



Analysis of tune modulations in the LHC

Analyse von Tune-Modulationen im LHC

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Declaration of Authorship

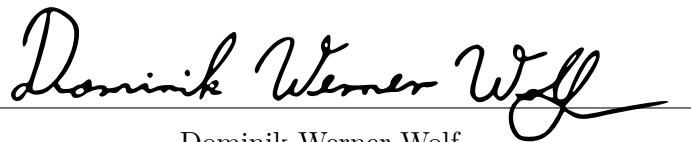
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Dominik Werner Wolf

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1 Introduction

After world war II, CERN was created to establish a research facility which unites Europe by trying to answer the fundamental question on how matter is built. Over the time, the accelerator facilities have become larger and larger to reach higher center of mass energies for the beam particles. In 2008 the Large Hadron Collider (LHC) run for the first time [40]. The circumference of the LHC is about 27km [40]. The left picture in figure 1 visualizes the dimensions of the LHC. In the spirit of the cosmopolitan attitudes of CERN the facilities were build on the border between Switzerland and France. The guiding dipoles as well as the experiments are placed underneath the surface. The inner life of the tunnels is illustrated on the right picture in figure 1. The design of the LHC was dimensioned to proof the existence of the Higgs-Boson. In 2012 this milestone has been achieved [40]. But there is still more to investigate about. Especially, the theory of supersymmetry needs to be evaluated by further measurements at higher beam energies. If the theory of supersymmetry is correct we should be able to find for each standard particle the corresponding superpartner. Consequently, there is a huge potential to find a large family of new, undiscovered particles. Therefore, further optimizations of the LHC will be undertaken to improve the production rate of such rare events. For realizing those improvements it is necessary to have a good knowledge about the machine parameters of the LHC. This includes the evaluation of the uncertainties of those parameters and even the uncertainties of the uncertainties. As a consequence, this thesis will elaborate on the stability of one of the most important quantities of an accelerator, the natural tune. As a result, this thesis contributes to the prerequisites for the high luminosity upgrade of the LHC [43].



Figure 1: On the left side, the LHC is indicated as a yellow line on the map [42]. On the right side, the LHC-tunnel is shown [41]. The blue tubes are superconducting electromagnetic dipoles which bend the particle beam.

2 Motivation and theoretical background

2.1 Operating cycle

The Large Hadron collider (LHC) has different phases to reach the collision plateau. In figure 2 the time dependency of the beam energy for the LHC is illustrated. The categorization of the beam phases corresponds to the current setting of the electromagnets. At injection a new particle beam is injected from the pre-accelerator (Super Proton Synchrotron (SPS)). If the new beam is stable the particles will be accelerated to higher energies wherefore the magnetic field of the electromagnets has to become stronger. Therefore, the magnet current has to rise. This phase is called ramp. If the necessary energy is reached the current stabilizes at the collision plateau. The beams are then at flattop energy. Due to the acceleration of the beam particles in radio-frequency cavities (RF-cavities) the particles are accumulated in particle bunches. When the beam bunches interact in one of the interaction points (IP's) then there is a certain possibility that the particles of the beam bunches collide. This possibility correlates to the cross section. If the particle density in the bunches gets higher the collision likelihood increases. Therefore, the extended cross section will lead to a rise of the production rate. For increasing the density of the particle bunches the beams are squeezed. The squeezing is reached by additional electromagnetic optic elements (triplet magnets), which shrink the beta function at the IP. In the LHC there are four IP's where different experiments are placed as visualized in figure 3. The average time for the flattop phase of the operating cycle is 5.9 min [31]. Comparing this value with the average time for the injection phase (72 min [31]) and the ramping phase (20 min [31]) gives an insight of the effort that is needed to prepare the beams for the collision plateau. In the actual setting of the LHC the flattop energy is 6.5 TeV. Consequently, the centre of mass energy is 13 TeV. By accelerating protons to a flattop energy of 6.5 TeV a velocity of $v_p \approx (1 - 1 \text{ ppm}) \cdot c$ is reached.

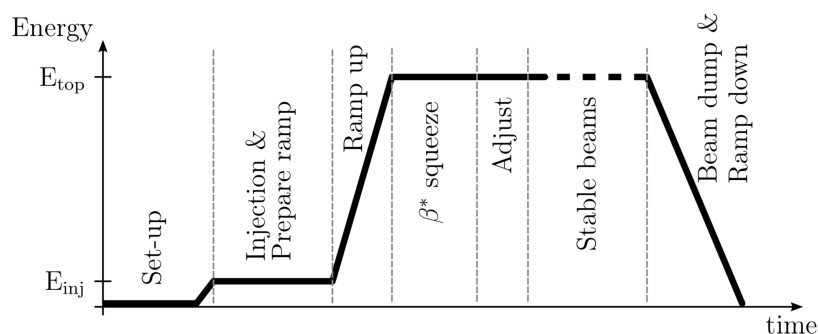


Figure 2: Time dependency of the beam energy in the LHC [32].

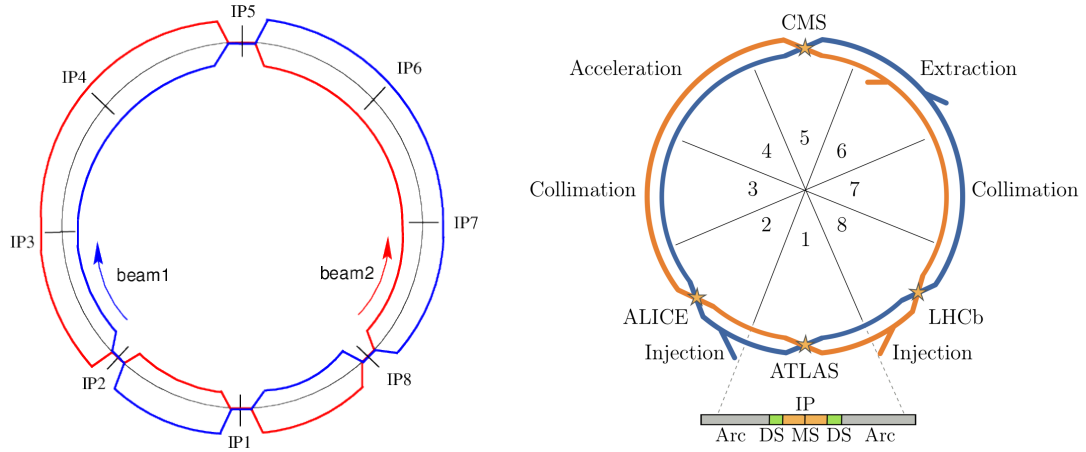


Figure 3: On the left side, a schematic layout of the LHC collision points and beams is shown [29]. On the right side, a sketch of the LHC ring is presented showing the locations of all experiments [32]. CMS and ATLAS are two general purpose experiments while ALICE and LHCb are dedicated for heavy-ion physics.

2.2 Frenet-Serret coordinate system

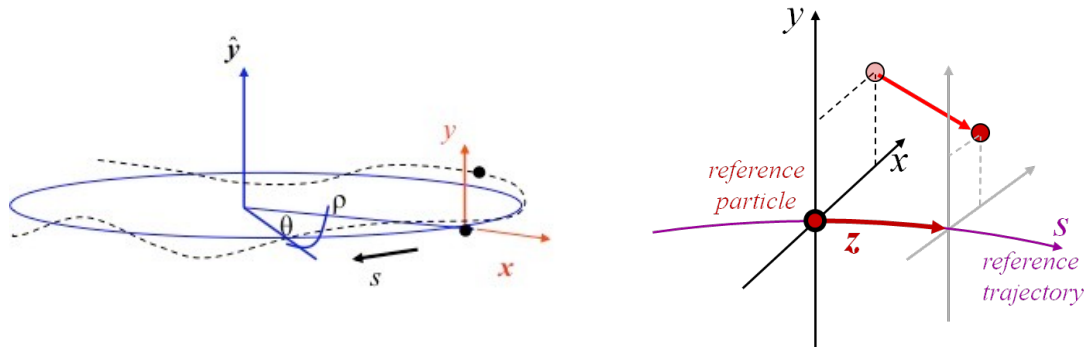


Figure 4: On the left, a sketch of a circular accelerator is shown [27]. In blue the reference trajectory of the particle beam is illustrated. An example for a single particle trajectory is provided as a dark dashed line. In orange the Frenet-Serret coordinate system is indicated. The tangential dimension with regard to the reference trajectory is symbolized with $\hat{s}$. The orthogonal, horizontal dimension with regard to $\hat{s}$ is abbreviated as $\hat{x}$. The orthogonal, vertical dimension in relation to $\hat{s}$ is defined as $\hat{y}$. On the right, a single particle is illustrated in relation to the reference particle [37]. The space vector of the Frenet-Serret coordinate system always describes the displacement between the position of the reference particle and the actual position of the real particle. The reference particle is an imaginary particle which follows the reference trajectory.

The schema of the Frenet-Serret coordinate system is illustrated in figure 4. In a circular accelerator the coordinate system is related to the resting frame of the particle bunch which follows the reference trajectory. A particle bunch is composed by an accumulation of beam particles and describes the average behaviour of the beam particles. The real trajectory of a single particle of the bunch can vary from the reference trajectory. To characterize the space position of the beam particles a right handed coordinate system is invented which is based on the resting frame of the particle bunch. Therefore, the origin of the Frenet-Serret coordinate system is always based on the reference trajectory.

2.3 Electromagnetic optics

The reference trajectory of the particles is determined by the homogeneous electromagnetic field generated by the dipole magnets of the LHC. The electric charged particles are effected by the Lorentz force of the electromagnetic field. The Lorentz force is defined in equation (1).

$$\vec{F}_L = q \cdot (\vec{E} + \vec{v} \times \vec{B}) \quad (1)$$

The Lorentz force will act as a centripetal force against the centrifugal force caused by the particle momentum. If the forces are in equilibrium the reference trajectory of the beam will be a cycle. The radius r of the cycle is determined by the equilibrium condition and results in equation (2).

$$r = q \cdot \frac{p}{B} \implies \Delta p \propto \Delta B \quad (2)$$

As a consequence, the radius r of the accelerator and the strength of the magnetic field B of the dipoles are the physical limitations regarding the momentum p of the particles. The LHC was built with a circumference of 27 km and can reach a kinetic energy of 6.5 TeV for protons in the current stage.

To accelerate the particles with radio frequency cavities (RF-cavities) the injected particles are accumulated as beam bunches. It is the goal to control those bunches as accurate as possible but a single particle can always show a random disturbance. For example if a particle shows an injection angle with regard to the tangential vector $\hat{s}$ of the reference trajectory then this particle would be lost without further disturbance handling. To keep the proportions of the beam bunch stable it is necessary to focus the beam continuously. Otherwise, a perturbation of the particle trajectory would lead to a lose of beam intensity. To ensure a stable reference trajectory a restoring force has to be installed. This is done by electromagnetic lenses and achieves an average movement of the particle bunch along the reference trajectory. In first order, quadrupoles are used as electromagnetic

lenses. According to the electromagnetic field properties, quadrupoles focus either in the horizontal or in the vertical plane and defocus at the same time in the perpendicular plane. This behaviour is illustrated in figure 5. Consequently, the quadrupole strength k ($1/r$ in equation (2) for the dipole strength) depends on the partial derivative of the vertical magnetic field component B_y with regard to the horizontal dimension x [32]. The entanglement of the field components is mathematically expressed by equation (3).

$$k := \frac{q}{p} \cdot \left. \frac{\partial B_y}{\partial x} \right|_{x=y=0} \quad (3)$$

The partial derivative in equation (3) is determined by the magnet structure of the single quadrupole. Considering, that the quadrupoles are arranged along the circular accelerator the partial derivative becomes periodic with regard to the $\hat{s}$ -axis. Consequently, the quadrupole strength k is periodic which leads to the Hill's differential equation presented in equation (5).

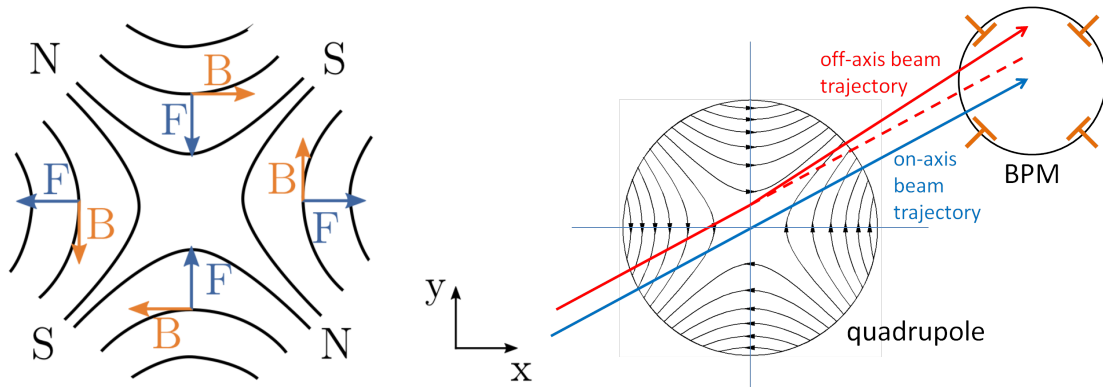


Figure 5: On the left, the magnetic field lines of a quadrupole magnet are illustrated [32]. The resulting Lorentz-force is indicated for a positively charged particle moving perpendicular into the drawing plane. Particles which show a transverse offset with regard to the quadrupole center will be focused in the vertical plane and defocused in the horizontal plane. On the right, the beam alignment with a quadrupole is visualised [37]. As a blue line the reference trajectory is indicated. The reference trajectory is not affected by the Lorentz-force because of the force equilibrium in the center of the quadrupole. A trajectory of an electron beam with a vertical offset is sketched as a red line. Due to the Lorentz-force the electron trajectory is bent and defocused in the vertical plane. Furthermore, a beam position monitor (BPM) is shown which is capable to measure the relative position of the real particle with respect to the reference particle.

In a nutshell, if quadrupoles are used to focus the particle beam in a circular accelerator it is indispensable to have an alternating structure of quadrupoles where the quadrupole strength k is negated pairwise. The alternating sequence of quadrupoles with a certain distance L and same absolute strength $|k(s)|$ is commonly known as a FODO-cell. The schema of a FODO-cell structure is visualized in figure 6. In general, for electromagnetic optics the same physical concept can be used as for photonic optics. Therefore, the single lenses in the FODO-cell structure are visualized as convex and concave lenses like in ray optics. For the mathematical description of the electromagnetic lens system in the LHC matrix optics is used to characterize the beam trajectory.

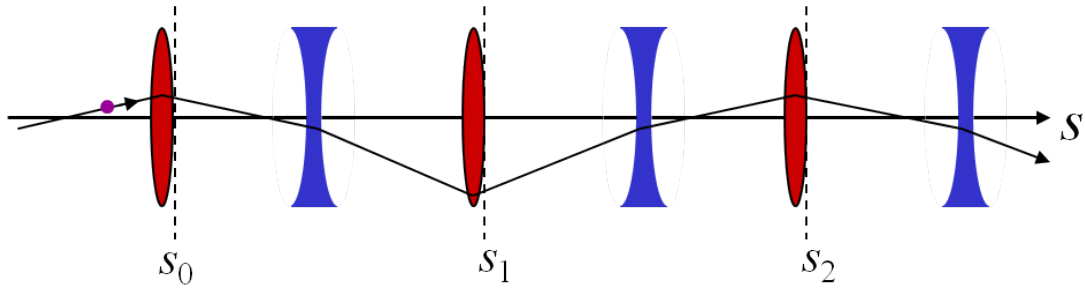


Figure 6: Structure of a FODO-cell. Focusing and defocusing electromagnetic optics alternate within a certain length L of free propagation [37].

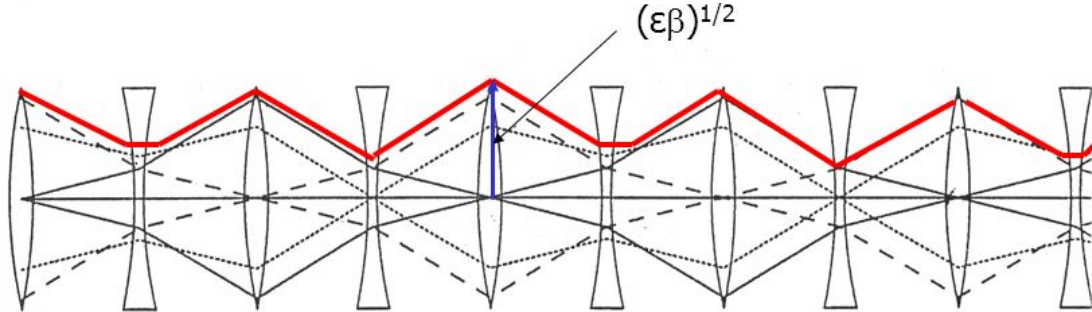


Figure 7: As black lines, examples for single particle trajectories in a FODO-cell structure are illustrated [38]. The elongation of the particles from the reference trajectory is determined by equation (6). The ensemble of all single particle trajectories characterizes a ceiling function for stable particle beams. The ceiling function is given by the square root of the product of the β -function and the emittance ϵ .

With the aid of the FODO-cells the bunch trajectory can be well controlled. Despite that, the trajectories of the single particles vary from each other within the limits of

the square root of the β -function times the emittance. Therefore, the β -function is also called the envelope function. Figure 7 visualizes this behaviour.

2.4 Hill's differential equation

In a periodic arrangement of electromagnetic quadrupole lenses like a FODO structure the derivation of the particle position from the reference trajectory is described by equation (4).

$$dy' = -k(s) \cdot y \cdot ds \quad (4)$$

Where k correlates to the focusing strength of the quadrupole. Considering that the periodic structure is closed the focusing strength of the quadrupoles k is a periodic function with a periodicity equal to the circumference of the accelerator. Keeping this fact in mind and rewriting equation (4) yields us to Hill's differential equation which is presented by relation (5).

$$\frac{dy'}{ds} + k(s) \cdot y = 0 \quad (5)$$

In a nutshell, Hill's differential equation describes a harmonic oscillator with a spring constant that follows a periodic function.

The solution for the differential equation (5) is given by relation (6).

$$y = \sqrt{2 \cdot J_y \cdot \beta_y} \cdot \cos(\phi_y) \quad (6)$$

Where J is the action in the vertical plane and β is the envelope function of the trajectory. The phase ϕ and β follow the same periodicity as the focusing strength function $k(s)$.

