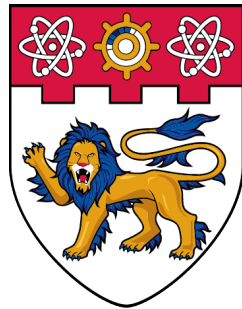


GLOBAL ANALYSIS OF DARK MATTER SIMPLIFIED MODELS (DMSMs)
WITH LEPTOPHILIC SPIN-ONE MEDIATORS USING
THE MasterCode FRAMEWORK



**NANYANG
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GLOBAL ANALYSIS OF DARK MATTER SIMPLIFIED MODELS
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Abstract

Recent cosmological and astrophysical studies have shown that ordinary matter, which can be observed directly, only constitutes 15% of the total matter content of Universe. The remaining 85% is what cosmologists and astrophysicists termed as exotic matter. Exotic matter is any form of matter whose properties, behaviour and interactions differ from those observed in ordinary matter. Since they cannot be detected directly, anomalies in astrophysical observations can be evidence for their existence. These anomalies seem to favour a form of matter called Dark Matter (DM) as one of the highly plausible candidates for exotic matter. Hence, a collective group of theories known as Dark Matter Simplified Models (DMSMs) were developed to study this form of matter in greater detail. One thing to note about these theories is that they do not provide the identity of the DM particle. However, one of the objectives of these theories is to provide limits on the masses of DM particles. Hence, this would be one of the key results that would be presented in this thesis.

Being theoretical models, the feasibility of DMSMs still needs to be validated by experimental results. Although a range of experiments have published a number of papers with regards to these limits, a global fit has yet to be conducted so as to gain a better understanding of these models. Although the **MasterCode** framework, which is actually a code package, was previously used to conduct global fits of Supersymmetric (SUSY) models to data from a variety of experiments, it can also be used to conduct global fits for a multitude of other models, including DMSMs. For these reasons, the **MasterCode** framework was employed to conduct these global fit of DMSMs.

Preface

This thesis is built upon a research paper that was submitted onto the `arXiv` e-print archive system in May 2019 and published in November 2019 [1]. In particular, this thesis is actually an extension of the work done in [1]. However, with the understanding that the reader may not already be in the field, instead of jumping straight into the main discussion of the thesis, the layout of this thesis will be as following:

- 1) Chapters 1 and 2 will be covering the basics of particle physics
- 2) In Chapter 3, the purpose of the research in [1] will be motivated. It will then be followed by a discussion of [1] in preparation for the discussion of current research effort in the following chapters.
- 3) In Chapter 4, the focus of the discussion will be on the `MasterCode` framework.
- 4) In Chapter 5, the current research effort as an extension to the research done in [1] will be discussed.
- 5) Chapter 6 concludes this thesis and discusses potential further areas of research which may extend the scope of the current research project

A complete set of derivations for calculations done in Chapter 1 and 2 will also be included in the appendix as a form of reference for the reader.

Acknowledgements

I would like to firstly thank the CN Yang Scholar's Programme for giving me this rare opportunity to complete my Final Year Project (FYP) overseas in Switzerland and for the financial support that allowed this Overseas FYP possible. I would also like to thank my local supervisor, Associate Professor Chew Lock Yue, for connecting me to Dr Albert de Roeck, who is my overseas supervisor in CERN. I would also like to thank Dr Albert de Roeck for making this trip to CERN possible, from the registration phase to the day when I arrived at CERN.

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Last but not the least, I would like to thank my family for being such an emotional support during the time when I was overseas. I am extremely grateful for the moments that I got to spend with my dad and my sister when we were travelling around Europe. I am also very thankful that despite the time difference between Switzerland and Singapore my family members will still keep in touch with me via WhatsApp to check how I am doing and how is everything.

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Chapter 1

Introduction

Before diving into the `MasterCode` framework and the details of the Dark Matter Simplified Models (DMSMs), we will first look at an overview of particle physics, both qualitatively ([Standard Model \(SM\)](#)) and quantitatively ([Feynman Calculus](#)). In a nutshell, the Standard Model plays a crucial role in describing elementary particles and their interactions with one another while Feynman calculus, containing a set of Feynman rules, provides the mathematical formalism that governs these interactions. The details of these 2 fundamental formalisms in particle physics will be discussed in further detail in the subsequent sections.

1.1 Standard Model (SM)

The Standard Model (SM) of particle physics was developed in the early part of the 1970s and has been one of the most successful theories in particle physics ever since. Many theoretical predictions by the Standard Model has been verified by experimental results, mainly from collider experiments. Some of the most well known successes of the Standard Model verified by experiments were: 1) The 1983 discovery of the W and Z bosons at SPS¹, a proton-antiproton collider, in CERN; 2) The 1995 discovery of the massive top quark by the CDF² and the DØ experiments from the proton-anti-proton collisions at Tevatron located in Fermilab; and 3) The ground-breaking discovery of the Higgs boson in 2012 announced by the CMS³ and ATLAS⁴ experiments from the high energy proton-antiproton collisions at the LHC in CERN.

