81 ± 6.60	-	-	0.40 ± 0.20	2.03 ± 0.10
N_{bkg}	41.6 ± 8.5	129.7 ± 4.9	56.6 ± 3.3	53.3 ± 0.8	36.6 ± 0.9

Table 6.4: The number of selected and background events determined for each analysis channel. The systematic uncertainty on the total number of background events is the sum in quadrature of the uncertainties on each individual background contribution.

certainties are tabulated in Table 6.4. The visible mass spectra for each final state are shown in Figure 6.16, with all background shapes normalised to the calculated number of events. The signal shapes are obtained from $Z \rightarrow \tau\tau$ simulation and normalised to the remaining number of events.

6.6.4 Acceptance

In order to compare the results with theoretical predictions, which only apply requirements on the direct leptonic products of the Z , the cross-section is quoted in a fiducial region determined by phase space requirements on the Z and tau leptons. For ease of comparison with LHCb’s measurements in the $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ channels, the fiducial region is chosen to be

$$\sigma_{Z \rightarrow \tau\tau} (p_T^\tau > 20 \text{ GeV}, 60 < M_Z < 120 \text{ GeV}, 2.0 < \eta^\tau < 4.5). \quad (6.15)$$

As only the the visible products of the tau are measured, and not the taus themselves, an acceptance factor, $\mathcal{A}$, is required to convert from the kinematic requirements placed on the visible tau decay products to the measurement fiducial. In each of the five analysis channels, p_T requirements of 20 GeV and 5 GeV are applied to the first and second decay products respectively, with the visible mass of the pair required to be greater than 20 GeV. In the $\tau_\mu\tau_\mu$ channel, the visible mass region 80 - 100 GeV

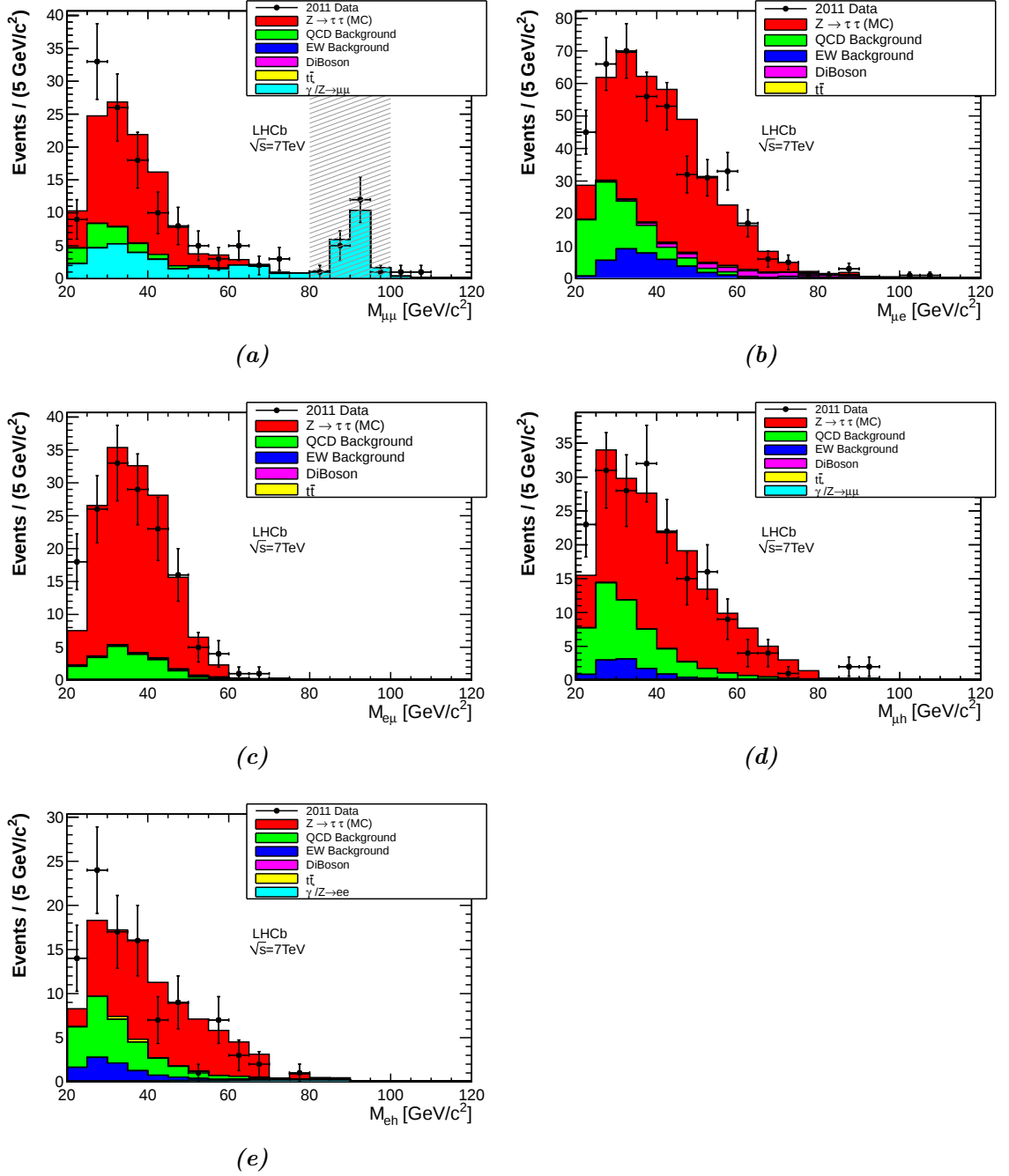


Figure 6.16: The visible mass distributions for the (a) $\tau_\mu\tau_\mu$, (b) $\tau_\mu\tau_e$, (c) $\tau_e\tau_\mu$, (d) $\tau_\mu\tau_h$, and (e) $\tau_e\tau_h$ final states.

	Acceptance
$\tau_\mu\tau_\mu$	0.405 ± 0.006
$\tau_\mu\tau_e$	0.248 ± 0.004
$\tau_e\tau_\mu$	0.152 ± 0.002
$\tau_\mu\tau_h$	0.182 ± 0.002
$\tau_e\tau_h$	0.180 ± 0.002

Table 6.5: The acceptance factors calculated for the five $Z \rightarrow \tau\tau$ final states using Pythia, Herwig and Powheg. The $\tau_\mu\tau_\mu$ channel has the largest factor as either muon can act as the leading decay product, while the $\tau_e\tau_\mu$ channel has the lowest acceptance due to the additional p_T cut on the muon.

is excluded, and in the $\tau_e\tau_\mu$ channel, the p_T of the secondary decay product must be below 20 GeV. The acceptance factor is calculated using simulation, and dividing the number of events generated which satisfy the kinematic requirements on the tau decay products, by the number where the taus fall within the quoted fiducial. This corrects back to the Born level cross-section, before final state radiation, to facilitate comparisons with fixed order theoretical predictions.

To calculate the acceptance factors, two different Monte Carlo Generators are used, Pythia 8.5.1 and Herwig 2.5.0, both of which use fully spin correlated tau decays and perform LO calculations, while Herwig additionally produces an NLO calculation through the use of the Powheg method [127]. These generators are chosen as they use different models for the treatment of initial and final state radiation and tau decays, and perform the calculations using different PDF sets; all of which contribute to the uncertainty on the acceptance factor. An acceptance factor is calculated for each generated sample, and the midpoint between the maximum and minimum values is taken as the overall acceptance factor, with half the maximum difference of the three results taken as a systematic. The factors and their associated uncertainties are given in Table 6.5.

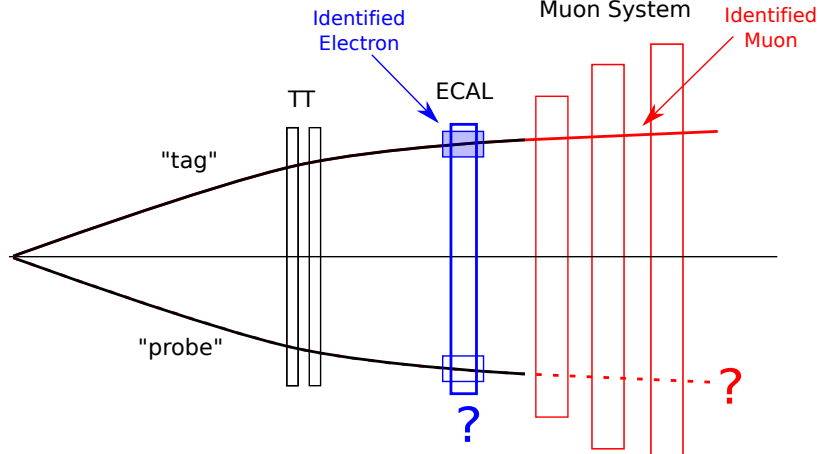


Figure 6.17: Illustration of the tag-and-probe method for calculating the identification efficiency. The tag is an identified muon or electron, while the probe is a long track.

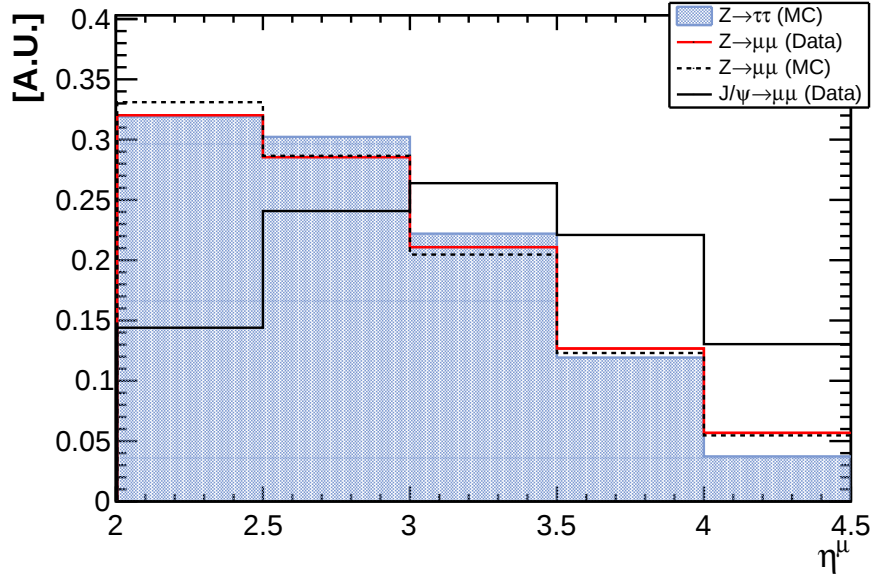


Figure 6.18: The pseudorapidity distributions for the $Z \rightarrow \tau\tau$ (simulation), $J/\psi \rightarrow \mu\mu$ (data) and $Z \rightarrow \mu\mu$ (data and simulation) events.

