



國立中央大學

物理學系
碩士論文

Search for pair production of a heavy
quark decaying into top quark and photon
in semi-leptonic channel with the CMS
detector in the LHC

研究生：張祐祥

指導教授：余欣珊

中華民國一百零三年六月

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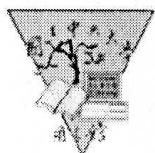
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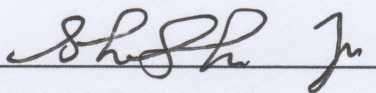
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Search for pair production of a heavy quark decaying into top quark and photon in semi-leptonic channel with the CMS detector in the LHC

Abstract:

Using the data collected by the CMS detector in 2012, corresponding to a luminosity of 19.7 / fb of proton-proton collisions at LHC center-of-mass energy of 8 TeV, we search for the excited top quark, T^* . The decay process and the final states studied in our analysis are: $T^* T^{*\bar{}} \rightarrow t \gamma t \bar{b} \gamma \rightarrow b W^+ \gamma b \bar{b} W^- \gamma$, where one W boson decays hadronically and the other leptonically, so we define a signal region by requesting events with 2 photons, 1 lepton and ≥ 4 jets. To reconstruct the invariant mass of T^* , a χ^2 -sorting method is used and 12 events are observed in the signal region from data. The matrix method is used to estimate the background contribution among the observed 12 events in the di-photon channel. As a result of this study, no significant excess is observed over expectations and a lower limit is set on a t^* quark mass of 969 GeV/c² at 95% confidence level.

以大型強子對撞機裡的緊湊渺子線圈偵測器尋找重夸克在半輕子頻道衰變成頂夸克和光子

摘要：

我們使用 CMS 在 2012 年所蒐集的積分亮度為 19.7^{-1} fb、LHC 質子對撞質心能量為 8 TeV 的數據來找尋激發態頂夸克。在我們分析中的衰變過程和最終產物是：激發態頂夸克、反激發態頂夸克→頂夸克、光子、反頂夸克、光子→底夸克、W 玻色子、光子、反底夸克、W 玻色子、光子，然後其中一個 W 玻色子以輕子的方式衰變而另一個 W 玻色子以夸克的方式衰變，所以我們藉著要求事件中有兩個光子、一個輕子和至少四個噴流來定義訊號區域。 χ^2 排序法被用來重建激發態頂夸克的質量，並且從數據中有十二個事件在訊號區域被觀察到。矩陣法被用在兩個光子的頻道裡來估計這十二個事件中的背景貢獻。這個研究的結果是沒有顯著的超過預期的多餘事件被觀察到，所以對於激發態頂夸克的質量我們以 95% 的信心水準設了一個低限在 $969 \text{ GeV}/c^2$ 。

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Contents

Abstract	i
中文提要	ii
感謝	iii
Contents	iv
List of table	v
List of figure	vi
Chapter 1 Introduction	1
Chapter 2 LHC and CMS detector	3
2.1 The Large Hadron Collider	3
2.2 The Compact Muon Solenoid Detector	5
2.2.1 Tracker	6
2.2.2 Electromagnetic Calorimeter	9
2.2.3 Hadronic Calorimeter	10
2.2.4 Muon Chamber	12
2.2.5 Magnet system	13
Chapter 3 CMS Trigger	14
3.1 Level-1 trigger	15
3.1.1 Muon triggers system in L1	17
3.1.2 Calorimeter triggers system in L1	17
3.1.2.1 The Electromagnetic Candidate Algorithm	19
3.1.2.2 The Jet Candidate Algorithm	21
3.1.2.3 Other calorimeter trigger tasks	23
3.1.3 Global trigger in L1	24
3.2 High Level trigger	25
3.2.1 Electrons/photons	25
3.2.2 Muon	35
3.2.3 Jet	36
3.2.4 Missing transverse energy	37
3.3 The triggers used in our analysis	38
Chapter 4 Event reconstruction in offline and our event selection	39
4.1 The anti-kt jet algorithm	39
4.2 The particle flow (PF) algorithm	41
4.3 The r9 variable and the missing hits	43

4.4	The shower shape variable σ_{ini}	44
4.5	The isolation variables	45
4.6	The event selection in our analysis	46
Chapter 5	Data and the simulated sample	51
Chapter 6	Mass Reconstruction	60
6.1	The χ^2 -sorting method	60
6.2	The neutrino's p_z solution	62
Chapter 7	Background estimation: the matrix method	63
7.1	The introduction of matrix method	63
7.2	The photon fake rate from lepton ϵ_l	66
7.3	The photon fake rate from jet ϵ_j	68
7.4	The photon signal efficiency ϵ_s	72
7.5	The ratio factor	74
Chapter 8	Result	75
Chapter 9	Conclusion	79
Reference		80
Appendix A	The derivation of Matrix method	85

List of tables

Table 3.1	the list of the objects sent from GCT to GT	18
Table 3.2	tau decay branching ratio	22
Table 3.3	global trigger menu	24
Table 3.4	parameter of hybrid algorithm	28
Table 3.5	trigger path of our analysis	38
Table 5.1	dataset of our analysis	51
Table 5.2	MC samples used in our analysis	52
Table 5.3	event yields before/after scale factors	53
Table 5.4	event yields in the loose region	59
Table 7.1	fit result of fake rate from jets	71
Table 8.1	event yields in the signal region	77

List of tables

Figure 1.1	The diagram of t^* decay	2
Figure 2.1	The CERN accelerator complex	3
Figure 2.2	The main accelerator rings and their beam particles	4
Figure 2.3	The CMS detector	5
Figure 2.4	The CMS coordinate	6
Figure 2.5	The pixel detector	6
Figure 2.6	The pixel material budget	7
Figure 2.7	The layout of tracker	8
Figure 2.8	The layout of tracker	9
Figure 2.9	The layout of ECAL	10
Figure 2.10	The relative positions of ECAL and HCAL	11
Figure 2.11	The relative positions of muon chambers	13
Figure 3.1	The overview of the L1-trigger and the HLT	15
Figure 3.2	The schematic diagram of the data flow in L1-trigger	16
Figure 3.3	The L1-trigger	16
Figure 3.4	The strip, the trigger tower and the calorimeter region grouping	18
Figure 3.5	Illustration of the calorimeter trigger e/γ algorithm	20
Figure 3.6	Illustration of the calorimeter trigger jet algorithm	22
Figure 3.7	Illustration of the Island clustering algorithm	26
Figure 3.8	Illustration of the super-clusters	27
Figure 3.9	Domino construction step of Hybrid algorithm	27
Figure 3.10	Illustration of the crystal off-pointing and the schematic diagram	30
Figure 3.11	Distribution of $E_{\text{meas}}/E_{\text{true}}$	31
Figure 3.12	$E_{\text{meas}}/E_{\text{true}}$ as a function of the number of crystals	32
Figure 3.13	Matching the hits in pixel detector	33
Figure 4.1	Anti-kt clustered jets	40
Figure 4.2	Consideration of the Bremsstrahlung photon emission of electron	42
Figure 4.3	The schematic diagram of particle flow algorithm	43
Figure 4.4	Missing Hits	44
Figure 4.5	Sketch of the photon isolation	45
Figure 5.1	The number of vertices before/after applying scale factors	54
Figure 5.2	The objects' multiplicities	55
Figure 5.3	The kinematic distributions	56

Figure 5.4	Four leading jets p_T distributions	57
Figure 5.5	Jets' p_T distributions for run dependences	58
Figure 6.1	The invariant mass of a top and a photon using the χ^2 -sorting method	61
Figure 6.2	The t^* mass spectra with different neutrino's solutions	62
Figure 7.1	The schematic of 3 levels of the events with two photons	64
Figure 7.2	The schematic of the tight, the loose and the FO selection of photon	64
Figure 7.3	Electron-Photon invariant mass fit for the no-electron-veto selection and the tight ID selection	67
Figure 7.4	The object has a photon associated with a jet within the cone of size 0.5	68
Figure 7.5	The schematic of the N_j^{FO} and N_j^T	69
Figure 7.6	Results of the template fitting for the various categories	71
Figure 7.7	Muon-Muon-Photon invariant mass fit for the no-electron-veto selection and the tight ID selection	73
Figure 8.2	The mass spectrum in the signal region	76
Figure 8.3	The exclusion limit	78
Figure A-1	Figure A-1	82
Figure A-2	Figure A-2	83
Figure A-3	Figure A-3	86
Figure A-4	Figure A-4	87

Chapter 1 Introduction:

The top quark, discovered in 1995, with its large mass $173.5 \pm 0.6 \pm 0.8 \text{ GeV}/c^2$, is unique among the fundamental particles. Many models have emerged surmising that it may be a composite particle rather than an elementary particle [1–5].

A direct test of this possibility would be to show the existence of its excitation state. Several theories have hinted the existence of this excited top quark (t^*) using different approaches. In [6] and [7], a heavy spin-3/2 excitation of a heavy spin-1/2 quark is introduced using "Rarita-Schwinger" [8] vector spinor Lagrangian. In [9], this t^* is further elaborated to be a spin-3/2 Regge excitation in Randall-Sundrum [10], [11] model by a string theory inspired context. Among other excitation tower in the wrapped extra dimension, the right handed top quark excitation (t^*) is expected to be the lightest among other spin-3/2 fermion resonances. It is described by an effective Rarita-Schwinger Lagrange in four dimensions.

$$\mathcal{L}_4 = i\bar{\psi}_\mu \gamma^{\mu\nu\rho} D_\nu \psi_\rho + M\bar{\psi}_\mu \gamma^{\mu\rho} \psi_\rho$$

Where $D_\mu = \partial_\mu - igA_\mu$, which contains the leading order interaction responsible for the dominant production of t^* at LHC. The coupling between spin-1/2 and spin-3/2 states via strong, weak or e.m. interaction via five dimension operators are of the form

$$\mathcal{L}_5 = i\frac{a}{\Lambda}\bar{\psi}_\mu(\eta^{\mu\alpha} + z\gamma^{\mu\alpha}\gamma^\alpha)F_{\alpha\beta}\gamma^\beta P_{R,L}\chi + h.c.$$

Where $P_{R,L}\chi$ denotes the chiral spin-1/2 state. Based on [9], cross sections are calculated in [11] which predicted a cross section of order one for $500 \text{ GeV}/c^2$ t^* . The base line of this analysis will follow the model invented within these two references.

For this analysis, we assume a 100% branching fraction for $t^* \rightarrow t\gamma$. We consider only pair production of t^* because it has a higher production cross section than single production of t^* at the LHC. This is due to the fact that mixing between spin-3/2 and spin-1/2 states is suppressed [6, 9]. Although we focus on a spin-3/2 RS resonance,

the analysis is performed in a model independent way. We consider the semi-leptonic decay channel of the system, i.e. $t \rightarrow bW$ with $W \rightarrow qq$ on one side and $W \rightarrow \nu l$ on the other side, where l may be either an electron or a muon. (see Fig. 1)

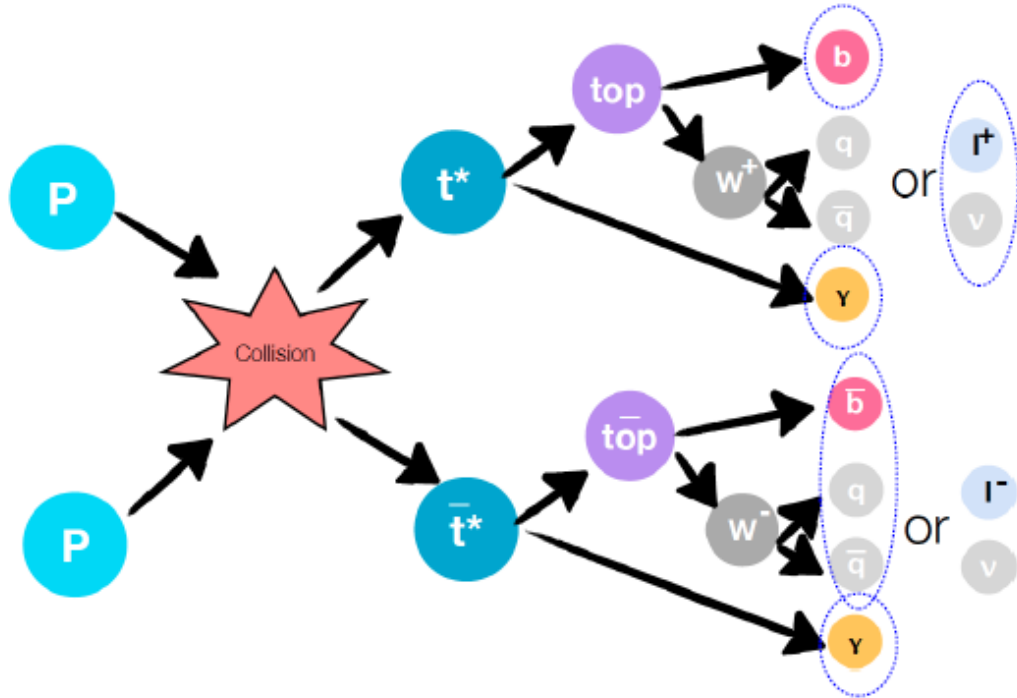


Figure 1.1: The diagram of t^* decay and the signal region: 2 photons, 1 lepton and at least 4 jets.

Our group member includes Ece Asilar¹, Marco Cardaci², Kai-Feng Chen³, Murat Ali Guler¹, Yeng-Ming Tzeng³, Shin-Shan Yu² and Fang-Ying Tsai². I join this group in 2012 summer. Before my participation, our group has investigated the t^* exclusion limit up to 990 GeV.[12] In our team, my task is to estimate the photon fake rate from jet (see section 7.3), and the rest parts are done by other members.

The following pages will introduce how we perform the analysis, including the introduction of CMS detector, Event reconstruction, mass reconstruction, background estimation, limit calculation and the result.

Note: ¹Middle East Technical University, ²National Central University and ³ National Taiwan University.

Chapter 2 LHC and CMS detector

In Chapter 2 I will introduce the experimental facility that we utilize, the accelerator Large Hadron Collider (LHC) and the particle detector Compact Muon Solenoid (CMS). They were built by the European Organization for Nuclear Research (CERN) and thousands of scientists around the world. Our team has participated the CMS so we can use the CMS collected data to do our analysis.

2.1 The Large Hadron Collider

The accelerator complex at CERN is composed of several sections.(see Fig 2.1 , Fig 2.2) In each section the accelerator machine accelerates the particle beam to a certain energy level. Then the beam will be injected to the next section and be accelerated to another higher energy level. It will be accelerated to the highest energy level in the last section, the Large Hadron Collider (LHC).

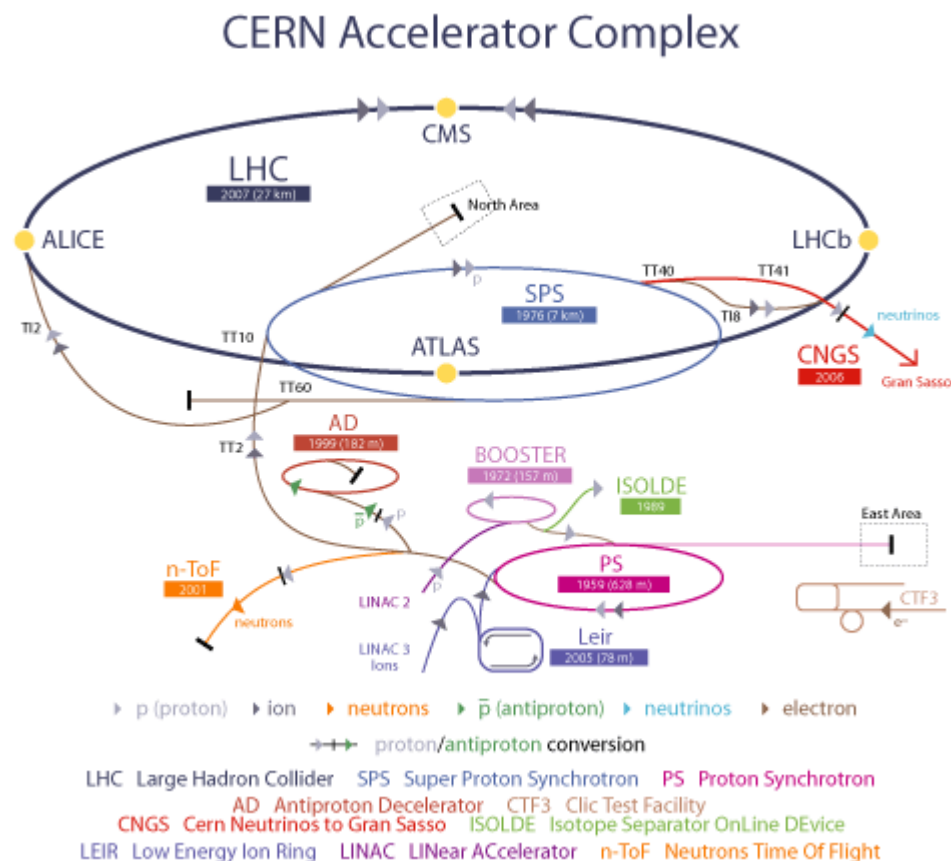


Fig 2.1: The CERN accelerator complex. Here shows the series of accelerators, the beam particles, the beam paths. The year marked below the name is the beginning year of the accelerator.

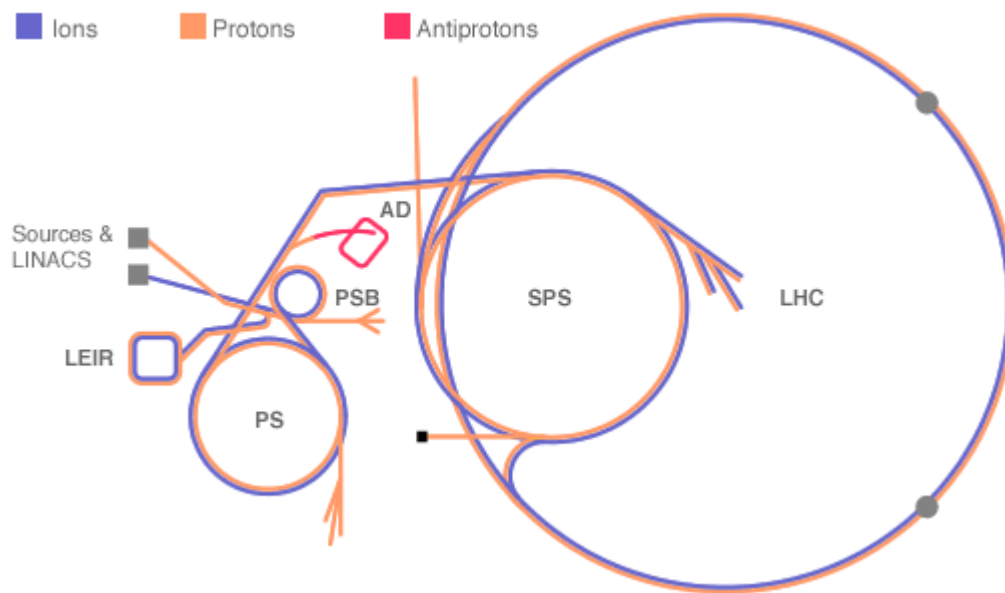


Fig 2.2: The main accelerator rings and their beam particles.

In the accelerator complex, the accelerating particles can be proton or other ions, like lead ions for different purpose. For protons, the beam source is a simple bottle of hydrogen gas, and an electric field is used to strip hydrogen atoms to yield protons. Linac 2, the first linear accelerator can accelerate the beam up to 50 MeV. The proton beam is then injected into the Proton Synchrotron Booster (PSB) and accelerated to 1.4 GeV. Proton Synchrotron (PS), the following machine pushes the beam to 25 GeV and then the protons are sent to the Super Proton Synchrotron (SPS) where the beam is up to 450 GeV. Finally in the last element of chain, Large Hadron Collider (LHC), it is accelerated energy up to 4 TeV per beam. In this time the two beams travel at the speed of $0.99999991c$. These two beams are brought into collision inside four detectors in LHC – ALICE, ATLAS, CMS and LHCb – where the center of mass energy at the collision point is equal to 8 TeV

Lead ions for the LHC start from a source of vaporized lead and enter Linac 3 before being collected and accelerated in the Low Energy Ion Ring (LEIR). They then follow the same route to maximum energy as the protons.

In addition to LHC, most of the other accelerators in the chain have their own experimental halls where beams are used for experiments at lower energies.

2.2 The Compact Muon Solenoid Detector

The Compact Muon Solenoid (CMS) is a particle detector designed to explore physics at the TeV scale, to discover the Higgs boson and to find the physics beyond the standard model. The complete detector is 21.6 m long, 15 m in diameter and 12500 ton weight. It is located in 100 m underground and near Cessy in France.

The CMS detector is shown in Figure 2.3. It contains several subsystems, the tracker, the electromagnetic calorimeter (ECAL), the hadronic calorimeter (HCAL), the magnetic system and the muon chamber. Each subsystem takes responsibility for a certain purpose so that one can reconstruct the collision event by combining information of all subsystem. I will introduce each subsystem in detail later.

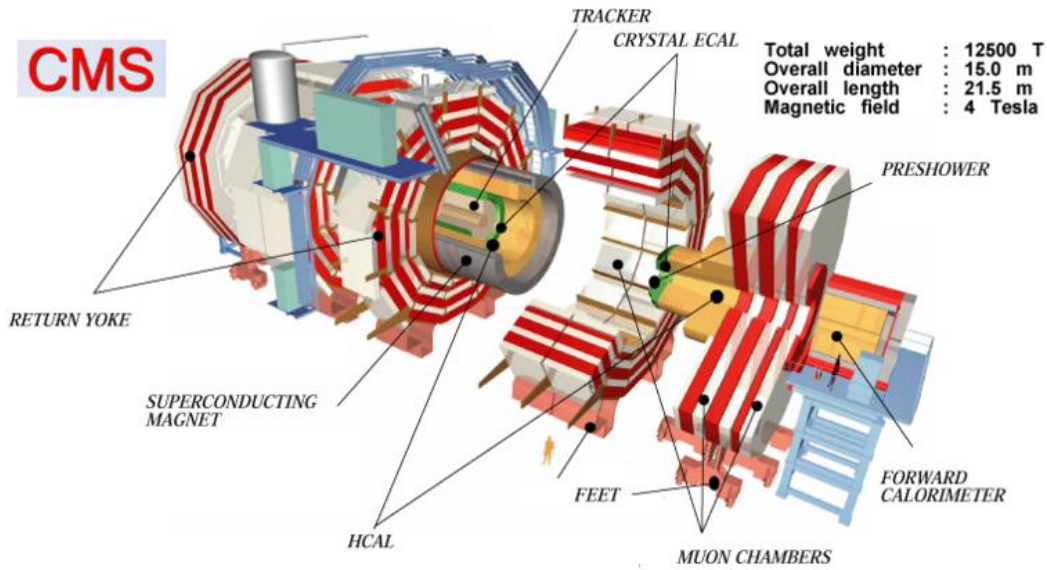


Fig. 2.3: The CMS detector and its sub-systems.

In CMS right-handed coordinate is used (see Fig 2.4), with the origin at the nominal interaction point (the center of detector, or called “nominal vertex”), the x-axis pointing to the center of the LHC, the y-axis pointing up (perpendicular to the LHC plane), and the z- axis along the anticlockwise-beam direction. The polar angle, θ is measured from the positive z-axis and the azimuthal angle ϕ is measured in the x-y plane. The pseudorapidity η is defined as $-\log \tan \frac{\theta}{2}$. The pseudorapidity is also used to define the different region of CMS detector – barrel ($|\eta| < 1.479$), endcap ($1.479 < |\eta| < 3.0$), and forward ($3.0 < |\eta| < 5.0$).

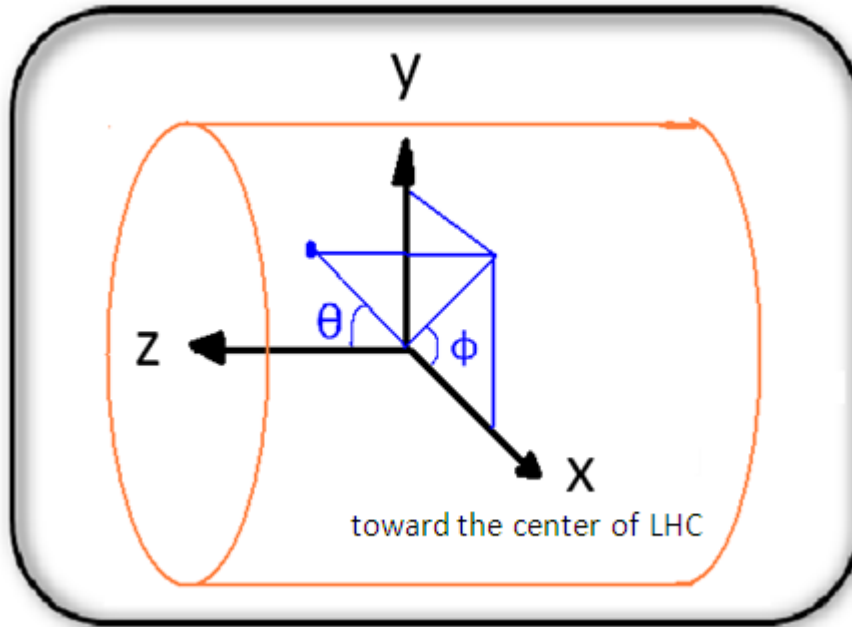


Fig 2.4: The CMS right-handed coordinate.

2.2.1 Tracker

The most inner part of the CMS detector is the silicon tracker[13,14]. The layout of CMS Tracker is shown in Figure 2.5.

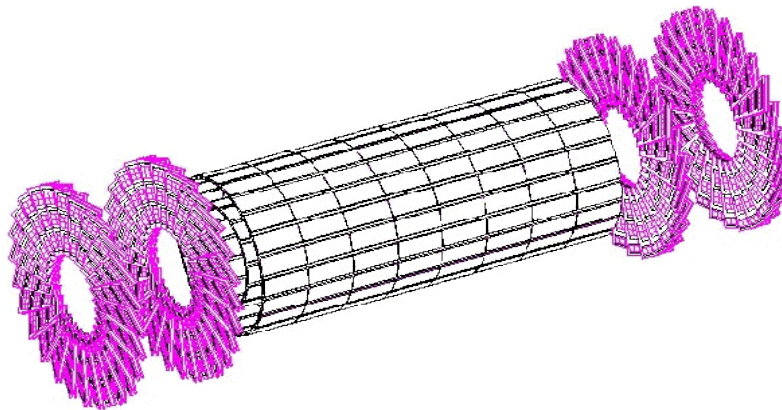


Fig 2.5: The pixel detector.

One of its purposes is to measure the curvature of trajectories then to derive charged particle's momentum as precisely as possible. The Lorentz Force bends the tracks in the $r\phi$ -plane because of the solenoidal magnetic field.

$$\vec{F}_L = \frac{q}{m} \vec{p} \times \vec{B}.$$

Meanwhile, the material in the tracker will affect photons to convert to electron pair, so the minimization of material budget, which is the effect of the material length to photon converting, is required. The photon conversion probability P is

$$P = (1 - e^{-\frac{x}{\frac{7}{9}x_0}})$$

, where x is the photon traveling length in material, and x_0 is the radiation length of that material.

Figure 2.6 shows the material budget in fractional radiation length (X/X_0) as a function of pseudo-rapidity.