As briefly described in the introduction, the Standard Model describes elementary particles and the 3 fundamental forces, excluding gravity, that governs the interactions between these elementary particles. Since the Standard Model is a *Quantum Field Theory* (QFT), the 3 fundamental interactions (the strong, weak⁵ and electromagnetic (EM)⁵ interactions) can be described using *Quantum Electrodynamics* (QED) for *electroweak*⁵ interactions and *Quantum Chromodynamics* (QCD) for *strong* interactions. In the Standard Model, elementary particles can be classified into two broad categories, *fermions* and *bosons* (the gauge bosons and the Higgs boson). According to the spin-statistics the-

¹ Super Proton Synchrotron

² Collider Detector at Fermilab

³ Compact Muon Solenoid

⁴ A Toroidal Large Hadron Collider (LHC) Apparatus

⁵ Both the weak and electromagnetic interactions are unified to form the electroweak (EW) interaction at high energies which will play an important role in the next chapter on [Weak Interactions](#)

orem, fermions are defined as particles with half-integer spin while bosons are particles with integer spins⁶. Fermions in the Standard Model can be further classified into *quarks* which experiences strong interactions and *leptons* which do not. The classification of the elementary particles in the Standard Model is shown below in Figure 1.1.

STANDARD MODEL OF ELEMENTARY PARTICLES



Figure 1.1: Standard Model of Particle Physics [2]

As seen in Figure 1.1, there are 3 generations of fermions, regardless whether they are quarks or leptons, with the masses of the fermions increasing with each successive generation. Within the quark family, the quarks in the first row are known as “*up-type*” quarks (u , c , t quarks), where they share a common charge of $Q = \frac{2}{3}$, while the quarks in the second row are known as “*down-type*” quarks (d , s , b quarks), with a common charge of $Q = -\frac{1}{3}$. The same can be said for the lepton family where the leptons in the first row are known as charged leptons, since they carry a common charge of $Q = -1$ whereas the leptons in the second row are known as neutrinos (neutral leptons) since they are neutral ($Q = 0$). Since quarks and charged leptons are charged, for each particle there is an antiparticle associated with it, whose mass is the same as the original particle but with a charge that is the opposite from the original particle. However, for neutrinos, since they have no charge, they are their own anti-particles and they are known as “*Majorana fermions*” [3].

The next class of particles in the Standard Model is the bosons, which can be further

⁶ This can be clearly seen in Figure 1.1

categorized into the spin-1 gauge bosons and the scalar Higgs boson, whose spin is 0. Similar to neutrinos, since gauge bosons are chargeless other than the charged $W^\pm$ bosons, they are also their own anti-particles. The gauge bosons are also known as the force carriers or mediators for the 3 fundamental forces, where the gluon mediates the strong interaction, the photon mediates the electromagnetic interaction, and the $W^\pm$ and Z^0 bosons⁷ mediate the weak interaction. For the Higgs boson, however, being the only scalar particle in the Standard Model does not have an anti-particle associated with it. Unlike the gauge bosons, the Higgs boson mediates the *Higgs field* which interacts with massless particle fields and causes these particles to acquire mass through the *Higgs Mechanism*⁸.

As previously mentioned, since the Standard Model is a Quantum Field Theory, in particular a gauge quantum field theory, it contains internal gauge symmetries describing the 3 fundamental interactions. These internal gauge symmetries can be expressed mathematically in Group theory language as

$$SU(3) \otimes SU(2) \otimes U(1) \quad (1.1)$$

where $SU(3)$ represents the symmetry group of color which contains the strong interaction, as described by Quantum Chromodynamics (QCD), with the 3 representing the number of colored states⁹ in the theory, and $SU(2) \otimes U(1)$ represents the gauge symmetry group of the electroweak (EW) sector of the Standard Model, where the weak isospin T and the weak hypercharge Y acts as generators for the symmetry group. For the $SU(2) \otimes U(1)$ gauge group, the sum of the third component of the weak isospin T^3 and the weak hypercharge Y gives rise to the electric charge of fermions [4]:

$$\begin{aligned} Q &= T^3 + \frac{1}{2}Y \text{ where } T^3 = \frac{\tau^3}{2} \text{ (for left-handed fermions¹⁰),} \\ &= 0 \text{ (for right-handed fermions¹¹)} \\ \tau^3 &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \text{ being the third Pauli matrix} \end{aligned} \quad (1.2)$$

The weak hypercharge in the above equation depends on the electric charge, which depends on the species¹² of the fermion, and the chirality¹³ of the fermion in question. The concept of chirality will play a very influential role in the electroweak sector of the Standard Model, which will be elaborated further in the following chapter on [Weak](#)

⁷ Also known as massive intermediate vector bosons

⁸ Details will be elaborated further in the following chapter on [Weak Interactions](#)

⁹ Namely red(r), green(g) and blue(b)

¹⁰ Whose spin is in the *opposite* direction from its motion

¹¹ Whose spin is in the *same* direction as its motion

¹² Or the family in which the fermion belongs to

¹³ Whether it is left-/right-handed

Interactions.

Another way the Standard Model can be represented mathematically is using the Lagrangian formulation. The Lagrangian of the Standard Model can be summarized in the following manner

$$\begin{aligned}
 \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
 & + i\bar{\psi}\not{D}\psi + \text{h.c.} \\
 & + \psi_i Y_{ij} \bar{\psi}_j \phi + \text{h.c.} \\
 & + |D_\mu \phi|^2 - V(\phi)
 \end{aligned} \tag{1.3}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, ψ refers to the fermionic field, $\bar{\psi} = \psi^\dagger \gamma^0$ is the Dirac adjoint of the fermionic field (can also be interpreted as the anti-fermionic field), $\not{D} = \gamma^\mu D_\mu$ using the Feynman-slash notation, h.c. refers to the hermitian conjugate, Y_{ij} refers to the Yukawa couplings between the fermionic fields and the Higgs field ϕ , $|D_\mu \phi|^2 = (D_\mu \phi)^\dagger (D^\mu \phi)$, $D_\mu = \partial_\mu + ieA_\mu$ is the covariant derivative, and $V(\phi)$ is the *Higgs potential*.