6.6.5 Reconstruction Efficiency

The reconstruction efficiency, ε_{rec} , refers to the efficiency for reconstructing signal and background events, and can be broken down into the components

$$\varepsilon = \varepsilon_{GEC} \cdot \varepsilon_{reso} \cdot \varepsilon_{trig} \cdot \varepsilon_{ID}^{(1)} \cdot \varepsilon_{ID}^{(2)} \cdot \varepsilon_{track}^{(1)} \cdot \varepsilon_{track}^{(2)} \quad (6.16)$$

where the global event cut efficiency, ε_{GEC} , is the efficiency for the event to pass specific global event requirements placed on the event at the trigger stage, the resolution efficiency, ε_{reso} , is the efficiency for true particles in the kinematic range of the measurement to be reconstructed in that range, and the trigger efficiency, representing the efficiency of triggering on the final state particles, is given by ε_{trig} . The efficiency for reconstructing the particles as long tracks, known as the track finding efficiency, is given by $\varepsilon_{track}^{(1)}$ and $\varepsilon_{track}^{(2)}$ for the tracks corresponding to the first and second decay products. When the long tracks have been reconstructed, the efficiencies for identifying these tracks as a given particle type are known as the identification efficiencies, given by $\varepsilon_{ID}^{(1)}$ and $\varepsilon_{ID}^{(2)}$.

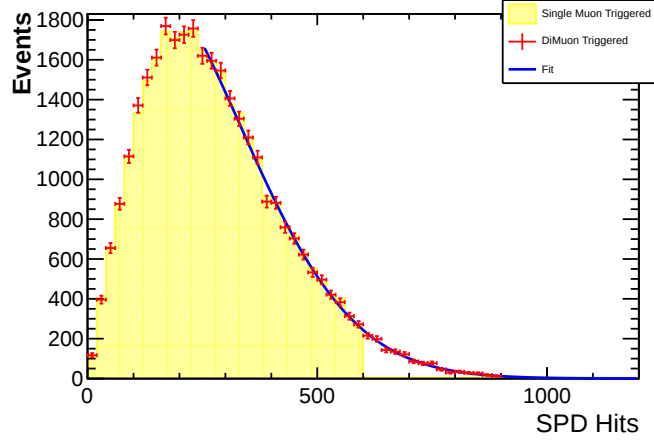
Where possible, the reconstruction efficiencies are calculated using data-driven methods, in particular using a tag-and-probe method. A tag-and-probe method, illustrated in Figure 6.17, is a data-driven method used to determine reconstruction efficiencies. Well known mass resonances can be used to identify di-muon or di-electron final states by selecting pairs of oppositely signed tracks with an invariant mass in the region of the resonance where one of the tracks (the “tag”) is required to have triggered the event and pass tight particle identification requirements. The second track (the “probe”) then provides an unbiased sample which can be used to measure various reconstruction efficiencies. The reconstruction efficiencies typically show dependencies on the momentum of the particle and its flight direction, as different regions of the detector may have different efficiencies and so are evaluated as a function of however many variables they show dependence on.

In this analysis the Z and J/ψ mass resonances are used to probe the high and low momentum regions respectively. The reconstruction efficiency is corrected on an event by event basis, where the efficiency is determined independently for each event, and the event weighted as $1/\varepsilon_{rec}$. In cases where the sample used to calculate the efficiency has the same event topology as the signal, any angular dependencies are

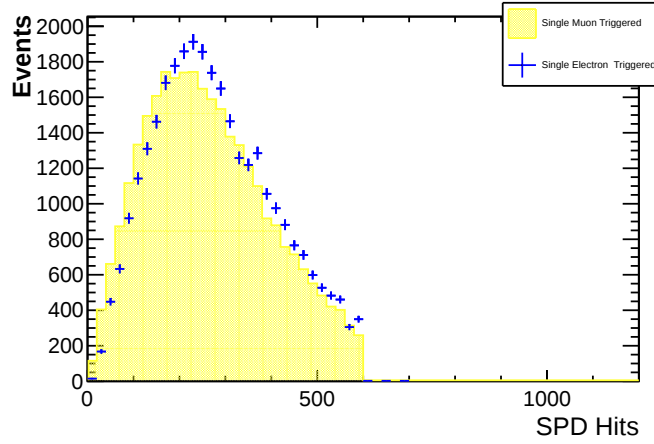
accounted for in the overall distribution. While the $Z \rightarrow \mu\mu$ and $Z \rightarrow \tau\tau$ angular distributions are almost identical, the $J/\psi \rightarrow \mu\mu$ distributions have significantly different shapes. For $Z \rightarrow \tau\tau$ signal events where $Z \rightarrow \ell\ell$ events are used to determine the efficiencies, the efficiencies need only be evaluated as a function of momentum. In J/ψ events, which have a significantly different event shape to $Z \rightarrow \tau\tau$ events, the only significant dependence observed is as a function of momentum and pseudorapidity, and so the efficiencies are evaluated with respect to these two variables. The pseudorapidity distributions for simulated $Z \rightarrow \tau\tau$ events, $J/\psi \rightarrow \mu\mu$ events from data, and $Z \rightarrow \mu\mu$ events from data and simulation are shown in Figure 6.18. The $Z \rightarrow \mu\mu$ and $J/\psi \rightarrow \mu\mu$ samples can be combined to produce efficiencies spanning the whole momentum range by reweighting the J/ψ efficiencies in pseudorapidity to match the Z distribution. Agreement in any overlap bins acts as a cross-check of the method.

Global Event Cut Efficiency

At L0 level in the trigger, certain global event cuts are performed in order to reject events which will require a large processing time. Specifically, for the L0Muon and L0Electron trigger, the number of SPD clusters in the event is required to be less than 600. The efficiency of this requirement can be evaluated by using Z events triggered using di-muon triggers at L0 and Hlt Level, as these triggers have a looser cut of 900 on the SPD multiplicity. This still rejects a very small fraction ($\sim 0.2\%$) of events above 900, however, as the tail of the distribution is observed to follow the shape of a Γ -function, the tail is fitted to this shape to retrieve the number of events with a multiplicity greater than 900. The SPD multiplicity in di-muon triggered events fit to a Γ -function is illustrated in Figure 6.19a. The efficiency is then given by the number of di-muon triggered events with a multiplicity less than 600 divided by the total number of di-muon triggered events plus the integral of the fit above 900. The SPD multiplicity for single muon and electron triggered $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ events is shown in Figure 6.19b, and the SPD multiplicity of the $Z \rightarrow ee$ events is observed to be shifted upwards by 20, due to early showering of the electrons, and so the efficiency for the electron trigger is evaluated with the di-muon triggered $Z \rightarrow \mu\mu$ events shifted by this amount. The efficiencies are evaluated to be 0.960 ± 0.001 for



(a)



(b)

Figure 6.19: The SPD Multiplicity for (a) single and di-muon triggered $Z \rightarrow \mu\mu$ events, where the distribution is fitted with a Γ -Function to evaluate the number of events with a multiplicity greater than 900 and (b) single muon and electron triggered $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ events, where the $Z \rightarrow ee$ distribution is observed to be shifted to the left of the $Z \rightarrow \mu\mu$ distribution by 20 SPD hits.

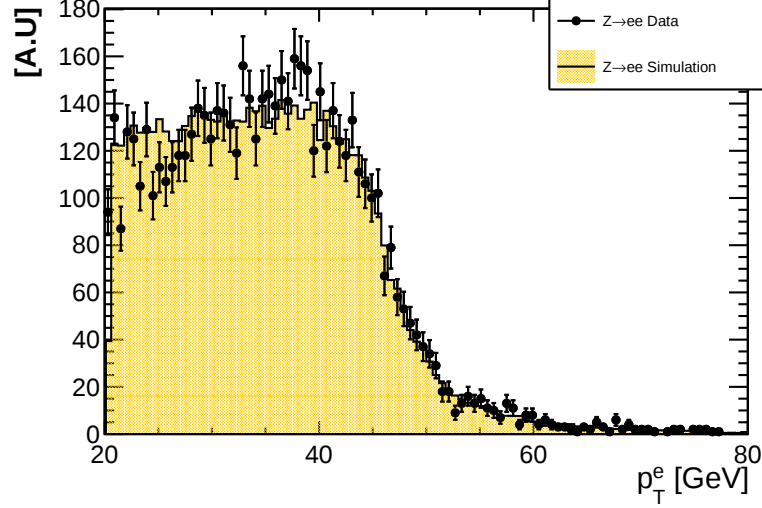


Figure 6.20: The transverse momentum distribution for electrons from $Z \rightarrow ee$ events in data and scaled simulation.

the muon trigger and 0.955 ± 0.001 for the electron trigger.

Resolution Efficiency

The resolution efficiency is a correction due to the finite resolution of the detector, where the reconstructed momentum and direction of a particle differ from their true values due to resolution effects. When a measurement is quoted within a kinematic region, migrations can occur, where particles produced within the kinematic region are reconstructed with values outside of the requirements, and particles produced outside of the kinematic region are reconstructed with values inside the kinematic region. Where the net migration is not consistent with zero, the effect of the migrations must be taken into account in order to correctly identify the number of events in the kinematic region. The resolution efficiency for muons and hadrons is expected to be consistent with one, corresponding to a negligible net migration, but is expected to be low for final states involving electrons due to the saturation of the ECAL, which prevents full recovery of Bremsstrahlung photons and results in the momentum spectrum of electrons being smeared to lower values than their true values.