In the barrel region, the tracking system consists of the Tracker Inner Barrel (TIB) and Tracker Inner Disks (TID) in 2 sides, and each of them is composed of three small discs. The outer Barrel (TOB) with six concentric layers is closed by the Tracker End Caps (TEC), which are the two endcaps detectors on either side of the detector as shown in Figure 2.7 and Figure 2.8. The pixel detectors is located at the CMS detector's center interaction region, it is used to reconstruct the primary and secondary vertices.

Tracker Material Budget

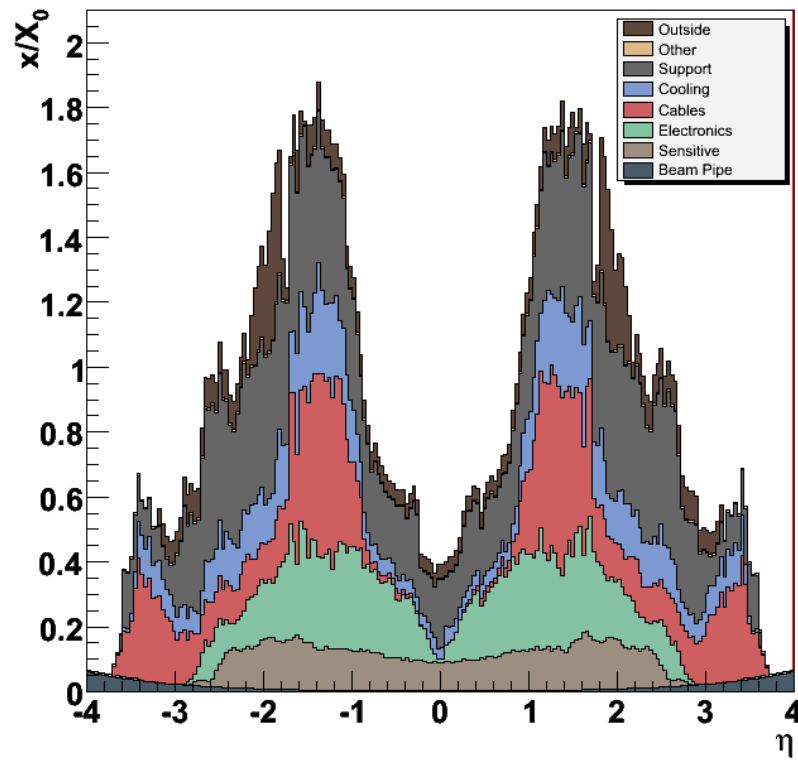


Fig 2.6: the tracker material budget in fractional radiation length(X/X_0) as a function of pseudo-rapidity.

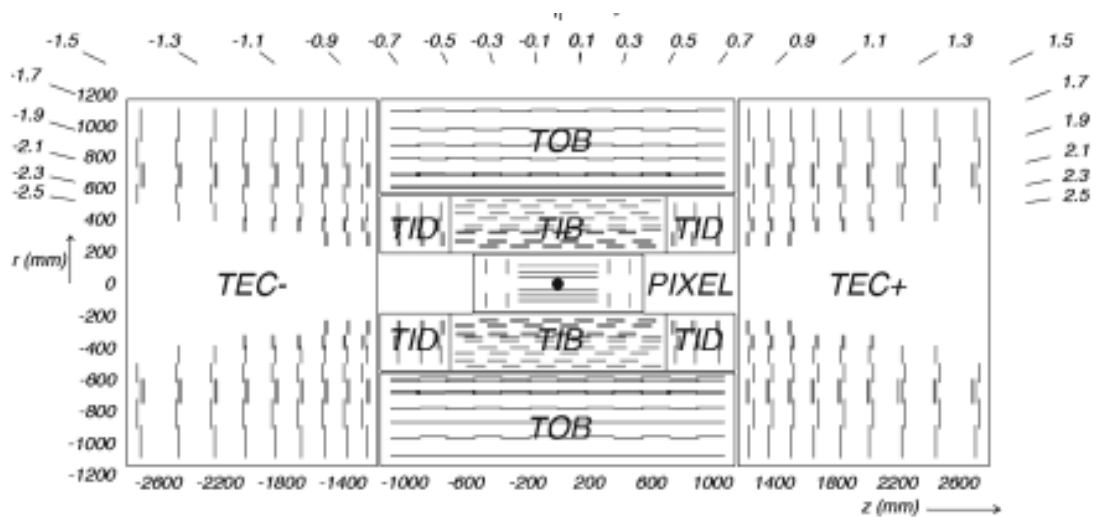


Fig 2.7: the layout of tracker.

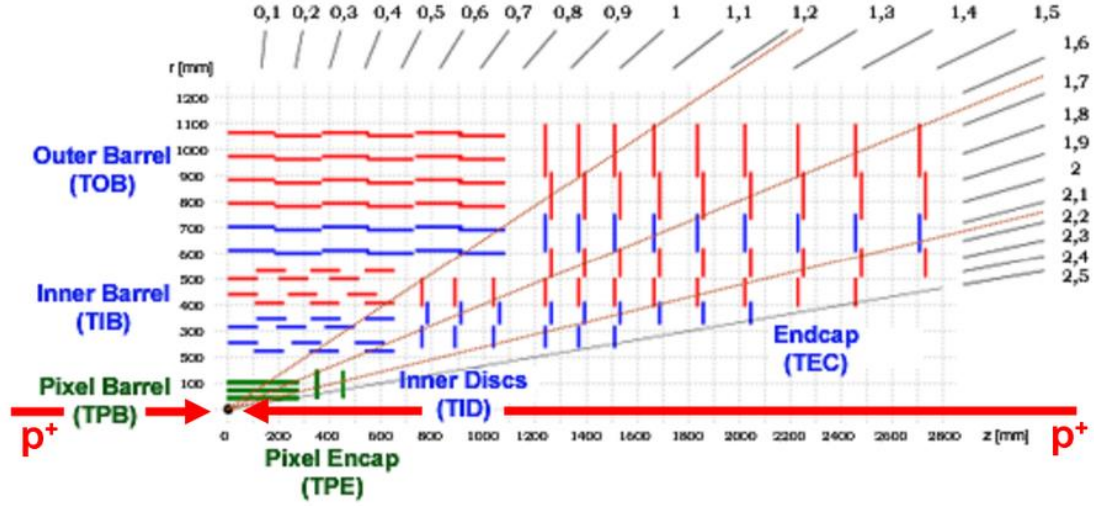


Fig 2.8: the layout of tracker.

Tracking system can improve the overall jet energy measurement by matching associated tracks to energy reconstruction in the calorimeter. It is also used to suppress some background processes by finding the secondary vertex of particle with long lifetime such as B-hadrons.

2.2.2 Electromagnetic Calorimeter

The Electromagnetic Calorimeter (ECAL) (see figure 2.9) is outside the tracker[15,16]. The ECAL is used to measure photon's and electron's energy by completely stopping them. These high energetic particles will create showers, which is the mutual induction of electron pair and photon, due to the electromagnetic field in material. The induced particles will inherit parts of the mother particle's energy. Repeating this process hundreds of times, the resulting particles possess quite low energy then it will be absorbed by material and stop finally. During their travel in material, the ECAL material will be stimulate and produce light. Absorbing the lights by photo-detector and calibrating it, we can reconstruct the energy of original particles. The ECAL material is the lead tungstate crystals (PbWO₄), which is a transparent crystal so that the emitted light can pass it.

The ECAL is also separates to 2 regions, barrel (EB) corresponding to the range $|\eta| < 1.49$ and endcap (EE) corresponding the range $1.6 < |\eta| < 2.6$. There are about 61000 crystals in the EB and about 7300 crystals in the EE.

The energy resolution of the ECAL can be calculated as follows[17]:

$$\left(\frac{\sigma}{E}\right)^2 \approx \left(\frac{N}{E}\right)^2 + \left(\frac{S}{\sqrt{E}}\right)^2 + C^2$$

The N is the term of electronic noise and event pile-up. The S is the term of the stochastic and C represents the constant term. The value of the three parameters were determined by a electron test beam measurement to be $S = 0.028 \text{ GeV}^{1/2}$, $N = 0.12 \text{ GeV}$ and $C = 0.003$.

In the endcap regions, there are preshower detectors before the ECAL, which is a tracker detector and made of silicon strip. It is used to reject the fake photons from π^0 decay thus improves the photon reconstruction in ECAL.

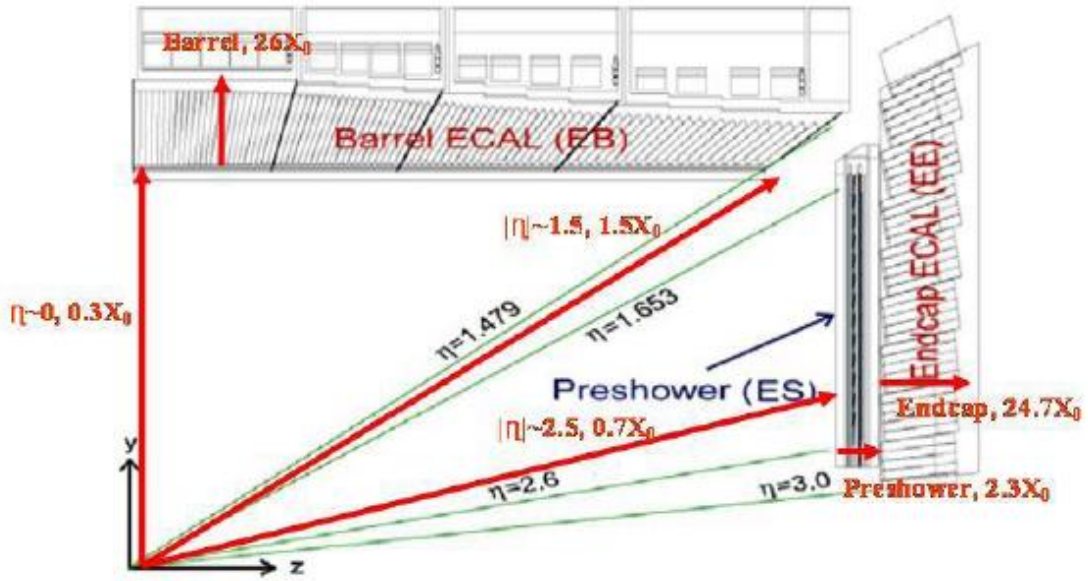


Fig 2.9 the layout of ECAL and the preshower. The Preshower includes $2.3X_0$ of Pb absorber. Each Endcap calorimeter is made about $24.7X_0$ in depth and Barrel is about $26X_0$ in depth.

2.2.3 Hadronic Calorimeter

The hadron calorimeter (HCAL)[18] is used to measure the energy of particles which will undergo the strong interaction. It means to measure the energy of hadrons, or so called jets, which is a jet-like energy cluster and it contains many hadrons coming from a series of hadronization. A jet can come from a quark or a gluon originally.

The mechanism of HCAL is similar to ECAL except the shower is due to strong interaction instead of EM interaction. To measure the jet's energy, it needs to combine the information from both ECAL and HCAL due to the charge hadron will deposits energy in ECAL too.

In HCAL, the absorber material is made of copper alloy and the sensitive material is made from plastic scintillator.

The HCAL can be separate to different parts. The Barrel Hadronic Calorimeter(HB) and the Endcap Hadronic Calorimeter(HE) are inside the magnetic coil. The former one covers the central pseudo-rapidity up to $|\eta| = 1.3$, and the latter one extends the angular coverage in $1.3 < |\eta| < 3$ (See Figure 2.10). The Forward Hadronic Calorimeter (HF) located 11 m either side of the interaction point. Its pseudo-rapidity covers up to $|\eta|=5$. The Outer Hadron Calorimeter (HO) sit outside the magnetic coil, ensuring no energy leaks out the back of the HB undetected.

The energy resolution of HCAL is given by the formula[19,20]:

$$\left(\frac{\sigma}{E}\right)^2 \approx \left(\frac{0.71}{E}\right)^2 + \left(\frac{0.97}{\sqrt{E}}\right)^2 + 0.08^2$$

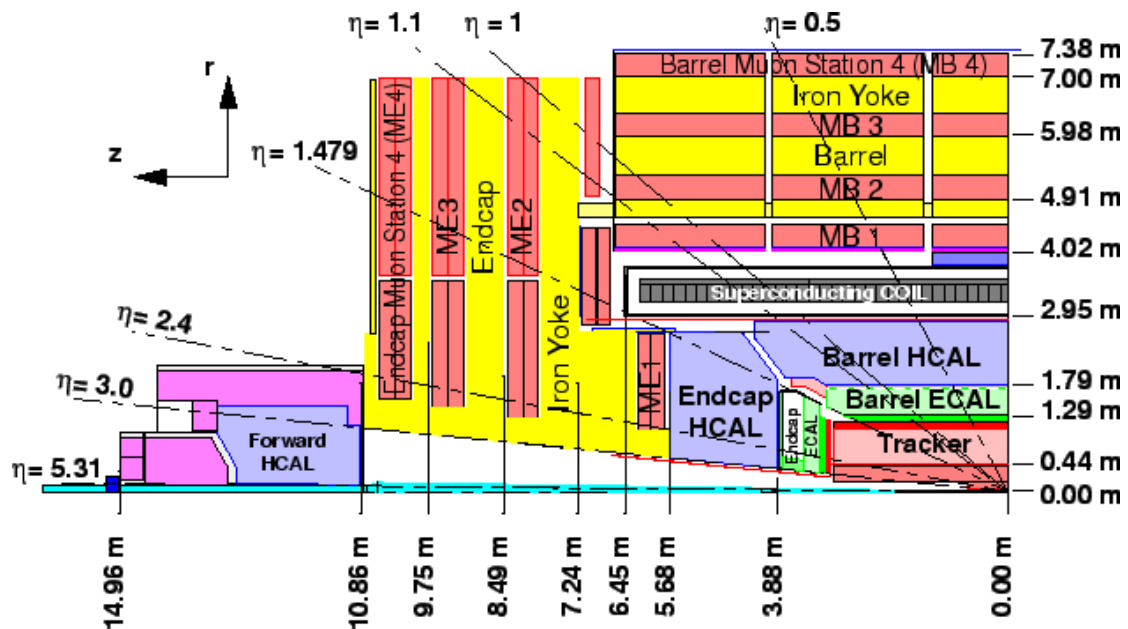


Fig 2.10 The relative positions of each sub-detector including the barrel Ecal (EB), the endcap Ecal (EE), the barrel Hcal (HB), the endcap Hcal (HE), and the forward Hcal (HF).

2.2.4 Muon Chamber

The Muon Chamber is design to measure the momentum of muons. In the presence of magnetic field, the curvature of moun trajectories can derive its momentum. Because of muons' characteristic, the relatively long lifetime and passing without interaction through the detector, this system is placed in the outermost of the detector. Thus only muons can flight into this region. The complete reconstruction of muon also uses the information of Tracker.

In order to have strong enough magnetic field to bending the muon's track, the Muon Chamber is embedded in the return yoke, which is a iron shell that increases the number of magnetic lines in this outside coil region.

The muon system enables good identification by using three types of detector: Drift Tubes (DT), Cathode Strip Chambers (CSC) and Resistive Plate Chambers (RPC). The DTs are used to measure position in the barrel region with the $|\eta| < 1.2$ range. The CSCs are chosen for measuring the endcaps region position with coverage up to $|\eta| < 2.4$, where the particle rate is higher. The RPC are fast gaseous detectors which are both in the barrel and in the endcaps. And it also provides accurate time measurements. The schematic layout is shown in Figure 2.11.

The DT's position resolution and CSC's position resolution are up to 100 μm and 200 μm separately. The DT is approximately 1 mrad in ϕ direction and CSC is about 10 mrad. And the time resolution with the combination of good spatial resolution is up to three nanoseconds.

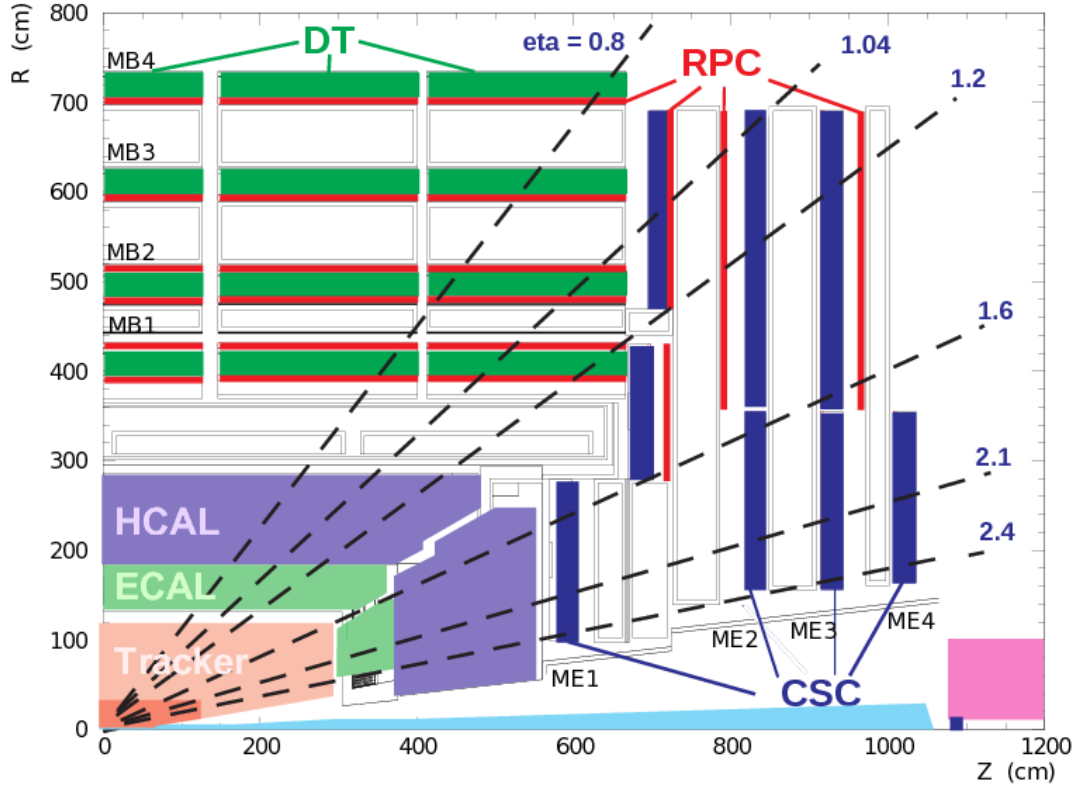


Fig 2.11 The relative positions of DT, CSC and RPC in the muon chamber.

2.2.5 Magnet system

In CMS, it needs a strong magnet field to bend the trajectories for the charged particle momentum measurement. The relative uncertainty of momentum measurement is

$$\frac{\Delta P}{P} \propto \frac{P}{B}$$

In order to have a unambiguous sign ($\Delta P/P \sim 10\%$) for 1 TeV muons, CMS uses a superconducting solenoid to provide the magnetic field of 3.8 Tesla at the interaction point. The same field strength is also in the other place inside the coil, such as the tracker system, ECAL and HCAL. Outside the coil, the return yoke collects the magnetic lines and provides enough fields for the muon chamber.

Chapter 3 CMS Trigger

In this chapter I will introduce how to reconstruct a collision event using the information collected from each sub-detector as described in chapter 2.

When collision happens in LHC, the produced particles fly in the speed near the speed of light c and arrive at each detector almost at the same time. Therefore, the information in the same small time interval (10^{-8} sec) from all detector-channels (millions in Tracker, 76,832 Ecal crystals, 15,096 Hcal cells and 1,400 muon chambers) belongs to the same event. From the information, CMS uses algorithms to reconstruct the physical objects (electrons, muons, photons and jets), i.e the event reconstruction.

However, in LHC the frequency of proton beam bunch crossing is 20 MHz (in 2012), while the frequency of storing data on disks in CMS is only about 100 Hz. Therefore, CMS needs a trigger system to reduce the amount of data before storing them. Only the events that meet the trigger requirements will be saved.

Since the purpose of the trigger system is to have a quick way to tag the event that is valuable to be recorded, it doesn't use the full information to reconstruct events and it only performs the partial event reconstruction. The full event reconstruction will be done after the data storage. The stage of trigger-level and the reconstruction after storage are also called "online" and "offline", respectively. Compared to the fast and simple online algorithms, the offline algorithms are more complicated, complete and time-consuming.

The CMS trigger system is composed of the level-1 trigger(L1-trigger) and the high-level-trigger(HLT) (Fig3.1). The L1-trigger is based on custom hardware processors. For the purpose of quickly rejecting large amount of un-interesting events, the L1-trigger uses only coarse information from the calorimeters and muon detectors. The events passing L1-trigger will be piped into the HLT processor farm. The HLT uses the information from all detectors, and its algorithms are more similar to the ones in offline. Through the trigger system, the event rate is reduced from 20 MHz to 100 kHz in L1-trigger and further to 500 Hz in HLT, which is the order of the data storage rate in CMS.

In the following sections I will introduce the reconstruction in the L1-trigger and the

HLT.

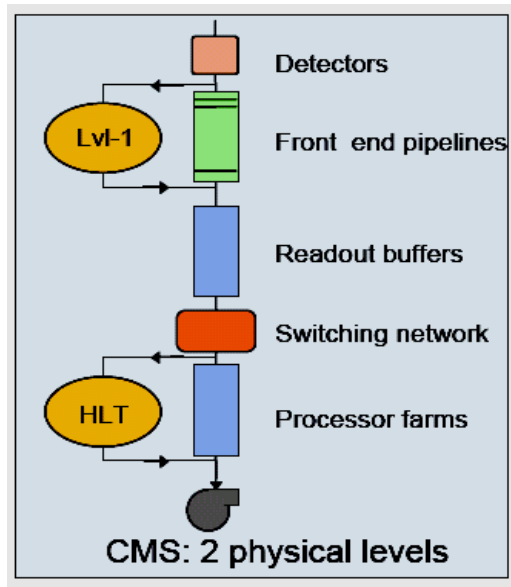


Figure 3.1 the overview of the L1-trigger and the HLT (online). After the HLT, the output data are saved in disks for further study (offline).

3.1 Level-1 trigger

In every bunch-crossing, the full-granularity data from all sub-detectors are digitized first, and then the data are stored temporarily ($\sim 3 \mu\text{s}$) in on-detector pipelined buffers to be read out at a maximum rate of 100 kHz. Meanwhile, the coarse-grained information is read out and used for L1-triggers [21-23]. The L1 triggers choose the best candidates of electrons, photons, muons and jets to the global trigger (GT) and pass them to determine whether the event is accepted or not. If no trigger decision is received before the data reaches the end of the buffer, the data are irretrievably lost (Fig 3. 2).

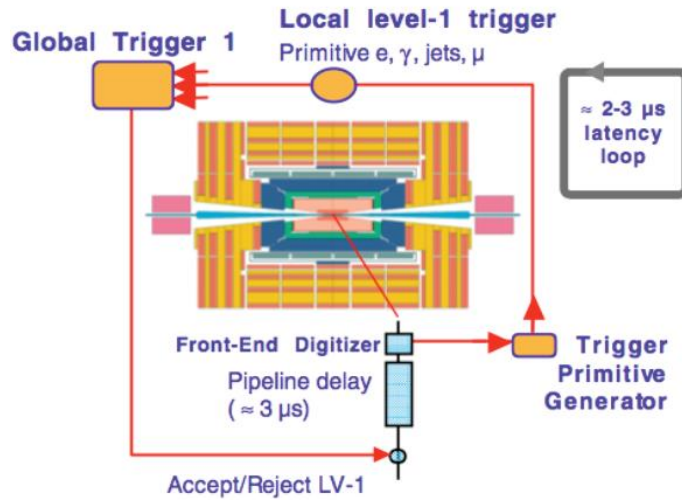


Figure 3.2 the schematic diagram of the data flow in L1-trigger.

An overview of L1-trigger is shown in Fig 3.3. Both calorimeter global trigger and the muon global trigger will send the best candidates to the global trigger (GT) that determines the event to be accepted or rejected in L1-trigger. The following text describes the algorithms and the criteria used to determine the best candidates.

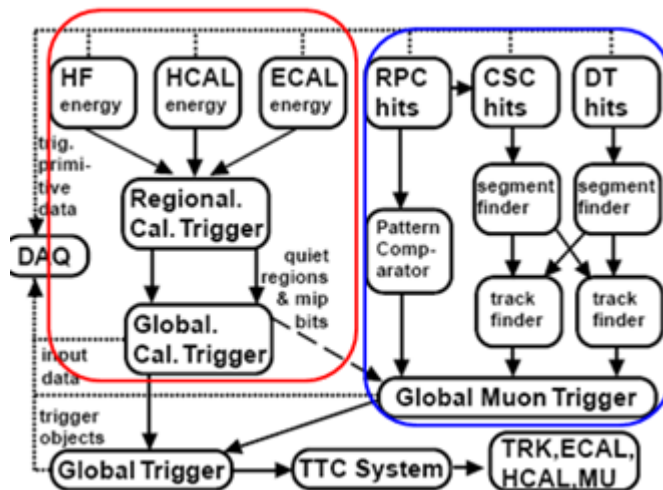


Figure 3.3 the L1-trigger is composed of the calorimeter trigger system (red) and the muon trigger system (blue). In both systems, their global triggers (GCT and GMT) send the best candidates to the global trigger (GT).

3.1.1 Muon triggers system in L1

In the L1 Trigger, candidate muons are identified in each of the DT, CSC and RPC detectors separately. In the DT and CSC detectors, track stubs are identified at the chamber/sector level, and are forwarded to the individual track-finder of each sub-detector (DTTF and CSCTF). Track stubs are shared between these systems to ensure full coverage of the barrel-endcap transition region. The identified candidates from all three systems are sent to the Global Muon Trigger (GMT), where they are combined and the four best muon candidates (according to their transverse energy and quality, the transverse energy E_T is defined in Section 3.1.2.3) of barrel and of endcap are forwarded to the Global Trigger (GT).

3.1.2 Calorimeter triggers system in L1

The main goal of the calorimeter trigger is to choose the best candidates of the electrons/photons and the jets. This is achieved in two stages; first, by summing groups of the 5×5 ECAL crystals into trigger-towers (Fig 3.4 and Fig 3.5), which have a one-to-one mapping to the HCAL towers and, secondly, summing these towers into regions whose size is closer to the expected size of electrons/photons or jets. These objects are classified to different regions according to their pseudorapidity η , and each type is sorted by its rank (a measure of its transverse energy and 'quality') and the top four of each type forwarded to the global trigger (GT) for further processing. The detail of algorithms for electron/photon (e/γ) and for jets will be mentioned in the following text.

The list of object sent to the global trigger (GT) is shown in table 3.1, and I will explain the meaning of each object later.