2.5 Natural tune and phase

The phase ϕ and the envelope function β are linked according to equation (7).

$$\frac{d\phi}{ds} = \frac{1}{\beta(s)} \quad (7)$$

The phase difference over one turn in the accelerator (with an explicit circumference L) is defined as the natural tune Q . Therefore, the phase difference is directly connected to the tune which expresses equation (8).

$$Q = \frac{\Delta\phi}{2\pi} = \frac{1}{2\pi} \cdot \int_0^L \frac{1}{\beta(s)} ds \quad (8)$$

2.6 Luminosity and natural tune jitter

In future stages of the Large Hadron Collider (LHC) it is the goal to increase the luminosity $\mathcal{L}$ of the accelerator to achieve a higher event rate. The luminosity is the quantity that measures the ability of a particle accelerator to produce the required number of interactions [20]. As a consequence, the luminosity is the proportionality factor between the interaction rate and the production cross section σ as shown in equation (9).

$$\frac{dN}{dt} = \mathcal{L} \cdot \sigma \quad (9)$$

The unit of the luminosity is $\text{cm}^{-2} \cdot \text{s}^{-1}$ and therewith equivalent to the unit of a flux density. The luminosity of two head-on colliding gaussian beams depends reciprocally on the width of the interacting beams as shown in equation 10.

$$\mathcal{L} = \frac{N_1 \cdot N_2 \cdot f \cdot N_B}{2\pi \cdot \sqrt{\sigma_{B1H}^2 + \sigma_{B2H}^2} \cdot \sqrt{\sigma_{B1V}^2 + \sigma_{B2V}^2}} \quad (10)$$

Where N_i describes the number of particles per bunch, f represents the revolution frequency and N_B is defined as the number of bunches. The width σ_i of the gaussian shaped bunches can be different for the horizontal and the vertical plane.

To detect very rare events significantly a higher event rate is necessary. For realizing that, the bunch width has to become smaller and consequently the beta function too. This will be realized by using stronger magnets in the LHC. As a result, for the high luminosity upgrade of the LHC it will be mandatory to determine the beta function very accurate at the interaction points. Those values are calculated by using k-modulation (for further details regarding k-modulation see [21]) which uses in principle the relation between natural tune and the beta function from equation (6). Therewith, it will become indispensable to determine the natural tune with an accuracy down to $\pm 10^{-5}$ [21] before the upgrade can be realized. As a consequence, the stability of the natural tune Q will be the main important aspect of this thesis. The quantity which characterizes the fluctuations of the natural tune is called the natural tune jitter (RMS-jitter) and is defined as the standard derivation of the natural tune. The RMS-jitter ξ is defined according to formula (11).

$$\xi := \sqrt{\sum_{i=1}^N \frac{(x_i - \langle x \rangle)^2}{N-1}} \quad (11)$$

2.7 Amplitude detuning

Previous studies of Courant and Snyder [14] resulted in the knowledge of the linear dependency of the natural tune Q from the action J . According to equation (6) the action J has a quadratic dependency from the elongation y of the reference trajectory. Consequently, the natural tune depends on the elongation itself which is called amplitude detuning. Without crossing angles the cosine function reduces to one what makes it possible to give a simplified version of the relation in equation (12).

$$Q \propto J = \frac{\langle y \rangle^2}{2 \cdot \beta^*} = \frac{\langle \hat{y} \rangle^2}{8 \cdot \beta^*} \quad (12)$$

Where $\hat{y}$ describes the peak to peak elongation and beta star the beta function at the interaction point.

Since 2017, the impact of amplitude detuning on the natural tune is compensated automatically by a post analysis. Therefore, the measurement files already contain the values levelled on the same reference amplitude. But for analyzing data sets from 2016 and earlier it will be necessary to compensate the tune shift manually. Therefore, it is important to determine the coefficient of the quadratic dependency of the natural tune from the peak to peak elongation for those measurements. This will be done by a linearisation of the problem and finding the optimal parameters for the line of best fit (the method is described in the appendix - chapter 8.1).

2.8 β -function at the interaction point

The β -function at the interaction point is usually abbreviated as β^* . Electromagnetic optics with a positive k -value act as focusing lenses. On the contrary, electromagnetic lenses with a negative k -value act as defocusing lenses. As explained in section 2.3, quadrupoles focus in one direction and defocus at the same time in the perpendicular plane. The principle of the FODO-cell structure can also be noticed in the upper diagram of figure 8. Focusing and defocusing lenses alternate within an equidistant distance L . As a consequence, an oscillation of the beam with regard to the beta function is observable. Whereby, the oscillation movement of the horizontal and vertical plane is displaced by a phase shift of π due to the fact that the quadrupole lenses focus in one plane and defocus at the same time in the perpendicular plane. The FODO-cells are used in almost every part of the LHC to stabilize the beam around the reference trajectory except at the interaction points. At the interaction points it is the goal to

collide the beams. For reaching a high event rate it is important to influence the beam in a way that the luminosity is maximized (see equation (9)). To maximize the luminosity of the beam a small beam width is necessary wherefore a small envelope function must be parameterized (see equation (10)). In the lower plot of figure 8 it is shown that a small beta function at the interaction point is reached by using a unique combination of electromagnetic lenses which fulfill the goal for both planes.

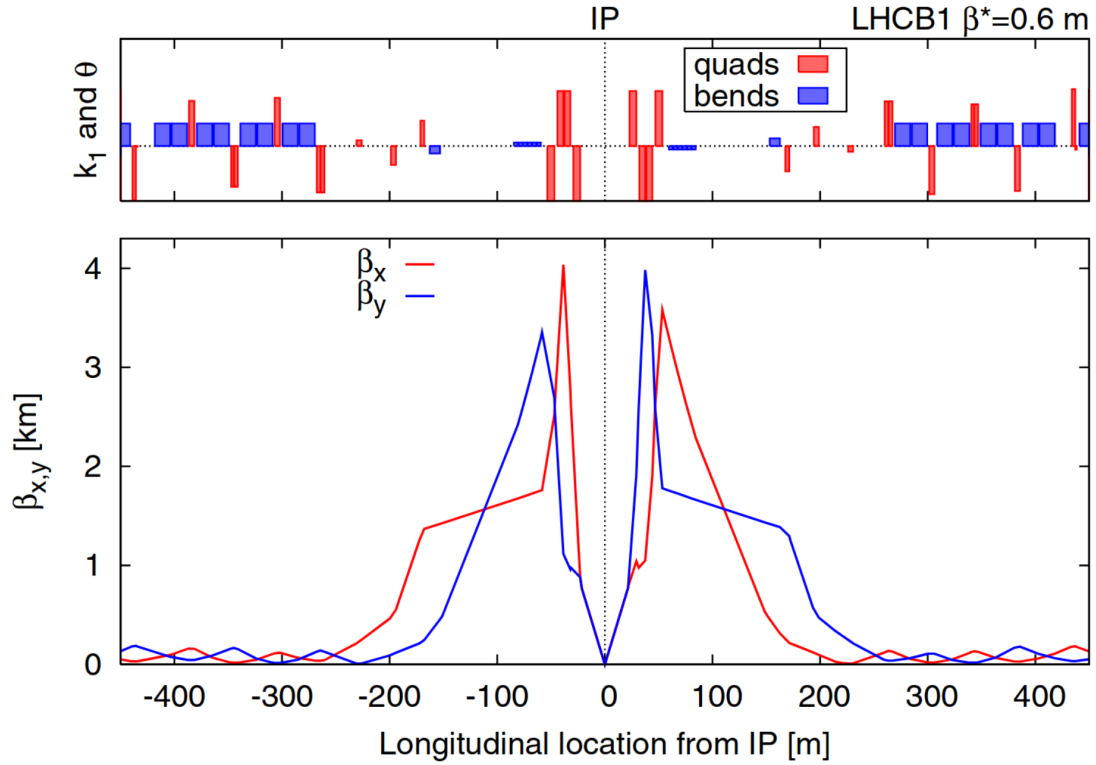


Figure 8: In the upper plot, the lattice structure of the LHC is shown. As red bars are the quadrupoles indicated. In the lower plot, the dependency of the beta-function from the longitudinal position around the interaction point is shown. Whereby, the red line represents the horizontal dimension and the blue line indicates the vertical dimension of the two-dimensional beta-function [26].

2.9 Chromaticity

Equation 3 defines the correlation between the focusing strength $k(s)$ of a quadrupole and the particle momentum p . As a consequence, particles which show a momentum deviation will experience a different focusing strength. This effect is in general known as dispersion. Therefore, a momentum spread in the bunch causes a position offset and influences the beta-function. As a result, the natural tune Q will also be shifted by the momentum spread Δp . This phenomena is called chromaticity. The relation between the natural tune shift ΔQ and the momentum shift Δp is expressed in equation (13).

$$\frac{\Delta p}{p} = 4\pi \cdot \Delta Q \cdot \left[\int_0^L \beta(s) \cdot k(s) ds \right]^{-1} \quad (13)$$

3 Measurements

For analyzing the dependency of the natural tune jitter ξ from the optics parametrization several data sets have been created. The AC-dipole excitations (Kicks) have been clustered in data sets of the same measurement day. In the majority of cases the optics parametrization is constant for a data set. In table 1 are the key settings of the individual data sets presented. The data files for each data set are listed in the references (Ref.). E_B is defined as the beam energy and the abbreviation Fill describes the fill number. The fill number is the name of the LHC logbook folder for the corresponding machine development (MD) session. Δt correlates to the measurement time window.

Table 1: Definition of measurement sets according to the measuring day.

Data set	Optics	Beam	Status	E_B [TeV]	β^* [m]	Fill	Δt [min]	Kicks [#]	Ref.	Date [dd:mm:yyyy]
A	round optics	LHCB1	flattop	6.5	0.65	6800	~ 27	18	[1]	15:06:2018
	round optics	LHCB2	flattop	6.5	0.25	6800	~ 19	8	[1]	
B	CTPPS2	LHCB1	flattop	6.5	0.4	5624	~ 5	4	[2]	08:05:2017
	CTPPS2	LHCB2	flattop	6.5	0.4	5624	~ 13	9	[2]	
C	ballistic optics	LHCB2	flattop	6.5	—	5600	~ 15	6	[3]	04:05:2017
D	ballistic optics	LHCB1	flattop	6.5	—	4734	~ 9	8	[4]	28:03:2016
	ballistic optics	LHCB2	flattop	6.5	—	4734	~ 10	8	[4]	
E	ATS	LHCB2	squeeze	6.5	0.4	5138	~ 14	10	[5]	30:07:2016
F	flat orbit	LHCB1	squeeze	6.5	0.3	6811	~ 62	34	[6]	16:06:2018
	flat orbit	LHCB2	squeeze	6.5	0.3	6811	~ 67	32	[6]	
G	high β^*	LHCB1	flattop	6.5	90	6739	~ 54	9	[7]	29:05:2018
	high β^*	LHCB2	flattop	6.5	90	6739	~ 73	11	[7]	
H	injection nominal	LHCB1	injection	0.45	11	7002	~ 30	18	[8]	30:07:2018
	injection nominal	LHCB2	injection	0.45	11	7002	~ 7	7	[8]	
I	flat optics	LHCB1	flattop	6.5	0.60/0.15	6445	~ 92	10	[9]	01:12:2017
J	flattop nominal	LHCB1	flattop	6.5	1.0	6496	~ 22	10	[10]	02:04:2018
	flattop nominal	LHCB2	flattop	6.5	1.0	6496	~ 7	5	[10]	
K	ATS (600m arcs)	LHCB1	flattop	6.5	0.25	5893	~ 4	4	[11]	30:06:2017
	ATS (600m arcs)	LHCB2	flattop	6.5	0.25	5893	~ 3	4	[11]	
L	CTPPS2 (200m arcs)	LHCB1	flattop	6.5	0.3	5708	~ 35	11	[12]	25:05:2017
	CTPPS2 (200m arcs)	LHCB2	flattop	6.5	0.3	5708	~ 37	13	[12]	
M	high β^*	LHCB1	injection	0.45	100	6327	~ 7	4	[13]	26:10:2017
	high β^*	LHCB2	injection	0.45	100	6327	~ 10	7	[13]	

4 Spectrum analysis

The machine nonlinearities of the accelerator can impact the key parameters and result in performance limitations. A nondestructive way to characterize the nonlinearities is the amplitude detuning measurement with AC-dipole excitations [36]. The turn by turn data [34] of the beam position monitors (BPM's) are available as sdds-files. Those files can be analyzed with a smart python algorithm [39] which uses fast Fourier transformation (FFT) [24] and singular value decomposition (SVD) [35] to evaluate the spectrum data for the regarding AC-dipole measurement [23]. Therefore, the amplitude and frequency values can be used to create a spectrum for the individual data set.

4.1 Round optics at flattop energy (data set A) - 15th of June in 2018

The AC-dipole spectra for the measurement set A [1] is shown in figure 9.

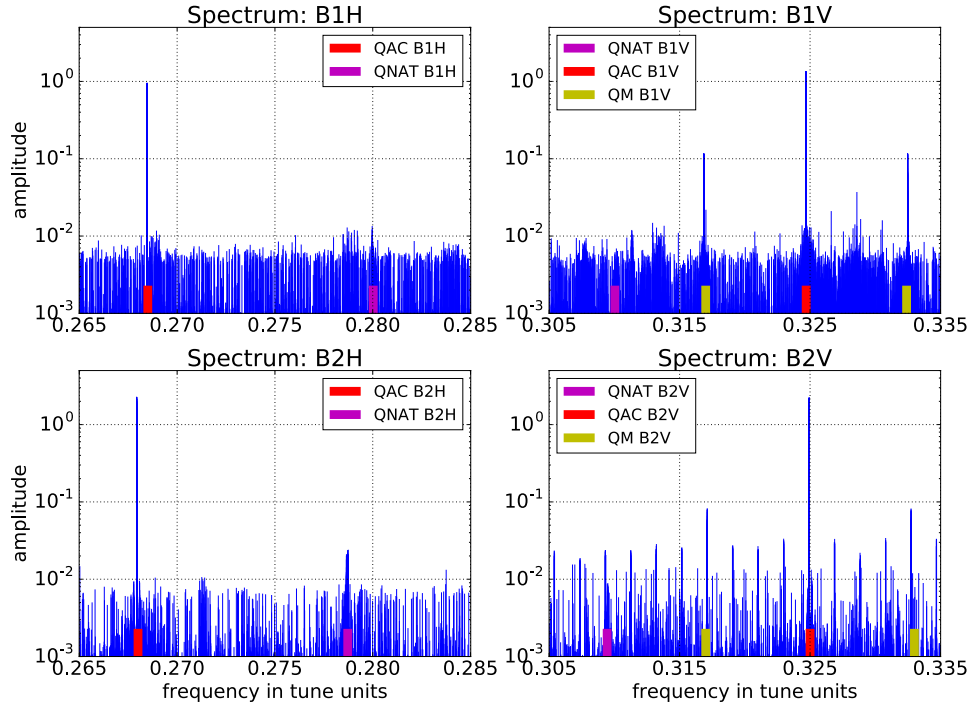


Figure 9: The red line is the AC-dipole kick (QAC) in the corresponding plane and the magenta line is the natural tune (QNAT) in the corresponding plane. The yellow lines in the vertical planes are sidebands (QM) which will be the subject of further analysis's.

The domain of definition for the frequency is $[0, 1]$. The spectra are mirror-symmetric at $f=0.5$. The marked lines are a forecast for the results and can not be characterized by only taking a closer look to the spectrum. Although a noise cleaning has been applied, which uses an amplitude threshold as criterion, there is still very much noise which covers a lot of information. The question could be asked why the threshold was chosen so small. With regard to the average noise level the threshold is definitely too low. But if the average noise level would have been chosen as the threshold, a lot of information about the significant peaks would be lost and all measurements for the natural tune would be withdrawn.

The previous amplitude filter was not sufficient enough for extracting the significant peaks in the spectrum. Therefore, a density cleaning was applied to the spectrum data. In a two dimensional histogram the data density can be visualized. A threshold of $N=5$ measurements for each bin area was set as a density threshold. If the threshold is exceeded the signal will be classified as significant. In addition to the density threshold there is also a second step of cleaning the data. A signal is defined as reliable if more than 50% of all BPM's measured a significant signal. If a signal is not reliable, it will be handled as it would not have been passed the density threshold. The scale for the data density was chosen as logarithmic to guarantee that even small densities can be visualized clearly.

In figure 10 are the 2D-histograms for each plane and beam shown. The y-axis carries the frequency information and the x-axis shows the longitudinal position of the BPM in the LHC. In the horizontal plane of beam 1 two lines are visible which are close together. This is against the expectations and will be a key question in this Bachelor-thesis. Three significant lines can be observed in the vertical plane of beam 1. Only one of the lines can be the natural tune. The line with the lowest frequency value shows a low information density wherefore it is high likely that this line could be caused by a faulty measurement. The other two lines contain much more data points. None of them can be explained by higher order harmonics. As seen for the vertical plane of beam 1 there is also in the vertical plane of beam 2 more than one line observable. This is contrary to all expectations.

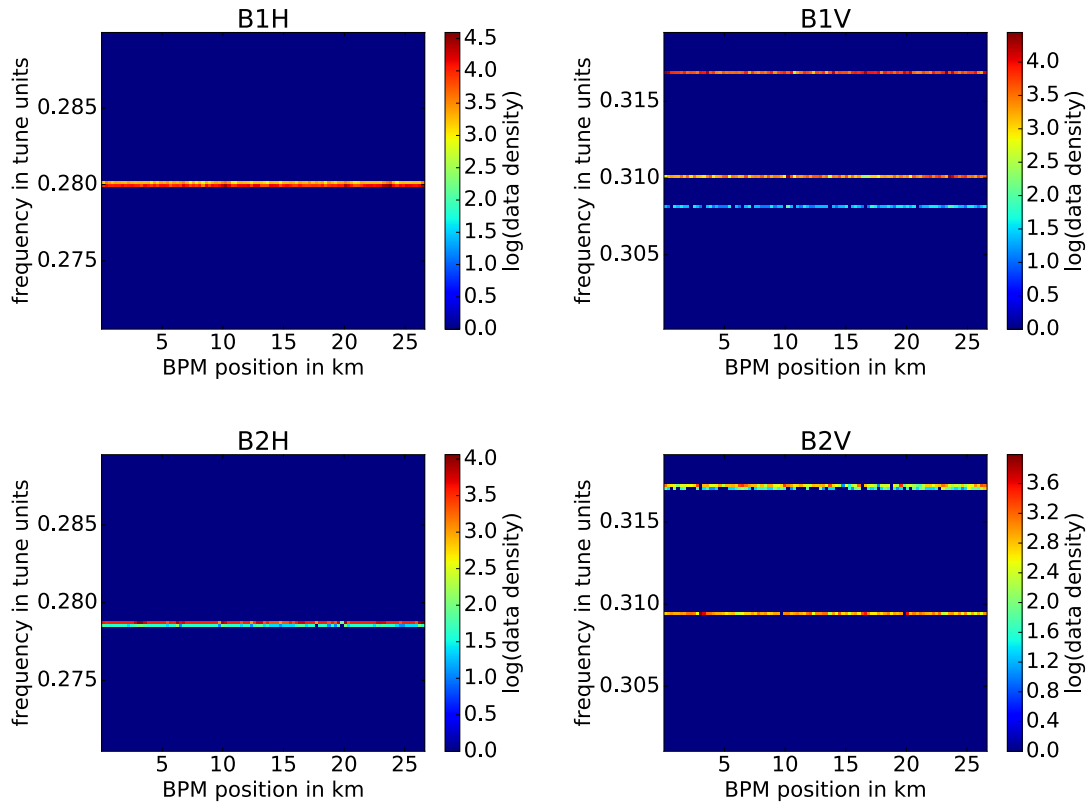


Figure 10: Two-dimensional histograms of the frequency data for measurement set A.