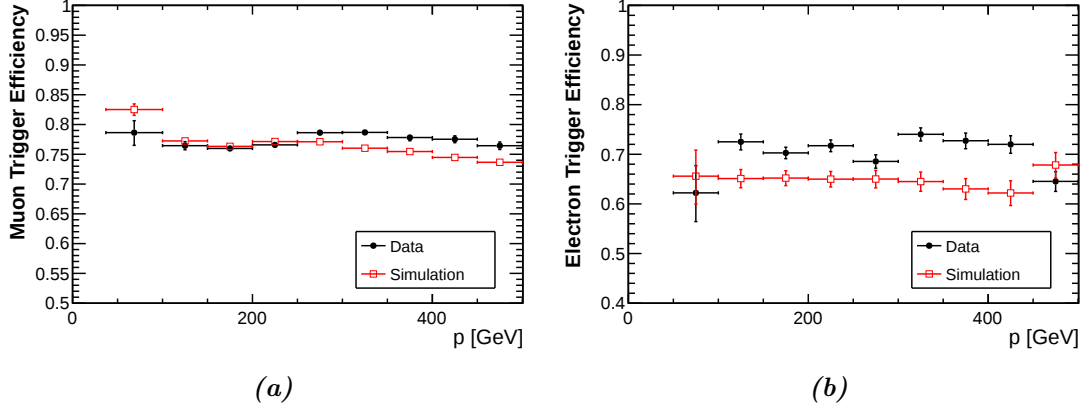


Figure 6.21: The trigger efficiencies obtained for (a) muon and (b) electron plotted as a function of momentum and compared with simulation.

The efficiency is determined using simulation and relies on an accurate description of the reconstruction of electrons by the detector. A comparison of the transverse momentum spectrum of electrons in $Z \rightarrow ee$ events in data and simulation is shown in Figure 6.20, where the simulation was determined to be shifted by a factor of 1.02 ± 0.01 with respect to the data and scaled by this amount. An uncertainty reflecting how well the simulation models the electron momentum was determined by re-calculating the efficiency with the scaling factor varied within its uncertainty, and the difference between the calculated efficiencies taken as the uncertainty. This uncertainty is then added in quadrature with the statistical uncertainty of the determination to produce a systematic uncertainty. The efficiency is verified to be consistent with one within statistical uncertainty for the $\tau_\mu\tau_\mu$ and $\tau_\mu\tau_h$ analysis channels, and determined to be $94.3 \pm 0.7\%$, $64.9 \pm 1.9\%$, and $63.6 \pm 1.1\%$ for the $\tau_\mu\tau_e$, $\tau_e\tau_\mu$ and $\tau_e\tau_h$ analysis channels respectively.

Muon Trigger Efficiency

The Muon Trigger Efficiency is defined as the efficiency of any true muon passing the muon trigger requirements at L0 and Hlt level. This can be evaluated using a tag-and-probe method on $Z \rightarrow \mu\mu$ events. Pairs of tracks are selected which satisfy the muon identification requirements, have transverse momenta greater than 10 GeV

and are within the mass range $60 < M_{\mu\mu} < 120$ GeV. The efficiency is then given by the number of events triggered on the tag and the probe divided by the number triggered on the tag. The efficiency is calculated in ten bins of momentum between 0 and 500 GeV, and the results are shown in shown in Figure 6.21a and compared to simulation. The systematic error on the determination of the efficiencies is taken to be the statistical error on each bin, and varies between 0.7 and 0.9% as a function of momentum.

Electron Trigger Efficiency

The electron trigger efficiency is calculated in a similar way to the muon trigger efficiency, this time using a tag-and-probe on $Z \rightarrow ee$ events. Due to higher backgrounds and the smeared nature of the $Z \rightarrow ee$ mass peak, the mass range is chosen to be $70 < M_{ee} < 120$ GeV, and the transverse momenta are required to be greater than 20 GeV. The tracks are also required to be isolated, satisfying $I_{pT} < 2$ GeV. No bias is observed in simulation from the isolation requirement and so the efficiency is again calculated in ten bins of momentum from 0 to 500 GeV. The distribution is shown in Figure 6.21b, and the systematic error of the determination is taken to be the statistical uncertainty on each bin, and varies between 2 and 3%.

Muon Identification Efficiency

To cover the full momentum range of the $Z \rightarrow \tau\tau$ signal, the muon identification efficiency is calculated using a tag-and-probe method on $Z \rightarrow \mu\mu$ and $J/\psi \rightarrow \mu\mu$ events. In the case of $Z \rightarrow \mu\mu$ events, the tag is an identified muon passing the trigger requirements, and the highest p_T oppositely signed long track in the event acts as the probe. To access the low momentum regions, $J/\psi \rightarrow \mu\mu$ events are selected where again an identified and triggered muon acts as the tag and a long track acts as the probe. The requirements placed on both samples are given in Table 6.6. As a large background is admitted to the J/ψ sample, the mass spectrum is fitted to a signal crystal ball plus linear background shape, and the number of signal events taken as the integral of the crystal ball in the region of the J/ψ mass peak. The fitting procedure and error determination is described in more detail in Appendix A.1. The efficiency is given by the number of signal

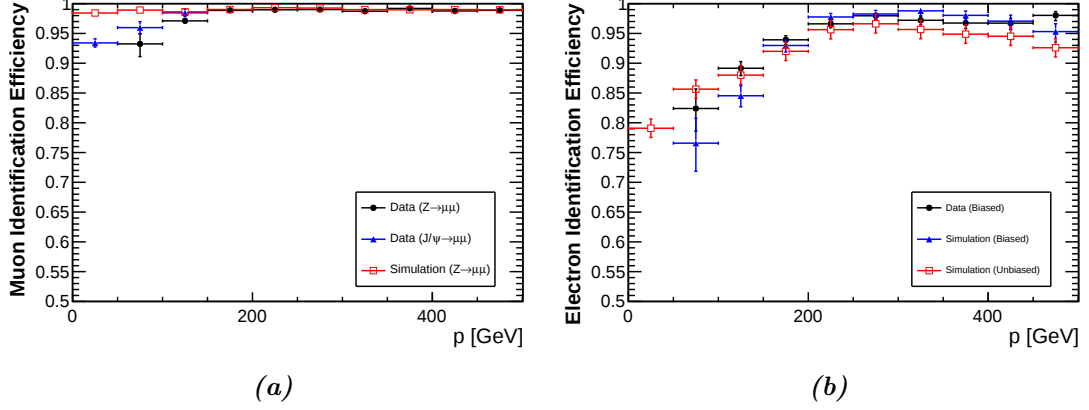


Figure 6.22: The identification efficiencies obtained for (a) muon and (b) electron plotted as a function of momentum and compared with simulation. In the case of the electron, as good agreement is observed between data and simulation in the biased distributions, the efficiency is taken from the red unbiased Monte Carlo distribution.

	$J/\psi \rightarrow \mu\mu$	$Z \rightarrow \mu\mu$
Tag	$p_T > 1.5 \text{ GeV}, P > 6 \text{ GeV}$ $IP > 25$	$p_T > 20 \text{ GeV}$ $I_{pT} < 2 \text{ GeV}$
Probe	$p_T > 0.8 \text{ GeV}, P > 3 \text{ GeV}$ $IP > 10$	$p_T > 20 \text{ GeV}$ $I_{pT} < 2 \text{ GeV}$
Event	Vertex $\chi^2 < 8 \text{ GeV}$ $3.0 < M_{\mu\mu} < 3.2$ $IP_{\mu\mu} > 3.0$	$60 < M_{\mu\mu} < 120$

Table 6.6: The requirements put in place on the $J/\psi \rightarrow \mu\mu$ and $Z \rightarrow \mu\mu$ samples used to calculate the muon identification efficiency. IP refers to the impact parameter significance of a single track with respect to the primary vertex, while $IP_{\mu\mu}$ refers to the impact parameter significance of the di-muon system.

events in the distribution after the matching requirement divided by the number of signal events in the distribution before the matching requirement, as determined from the fit. For both Z and J/ψ samples, the statistical uncertainty is taken as the systematic uncertainty for all bins, with a further uncertainty applied when using the J/ψ sample to account for the uncertainty of the fit. Good agreement is observed in the overlap bin between the efficiencies obtained from the two samples. Again, the efficiency is corrected in ten bins of momentum from 0 to 500 GeV with the distribution given in Figure 6.22a. The uncertainty on the efficiencies calculated using the J/ψ sample are approximately 1% , while the uncertainty on those calculated with the Z sample vary between 0.2 and 0.3%.

Electron Identification Efficiency

The electron identification efficiency is calculated using a tag-and-probe method on $Z \rightarrow ee$ events in a similar fashion to the muon identification efficiency. The same p_T and I_{p_T} requirements are put in place, with the tracks additionally required to have an azimuthal separation, $|\Delta\phi|$, greater than 3.0 radians to further reduce the background. In this case, unlike the muon identification procedure, a bias is observed from the probe isolation requirement. The efficiency obtained in biased data (black points), biased simulation (blue points) and unbiased simulation (red points) as a function of momentum is shown in Figure 6.22b. When electrons are required to be isolated, in the biased distribution, they are identified with a higher efficiency than in the unbiased distribution, where no event selection requirements are applied. In addition, due to the differing identification requirements for high and low momentum electrons at LHCb, no suitable $J/\psi \rightarrow ee$ sample exists to determine the efficiency of identifying low momentum electrons. By comparing the biased data and biased simulation distributions, the simulation is observed to reflect the data well, and so the efficiency is determined from the unbiased simulation distribution. The efficiency is corrected in ten bins of momentum from 0 to 500 GeV and the systematic uncertainty is taken as the difference between the biased data and biased simulation shapes, varying between 1.5 and 2% as a function of momentum.