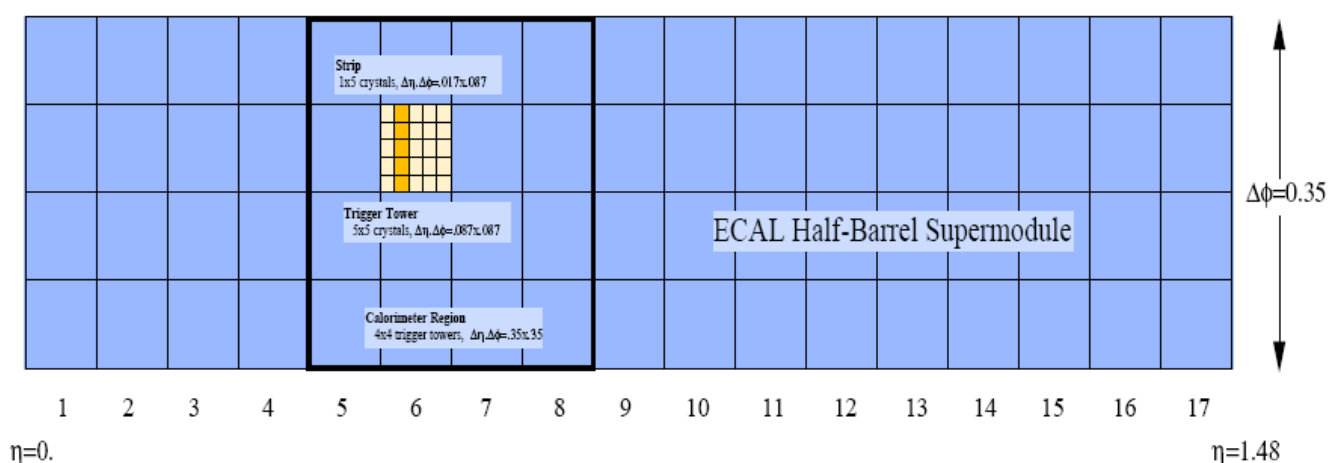


Figure 3.4 The strip, the trigger tower and the calorimeter region grouping. The size ($\Delta\eta \times \Delta\phi$) of a Ecal crystal and a Hcal cell are $\Delta\eta \times \Delta\phi=0.017 \times 0.017$ and $\Delta\eta \times \Delta\phi=0.087 \times 0.087$, respectively. The 1x5 Ecal crystals form a strip. The 5x5 Ecal crystals plus a Hcal cell (near size of the 5x5 Ecal crystals) behind form a trigger tower. The 4x4 trigger towers form a calorimeter region. The energy sum of the strip, the trigger tower or the calorimeter region is used in calorimeter algorithms of L1-trigger.

The list of objects sent from GCT to GT
4 isolated e/ γ candidates
4 non-isolated e/ γ candidates
4 forward-jets
4 central-jets
4 τ -jets
12 jet counts
Total transverse energy
Total transverse energy in jets
Missing transverse energy
Missing transverse energy in jets
Energy in forward hadronic rings

Table 3.1 The list of the objects, which are sent by the global calorimeter trigger (GCT) to the global trigger (GT) of L1-trigger.

3.1.2.1 The Electromagnetic Candidate Algorithm

Description of the object shape:

The electromagnetic objects are ‘compact’ objects, being well contained within a single trigger tower (5×5 ECAL crystals). Furthermore their size in the η direction is even more tightly constrained, being typically as wide as just two crystals, while the extent in the ϕ direction is larger (typically 5 crystals) due to pair-production and the dispersive effect of the 3.8T magnetic field. The ECAL crystals are also 26 radiation-lengths in depth, resulting in near-complete longitudinal containment of the shower.

Fine-grain veto:

Pattern matching is performed to determine whether the ratio of the highest energy pair of adjacent strips (where a strip is defined to be the 5×1 crystals in the $\phi \times \eta$ directions), to the total energy of the tower is lower than some programmable fraction, R . For tower energies below 10 GeV, R is set to 0.95 in order to improve rejection of fake signatures and above 10 GeV, $R=0.9$ to improve acceptance of high energy candidates. If the ratio is lower than R , the so-called fine-grain veto bit is set, otherwise it is unset.

H/E veto:

As the ECAL crystals offer nearly complete longitudinal containment of the shower, for each tower a hadronic veto flag is set if the ratio of transverse energy in the HCAL to that in the ECAL is greater than a programmable threshold (nominally 0.05): such a flag identifies calorimeter towers whose signals are likely to have been produced by hadronic interactions.

e/γ candidates:

The e/γ algorithm is based on windows of 3×3 trigger tower (figure 3.5) tiled repeatedly across the entire span of the ECAL ($|\eta| < 2.5$). In each window it is required that the sum of the energy of the central tower and that of its highest energy neighbour (excluding diagonal neighbours) is greater than a programmable threshold (nominally 15 GeV at low luminosity and 30 GeV at high luminosity). By doing so, candidates of interest are identified even if they are centred on the boundary between towers. It is further required that neither the fine-grain veto bit nor the hadronic veto bit is set for the central tower; this provides restrictions on both the lateral and longitudinal shape of the shower.

Isolation:

If a trigger-tower window passes the above criteria, it is then classified according to the isolation of this trigger-tower window: (i) each of the 8 trigger towers surrounding the central tower must pass the fine-grain and hadronic vetos. (ii) at least one of the four 5-tower corners (marked as grey 'L' shapes in figure 3.5) contains energies below a programmable threshold for each tower. The candidate passing both (i) and (ii) is considered to be an isolated electron/photon, otherwise non-isolated.

Finally, the e/γ candidates are sorted by rank (a measure of its transverse energy and 'quality') and the four highest of isolated/non-isolated types are forwarded to the global trigger (GT).

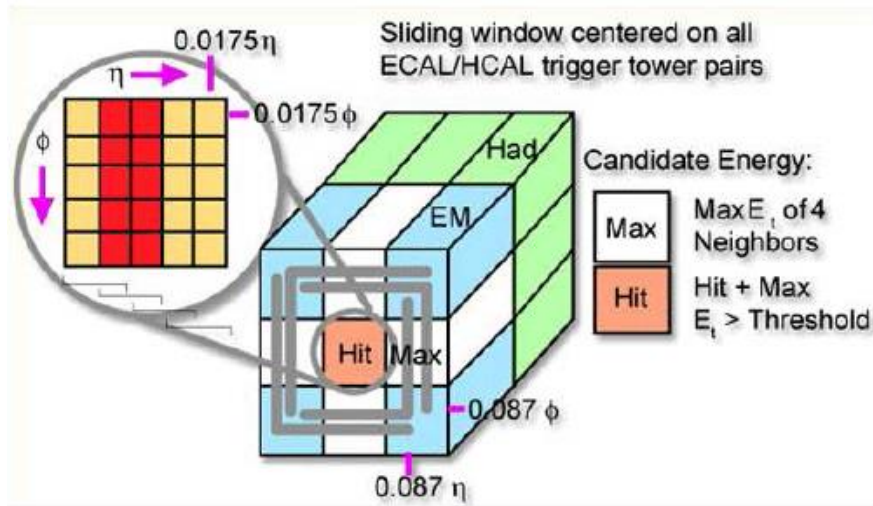


Figure 3.5 illustration of the calorimeter trigger e/γ algorithm. The 1x5 Ecal crystals forms a strip (red). The 5x5 front Ecal crystals (EM) plus a back Hcal cell (Had) form a trigger tower.

3.1.2.2 The Jet Candidate Algorithm

Description of the object shape:

Whereas e/γ objects are compact and well contained, hadronic jets are much larger in both the η and the ϕ directions and must be dealt with differently. Groups of 4×4 trigger towers (ie the 20×20 ECAL crystals) are summed into a calorimeter region. These 4×4 regions have the same size as a region in the forward hadronic calorimeter.

Jet candidates:

Jet candidates are defined by the criterion that the transverse energy in a region is greater than or equal to the transverse energy in each of the eight surrounding regions (as shown in figure 3.6). If the region meets this criterion, then the energy of the jet is defined to be the sum of the transverse energy of this region and its 8 neighbours. Depending on the location of the central region in η , the candidate is labelled as being either a forward-jet ($|\eta| > 3$) or central-jet ($0 < |\eta| < 3$).

τ -jet:

The τ -jet [24] is a jet resulting from the fully hadronic decay of τ lepton (shown in table 3.2), because τ leptons are relatively light and likely to be highly boosted, the hadronic decay of τ leptons results in jets which are more tightly collimated than jets produced by QCD processes. By requiring that at least one of the regions within the jet candidate (3×3 regions) matches one of the predefined patterns shown in Figure 3.6, the lateral extent of the jet can be inferred to be small and the jet is labeled as a τ -jet.

$\tau \rightarrow e \nu_e \nu_\tau,$	17.8 %
$\tau \rightarrow \mu \nu_\mu \nu_\tau$	17.4 %
$\tau \rightarrow \pi^\pm \nu_\tau$	11.1 %
$\tau \rightarrow \pi^0 \pi^\pm \nu_\tau$	25.4 %
$\tau \rightarrow \pi^0 \pi^0 \pi^\pm \nu_\tau$	9.19 %
$\tau \rightarrow \pi^0 \pi^0 \pi^0 \pi^\pm \nu_\tau$	1.08 %
$\tau \rightarrow \pi^\pm \pi^\pm \pi^\pm \nu_\tau$	8.98 %
$\tau \rightarrow \pi^0 \pi^\pm \pi^\pm \pi^\pm \nu_\tau$	4.30 %
$\tau \rightarrow \pi^0 \pi^0 \pi^\pm \pi^\pm \pi^\pm \nu_\tau$	0.50 %
$\tau \rightarrow \pi^0 \pi^0 \pi^0 \pi^\pm \pi^\pm \pi^\pm \nu_\tau$	0.11 %
$\tau \rightarrow K^\pm X \nu_\tau$	3.74 %
$\tau \rightarrow (\pi^0) \pi^\pm \pi^\pm \pi^\pm \pi^\pm \pi^\pm \pi^\pm \nu_\tau$	0.10 %
others	0.03 %

Table 3.2 Most relevant τ -decay branching ratios.

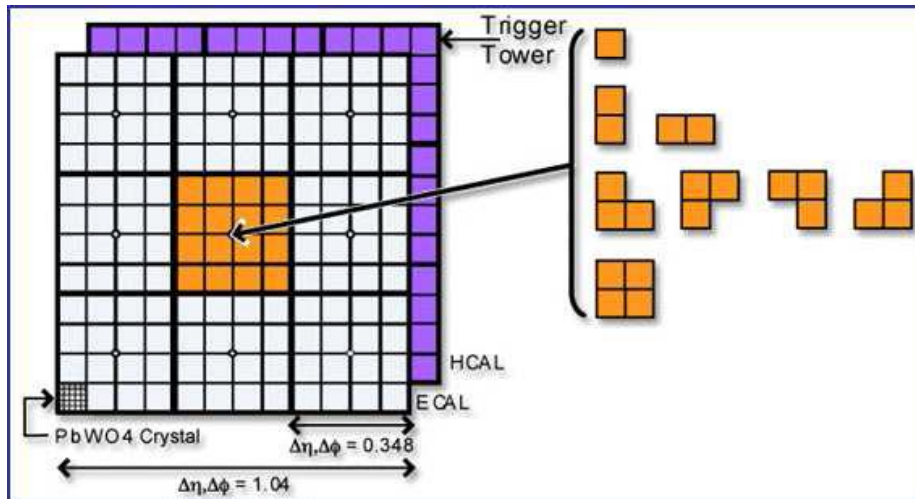


Figure 3.6 illustration of the calorimeter trigger jet algorithm. The hatched element at the lower left corner is an example of trigger tower, which contains 5x5 Ecal crystals (the PbWO4 crystal). The trigger tower is formed by front Ecal crystals (white) and by back Hcal cell (purple). A group of 4x4 trigger towers form a calorimeter region (orange). The patterns in the right are used for the identification of τ -jet.

Finally, the jet candidates are sorted and the four candidates with highest transverse energy of each type, central-, forward- and τ -, are forwarded to the global trigger (GT).

3.1.2.3 Other calorimeter trigger tasks

Jet count - In order to improve the trigger efficiency of rare multi-jet events, the calorimeter trigger counts the number of central, forward and τ -jet objects fulfilling the position criteria and sends twelve counts to the Global Trigger.

Total transverse energy - The transverse energy (E_T) is the energy projecting in the transverse plane, ie $E_T = E \sin\theta$, where the θ is the angle between position vector and positive z-axis in spherical coordinate (see Figure 2.4), and the origin is chosen to be the nominal vertex (the center of detector) before the primary vertex has been estimated. (the transverse momentum, p_T , has the similar definition)

The transverse energies of all calorimeter regions are added as scalar quantities to calculate the **total transverse energy** in an event.

Total transverse energy in jets – the **total transverse energy** that is carried by jets.

Missing transverse energy - The transverse energies of all calorimeter regions are also added as vector quantities in order to calculate the asymmetry in the transverse energy which may be used to infer the presence of neutrinos or other ‘invisible’ particles.

Missing transverse energy in jets – the **missing transverse energy** that is calculated by using jet energy.

Energy in forward hadronic rings (ring triggers) - In order to understand the detector, it is desirable to obtain a large sample of background (minimum bias) data, for which the low-luminosity start-up conditions are ideal. Such events are dominated by low momentum particles in the forward region and, as such, forward- (rather than transverse-) energy must be used to identify them. The ring triggers calculate the total energy in concentric rings of the forward HCAL and the number of towers over threshold within each ring. Both values are sent to the global trigger. Under normal running conditions this trigger is not used.

3.1.3 Global trigger in L1

With the best candidates and objects coming from the global muon trigger (GMT) and the global calorimeter trigger (GCT), the global trigger (GT) decides whether the event will be accepted or rejected. The event meeting one of the GT requirements (a example [22] shown in table 3.3) will be tagged and be forwarded to the HLT for further processing.

Trigger	Threshold (GeV)	Rate (kHz)
Single μ ($\eta < 2.1$)	14	7
Double μ ($\eta < 2.4$)	10, 0	6
Single e/γ	20	13
Double e/γ	13, 7	8
$e/\gamma + \mu$	12, 3.5	3
$\mu + e/\gamma$	12, 7	1.5
Single jet	128	1.5
Quad jet	36	5
H_T	150	5
E_T^{miss}	40	8

Table 3.3 A example of the global trigger (GT) menu and the rates at luminosity $L = 6.6 \times 10^{33} \text{ cm}^2 \text{ s}^{-1}$.

In our analysis, we choose 20GeV and 22GeV for the L1 trigger in the electron channel, and 14GeV and 16GeV for the L1 trigger in the muon channel.

3.2 High Level trigger

In this section I will introduce the high level trigger (HLT) for electrons/photons, for muons and for jets (also for missing transverse energy). The main reference of this part is from [25](2005), and the algorithms and the threshold of HLT may vary with time.

In the following a convention of level 2.0, level 2.5 and level 3.0 will be used, and they mean reconstruction with only calorimeter/moun chamber information, with matching hits in pixel detector, and with full tracker information respectively.

3.2.1 electrons/photons

Electrons and photons are reconstructed using similar algorithms. In the following text, the electron algorithm will be introduced first.

In the beginning, I explain some important issues of electron reconstruction in calorimeters: the clustering in calorimeter [26], the position measurement, and the energy measurement. Then I mention the reconstruction at level-2.0, level 2.5 and level 3.0 for electrons.

Clustering in calorimeter

The first step in the electron reconstruction in the High-Level Trigger is the clustering of the energy deposits in the ECAL and the estimate of the electron's energy and position from this information. The electron energy can be collected by forming a group of clusters along the ϕ direction. This cluster of base clusters is called a "super-cluster" (SC). For the clustering of base clusters and the super-cluster, the island algorithm, the hybrid algorithm, and the endcap reconstruction with the Pre-shower detector are introduced.

The island algorithm starts by searching for seeds which are defined as crystals with energy above a certain threshold. Starting from the seed position, adjacent crystals are examined, scanning first in the ϕ direction and then in the η direction. Along each scan, crystals are added until either a rise in energy or a crystal that has no read-out is encountered. The clustering starts from the seed with the highest

transverse energy ET in the list of seeds, and the crystals included in the cluster are marked after being used so to be included only in a single cluster. The sequence is sketched in Figure. 3.7.

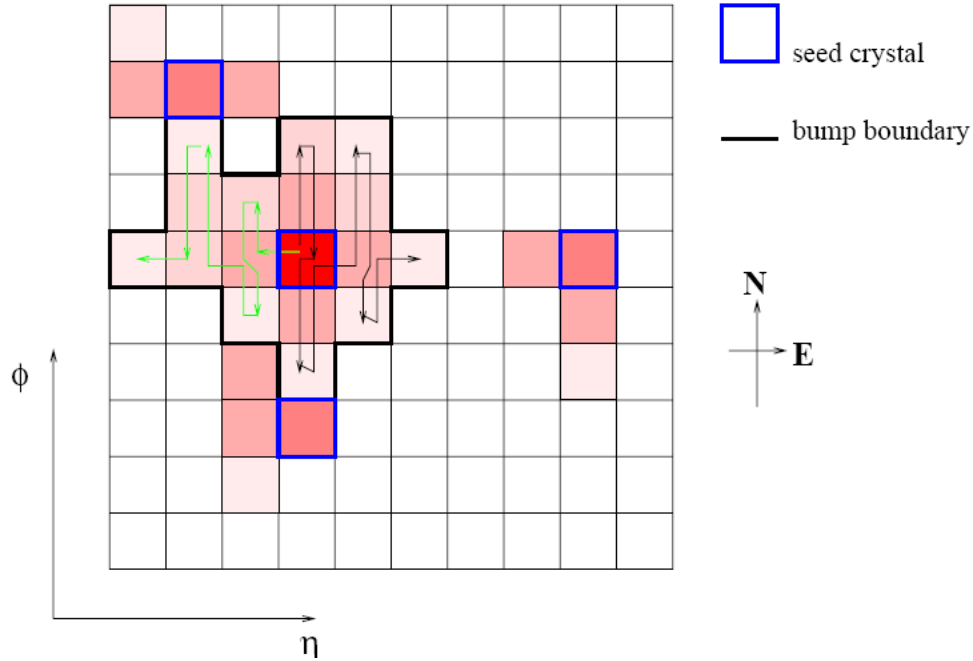


Figure 3.7 Illustration of the Island clustering algorithm in the Barrel ECAL. Each square represents an Ecal crystal. The darkness of red color in a crystal represents the energy density. The crystal surrounding with blue line is the seed crystal. The arrow line shows the scan is first in the ϕ direction and then in the η direction. After scanning, crystals within the broad black line form a cluster.

For the super-cluster, the procedure is seeded by searching for the most energetic cluster and then collecting all the nearby clusters in a very narrow η -window and a much wider ϕ -window (see figure 3.8).

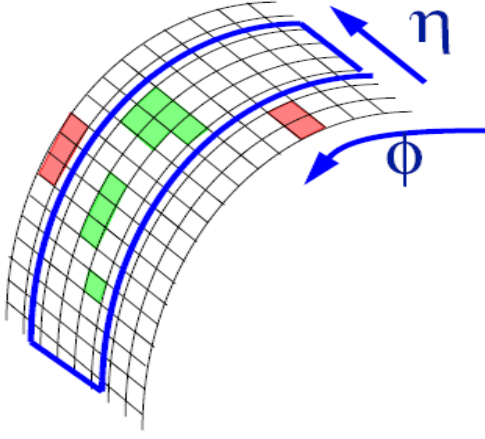


Figure 3.8 Illustration of the super-clusters. The colored elements form clusters. The blue window is the range of a super-cluster.

The hybrid algorithm starts from a seed crystal. If the transverse energy of the seed crystal is higher than the threshold ($E_T > E_{T \text{ hybseed}}$), the 1x3 crystal dominoes are made, each with their central crystal aligned in the η direction with the seed crystal. If the energy of the central crystal of a domino is greater than E_{wing} then a 1x5 domino is used. This making of dominoes proceeds N_{step} crystals in each ϕ direction from the original seed. Dominoes with energy less than E_{thresh} are eliminated. The domino construction step of the algorithm is illustrated in Figure 3.9.

The dominoes are then clustered in the ϕ direction. Each distinct cluster of dominoes is required to have a seed domino with energy greater than E_{seed} . We thus end up with a cluster of clusters, entirely analogous to super-clusters of Island clusters. The default values of the control parameters are shown in Table 3.4.

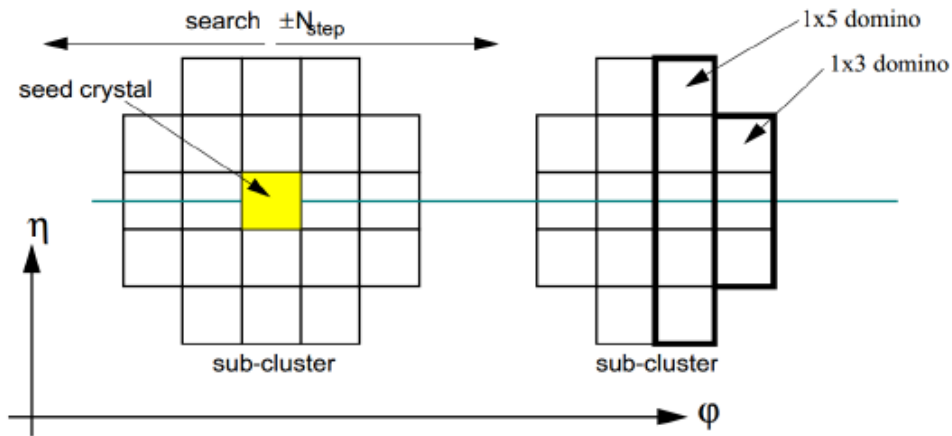


Figure 3.9 Domino construction step of Hybrid algorithm.

Parameter description	label used in text	default value
Minimum E_T for Hybrid super-cluster seed crystal	E_T^{hybseed}	1 GeV
Number steps (crystals) for search in ϕ (in each direction)	N_{step}	17
Threshold for using 1x5 crystals (rather than 1x3)	E_{wing}	1 GeV
Threshold for using domino	E_{thresh}	0.1 GeV
Minimum domino to make a disconnected subcluster	E_{seed}	0.35 GeV

Table 3.4 Default values of control parameters for Hybrid algorithm.

The endcap reconstruction with the Pre-shower detector: Much of the endcap is covered by a preshower detector with two planes of silicon strip readout. The energy deposited in the preshower detector (which is about 3 radiation lengths X_0 thick) needs to be added to the crystal clusters. The energy in the crystals is clustered using the Island algorithm and the clusters are associated to form super-clusters. A preshower cluster is constructed in each plane, in front of each crystal cluster of the super-cluster. The search area in the preshower is centered on the point determined by extrapolating the crystal cluster position to the preshower plane in the direction of the nominal vertex position.

Position measurement

For measuring the shower position, two features need to be addressed in more detail in order to obtain a precise position measurement.

The first feature is the calculation of the mean of crystal position. Since the energy density does not fall linearly with distance from the shower axis, but rather exponentially, the logarithmic energy fraction is used for the weight in the average of position. The weighted average formula is:

$$x = \frac{\sum x_i \cdot W_i}{\sum W_i}$$

with the sum over all crystals of the cluster, and where x_i is the position of crystal i , and W_i is the log weight of the crystal - the logarithm of the fraction of the cluster energy contained in the crystal, calculated with the formula:

$$W_i = W_0 + \ln \frac{E_i}{\sum E_j}$$

where the weight is constrained to be positive, or is otherwise set to zero. The W_0 controls the smallest fractional energy that a crystal must have to contribute to the position measurement [26].

The second feature is the precise definition of the "crystal position". Because the crystals are "off-pointing" (do not exactly point to the nominal interaction vertex), the lateral position (η, ϕ) of the crystal depends upon the depth of shower on its axis (see Fig 3.10). The shower depth is approximate to the longitudinal gravity center of shower, and its optimal mean value varies logarithmically with the shower energy. The energy dependence is accounted for with a logarithmic parametrization [26].

Practically in hybrid and island algorithm, the depth t_{max} (see Fig 3.10) is measured relative to the front face of the crystals along the direction from the nominal vertex position, O, to the approximate shower entering position, P, which is calculated using the arithmetic energy weighted mean of the shower front face centers. Projecting the t_{max} to the crystal axis, the real depth on the axis is determined, and so does the lateral position (η, ϕ).

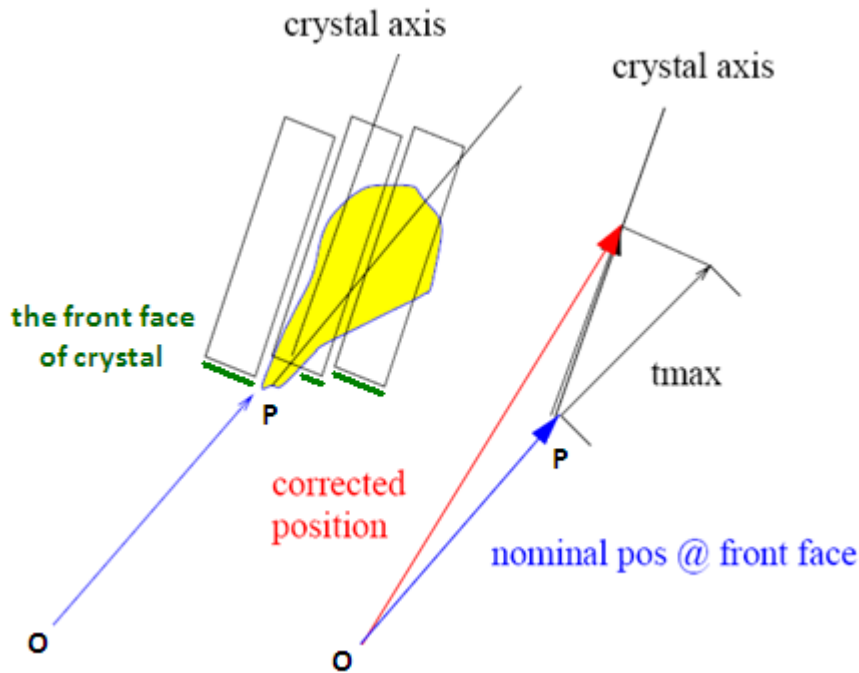


Figure 3.10 Illustration of the crystal off-pointing (the left) and the schematic diagram (the right). The nominal position (blue) and the corrected position (red) have different (η, ϕ) . The depth t_{max} is measured relative to the front face of crystal along the direction, O to P.

Energy measurement and corrections

The energy measurement of an electron is obtained by simply addition of the energy deposit in crystals, which are clustered by the island algorithm or the hybrid algorithm. However, in simulation one can find that the energy ratio at the reconstruction level (simulation of the detector response and the reconstruction algorithms) over the generator level (simulation of the production of particles in the collision), E_{meas}/E_{true} , peaks at few percent less than unity (see Figure 3.11). Thus a correction for the energy measurement is needed.

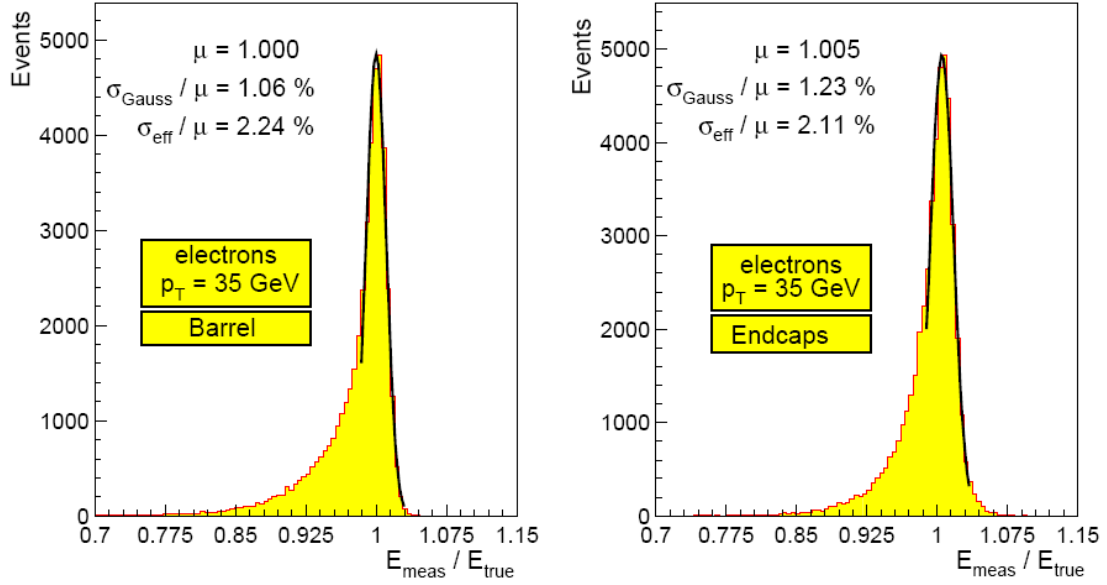


Figure 3.11 Distribution of $E_{\text{meas}}/E_{\text{true}}$ for the $p_T = 35$ GeV/c electron sample. Electrons in the barrel Ecal, which are reconstructed with the Hybrid super-clustering algorithm, is shown on the left. Electrons in the endcap Ecal, which are reconstructed with the Island super-clustering algorithm and with preshower energy included, is shown on the right. The variable μ shown in the curve is the mean value of the Gaussian fit.