The Lagrangian in Eq (1.3) can be broken down into the following terms:

1. First line $\mathcal{L}_{kin}$ ($\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$): The kinetic term of the Lagrangian in the $U(1)$ gauge group.
2. The second line $\mathcal{L}_f$: The term describing the interactions between fermions of the same species¹⁴ and the gauge field, as mentioned in the previous paragraph.
3. The third line $\mathcal{L}_{Yuk}$: The Yukawa term which describes the interactions between the different species of fermions and the Higgs field.
4. The last line $\mathcal{L}_{Higgs}$: The Higgs term which describes the Higgs Mechanism. This term will be discussed in greater detail under a later section called [Higgs Mechanism](#).

The Lagrangian formulation is very useful in the sense that Feynman rules and Feynman diagrams describing the different interactions in the Standard Model can immediately be derived from the Lagrangian in Eq (1.3). Further details on how this can be done will be discussed in the following section on [Feynman Calculus](#).

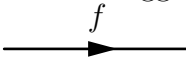
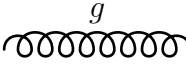
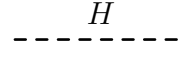
1.2 Feynman Calculus

As discussed in the previous section, interactions between different particles in the Standard Model can be derived from the Lagrangian in Eq (1.3) using Feynman diagrams.

¹⁴ Quark-quark (qq) or lepton-lepton (ll), interactions

Feynman diagrams are essentially *pictorial representations* of the mathematics that describes these interactions within the Standard Model, as briefly introduced in the [Introduction](#). They can be used to describe different processes in the Standard Model, mainly scattering, production and annihilation processes, as a result of these interactions. A very useful feature about Feynman diagrams is that they include intermediate processes, including intermediate states, rather than merely showing only the initial and final states. These intermediate states are also known as virtual states, since they cannot be detected by experiments, and they can come in the form of massless mediators, that mediate either the electromagnetic (EM) or the strong force, or short-lived particles, including the heavy gauge bosons that mediate the weak force.

Each particle, whether it is an initial or intermediate or final state, can be represented using a line and the style of the line represents the type of particle, whether it is fermion or photon or Higgs boson etc.

- a)  : Represents a Standard Model fermion, whether it is a quark q or a lepton l
- b)  : Represents a photon γ , the mediator for the EM interactions, or a massive intermediate vector boson $W^\pm/Z^0$, the mediator for the weak interactions
- c)  : Represents a gluon, the mediator for the strong interactions
- d)  : Represents a Higgs boson

Since the Standard Model is a quantum field theory, these interactions are hence probabilistic in nature and the probability of an interaction occurring is the absolute square of the sum of all probability amplitudes $\mathcal{M}$, involved in the interaction, *averaged over all possible spin orientations*¹⁵ [4].

$$P(\text{interaction}) = \langle |\mathcal{M}|^2 \rangle \quad (1.4)$$

In Feynman diagrams, different particles interact with one another at interaction points known as *vertices*. Each vertex has a factor that is associated with it, known as the *vertex factor*, and this factor depends on the type of interactions involved. Each vertex factor is related to a dimensionless coupling constant that determines the strength of the interaction involved. The vertex factors for the 3 different types of interactions are shown below and in the subsequent pages [4]:

- i) Quantum Electrodynamics (for photons and charged fermions): $ig_e\gamma^\mu$, where $g_e = \sqrt{4\pi\alpha_{\text{EM}}}$, $\alpha_{\text{EM}} \approx \frac{1}{137}$ is the electromagnetic (EM) fine structure constant, and γ^μ 's are the 4×4 gamma matrices, also known as the *Dirac matrices*

¹⁵ The $\langle |\mathcal{M}|^2 \rangle$ here is calculated directly, without actually computing each of the individual probability amplitudes

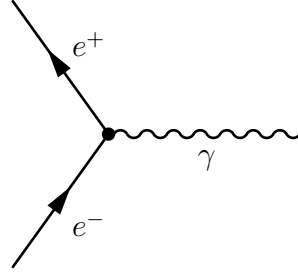


Figure 1.2: A typical QED vertex

ii) Quantum Chromodynamics (for quarks¹⁶ and gluons):

- (a) Quark-gluon vertex: $-\frac{ig_s}{2} \lambda^\alpha \gamma^\mu$, where $g_s = \sqrt{4\pi\alpha_s}$, α_s is the "strong analogue" of the EM fine structure constant and is related to the theory of asymptotic freedom¹⁷ observed in QCD studies, and λ^α are the 3×3 Gell-Mann "λ-matrices"¹⁸, which are the $SU(3)$ analogues of the Pauli spin matrices, σ^1 , σ^2 and σ^3 , in the $SU(2)$ group