The task will be to distinguish between the different spectral lines and to decide which line represents the natural tune. Furthermore, it is also important to find out what causes the unexpected lines. To analyze the shape of the profile the dimension of the longitudinal position was eliminated and a projection on the tune axis was made. Figure 11 shows the frequency distribution of the horizontal natural tune for different measurements with the same setting as well as for beam 1 and beam 2. The reason for the second line in the histogram can already be explained with this figure. The natural tune in the horizontal plane drifted over time. Figure 12 shows the frequency distribution of the vertical natural tune for different measurements with the same setting as well as for beam 1 and beam 2. For beam 1 three lines were observed in the 2d-histogram. The first line is caused by only one data file, wherefore this event can be declared as a faulty measurement. The shapes of the accumulated peaks for beam 1 and beam 2 share almost the same shape. Especially, the profile of the line with the highest frequency value seems to follow a Lorentzian distribution (Lorentzian fit in red).

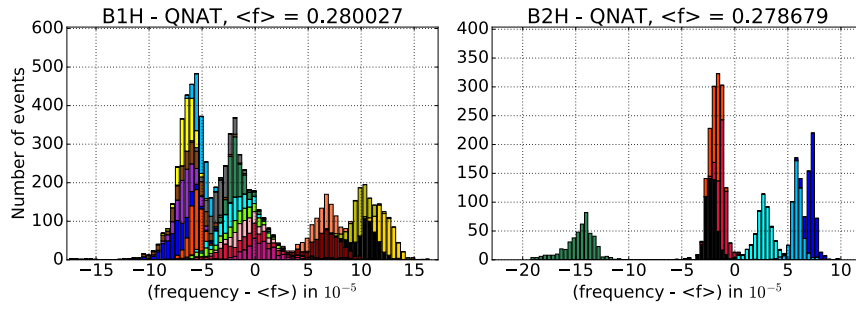


Figure 11: Frequency spectrum for the horizontal plane of measurement set A.

The third spectral line appears symmetric around the driven frequency of the AC-dipole. Therefore, those lines are called sidebands. According to the exclusion principle the second line is indicated as the natural tune. From a physical point of view, the frequency profile of the natural tune describes the interference of all single natural tune distributions. Those single distributions are shifted by the tune jitter. Therefore, the shape of the envelope function of the frequency profile strongly corresponds to the jitter of the natural tune. Consequently, the stacked frequency profile for the natural tune will appear as a smudged convolution of the single AC-dipole measurements.

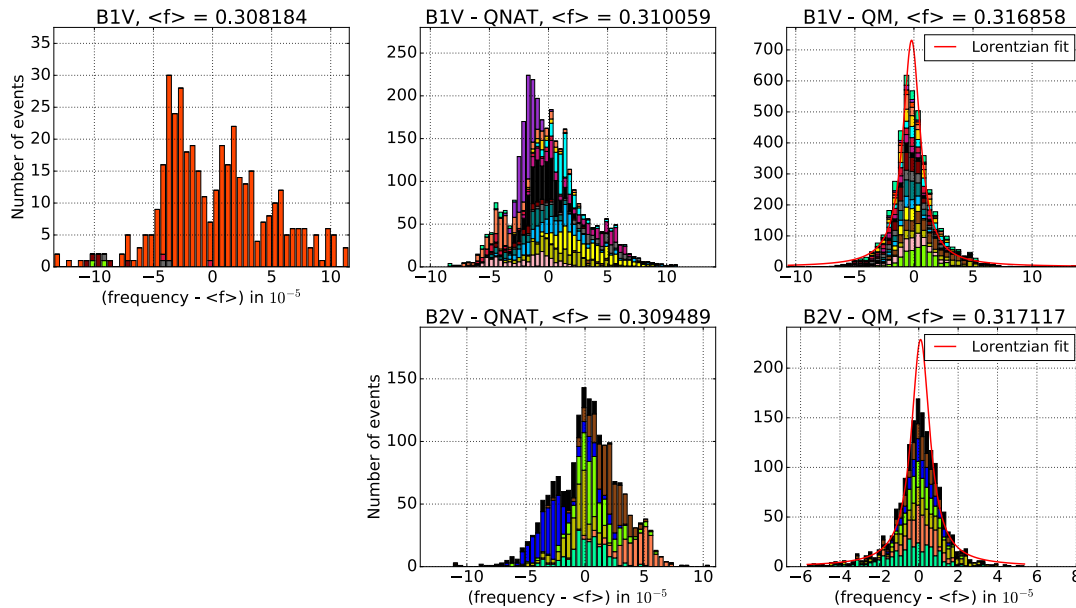


Figure 12: Frequency spectrum for the vertical plane of measurement set A.

The shape of the sideband in figure 12 seems to be characteristic, wherefore a further profile analysis was made with regard to the AC-dipole kick and the sideband on the other side of the symmetric spectrum. This analysis is visualized in figure 13. As expected, the tune separation (difference of frequency between AC-dipole kick and sideband) is in light of the parameter uncertainties the same for both sidebands. The tune separation is $\Delta Q = 0.00781 \pm 10^{-5}$. Unfortunately, neither the shape of the sideband nor the shape of the AC-dipole can be classified as a classical Lorentzian or Gaussian curve. For further studies about the envelope function of the frequency profile of the sideband a linear combination of both functions should be assumed.

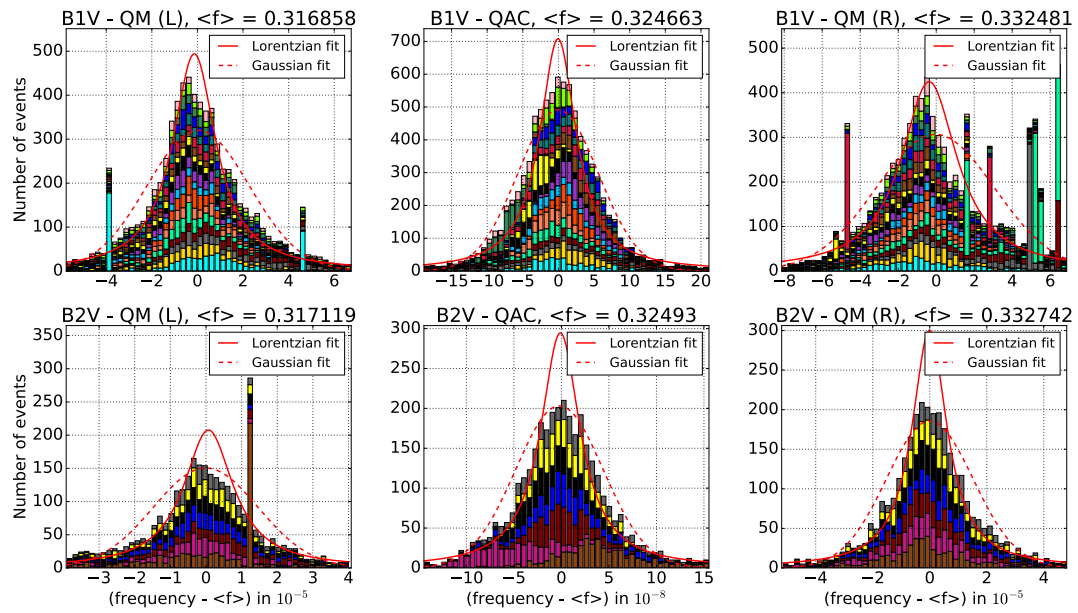


Figure 13: Profile analysis of the tune measurements from data set A. The analysis was focused on the sidebands (leftsided and rightsided) and the AC-dipole kick (in the middle).

As a result, a characteristic probability density function (pdf) for the frequency profile of the AC-dipole kick and the sideband could not be determined. Therefore, the similarity of the shape of the frequency profile between the sideband and the AC-dipole kick can not be quantified. From this perspective, the AC-dipole could not be concluded as the reason for the sideband.

4.2 CTPPS2 optics (data set B) - 8th of May in 2017

It is of utmost interest to find out if the observed sidebands are reproducible or not. Therefore, an analysis of a measurement set B [2] was made. The spectra of measurement set B is shown in figure 14.

As for measurement set A the data density analysis was executed to extract the significant and reliable lines out of the spectra. Those lines are marked in the spectra.

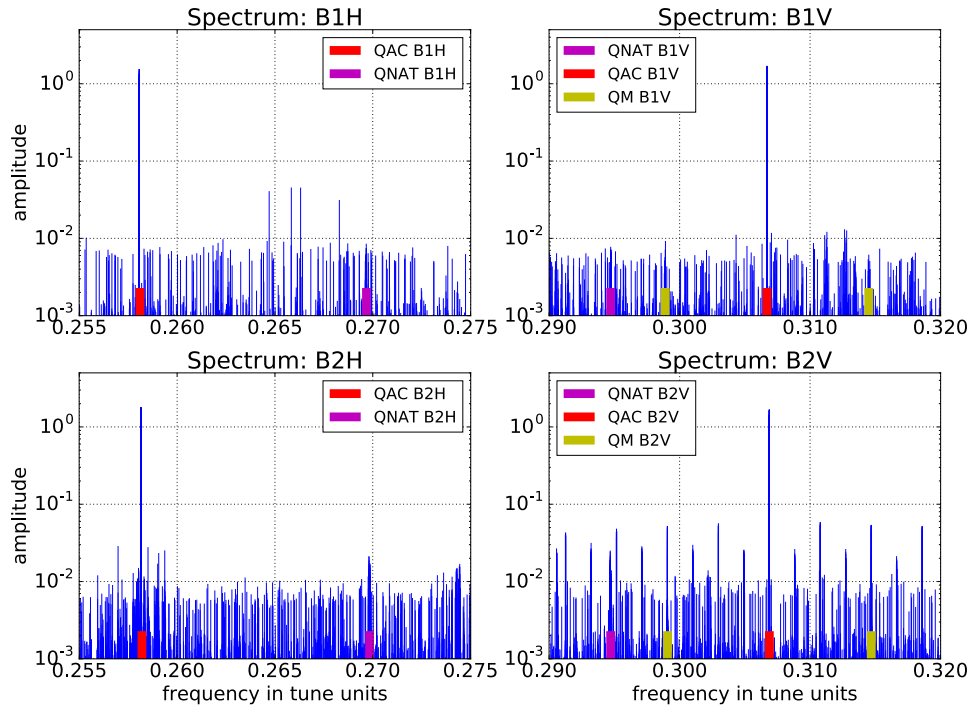


Figure 14: The red line is the AC-dipole kick (QAC) in the corresponding plane and the magenta line is the natural tune (QNAT) in the corresponding plane. The yellow lines in the vertical planes are sidebands (QM) which will be the subject of further analysis's.

In conclusion, the sidebands are also visible in the spectra of measurement set B. Therefore, the reason for the sidebands must be a setting which was the same in May 2017 and in June 2018. That is remarkable keeping in mind that the operational settings of the LHC has been completely different for data set A and B. This points to a problem with the AC-dipole. Therefore, further analysis's are needed to check the waveform of

the AC-dipole excitations. This is elaborated in section 5.3.

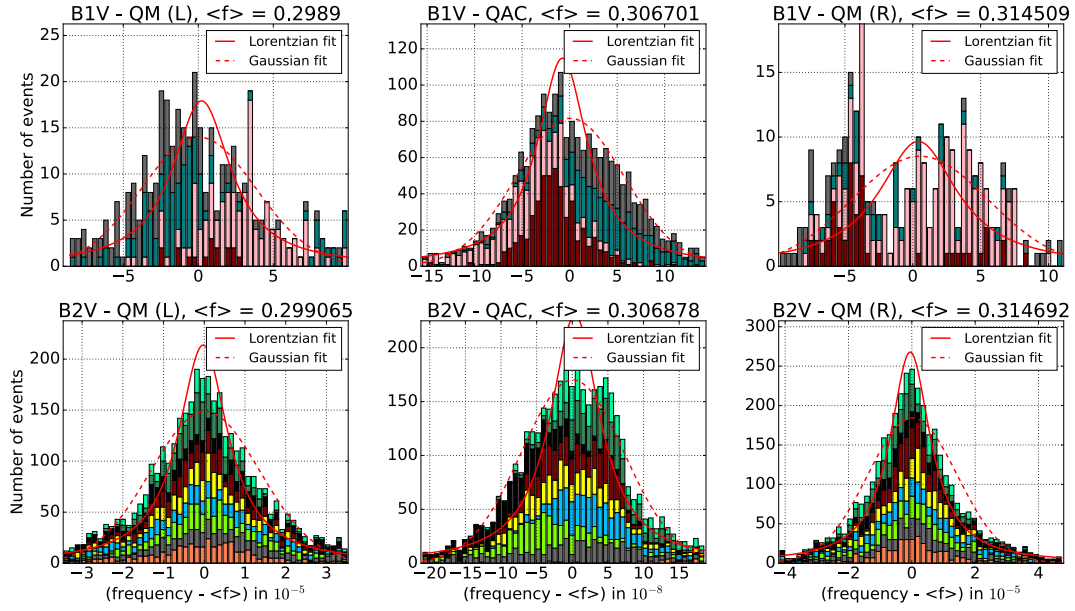


Figure 15: Profile analysis of the tune measurements from data set B. The analysis was focused on the sidebands (leftsided and rightsided) and the AC-dipole kick (in the middle).

In figure 15, the profile analysis of the sidebands and the AC-dipole kick is shown for the measurement set B. As seen in the profile analysis for measurement set A the distribution of the sidebands can not be described uniformly by a Gaussain or a Lorentzian curve. But both measurement sets support the quantity for the tune separation of the sidebands with a value of $\Delta Q = 0.00781 \pm 10^{-5}$.

At this point it has to be emphasized, that in some files the sidebands are not significantly noticeable as for example the rightsided sideband in measurement set B (compare to figure 15, upper right corner).

5 Sidebands

As sidebands are those symmetric peaks defined which can not be explained by the excitation frequency (QAC) or the natural tune (QNAT) nor higher harmonics of the linear combination of QAC and QNAT. Such sidebands were noticeable in the previous spectrum analysis of chapter 4. The properties of those sidebands need to be characterized to elaborate on possible reasons for the appearance of the sidebands. The sideband frequency is abbreviated as QM.

5.1 Overview of observation

In Chapter 6 several measurement sets for different optics have been analyzed. In some of those files there are also sidebands provable. In table 2 all observations are condensed.

Table 2: Tune separation between sideband and QAC.

Plane	Optics	QM left	QAC	QM right	QNAT	∂Q	$\leftarrow \Delta Q $	$ \Delta Q \rightarrow$
B1V	Round 65	0.316858	0.324663	0.332481	0.310053	0.015	0.007805	0.007818
B2V	Round 25	0.317119	0.324930	0.332742	0.309493	0.015	0.007811	0.007812
B1V	CTPPS2 40	0.298900	0.306701	0.314509	0.294742	0.012	0.007801	0.007808
B2V	CTPPS2 40	0.299065	0.306878	0.314692	0.294706	0.012	0.007813	0.007814
B1V	Flat orbit	0.322007	0.329818	0.337635	0.320000	0.010	0.007811	0.007818
B2V	Flat orbit	0.322007	0.329821	0.337640	0.319909	0.010	0.007815	0.007819
B2V	Ballistic	0.317202	0.325008	0.332815	0.310465	0.015	0.007806	0.007807
B2V	ATS 40	0.313736	0.321552	0.329365	0.309641	0.012	0.007816	0.007813

In table 2, the frequency difference between QM left and QAC is abbreviated as $\leftarrow |\Delta Q|$. Furthermore, the frequency difference between QAC and QM right is abbreviated as $|\Delta Q| \rightarrow$. In the column " ∂Q " is the tune separation between the vertical natural tune Q_y and the horizontal natural tune Q_x indicated. Without coupling of the horizontal and vertical tune the relation is given by $\partial Q = Q_y - Q_x$.

According to the measured tune separation ΔQ between the sideband and the excitation frequency QAC the sidebands appear symmetric around QAC with a tune separation of $\Delta Q = 0.00781 \pm 10^{-5}$. In some files the sidebands are very strongly pronounced wherefore even sub-sidebands are observable. As an example, with the ATS 40 cm optics there

also appeared sub-sidebands with a tune separation of $\Delta Q = 0.00391 \pm 10^{-5}$ (compare to table 3). This is barely half of the frequency of the sidebands.

Table 3: Tune separation between sub-sideband and QAC.

Plane	Optics	QM left	QAC	QM right	QNAT	∂Q	$\leftarrow \Delta Q $	$ \Delta Q \rightarrow$
B2V	ATS 40	0.317645	0.321552	0.325458	0.309641	0.012	0.003907	0.003906

5.2 Phase

The analysis algorithm which is used to extract the spectrum out of the raw data also returns the phase ϕ . The phase is related to the beta function according to equation (7). Figure 16 shows that the absolute value of the phase advance of QM right and the absolute value of the phase advance of QM left share a common range with respect to the statistical uncertainties. Therefore, it is possible that they share the same expectation value, which would be reasonable having in mind that the sidebands are symmetric around the AC-dipole frequency. Figure 17 visualizes the phase differences for beam 2 and underpins the thesis that the sidebands share the same absolute phase value in light of the statistical uncertainties. The quantities regarding the phase analysis of the sidebands are listed in table 4.

Table 4: Phase analysis regarding data set A.

Plane	$\Delta\phi$ uncertainty		$\Delta\phi$ uncertainty		Optics	β^*
	QM left	QM left	QM right	QM right		
B1V	-0.09	± 0.03	0.15	± 0.02	round	65cm
B2V	-0.11	± 0.01	0.13	± 0.01	round	25cm

In a nutshell, the absolute phase advance $|\Delta\phi|$ between AC-dipole and sideband is given by approximately $|\Delta\phi| \approx 0.12 \pm 0.01 \hat{=} 43^\circ \pm 4^\circ$ [68%].