In the distribution of the ratio ($E_{\text{meas}}/E_{\text{true}}$), the long tail on the low side is due to non-recovered bremsstrahlung energy, which is induced by the tracker material. The amount of tracker material varies strongly with η , so a variation in the fraction of events in the tail as a function of η is expected. The corrections are parametrized in terms of the number of crystals in the cluster ($f(N_{\text{cry}})$ corrections) (see Figure 3.12). This helps to minimize the residual dependence on both E and η of the energy scale.

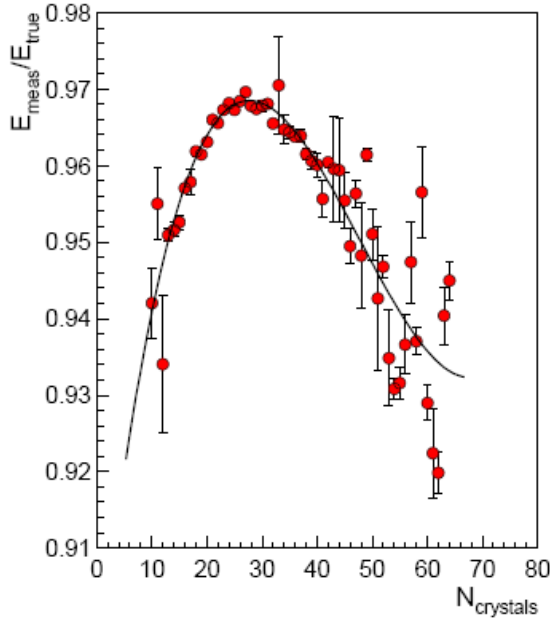


Figure 3.12 $E_{\text{meas}}/E_{\text{true}}$ as a function of the number of crystals in a Hybrid super-cluster together with a fitted polynomial function.

Level-2.0 of the electron trigger

The first step of the HLT, using only calorimeter information, is to reconstruct an ECAL super-cluster in a region specified by the Level-1 trigger. The super-cluster is first required to fall within the fiducially region of the ECAL(remove the barrel/endcap transition region $1.4442 < |\eta| < 1.5660$).

The super-cluster is then required to have ET above a threshold (26 GeV/14.5 GeV for the single/double electron trigger at $L=2 \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, and 31 GeV/16.9 GeV for the single/double electron trigger at $L=10^{34} \text{ cm}^{-2} \text{ s}^{-1}$).

Level-2.5: matching of super-clusters to hits in the pixel detector

After level-2.0, the level-2.5 performs a matching between the super-cluster and the hits, which is left by electrons (or by few converted photons), in the pixel detector. In most cases, at least one hit will be left in either barrel layers or endcap disks of the pixel detector (see Figure 3.13). It takes advantage of the fact that the energy-weighted average impact point of an electron and the bremsstrahlung photons that it has radiated, is precisely where a non-radiating electron would have impacted in the calorimeter.

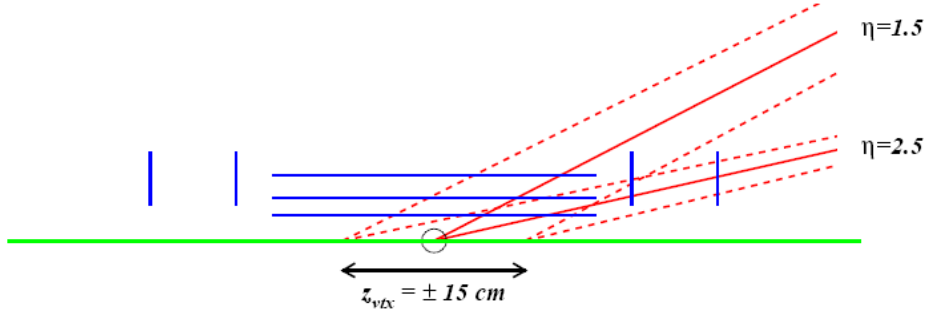


Figure 3.13 Full pixel detector with continuous lines pointing from the nominal vertex to the edges of the ECAL barrel and endcap, and dashed lines pointing from $z = \pm 15$ cm.

The method proceeds as follows:

- (i) The electromagnetic cluster gives the energy and the position of the electron candidate. The transverse momentum is computed from the measured energy and position of the cluster. The expected hit position on the pixel layers is estimated by propagating the electron inwards to the nominal vertex using the magnetic field map. A search area is defined in the innermost pixel layer. It is unrestricted in z , and its width in ϕ (typically 40 mrad) is the main parameter used to control the tightness of the pixel matching.
- (ii) If a compatible hit is found within the search area on the innermost pixel layer, a better estimate of the longitudinal (z) vertex position is obtained by interpolating the line from the cluster through the corresponding hit to the beam line. Nominal values (0,0) for x and y coordinates of the vertex are assumed. If no hit is found in the search area of the innermost pixel layer, the search is repeated in the next layer.
- (iii) The track is propagated from the newly estimated vertex to the next pixel layer through the compatible hit in the first (or second) layer.
- (iv) If another compatible hit is found, the cluster is identified as an electron, if not, it is rejected as a jet. If there are no compatible hits in the current layer, there may be one more pixel layer (outward) left, and the search is repeated there.

The search (i) ~ (iv) is made twice, once for each charge ("-" of electron and "+" of

positron).

Level-3.0 Inclusion of Full Tracking Information

In Level-3, all tracker information is available, so it starts with electron track-finding,, seeded by the Level-2.5 pixel match. The detail of the track-finding can be found in [27].

After the electron track is found, loose cuts can be applied to maintain the high signal efficiency. These cuts includes the ratio of energy/momentum (E/P), the distance between the super-cluster position and the extrapolated track position in the ECAL in the longitudinal coordinate $\Delta\eta(\text{track} - \text{cluster})$, the energy deposit ratio of Hcal and Ecal (H/E), the number of hits associated in the tracks, and the isolation cuts. The isolation cuts will be introduced in Section 4.5.

The text above describes the HLT selection of **electrons**. For **photons**, the super-cluster of single and double triggers that fail the level-2.5 pixel matching must satisfy E_T thresholds, higher than those applied at level-2.0 for electrons. The events passing these cuts form the photon stream. The background of photon can be rejected by applying the tracker isolation cuts and the lateral shower shape cut $\sigma_{\text{in}}^{\text{in}}$. The lateral shower shape cut will also be introduced in Section 4.4.

3.2.2 Muon

Muon reconstruction in HLT is composed of 2 parts; the level-2, which is the track reconstruction with only the information from the muon system, and the level-3, which is the track reconstruction with the information including both tracker and muon system. The algorithms in the muon HLT are the same as those in offline. Both in level-2 and in level-3, the isolation cuts (introduced in Section 4.5) can be applied to provide the further rejection of background.

Level-2: reconstructing tracks within the muon system (the standalone muon)

At the level-2 of muon reconstruction, track segments are built within individual muon chambers using a linear fit to the positions of the hits reconstructed in each of the 8-12 (in case of DT) or 6 (in case of CSC) layers of the chamber. These segments are used to generate “seeds” consisting of position, direction vectors and an initial estimate of the muon transverse momentum. The seeds serve as starting points for the track fits in the muon system, which are performed using segments and hits from DTs, CSCs, and RPCs, and are based on the Kalman-filter technique [28]. The result is a collection of tracks reconstructed using only information from the muon system, which are referred to as “**standalone muons**”. The standard reconstruction algorithm for standalone muons is described in section 9.1.1 of ref. [29].

Level-3: reconstructing tracks including the tracks in the tracker (the global muon)

At the level-3 of muon reconstruction, each standalone-muon track is compared with the tracks reconstructed in the tracker (referred to as “tracker tracks”), and the best-matching tracker track is selected. For each “tracker track” — “standalone muon” pair, the track fit using all hits in both tracks is performed, again based on the Kalman-filter technique and taking into account the average expected energy losses, the magnetic field, and multiple scattering in the detector materials. The result is a collection of tracks referred to as “**global muons**”. More details on the standard reconstruction of global muons can be found in section 9.1.2 of ref. [29].

3.2.3 Jet

To identify and select a jet at HLT, a simple and fast algorithm, the iterative seed cone algorithm (in section 11.2.1 of ref. [29]), is used. Besides, the jet energy correction is needed to be applied at HLT. Both issues will be mentioned in following text.

Iterative seed cone algorithm

In the algorithm, all calorimeter towers are used, and the jet and its constituents are consider as massless particles so that the transverse energy is equating to the transverse momentum ($E_T = c.p_T$) and the direction is determined by the momentum.

The algorithm proceeds as follows:

(i) An E_T -ordered list of input objects (particles or calorimeter towers) is created. The input objects having transverse energy above the threshold form the seeds. The seed are also ordered by transverse energy E_T .

(ii) From the first seed, a cone of size R in the η - ϕ space

$$R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$$

is cast around the seed.

(iii) From all input objects inside the cone, the one having maximum energy is chosen to add with the seed, and they form a new proto-jet.

(iv) From the rest of input objects inside the cone, the next one (ordered by E_T) is chosen to add with the present proto-jet to form a new proto-jet again.

(v) The (iv) is repeated until the energy of the proto-jet changes by less than 1% between iterations and the direction of the proto-jet changes by $\Delta R < 0.01$, then the proto-jet becomes the stable proto-jet.

(vi) When a stable proto-jet is found, all objects in the proto-jet are removed from the list of input objects and the stable proto-jet is added to the list of jets. The procedure (ii)~(v) is repeated on the next seed until all seeds finish.

The two parameters, the cone size and the seed threshold, can be optimized using the simulation sample, where the matching between the generator-level jet and the reconstruction-level jet is performed.

Jet energy correction

The CMS calorimeter system is optimized for the precision measurement of electrons and photons. Since it has a non-linear response to jets (main composed of pions), the correction for jet energy measurement is needed. The jet energy also has to be corrected for noise in the electronics and for pileup energy. More detail of the jet energy correction can be found in [30].

3.2.4 Missing transverse energy

The missing transverse energy at HLT is the same definition as at level-1, but the contribution from jet is corrected with the jet energy correction.

3.3 The triggers used in our analysis

The triggers used in our analysis are list in table 3.5. One can find the whole trigger path of the certain run range for both the muon channel and the electron channel, and the detail of each trigger in the path can be found in the ref. [31].

Trigger path	Run range	$\mathcal{L}$ (pb^{-1})
Muon Channel		
HLT_IsoMu17_eta2p1_TriCentralPFJet30	190645–193621	876.2
HLT_IsoMu17_eta2p1_TriCentralPFNoPUJet30	193834–194225	555.1
HLT_IsoMu17_eta2p1_TriCentralPFNoPUJet30_30_20	194250–199648	5590.6
HLT_IsoMu17_eta2p1_TriCentralPFNoPUJet45_35_25	199698–208686	12690.2
Electron Channel		
HLT_Ele25_CaloIdVT_CaloIsoT_TrkIdT_TrkIsoT_TriCentralPFJet30	190645–191411	484.8
HLT_Ele25_CaloIdVT_CaloIsoT_TrkIdT_TrkIsoT_TriCentralPFNoPUJet30	191691–194225	946.5
HLT_Ele25_CaloIdVT_CaloIsoT_TrkIdT_TrkIsoT_TriCentralPFNoPUJet30_30_20	194250–199648	5590.6
HLT_Ele25_CaloIdVT_CaloIsoVL_TrkIdVL_TrkIsoT_TriCentralPFNoPUJet45_35_25	199698–208686	12690.2

Table 3.5 Summary of the trigger paths used for data selection, including valid run ranges and integrated luminosity.

Taking the HLT trigger

path “HLT_Ele25_CaloIdVT_CaloIsoT_TrkIdT_TrkIsoT_TriCentralPFNoPUJet30_30_20”

and the L1 trigger “L1_SingleEG20” as two examples, the meanings of terms are:

Ele25= single electron, whose transverse energy threshold is the 25GeV.

CaloIdVT = calorimeter ID at very tight level.

CaloIsoT = calorimeter isolation at tight level.

TrkIdT = tracker ID at tight level.

TrkIsoT = tracker isolation at tight level.

TriCentralPFNoPUJet30_30_20 = three central Particle flow jets without pileup, whose transverse energy threshold are the 30, 30 and 20GeV respectively.

SingleEG20= single electron/photon, whose transverse energy threshold is the 20GeV.

The ID/isolation variables of calorimeter/tracker and the Particle flow algorithm will be introduced in detail in Chapter 4.

Chapter 4 Event reconstruction in offline and our event selection

In this chapter, I will introduce the reconstruction in offline. The data, which are saved in disks after HLT, are used for reconstruction in offline. The basic techniques of reconstruction in offline are the same of those in online (see chapter 3), like the super-clustering, the matching SC to hits in pixel, the track reconstruction, the standalone muon & the global muon, but the algorithms, which are more complicated and time-consuming than those in online, are used for the full event reconstruction.

The offline algorithms and selection criteria including the anti-kt jet algorithm, the particle flow (PF) algorithm, the r_9 variable, the missing hits, the shower shape variable $\sigma_{\text{in}}^{\text{in}}$ and the isolation variables will be introduced in the following text. Finally, the event selection (the signal region) in our analysis is described.

4.1 The anti-kt jet algorithm

The anti-kt algorithm[32], which is associated to the kt algorithm[33] and the Cambridge/Aachen algorithm[34], is based on the two distance definitions, d_{ij} and d_{iB} .

d_{ij} : the distance between two particles i and j .

$$d_{ij} = \min(k_{Ti}^{2p}, k_{Tj}^{2p}) \frac{\Delta y^2 + \Delta \phi^2}{R^2}$$

with

$$\Delta y^2 = (y_i - y_j)^2$$

and

$$\Delta \phi^2 = (\phi_i - \phi_j)^2,$$

where k_{Ti}, y_i and ϕ_i are the transverse momentum, rapidity and azimuth of particle i respectively. The jet radius parameter R , which determines the jet size, is set to be 0.5 and 0.7 in CMS. The parameter p , which sets the power of the transvers momentum scale, is set to be -1 in the anti-kt algorithm.

d_{iB} : the distance from particle i to the beam. It is defined to be

$$d_{iB} = k_{Ti}^{2p}$$

The anti-kt algorithm proceeds as follows:

- (i) For each pair of particles i and j, it work out the anti-kt distance d_{ij} , and for each particle i it also work out the beam distance d_{iB} .
- (ii) Find the minimum $d_{\min}$ of all the d_{ij} and d_{iB} . If $d_{\min}$ is a d_{ij} , merge particles i and j into a single particle, summing their four-momenta; if it is a d_{iB} , then declare particle i to be a final jet and remove it from the list.
- (iii) Repeat from step 1 until no particles are left.

Figure 4.1 shows the anti-kt algorithm result.

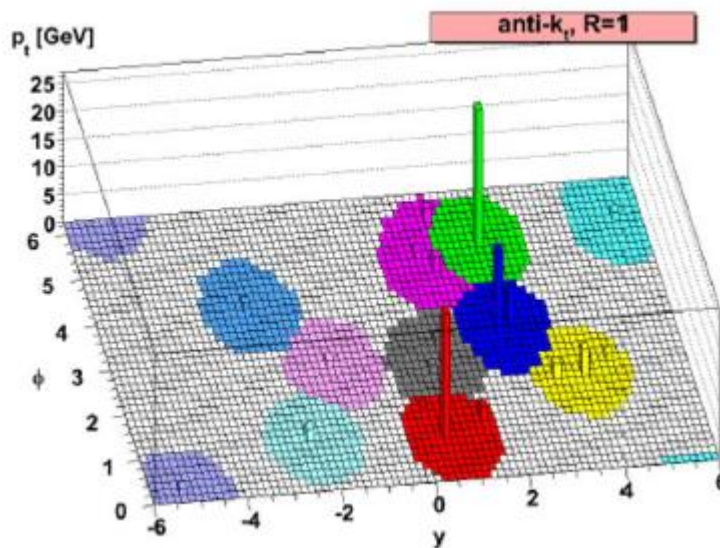


Figure 4.1 Colored areas represent anti-kt clustered jets.

4.2 The particle flow (PF) algorithm

The particle flow (PF) algorithm [35-37] can reconstruct an event to the level of individual particles, and its success relies on an efficient and pure track reconstruction, on a clustering algorithm able to disentangle overlapping showers, and on an efficient link procedure to connect together the deposits of each particle in the sub-detectors. The algorithm follows two principles: (1) separating charged particles and the neutral particles, (2) starting the reconstruction with the order from the easiest object, ie muon 1st, electron 2nd, charged hadron 3rd and so on.

The algorithm proceeds as follows:

- (i) Reconstruct the muon object, which includes the track in tracker and the track in muon system. The tracks of muon will not give rise to other object's tracks in later steps.
- (ii) Extrapolate the track in tracker to clusters in calorimeter to reconstruct the the charged hadrons (the cluster in Ecal+Hcal). The sets of track and cluster(s), which constitute a charged hadron, are not considered anymore in the rest of the algorithm.
- (iii) Similar to (ii) to reconstruct the electron (the cluster in Ecal). Due to the frequent Bremsstrahlung photon emission of electron, a specific track reconstruction [38] is needed as well as a dedicated treatment to properly attach the photon clusters to the electron and avoid energy double counting. (see Figure 4.2)
- (iv) Once all the tracks are treated, the remaining clusters result in photons in case of the electromagnetic calorimeter (ECAL) and neutral hadrons in the hadron calorimeter (HCAL).
- (v) For the particle found in (iv), combine the information of the sub-detectors to determine optimally its four-momentum. If the calibrated calorimeter energy of the clusters associated to a track is found to be in excess with respect to the track momentum at more than one sigma, the excess is attributed to an overlapping neutral particle (photon or hadron), carrying an energy corresponding to the difference of the two measurements.
- (vi) The resulting list of particles, namely charged hadrons, photons, neutral hadrons, electrons and muons, is then used to reconstruct the jets, the missing transverse energy (E_{miss}^T), to reconstruct and identify the taus from their decays products and to measure the isolation of the particles.

The schematic of particle flow algorithm shows in figure 4.3.

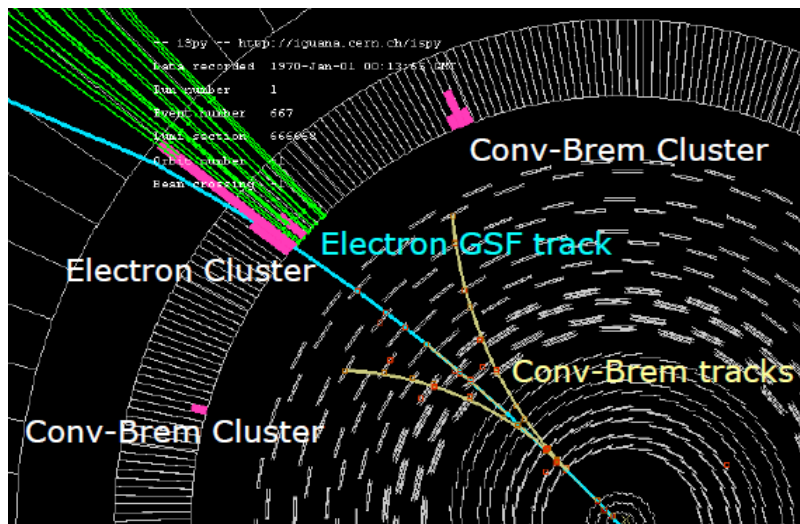


Figure 4.2 The consideration of the Bremsstrahlung photon emission of electron. The red spots are the hits in tracker. The pink-colored blocks are the energy deposit in calorimeter that forms the cluster. The blue line shows an electron track that associated to an electron cluster after extrapolating. The two yellow lines show the possible track of Bremsstrahlung photons (no red spot on it) and the track of the secondary electrons (with red spot on it), which are converted from Bremsstrahlung photons. The yellow lines are end with the extrapolating clusters (marked with Conv-Brem Cluster).

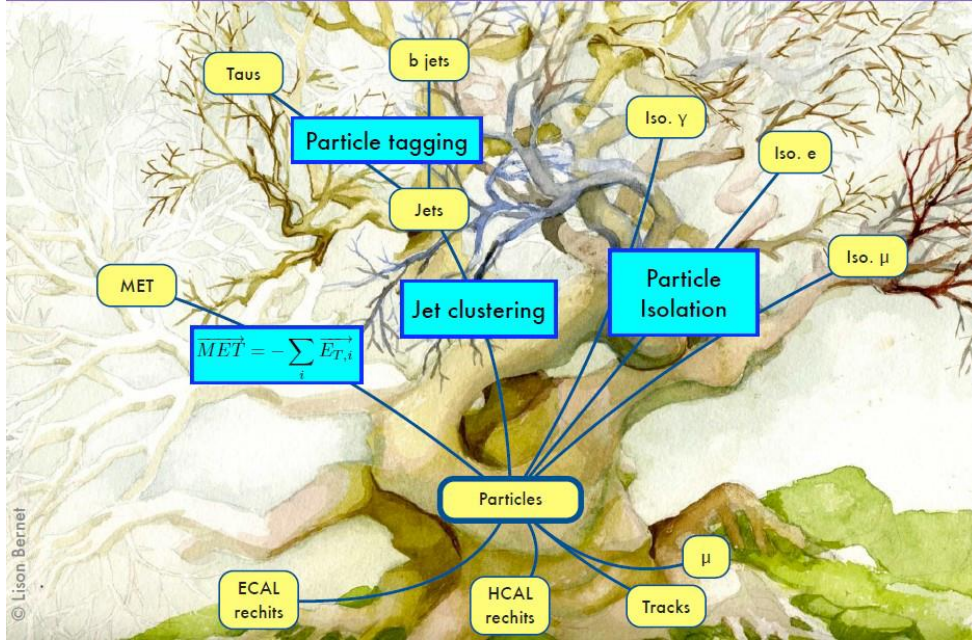


Figure 4.3 The schematic diagram of particle flow algorithm. The tree represents the object flow in the algorithm. The inputs (in root) are the energy deposits in Ecal&Hcal, the tracks, and the muon object. All inputs (in stem) are reconstructed to individual particles, and they are sent to perform different procedures and calculations (in branches): the missing E_T calculation, the jet clustering following the particle tagging and the isolation calculation.

4.3 The r9 variable and the missing hits

r9 is one of the photon ID variables. The definition is $r9 = E_{3 \times 3} / E_{SC}$, the energy ratio of the energy in the 3×3 crystal window around the super-cluster seed divided by the total energy of the super-cluster. Due to the energy of the converted photon, which becomes the electron pair, is spread along ϕ due to the magnetic field, the small values of r9 are typical for the converted photons. The candidates having r9 more than 0.94 (0.95) in EB (EE) are regard as the unconverted photon. The fake photon (from π^0) has the lower r9 value than isolated photons'.

Missing hits, the number of missing hits in the inner most layer of tracker, is one of the electron ID variables. In the contrary to the prompt electron, which is generated at the primary vertex and will leave hits in the inner most layer of tracker, the electrons coming from photon conversion may not leave hits in the inner most layer of tracker (see figure 4.4). Therefore, the missing hits can be used to reject the electrons from photon conversion.

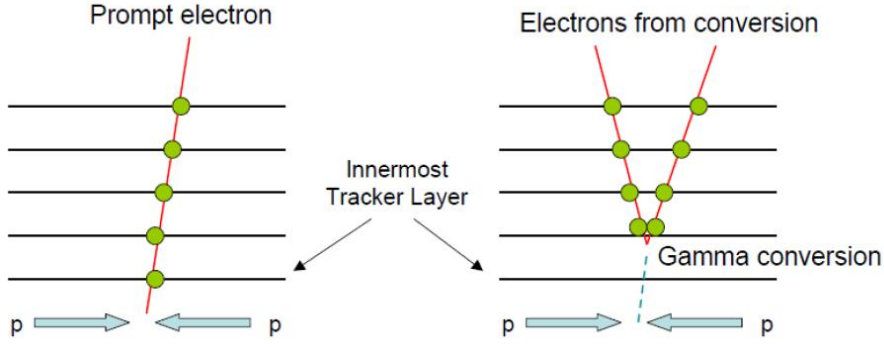


Figure 4.4 Missing Hits. The left hand side shows the track of prompt electron, which frequently has a hit in the innermost track layer. The right hand side shows the track of the converted photon, which often does not have hits in the innermost track layer.

4.4 The shower shape variable $\sigma_{\eta\eta}$

$\sigma_{\eta\eta}$ is the average width of the electromagnetic cluster measured along the η direction and computed with logarithmic weights[39]:

$$\sigma_{\eta\eta}^2 = \frac{\sum (\eta_i - \bar{\eta})^2 \omega_i}{\sum \omega_i}$$

$$\bar{\eta} = \frac{\sum \eta_i \omega_i}{\sum \omega_i}$$

$$\omega_i = \max(0, 4.7 + \log\left(\frac{E_i}{E_{5 \times 5}}\right))$$

Where E_i and η_i are the energy and pseudorapidity of the i -th crystal within a matrix of 5×5 crystals cluster seed. The $E_{5 \times 5}$ and $\bar{\eta}$ are the sum of the energy and the weighted average of the pseudorapidity in the matrix. The 4.7 value controls the smallest fractional energy that contributes to the position measurement.

Because the energy deposits spread in the ϕ direction, the width of the η direction of the cluster does not be strongly affected by the magnetic field, and the value of $\sigma_{\eta\eta}$ of an isolated photon is typically smaller than the fake photons' (from π^0 mainly).

In our analysis, the $\sigma_{\eta\eta}$ shape is used to distinguish the signal photon and the background photon in the template fitting.

4.5 The isolation variables

The isolation variable is defined to measure the degree of isolation of an object, ie the jet activity around the object. There are several isolations used for electron/photon and muon, all of them are similar definition: calculate the energy/momentum within a cone around the object (electron, photon or muon), but remove the inner cone (sometimes plus a rectangular strip) component that contains the energy/momentum of the object. Three photon isolations are described as below:

IsoTrk is the scalar sum of the transverse momentum of track originating from the primary interaction vertex and lying in a hollow cone of size $\Delta R=0.4$ around the photon momentum direction. An inner cone $\Delta R = 0.02$ and a rectangular strip of $\Delta\eta \times \Delta\phi = 0.015 \times 0.4$ are excluded to avoid including the momentum of conversion tracks.

IsoECAL is the sum of the transverse energy collected by the electromagnetic calorimeter crystals in a cone $0.06 < \Delta R < 0.4$ around the photon direction. The contribution due to conversion is also removed by excluding the sum of transverse energy in a $\Delta\eta \times \Delta\phi = 0.04 \times 0.4$ strip. In addition, the energy in a crystal must satisfy $|E| > 0$ MeV to reduce electronics noise.

isoHCAL is the sum of the energy collected in the HCAL towers in a cone $\Delta R < 0.4$ around the photon momentum direction, excluding an inner cone of radius $\Delta R < 0.15$, which corresponds to the region used to measure the H/E ratio.