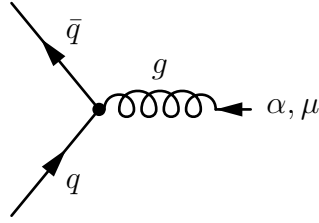


Figure 1.3: Quark-gluon vertex

- (b) Three-gluon vertex: $-g_{sJ}^{\alpha\beta\gamma} [g_{\mu\nu}(k_1 - k_2)_\lambda + g_{\nu\lambda}(k_2 - k_3)_\mu + g_{\lambda\mu}(k_3 - k_1)_\nu]$, where $g_{sJ}^{\alpha\beta\gamma}$ is the tri-coupling that is symmetric under the interchange of indices ($\alpha \leftrightarrow \beta$, $\beta \leftrightarrow \gamma$ and $\gamma \leftrightarrow \alpha$) and k_1 , k_2 and k_3 are the momenta of the gluons

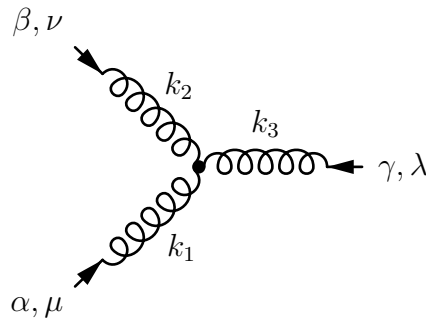


Figure 1.4: Three-gluon vertex

- (c) Four-gluon vertex: $-ig_s^2 [f^{\alpha\beta\eta} f^{\gamma\delta\eta} (g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}) + f^{\alpha\delta\eta} f^{\beta\gamma\eta} (g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho}) + f^{\alpha\gamma\eta} f^{\delta\beta\eta} (g_{\mu\rho} g_{\nu\lambda} - g_{\mu\nu} g_{\lambda\rho})]$, where $f^{\alpha\beta\eta}$ ¹⁹ are the "structure constants" of $SU(3)$ group and the summation over the index η is implied

¹⁶ Since they are colored

¹⁷ I will not elaborate on the details since this has little relevance to the project

¹⁸ There are 8 of these matrices in total, each corresponding a particular gluon color charge

¹⁹ $[\lambda^\alpha, \lambda^\beta] = 2if^{\alpha\beta\eta}\lambda^\eta$ where λ^α , λ^β and λ^η are the Gell-Mann "λ- matrices"

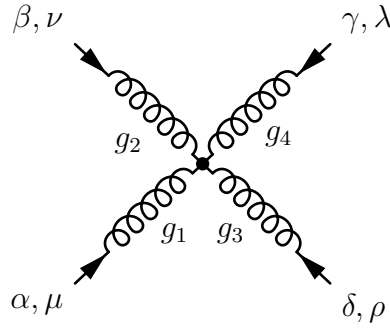


Figure 1.5: Four-gluon vertex

iii) Weak Interactions (for fermions and the massive intermediate vector bosons $W^\pm$ and Z^0):

1) $W^\pm$ vertex:

$$-\frac{ig_w}{2\sqrt{2}}\gamma^\mu(1-\gamma^5) \quad \text{where } g_w = \frac{g_e}{\sin\theta_w} \quad (1.5)$$

where g_e and g_w are the EM and weak coupling constants respectively, $\theta_w \approx 28.7^\circ$ is the *weak mixing angle*²⁰, also known as the Weinberg angle, and $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ is the fifth γ -matrix, which anti-commutes with all other γ -matrices, γ^μ

2) Z^0 vertex:

$$-\frac{ig_z}{2}\gamma^\mu(c_V^f - c_A^f\gamma^5) \quad \text{where } g_z = \frac{g_w}{\cos\theta_w} \quad (1.6)$$

where g_z is the neutral coupling constant, c_V is the vector coupling to the “weak charge”, c_A is the axial-vector coupling to the “weak charge”, and the superscript f refers to the fermion that is involved in the interaction, be it a quark or lepton. The vector c_V and the axial-vector c_A couplings depend on the species of the fermions, whether it is a neutrino, a lepton, a “up-type” or a “down-type” quark. The values for these couplings for the different species of fermions are given in Table 2.1.

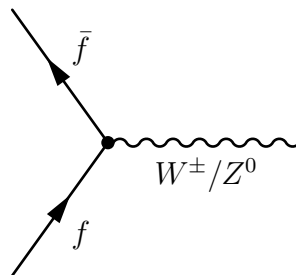


Figure 1.6: A typical weak vertex

²⁰ The weak mixing angle θ_w will be elaborated further in the following chapter on [Weak Interactions](#)

For each vertex, to conserve energies and momenta of the particles involved in the interaction, a delta function with a factor of $(2\pi)^4$ needs to be included

$$(2\pi)^4 \delta^4(\mathbf{k}_{total}), \text{ where } \mathbf{k}_{total} = \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \dots \quad (1.7)$$

where $\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \dots$ are the individual four-momenta approaching the vertex. For a four-momentum leaving the vertex, whose vector is pointing outwards from the vertex as shown in Figures 1.2, 1.3 and 1.6, its sign will be flipped from positive to negative. Note that the delta function in Eq (1.7) is zero unless the sum of incoming four-momenta is equal to the sum of outgoing momenta, hence ensuring energy and momentum conservation at the vertex.