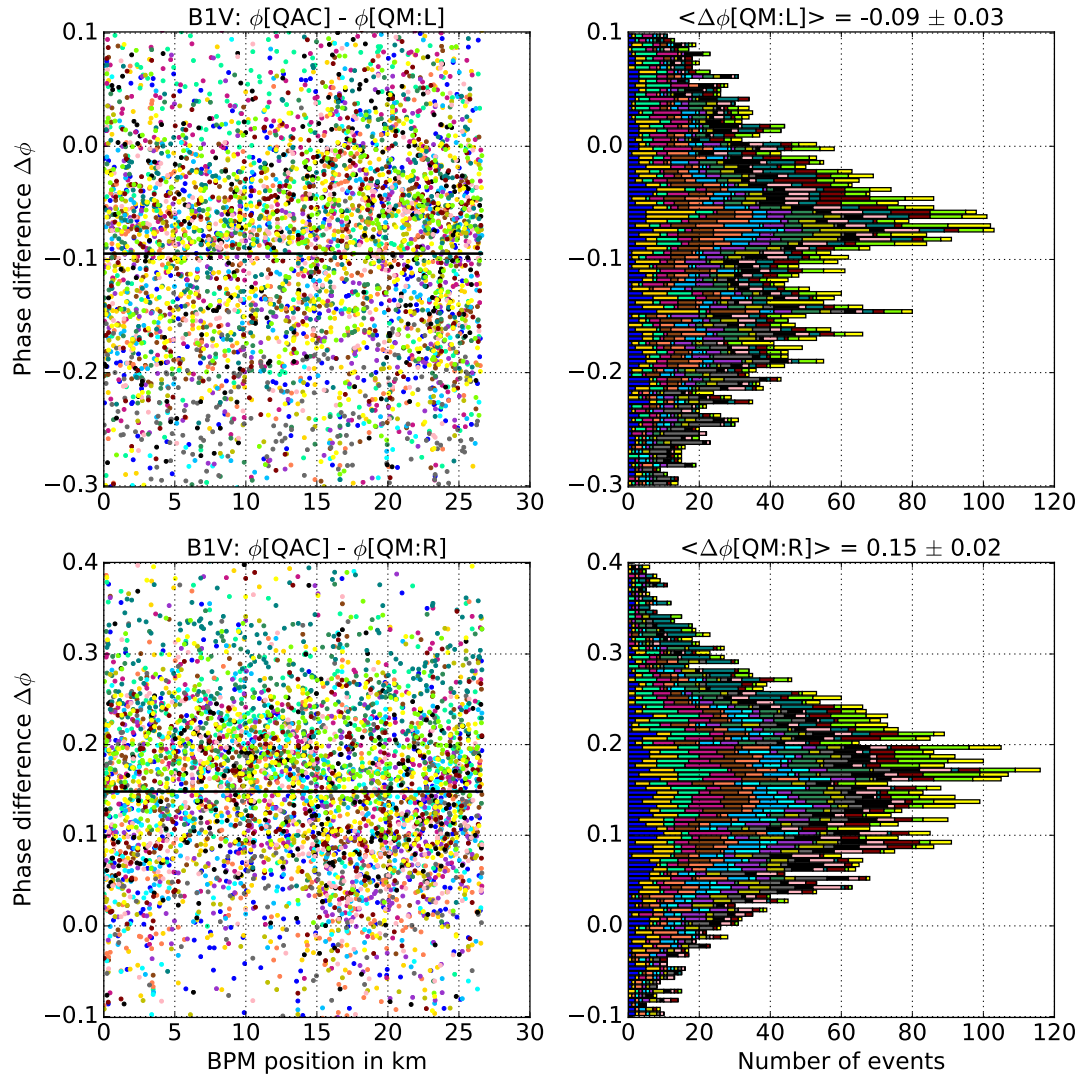


Figure 16: Phase difference between sideband and driven frequency for beam 1 in the vertical plane. Each colour represents a single AC-dipole measurement file illustrating the different AC-dipole kicks. In the plot on the upper left side are the phase differences between AC-dipole kick (QAC) and leftsided sideband (QM:L) shown. On the upper right site are the leftsided phase differences visualized in a histogram. In the plot on the lower left side are the phase differences between AC-dipole kick (QAC) and rightsided sideband (QM:R) shown. The histogram on the lower right side visualizes the distribution of the rightsided phase differences.

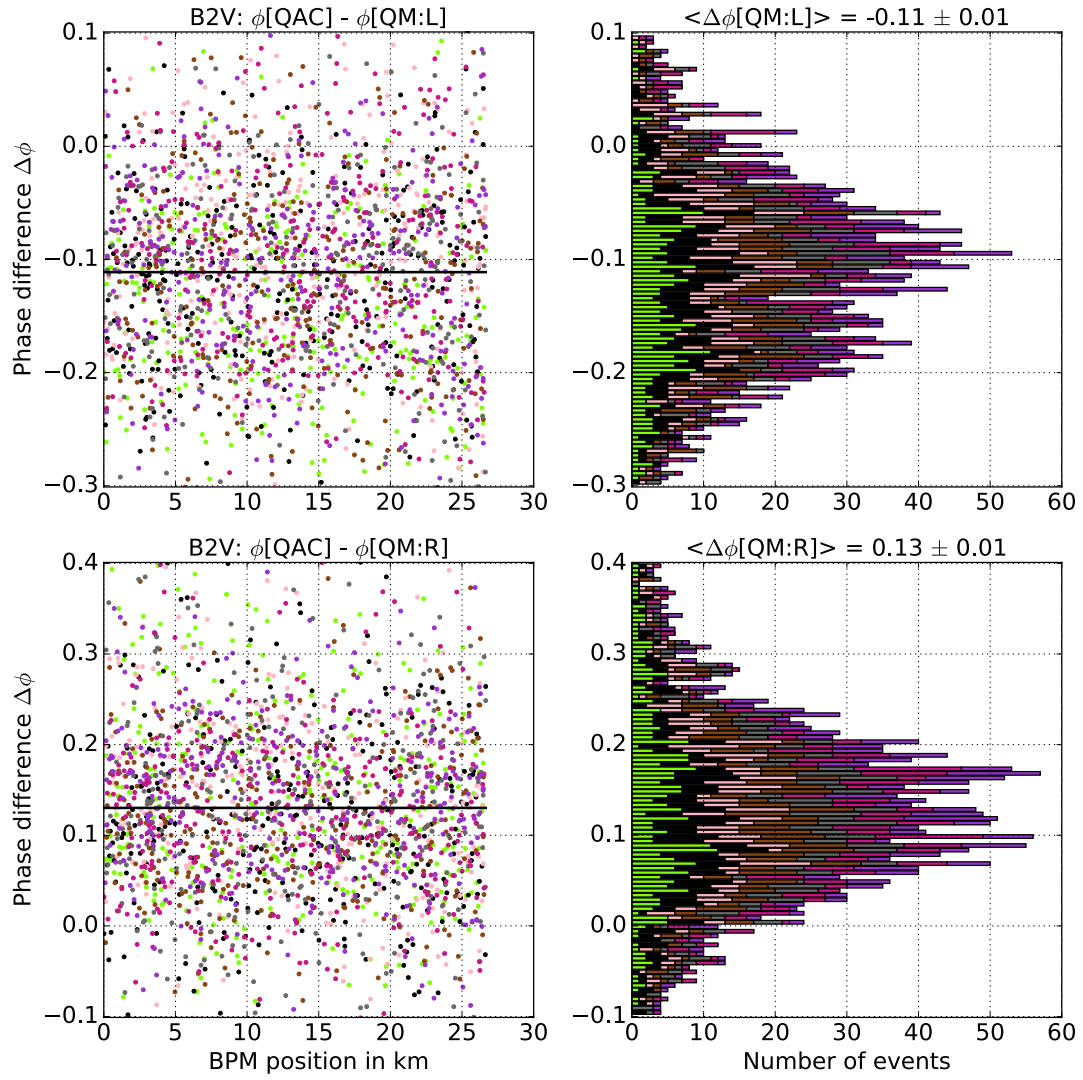


Figure 17: Phase difference between sideband and driven frequency for beam 2 in the vertical plane. Each colour represents a single AC-dipole measurement file illustrating the different AC-dipole kicks. In the plot on the upper left side are the phase differences between AC-dipole kick (QAC) and leftsided sideband (QM:L) shown. On the upper right site are the leftsided phase differences visualized in a histogram. In the plot on the lower left side are the phase differences between AC-dipole kick (QAC) and rightsided sideband (QM:R) shown. The histogram on the lower right side visualizes the distribution of the rightsided phase differences.

5.3 AC-dipole waveform check

A possible reason for the occurrence of the sidebands in the frequency spectra could be a malfunctioning in the driven waveform of the AC-dipole kick. Therefore, the time dependency of the current of the AC-dipole for the 15th of June 2018 was analysed. To do that, fast Fourier transformation (FFT) was used. The spectrum of the flattop waveform for beam 1 is shown in figure 18 and for beam 2 in figure 19.

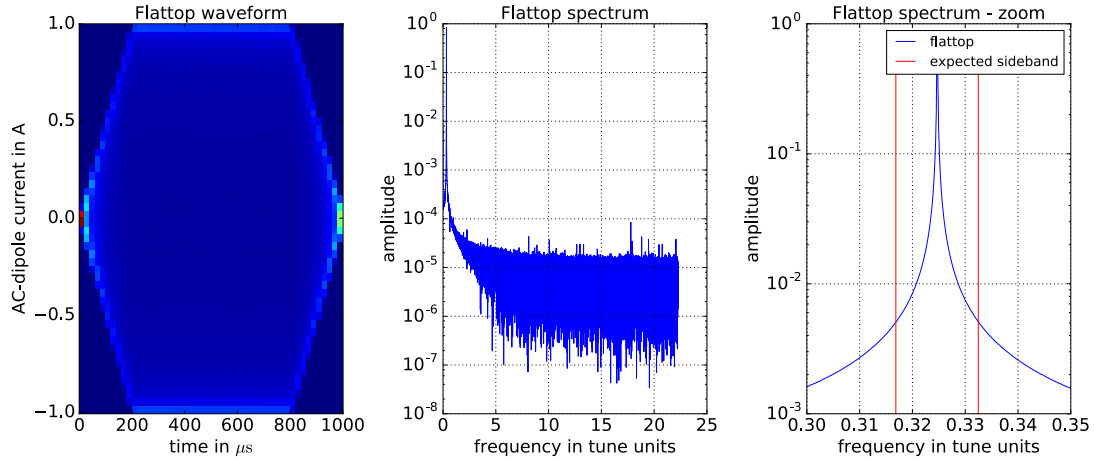


Figure 18: On the left side, the time dependency of the current of the AC-dipole is shown for beam 1. It includes the ramping-process. For the FFT analysis only the flattop of the AC-dipole current was used. In the middle, the spectrum of the driven current is shown. The amplitude scale was chosen as logarithmic to identify sidebands more easily. But there are no sidebands visible. On the right side, the observed sidebands from the profile analysis are marked as red vertical lines in the figure.

Figure 18 manifests that the AC-dipole current has only one main frequency. Therefore, an artifact in the power supply of the AC-dipole can be excluded as a reason for the appearance of the sidebands in beam 1.

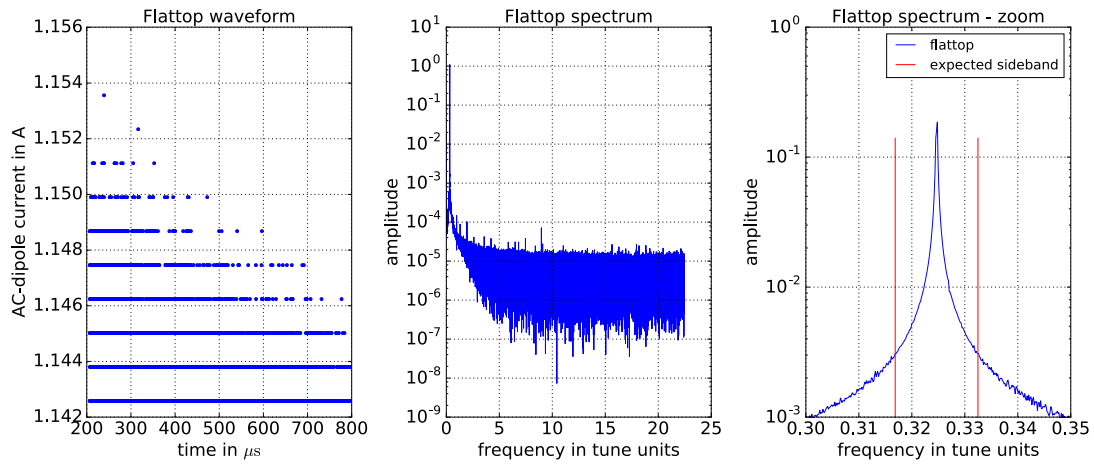


Figure 19: On the left side, the time dependency of the current of the AC-dipole is shown for beam 2 with a zoom into the flattop area. The AC-dipole current shows in the flattop area a clearly noticeable drift. In the middle, the spectrum of the driven current is shown. As for beam 1, there are no sidebands visible in the spectrum for beam 2. On the right side, the observed sidebands from the profile analysis are marked as red vertical lines in the figure.

Despite the fact that the flattop current is showing a drift (for both beams) the spectrum analysis for beam 2 also does not show any sideband. Wherefore the AC-dipole power supply can be excluded as a reason for the sidebands.

6 Natural tune jitter

As elucidated the implemented algorithm uses raw data from AC-dipole excitations to execute a spectrum analysis of the tune. In addition, there is also an alternative approach. One of the beam position monitors (BPM's) in the LHC is also able to measure the tune directly. Those measurements are called base band tunes (BBQ).

6.1 Analysis algorithm

The BBQ-data for data set A - beam 1 - horizontal plane - is shown in figure 20. The BBQ-measurements are totally independent from the AC-dipole method. Therefore, it is possible to confirm or to reject conclusions which are only based on AC-dipole data.

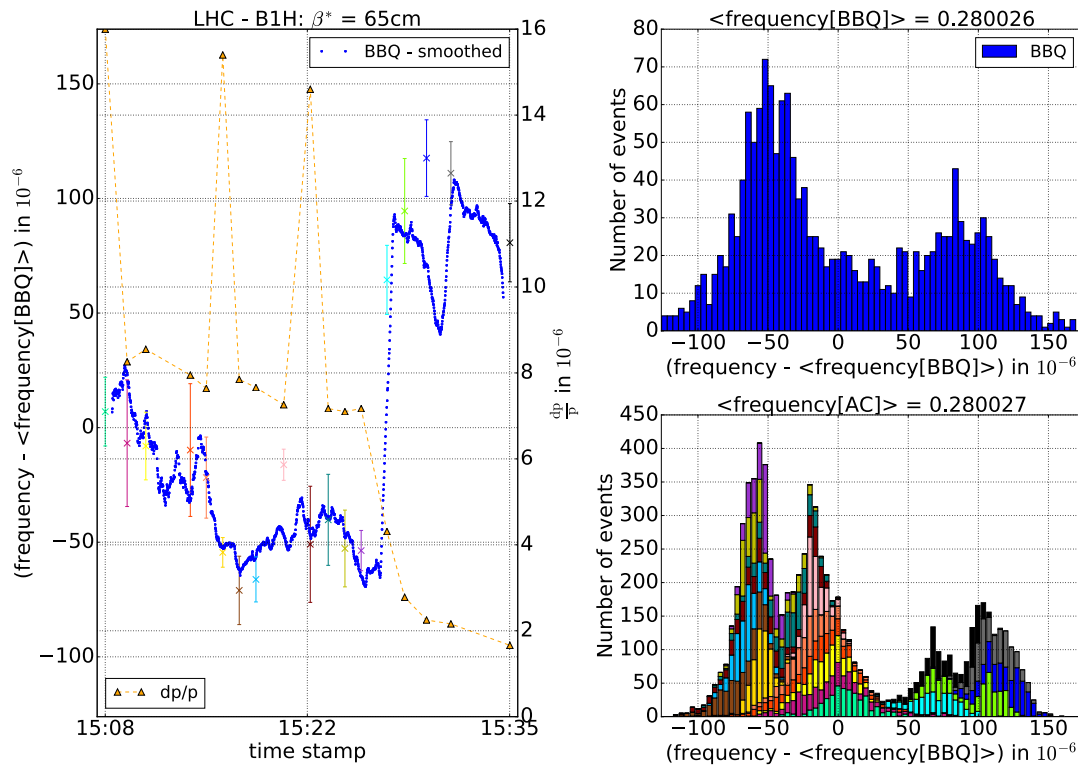


Figure 20: On the left, the BBQ-data and the AC-dipole data are plotted for the natural tune regarding measurement set A. Furthermore, the dp/p values are shown for each AC-dipole measurement quantifying the chromaticity impact on the natural tune (compare to section 2.9). On the upper right, the BBQ-frequency spectrum of the natural tune is shown for the horizontal plane of beam 1 (B1H). On the lower right, the AC-dipole frequency spectrum of the natural tune is shown for B1H. Each colour represents a single AC-dipole kick.

Figure 20 unifies the previous spectrum analysis regarding the horizontal plane of beam 1 on the 15th of June 2018 (compare to figure 10) with the corresponding BBQ-measurements. As a result, the BBQ-data and the AC-dipole measurements agree regarding the natural tune shift observed in the frequency domain.

For the BBQ-data an average smoothing was used to reduce the noise and the AC-dipole excitations were cleaned. The AC-dipole data points are the mean values of the individual tune distributions from each measurement file. As shown in the previous profile analysis, each file contains one peak for the natural tune. The peak of the single measurement file is not strongly influenced by the natural tune jitter because the timescale for a single excitation measurement is very small compared to the effect of tune jitter. Therefore, a Gaussian bell curve can describe the fluctuations of the measurements around the natural tune. To get the best set of parameters, a profile likelihood algorithm was implemented. Furthermore, it is important to determine the statistical uncertainties of the fitting parameters, wherefore a toy Monte-Carlo study was done. For accelerating the algorithm a multiscale gradient descent procedure was used to find the minimum of the multivariate profile likelihood. In order to execute the toy Monte-Carlo study bootstrapping is used to select a random sample from the data for which the profile likelihood algorithm will be executed. The cardinality of the bootstrapping sample is chosen as 25% of the cardinality of the raw data sample to accelerate the algorithm. For each of the 5000 bootstrapping samples results an optimal parameter set for the Gaussian bell curve. In figure 21 those results are shown for file A16, regarding measurement set A [1]. In theory, the mean value and the standard derivation of a Gaussian bell curve are completely independent parameters. As a consequence, the resulting covariance of the toy Monte-Carlo study should be negligible. If this is not the case the suspicion of selecting the wrong model would arise. At this point it has to be mentioned that the chosen cardinality of the bootstrapping sample strongly influences the uncertainty of the parameters if the ratio between cardinality of the bootstrapping sample and cardinality of the raw data falls short of 25%. This threshold was determined in a verification check of the algorithm comparing the resulting uncertainty of the mean value with the parameter uncertainty given by the scipy package curve fit. The verification check is illustrated in figure 22. The standard derivation of the Gaussian bell curve is an estimator for the statistical uncertainty of the observed measurement for the natural tune. Therefore, the value for sigma can be used in figure 20 as the value of the error-bars.