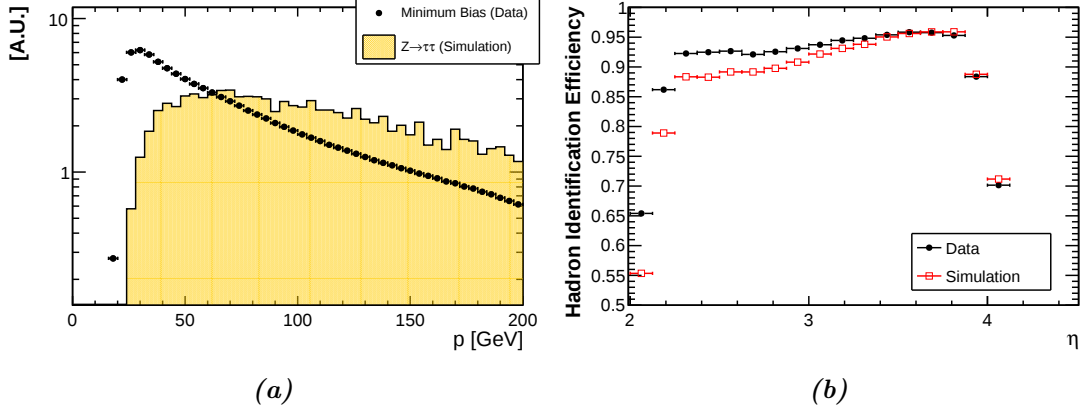


Figure 6.23: (a) The momentum spectrum for hadrons in minimum bias data and $Z \rightarrow \tau\tau$ simulation and (b) The efficiency of identifying hadrons as a function of pseudorapidity in data and simulation.

Hadron Identification Efficiency

To identify hadrons, a sample of minimum-bias events is used, which is observed from simulation to consist mostly of hadrons. All tracks in the event with a transverse momentum exceeding 5 GeV are selected, and the momentum distribution for hadrons in minimum bias events (data) and $Z \rightarrow \tau\tau$ events (simulation) is shown in Figure 6.23a. Hadrons in minimum bias events have a softer spectrum than $Z \rightarrow \tau\tau$ events, where the distribution is spread to larger values. The efficiency is evaluated by observing the number which pass the hadron identification requirements and shown as a function of pseudorapidity in Figure 6.23. The efficiency drops to very low values in the edge of the detector due to the HCAL acceptance. No significant momentum dependence is observed, and so the efficiency correction is applied as a function of pseudorapidity, and to avoid large systematic uncertainties caused by bin-to-bin migrations in these edge regions, only hadrons in the region $2.25 < \eta^\pi < 3.75$ are considered. The uncertainty is taken to be the statistical uncertainty, and varies between 0.1 and 0.2% as a function of pseudorapidity.

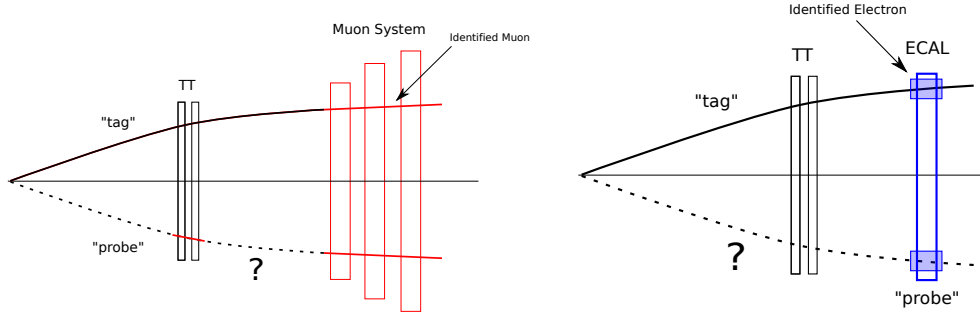


Figure 6.24: Illustration of the tag-and-probe method for calculating the tracking efficiency. The tag is an identified muon or electron, while the probe is either a Muon-TT track or an ECAL cluster.

	$J/\psi \rightarrow \mu\mu$	$Z \rightarrow \mu\mu$
Tag	$p_T > 1.3 \text{ GeV}, IP > 0.5$	$p_T > 20 \text{ GeV}$
Probe	-	$p_T > 20 \text{ GeV}$
Event	$p_T^{\mu\mu} > 1 \text{ GeV}, \text{Vertex } \chi^2 < 5$ $3.0 < M_{\mu\mu} < 3.2 \text{ GeV}$ $IP_{\mu\mu} > 3.0$	$\text{Vertex } \chi^2 < 5$ $60 < M_{\mu\mu} < 120 \text{ GeV}$ $ \Delta\phi > 1.0 \text{ rads}$

Table 6.7: The requirements put in place on the $J/\psi \rightarrow \mu\mu$ and $Z \rightarrow \mu\mu$ samples used to calculate the muon track finding efficiency

Muon Track Finding Efficiency

The muon track finding efficiency refers to the efficiency of reconstructing a long track satisfying the track quality requirement, $P(\chi^2) > 0.001$ in the pseudorapidity region $2 < \eta^\mu < 4.5$, when a muon is produced in that region. This efficiency is evaluated by repeating the analysis performed in Reference [128] for 2011 data. A tag-and-probe method, illustrated in Figure 6.24, is performed on both $J/\psi \rightarrow \mu\mu$ and $Z \rightarrow \mu\mu$ events, where again the tag is an identified muon satisfying the trigger requirements. The procedure for reconstructing long tracks was outlined in Section 4.1, and the pattern recognition procedure does not include information from the muon chambers or the TT sub detector. An unbiased measure of the track finding efficiency can thus be performed by combining information in the muon chambers with hits from the TT to produce a Muon-TT track which acts as a probe. The requirements in place for both samples are tabulated in Table 6.7. In both cases the reduced χ^2 of the vertex of the two tracks is required to be less than 5 to ensure both tracks originate from a common vertex. For the $Z \rightarrow \mu\mu$ sample, they are also required to satisfy $|\Delta\phi| > 1.0$ radians, to ensure that the same muon cannot act as both the tag and probe in the case of the charge misidentification of the Muon-TT track. The Z mass peak can be seen in Figure 6.25a, where the broadening of the peak is due to the lower momentum resolution of the Muon-TT track. The Muon-TT track is then judged to be matched to a long track in the event if there exists a track which shares more than 40% of its hits with the Muon-TT track. In the case of $J/\psi \rightarrow \mu\mu$ events, the same fitting procedure is performed as for the muon identification efficiency and both samples are merged to obtain an efficiency correction in ten bins of momentum from 0 to 500 GeV. This distribution is shown in Figure 6.25b. The systematic uncertainty varies between 0.5 and 1% as a function of momentum.

Hadron Track Finding Efficiency

The efficiency for reconstructing a long track corresponding to a charged hadron is expected to be lower than that of a muon due to nuclear interactions which result in the hadron interacting with the detector material before it reaches the last tracking station [129]. This effect is evaluated by comparing the tracking efficiency as a function

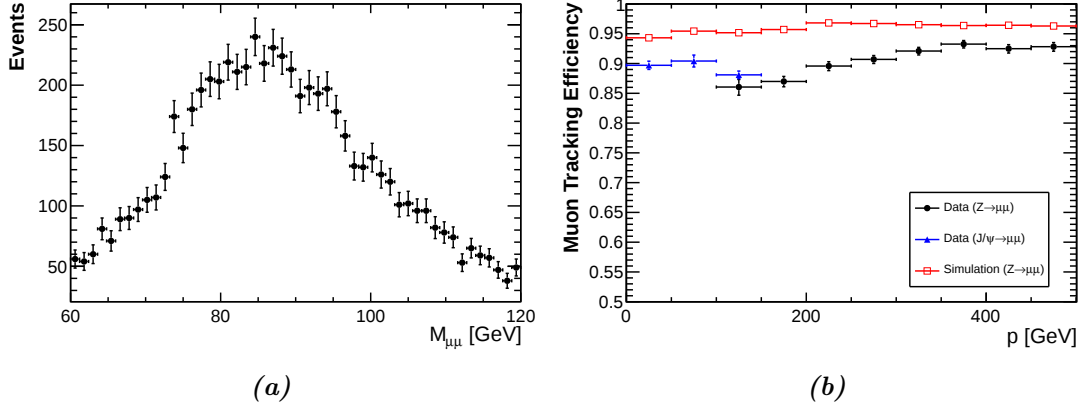


Figure 6.25: (a) The Z mass peak obtained by combining a long track identified as a muon and a Muon-TT track and (b) the track finding efficiency as a function of momentum.

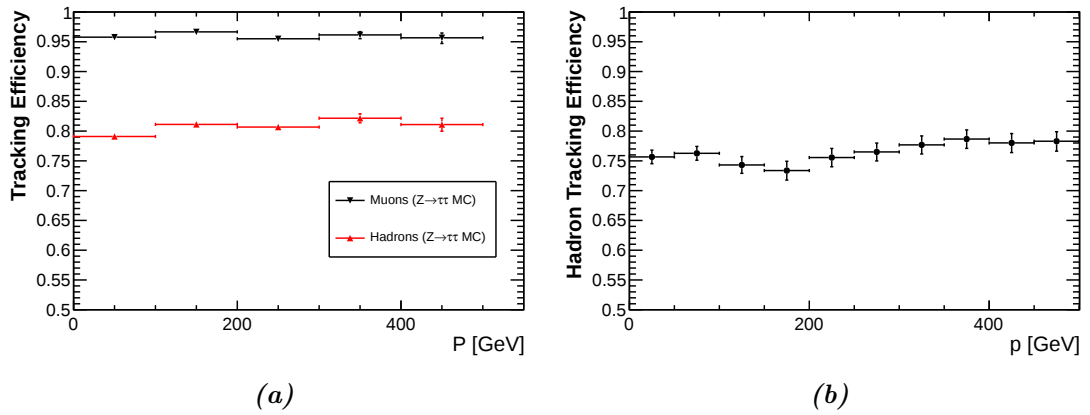


Figure 6.26: (a) The track finding efficiency as a function of momentum for muons and hadrons in simulated $Z \rightarrow \tau\tau$ events and (b) the tracking efficiency for hadrons obtained from data by correcting the muon tracking efficiency for nuclear interactions.