Connecting one vertex to another is an internal line or a propagator representing a particle, which is normally a mediator boson. Each propagator carries a factor

$$\frac{i}{q^2 - (mc)^2}, \quad q^2 \neq (mc)^2 \quad (1.8)$$

where $q^2 = q^\mu q_\mu$ is the square of the propagator's four-momentum, or the energy of the propagator, and m is the mass of the particle that the propagator represents. For a massless particle such as a photon or gluon, since $m = 0$, the denominator in Eq (1.8) becomes q^2 . For a photon, the numerator in Eq (1.8) acquires a factor of $-g_{\mu\nu}$ while the numerator for a gluon acquires an additional factor of $\delta^{\alpha\beta}$ on top of the factor $-g_{\mu\nu}$, hence Eq (1.8) becomes $-\frac{ig_{\mu\nu}}{q^2}$ for the photon and $-\frac{ig_{\mu\nu}\delta^{\alpha\beta}}{q^2}$ for the gluon. For fermions, whether they are quark-antiquark or lepton-antilepton internal lines, their associated propagators carry an additional factor of $\not{q} + mc$, where $\not{q} = \gamma^\mu q_\mu$ using the Feynman-slash notation, thus Eq (1.8) becomes $\frac{i(\not{q} + mc)}{q^2 - (mc)^2}$. For the massive intermediate vector bosons $W^\pm$ and Z^0 , their associated propagators acquire a rather complicated factor of $-[g_{\mu\nu} - \{q_\mu q_\nu / (Mc)^2\}]$, where $M = M_W/M_Z$ is the mass of the $W^\pm$ or Z^0 boson respectively, and as a result Eq (1.8) transforms into the following form:

$$-\frac{i(g_{\mu\nu} - \frac{q_\mu q_\nu}{(Mc)^2})}{q^2 - (Mc)^2} \quad (1.9)$$

for energies, *i.e.* q^2 , comparable to the rest mass energy of the $W^\pm/Z^0$ boson. For $q^2 \ll (Mc)^2$, Eq (1.9) becomes $\frac{ig_{\mu\nu}}{(Mc)^2}$.

For each and every propagator, an additional factor $\frac{d^4 q}{(2\pi)^4}$ needs to be included, which should then be followed by the integration over all internal momenta.

Using the Feynman rules above, the probability amplitudes $\mathcal{M}$ in Eq (1.4) can be obtained. From these computed probability amplitudes, the cross-sections of these interactions can then be obtained. Cross sections are important observables that experiments aim to measure so as to better understand the processes underlying these interactions.

They can be expressed into two main forms, one being the differential cross section $\frac{d\sigma}{d\Omega}$ and the other being the total cross section σ_{total} . Here, the total cross section σ_{total} can be obtained by integrating the differential cross section over all solid angles $d\Omega$. The relationship between the probability of interaction (Eq (1.4)) and the cross section, $\frac{d\sigma}{d\Omega}$, can be expressed in terms of a general formula as shown below:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi} \right)^2 \left[\frac{\mathcal{S} \langle |\mathcal{M}|^2 \rangle}{(E_{total_incoming})^2} \right] \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} \quad (1.10)$$

where $\mathcal{S}$ refers to the combined spin configuration of all particles involved in the interaction, $E_{total_incoming} = E_1 + E_2 + E_3 + \dots$ refers to the sum of all incoming particles' energies, $|\mathbf{p}_f|$ and $|\mathbf{p}_i|$ refer to the magnitudes of any of the outgoing (final) and incoming (initial) momenta respectively. Note that Eq (1.10) is just a general formula and may not be applicable for some interactions.

To put what was discussed above into perspective, let's look at a simple and standard example in QED, the electron-muon scattering.

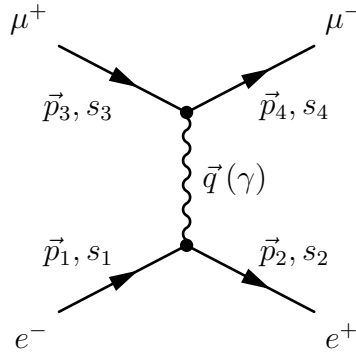


Figure 1.7: Electron-Muon scattering mediated by a photon γ

Using the Feynman rules mentioned above, labelling the fermions, e^- and μ^- , as $u(1)$ ²¹ and $\bar{u}(4)$ ²¹, and the anti-fermions, e^+ and μ^+ as $\bar{v}(2)$ ²² and $v(3)$ ²², the following equation is obtained:

$$(2\pi)^4 \int [u(1)(ig_e\gamma^\mu\bar{v}(2)) \left(-\frac{ig_{\mu\nu}}{q^2} \right) [v(3)(ig_e\gamma^\nu)\bar{u}(4)] \times \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{q})\delta^4(\vec{q} - \vec{p}_3 - \vec{p}_4) \quad (1.11)$$

Extracting the probability amplitude $\mathcal{M}_\gamma$ and letting $q = p_1 + p_2$,

$$\mathcal{M}_\gamma = -\frac{g_e^2}{(p_1 + p_2)^2} [u(1)\gamma^\mu\bar{v}(2)] [v(3)\gamma_\mu\bar{u}(4)] \quad (1.12)$$

²¹ The convention used here: An incoming fermion is labelled as u while an outgoing fermion is labelled as $\bar{u}$

²² The convention for anti-fermions is exactly the opposite of that for fermions: An incoming anti-fermion (pointing outwards from the vertex) is labelled as $\bar{v}$ while an outgoing anti-fermion (pointing inwards into the vertex) is labelled as v