The uncertainty of the RMS value can be obtained by doing a toy-Monte Carlo study using the determined statistical uncertainty for each mean value and the corresponding systematical uncertainty. At this point, the systematical uncertainties for the frequency

measurements must be estimated. A systematical uncertainty for the AC-dipole measurements of 5 ppm seems to be reasonable and a systematical uncertainty for the BBQ measurements of 10 ppm was taken into account.

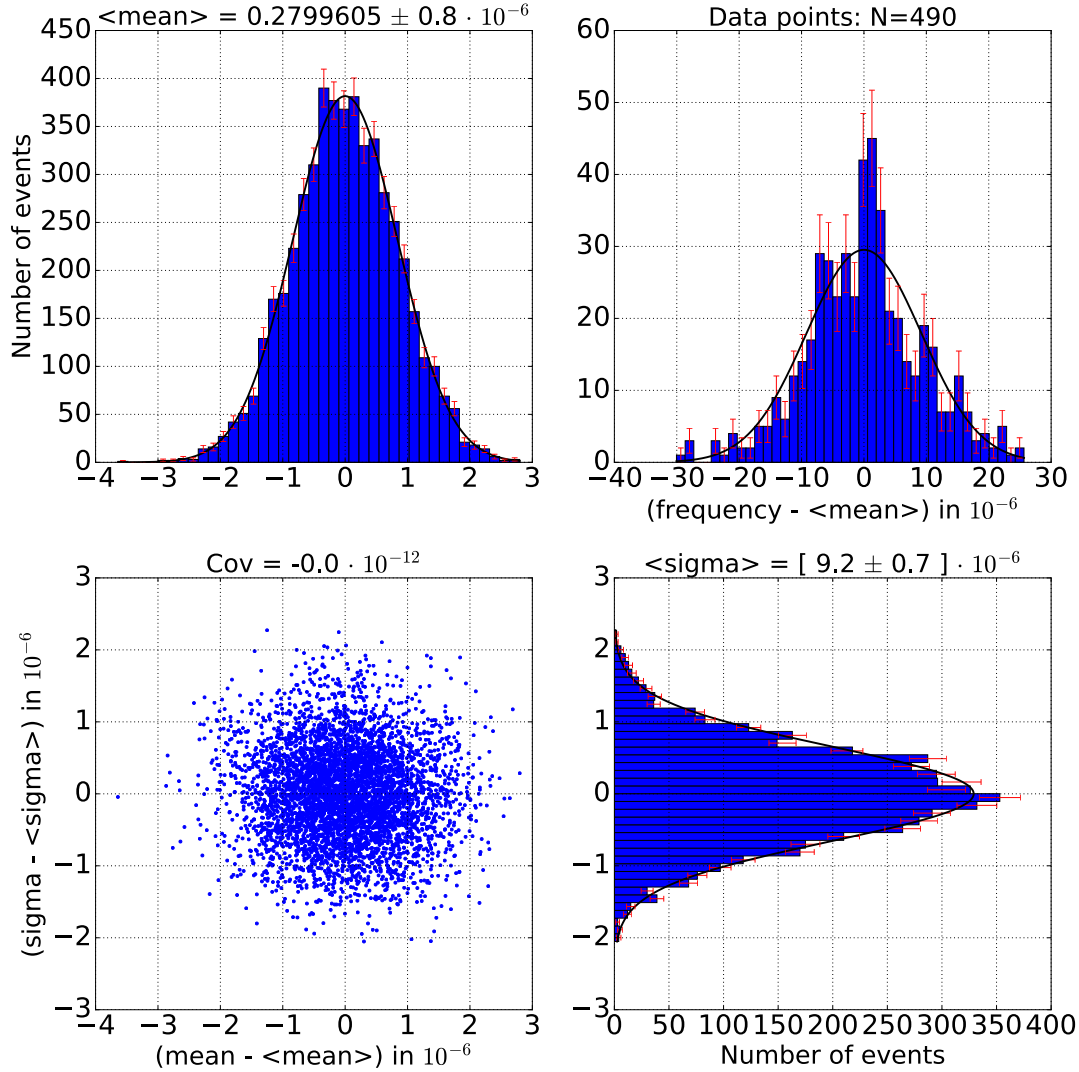


Figure 21: Toy Monte-Carlo study of the single natural tune peak from measurement file A16. 5000 random samples were taken from the data and for each of them a profile likelihood analysis has been executed. As a result, the distribution of the expectation value of the mean $\langle \text{mean} \rangle$ and the distribution of the expectation value of the standard deviation $\langle \sigma \rangle$ can also be visualized. Furthermore, the covariance between the two fitting parameters is visualized in the scatter plot on the lower left side.

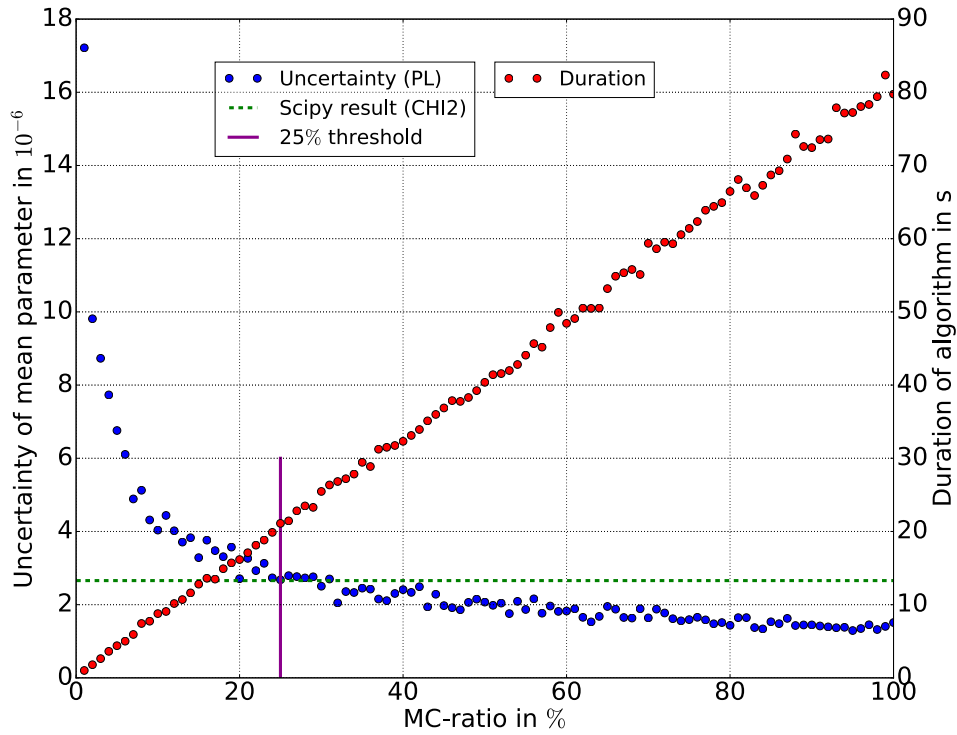


Figure 22: The blue dotted line represents the uncertainty dependency of the mean parameter from the chosen cardinality ratio (MC-ratio). The red dotted line visualizes the dependency of the algorithm duration from the chosen MC-ratio. The green dashed line shows the fitting uncertainty of the mean parameter by using the scipy package from python. The violet line symbolizes a reasonable minimum threshold of 25% for the MC-ratio.

Consequently, all single natural tune measurements carry a statistical uncertainty according to the fluctuations around the mean parameter. Therefore, the data set of all natural tune measurements, composed by the single AC-dipole kicks, can vary according to the statistical and systematical uncertainties. Those fluctuations also influence the expectation value for the natural tune, which is the mean value of all natural tune values from the individual measurement files.

For the error propagation a Monte-Carlo simulation was combined with bootstrapping to cover all uncertainty sources of the RMS-jitter. On the one hand, the single AC-dipole tune measurements are contaminated by the statistical and systematical uncertainty. On the other hand, the natural tune measurements show a linear drift which biases the RMS-jitter and sometimes there even occur natural tune jumps. Furthermore, for the data set E the amplitude detuning had to be corrected manually by subtracting the

quadratic dependency of the natural tune from the kick amplitude. To handle all this uncertainty sources the error propagation simulation was structured in two levels. In the first step, a new data set was generated from the raw data by letting the fluctuations act on the single measurement points. The fluctuations are simulated by random Gaussian numbers with the properties of mean equal one and sigma equal to the relative statistical respectively systematical uncertainty. The second step is a bootstrapping method, which will be different for the AC-dipole and the BBQ-data. Before we go into more detail, bootstrapping is theoretically a method where a random sample is chosen from the data with replacement to avoid correlations [16]. If the cardinality of the bootstrap sample is set as equal to the cardinality of the measurement set a random sample without replacement would automatically let to the raw data set. But for the case of the RMS-jitter it is important to reconsider that this value describes the fluctuation of the natural tune and therefore a random sample with replacement would result in an unobserved small RMS-jitter. Consequently, a bootstrapping method with replacement would bias the distribution of the RMS-jitter to smaller values. Therefore, the question must be asked if there is an alternative way to extract more information out of the raw data without biasing the results. A new approach in this thesis is to reduce the cardinality of the random sample in comparison to the raw data and to execute the drawing without replacement. As in the initial idea of bootstrapping the random sample space is also limited for this approach. The cardinality of the random sample space $|\Omega|_A$, for the bootstrapping method with replacement and equal cardinality N of the bootstrapping sample and the raw data set, is given by equation (14).

$$|\Omega|_A = N^N \quad (14)$$

For the new approach all permutations, without replacement and without taking care of the order of the bootstrapping sample, will be considered as possible choices. In this case, it is important to emphasize that the cardinality of the bootstrapping sample must be variable otherwise the result would be the initial data set. Therefore, all sub permutations with a cardinality smaller than the cardinality of the raw data will also be taken into account. At this point, an additional constraint was introduced regarding a minimum cardinality for the bootstrapping sample of three. This seems to be reasonable for an RMS-value. The cardinality of the random sample space $|\Omega|_B$ for the new bootstrapping approach is given by equation (15).

$$|\Omega|_B = \sum_{q=3}^N \binom{N}{q} \stackrel{\text{def}}{=} \sum_{q=3}^N \frac{N!}{q! \cdot (N-q)!} \quad (15)$$

In equation (15), N represents the number of natural tune measurements and q is defined as the cardinality of the chosen bootstrapping sample. The question might be asked if this method is able to extract as much information out of the data as the initial version of bootstrapping. To answer this question it is advisable to have a look to the entropy definition of Shannon [17] who described the information richness of a data sample M by formula (16).

$$H = - \sum_{i=1}^n p_i \cdot \log_2(p_i) \quad (16)$$

In formula 16, n describes the number of realizations whereby each realization has a certain probability p_i . Bootstrapping methods need to fulfill the Laplace assumption of equal probability in the bootstrapping sample space to avoid a bias of the raw data [18]. In the introduced bootstrapping method the requirement of a uniform distribution for the random sample probability is already ensured for a fix cardinality. Usually, $|\Omega|_B$ is too large to cover all permutations with a bootstrapping algorithm. Therefore, the bootstrapping samples have to be chosen randomly from the space. To fulfill the requirement of uniformity for the whole bootstrapping sample space the cardinality has to be chosen first with a none uniform probability density distribution (pdf). The probability of each cardinality is chosen according to the cardinality of the subset with constant cardinality. For example, if we consider a raw data sample with cardinality five then the probability for choosing a bootstrapping sample with cardinality four is five over $|\Omega|_B$. In a nutshell, the ratio of the binomial coefficient and $|\Omega|_B$ represents the pdf for choosing a cardinality of a bootstrapping sample in the implemented algorithm. Therewith, the Laplace assumption is also totally fulfilled by the introduced approach. By using the Laplace assumption it is possible to rewrite equation (16) as relation (17).

$$H = - \sum_{i=1}^{|\Omega|} p_i \cdot \log_2(p_i) \stackrel{\Delta}{=} \log_2(|\Omega|) \quad (17)$$

Figure 23 shows the information entropy dependency from the cardinality of the raw data for the initial concept of bootstrapping and for the new approach.

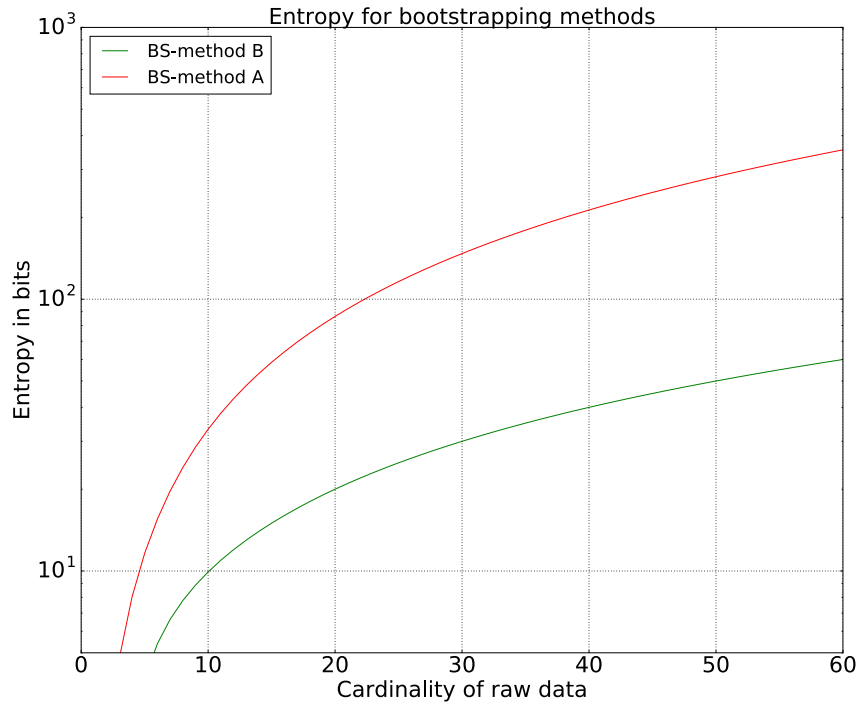


Figure 23: Information entropy for bootstrapping method A which is related to the initial idea of bootstrapping and for bootstrapping method B which is related to the new introduced approach regarding bootstrapping of a RMS-quantity.

As a result, the new approach of bootstrapping for the RMS-jitter of the natural tune will be able to extract information efficiently from the raw data. For the avoidance of doubt, despite the information technology point of view there might also appear the question of how this approach of bootstrapping reduces the significance of the raw data. For extracting more information about the natural tune jitter from the raw data it was necessary to vary the cardinality of the bootstrapping sample. Therefore, the significance is decreased for the single random bootstrapping sample but it is compensated by an increased information content of the stacked array composed by all bootstrapping samples. As mentioned before, the natural tune shows almost always a linear drift behaviour and sometimes there even appear frequency jumps. A smaller sampling rate increases the sensitivity for this effects and includes those into the distribution of the RMS-jitter for the natural tune. For a study with infinity calculations of the RMS-jitter the result would be the truth distribution of the RMS-jitter. But with respect to the limited computation capacity the toy Monte-Carlo study evaluates for 1k simulated data sets, 10k bootstrapping samples and therewith 10M values for the natural tune RMS-jitter

of the LHC. On account of this data acquisition it is possible to analyze the pdf of the RMS-jitter which is shown as an example for data set A in figure 24 for the AC-dipole data and in figure 25 for the corresponding BBQ-data.

At this point it is important to elaborate the differences of the bootstrapping algorithm between the AC-dipole and the BBQ-data. All previous explanations were related to the AC-dipole data. In case of the BBQ-data the cardinality of the raw data is mostly between 200 and 3000 samples. To get the same sampling rate and sensitivity for the RMS-jitter with the BBQ-data as for the AC-dipole data it is appropriate to use the same maximum cardinality as for the corresponding AC-dipole data. Consequently, the same algorithm was applied to the BBQ-data whereby the number of natural tune measurements in relation (15) was changed to the corresponding number of measurements with the AC-dipole method.

Furthermore, the ATS-optics measurements from 2016 had to be corrected regarding the effect of amplitude detuning. For that reason, the quadratic dependency of the natural tune from the excitation amplitude was fitted with linear regression (quadratic abscissa). The optimal set of parameters which minimizes the residual sum of squares are fraught with additional statistical uncertainties. The upcoming statistical uncertainty can be characterized by the remaining residual sum of squares according equation (26). This error will be intercepted by the toy Monte-Carlo study in the implemented algorithm because the variation of the raw data set will also result in different parameter sets for the linear regression. Therefore, the error propagation of the fitting parameters is considered. It is significant noticeable that the pdf's of the natural tune jitter mostly show an inclination wherefore the expectation value is not equal to the mode value. Furthermore, in case of a pdf with an inclination the width, defined as the 68% confidence interval, is asymmetric. To characterize the individual pdf's of the RMS tune jitter the equations in 18 have been applied for the expectation value $\langle x \rangle$, the left handed width σ_- and the right handed width σ_+ .

$$\langle x \rangle \stackrel{!}{=} \int_{-\infty}^{+\infty} x \cdot \text{pdf}(x) dx, \quad 0.34 \stackrel{!}{=} \int_{\sigma_-}^{\langle x \rangle} \text{pdf}(x) dx, \quad 0.34 \stackrel{!}{=} \int_{\langle x \rangle}^{\sigma_+} \text{pdf}(x) dx \quad (18)$$

In conclusion, the profile likelihood analysis's of the single natural tune peaks combined with a toy Monte-Carlo study which uses a new approach of bootstrapping returns the best estimator for the expectation value of the RMS-jitter and furthermore it provides a reliable quantity for the common uncertainty of the RMS-jitter of the LHC.