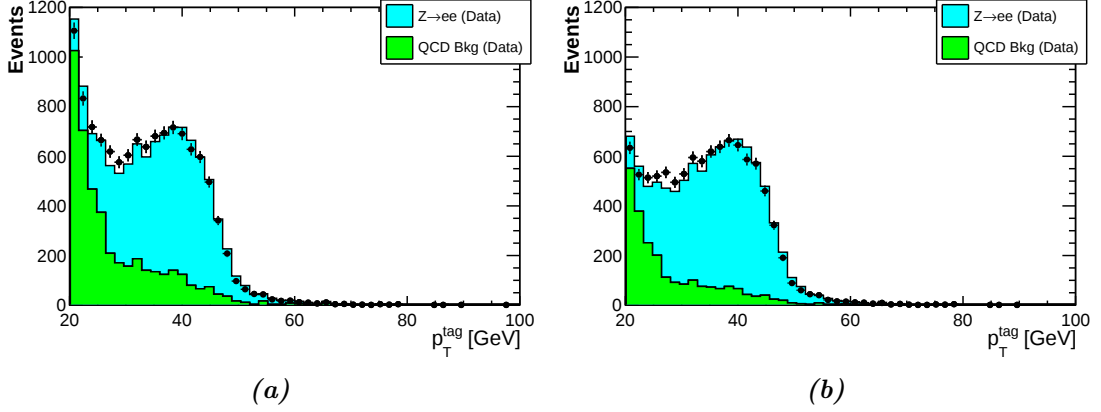


Figure 6.27: The p_T spectrum of the tag particle fitted to signal and background templates for the sample used to determine the electron tracking efficiency (a) before and (b) after the track matching requirement.

of momentum for muons and hadrons in simulated $Z \rightarrow \tau\tau$ events in Figure 6.26. The tracking efficiency for hadrons shown to be a factor of 16% lower than for muons, with no significant momentum dependence. This factor is applied as a correction to the muon tracking efficiency obtained from data in order to obtain an estimate of the hadron tracking efficiency, as shown in Figure 6.26b. As recommended in Reference [128], an uncertainty of 1.5% on the correction factor, reflecting the 10% uncertainty on the detector material budget, is added in quadrature with the statistical uncertainty of the efficiency determination. The total systematic uncertainty varies between 1.6 and 1.8% as a function of momentum.

Electron Track Finding Efficiency

The electron track finding efficiency is determined by using a tag-and-probe method on the $Z \rightarrow ee$ sample as illustrated in Figure 6.24. In this case, the tag is a long track identified as an electron which has triggered the event, while the probe is a high energy ECAL cluster. As a number of different backgrounds can produce an ECAL cluster, this method requires a number of requirements to ensure a high purity of $Z \rightarrow ee$ events. The p_T of the tag electron is required to be greater than 20 GeV and the transverse energy, E_T , of the probe cluster is required to be greater

than 5 GeV, with the transverse energy defined as

$$E_T = \frac{E}{\cosh \eta} \quad (6.17)$$

where E is the energy deposited in the calorimeter and η is the pseudorapidity of the cluster, where it is assumed to originate from the primary vertex. The tag is required to be isolated from charged tracks, satisfying $I_{p_T} < 2\text{GeV}$ and the energy deposited in the HCAL divided by that deposited in the ECAL in an $\eta - \phi$ cone of 0.5 around the probe cluster is required to be less than 0.5. The ECAL deposits of the tag and probe clusters are also required to be balanced, satisfying $|E_{\text{ECAL}}^{\text{tag}} - E_{\text{ECAL}}^{\text{probe}}|/E_{\text{ECAL}}^{\text{tag}} < 0.2$. The tag and probe clusters must also have an angular separation of 3.0 radians in the transverse plane. As a result of inefficiencies in the matching of a track to an ECAL cluster, the track matching requirement simply requires the presence of a high momentum, oppositely signed track with $|\Delta\phi| > 2.4$ with respect to the tag particle. In data, even after all these requirements are put in place, a large number of background events are admitted. As the efficiency is calculated by comparing the number of signal events before and after the track matching requirement, the signal contribution to both samples is estimated using a template fit to the p_T spectrum of the tag particle. A signal template is taken from identified $Z \rightarrow ee$ events and a QCD background template taken from events where the probe cluster is matched to a track of the same charge as the probe. In Figure 6.27, the template shapes are fitted to the distribution, again using the method in Reference [126], before and after the track matching requirement. As no momentum information is available from the probe cluster, an efficiency correction cannot be applied as a function of momentum as in the case of the muon tracking efficiency. However, good agreement is observed in the efficiencies obtained in simulated $Z \rightarrow \tau\tau$ and $Z \rightarrow ee$ events, determined to be 0.857 ± 0.002 and 0.852 ± 0.049 respectively, and so an overall efficiency number is deemed sufficient. A systematic uncertainty on the efficiency is calculated by adding the statistical uncertainty of the efficiency determination in quadrature with the fit uncertainty. The value obtained is 0.83 ± 0.03 .

6.6.6 Selection Efficiency

The efficiency for selecting the events can be represented as ε_{sel} and parameterised as

$$\varepsilon_{sel} = \varepsilon_{I_{pT}} \cdot \varepsilon_{IPS} \cdot \varepsilon_{|\Delta\phi|} \cdot \varepsilon_{A_{pT}} \quad (6.18)$$

where the factors correspond to the efficiency of the event selection requirements, namely those on the azimuthal angle separation, $\varepsilon_{|\Delta\phi|}$, the p_T asymmetry, $\varepsilon_{A_{pT}}$, the impact parameter sum, ε_{IPS} , and the isolation of the candidates, $\varepsilon_{I_{pT}}$. The efficiency of the $|\Delta\phi|$ and A_{pT} requirements are directly related to the kinematics of the hard matrix element and, as described in Sections 6.5.5 and 6.5.6, are well described by simulation. The efficiencies of these two requirements are taken directly from simulation, and a systematic uncertainty is taken to be the difference between the efficiency of the requirement as applied to $Z \rightarrow \mu\mu$ events in data and simulation. The uncertainty on the efficiency of the $\Delta\phi$ requirement contributes an uncertainty of between 0.6 and 1.1% to each channel, while the uncertainty on the efficiency of the A_{pT} requirement contributes an uncertainty of 2% in the $\tau_\mu\tau_\mu$ channel. As described in Section 6.5.4, the IPS distribution, which relies on a faithful modelling of the interaction of the tracks with the detector elements, is found to be a factor of 1.12 ± 0.01 worse in data compared to simulation. As a result, the efficiency of the IPS requirement is obtained by applying the requirement to the scaled $Z \rightarrow \tau\tau$ simulated distributions. A systematic is determined by varying the scale factor within its uncertainty, and re-evaluating the efficiency of the requirement, with the difference taken as the systematic uncertainty. This uncertainty varies between 1.9 and 2.9% depending on the channel in which the requirement is applied. The isolation variable, I_{pT} is largely dependent on underlying event activity and is not well described in simulation, as was described in Section 6.5.3. As a result, the efficiency of this requirement is taken from simulation for each analysis channel, but calibrated to data by multiplying the ratio of the efficiencies obtained by applying the requirement to $Z \rightarrow \mu\mu$ data and $Z \rightarrow \tau_\mu\tau_\mu$ simulation. The systematic uncertainty is evaluated by comparing the efficiency of the requirement as applied to $Z \rightarrow \mu\mu$ and $Z \rightarrow \tau_\mu\tau_\mu$ events in simulation, and varies between 1.8 and 3.2% depending on the channel in question. The selection efficiencies and their uncertainties are tabulated in Table 6.8. No significant correlation between the selection requirements

	$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$
$\varepsilon_{I_{pT}}$	0.660 ± 0.012	0.613 ± 0.012	0.623 ± 0.020	0.413 ± 0.007	0.386 ± 0.011
$\varepsilon_{\Delta\phi}$	0.848 ± 0.009	0.850 ± 0.009	0.827 ± 0.015	0.845 ± 0.005	0.838 ± 0.008
ε_{IPS}	0.414 ± 0.011	-	-	0.387 ± 0.007	0.379 ± 0.011
$\varepsilon_{A_{pT}}$	0.597 ± 0.012	-	-	-	-
ε_{sel}	0.138 ± 0.006	0.522 ± 0.010	0.516 ± 0.013	0.135 ± 0.004	0.123 ± 0.005

Table 6.8: A table of the selection efficiencies obtained for each $Z \rightarrow \tau\tau$ final state is observed in simulation.

6.6.7 Measurement Uncertainties

The measurement uncertainties for each analysis channel are split into the statistical, systematic and luminosity uncertainties and summarised for each source and channel in Table 6.9. The total systematic uncertainties are evaluated as the sum in quadrature of all individual contributions. The uncertainties are expressed as a percentage of the cross-section evaluated for each analysis channel in Table 6.9. The precision of each analysis channel is statistically limited, and in addition, many of the systematic uncertainties, in particular the background and efficiency determinations, are statistical in nature and so the measurement precision will significantly improve with a larger dataset.

6.6.8 Results

The final results obtained constitute a measure of the $Z \rightarrow \tau\tau$ cross-section at LHCb in the measurement fiducial region in five different di-tau analysis channels corresponding to four distinct final states. The results obtained are

$$\sigma_{Z \rightarrow \tau\tau} = 77.4 \pm 10.4 \pm 8.6 \pm 2.7 \text{ pb } (\tau_\mu\tau_\mu)$$

$$\sigma_{Z \rightarrow \tau\tau} = 75.2 \pm 5.4 \pm 4.1 \pm 2.6 \text{ pb } (\tau_\mu\tau_e)$$

$$\sigma_{Z \rightarrow \tau\tau} = 64.2 \pm 8.2 \pm 4.9 \pm 2.2 \text{ pb } (\tau_e\tau_\mu)$$

$$\sigma_{Z \rightarrow \tau\tau} = 68.3 \pm 7.0 \pm 2.6 \pm 2.4 \text{ pb } (\tau_\mu\tau_h)$$

$$\sigma_{Z \rightarrow \tau\tau} = 77.9 \pm 12.2 \pm 6.1 \pm 2.7 \text{ pb } (\tau_e\tau_h)$$

$\Delta\sigma_{Z\rightarrow\tau\tau}$ (%)						
		$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$
	N_{bkg}	10.03	1.75	3.44	0.61	1.32
	$\mathcal{A}$	1.48	1.61	1.32	1.10	1.11
	BR	0.46	0.32	0.32	0.32	0.33
ε_{sel}	$\varepsilon_{I_{p_T}}$	1.79	1.91	3.19	1.65	2.75
	ε_{IPS}	2.70	-	-	1.92	2.85
	$\varepsilon_{ \Delta\phi }$	1.08	1.03	1.86	0.60	0.97
	$\varepsilon_{A_{p_T}}$	2.03	-	-	-	-
ε_{rec}	ε_{GEC}	0.10	0.10	0.10	0.10	0.10
	ε_{reso}	-	1.04	2.89	-	1.91
	ε_{trig}	0.88	0.71	2.29	0.72	4.30
	ε_{id1}	0.38	0.28	1.72	0.29	1.73
	ε_{id2}	0.78	0.18	0.56	0.03	0.09
	ε_{track1}	0.71	0.74	3.67	0.79	3.67
	ε_{track2}	0.34	3.67	0.61	1.76	1.68
Statistical		13.44	7.18	12.77	10.24	15.66
Systematic		11.13	5.41	7.56	3.88	7.88
Luminosity		3.50	3.50	3.50	3.50	3.50