Taking the absolute square of the probability amplitude $\mathcal{M}_\gamma$,

$$|\mathcal{M}_\gamma|^2 = \frac{g_e^4}{(p_1 + p_2)^4} [u(1)\gamma^\mu \bar{v}(2)] [v(3)\gamma_\mu \bar{u}(4)] [u(1)\gamma^\mu \bar{v}(2)]^* [v(3)\gamma_\mu \bar{u}(4)]^* \quad (1.13)$$

Extracting the first and third terms from Eq (1.13), naming the factor G_1 , and relabelling γ^μ and γ_μ as Γ_1 and Γ_2 ,

$$G_1 = [u(1)\Gamma_1 \bar{v}(2)] [u(1)\Gamma_2 \bar{v}(2)]^* = [u(1)\Gamma_1 \bar{v}(2)] [u(1)\Gamma_2 v(2)^\dagger \gamma^0]^* \quad (1.14)$$

After re-arranging the terms, the following expression is obtained

$$G_1 = [u(1)\Gamma_1 \bar{v}(2)] [v(2)\bar{\Gamma}_2 \bar{u}(1)] \quad (1.15)$$

Summing G_1 over all spins (s_1 and s_2) and using “Casimir’s trick”²³,

$$\sum_{s_1, s_2} G_1 = \text{Tr}\{\Gamma_1(\not{p}_2 - m_2 c) \bar{\Gamma}_2(\not{p}_1 + m_1 c)\} \quad (1.16)$$

Applying the above procedure from Eq (1.14) to Eq (1.16) to the second and fourth terms in Eq (1.13), the final expression obtained is shown below

$$\sum_{s_3, s_4} G_2 = \text{Tr}\{\Gamma_3(\not{p}_4 + m_4 c) \bar{\Gamma}_4(\not{p}_3 - m_3 c)\} \quad (1.17)$$

Combining Eqs (1.16) and (1.17), $\Gamma_1 \leftrightarrow \gamma^\mu$, $\bar{\Gamma}_2 \leftrightarrow \gamma^\nu$, $\Gamma_3 \leftrightarrow \gamma_\mu$, $\bar{\Gamma}_4 \leftrightarrow \gamma_\nu$, inserting all these back into Eq (1.13), and averaging over all possible spin orientations, the following equation is obtained

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{g_e^4}{4(p_1 + p_2)^4} \text{Tr}\{\gamma^\mu(\not{p}_2 - m_e c) \gamma^\nu(\not{p}_1 + m_e c)\} \\ &\quad \times \text{Tr}\{\gamma_\mu(\not{p}_4 + m_\mu c) \gamma_\nu(\not{p}_3 - m_\mu c)\} \end{aligned} \quad (1.18)$$

Using the relevant trace theorems²⁴ and the relevant trace operations, the first trace, *i.e.* $\text{Tr}\{\gamma^\mu(\not{p}_2 - m_e c) \gamma^\nu(\not{p}_1 + m_e c)\}$, can be re-expressed in the following form

$$\begin{aligned} \text{Tr}\{\gamma^\mu(\not{p}_2 - m_e c) \gamma^\nu(\not{p}_1 + m_e c)\} &= \text{Tr}\{\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1\} \\ &\quad + m_e c [\text{Tr}\{\gamma^\mu \not{p}_2 \gamma^\nu\} - \text{Tr}\{\gamma^\mu \gamma^\nu \not{p}_1\}] \\ &\quad - (m_e c)^2 \text{Tr}\{\gamma^\mu \gamma^\nu\} \\ &= 4[p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu} \{(m_e c)^2 + (p_2 \cdot p_1)\}] \end{aligned} \quad (1.19)$$

In a similar fashion, replacing the upper indices with lower indices, m_e with m_μ , $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$, the second trace, *i.e.* $\text{Tr}\{\gamma_\mu(\not{p}_4 + m_\mu c) \gamma_\nu(\not{p}_3 - m_\mu c)\}$, can be re-written as

²³ For the details on this mathematical procedure, please refer to [Electron-Muon Scattering Cross Sections](#) in Appendix A

²⁴ For explicit details of the trace theorems, please refer to [Trace Theorems](#) in Appendix A

the following expression

$$\text{Tr}\{\gamma_\mu(\not{p}_4 + m_\mu c)\gamma_\nu(\not{p}_3 - m_\mu c)\} = 4[p_{4\mu}p_{3\nu} + p_{3\mu}p_{4\nu} - g_{\mu\nu}\{(m_\mu c)^2 + (p_4 \cdot p_3)\}] \quad (1.20)$$

Re-expressing Eq (1.18) in terms of expressions (1.19) and (1.20) above,

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{4g_e^4}{(p_1 + p_2)^4} [p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}\{(m_e c)^2 + (p_2 \cdot p_1)\}] \\ &\quad \times [p_{4\mu}p_{3\nu} + p_{3\mu}p_{4\nu} - g_{\mu\nu}\{(m_\mu c)^2 + (p_4 \cdot p_3)\}] \end{aligned} \quad (1.21)$$

After expanding and simplifying the above expression, and using the fact that $p^\mu p_\mu = p^\nu p_\nu = p \cdot p$,

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{8g_e^4}{(p_1 + p_2)^4} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \\ &\quad + (m_e c)^2(p_3 \cdot p_4) + (m_\mu c)^2(p_1 \cdot p_2) + 2(m_e m_\mu c^2)^2] \end{aligned} \quad (1.22)$$