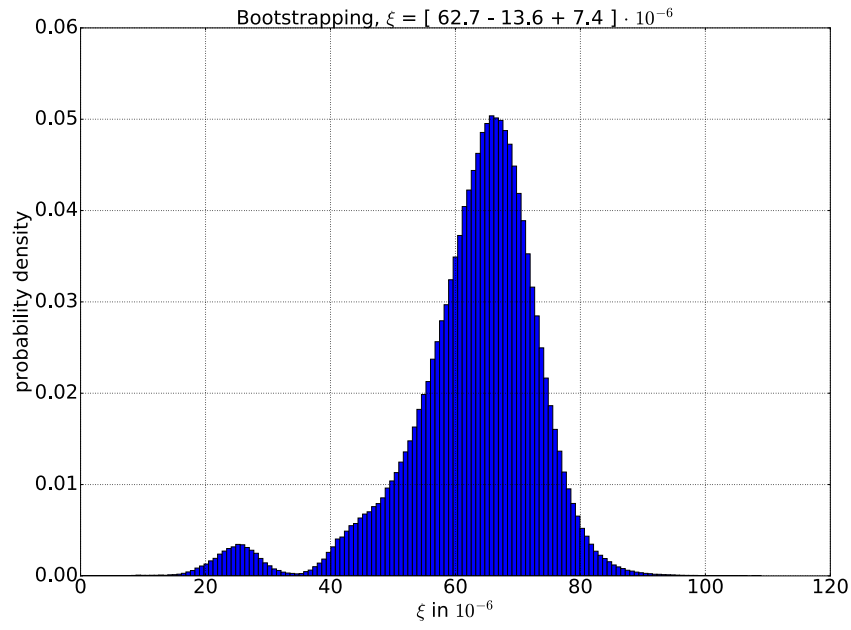


Figure 24: Distribution of the AC-dipole RMS-jitter regarding data set A - beam 1 - horizontal plane. Elicited by the toy Monte-Carlo algorithm.

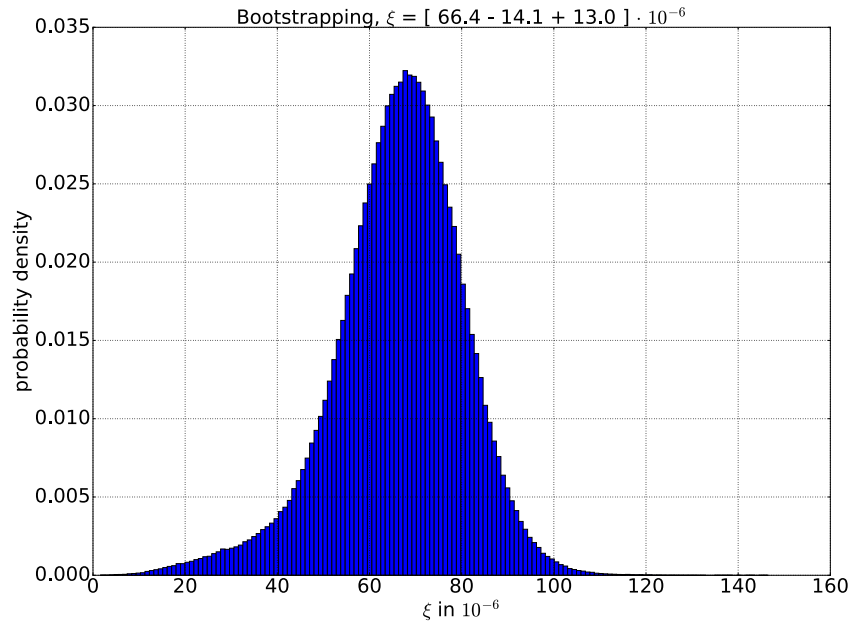


Figure 25: Distribution of the BBQ RMS-jitter regarding data set A - beam 1 - horizontal plane. Elicited by the toy Monte-Carlo algorithm.

6.2 Results overview

In a nutshell, the RMS-jitter and the corresponding uncertainty can be quantified. Tables 5 till 15 show the results for each beam and plane as well as for different optics. The names of the raw data files are listed in the references for each data set.

During analyzing the measurement files it was noticeable that most of the data is influenced by a linear drift. Therefore, the natural tune jitter ξ was also calculated for the drift corrected data. The ratio between the corrected natural tune jitter and the natural tune jitter without drift correction is accounted in the column "lin. drift". The left sided value is related to the AC-dipole data and the right sided value to the BBQ-data.

Moreover, some data sets showed a frequency shift in the natural tune at some point. For those measurements, the absolute shift of the natural tune was related to the natural tune jitter $\xi(\text{BBQ})$. A natural tune shift has been classified as significant if the ratio was larger than 0.5 and the jump was succeeded under a minute. The shift ratio is accounted in the column ΔQ . The shift ratio has been based on the natural tune jitter of the BBQ-data because the BBQ-data is taken with a higher measurement clock frequency than the AC-dipole data. It is important to emphasize that both methods have to show the jump-phenomena. As a consequence, the tune jump will be noticeable in the BBQ-spectrum and in the AC-dipole spectrum as a second accumulated peak. For data set A, a tune jump in the horizontal plane of beam 1 has been observed which can be seen in the spectra of diagram 20.

Table 5: RMS-jitter for the natural tune, regarding round flattop optics [DS: A].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	63	-14 +7	66	-14 +13	65	19 12	↗ [+2.3]
B1V	26	-5 +3	24	-7 +13	65	19 0	--
B2H	64	-37 +21	41	-16 +32	25	4 6	--
B2V	25	-9 +6	20	-8 +11	25	12 0	--

Table 6: RMS-jitter for the natural tune, regarding CTPPS2 optics [DS: B].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	20	-13 +3	18	-8 +12	40	28 0	--
B1V	23	-12 +4	15	-6 +9	40	45 13	--
B2H	26	-10 +6	28	-10 +12	40	52 23	--
B2V	41	-8 +6	29	-11 +9	40	63 50	--

Table 7: RMS-jitter for the natural tune, regarding ballistic optics.

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	DS	lin. drift [%]	ΔQ
B2H	7	-2 +2	10	-4 +5	C	29 17	--
B2V	16	-4 +3	17	-8 +14	C	13 11	--
B1H	11	-5 +4	8	-3 +5	D	0 0	--
B1V	7	-2 +2	16	-7 +14	D	29 5	--
B2H	10	-3 +3	7	-2 +4	D	0 0	--
B2V	12	-7 +2	11	-4 +7	D	8 8	--

Table 8: RMS-jitter for the natural tune, regarding ATS optics [DS: E].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
<i>B2H</i>	74	-25 +16	48	-16 +15	40	-- --	--
<i>B2V</i>	128	-29 +18	48	-16 +16	40	-- --	--
B2H	34	-10 +6	48	-16 +15	40	6 38	--
B2V	28	-5 +4	48	-16 +16	40	7 12	--

Remark: The rows which are marked cursively in table 8 are influenced by the effect of amplitude detuning (see section 2.7). To make the results comparable this effect has to be corrected which results in the not cursive rows for the same data set.

Table 9: RMS-jitter for the natural tune, regarding flat orbit [DS: F].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	51	-11 +8	46	-8 +10	30	23 35	$\searrow$ [-1.0]
B1V	47	-7 +5	52	-8 +8	30	8 2	$\nearrow$ [+3.1]
B2H	54	-6 +5	51	-9 +10	30	0 4	$\nearrow$ [+2.1]
B2V	52	-9 +7	60	-9 +8	30	23 57	$\searrow$ [-0.8]

Table 10: RMS-jitter for the natural tune, regarding high β^* measurements at flattop energy [DS: G] and high β^* measurements at injection [DS: M].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_{\xi}(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_{\xi}(\text{BBQ})$ [10^{-6}]	β^* [m]	DS	lin. drift [%]	ΔQ
B1H	28	-8 +6	58	-36 +28	90	G	19 0	--
B1V	27	-7 +6	62	-28 +31	90	G	53 2	--
B2H	39	-11 +6	43	-16 +16	90	G	1 0	--
B2V	50	-13 +16	60	-25 +26	90	G	27 10	--
B1H	281	-116 +66	233	-104 +114	100	M	61 63	--
B1V	338	-152 +20	237	-114 +108	100	M	80 43	--
B2H	276	-118 +54	309	-114 +110	100	M	69 62	--
B2V	316	-93 +101	241	-92 +95	100	M	53 48	--

Table 11: RMS-jitter for the natural tune, regarding injection nominal [DS: H].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_\xi(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_\xi(\text{BBQ})$ [10^{-6}]	β^* [m]	lin. drift [%]	ΔQ
B1H	171	-56 +47	315	-99 +88	11	5 66	--
B1V	424	-59 +45	306	-68 +64	11	69 64	--
B2H	159	-80 +33	122	-52 +73	11	1 20	--
B2V	219	-59 +50	201	-78 +91	11	69 45	--

Table 12: RMS-jitter for the natural tune, regarding flat optics [DS: I].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_\xi(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_\xi(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	114	-28 +15	69	-23 +43	60/15	56 19	$\nearrow$ [+1.6]
B1V	80	-22 +14	70	-50 +20	60/15	40 33	$\searrow$ [-1.8]

Table 13: RMS-jitter for the natural tune, regarding flattop nominal [DS: J].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_\xi(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_\xi(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	90	-22 +17	98	-31 +37	100	55 34	--
B1V	42	-11 +11	45	-15 +21	100	38 11	--
B2H	26	-16 +3	22	-8 +12	100	79 27	--
B2V	22	-9 +5	30	-14 +29	100	71 10	--

Table 14: RMS-jitter for the natural tune, regarding 600m arcs [DS: K].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_\xi(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_\xi(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	34	-19 +7	27	-11 +20	25	13 1	--
B1V	44	-22 +2	25	-11 +14	25	11 20	--
B2H	23	-16 +2	31	-12 +29	25	15 0	--
B2V	27	-9 +4	45	-21 +45	25	0 1	$\searrow$ [-0.6]

Table 15: RMS-jitter for the natural tune, regarding 200m arcs [DS: L].

Plane	$\xi(\text{AC})$ [10^{-6}]	$\sigma_\xi(\text{AC})$ [10^{-6}]	$\xi(\text{BBQ})$ [10^{-6}]	$\sigma_\xi(\text{BBQ})$ [10^{-6}]	β^* [cm]	lin. drift [%]	ΔQ
B1H	81	-57 +12	72	-23 +19	30	82 62	--
B1V	62	-33 +9	63	-18 +18	30	73 56	--
B2H	35	-9 +6	49	-12 +19	30	17 34	--
B2V	114	-33 +16	100	-32 +40	30	68 40	--

Finally, it is convenient to have an overview of all results with the corresponding theoretical value for each beam optic. The resulting overview is provided in figure 26. For each plane is the RMS-jitter of the natural tune shown. The tune jitter of the BBQ-data is marked as a blue cross and the tune jitter of the AC-dipole measurements is marked as a red cross. The simulated values are visualized as magenta dots.

For the simulated values of the RMS-jitter an error of 1 ppm regarding the magnet current is applied to the computation in MAD-X (software for beam optic calculations at CERN - for further information on MAD-X see [33]). Consequently, the error will be propagated circuit by circuit and the impact on tune, orbit and beta function results. Knowing the impact of each circuit and assuming that everything is Gaussian, linear and uncorrelated, one can add in quadrature all effects to obtain the expected value for the RMS-jitter [19].

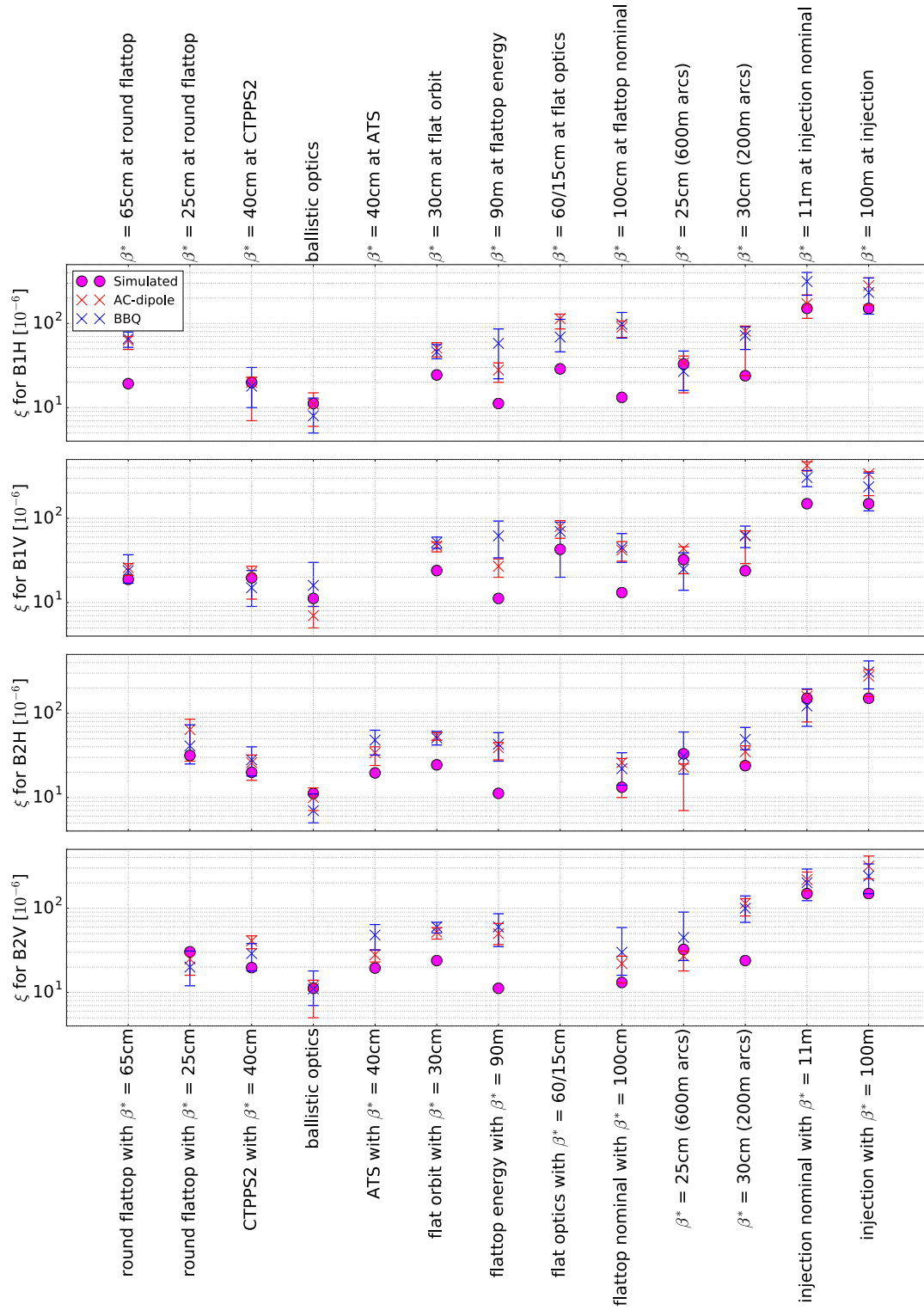


Figure 26: Overview of the natural tune jitter dependency from the chosen beam optics.

7 Achievements

The major goal of this thesis was to evaluate the stability of the natural tune. For this reason several data sets have been analyzed regarding the natural tune jitter. During the studies tune jumps were observed which are not considered in the theory of the existing simulations. Actually, it is under investigation if a movement of single quadrupoles of approximately $1\mu\text{m}$ could have such an impact on the natural tune. Because of external civil activities the quadrupole positions can vary in the ballpark of less than $1\mu\text{m}$. Furthermore, the analysis of the natural tune has shown that in many cases there is a non negligible linear drift in the data. As a consequence, it would be recommendable to run further studies about the question if the effect of linear drift depends on the time since the accelerator phase was reached. In figure 27 are the impacts of those effects on the results for the RMS-jitter visualized. For each plane the corresponding fraction of linear drift is shown in a bar chart below the RMS-jitter value. In light green the linear drift fraction for the AC-Dipole jitter is plotted and in dark green the linear drift fraction for the BBQ-data is visualized. If both values are above 25% a machine drift might be considered as a reason for a biased tune jitter. Therefore, the corresponding values for the RMS-jitter are highlighted in yellow to indicate the bias. Furthermore, the observed tune jumps are also visualized in the bar chart as dark red trapeziums. If a tune jump was observed it can be assumed that an external event forced the tune to change to a different frequency level. As a consequence, the corresponding values for the RMS-jitter are also considered as biased and were highlighted in the same way. The additional effects of the highlighted values are not included in the assumptions for the simulation. Therefore, only the non highlighted values can be compared with the simulated values. For data set G (high β^* at flattop energy) as well as for data set J (flattop nominal) a significant difference between simulation and measurement have become manifest. A reason for this derivation can not be given at the moment. Either there are additional effects which influence the RMS-jitter but are not included in the model for the simulation yet or the assumed uncertainties regarding the magnet current are not representative enough. In a nutshell, for clarifying the unresolved issues further investigations will be necessary. This research is of paramount importance for identifying the influencing factors of the tune jitter and for reaching the prerequisites of the high luminosity upgrade of the LHC. A remarkable byproduct of this thesis is the insight into the sidebands quantities. The first presumption of an AC-dipole malfunctioning has been disproved by an FFT-analysis of the AC-dipole current. For verifying the validity of the existing analysis routine it is indispensable to identify the reason for the sidebands.