Table 6.9: Table of the uncertainties in each of the five tau analysis channels expressed as a percentage of the cross-section in each channel.

where the first uncertainty is statistical, the second is systematic and the third is due to the luminosity determination. The measurements in the five different final states represent independent measurements of the same physical observable, and as such can be combined to produce one final result for the $Z \rightarrow \tau\tau$ cross-section at LHCb. This is done using the Best Linear Unbiased Estimator (BLUE) [130] method which is described in more detail in Appendix A.2. Covariance matrices are constructed for each measurement, with each of the sources of systematic uncertainty taken into account as well as any correlations between them which may exist. Any shared reconstruction or selection efficiencies are assumed to be correlated, the statistical errors to be uncorrelated as independent datasets are used, while the luminosities are assumed to be fully correlated. More details are also given in Appendix A.2. The

final result is

$$\sigma_{Z \rightarrow \tau\tau} = 71.4 \pm 3.5 \pm 2.8 \pm 2.5 \text{ pb}$$

with a χ^2 per degree of freedom of 0.43. The individual measurements and their combined result are compared to theoretical predictions produced with the package DYNNLO [131] using the PDF set MSTW08 [132] and previous measurements made by LHCb in the $Z \rightarrow ee$ [108] and $Z \rightarrow \mu\mu$ [109] channels in Figure 6.28. The overall precision of 7.2% on the final measurement also compares favourably with the precision of 10.0% and 10.2% achieved by ATLAS and CMS in the same decay mode [101, 102]. Lepton universality is also tested by comparing the ratio of the $Z \rightarrow \tau\tau$ cross-section measurement to the $Z \rightarrow \mu\mu$ measurement. The ratio is determined to be

$$\frac{\sigma_{pp \rightarrow Z \rightarrow \tau\tau}}{\sigma_{pp \rightarrow Z \rightarrow \mu\mu}} = 0.93 \pm 0.09$$

All measurements are in agreement and consistent with lepton universality.

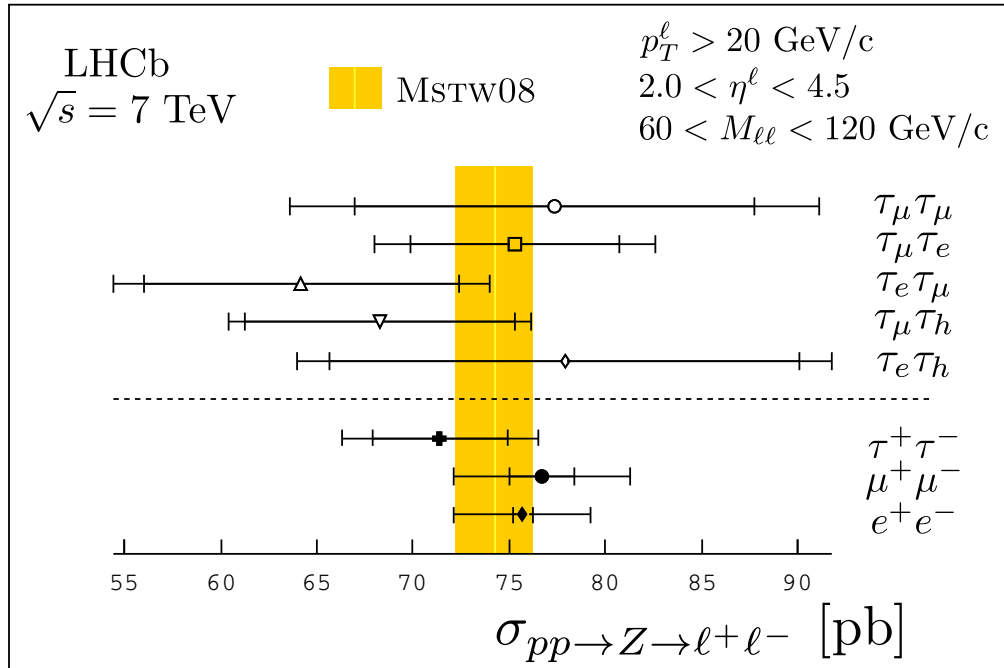


Figure 6.28: The $Z \rightarrow \tau\tau$ cross-section measurements and their combined result are compared to the $Z \rightarrow \mu\mu$ and $Z \rightarrow ee$ measurements made by LHCb with 2010 and 2011 data respectively, and with theoretical predictions. The results are in good agreement with each other and with the theoretical predictions.

CHAPTER 7

CONCLUSIONS

This thesis detailed a measurement of the cross-section times branching ratio for the production of Z bosons decaying to tau pairs using 1 fb^{-1} of data collected by the LHCb detector in 2011. The selection of $Z \rightarrow \tau\tau$ events exploited the lifetime of the tau leptons, which allowed decay products of taus to be identified as those with large impact parameters with respect to the primary vertex. The impact parameter was determined using information from the VELO subdetector, which precisely measures the trajectory of charged particles near the interaction region. To understand the precision with which the impact parameters were determined, resolution studies of the VELO were also performed.

Three independent VELO resolution studies were presented in Chapter 5. The hit position of a track in a VELO sensor is calculated by assuming a linear distribution of charge among the strips in which it deposits charge. The validity of this assumption was tested, and a method for modelling it more accurately presented. This was shown to give an improvement of up to 30% in hit resolution at low projected angles. The single hit resolution of the VELO sensors as a function of strip pitch was also measured using residual distributions. The best resolution, achieved at a pitch of $40 \mu\text{m}$ and a projected angle between 10° and 14° was determined to be $3 \mu\text{m}$. Finally the resolution of the impact parameter of tracks with respect to the primary vertex was calculated using a minimum bias sample as a function of inverse transverse momentum and found to be approximately $15 \mu\text{m}$ in the x and y

components.

A measurement of the cross-section times branching ratio for the $Z \rightarrow \tau\tau$ process was presented in Chapter 6. A measurement was performed in five different analysis channels corresponding to four different final states, $\tau_\mu\tau_\mu$, $\tau_\mu\tau_e$, $\tau_e\tau_\mu$, $\tau_\mu\tau_h$ and $\tau_e\tau_h$, where the subscripts indicate the visible product of the tau decay. The final state including a muon and an electron was split into two separate non-overlapping analysis channels based on the kinematic requirements imposed. Particle and event selection schemes were developed, and the background contributions and efficiencies measured using a combination of data-driven method and simulation. The measurements were combined to give an overall cross-section measurement of

$$\sigma_{Z \rightarrow \tau\tau} = 71.4 \pm 3.5 \pm 2.8 \pm 2.5 \text{ pb}$$

where the first uncertainty is statistical, the second is systematic, and the third is due to the luminosity determination. The measurement was compared to measurements made by LHCb in the $Z \rightarrow \mu\mu$ decay mode, as well as theoretical predictions and found to be in agreement. Currently, all measurements are statistically limited, and additionally many of the systematic uncertainties due to the determination of the efficiencies are statistical in nature. With a larger dataset, a more accurate measurement of the $Z \rightarrow \tau\tau$ cross-section at LHCb can be performed which can precisely test lepton universality and search for new physics processes at the LHC.

APPENDIX A

APPENDIX A

A.1 Efficiency determination with J/ψ events

In the determination of efficiencies using the tag-and-probe method, two different distributions are obtained, the *total* and *pass* distributions, where the events in the pass distribution are the subset of the events in the total distribution which have passed a specific requirement. For a pure sample, the efficiency is calculated as the number of events in the pass distribution divided by the number in the total distribution. Where a background is admitted, the background must be subtracted from the total and pass distributions to obtain the correct efficiency. As outlined in Sections 6.6.5 and 6.6.5, the low momentum muon identification and track finding efficiencies are determined using a tag-and-probe method applied to $J/\psi \rightarrow \mu\mu$ events, with a large background admitted. The number of signal events in each bin is determined by fitting the mass spectrum in the region of the J/ψ mass peak with

a crystal ball plus linear background fit, given by

$$f(x) = \underbrace{p_0 + p_1 x}_{\text{Linear Background}} + p_2 \underbrace{\begin{cases} e^{\frac{(x-p_3)^2}{2p_4^2}} & \text{if } \frac{x-p_3}{p_4} < -p_6 \\ \left(\frac{p_5}{|p_6|}\right)^{p_5} e^{-\frac{p_6^2}{2}} \left(\frac{p_5}{|p_6|} - |p_6| - \frac{x-p_3}{p_4}\right) & \text{if } \frac{x-p_3}{p_4} \geq -p_6 \end{cases}}_{\text{Crystal Ball Function}} \quad (\text{A.1})$$

where the parameters $\{p_i\}_{i=0-6}$ are determined by fitting the function $f(x)$ to the mass distribution in different bins of phase space. Values can be obtained from an iterative fitting procedure, with the distribution first fitted to a linear distribution, and then to a linear distribution plus a gaussian, and finally to the function $f(x)$. The fits are presented here, with the distribution used to determine the number of signal events in the J/ψ sample for the determination of the muon identification efficiency shown in Figures A.1 and A.2, and the distributions used to determine number of signal events in the J/ψ sample used for the tracking efficiency shown in Figures A.3 and A.4. In order to assign a systematic error associated with the fitting procedure to the efficiency determination, the fit is performed in an alternative way to obtain a second estimate of the number of signal events. The region of the mass peak is excised and a linear background function is fitted to the sideband regions of the mass distributions, and the number of signal events is defined as the total number of events minus the integral of the background function in the region of the mass peak. The efficiency in each bin is evaluated using the number of signal events obtained with both fits, and half the difference is taken as the systematic error.