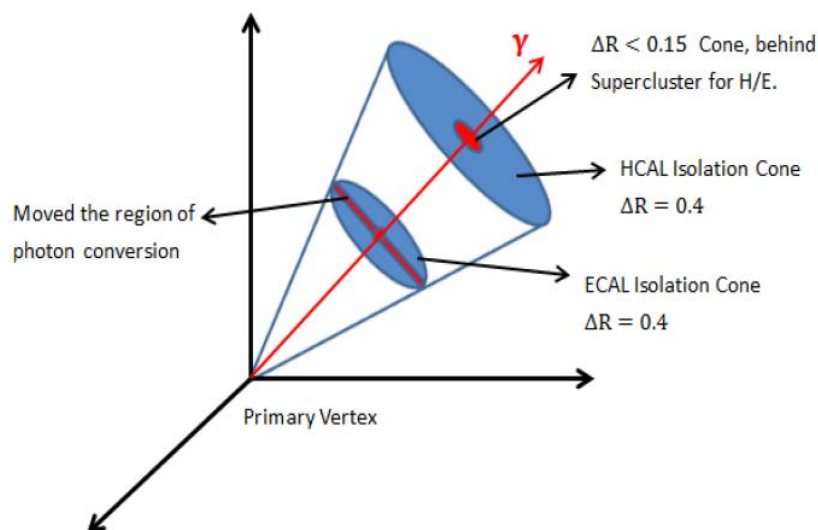


Figure 4.5 Sketch of the photon isolation (IsoECAL and isoHCAL).

4.6 The event selection in our analysis

In this section the event selection of our analysis is described. Our requirements for each object, the primary vertex, muon, electron, photon and jet, are described first, and then dose the event selection of all objects (the signal region).

The primary vertex

The primary vertex is required as follows:

- more than 10 tracks originating from it, with at least 25% of those being high purity;
- at least 4 degrees of freedom;
- an impact parameter with respect to the beamspot in z , d_z , that satisfies $|d_z| < 24$ cm;
- an impact parameter with respect to the beamspot in the xy plane, d_{xy} , that satisfies $d_{xy} < 2$ cm.

The detail of primary vertex reconstruction can be found in ref. [40].

Muon

Muon candidates are required to have $p_T > 20\text{GeV}/c$, be within $|\eta| < 2.1$, and pass the “Tight Muon” selection criteria [41]:

- reconstructed as a global muon;
- particle-flow muon identification;
- χ^2/ndof of the global muon track fit < 10 ;
- at least one muon chamber hit included in the global-muon track fit;
- muon segments is in at least two muon stations;
- its tracker track has transverse impact parameter $d_{xy} < 2\text{mm}$ with respect to the primary vertex;
- its tracker track has a longitudinal distance $d_z < 5\text{mm}$ with respect to the primary vertex;
- at least one pixel hit;
- and more than 5 tracker layers that have hits.

- have a combined-relative-particle-flow isolation [41]

$$I_{\text{rel}}^{\mu} = \frac{\sum p_T^{\text{ch. had.}} + \max(0.0, \sum p_T^{\text{neu. had.}} + \sum p_T^{\gamma} - 0.5 \times \sum p_T^{\text{PU}})}{p_T^{\mu}} < 0.12,$$

where the $\sum p_T^{\text{ch. had., neu. had., } \gamma}$ is the summed p_T of particles (charged hadrons, neutral hadrons, or photons, respectively) in a cone of $\Delta R < 0.4$ around the muon, the $\sum p_T^{\text{PU}}$ is the summed p_T of charged particles not originating from the primary vertex in the same cone, and the p_T^{μ} is the muon p_T .

Events are removed from consideration that have an additional muon having:

- The $p_T > 10\text{GeV}/c$,
- The $|\eta| < 2.5$,
- $I_{\text{rel}}^{\mu} < 0.2$,
- passing the “LooseMuon” criteria [41]: reconstructed as a global muon or tracker muon and having particle-flow muon ID.

Electron

Electron candidates are required to have $p_T > 30\text{GeV}/c$, be within $|\eta| < 2.4$ (not excluding the barrel-endcap transition region $1.4442 < |\eta| < 1.566$), and pass the “tight electron” selection criteria defined in [42]:

- $\Delta\eta(\text{SC, track})$: An electron will leave a track when it goes through the tracker, and deposit most of its energy in ECAL. The difference, $\Delta\eta$, between the energy weighted mean position and the extrapolated position from the associated GSF track is required to be less than 0.004 and 0.005 for the barrel and endcap, respectively.
- $\Delta\phi(\text{SC, track})$: Similar to the $\Delta\eta(\text{SC, track})$, this variable is defined in the ϕ direction, and required to be less than 0.03 for the barrel and less than 0.02 for the endcap.
- $\sigma_{\eta\eta}$: This shape variable, measured in the η direction in ECAL, can discriminate strongly against jets. We require $\sigma_{\eta\eta}$ to be less than 0.01 (0.03) in the barrel (endcap).
- $E_{\text{HCAL}}/E_{\text{ECAL}}$: Because an electron will deposit most of energy in ECAL rather than HCAL, the ratio, $E_{\text{HCAL}}/E_{\text{ECAL}}$, is required to be less than 0.12 (0.10) in the barrel (endcap).

- d_{xy} : Its Gaussian Sum Filter (GSF) track has a transverse impact parameter $d_{xy} < 0.2\text{mm}$ with respect to the primary vertex.
- d_z : Its GSF track has a longitudinal impact parameter $d_z < 1\text{mm}$ with respect to the primary vertex.
- $|1/E - 1/p|$: The difference between inverse of its ECAL energy and inverse of its track momentum at an associated vertex should be less than 0.05.
- A combined-relative-particle-flow isolation with applying a p correction,

$$I_{\text{rel}}^e = \frac{\sum p_T^{\text{ch. had.}} + \max(0.0, \sum p_T^{\text{neu. had.}} + \sum p_T^\gamma - A_{\text{eff.}} \times \rho)}{p_T^e} < 0.1,$$

where $\sum p_T$ now represents the summed p_T of particles within a cone of $\Delta R < 0.3$ around the electron and p^e is the electron p_T . The effect of neutral hadrons and photons from pile-up is corrected for by the factor $A_{\text{eff.}} \times \rho$, where the ρ is the average energy density due to pile-up, obtained from simulation using FastJet, and the $A_{\text{eff.}}$ is the effective area, chosen such that the contribution is independent of the number of pile-up events occurring in a bunch crossing.

- Conversion rejection: It is not identified as originating from a conversion photon by considering a vertex fit probability less than $1e-6$ and no missing hits when its track was reconstructed in the pixel sub-detector.

In addition, electron candidates overlapped with muon candidates within a cone size of 0.3 are removed.

Events having a second electron that meets the “Loose Electron” criteria [42] below are rejected from consideration:

- have $p_T > 20\text{GeV}/c$;
- be within $|\eta| < 2.4$;
- have $\Delta\eta(\text{SC, track})$ less than 0.007 (0.009) in the barrel (endcap);
- have $\Delta\phi(\text{SC, track})$ less than 0.15 (0.10) in the barrel (endcap);
- have the same requirements on $\sigma_{i\eta i\eta}$, $E_{\text{HCAL}}/E_{\text{ECAL}}$, d_{xy} , $|1/E - 1/p|$, and its vertex fit probability;
- have d_z less than 2mm;
- have $I_{\text{rel}}^e < 0.15$.
- be allowed with one missing hit.

Photon

Photon candidates are required to have $p_T > 30\text{GeV}/c$, be within $|\eta| < 2.5$ (not including the barrel-endcap transition region $1.4442 < |\eta| < 1.566$), and pass the “Tight Photon” selection criteria defined by [44]:

- pass a requirement of conversion safe electron veto;
- have a single tower H/E less than 0.05;
- have a $\sigma_{\eta\eta}$ less than 0.011 (0.031) in the barrel (endcap);
- have a ρ corrected PF charged hadron isolation

$$I_{\text{ch}}^{\rho \text{ corrected}} = \max(0.0, \sum p_T^{\text{ch. had.}} - A_{\text{eff.}} \times \rho) < 0.7 \text{ (0.5)}$$

in the barrel (endcap), where definitions of $p_T^{\text{ch. had.}}$, $A_{\text{eff.}}$, and ρ are as same as electron’s.

- have a ρ corrected PF neutral hadron isolation,

$$I_{\text{neu}}^{\rho \text{ corrected}} = \max(0.0, \sum p_T^{\text{neu. had.}} - A_{\text{eff.}} \times \rho) < 0.4 \text{ (1.5)} + 0.04 \times p_T$$

in the barrel (endcap), where definitions of $p_T^{\text{ch. had.}}$, $A_{\text{eff.}}$, and ρ are as same as electron’s.

- have a ρ corrected PF photon isolation,

$$I_{\text{pho}}^{\rho \text{ corrected}} = \max(0.0, \sum p_T^{\text{pho.}} - A_{\text{eff.}} \times \rho) < 0.5 \text{ (1.0)} + 0.005 \times p_T$$

in the barrel (endcap), where definitions of $p_T^{\text{ch. had.}}$, $A_{\text{eff.}}$, and ρ are as same as electron’s.

In addition, photon candidates overlapped with lepton candidates within a cone size of 0.3 are dropped.

Jet

Jets reconstructed using particle-flow information and an anti-kT clustering algorithm (cone of $\Delta R < 0.5$) [45] are considered for the analysis. Jet candidates are required to have $p_T > 25\text{GeV}/c$ and be within $|\eta| < 2.5$. Jets are also required to pass a loose PF jet ID [46]:

- neutral hadron fraction < 0.99 ;

- neutral electromagnetic fraction < 0.99 ;
- number of constituents > 1 ;

and if $|\eta| < 2.4$:

- charged hadron fraction > 0 ;
- charged multiplicity > 0 ;
- charged electromagnetic fraction < 0.99 .

To be fully efficient given our triggers, we further require the three leading jets to have p_T greater than 55GeV/c, 45GeV/c, and 35GeV/c, respectively, for data recorded during Runs C and D. The three leading jets must all have p_T greater than 45GeV/c for Runs A and B.

Jet candidates within a cone $\Delta R < 0.5$ of lepton or photon candidates are rejected.

The event selection of all objects (the signal region)

In our analysis, the semi-leptonic decay of the $t^*t^*\bar{b}$ system is considered. Assuming t^* decays to t gamma with the branch ratio 100%, we are looking for $t^*t^*\bar{b} \rightarrow t$ gamma $t\bar{b}$ gamma $\rightarrow W$ b gamma W $\bar{b}$ gamma with one W boson decaying leptonically and the other decaying hadronically (see Figure 1.1). Therefore, the event selection of all objects, i.e. **the signal region**, is to select events having:

- exactly one muon or electron (from the leptonically-decaying W boson) passing the selections defined in the above text;
- no additional muons or electrons passing the looser criteria also defined in the above text;
- exactly two photons passing the selections defined in the above text;
- at least four jets (two from the hadronically decaying W boson, and two b quarks from top decays) passing the selection in the above text.

Chapter 5 data and the simulated sample

The real data (data) used in our analysis is collected by the CMS detector during the 2012 run corresponding to a luminosity of 19.7 / fb of proton-proton collisions at LHC center-of-mass energy of 8 TeV. The processed datasets are listed in table 5.1.

Dataset	Run Range	$\mathcal{L}$ (pb ⁻¹)
/MuHad/Run2012A-22Jan2013-v1	190645–193621	876.2
/SingleMu/Run2012B-22Jan2013-v1	193834–196531	4411.7
/SingleMu/Run2012C-22Jan2013-v1	198049–203002	7055.2
/SingleMu/Run2012D-22Jan2013-v1	203709–208686	7369.0
/ElectronHad/Run2012A-22Jan2013-v1	190645–193621	876.2
/SingleElectron/Run2012B-22Jan2013-v1	193834–196531	4411.7
/SingleElectron/Run2012C-22Jan2013-v1	198049–203002	7055.2
/SingleElectron/Run2012D-22Jan2013-v1	203709–208686	7369.0

Table 5.1 Summary of 8 TeV collision data streams used in this analysis, along with their run ranges and integrated luminosity.

For the event simulation, several packages are used to simulate the data. The MADGRAPH [46] event generator is used for the event simulation in matrix element level with the CTEQ611 [47] parton distribution function (PDF). The PYTHIA [48] is used for the process of hadronization. The POWHEG [49-50] is used for the simulation of signal top sample. The GEANT4 [51] is used to simulate the response of CMS detector. The simulated Monte Carlo samples (MC sample) including both signal and background are shown in table 5.2.

Process	Cross-Section (pb)	Equivalent $\mathcal{L}$ (pb ⁻¹)	Generator
$t^* \bar{t}^*, M(t^*) = 700 \text{ GeV}/c^2$	0.295 (LO)	334865	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 750 \text{ GeV}/c^2$	0.160 (LO)	625400	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 800 \text{ GeV}/c^2$	0.0889 (LO)	1095888	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 850 \text{ GeV}/c^2$	0.0506 (LO)	1942075	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 900 \text{ GeV}/c^2$	0.0294 (LO)	3271183	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 950 \text{ GeV}/c^2$	0.0174 (LO)	5610029	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1000 \text{ GeV}/c^2$	0.0104 (LO)	9155708	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1100 \text{ GeV}/c^2$	0.0039 (LO)	24259969	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1200 \text{ GeV}/c^2$	0.0015 (LO)	61832458	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1300 \text{ GeV}/c^2$	0.0006 (LO)	158954317	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1400 \text{ GeV}/c^2$	0.0002 (LO)	389744156	MADGRAPH
$t^* \bar{t}^*, M(t^*) = 1500 \text{ GeV}/c^2$	0.0001 (LO)	941297265	MADGRAPH
Z + jets (M-50)	3512.1 $\pm$ 70.1 (CMS)	5418	MADGRAPH
Z + jets (M-10to50)	762.5 (LO)	8992	MADGRAPH
W + jets	35640 $\pm$ 666.1 (CMS)	1529	MADGRAPH
WW	69.9 $\pm$ 6.3 (CMS)	43199	PYTHIA
WZ	33.2 $\pm$ 0.51 (NLO)	252226	PYTHIA
ZZ	8.4 $\pm$ 1.2 (CMS)	1076497	PYTHIA
WWW+ jets	0.082170 (LO)	1943751	MADGRAPH
$W\gamma$	461.6 (LO)	10260	MADGRAPH
$WW\gamma$ + jets	0.528 (LO)	405907	MADGRAPH
$Z\gamma$	132.6 (LO)	39804	MADGRAPH
γ + jets (HT-40to100)	20930 (LO)	946	MADGRAPH
γ + jets (HT-100to200)	5212 (LO)	1774	MADGRAPH
γ + jets (HT-200to400)	960.5 (LO)	55211	MADGRAPH
γ + jets (HT-400toInf)	107.5 (LO)	362539	MADGRAPH
$\gamma\gamma$ + jets	75.39 (LO)	15286	MADGRAPH
$t\bar{t}$ + jets	234.2 $\pm$ 11.3 (CMS)	29320	MADGRAPH
Single t (s-channel)	3.79 $\pm$ 0.07 (NNLO)	68385	POWHEG
Single t (t-channel)	56.4 ^{+2.1} _{-0.3} (NNLO)	66103	POWHEG
Single t (tW-channel)	11.1 $\pm$ 0.3 (NNLO)	44646	POWHEG
Single $\bar{t}$ (s-channel)	1.76 $\pm$ 0.01 (NNLO)	79320	POWHEG
Single $\bar{t}$ (t-channel)	30.7 $\pm$ 0.7 (NNLO)	62223	POWHEG
Single $\bar{t}$ (tW-channel)	11.1 $\pm$ 0.3 (NNLO)	44277	POWHEG
$t\bar{t}\gamma$	2.187 $\pm$ 0.347 (CMS)	32483	MADGRAPH
$t\bar{t}W$	0.217 $\pm$ 0.109 (LO)	837203	MADGRAPH
$t\bar{t}Z$	0.172 $\pm$ 0.086 (LO)	1194022	MADGRAPH
$t\bar{t}H$	0.130 (LO)	7118254	MADGRAPH
QCD (p_T 15to30)	988287420 (LO)	0.0004	PYTHIA
QCD (p_T 30to50)	66285328 (LO)	0.006	PYTHIA
QCD (p_T 50to80)	8148778 (LO)	0.05	PYTHIA
QCD (p_T 80to120)	1033680 (LO)	0.8	PYTHIA
QCD (p_T 120to170)	156293 (LO)	5.3	PYTHIA
QCD (p_T 170to300)	34138 (LO)	6.9	PYTHIA
QCD (p_T 300to470)	1760 (LO)	178.5	PYTHIA
QCD (p_T 470to600)	114 (LO)	2317	PYTHIA
QCD (p_T 600to800)	27.0 (LO)	9759	PYTHIA
QCD (p_T 800to1000)	3.55 (LO)	87151	PYTHIA

Table 5.2 Summary of the Summer12 simulated samples used in this analysis.

To cross check that the MC samples can simulate the data well, several comparisons between data and the MC samples. A loose region, which includes at least one photon, exactly one lepton (electron or muon), and at least four jets, is defined to perform the cross check (the selection of each object is defined in Section 4.6). The figure 5.1 shows the number of vertex distributions of gradually applying different scale factors, which are used to correct the difference between data and MC sample. The table 5.3 shows the number of event (event yields) before and after applying different scale factor gradually.

Under the same requirement, the loose region, Figure 5.2 shows the object multiplicity plots. The kinematic distributions are displayed in Figure 5.3, and four leading jets' p_T are presented in Figure 5.4, separately. A check for run dependences is also performed by considering jet's p_T distribution shown in Figure 5.5.

After applying all scale factors, the difference between data and MC is less than 4%, and the detailed numbers are listed in Table. 5.4.

	Yields				
	without scale factors	pile-up	trigger	lepton Id	photon Id
Total background (MC)	3160 ± 51	3177 ± 55	2917 ± 50	2886 ± 50	2829 ± 49
data	2733				

Table 5.3 Event yields in a loose region before and after applying scale factors.

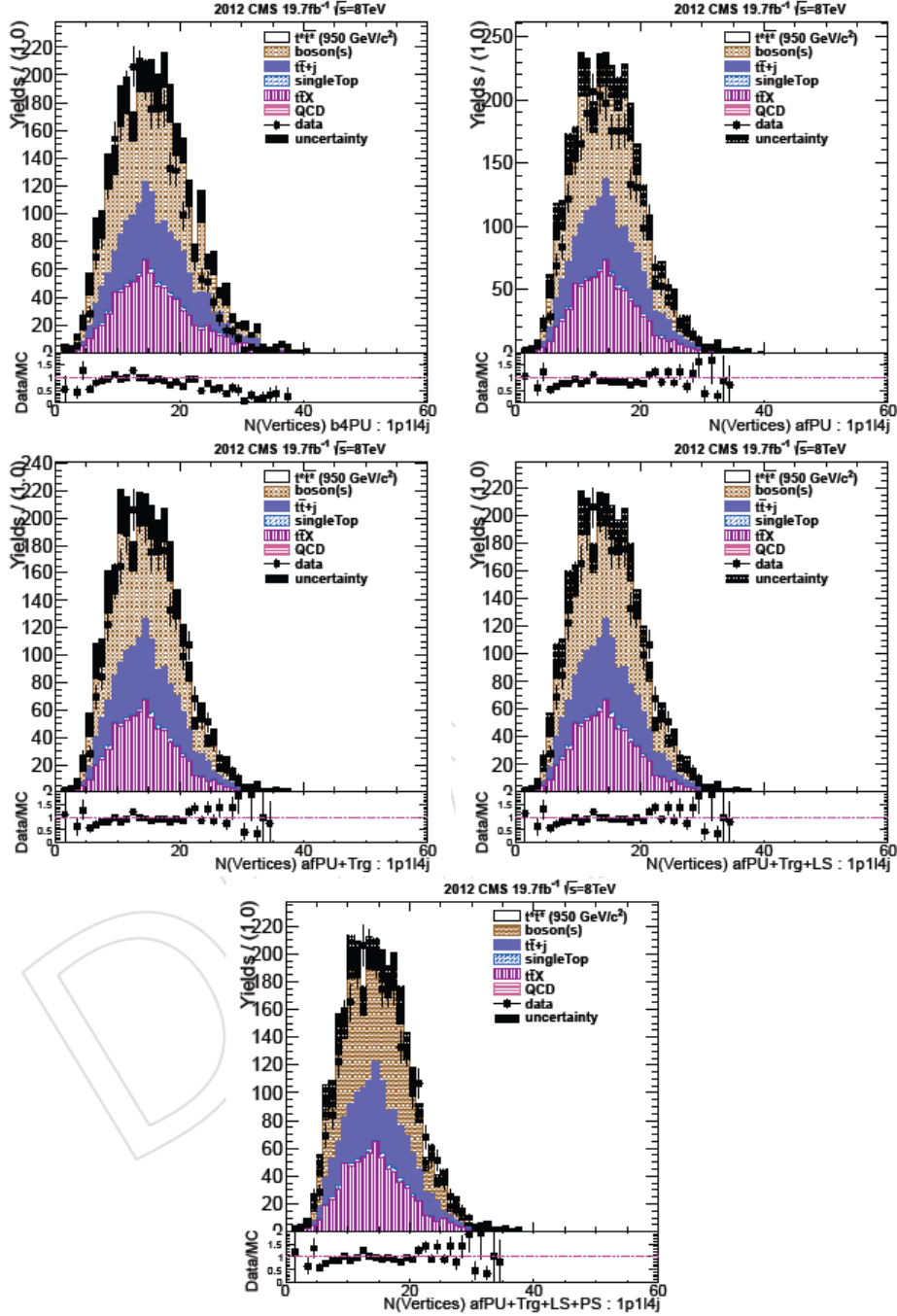


Figure 5.1 The number of reconstructed primary vertices before (top-left) and after (top-right) applying the pileup reweighting procedure. The middle-left plot to the bottom one shows the number of reconstructed primary vertices with applying trigger efficiency, lepton identification scale factor, and photon identification scale factor on simulated samples one by one in order. The open histogram stands for the t^* signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

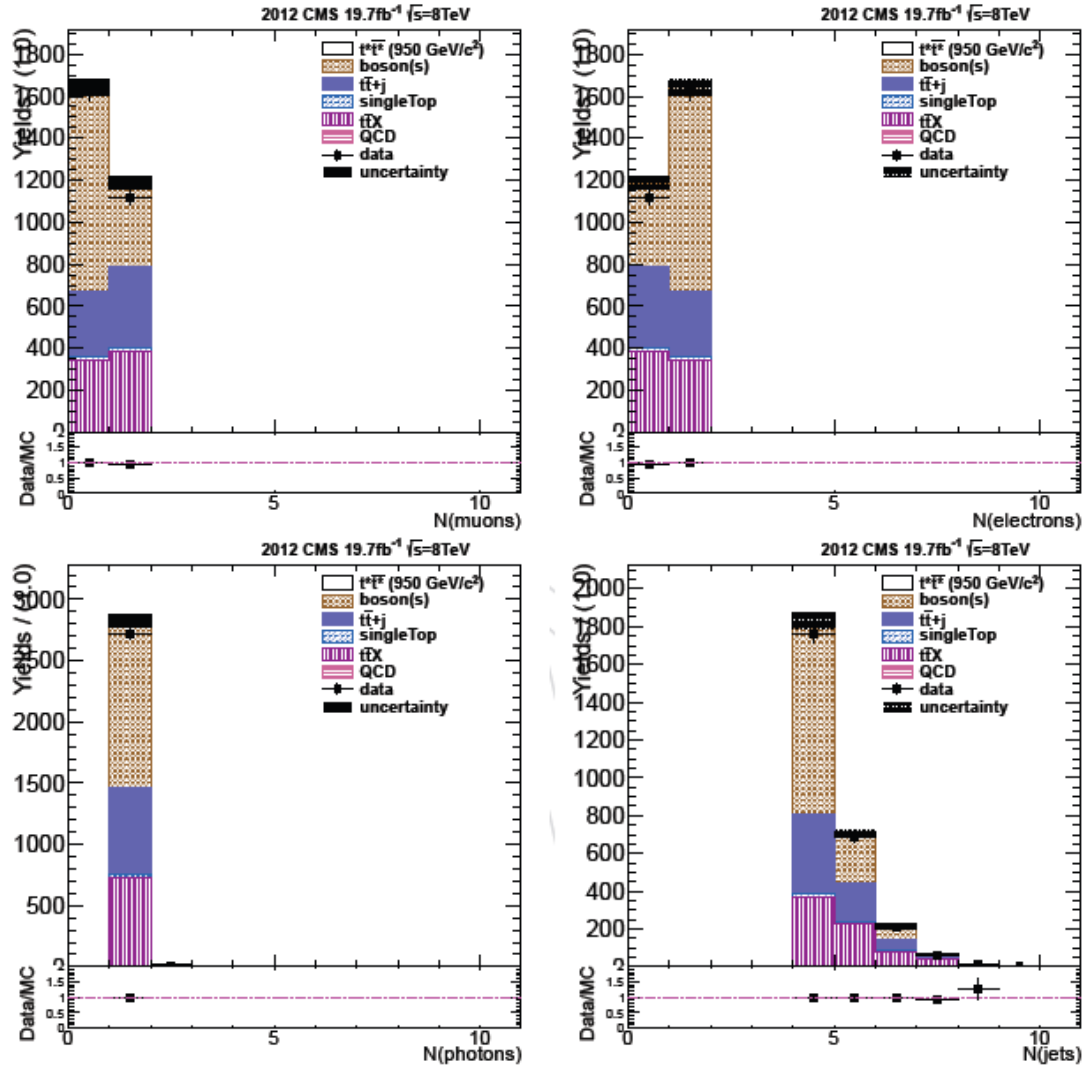


Figure 5.2 the objects' multiplicities. The plots from top-left one to bottom-right one show the multiplicity distribution for muon, electron, photon, and jet, respectively. The open histogram stands for the $t\bar{t}$ signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