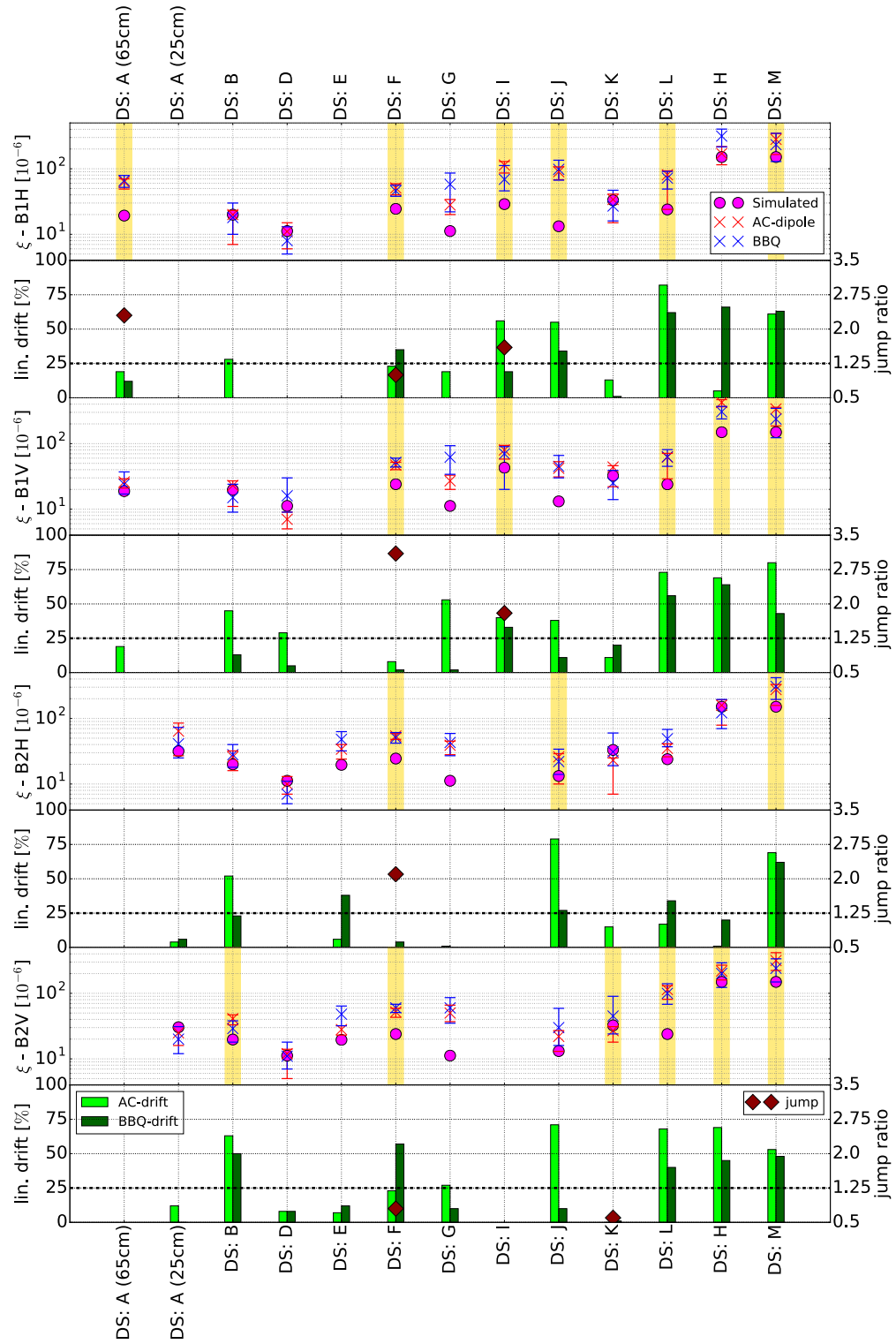


Figure 27: Impact of lin. drift and tune jumps on the natural tune jitter results.

8 Appendix

8.1 Linear least square fit

For calculating the parameters of the fitted straight line the method of minimizing the residual sum of squares was used. For gaussian distributed measurements around the model this procedure is equivalent to the likelihood principle. The model function is in this case a straight line and can be parameterized according to relation 19.

$$f(x) := m \cdot x + b \quad (19)$$

For evaluating the best parameter set for the model function $f(x)$ the residual function $R(m,b)$ has to be defined. This was made in equation 20 and follows directly from the likelihood principle [22].

$$-2 \cdot \log(\mathcal{L}[\vec{y} | (m, b)]) \iff R(m, b) := \sum_{i=1}^N \frac{(y_i - f(x_i))^2}{\sigma_i^2} = \sum_{i=1}^N \frac{(y_i - m \cdot x_i - b)^2}{\sigma_i^2} \quad (20)$$

The residual function shall be minimal for the best parameter set. Therefore, it is important to evaluate the local minimum of the residual function regarding the parameters m and b . As a result, the parameters which let disappear the partial derivatives in relation 21 are the best parameter set for the model function.

$$\Leftrightarrow \frac{\partial R(m, b)}{\partial m} = \frac{\partial R(m, b)}{\partial b} = 0 \quad (21)$$

If the uncertainties are evenly distributed the result is a linear equation system as shown in formula 22.

$$\sum_{i=1}^N \begin{cases} x_i \cdot y_i - m \cdot x_i^2 - b \cdot x_i = 0 \\ y_i - m \cdot x_i - b = 0 \end{cases} \quad (22)$$

By using the sum convention of Gauß and methods of linear algebra it is possible to rewrite the linear equation system in a much more convenient way. This results in equation 23.

$$\begin{pmatrix} N & [x] \\ [x] & [x^2] \end{pmatrix} \cdot \begin{pmatrix} b \\ m \end{pmatrix} = \begin{pmatrix} [y] \\ [x \cdot y] \end{pmatrix} \quad (23)$$

The best parameter set can now easily determined by applying the Cramer procedure.

$$b = \frac{\begin{vmatrix} [y] & [x] \\ [x \cdot y] & [x^2] \end{vmatrix}}{\begin{vmatrix} N & [x] \\ [x] & [x^2] \end{vmatrix}} = \frac{[y] \cdot [x^2] - [x] \cdot [x \cdot y]}{N \cdot [x^2] - [x]^2} \quad (24)$$

$$m = \frac{\begin{vmatrix} N & [y] \\ [x] & [x \cdot y] \end{vmatrix}}{\begin{vmatrix} N & [x] \\ [x] & [x^2] \end{vmatrix}} = \frac{N \cdot [x \cdot y] - [x] \cdot [y]}{N \cdot [x^2] - [x]^2} \quad (25)$$

As a consequence, we receive the conditional equations 24 and 25 for the best parameter set. In a nutshell, those parameters minimize the residual function and they fully describe the model function of the measurement data set.

It is important to emphasize that even the best set of parameters for the model function carries a statistical uncertainty which is related to the remaining residual sum of squares. Therefore, the statistical variance of the resulting quantity y is determined by the remaining residua. Keeping in mind that two quantities have already been evaluated out of the measurement data set the amount of degrees of freedom has to be reduced about two degrees [15].

$$\sigma(y)_{stat}^2 = \frac{R(m, b)}{N - 2} = \frac{1}{N - 2} \cdot \sum_{i=1}^N (y_i - m \cdot x_i - b)^2 \quad (26)$$

The variance of the resulting quantity y in formula 26 characterizes the goodness of fit and provides an estimator for the uncertainty of the amplitude detuning correction.

References

[1] Measurement set A is a cluster of the following files:

Beam 1	
Kick group name:	b1_afterGlobalHighTeleCorrAndGlobalCoup
Optics:	R2018aT200_A65C65A10mL300
Optics model:	/user/slops/data/LHC_DATA/OPDATA/Betabeat/2018-06-15/models/LHCB1/b1_round_flattop
A1: Beam1@Turn@2018:06:15@13:08:23:289.sdds	A2: Beam1@Turn@2018:06:15@13:09:52:018.sdds
A3: Beam1@Turn@2018:06:15@13:11:08:998.sdds	A4: Beam1@Turn@2018:06:15@13:14:09:681.sdds
A5: Beam1@Turn@2018:06:15@13:15:14:688.sdds	A6: Beam1@Turn@2018:06:15@13:16:21:108.sdds
A7: Beam1@Turn@2018:06:15@13:17:28:826.sdds	A8: Beam1@Turn@2018:06:15@13:18:36:290.sdds
A9: Beam1@Turn@2018:06:15@13:20:29:969.sdds	A10: Beam1@Turn@2018:06:15@13:22:17:759.sdds
A11: Beam1@Turn@2018:06:15@13:23:30:329.sdds	A12: Beam1@Turn@2018:06:15@13:24:38:123.sdds
A13: Beam1@Turn@2018:06:15@13:25:44:297.sdds	A14: Beam1@Turn@2018:06:15@13:27:29:839.sdds
A15: Beam1@Turn@2018:06:15@13:28:41:662.sdds	A16: Beam1@Turn@2018:06:15@13:30:10:984.sdds
A17: Beam1@Turn@2018:06:15@13:31:49:739.sdds	A18: Beam1@Turn@2018:06:15@13:35:49:115.sdds
Beam 2	
Kick group name:	b2_round_25cm
Optics:	R2018aT77_A25C25A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-06-15/models/LHCB2/b2_round_25cm
A19: Beam2@Turn@2018:06:15@15:54:08:166.sdds	A20: Beam2@Turn@2018:06:15@16:03:55:291.sdds
A21: Beam2@Turn@2018:06:15@16:05:32:742.sdds	A22: Beam2@Turn@2018:06:15@16:07:01:184.sdds
A23: Beam2@Turn@2018:06:15@16:08:27:887.sdds	A24: Beam2@Turn@2018:06:15@16:10:16:120.sdds
A25: Beam2@Turn@2018:06:15@16:11:38:384.sdds	A26: Beam2@Turn@2018:06:15@16:13:20:767.sdds

[2] Measurement set B is a cluster of the following files:

Beam 1	
Kick group name:	b1_40cm_forOptics
Optics:	R2017a_A40C40A10mL300_CTPPS2
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-05-08/models/LHCB1/b1_40cm_forCoupling
B1: Beam1@Turn@2017:05:08@20:13:31:521.sdds	B2: Beam1@Turn@2017:05:08@20:15:21:922.sdds
B3: Beam1@Turn@2017:05:08@20:16:43:577.sdds	B4: Beam1@Turn@2017:05:08@20:18:02:833.sdds

Beam 2	
Kick group name:	ForBetaBeatMeasurementFlatMachine40cm
Optics:	R2017a_A40C40A10mL300_CTPPS2
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-05-08/models/LHCB2/b2_40cm_correctOptics_1
B5: Beam2@Turn@2017:05:08@20:05:13:200.sdds	B6: Beam2@Turn@2017:05:08@20:08:16:974.sdds
B7: Beam2@Turn@2017:05:08@20:09:49:270.sdds	B8: Beam2@Turn@2017:05:08@20:11:26:487.sdds
B9: Beam2@Turn@2017:05:08@20:12:32:573.sdds	B10: Beam2@Turn@2017:05:08@20:13:39:842.sdds
B11: Beam2@Turn@2017:05:08@20:14:46:308.sdds	B12: Beam2@Turn@2017:05:08@20:17:19:751.sdds
B13: Beam2@Turn@2017:05:08@20:18:30:549.sdds	

[3] Measurement set C is a cluster of the following files:

Beam 2	
Kick group name:	Kick_for_Amplitude_detuning_Beam_2
Optics:	RAMP-6.5TeV-BallisticOptics-2017_V1
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-05-04/models/LHCB2/b2_ballistic_from_update_model
C1: Beam2@Turn@2017:05:04@07:47:32:674.sdds	C2: Beam2@Turn@2017:05:04@07:49:26:422.sdds
C3: Beam2@Turn@2017:05:04@07:51:24:705.sdds	C4: Beam2@Turn@2017:05:04@07:53:43:348.sdds
C5: Beam2@Turn@2017:05:04@07:58:27:766.sdds	C6: Beam2@Turn@2017:05:04@08:02:12:362.sdds

[4] Measurement set D is a cluster of the following files:

Beam 1	
Kick group name:	Beam_1_after_global_corrections
Optics:	2016_ballistic_IR278localcorr
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2016-03-28/models/LHCB1/Ballistic_ft
D1: Beam1@Turn@2016:03:28@23:02:35:609.sdds	D2: Beam1@Turn@2016:03:28@23:04:03:315.sdds
D3: Beam1@Turn@2016:03:28@23:05:13:896.sdds	D4: Beam1@Turn@2016:03:28@23:06:25:236.sdds
D5: Beam1@Turn@2016:03:28@23:07:51:869.sdds	D6: Beam1@Turn@2016:03:28@23:08:59:616.sdds
D7: Beam1@Turn@2016:03:28@23:10:25:738.sdds	D8: Beam1@Turn@2016:03:28@23:11:53:935.sdds

Beam 2	
Kick group name:	Kicks_Beam2_after_global_corrections
Optics:	2016_ballistic_IR278localcorr
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2016-03-28/models/LHCB2/Ballistic_ft

D9:	Beam2@Turn@2016:03:28@23:02:24:580.sdds	D10:	Beam2@Turn@2016:03:28@23:04:35:629.sdds
D11:	Beam2@Turn@2016:03:28@23:06:04:282.sdds	D12:	Beam2@Turn@2016:03:28@23:07:35:528.sdds
D13:	Beam2@Turn@2016:03:28@23:08:44:381.sdds	D14:	Beam2@Turn@2016:03:28@23:10:16:191.sdds
D15:	Beam2@Turn@2016:03:28@23:11:29:013.sdds	D16:	Beam2@Turn@2016:03:28@23:12:34:815.sdds

[5] Measurement set E is a cluster of the following files:

Beam 2	
Kick group name:	BEAM_2_40CM
Optics:	SQUEEZE-6.5TeV-ATS-3m-40cm-2016_V1
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2016-07-30/models/LHCB2/ATS_40cm

E1:	Beam2@Turn@2016:07:30@12:15:01:488.sdds	E2:	Beam2@Turn@2016:07:30@12:16:17:568.sdds
E3:	Beam2@Turn@2016:07:30@12:17:32:565.sdds	E4:	Beam2@Turn@2016:07:30@12:18:52:097.sdds
E5:	Beam2@Turn@2016:07:30@12:20:15:329.sdds	E6:	Beam2@Turn@2016:07:30@12:21:27:545.sdds
E7:	Beam2@Turn@2016:07:30@12:23:25:871.sdds	E8:	Beam2@Turn@2016:07:30@12:26:24:850.sdds
E9:	Beam2@Turn@2016:07:30@12:27:33:202.sdds	E10:	Beam2@Turn@2016:07:30@12:28:44:125.sdds

[6] Measurement set F is a cluster of the following files:

Beam 1			
Kick group name:		b1_flatorbit_ampdet	
Optics:		R2017aT_A30C30A10mL300_CTPPS2	
Optics model:		/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-06-16/models/LHCB1/b1_squeeze_collQ	
F1:	Beam1@Turn@2018:06:16@17:21:58:964.sdds	F2:	Beam1@Turn@2018:06:16@17:23:15:867.sdds
F3:	Beam1@Turn@2018:06:16@17:24:25:704.sdds	F4:	Beam1@Turn@2018:06:16@17:26:04:577.sdds
F5:	Beam1@Turn@2018:06:16@17:30:47:667.sdds	F6:	Beam1@Turn@2018:06:16@17:32:32:233.sdds
F7:	Beam1@Turn@2018:06:16@17:33:53:240.sdds	F8:	Beam1@Turn@2018:06:16@17:35:41:330.sdds
F9:	Beam1@Turn@2018:06:16@17:37:23:562.sdds	F10:	Beam1@Turn@2018:06:16@17:39:38:500.sdds
F11:	Beam1@Turn@2018:06:16@17:41:05:055.sdds	F12:	Beam1@Turn@2018:06:16@17:42:26:736.sdds
F13:	Beam1@Turn@2018:06:16@17:44:06:724.sdds	F14:	Beam1@Turn@2018:06:16@17:45:29:998.sdds
F15:	Beam1@Turn@2018:06:16@17:48:24:925.sdds	F16:	Beam1@Turn@2018:06:16@17:49:57:489.sdds
F17:	Beam1@Turn@2018:06:16@17:51:28:289.sdds	F18:	Beam1@Turn@2018:06:16@17:52:56:386.sdds
F19:	Beam1@Turn@2018:06:16@17:54:35:023.sdds	F20:	Beam1@Turn@2018:06:16@17:56:50:762.sdds
F21:	Beam1@Turn@2018:06:16@18:00:08:493.sdds	F22:	Beam1@Turn@2018:06:16@18:01:42:542.sdds
F23:	Beam1@Turn@2018:06:16@18:03:00:822.sdds	F24:	Beam1@Turn@2018:06:16@18:04:27:984.sdds
F25:	Beam1@Turn@2018:06:16@18:06:02:679.sdds	F26:	Beam1@Turn@2018:06:16@18:07:26:926.sdds
F27:	Beam1@Turn@2018:06:16@18:08:59:823.sdds	F28:	Beam1@Turn@2018:06:16@18:11:00:853.sdds
F29:	Beam1@Turn@2018:06:16@18:14:38:390.sdds	F30:	Beam1@Turn@2018:06:16@18:16:19:652.sdds
F31:	Beam1@Turn@2018:06:16@18:18:39:218.sdds	F32:	Beam1@Turn@2018:06:16@18:20:31:404.sdds
F33:	Beam1@Turn@2018:06:16@18:22:38:587.sdds	F34:	Beam1@Turn@2018:06:16@18:23:55:135.sdds

Beam 2			
Kick group name:		b2_flatorbit_ampdet	
Optics:		R2017aT_A30C30A10mL300_CTPPS2	
Optics model:		/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-06-16/models/LHCB2/b2_flatorbit_ampdet	
F35:	Beam2@Turn@2018:06:16@17:22:11:103.sdds	F36:	Beam2@Turn@2018:06:16@17:23:30:195.sdds
F37:	Beam2@Turn@2018:06:16@17:24:41:752.sdds	F38:	Beam2@Turn@2018:06:16@17:30:31:292.sdds
F39:	Beam2@Turn@2018:06:16@17:33:33:960.sdds	F40:	Beam2@Turn@2018:06:16@17:35:31:287.sdds
F41:	Beam2@Turn@2018:06:16@17:37:14:214.sdds	F42:	Beam2@Turn@2018:06:16@17:39:12:606.sdds
F43:	Beam2@Turn@2018:06:16@17:40:40:972.sdds	F44:	Beam2@Turn@2018:06:16@17:42:18:231.sdds
F45:	Beam2@Turn@2018:06:16@17:44:21:105.sdds	F46:	Beam2@Turn@2018:06:16@17:45:49:716.sdds
F47:	Beam2@Turn@2018:06:16@17:47:18:725.sdds	F48:	Beam2@Turn@2018:06:16@17:48:45:282.sdds
F49:	Beam2@Turn@2018:06:16@17:50:17:686.sdds	F50:	Beam2@Turn@2018:06:16@17:51:46:237.sdds
F51:	Beam2@Turn@2018:06:16@17:53:12:492.sdds	F52:	Beam2@Turn@2018:06:16@17:54:51:504.sdds
F53:	Beam2@Turn@2018:06:16@17:56:26:784.sdds	F54:	Beam2@Turn@2018:06:16@18:06:41:584.sdds
F55:	Beam2@Turn@2018:06:16@18:08:05:432.sdds	F56:	Beam2@Turn@2018:06:16@18:09:27:846.sdds
F57:	Beam2@Turn@2018:06:16@18:10:49:009.sdds	F58:	Beam2@Turn@2018:06:16@18:12:14:352.sdds
F59:	Beam2@Turn@2018:06:16@18:13:37:598.sdds	F60:	Beam2@Turn@2018:06:16@18:15:11:194.sdds
F61:	Beam2@Turn@2018:06:16@18:19:42:663.sdds	F62:	Beam2@Turn@2018:06:16@18:22:49:349.sdds
F63:	Beam2@Turn@2018:06:16@18:24:16:830.sdds	F64:	Beam2@Turn@2018:06:16@18:25:43:899.sdds
F65:	Beam2@Turn@2018:06:16@18:27:20:942.sdds	F66:	Beam2@Turn@2018:06:16@18:28:55:247.sdds