A.2 BLUE Method

The **B**est **L**inear **U**nbiased **E**stimate method refers to a technique of combining measurements of a common observable into a single value with an associated uncertainty. Given a set y of N independent measurements of an observable, it combines them into an estimator, $\hat{y}$, as

$$\hat{y} = \sum_i^N \alpha_i y_i \quad (\text{A.2})$$

where α is a vector constrained to be unbiased by requiring

$$\sum_i^N \alpha_i = 1 \quad (\text{A.3})$$

The associated uncertainty for this estimator is given by

$$\sigma^2 = \alpha^T V \alpha = \sum_i^N \sum_j^N V_{ij} \alpha_i \alpha_j \quad (\text{A.4})$$

where V is the covariance matrix. The BLUE technique minimises this uncertainty subject to the constraint of Equation (A.3). Using the method of Lagrangian multipliers, this gives

$$\alpha = \frac{V^{-1}U}{U^T V^{-1}U} \quad (\text{A.5})$$

where U is a vector of length N populated with unity values. The χ^2 of the fit is then given by its variance as

$$\chi^2 = \sum_i^N \sum_j^N (\hat{y} - y_i)(\hat{y} - y_j)(E^{-1}) \quad (\text{A.6})$$

where the fit has $N - 1$ degrees of freedom. The method is described in detail in Reference [130].

In this analysis, the different measurements correspond to the measurement of the $Z \rightarrow \tau\tau$ cross section in the five analysis channels. A covariance matrix is constructed which takes into account the correlations between the different sources of uncertainties in the measurements. This is done by first constructing a 5×5 correlation matrix, $\mathcal{C}$, for each source of uncertainty tabulated in Table 6.9, where the rows and columns of the matrix correspond to the five analysis channels. A single correlation matrix exists for the statistical and luminosity uncertainties, with independent correlation matrices for each source of systematic uncertainty. Non-zero diagonal elements in the correlation matrices represent correlations between the analysis channels. In the case of statistical uncertainties, this is a diagonal

matrix of ones, as no correlations exist between the channels. For the luminosity uncertainty, all the errors are fully correlated, and so the correlation matrix is fully populated with unity values. In the case of the systematic uncertainties, all shared reconstruction efficiencies are assumed to be fully correlated, and all independent uncertainties to be uncorrelated. The covariance matrix, V , for a particular source or uncertainty, is then given by

$$V_{ij} = \mathcal{C}_{ij} \delta_i \delta_j \quad (\text{A.7})$$

where V_{ij} and $\mathcal{C}_{ij}$ represent the components of the matrices, and δ_i and δ_j represent the uncertainty for channels i and j , where i and j are indices running over the five analysis channels. The covariance matrices are given in Table A.1, where the systematic covariance matrix is given as the sum of the covariance matrices for the individual sources of systematic uncertainty. The sum of the statistical, systematic and luminosity covariance matrices is used as the covariance matrix, V , in Equation A.5 to determine α .

Systematic Covariance Matrix [pb ²]						Statistical Covariance Matrix [pb ²]					
	$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$		$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$
$\tau_\mu\tau_\mu$	72.3	3.4	4.2	5.7	8.6	$\tau_\mu\tau_\mu$	108.0	0	0	0	0
$\tau_\mu\tau_e$	3.4	15.0	3.9	2.6	3.7	$\tau_\mu\tau_e$	0	32.5	0	0	0
$\tau_e\tau_\mu$	4.2	3.9	23.8	3.3	19.0	$\tau_e\tau_\mu$	0	0	84.6	0	0
$\tau_\mu\tau_h$	5.7	2.6	3.3	5.9	7.2	$\tau_\mu\tau_h$	0	0	0	33.6	0
$\tau_e\tau_h$	8.6	3.7	19	7.2	37.0	$\tau_e\tau_h$	0	0	0	0	112.0
(a)						(b)					
Luminosity Covariance Matrix [pb ²]											
	$\tau_\mu\tau_\mu$	$\tau_\mu\tau_e$	$\tau_e\tau_\mu$	$\tau_\mu\tau_h$	$\tau_e\tau_h$						
$\tau_\mu\tau_\mu$	7.4	7.2	6.1	6.5	7.4						
$\tau_\mu\tau_e$	7.1	6.9	5.9	6.3	7.2						
$\tau_e\tau_\mu$	6.1	6.0	5.1	5.4	6.1						
$\tau_\mu\tau_h$	6.5	6.3	5.4	5.7	6.5						
$\tau_e\tau_h$	7.4	7.2	6.1	6.5	7.4						
(c)											

Table A.1: The (a) systematic, (b) statistical and (c) covariance matrices obtained and the sum of which is used as the covariance matrix in the determination of the best linear unbiased estimator.

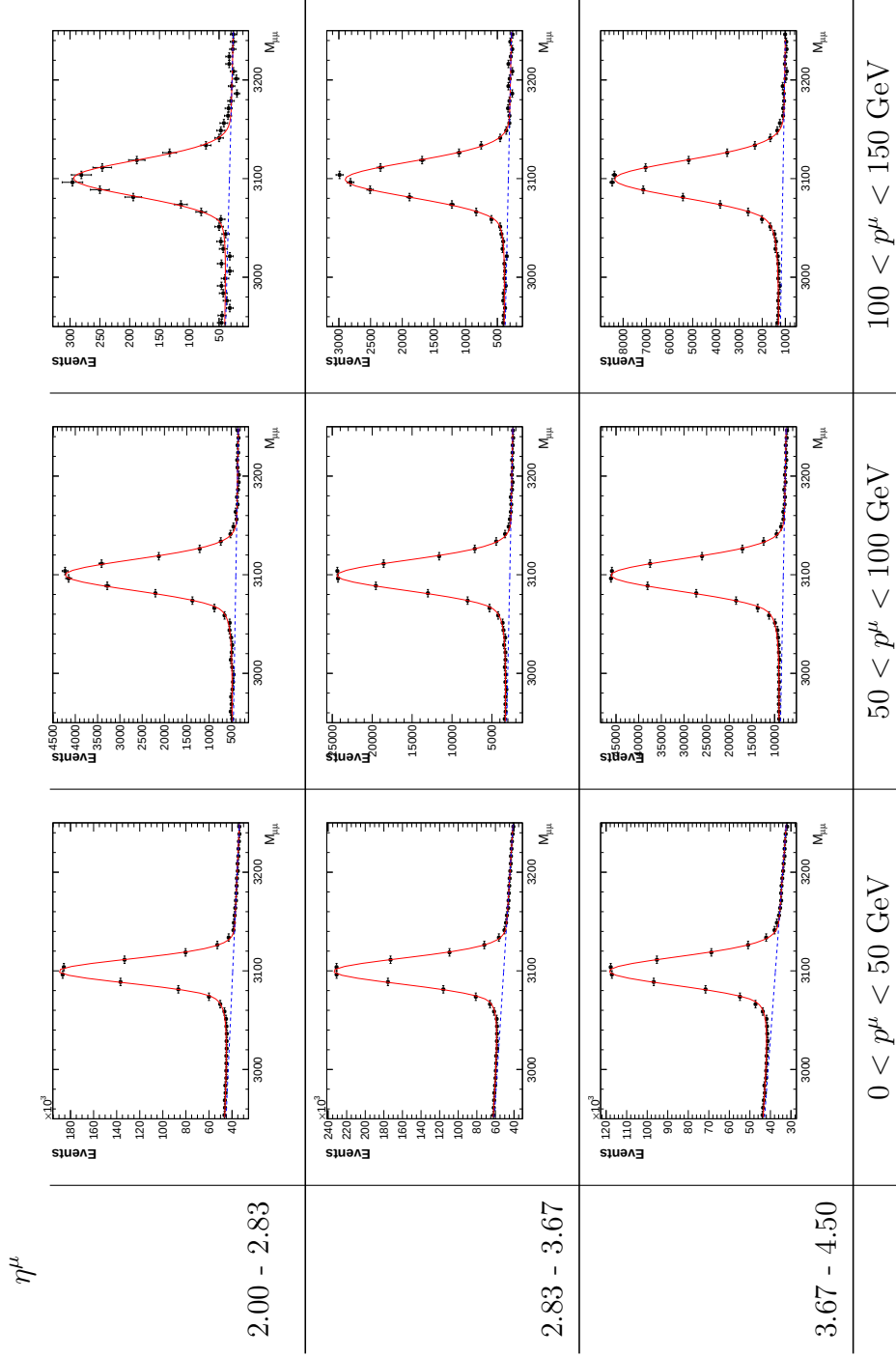


Figure A.1: The fits used to determine the number of J/ψ signal events before the tag identification requirements for the muon identification efficiency