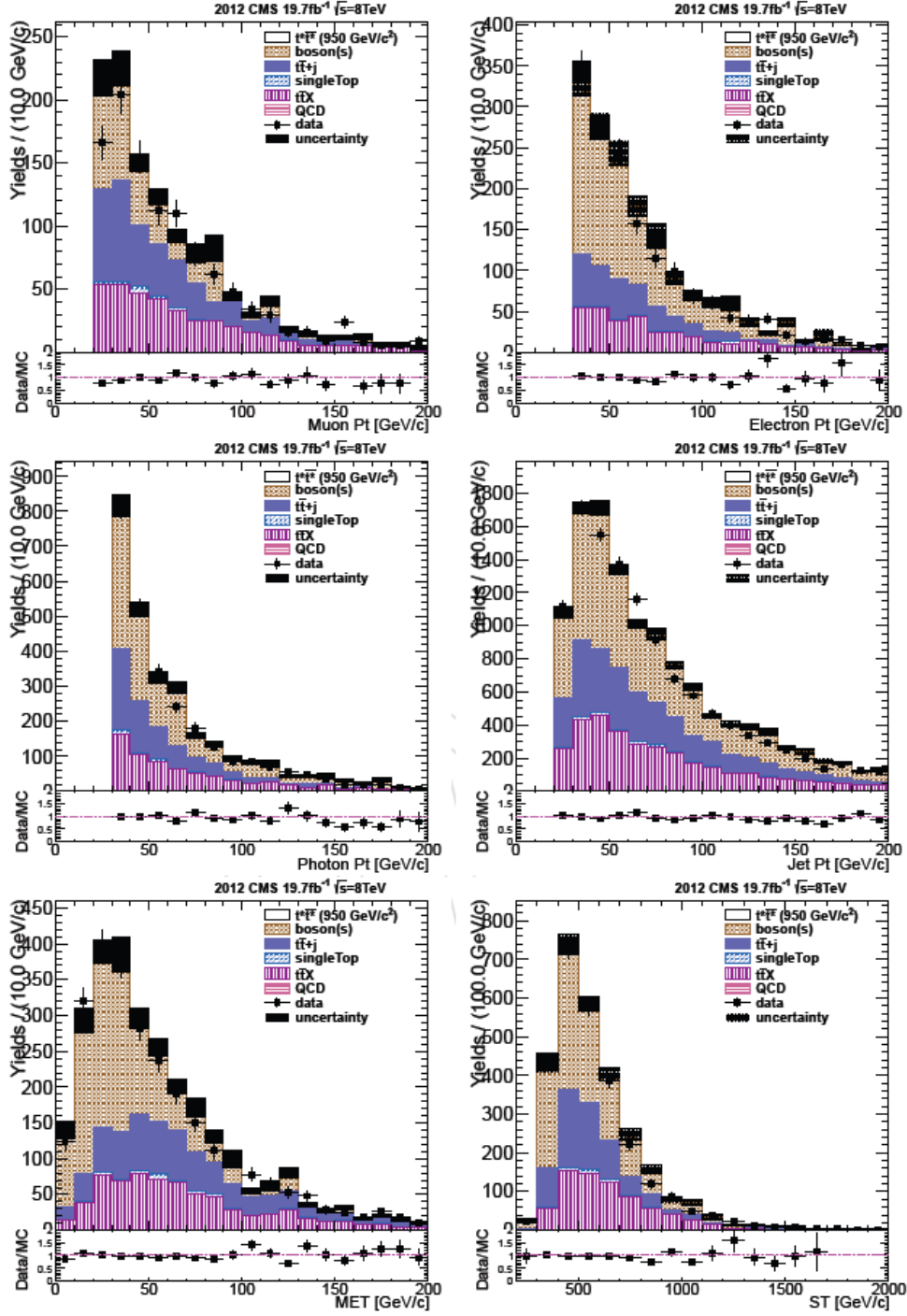


Figure 5.3 the kinematic distributions. The plots from top-left one to bottom-right one show the kinematic distribution for muon's p_T , electron's p_T , photon's p_T , jet's p_T , Missing E_T (MET), and ST, respectively. ST is defined as sum of objects' transverse momenta. The open histogram stands for the t^* signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

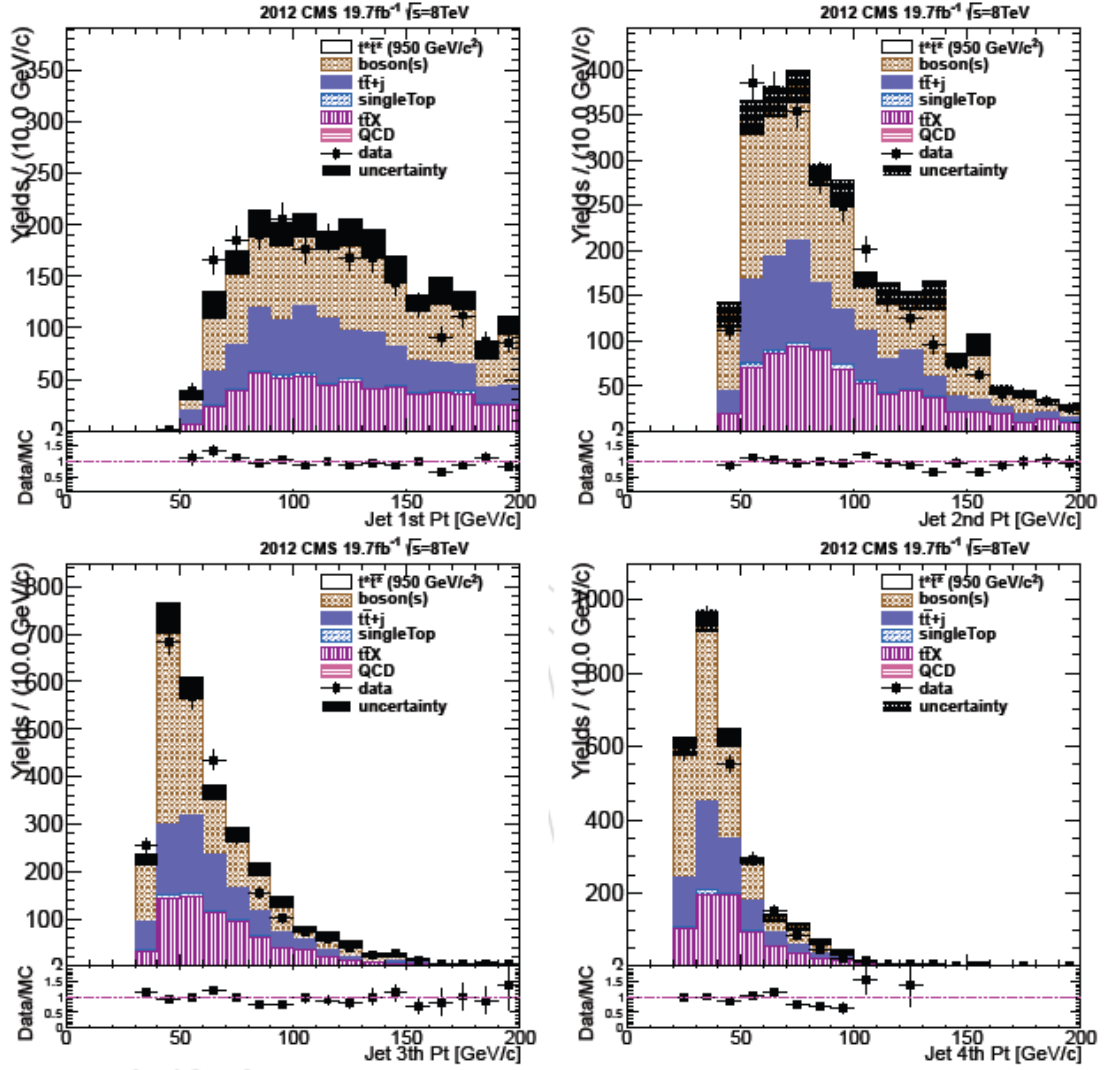


Figure 5.4 Four leading jets p_T distributions. The plots from top-left one to bottom-right one show the jet's p_T distribution for the first, second, third, and fourth leading jet, respectively. The open histogram stands for the $t\bar{t}$ signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

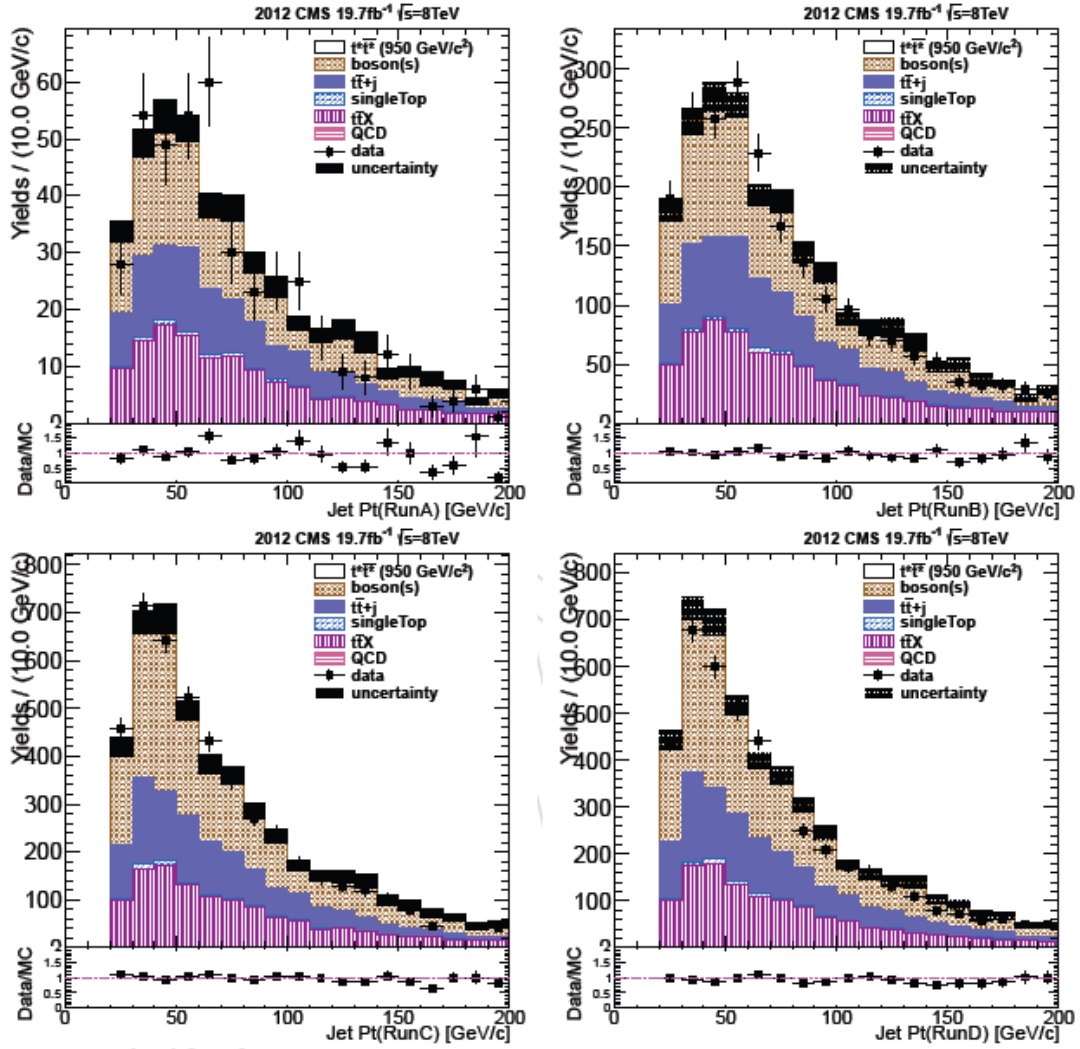


Figure 5.5 Jets' p_T distributions for run dependences. The plots from top-left one to bottom-right one show the jet's p_T distribution for RunA, RunB, RunC, and RunD, respectively. The open histogram stands for the t^* signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

Process		Yields
Signal	$t^*t^*, M(t^*) = 700 \text{ GeV}/c^2$	507.8 ± 3.2
	$t^*t^*, M(t^*) = 750 \text{ GeV}/c^2$	277.8 ± 1.7
	$t^*t^*, M(t^*) = 800 \text{ GeV}/c^2$	152.3 ± 1.0
	$t^*t^*, M(t^*) = 850 \text{ GeV}/c^2$	85.65 ± 0.54
	$t^*t^*, M(t^*) = 900 \text{ GeV}/c^2$	48.26 ± 0.31
	$t^*t^*, M(t^*) = 950 \text{ GeV}/c^2$	28.05 ± 0.18
	$t^*t^*, M(t^*) = 1000 \text{ GeV}/c^2$	16.23 ± 0.11
	$t^*t^*, M(t^*) = 1100 \text{ GeV}/c^2$	5.962 ± 0.040
	$t^*t^*, M(t^*) = 1200 \text{ GeV}/c^2$	2.144 ± 0.015
	$t^*t^*, M(t^*) = 1300 \text{ GeV}/c^2$	0.803 ± 0.006
	$t^*t^*, M(t^*) = 1400 \text{ GeV}/c^2$	0.316 ± 0.002
	$t^*t^*, M(t^*) = 1500 \text{ GeV}/c^2$	0.122 ± 0.001
Boson(s)	Z + jets (M-50)	630.7 ± 28.3
	Z + jets (M-10to50)	15.04 ± 3.26
	W + jets	230.4 ± 32.2
	WW	8.321 ± 0.664
	WZ	10.42 ± 0.49
	ZZ	3.730 ± 0.149
	WWW + jets	0.302 ± 0.031
	$W\gamma$	322.9 ± 14.49
	$WW\gamma$ + jets	13.47 ± 0.47
	$Z\gamma$	91.38 ± 3.92
	γ + jets (HT-40to100)	< 20.84
	γ + jets (HT-100to200)	< 11.11
	γ + jets (HT-200to400)	15.63 ± 1.37
	γ + jets (HT-400toInf)	19.28 ± 0.60
	$\gamma\gamma$ + jets	3.233 ± 1.101
$t\bar{t}$ + jets		704.6 ± 12.7
Single top	Single t (s-channel)	0.555 ± 0.220
	Single t (t-channel)	6.202 ± 0.804
	Single t (tW-channel)	11.82 ± 1.30
	Single $\bar{t}$ (s-channel)	< 0.249
	Single $\bar{t}$ (t-channel)	3.370 ± 0.593
	Single $\bar{t}$ (tW-channel)	14.27 ± 1.54
$t\bar{t}$ + X	$t\bar{t} \gamma$	711.3 ± 12.1
	$t\bar{t} W$	4.475 ± 0.190
	$t\bar{t} Z$	4.390 ± 0.170
	$t\bar{t} H$	3.103 ± 0.053
Total background		2829 ± 49
Data		2733

Tab. 5.4 Event yields in a loose region, including at least one photon, exactly one lepton (electron or muon), and at least four jets.

Chapter 6 Mass Reconstruction

In this chapter the χ^2 -sorting method used to reconstruct t^* mass from data is described, and the neutrino's p_z solution is mentioned.

6.1 The χ^2 -sorting method

In our analysis, we are looking for $t^*t^*\text{bar} \rightarrow t \text{ gamma } t\text{bar gamma} \rightarrow W b \text{ gamma } W \text{ bbar gamma}$ with one W boson decaying leptonically and the other decaying hadronically (see Figure 1.1), and thus the signal region, exactly two photons, exactly one lepton, at least four jets (see section 4.6), is defined.

However, once the event satisfying the requirement of signal region is found in data, the determination of correct object combination is needed to reconstruct the t^* mass, ie to choose two jets to form the (hadronic) W boson, to choose one jet with the (hadronic) W boson to form a top, to choose one jet with the leptonic W boson to form a top, choose one photon with a top to form a t^* . There are 24 combinations if no constraint is set.

The correct object combination is determined by minimizing the χ^2 (the χ^2 -sorting method). The χ^2 is defined as:

$$\chi^2 = \frac{|W_{jj} - M(W)|^2}{\sigma(W_{jj})^2} + \frac{|t_{W_{jj}+b} - M(top)|^2}{\sigma(t_{W_{jj}+b})^2} + \frac{|t_{W_{lv}+b} - M(top)|^2}{\sigma(t_{W_{lv}+b})^2} + \frac{|t^*(t_{W_{jj}+b} + \gamma) - t^*(t_{W_{lv}+b} + \gamma)|^2}{\sigma(t^*_{lep})^2 + \sigma(t^*_{had})^2}$$

where W_{jj} (W_{lv}) is the mass reconstructed (by adding their four momentum) with two jets (a lepton and a neutrino), $t_{W_{jj}+b}$ ($t_{W_{lv}+b}$) is the mass reconstructed with W_{jj} (W_{lv}) and an additional jet, t^* is the mass reconstructed by the W_{jj} (W_{lv}), an additional jet and a photon, M indicates an object's mass, and σ stands for the corresponding mass resolution. $M(W)$ is the W boson mass $80.398\text{GeV}/c^2$, and $M(top)$ is the top mass $172.9\text{GeV}/c^2$.

Therefore, by the χ^2 definition, the combination that choose the correct objects to reconstruct its mother object (Ex. choose the two jets that really come from a W boson to reconstruct the W boson) should have the minimum χ^2 value.

According to the MC truth in simulation, the mass resolutions can be obtained as below:

$$\begin{aligned}
\sigma(W_{jj}) &= 9.296 \text{ GeV}/c^2 \\
\sigma(t_{W_{jj}+b}) &= 16.49 \text{ GeV}/c^2 \\
\sigma(t_{W_{lv}+b}) &= 22.43 \text{ GeV}/c^2 \\
\sigma(t^*_{lep}) &= 31.63 \text{ GeV}/c^2 \\
\sigma(t^*_{had}) &= 31.59 \text{ GeV}/c^2.
\end{aligned}$$

The neutrino's 4 momentum is obtained by the missing E_T (MET) and considering the lepton's kinematics, which will be discussed in section 6.2.

Figure 6.1 shows the result of applying the χ^2 -sorting method on the t^* signal sample to reconstruct the t^* mass, and the t^* mass resonances (peaks) can be reconstructed well.

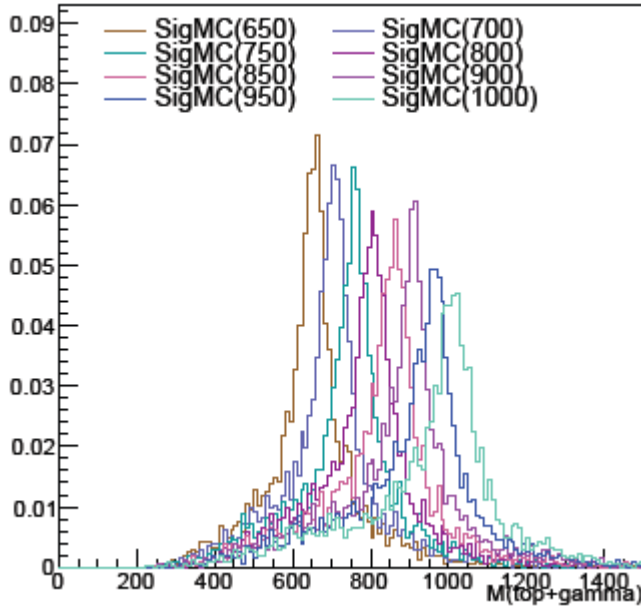


Figure 6.1 The invariant mass of a top and a photon (gamma) using the χ^2 -sorting method. Each mass spectrum is corresponding to a t^* signal with a different mass, ranging between $650 \text{ GeV}/c^2$ and $1000 \text{ GeV}/c^2$.

6.2 The neutrino's p_z solution

The neutrino's transverse momentum $p_T (P_{\perp}^{\nu})$ can be obtained by the missing E_T (MET). For neutrino's longitudinal momentum $p_z (P_{\parallel}^{\nu})$, considered the invariant mass of the W boson that has leptonic decay:

$$P_W^2 = M_W^2 = (P_{\nu} + P_{\ell})^2 = P_{\nu}^2 + P_{\ell}^2 + 2P_{\nu}P_{\ell}$$

where M_W is the W boson mass, the P_{ν} (P_l) is the momentum (vector) of the neutrino (lepton). Inserting the M_W , P_l and $P_{\perp}^{\nu}$, the equation becomes an equation with one unknown $P_{\parallel}^{\nu}$ of power 2. Therefore, there are two solutions for the neutrino's p_z solution, and they can be solved.

From the two solution of neutrino's p_z , the comparison of mass reconstruction in the t^* signal sample with the larger solution, the smaller solution and the solution having lower χ^2 are performed (shown in figure 6.2). One can find the solution having lower χ^2 has the best performance, so we use the neutrino's p_z solution having lower χ^2 in our analysis.

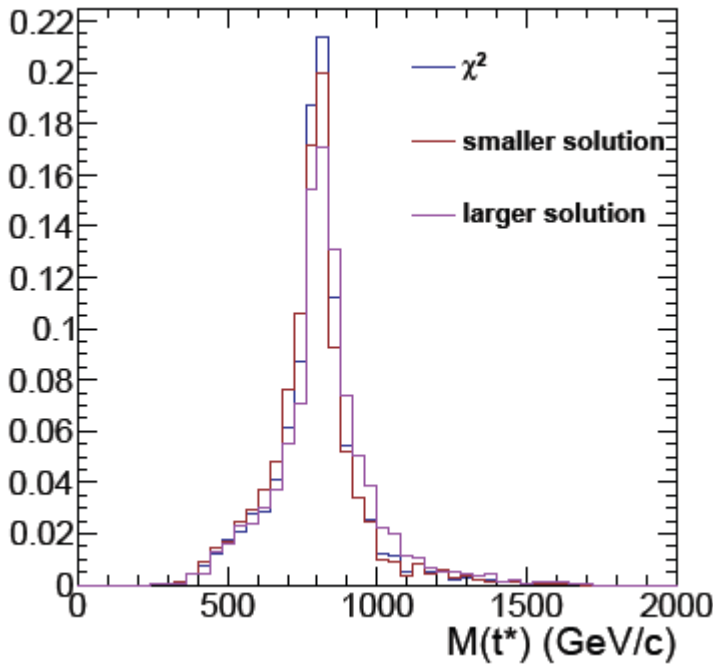


Figure 6.2 The t^* mass spectra with different neutrino's solutions, including the smaller solution, the larger solution, and a solution chosen by getting the minimum of χ^2 value.

Chapter 7 background estimation: the matrix method

In this chapter, the background estimation is described. The matrix method, which is used for background estimation in our analysis, is introduced first. The coefficients used in the matrix method, the photon fake rates (from lepton and jet), the photon signal efficiency, and the ratio factors, are estimated then.

7.1 The introduction of matrix method

From chapter 4 to chapter 6, the signal region selection, the data and MC samples and the method to reconstructing t^* mass have been mentioned. Once applying the signal region selection on data, one can obtain the number of events that passing our signal region selection. Among the events passing signal region, the number of events coming from background is needed to be estimated by a background estimation method.

From the objects used in the signal region, which have exactly two photons, exactly one lepton and at least four jets, the di-photon channel (two photons) is considered to estimate the background contribution due to that compared to leptons and jets, photons has the largest portion of fake object (in photon case, something that is identified as a photon but it is not a photon indeed). Among the events passing signal region, which have 2 photons passing the selection defined in section 4.6, the matrix method is applied to estimate the number of events that doesn't have two real photons, i.e. the event whose photon comes from other object but not photon.

The matrix method starts with classifying the events with two photons to three levels (the levels here is not associated with the trigger level in chapter 3). Events have two loose photons, at least one tight photon and two tight photons belong to the level-1, the level-2 and the level-3, respectively. The tight photon is the photon passing the selection defined in section 4.6. The loose photon is the photon passing a selection looser than the tight photons'. The selection between the tight photon and the loose photon is called fakeable object (FO). The schematic of the three levels and the tight/loose photon are shown in Figure 7.1 and Figure 7.2 respectively. Besides the requirement on photon, the three levels have the same requirement to the signal region in the rest objects.

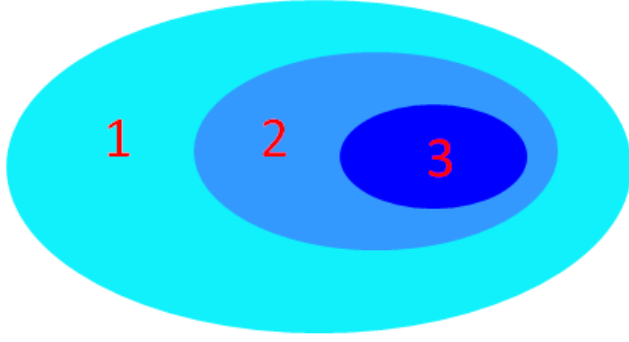


Figure 7.1 the schematic of 3 levels of the events with two photons.

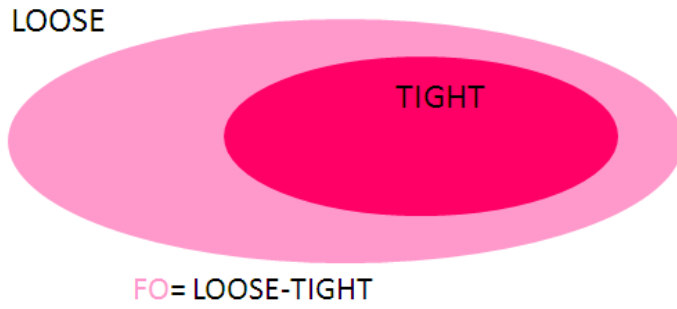


Figure 7.2 the schematic of the tight, the loose and the FO selection of photon.

The events with two photons can be further classified to three kinds as shown in below:

$$\begin{aligned} N_1 &= N_1^{ff} + N_1^{sf} + N_1^{ss} \\ N_2 &= N_2^{ff} + N_2^{sf} + N_2^{ss} \\ N_3 &= N_3^{ff} + N_3^{sf} + N_3^{ss} \end{aligned}$$

eqn (1)

where $N_{1,2,3}$ stands for the number of events of level-1, level-2 and level-3, respectively, and the $N_{1,2,3}^{ff, sf, ss}$ stand for the number of events (of each level) of two fake photons (ff), one fake photon plus one signal photon (sf), and two signal photons (ss), respectively. The N_3 is the events in the signal region observed in data. **The N_3^{ff} and N_3^{sf} are the background contribution that we want to estimate.**

The $N_{2,3}^{ff,sf,ss}$ can be written by the $N_1^{ff,sf,ss}$, then the eqn(1) becomes

$$\begin{aligned} N_1 &= 1 \cdot N_1^{ff} + 1 \cdot N_1^{sf} + 1 \cdot N_1^{ss} \\ N_2 &= \epsilon_{ff}^{1 \rightarrow 2} \cdot N_1^{ff} + \epsilon_{sf}^{1 \rightarrow 2} \cdot N_1^{sf} + \epsilon_{ss}^{1 \rightarrow 2} \cdot N_1^{ss} \\ N_3 &= \epsilon_{ff}^{1 \rightarrow 3} \cdot N_1^{ff} + \epsilon_{sf}^{1 \rightarrow 3} \cdot N_1^{sf} + \epsilon_{ss}^{1 \rightarrow 3} \cdot N_1^{ss} \end{aligned} \quad \text{eqn(2)}$$

where $\epsilon_{ab}^{1 \rightarrow 2,3} = \frac{N_{2,3}^{ab}}{N_1^{ab}}$ eqn(3),

ab stand for s or f.

In eqn(2), the $N_{1,2,3}$ (green) is obtain after applying selections of three levels on the data. Therefore, once **the six efficiencies** (orange $\epsilon_{ff,sf,ss}^{1 \rightarrow 2,3}$) are estimated, the three equations with 3 unknowns (red $N_1^{ff,sf,ss}$) can be solved by reversing the coefficient matrix (that why it's called matrix method), and **the desired $N_3^{ff,sf}$** can be obtained by the $N_1^{ff,sf}$ times the $\epsilon_{ff,sf}^{1 \rightarrow 3}$.

The six efficiencies ($\epsilon_{ff,sf,ss}^{1 \rightarrow 2,3}$) can be transformed to be **4 per-object efficiencies**, ϵ_l , ϵ_q , ϵ_g , and ϵ_s , and **7 ratio factors**, $R^{sl/sq}$, $R^{sl/sg}$, $R^{qg/lq}$, $R^{qg/lg}$, $R^{qg/qq}$, $R^{qg/gg}$, $R^{qg/ll}$. The suffixes l, q and g stand for the three sources of fake photon, the lepton, the quark-like jet and the gluon-like jet, respectively, and the suffix s stand for the signal photon (real photon). The full derivation, from the **6 efficiencies** to the **4 per-object efficiencies** and **7 ratio factors**, can be found in the Appendix A.