[7] Measurement set G is a cluster of the following files:

Beam 1			
Kick group name:		b1_90m	
Optics:		R2018h_A90mC90mA10mL10m	
Optics model:		/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-05-29/models/LHCB1/b1_90m_right_tunes	
G1:	Beam1@Turn@2018:05:29@17:48:36:697.sdds	G2:	Beam1@Turn@2018:05:29@17:49:45:260.sdds
G3:	Beam1@Turn@2018:05:29@17:50:59:654.sdds	G4:	Beam1@Turn@2018:05:29@17:53:07:186.sdds
G5:	Beam1@Turn@2018:05:29@18:36:02:041.sdds	G6:	Beam1@Turn@2018:05:29@18:37:33:025.sdds
G7:	Beam1@Turn@2018:05:29@18:38:59:013.sdds	G8:	Beam1@Turn@2018:05:29@18:40:13:535.sdds
G9:	Beam1@Turn@2018:05:29@18:42:21:016.sdds		

Beam 2	
Kick group name:	b2_90m
Optics:	R2018h_A90mC90mA10mL10m
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-05-29/models/LHCB2/b2_90m
G10:	Beam2@Turn@2018:05:29@17:34:27:192.sdds
G11:	Beam2@Turn@2018:05:29@17:38:59:700.sdds
G12:	Beam2@Turn@2018:05:29@17:41:00:044.sdds
G13:	Beam2@Turn@2018:05:29@17:42:25:585.sdds
G14:	Beam2@Turn@2018:05:29@17:44:58:606.sdds
G15:	Beam2@Turn@2018:05:29@17:48:10:886.sdds
G16:	Beam2@Turn@2018:05:29@18:16:16:762.sdds
G17:	Beam2@Turn@2018:05:29@18:30:45:029.sdds
G18:	Beam2@Turn@2018:05:29@18:39:18:847.sdds
G19:	Beam2@Turn@2018:05:29@18:40:57:346.sdds
G20:	Beam2@Turn@2018:05:29@18:47:10:501.sdds

[8] Measurement set H is a cluster of the following files:

Beam 1	
Kick group name:	b1_amp_det_inj
Optics:	R2017a_A11mC11mA10mL10m
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-07-30/models/LHCB1/b1_injection
H1:	Beam1@Turn@2018:07:30@19:49:11:452.sdds
H2:	Beam1@Turn@2018:07:30@19:50:37:684.sdds
H3:	Beam1@Turn@2018:07:30@19:51:46:755.sdds
H4:	Beam1@Turn@2018:07:30@19:52:57:966.sdds
H5:	Beam1@Turn@2018:07:30@19:54:11:387.sdds
H6:	Beam1@Turn@2018:07:30@19:55:19:553.sdds
H7:	Beam1@Turn@2018:07:30@19:56:28:246.sdds
H8:	Beam1@Turn@2018:07:30@19:57:44:192.sdds
H9:	Beam1@Turn@2018:07:30@19:58:51:989.sdds
H10:	Beam1@Turn@2018:07:30@20:00:01:076.sdds
H11:	Beam1@Turn@2018:07:30@20:04:33:826.sdds
H12:	Beam1@Turn@2018:07:30@20:05:41:861.sdds
H13:	Beam1@Turn@2018:07:30@20:06:52:698.sdds
H14:	Beam1@Turn@2018:07:30@20:08:01:812.sdds
H15:	Beam1@Turn@2018:07:30@20:09:15:025.sdds
H16:	Beam1@Turn@2018:07:30@20:10:30:419.sdds
H17:	Beam1@Turn@2018:07:30@20:17:36:154.sdds
H18:	Beam1@Turn@2018:07:30@20:18:57:578.sdds

Beam 2	
Kick group name:	b2_ampdet_horizontal
Optics:	R2017a_A11mC11mA10mL10m
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-07-30/models/LHCB2/b2_injection
H19:	Beam2@Turn@2018:07:30@19:50:53:951.sdds
H20:	Beam2@Turn@2018:07:30@19:52:05:651.sdds
H21:	Beam2@Turn@2018:07:30@19:53:18:463.sdds
H22:	Beam2@Turn@2018:07:30@19:54:25:509.sdds
H23:	Beam2@Turn@2018:07:30@19:55:33:310.sdds
H24:	Beam2@Turn@2018:07:30@19:56:39:459.sdds
H25:	Beam2@Turn@2018:07:30@19:58:10:879.sdds

[9] Measurement set I is a cluster of the following files:

Beam 1	
Kick group name:	60_15_b1_after_couplingCorr
Optics:	R2017aT65_A60_15C15_60A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-12-01/models/LHCB1/60cm_15cm_beam1_ac
I1: Beam1@Turn@2017:12:01@15:49:40:440.sdds	I2: Beam1@Turn@2017:12:01@15:50:52:324.sdds
I3: Beam1@Turn@2017:12:01@15:52:08:665.sdds	I4: Beam1@Turn@2017:12:01@15:54:10:000.sdds
I5: Beam1@Turn@2017:12:01@15:55:35:907.sdds	I6: Beam1@Turn@2017:12:01@15:57:06:906.sdds
I7: Beam1@Turn@2017:12:01@15:59:02:994.sdds	I8: Beam1@Turn@2017:12:01@16:00:19:805.sdds
I9: Beam1@Turn@2017:12:01@17:20:00:800.sdds	I10: Beam1@Turn@2017:12:01@17:21:12:280.sdds

[10] Measurement set J is a cluster of the following files:

Beam 1	
Kick group name:	beam1_flattop
Optics:	R2017a_A100C100A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-04-02/models/LHCB1/b1_injection
J1: Beam1@Turn@2018:04:02@11:33:08:895.sdds	J2: Beam1@Turn@2018:04:02@11:34:59:926.sdds
J3: Beam1@Turn@2018:04:02@11:40:10:129.sdds	J4: Beam1@Turn@2018:04:02@11:43:45:882.sdds
J5: Beam1@Turn@2018:04:02@11:44:59:573.sdds	J6: Beam1@Turn@2018:04:02@11:47:08:135.sdds
J7: Beam1@Turn@2018:04:02@11:48:20:689.sdds	J8: Beam1@Turn@2018:04:02@11:50:15:479.sdds
J9: Beam1@Turn@2018:04:02@11:53:01:893.sdds	J10: Beam1@Turn@2018:04:02@11:55:08:244.sdds
Beam 2	
Kick group name:	Flattop_B2_2018_b1m
Optics:	R2017a_A100C100A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2018-04-02/models/LHCB2/Flattop_B2
J11: Beam2@Turn@2018:04:02@11:43:24:250.sdds	J12: Beam2@Turn@2018:04:02@11:45:05:032.sdds
J13: Beam2@Turn@2018:04:02@11:46:19:386.sdds	J14: Beam2@Turn@2018:04:02@11:48:39:657.sdds
J15: Beam2@Turn@2018:04:02@11:50:17:902.sdds	

[11] Measurement set K is a cluster of the following files:

Beam 1	
Kick group name:	b1cm_25cm_onMomentum
Optics:	R2017aT100_A25C25A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-06-30/models/LHCB1/b1_25cm
K1: Beam1@Turn@2017:06:30@18:58:19:067.sdds	K2: Beam1@Turn@2017:06:30@18:59:39:423.sdds
K3: Beam1@Turn@2017:06:30@19:00:45:238.sdds	K4: Beam1@Turn@2017:06:30@19:01:52:642.sdds
Beam 2	
Kick group name:	ATS-B2-25cm
Optics:	R2017aT100_A25C25A10mL300
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-06-30/models/LHCB2/b2_25cm
K5: Beam2@Turn@2017:06:30@18:59:13:609.sdds	K6: Beam2@Turn@2017:06:30@19:00:27:299.sdds
K7: Beam2@Turn@2017:06:30@19:01:34:751.sdds	K8: Beam2@Turn@2017:06:30@19:02:40:037.sdds

[12] Measurement set L is a cluster of the following files:

Beam 1	
Kick group name:	Beam1_30cm_log
Optics:	R2017aT_A30C30A10mL300_CTPPS2
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-05-25/models/LHCB1/Beam1_30cm_rightTunes
L1: Beam1@Turn@2017:05:25@19:07:04:535.sdds	L2: Beam1@Turn@2017:05:25@19:08:32:460.sdds
L3: Beam1@Turn@2017:05:25@19:10:10:898.sdds	L4: Beam1@Turn@2017:05:25@19:31:00:130.sdds
L5: Beam1@Turn@2017:05:25@19:32:11:172.sdds	L6: Beam1@Turn@2017:05:25@19:33:28:694.sdds
L7: Beam1@Turn@2017:05:25@19:34:38:530.sdds	L8: Beam1@Turn@2017:05:25@19:36:28:654.sdds
L9: Beam1@Turn@2017:05:25@19:38:35:608.sdds	L10: Beam1@Turn@2017:05:25@19:40:01:427.sdds
L11: Beam1@Turn@2017:05:25@19:41:38:239.sdds	
Beam 2	
Kick group name:	30cm_b2
Optics:	R2017aT_A30C30A10mL300_CTPPS20
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-05-25/models/LHCB2/30cm_correctTunes_b2
L12: Beam2@Turn@2017:05:25@19:06:39:344.sdds	L13: Beam2@Turn@2017:05:25@19:08:58:180.sdds
L14: Beam2@Turn@2017:05:25@19:10:34:547.sdds	L15: Beam2@Turn@2017:05:25@19:13:51:778.sdds
L16: Beam2@Turn@2017:05:25@19:30:50:177.sdds	L17: Beam2@Turn@2017:05:25@19:32:18:546.sdds
L18: Beam2@Turn@2017:05:25@19:33:37:937.sdds	L19: Beam2@Turn@2017:05:25@19:34:52:352.sdds
L20: Beam2@Turn@2017:05:25@19:36:36:132.sdds	L21: Beam2@Turn@2017:05:25@19:38:21:806.sdds
L22: Beam2@Turn@2017:05:25@19:39:52:733.sdds	L23: Beam2@Turn@2017:05:25@19:41:44:765.sdds
L24: Beam2@Turn@2017:05:25@19:43:40:521.sdds	

[13] Measurement set M is a cluster of the following files:

Beam 1	
Kick group name:	b1_highbeta_virgin
Optics:	R2017h_A100mC100mA10mL10m
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-10-26/models/LHCB1/multiturn_highbeta_colltunes
M1:	Beam1@Turn@2017:10:26@08:06:22:665.sdds
M2:	Beam1@Turn@2017:10:26@08:07:32:768.sdds
M3:	Beam1@Turn@2017:10:26@08:09:14:792.sdds
M4:	Beam1@Turn@2017:10:26@08:13:16:757.sdds
Beam 2	
Kick group name:	b2_highBeta_beforeCorrections_notCorrectModel
Optics:	R2017h_A100mC100mA10mL10m
Optics model:	/user/slops/data/LHC_DATA/OP_DATA/Betabeat/2017-10-26/models/LHCB2/highBetaInjectionB2
M5:	Beam2@Turn@2017:10:26@08:04:02:629.sdds
M6:	Beam2@Turn@2017:10:26@08:06:23:632.sdds
M7:	Beam2@Turn@2017:10:26@08:07:42:587.sdds
M8:	Beam2@Turn@2017:10:26@08:08:55:084.sdds
M9:	Beam2@Turn@2017:10:26@08:10:40:209.sdds
M10:	Beam2@Turn@2017:10:26@08:12:36:295.sdds
M11:	Beam2@Turn@2017:10:26@08:13:42:065.sdds

- [14] Courant E.D., Snyder H.S., Theory of the Alternating-Gradient Synchrotron, Annals of Physics (N.Y.)(1957), p. 382, formula 4.31.
- [15] Adunka F., Messunsicherheiten - Theorie und Praxis, 3rd edition, Vulkan-Verlag (2007), p. 156.
- [16] Davison A.C. and Hinkley D.V., Bootstrap Methods and their Applications, Cambridge Series in Statistical and Probabilistic Mathematics, Cambridge University Press (1997).
- [17] Shannon C. E., A mathematical theory of communication, The Bell System Technical Journal, vol. 27, no. 3, pp. 379-423, July 1948.
- [18] Engel J. and Grübel R., Bootstrap - oder die Kunst, sich selbst aus dem Sumpf zu ziehen, mathematical semester documentation of the university of Hannover (2008), page 8, weblink: <https://www.stochastik.uni-hannover.de/fileadmin/institut/pdf/MathSemBer2008preprint.pdf>.
- [19] Gamba D., Coello De Portugal J.M., Impact of noise in the main LHC circuits – orbit, β^* and tune ripple, in 99th HiLumi WP2 Meeting, July 2017. Weblink: <https://indico.cern.ch/event/655317/contributions/2668982/>.

- [20] Herr W., Muratori B., Concept of luminosity, CERN. Weblink: <https://cds.cern.ch/record/941318/files/p361.pdf>.
- [21] Carlier F., Tomas R., Accuracy and feasibility of the β^* measurement for LHC and High Luminosity LHC using k-modulation, Phys. Rev. Accel. Beams, Jan 2017, American Physical Society.
- [22] Quast G., Parameteranpassung mit der Likelihood-Methode, Rechnernutzung in der Physik, Karlsruhe Institute of Technology (KIT), lecture slides WS 2017/2018. Weblink: https://comp.physik.kit.edu/Lehre/Rechnernutzung/Vorlesungsfolien/V11_Likelihood.pdf
- [23] White S., Maclean E., Tomás R., Direct amplitude detuning measurement with ac dipole, Physical Review Special Topics-Accelerators and Beams 16, no. 7 (2013).
- [24] Calaga R., Tomás R., Statistical analysis of RHIC beam position monitors performance, Physical Review Special Topics-Accelerators and Beams 7, no. 4 (2004).
- [25] Sammut, Nicholas, Bottura, Luca, Deferne, Guy, Delsolaro, Walter, Mathematical formulation to predict the harmonics of the superconducting Large Hadron Collider magnets: III. Precycle ramp rate effects and magnet characterization, Physical Review Special Topics - Accelerators and Beams, no. 12 (2009).
- [26] Tomás R., Bach T., Calaga R., Langner A., Levinsen Y., Maclean E., Persson T., Skowronski P., Strzelczyk M., Vanbavinckhove G., Miyamoto M., Record low β^* beating in the LHC, Phys. Rev. ST Accel. Beams, American Physical Society, Sep. 2012.
- [27] Holzer B.J., Introduction to Particle Accelerators and their Limitations, CAS - CERN Accelerator School: Plasma Wake Acceleration, May 2017, CERN, Geneva, Switzerland. Weblink: http://cds.cern.ch/record/2203629/files/1418884_29-50.pdf?version=1.
- [28] Matteo Solfaroli Camillocci, LHC nominal cycle, CERN, Geneva, Switzerland. Weblink: https://indico.cern.ch/event/434129/contributions/1917195/attachments/1205096/1765722/Nominal_cycle.pdf.
- [29] Herr W. and Pieloni T., Beam-Beam Effects, CERN, Geneva, Switzerland. Weblink: https://www.researchgate.net/publication/291437012_Beam-Beam_Effects.

- [30] Singh A., Bhat P., Mokhov N., Beri S., Beam-induced radiation in the compact muon solenoid tracker at the Large Hadron Collider, PRAMANA - journal of physics, Indian Academy of Sciences, Vol. 74, No. 5 May 2010, pp. 719-729. Weblink: https://www.researchgate.net/publication/225436837_Beam-induced_radiation_in_the_compact_muon_solenoid_tracker_at_the_Large_Hadron_Collider.
- [31] Matteo Solfaroli Camillocci, LHC nominal cycle, CERN, Geneva, Switzerland. Weblink: https://indico.cern.ch/event/434129/contributions/1917195/attachments/1205096/1765722/Nominal_cycle.pdf.
- [32] Andy Sven Langner, A Novel Method and Error Analysis for Beam Optics Measurements and Corrections at the Large Hadron Collider, Universität Hamburg, dissertation (2016).
- [33] Laurent Deniau, Hans Grote, Ghislain Roy, Frank Schmidt, European laboratory for particle physics (CERN), The MAD-X Program - Methodical Accelerator Design, User's Reference Manual, Geneva, Switzerland, 3. October 2018.
- [34] Persson T., Carlier F., Coello de Portugal J., Garcia-Tabares Valdivieso A., Langner A., Maclean E. H., Malina L., Skowronski P., Salvant B., Tomás R., García Bonilla A. C., LHC optics commissioning: A journey towards 1% optics control, physical review accelerators and beams, June 2017, CERN, Geneva, Switzerland.
- [35] Malina L., Coello de Portugal J., Dilly, P.K. Skowroński J., Tomás R., Toplis M., performance optimisation of turn-by-turn beam position monitor data harmonic analysis, 9th International Particle Accelerator Conference - IPAC2018, Vancouver, Canada.
- [36] Malina L., Coello de Portugal J., Persson T., Skowroński P. K., Tomás R., Franchi A., Liuzzo S., Improving the precision of linear optics measurements based on turn-by-turn beam position monitor data after a pulsed excitation in lepton storage rings, Physical review accelerators and beams, CERN, Geneva, Switzerland, August 2017.
- [37] Wolski Andrzej, Low-emittance Storage Rings, CAS - CERN Accelerator School: Advanced Accelerator Physics Course, July 2017, CERN, Geneva, Switzerland. Weblink: <http://cds.cern.ch/record/1982424/files/245-294%20Wolski.pdf?version=1>.

- [38] Brandt D., Introduction to Accelerators, High School Teachers Programme 2009, CERN, Geneva, Switzerland.
- [39] Coello de Portugal J., Carlier F., Langner A., Persson T., Skowronski P., Tomás R., OMC software improvements in 2014, IPAC2015, Richmond, USA.
- [40] CERN public information, The Large Hadron Collider, weblink: <https://home.cern/science/accelerators/large-hadron-collider>.
- [41] Science X, CERN's Large Hadron Collider gears up for run two, December 12, 2014, CERN.
- [42] Science X, Upgrade to boost capacity of CERN's giant particle smasher (Update) June 15, 2018, CERN.
- [43] CERN public information, High-Luminosity LHC, weblink: <https://home.cern/science/accelerators/high-luminosity-lhc>.