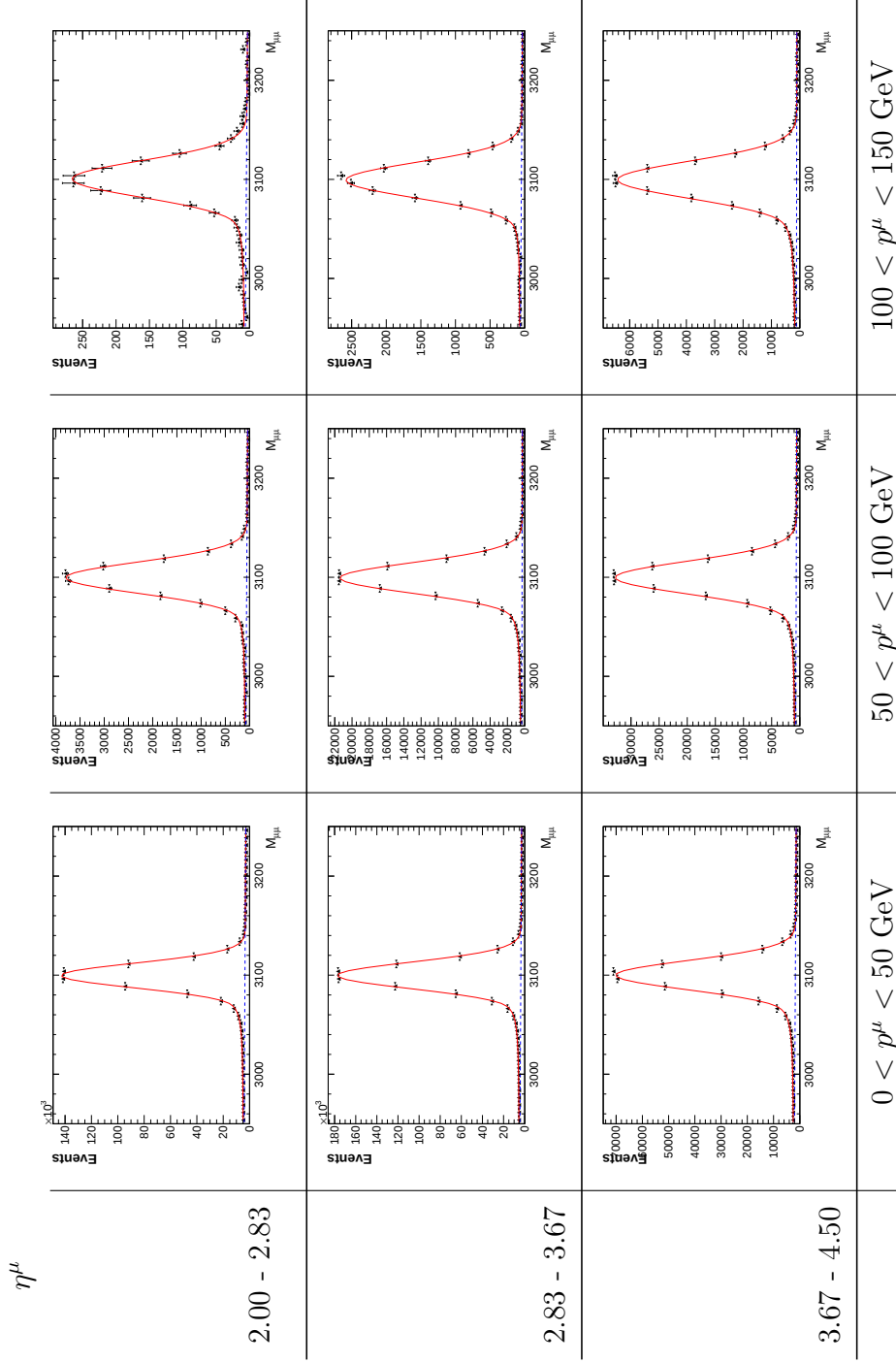


Figure A.2: The fits used to determine the number of J/ψ signal events after the tag identification requirements for the muon identification efficiency

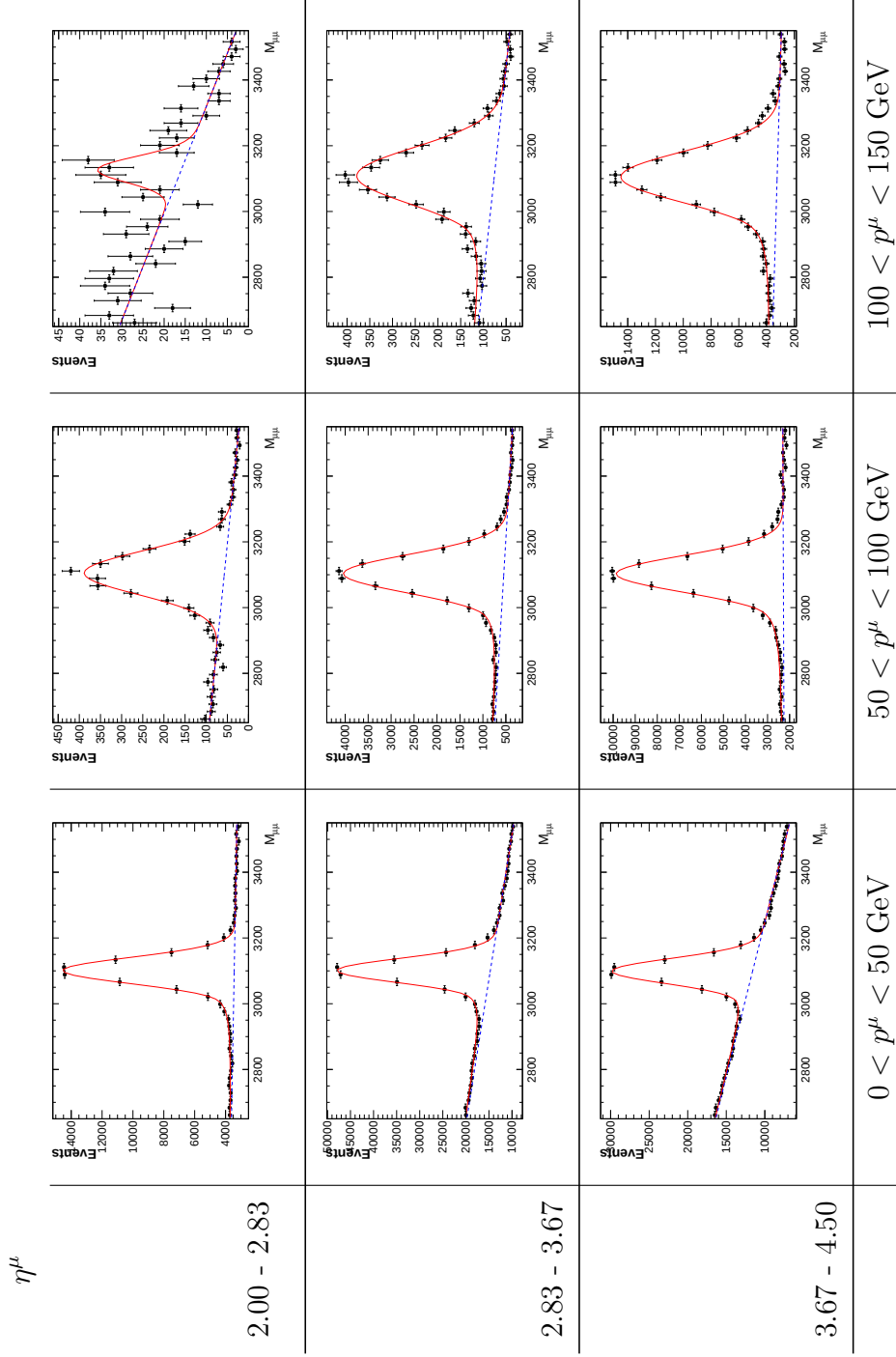


Figure A.3: The fits used to determine the number of J/ψ signal events before the tag identification requirements for the muon tracking efficiency

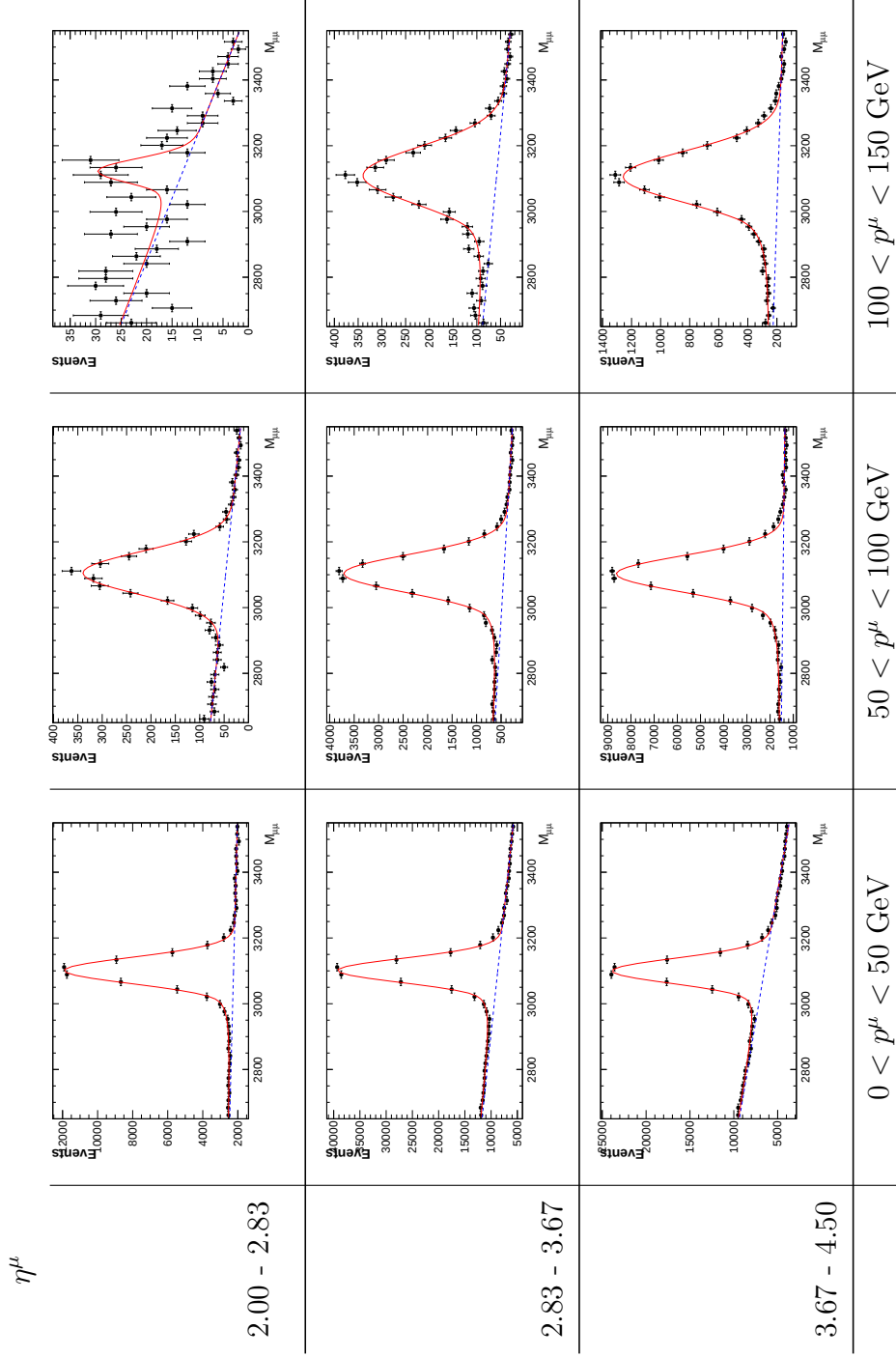


Figure A.4: The fits used to determine the number of J/ψ signal events after the tag identification requirements for the muon tracking efficiency

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