The per-object efficiency ϵ_i ($i=l,s,q,g$) is defined as

$$\epsilon_i = \frac{N_i^T}{N_i^L} \quad \text{eqn(4)}$$

where $N_i^{T(L)}$ is the number of events having a tight (loose) photon which comes from the i-object. For example, the N_q^T is the number of events having a fake photon from the quark-like jet, and that fake photon passes the tight selection.

The ratio factor $R^{xy/st}$ (x,y,s,t can be l,s,q,g) is defined as

$$R_{st}^{xy} = \frac{N_1^{xy}}{N_1^{st}} \quad \text{eqn(5)}$$

where $N_1^{xy(st)}$ is the same definition as in eqn(3).

Therefore, the next step of matrix method is to estimate **the 4 per-object efficiencies**

and the 7 ratio factors.

In practice we estimate ϵ_l from a Tag and Probe technique on an electron-photon couple of objects. We estimate ϵ_q , ϵ_g thanks to a template fitting procedure. We estimate ϵ_s thanks to a Tag and Probe technique, by fitting the invariant mass of the di-muon gamma system. Finally the ratio factors $R^{xy/st}$ will be estimated by counting the number of two object events in the signal region.

All the above tasks will be mentioned in the following sections, and among them I contribute to the part of the ϵ_q and ϵ_g estimation (see section 7.3).

7.2 the photon fake rate from lepton ϵ_l

To obtain the photon fake rate from lepton ϵ_l , first of all, selections of the tight photon and of the loose photon is defined. The tight (photon) selection is the photon tight ID, and the loose (photon) selection is the photon tight ID without the electron-veto cut.

The loose selection also has other parts (see section 7.3, 7.4) used in jet and signal photon channel (the loose selection should be the same in all channels), but that parts has no effect on the fake photon from lepton, so we neglect it and write only the necessary part in the lepton channel (in the case of jet and signal photon channel, we will also write the necessary part only, but the exact loose selection is the union of the loose selections used in three channels).

Once the tight and the loose selections are applied on the fake photon from lepton sample, the two numbers of events, N_l^T and N_l^L , are obtained. Inserting the N_l^T and N_l^L into eqn(4), the fake rate ϵ_l is found.

To obtain the pure fake photon from lepton sample, the Tag and Probe technology is used. By reconstructing the Z mass with an electron and a photon from data, the sample having large amount of fake photon from electron is obtained (Z $\rightarrow$ ee, and one electron becomes a fake photon). Fitting that invariant mass shape (e $\rightarrow$ γ) with the signal shape (Crstall Ball) and the background shape (CMSShape), the number of event of fake photon from lepton is found (the signal is the fake photon from electron, and the background is the photon that doesn't come from electron). The N_l^T (N_l^L) can be acquired by applied the tight/loose selection on that photon before fitting.

The fit is performed in the invariant mass range between 60 and 120 GeV. The results of the fit are presented in Figure 7.3.

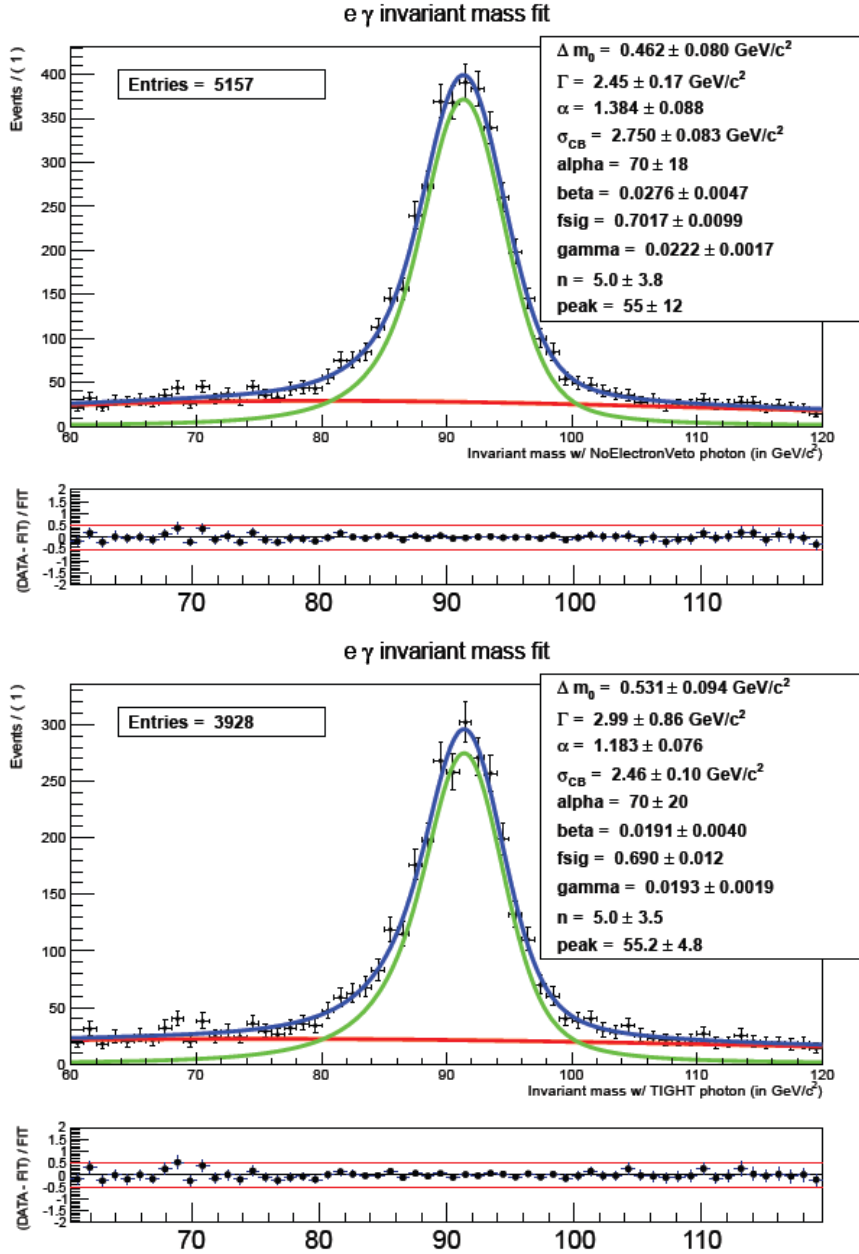


Figure 7.3 Electron-Photon invariant mass fit for the no-electron-veto selection (upper plot) and the tight ID selection (lower plot). The fit function is a BreitWigner convoluted thanks to a fast Fourier transformation to a Crystall Ball (for the signal) plus a CMSShape (for the background). The green line, the red line, the blue line and the black dot are the signal component, the background component, the fit and the data, respectively. The fsg is the signal fraction. The entries are the number of event in data. The plots of $(\text{data}-\text{fit})/\text{fit}$ are shown in below of both the upper plot and the lower plot.

7.3 The photon fake rate from jet ϵ_j

In the jet channel, the object considering for fake photon from jet is defined as an EM super-cluster (SC) and a jet, and they are within a cone of size 0.5 ($\Delta R=0.5$, see Figure 7.4), i.e. a photon associated with a jet (photon is reconstructed from SC, so a SC behaves like a photon). Therefore the two sides of object can be applied both photon selection and the jet selection.

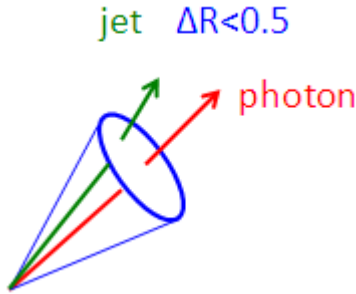


Figure 7.4 the object has a photon associated with a jet within the cone of size 0.5.

In the jet side of the object, jet selection is the same as in section 4.6, and the jet can be further separated to the quark-like jet and the gluon-like jet by applied the quark-gluon tagger. The jet has a QGTagsLikelihood smaller than 0.9 is called gluon-like jet, while the rest of the jet will be called quark-like jet.

In the photon side of the object, the tight selection and the loose selection can be applied on the SC. The tight is the same as in the lepton channel (section 7.2). The loose selection is defined by defining the FO selection (the loose = tight + FO, see Figure 7.2). The FO is defined as:

- The SC has to pass 5 times looser (with respect to the tight photon ID selection) cuts of the charge isolation, the neutral isolation and the photon isolation.
- The SC has to pass the inverted photon tight ID cuts, which have been multiplied by the 2, 2, 2, 3 (for the Gluon-like) and 0, 3, 1, 0 (for the Quark-like) respectively for 1/5 of the H/E tower threshold, the charge isolation, the neutral isolation and the photon isolation cuts.

The FO selection has been optimized so that the real photon contamination is less than 1% (and to increase statistics) in the MC simulation. Therefore, the N_j^{FO} , which is the number of events whose photon (SC) comes from a jet (fake photon from jet) and passes the FO selection, can be obtained by applied the FO selection on data.

Once the N_j^T , which is the number of events whose photon (SC) comes from a jet and passes the tight selection, the N_j^L can be found by adding N_j^{FO} and N_j^T ($N_j^L = N_j^{FO} + N_j^T$). Inserting the N_j^T and N_j^L into eqn(4), the fake rate ϵ_j is found.

The number of events having the object, whose photon comes from a jet and passes the tight selection, N_j^T , is obtained by applying the tight selection on the data and fitting them in the variable $\sigma_{i\eta i\eta}$ with the signal (real photon passing the tight selection) shape and the background (fake photon from jet passing the tight selection) shape.

The schematic of the N_j^{FO} and N_j^T shows in figure 7.5.

	FO	Tight
Fake photon from jet (background)	N_j^{FO}	N_j^T
Real photon (signal)	$N_{real}^{FO} \sim 0$	N_{real}^T

Figure 7.5 the schematic of the N_j^{FO} and N_j^T . The blue rectangle represents the loose selection of the fake photon from jet, which includes the FO selection and the tight selection. The red rectangle represents the data in the tight selection, which contains the signal component and the background component.

The background templates (fake photon from jet in tight selection) are taken directly from the data in the FO, due to that the FO has the negligible contamination of real photon, and the verification that the $\sigma_{i\eta i\eta}$ shapes of the fake photon from jet in the FO and in the tight are reasonably similar in MC simulation.

The signal (real photon in tight selection) templates are taken from the MC simulation (photon plus jet process) with the correction between MC and data for the $\sigma_{i\eta i\eta}$:

$$\begin{aligned} \sigma_{i\eta i\eta}^{EBcorr} &= (\sigma_{i\eta i\eta} - 0.0090405) \times 1.04 + 0.0089405 \\ \sigma_{i\eta i\eta}^{EEcorr} &= \sigma_{i\eta i\eta} \times 1.1 - 0.0025 \end{aligned} \quad \text{eqn(6)}$$

separately for the Barrel and Endcap photons.

Once obtained the shapes we can fit in the TIGHT selection the Barrel and Endcap $\sigma_{\text{in}} \eta$ distribution separately and for each FO definition (Quark-like and Gluon-like). We perform the fit on the entire $\sigma_{\text{in}} \eta$ range, but we integrate over the $\sigma_{\text{in}} \eta$ range of the tight selection to obtain the number of events and the signal fraction.

The χ^2 in fit is defined as

$$\chi^2 = \sum_i \left(\frac{y_i - f(x_i, a)}{\sigma_i} \right)^2,$$

where the summation is sum over all bins, the y_i is the data point, the model predicted value, $f(x_i, a)$, is

$$f(x_i, a) = \text{sig} * \text{signal fraction } a + \text{bkg} * (1 - \text{signal fraction } a),$$

the *sig* (*bkg*) is the signal (background) template value in the bin, the σ_i is the data uncertainty plus template uncertainty. The data, the signal template and the background template are normalized to unity.

The χ^2 is minimized by requiring

$$\frac{d\chi^2}{da} = 0.$$

The results are presented in Figure 7.6 and summarized in the table 7.1 with the systematic uncertainties from the fit and from the shape. The large (bad) χ^2/dof may result from the lack of knowledge of the signal and background shapes.

Therefore the shape uncertainty in the signal fraction, which can be obtained by assigning the discrepancy between the data points and the fit points to either the signal contribution or to the background contribution, can cover the possible improvement of the fit.

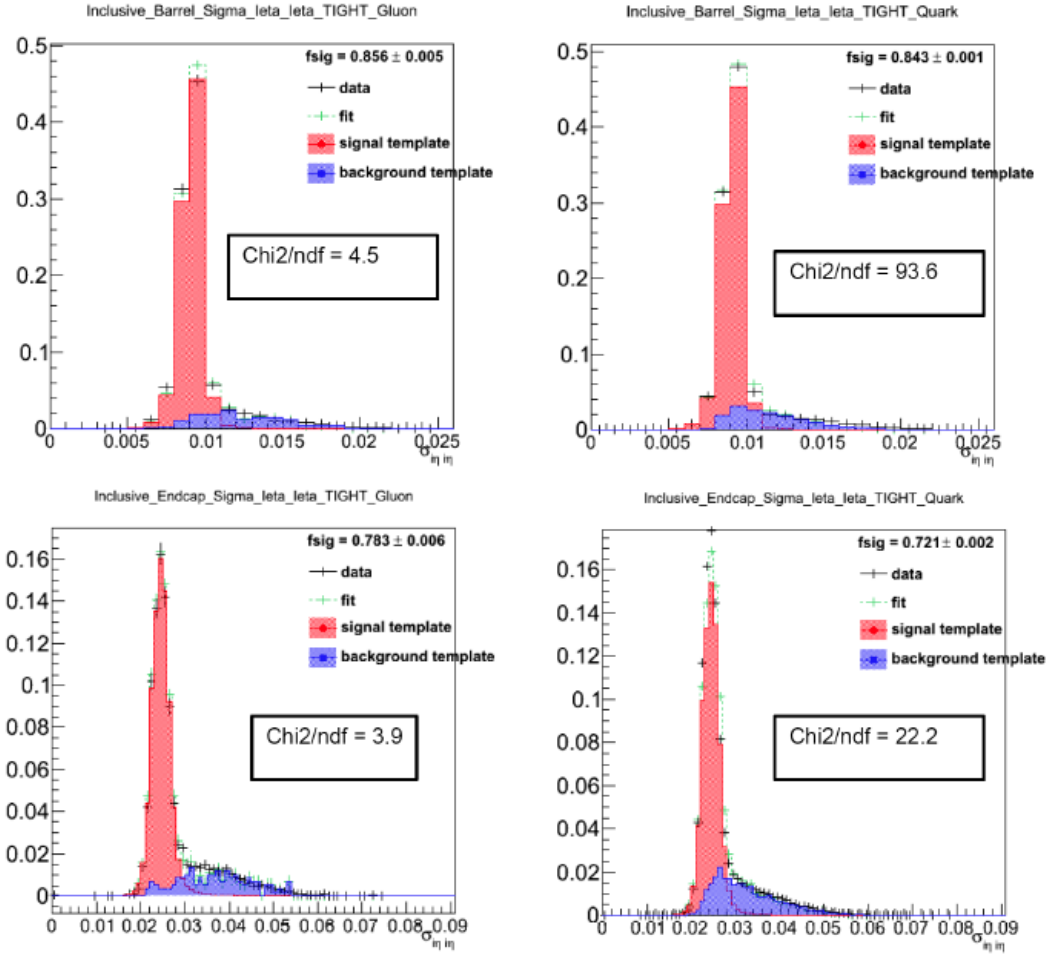


Figure 7.6 Results of the template fitting for the various categories: Barrel Gluon-like (upper-left), Endcap Gluon-like (upper-right), Barrel Quark-like (lower-left) and Barrel Quark-like (lower-right).

category	Signal fraction
Quark-like Barrel	$0.9162 \pm 0.0007(\text{sys. from fit}) +0.0185(\text{sys. from shape}) -0.0033$
Quark-like Endcap	$0.8354 \pm 0.0014(\text{sys. from fit}) +0.0530(\text{sys. from shape}) -0.0464$
Gluon-like Barrel	$0.9417 \pm 0.0022(\text{sys. from fit}) +0.0242(\text{sys. from shape}) -0.0181$
Gluon-like Endcap	$0.9306 \pm 0.0023(\text{sys. from fit}) +0.0269(\text{sys. from shape}) -0.0112$

Table 7.1 the summary of the fit result in each categories. The signal fraction is shown with the systematic uncertainty from the fit and from the shape.

7.4 The photon signal efficiency ϵ_s

As in the case of the fake rate from leptons, also in this case we use a Tag and Prone technique. We define the ϵ_s as the per-object ratio of the photon efficiency for the tight selection over the sum of the efficiencies for the no-electron-veto selection and the FO selection. However, the FO selection has been optimized to have negligible real photon contamination. Therefore it will not be taken into account. In order to count the number of fake photons for the tight and no-electron-veto selection, we use a simple Tag and Probe technique on the invariant mass of the muon-muon-photon system.

We fit the invariant mass with a BreitWigner convoluted thanks to a fast Fourier transformation to a Gaussian (for the signal) plus a CMSShape for the Background. This allows subtracting the non Drell-Yan component. The final number of fitted signal events is the number of real photons to be used for the ϵ_s calculation.

We apply no selection on the jet multiplicity to be at least 4, because we verified that the ratio of the efficiencies has no dependence on the jet multiplicity. We request the transverse momentum of the photon to be 30 GeV. On the leading muon transverse momentum we apply a tight selection and we lower the transverse momenta to 20 GeV in order to get enough statistics at the low end of the invariant mass distribution.

The fit is performed in the invariant mass range between 70 and 120 GeV. To increase the statistics we use both DoubleMu and SingleMu Primary Datasets, while we remove duplicated events. The results of the fit are presented in Fig. 7.2 .

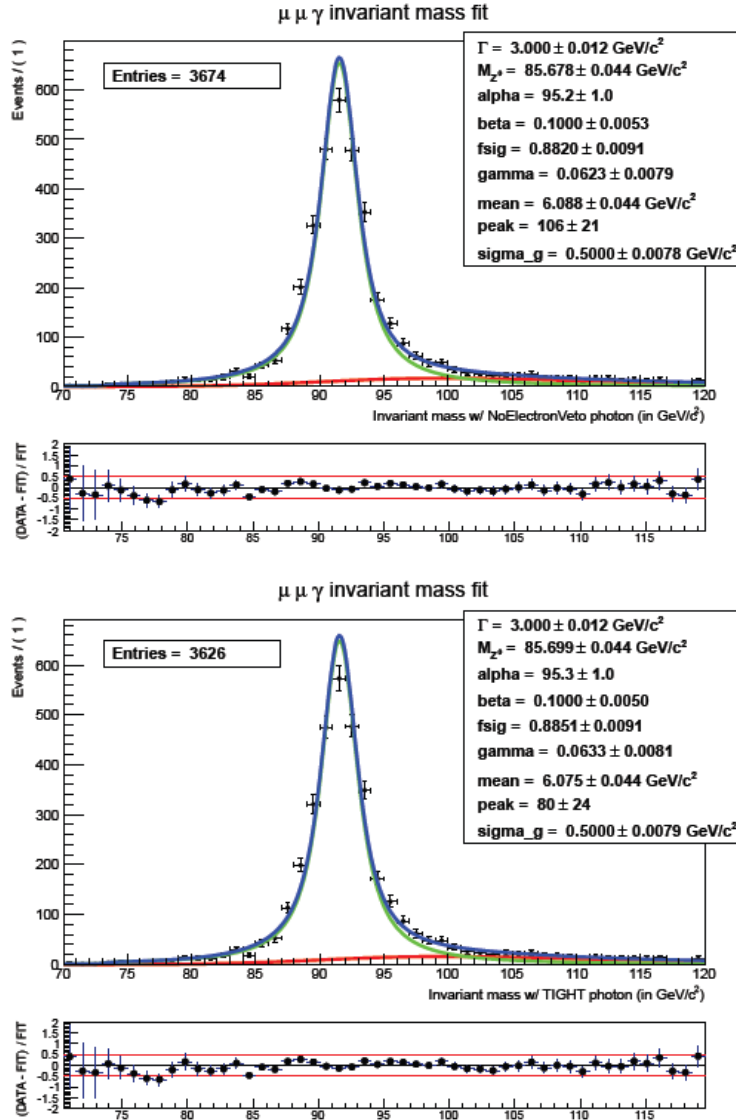


Fig. 7.7 Muon-Muon-Photon invariant mass fit for the no-electron-veto selection (upper plot) and the tight ID selection (lower plot). The fit function is a BreitWigner convoluted thanks to a fast Fourier transformation to a Gaussian (for the signal) plus a CMSShape (for the background).

7.5 The ratio factor

The ratio factors are determined from the data. To determine all the ratio factors is enough to determine the following number of events: $R^{sl/sq}$, $R^{sl/sg}$, $R^{qg/lq}$, $R^{qg/lg}$, $R^{qg/qq}$, $R^{qg/gg}$, $R^{qg/l}$.

In a good approximation the above number of events can be obtained by selecting two photons events in which as s, l, g, q photons, one can request a tight, anti-electron-veto, FO Quark-like and FO Gluon-like object, respectively. Any contamination due to this approximation can be subtracted by subtracting from MC the non-matching component (where in this case the matching is a MC matching to obtain true objects).

Given the FO definition used the number of two photon-like object events containing at least one FO is small. In order to increase the statistics another FO definition has been used. The alternative FO definition has been used to infer the ratio factors, after applying a correction factor which is derived from the data. The correction factor takes into account the ratio of the two FO definitions on a per-object base.

Chapter 8 Result

The result of our t^* analysis is shown in this chapter.

12 Events passing signal region

The signal region selection (defined in section 4.6) is applied on the data (described in chapter 5), and 12 events passing the signal region is observed.

Background estimation by the matrix method

Within the 12 events in the signal region, which have 2 photons passing the selection, the matrix method is applied to estimate the number of events that doesn't have two real photons, i.e. the event whose photon comes from other object but not photon.

After having estimated the ratio factors, real photon efficiency, the fake rates (from lepton and jets) and applying the level-1, 2 and 3 selections (defined section 7.1) on data, the matrix can be inverted in order to obtain the N_3^{ff} and the N_3^{sf} . The uncertainties are obtained by mean of 10 M pseudo-experiments and by rejecting solutions with negative number of events. In the pseudo-experiments the ratio factors and the efficiencies ϵ are allowed to vary within their uncertainties. The final results are that N_3^{sf} is to be 7.9 ± 0.4 and N_3^{ff} is to be 1.7 ± 0.08 . (as figure 8.1)

$$\begin{aligned} N_1 &= N_1^{ff} + N_1^{sf} + N_1^{ss} \\ N_2 &= N_2^{ff} + N_2^{sf} + N_2^{ss} \\ N_3 &= \underline{N_3^{ff}} + \underline{N_3^{sf}} + N_3^{ss} \end{aligned}$$

12 Events passing
signal region,
observed in data

Figure 8.1 the relationship between 12 events passing signal region, observed in data, and the N_3^{sf} and N_3^{ff} .

Compare the mass spectrum of data and directly MC

Besides the background estimation by matrix method, the 12 events in signal region also can be reconstructed the t^* mass by the χ^2 -sorting method (described in chapter 6) and be compared with the direct MC samples (shown in figure 8.2) (but this is not our background estimation method. Our background estimation relies on the data-driven matrix method described in chapter 7).

From the MC samples, the 12.1 signal yields with a mass of $950\text{GeV}/c^2$ is expected, and the total background contribution is 2.8 ± 0.7 (stat.). The detailed numbers are listed in Tab. 8.1. Both the event yields from MC and the peak around $950\text{GeV}/c^2$ (data and signal MC) shown in figure 8.2 hint that the t^* particle may be discovered.

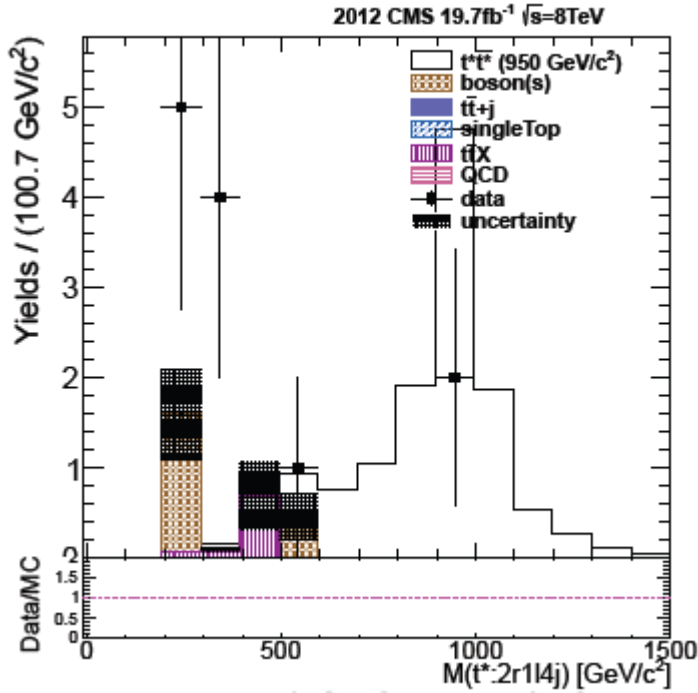


Figure 8.2 The mass spectrum in the signal region, having exactly two photons, exactly one lepton, and at least four jets. The open histogram stands for the t^* signal sample, the filled histograms represent for the standard model processes, the cross point indicates the observed events from data, and the hash marks show the statistic uncertainty.

Process		Yields
Signal	$t^* \bar{t}^*, M(t^*) = 700 \text{ GeV}/c^2$	212.1 ± 2.0
	$t^* \bar{t}^*, M(t^*) = 750 \text{ GeV}/c^2$	117.6 ± 1.1
	$t^* \bar{t}^*, M(t^*) = 800 \text{ GeV}/c^2$	64.19 ± 0.62
	$t^* \bar{t}^*, M(t^*) = 850 \text{ GeV}/c^2$	35.93 ± 0.35
	$t^* \bar{t}^*, M(t^*) = 900 \text{ GeV}/c^2$	20.95 ± 0.21
	$t^* \bar{t}^*, M(t^*) = 950 \text{ GeV}/c^2$	12.07 ± 0.12
	$t^* \bar{t}^*, M(t^*) = 1000 \text{ GeV}/c^2$	6.837 ± 0.070
	$t^* \bar{t}^*, M(t^*) = 1100 \text{ GeV}/c^2$	2.513 ± 0.026
	$t^* \bar{t}^*, M(t^*) = 1200 \text{ GeV}/c^2$	0.892 ± 0.010
	$t^* \bar{t}^*, M(t^*) = 1300 \text{ GeV}/c^2$	0.319 ± 0.004
	$t^* \bar{t}^*, M(t^*) = 1400 \text{ GeV}/c^2$	0.114 ± 0.001
	$t^* \bar{t}^*, M(t^*) = 1500 \text{ GeV}/c^2$	0.0339 ± 0.0005
Boson(s)	Z + jets (M-50)	< 3.638
	Z + jets (M-10to50)	< 2.192
	W + jets	< 12.89
	WW	< 0.165
	WZ	< 0.068
	ZZ	0.017 ± 0.010
	WWW+ jets	< 0.010
	$W\gamma$	< 1.921
	$WW\gamma$ + jets	0.015 ± 0.011
	$Z\gamma$	1.957 ± 0.557
	γ + jets (HT-40to100)	< 20.84
	γ + jets (HT-100to200)	< 11.11
	γ + jets (HT-200to400)	< 0.357
	γ + jets (HT-400toInf)	< 0.054
	$\gamma\gamma$ + jets	< 1.290
$t\bar{t}$ + jets		< 0.672
Single top	Single t (s-channel)	< 0.288
	Single t (t-channel)	< 0.298
	Single t (tW-channel)	< 0.442
	Single $\bar{t}$ (s-channel)	< 0.249
	Single $\bar{t}$ (t-channel)	< 0.317
	Single $\bar{t}$ (tW-channel)	< 0.445
$t\bar{t}$ + X	$t\bar{t} \gamma$	0.662 ± 0.377
	$t\bar{t} W$	< 0.024
	$t\bar{t} Z$	0.039 ± 0.016
	$t\bar{t} H$	0.151 ± 0.012
Total background		2.842 ± 0.673
Data		12

Table 8.1 Event yields in the signal region.