Expressing the momenta in the centre-of-mass (CM) frame in terms of their four-momenta, $p_1 = (\frac{E_e}{c}, \vec{p}_1)$; $p_2 = (\frac{E_e}{c}, -\vec{p}_1)$; $p_3 = (\frac{E_\mu}{c}, \vec{p}_3)$; $p_4 = (\frac{E_\mu}{c}, -\vec{p}_3)$, the first 2 terms in Eq (1.22) becomes

$$\begin{aligned} (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) &= 2 \left[\left(\frac{E_e}{c} \right)^2 \left(\frac{E_\mu}{c} \right)^2 + (\vec{p}_1 \cdot \vec{p}_3)^2 \right] \\ &= 2 \left[\left(\frac{E_e}{c} \right)^2 \left(\frac{E_\mu}{c} \right)^2 + |\vec{p}_1|^2 |\vec{p}_3|^2 \cos^2 \theta \right] \end{aligned} \quad (1.23)$$

In the high energy limit, the following approximations can be made: $E_e \approx E_\mu \approx E \approx pc$, $m_e c \approx m_\mu c \approx 0$, $\vec{p}_1 \approx \vec{p}_3$, $|\vec{p}_1| \approx |\vec{p}_3| \approx \frac{E}{c}$. Using these approximations, the last three terms in the second line of Eq (1.22) can be dropped and substituting the expression in Eq (1.23) back into Eq (1.22) will result in the following expression:

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{8g_e^4}{(2E)^4} \left\{ 2 \left[\left(\frac{E}{c} \right)^4 + \left(\frac{E}{c} \right)^4 \cos^2 \theta \right] \right\} \\ &= g_e^4 (1 + \cos^2 \theta) \\ &= (\sqrt{4\pi\alpha_{\text{EM}}})^4 (1 + \cos^2 \theta) \end{aligned} \quad (1.24)$$

Substituting Eq (1.24) into Eq (1.10), and since $|\mathbf{p}_i| = |\mathbf{p}_f|$ ($|\vec{p}_1| = |\vec{p}_3|$) and Eq (1.24) has already accounted for the spin configuration of the system, the resultant expression for the differential cross section is

$$\left(\frac{d\sigma}{d\Omega} \right)_\gamma = \left(\frac{\hbar c}{8\pi} \right)^2 \left[\frac{(\sqrt{4\pi\alpha})^4 (1 + \cos^2 \theta)}{(2E)^2} \right] = \frac{\hbar^2 c^2 \alpha^2}{16E^2} (1 + \cos^2 \theta) \quad (1.25)$$

Integrating over all solid angles $d\Omega$, the expression for the total cross section σ can be obtained

$$\begin{aligned}\sigma_\gamma &= \int \left[\frac{\hbar^2 c^2 \alpha^2}{16E^2} (1 + \cos^2 \theta) \right] d\Omega \\ &= \left(\frac{\hbar c \alpha}{4E} \right)^2 \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta (1 + \cos^2 \theta) = \frac{\pi}{3} \left[\frac{\hbar c \alpha}{E} \right]^2\end{aligned}\tag{1.26}$$

The above calculation for the cross section of the electron-muon scattering skips much of the mathematical details and procedures. For the detailed step-by-step derivation, the reader should refer to [Appendix A](#).

Chapter 2

Weak Interactions

In the previous chapter, we have discussed the three fundamental forces, namely the strong, weak and electromagnetic interactions, together with the fundamental particles in the Standard Model shown in Figure 1.1. We have also introduced different mathematical representations of the Standard Model, namely its representation in Group theory and the Lagrangian formulation. Furthermore, we have also discussed how the different interactions in the Standard Model Lagrangian can be expressed pictorially using Feynman diagrams and how Feynman diagrams describe the mathematics behind these interactions.

For this chapter, we will focus on one of the three fundamental interactions in the Standard Model, the *weak interaction*. As discussed in the section on the [Standard Model \(SM\)](#), we have seen that the weak interactions are mediated by the massive intermediate vector bosons, $W^\pm$ and Z^0 . Note that there is a key difference between the weak interaction and the other 2 fundamental interactions, namely the strong and electromagnetic (EM) interactions. For both the strong and EM interactions, their mediators (photons and gluons) are massless, as compared to very heavy gauge mediator bosons of the weak force. As a result of the internal symmetries and local gauge invariance in the Standard Model, all mediator gauge bosons in the Standard Model are expected to be massless. However, this is not the case for the $W^\pm$ and Z^0 bosons, thus implying the presence of a mechanism that leads to the breaking of this symmetry or gauge invariance which allows the $W^\pm$ and Z^0 bosons to acquire mass. In fact, this mechanism is called the Higgs Mechanism²⁵, developed upon the idea of spontaneous symmetry breaking. The Higgs Mechanism and the process of how spontaneous symmetry breaking gives rise to the mass of the $W^\pm$ and Z^0 bosons will be discussed more in depth in the following sections, [Higgs Mechanism](#) and [Spontaneous Symmetry Breaking \(SSB\)](#).

But first we will have a brief discussion about weak interactions, including a calculation of the cross section of the electron-muon scattering process mediated by the Z^0 boson instead of photon²⁶.