The result of t^* search

Although the MC estimation of background indicates a hint of the t^* signal, a complete background estimation using the data-driven method (fake photons estimated with matrix method: 9.6 ± 0.407 events) rules out the existence of t^* in our data.

The exclusion limit

Knowing that t^* ($950 \text{ GeV}/c^2$) is not discovered in our analysis, an exclusion limit on the $pp \rightarrow t^* \bar{t}^*$ cross section is estimated using an Asymptotic CLs method [44] with Poisson statistics. A log-normal prior is used in the integration given the background uncertainty. By comparing the result with the LO production cross section shown in Fig. 8.3, we set a limit on the t^* quark mass at $M(t^*) > 969 \text{ GeV}/c^2$ at 95% C.L. (The expected limit is $992^{+69}_{-68} \text{ GeV}/c^2$.)

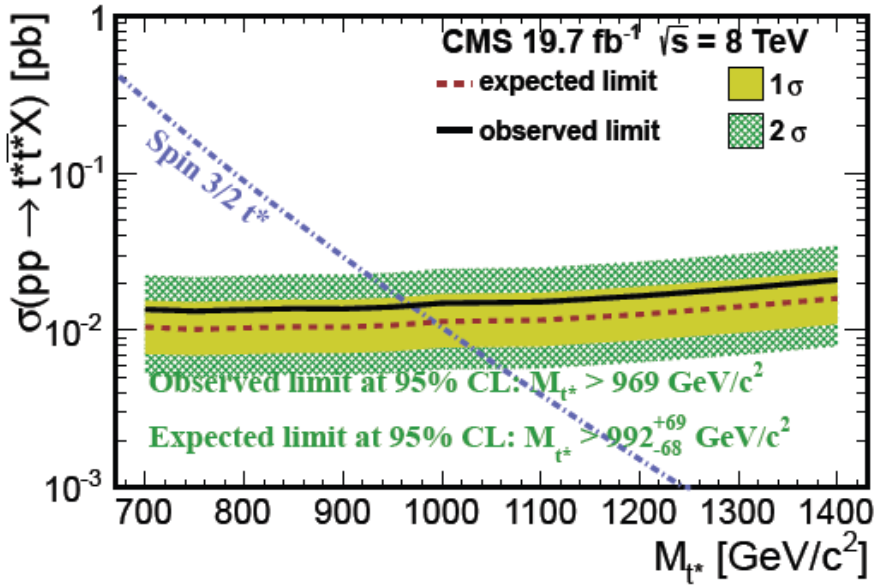


Fig. 8.3 The 95% confidence level (CL) upper limit on the cross section of the $pp \rightarrow t^* \bar{t}^*$ process, as a function of the t^* -quark mass. The branching fraction of $t^* \rightarrow t \gamma$ is assumed to be 100%. The solid line shows the observed limit. The dotted line corresponds to the expected limit under a background-only hypothesis. The solid (hatched) area shows the ± 1 (± 2) standard deviation uncertainties on the expected limit. The dot-dash line shows the value of the theoretical cross section for the spin-3/2 $t^* \bar{t}^*$ process.

Chapter 9 Conclusion

Using 2012 CMS data, in the semi-leptonic channel, with the χ^2 -sorting method to reconstruct t^* mass, the matrix method to estimate the background, we don't find a new particle $t^*(950\text{GeV}/c^2)$ and set an exclusion limit $M(t^*) > 969 \text{ GeV}/c^2$ by the CLs method.

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For example, the run range 196531, one can go to the link, in left window choose: online->collisions->2012->7e33->v2.5->HLT, and in the right window choose: A->SingleElectron, and find the HLT path "HLT_Ele25_CaloIdVT_CaloIsoT_TrkIdT_TrkIsoT_TriCentralPFNoPUJet30_30_20". The L1 triggers "L1_SingleEG20 or L1_SingleEG22" can also be found in the right most of trigger path.

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Appendix A the derivation of Matrix method

The Appendix A is the derivation of Matrix method. Firstly it starts from the eqn(2) in Section 7.1.

$$\begin{aligned} N_1 &= 1 \cdot N_1^{ff} + 1 \cdot N_1^{sf} + 1 \cdot N_1^{ss} \\ N_2 &= \epsilon_{ff}^{1 \rightarrow 2} \cdot N_1^{ff} + \epsilon_{sf}^{1 \rightarrow 2} \cdot N_1^{sf} + \epsilon_{ss}^{1 \rightarrow 2} \cdot N_1^{ss} \\ N_3 &= \epsilon_{ff}^{1 \rightarrow 3} \cdot N_1^{ff} + \epsilon_{sf}^{1 \rightarrow 3} \cdot N_1^{sf} + \epsilon_{ss}^{1 \rightarrow 3} \cdot N_1^{ss} \end{aligned} \quad \text{eqn (2)}$$

where $\epsilon_{ab}^{1 \rightarrow 2,3} = \frac{N_{2,3}^{ab}}{N_1^{ab}}$,

ab stand for s or f.

In eqn (2), the $\epsilon_{ff}^{1 \rightarrow 2}$ becomes

$$\begin{aligned} \epsilon_{ff}^{1 \rightarrow 2} &= \frac{N_2^{ff}}{N_1^{ff}} = \frac{N_2^{qq} + N_2^{gg} + N_2^{ll} + N_2^{qg} + N_2^{ql} + N_2^{gl}}{N_1^{ff}} \\ &= \epsilon_{qq}^{1 \rightarrow 2} + \epsilon_{gg}^{1 \rightarrow 2} + \epsilon_{ll}^{1 \rightarrow 2} + \epsilon_{qg}^{1 \rightarrow 2} + \epsilon_{ql}^{1 \rightarrow 2} + \epsilon_{gl}^{1 \rightarrow 2} \end{aligned}$$

eqn A.1,

where

$$\epsilon_{qq,gg,ll,qg,ql,gl}^{1 \rightarrow 2} = \frac{N_2^{qq,gg,ll,qg,ql,gl}}{N_1^{ff}}$$

eqn A.2.

In eqn A.1, the $\epsilon_{qq}^{1 \rightarrow 2}$ becomes

$$\begin{aligned}\epsilon_{qq}^{1 \rightarrow 2} &= \frac{N_2^{qq}}{N_1^{ff}} = \frac{N_2^{qq}}{N_1^{qq} + N_1^{gg} + N_1^{ll} + N_1^{qg} + N_1^{ql} + N_1^{gl}} \\ &= \frac{N_2^{qq}}{N_1^{qq}} \cdot \frac{1}{1 + \frac{N_1^{gg}}{N_1^{qq}} + \frac{N_1^{ll}}{N_1^{qq}} + \frac{N_1^{qg}}{N_1^{qq}} + \frac{N_1^{ql}}{N_1^{qq}} + \frac{N_1^{gl}}{N_1^{qq}}}\end{aligned}$$

eqn A.3.

And the N_2^{qq} / N_1^{qq} can be derived as in figure A-1. In figure A-1, the two circles represent the number of events with 2 photons, the hollowed circle represents the photon passing the loose selection but not passing the tight selection, the filled circle represents the photon passing the tight selection, and the label “q” above the circle represents that photon coming from quark.

In the 2nd line of figure A-1, the first term means that from all pairs in denominator to have no requirement (both loose and tight can be accepted) on the 1st photon and to have tight requirement on the 2nd photon, therefore, the 1st term equals to the per-object efficiency ϵ_q , the number of event with the tight photon over the number of events with the loose photons. The 2nd term is similar to the 1st term but with reverse roles of the 1st photon and the 2nd photon. The 3rd term means to require the tight photon on both the 1st one and the 2nd one in the two photons pair, thus the 3rd term equal to time the ϵ_q twice in the assumption that there is no correlation between the 1st photon and the 2nd photon.

$$\begin{aligned}
\frac{N_2^{qq}}{N_1^{qq}} &= \frac{\begin{array}{c} q \ q \quad q \ q \quad q \ q \\ \bigcirc \bullet + \bullet \bigcirc + \bullet \bullet \end{array}}{\left(\begin{array}{c} \bigcirc \bigcirc + \bigcirc \bullet + \bullet \bigcirc + \bullet \bullet \\ q \ q \quad q \ q \quad q \ q \quad q \ q \end{array} \right)} \leftarrow \text{denominator} \\
&= \frac{\begin{array}{c} q \ q \quad q \ q \\ \bigcirc \bullet + \bullet \bullet \end{array}}{\text{denominator}} + \frac{\begin{array}{c} q \ q \quad q \ q \\ \bullet \bigcirc + \bullet \bullet \end{array}}{\text{denominator}} - \frac{\begin{array}{c} q \ q \\ \bullet \bullet \end{array}}{\text{denominator}} \\
&= \mathcal{E}_q + \mathcal{E}_q - \mathcal{E}_q \times \mathcal{E}_q
\end{aligned}$$

Figure A-1

So the N_2^{qq}/N_1^{qq} becomes

$$\frac{N_2^{qq}}{N_1^{qq}} = 2\varepsilon_q - \varepsilon_q^2$$

eqn A.4.

Now the eqn A.3 becomes

$$\varepsilon_{qq}^{1 \rightarrow 2} = (2\varepsilon_q - \varepsilon_q^2) \cdot \frac{1}{1 + R \frac{gg}{qq} + R \frac{ll}{qq} + R \frac{qg}{qq} + R \frac{ql}{qq} + R \frac{gl}{qq}}$$

eqn A.5,

where

$$R \frac{gg, ll, qg, ql, gl}{qq} = \frac{N_1^{gg, ll, qg, ql, gl}}{N_1^{qq}}$$

eqn A.6.

In eqn A.1, the $\varepsilon_{gg}^{1 \rightarrow 2}$ and the $\varepsilon_{ll}^{1 \rightarrow 2}$ are similar to eqn A.5,

$$\varepsilon_{gg}^{1 \rightarrow 2} = (2\varepsilon_g - \varepsilon_g^2) \cdot \frac{1}{R \frac{qq}{gg} + 1 + R \frac{ll}{gg} + R \frac{qg}{gg} + R \frac{ql}{gg} + R \frac{gl}{gg}}$$

eqn A.7, and

$$\varepsilon_{ll}^{1 \rightarrow 2} = (2\varepsilon_l - \varepsilon_l^2) \cdot \frac{1}{R \frac{qq}{ll} + R \frac{gg}{ll} + 1 + R \frac{qg}{ll} + R \frac{ql}{ll} + R \frac{gl}{ll}}$$

eqn A.8.

In eqn A.1, the $\varepsilon_{qg}^{1 \rightarrow 2}$ becomes

$$\begin{aligned}\varepsilon_{qg}^{1 \rightarrow 2} &= \frac{N_2^{qg}}{N_1^{ff}} = \frac{N_2^{qg}}{N_1^{qq} + N_1^{gg} + N_1^{ll} + N_1^{qg} + N_1^{ql} + N_1^{gl}} \\ &= \frac{N_2^{qg}}{N_1^{qg}} \cdot \frac{1}{\frac{N_1^{qq}}{N_1^{qg}} + \frac{N_1^{gg}}{N_1^{qg}} + \frac{N_1^{ll}}{N_1^{qg}} + 1 + \frac{N_1^{ql}}{N_1^{qg}} + \frac{N_1^{gl}}{N_1^{qg}}}\end{aligned}$$

eqn A.9.

And the N_2^{qg} / N_1^{qg} can be derived as in figure A-2. The explanation of figure A-2 is similar to figure A-1.

$$\begin{aligned}\frac{N_2^{qg}}{N_1^{qg}} &= \frac{\begin{array}{ccc} q & g & \\ \bigcirc & \bullet & \end{array} + \begin{array}{ccc} q & g & \\ \bullet & \bigcirc & \end{array} + \begin{array}{ccc} q & g & \\ \bullet & \bullet & \end{array}}{\left(\begin{array}{cc} \bigcirc & \bigcirc \\ q & g \end{array} + \begin{array}{cc} \bigcirc & \bullet \\ q & g \end{array} + \begin{array}{cc} \bullet & \bigcirc \\ q & g \end{array} + \begin{array}{cc} \bullet & \bullet \\ q & g \end{array} \right)} \quad \leftarrow \text{denominator} \\ &= \frac{\begin{array}{cc} q & g \\ \bigcirc & \bullet \end{array} + \begin{array}{cc} q & g \\ \bullet & \bullet \end{array}}{\text{denominator}} + \frac{\begin{array}{cc} q & g \\ \bullet & \bigcirc \end{array} + \begin{array}{cc} q & g \\ \bullet & \bullet \end{array}}{\text{denominator}} - \frac{\begin{array}{cc} q & g \\ \bullet & \bullet \end{array}}{\text{denominator}} \\ &= \varepsilon_q + \varepsilon_g - \varepsilon_q \times \varepsilon_g\end{aligned}$$

Figure A-2

So the N_2^{qg} / N_1^{qg} becomes

$$\frac{N_2^{qg}}{N_1^{qg}} = \varepsilon_q + \varepsilon_g - \varepsilon_q \times \varepsilon_g$$

eqn A.10.

Now the eqn A.9 becomes

$$\varepsilon_{qg}^{1 \rightarrow 2} = (\varepsilon_q + \varepsilon_g - \varepsilon_q \times \varepsilon_g) \cdot \frac{1}{\frac{gg}{R^{qg}} + \frac{gg}{R^{qg}} + \frac{ll}{R^{qg}} + 1 + \frac{ql}{R^{qg}} + \frac{gl}{R^{qg}}}$$

eqn A.11.

In eqn A.1, the $\varepsilon_{ql}^{1 \rightarrow 2}$ and the $\varepsilon_{gl}^{1 \rightarrow 2}$ are similar to eqn A.11,

$$\varepsilon_{ql}^{1 \rightarrow 2} = (\varepsilon_q + \varepsilon_l - \varepsilon_q \times \varepsilon_l) \cdot \frac{1}{R \frac{gg}{ql} + R \frac{gg}{ql} + R \frac{ll}{ql} + R \frac{qq}{ql} + 1 + R \frac{gl}{ql}}$$

eqn A.12,

and

$$\varepsilon_{gl}^{1 \rightarrow 2} = (\varepsilon_g + \varepsilon_l - \varepsilon_g \times \varepsilon_l) \cdot \frac{1}{R \frac{gg}{gl} + R \frac{gg}{gl} + R \frac{ll}{gl} + R \frac{qq}{gl} + R \frac{ql}{gl} + 1}$$

eqn A.13

In eqn (2), the $\varepsilon_{sf}^{1 \rightarrow 2}$ becomes

$$\varepsilon_{sf}^{1 \rightarrow 2} = \frac{N_2^{sf}}{N_1^{sf}} = \frac{N_2^{sl} + N_2^{sq} + N_2^{sg}}{N_1^{sf}} = \varepsilon_{sl}^{1 \rightarrow 2} + \varepsilon_{sq}^{1 \rightarrow 2} + \varepsilon_{sg}^{1 \rightarrow 2}$$

eqn A.14,

where

$$\varepsilon_{sl,sq,sg}^{1 \rightarrow 2} = \frac{N_2^{sl,sq,sg}}{N_1^{sf}}$$

eqn A.15.

In eqn A.14, the $\varepsilon_{sl}^{1 \rightarrow 2}$ becomes

$$\varepsilon_{sl}^{1 \rightarrow 2} = \frac{N_2^{sl}}{N_1^{sf}} = \frac{N_2^{sl}}{N_1^{sl} + N_1^{sq} + N_1^{sg}} = \frac{N_2^{sl}}{N_1^{sl}} \cdot \frac{1}{1 + \frac{N_1^{sq}}{N_1^{sl}} + \frac{N_1^{sg}}{N_1^{sl}}}$$

eqn A.16.

In eqn A.16, the N_2^{sl} / N_1^{sl} is similar to eqn A.10,

$$\frac{N_2^{sl}}{N_1^{sl}} = \varepsilon_s + \varepsilon_l - \varepsilon_s \times \varepsilon_l$$

eqn A.17.

Now the eqn A.16 becomes

$$\varepsilon_{sl}^{1 \rightarrow 2} = (\varepsilon_s + \varepsilon_l - \varepsilon_s \times \varepsilon_l) \cdot \frac{1}{1 + R^{\frac{sq}{sl}} + R^{\frac{sl}{sq}}}$$

eqn A.18.

In eqn A.14, the $\varepsilon_{sq}^{1 \rightarrow 2}$ and the $\varepsilon_{sg}^{1 \rightarrow 2}$ are similar to eqn A.18,

$$\varepsilon_{sq}^{1 \rightarrow 2} = (\varepsilon_s + \varepsilon_q - \varepsilon_s \times \varepsilon_q) \cdot \frac{1}{R^{\frac{sl}{sq}} + 1 + R^{\frac{sq}{sl}}}$$

eqn A.19,

and

$$\varepsilon_{sg}^{1 \rightarrow 2} = (\varepsilon_s + \varepsilon_g - \varepsilon_s \times \varepsilon_g) \cdot \frac{1}{R^{\frac{sl}{sg}} + R^{\frac{sq}{sg}} + 1}$$

eqn A.20.

In eqn (2), the $\varepsilon_{ss}^{1 \rightarrow 2}$ is similar to eqn A.10,

$$\varepsilon_{ss}^{1 \rightarrow 2} = \frac{N_2^{ss}}{N_1^{ss}} = 2\varepsilon_s - \varepsilon_s^2$$

eqn A.21.

In eqn (2), the $\varepsilon_{ff}^{1 \rightarrow 3}$ becomes

$$\begin{aligned} \varepsilon_{ff}^{1 \rightarrow 3} &= \frac{N_3^{ff}}{N_1^{ff}} = \frac{N_3^{qq} + N_3^{gg} + N_3^{ll} + N_3^{qg} + N_3^{ql} + N_3^{gl}}{N_1^{ff}} \\ &= \varepsilon_{qq}^{1 \rightarrow 3} + \varepsilon_{gg}^{1 \rightarrow 3} + \varepsilon_{ll}^{1 \rightarrow 3} + \varepsilon_{qg}^{1 \rightarrow 3} + \varepsilon_{ql}^{1 \rightarrow 3} + \varepsilon_{gl}^{1 \rightarrow 3} \end{aligned}$$

eqn A.22.

In eqn A.22, the $\epsilon_{qq}^{1 \rightarrow 3}$ becomes

$$\begin{aligned}\epsilon_{qq}^{1 \rightarrow 3} &= \frac{N_3^{qq}}{N_1^{ff}} = \frac{N_3^{qq}}{N_1^{qq} + N_1^{gg} + N_1^{ll} + N_1^{qg} + N_1^{ql} + N_1^{gl}} \\ &= \frac{N_3^{qq}}{N_1^{qq}} \cdot \frac{1}{1 + \frac{N_1^{gg}}{N_1^{qq}} + \frac{N_1^{ll}}{N_1^{qq}} + \frac{N_1^{qg}}{N_1^{qq}} + \frac{N_1^{ql}}{N_1^{qq}} + \frac{N_1^{gl}}{N_1^{qq}}}\end{aligned}$$

eqn A.23.

And the N_3^{qq} / N_1^{qq} can be derived as in figure A-3. The explanation of figure A-3 is similar to figure A-1.

$$\begin{aligned}\frac{N_3^{qq}}{N_1^{qq}} &= \frac{\begin{array}{c} q \quad q \\ \bullet \quad \bullet \end{array}}{\left(\begin{array}{cc} \circ & \circ \\ q & q \end{array} + \begin{array}{cc} \circ & \bullet \\ q & q \end{array} + \begin{array}{cc} \bullet & \circ \\ q & q \end{array} + \begin{array}{cc} \bullet & \bullet \\ q & q \end{array} \right)} \quad \leftarrow \text{denominator} \\ &= E_q \times E_q\end{aligned}$$

figure A-3

So the N_3^{qq} / N_1^{qq} becomes

$$\frac{N_3^{qq}}{N_1^{qq}} = \epsilon_q^2$$

eqn A.24.

Now the eqn A.23 becomes

$$\epsilon_{qq}^{1 \rightarrow 3} = \epsilon_q^2 \cdot \frac{1}{1 + \frac{gg}{R^{qq}} + \frac{ll}{R^{qq}} + \frac{qg}{R^{qq}} + \frac{ql}{R^{qq}} + \frac{gl}{R^{qq}}}$$

eqn A.25.

In eqn A.22, the $\varepsilon_{gg}^{1 \rightarrow 3}$ and the $\varepsilon_{ll}^{1 \rightarrow 3}$ are similar to eqn A.25,

$$\varepsilon_{gg}^{1 \rightarrow 3} = \varepsilon_g^2 \cdot \frac{1}{\frac{qq}{R_{gg}} + 1 + \frac{ll}{R_{gg}} + \frac{qg}{R_{gg}} + \frac{ql}{R_{gg}} + \frac{gl}{R_{gg}}}$$

eqn A.26,

and

$$\varepsilon_{ll}^{1 \rightarrow 3} = \varepsilon_l^2 \cdot \frac{1}{\frac{qq}{R_{ll}} + \frac{gg}{R_{ll}} + 1 + \frac{qg}{R_{ll}} + \frac{ql}{R_{ll}} + \frac{gl}{R_{ll}}}$$

eqn A.27.

,

In eqn A.22, the $\varepsilon_{qg}^{1 \rightarrow 3}$ becomes

$$\begin{aligned} \varepsilon_{qg}^{1 \rightarrow 3} &= \frac{N_3^{qg}}{N_1^{ff}} = \frac{N_3^{qg}}{N_1^{qq} + N_1^{gg} + N_1^{ll} + N_1^{qg} + N_1^{ql} + N_1^{gl}} \\ &= \frac{N_3^{qg}}{N_1^{qg}} \cdot \frac{1}{\frac{N_1^{qq}}{N_1^{qg}} + \frac{N_1^{gg}}{N_1^{qg}} + \frac{N_1^{ll}}{N_1^{qg}} + 1 + \frac{N_1^{ql}}{N_1^{qg}} + \frac{N_1^{gl}}{N_1^{qg}}} \end{aligned}$$

eqn A.28.

And the N_3^{qg} / N_1^{qg} can be derived as in figure A-4. The explanation of figure A-4 is similar to figure A-1.

$$\begin{aligned} \frac{N_3^{qg}}{N_1^{qg}} &= \frac{\begin{array}{cc} q & g \\ \bullet & \bullet \end{array}}{\left(\begin{array}{cc} \circ & \circ \\ q & g \end{array} + \begin{array}{cc} \circ & \bullet \\ q & g \end{array} + \begin{array}{cc} \bullet & \circ \\ q & g \end{array} + \begin{array}{cc} \bullet & \bullet \\ q & g \end{array} \right)} \quad \leftarrow \text{denominator} \\ &= \varepsilon_q \times \varepsilon_g \end{aligned}$$

figure A-4

So the N_3^{qg} / N_1^{qg} becomes

$$\frac{N_3^{qg}}{N_1^{qg}} = \varepsilon_q \times \varepsilon_g$$

eqn A.29.

Now the eqn A.28 becomes

$$\varepsilon_{qg}^{1 \rightarrow 3} = \varepsilon_q \times \varepsilon_g \cdot \frac{1}{\frac{gg}{R^{qg}} + \frac{gg}{R^{qg}} + \frac{ll}{R^{qg}} + 1 + \frac{ql}{R^{qg}} + \frac{gl}{R^{qg}}}$$

eqn A.30.

In eqn A.22, the $\varepsilon_{ql}^{1 \rightarrow 3}$ and the $\varepsilon_{gl}^{1 \rightarrow 3}$ are similar to eqn A.30,

$$\varepsilon_{ql}^{1 \rightarrow 3} = \varepsilon_q \times \varepsilon_l \cdot \frac{1}{\frac{gg}{R^{ql}} + \frac{gg}{R^{ql}} + \frac{ll}{R^{ql}} + \frac{qg}{R^{ql}} + 1 + \frac{gl}{R^{ql}}}$$

eqn A.31,

and

$$\varepsilon_{gl}^{1 \rightarrow 3} = \varepsilon_g \times \varepsilon_l \cdot \frac{1}{\frac{gg}{R^{gl}} + \frac{gg}{R^{gl}} + \frac{ll}{R^{gl}} + \frac{qg}{R^{gl}} + \frac{ql}{R^{gl}} + 1}$$

eqn A.32.

In eqn (2), the $\varepsilon_{sf}^{1 \rightarrow 3}$ becomes

$$\varepsilon_{sf}^{1 \rightarrow 3} = \frac{N_3^{sf}}{N_1^{sf}} = \frac{N_3^{sl} + N_3^{sq} + N_3^{sg}}{N_1^{sf}} = \varepsilon_{sl}^{1 \rightarrow 3} + \varepsilon_{sq}^{1 \rightarrow 3} + \varepsilon_{sg}^{1 \rightarrow 3}$$

eqn A.33.

In eqn A.33, the $\varepsilon_{sl}^{1 \rightarrow 3}$ becomes

$$\varepsilon_{sl}^{1 \rightarrow 3} = \frac{N_3^{sl}}{N_1^{sf}} = \frac{N_3^{sl}}{N_1^{sl} + N_1^{sq} + N_1^{sg}} = \frac{N_3^{sl}}{N_1^{sl}} \cdot \frac{1}{1 + \frac{N_1^{sq}}{N_1^{sl}} + \frac{N_1^{sg}}{N_1^{sl}}}$$

eqn A.34.

And the N_3^{sl} / N_1^{sl} is similar to eqn A.29

$$\frac{N_3^{sl}}{N_1^{sl}} = \varepsilon_s \times \varepsilon_l$$

eqn A.35.

Now the eqn A.34 becomes

$$\varepsilon_{sl}^{1 \rightarrow 3} = \varepsilon_s \times \varepsilon_l \cdot \frac{1}{1 + R_{sl}^{sq} + R_{sl}^{sg}}$$

eqn A.35.

In eqn A.33, the $\varepsilon_{sq}^{1 \rightarrow 3}$ and the $\varepsilon_{sg}^{1 \rightarrow 3}$ are similar to eqn A.35,

$$\varepsilon_{sq}^{1 \rightarrow 3} = \varepsilon_s \times \varepsilon_q \cdot \frac{1}{R_{sq}^{sl} + 1 + R_{sq}^{sg}}$$

eqn A.36,

and

$$\varepsilon_{sg}^{1 \rightarrow 3} = \varepsilon_s \times \varepsilon_g \cdot \frac{1}{R_{sg}^{sl} + R_{sg}^{sq} + 1}$$

eqn A.37.

In eqn (2), the $\varepsilon_{ss}^{1 \rightarrow 3}$ is similar to eqn A.24,

$$\varepsilon_{ss}^{1 \rightarrow 3} = \frac{N_3^{ss}}{N_1^{ss}} = \varepsilon_s^2$$

eqn A.38.

Now all **efficiencies** ($\varepsilon_{ff, sf, ss}^{1 \rightarrow 2, 3}$) in eqn (2) have been transformed to be **4 per-object efficiencies**, ε_l , ε_q , ε_g , and ε_s , and **7 ratio factors**, $R^{sl/sq}$, $R^{sl/sg}$, $R^{qg/lq}$, $R^{qg/lg}$, $R^{qg/qq}$, $R^{qg/gg}$, $R^{qg/ll}$.