As previously mentioned in an earlier paragraph, the $W^\pm$ and Z^0 bosons mediate the weak

²⁵ Named after the 2013 Physics Nobel Laureate Peter Higgs who developed the theory behind the mechanism

²⁶This was done in the last part of the previous section on [Feynman Calculus](#)

interactions and are extremely heavy compared to the fermions in the Standard Model²⁷, excluding the top quark. Since the $W^\pm$ and Z^0 bosons are very heavy compared to most other fundamental particles in the Standard Model, they have short lifetimes, which can be considered as negligible. However, as seen in Eq (1.9), when the sum of energies from all incoming particles is very close to the rest mass energy of the Z^0 boson ($M_Z c^2$), the expression for the propagator blows up to infinity. In this scenario, the lifetime of the Z^0 boson becomes significant and the decay width Γ_Z of the Z^0 boson, the inverse of the Z^0 's lifetime τ_Z , needs to be accounted for in the propagator. Hence, near the rest mass energy of the Z^0 boson²⁸, the expression in (1.9) becomes [4]:

$$-\frac{i(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2})}{q^2 - (M_Z c)^2} \Rightarrow \frac{i(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2})}{q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z} \quad (2.1)$$

Since the expression for the propagator is a factor in the probability amplitude $\mathcal{M}$, the contribution to the cross section from the weak interaction dominates over the contributions from all other interactions near the Z^0 pole. This is especially significant for charged leptons which experience only the electromagnetic (EM) and weak interactions. As an example, we will have a look at the electron-muon scattering process in Figure 1.7. At the Z^0 pole, the cross section for the process if it was mediated by a Z^0 boson would be almost 167 times²⁹ of that for the same process if it was mediated by a photon γ . If mixing of the electromagnetic and weak interactions is considered, the mixing terms, $\mathcal{M}_\gamma \mathcal{M}_Z^*$ and $\mathcal{M}_Z \mathcal{M}_\gamma^*$, contributes to the absolute square of the sum of the probability amplitudes of processes mediated by the photon γ and the Z^0 boson separately, $|\mathcal{M}_\gamma|^2$ and $|\mathcal{M}_Z|^2$. In other words,

$$\langle |\mathcal{M}|^2 \rangle = |\mathcal{M}_\gamma + \mathcal{M}_Z|^2 = |\mathcal{M}_\gamma|^2 + |\mathcal{M}_Z|^2 + (\mathcal{M}_\gamma \mathcal{M}_Z^* + \mathcal{M}_Z \mathcal{M}_\gamma^*) \quad (2.2)$$

Since $\langle |\mathcal{M}|^2 \rangle$ is a factor in Eq (1.10) for the calculation of the cross section, mixing of the electromagnetic and weak interactions will enhance the cross section of the process even further³⁰, as compared to that if it was mediated by a photon γ only. The calculation for the mixing contribution to the cross section of the electron-muon scattering process will be discussed in the [Electron-Muon Scattering Cross Sections](#) section of Appendix A.

In 1956, a famous experiment conducted by C.S.Wu was the first experiment to show definitive evidence of parity violation in weak interactions [5]. Prior to this experiment, it was expected that all the 3 fundamental interactions in the Standard Model conserves parity, in other words mirror symmetry or parity invariance. Parity invariance is a property of axial-vectors, also known as pseudovectors, whose sign remains unchanged under parity transformation, or in other words when acted upon by a parity operator $\hat{P}$. How-

²⁷ As seen in Figure 1.1

²⁸ Instead of calling it the rest mass energy of the Z^0 boson, we shall call it the Z^0 pole from now onwards

²⁹ This figure is obtained using a formula that will be derived later in the chapter

³⁰ Up to 1000 times more

ever, vectors are not invariant under parity transformations where their signs flip when acted upon by $\hat{P}$. Mathematically: i) $\hat{P}(\vec{a}) = \vec{a}$ (for an axial-vector); ii) $\hat{P}(\vec{v}) = -\vec{v}$ (for a vector) [4] As seen in the expressions in Eqs (1.5) and (1.6), the vertex factors for the $W^\pm$ and Z^0 vertices contain a vector (γ^μ) component and an axial-vector ($\gamma^\mu\gamma^5$) component³¹. As a result of this mixing of the vector V and axial-vector couplings in the weak vertex factor, parity is not conserved in weak interactions. In the case of Z_0 vertex, in addition to the vector γ^μ and the axial-vector $\gamma^\mu\gamma^5$ couplings, each of the 2 couplings has an associated coefficient, explicitly c_V^f for the vector coupling and c_A^f for the axial-vector coupling. As mentioned in the previous section, the coefficients depend on the species of the fermion involved in the interaction, which have been neatly categorised in the table below:

fermion f	c_V	c_A
ℓ^- (Charged leptons), where $\ell = e, \mu, \tau$	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
ν_ℓ (Neutrinos), where $\ell = e, \mu, \tau$	$\frac{1}{2}$	$\frac{1}{2}$
u, c, t (Up-type quarks)	$\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d, s, b (Down-type quarks)	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

Table 2.1: Vector c_V and axial-vector c_A couplings for the different species of fermions f

The θ_w in the vector coupling column c_V refers to the weak mixing angle, but the weak mixing angle by itself is not very useful. However, when acted upon by a trigonometric function, the weak mixing angle directly relates the weak coupling constant g_w to the electromagnetic (EM) coupling constant g_e , as shown in Eq (1.5). In particular, $\sin^2\theta_w$ has a value of 0.23122 and the value of $\cos^2\theta_w$ can be obtained easily using the following trigonometric relation: $\cos^2\theta_w = 1 - \sin^2\theta_w$. Taking the square root of the value of $\cos^2\theta_w$, the ratio of the mass of the $W^\pm$ boson (M_W) to the mass of the neutral Z^0 boson (M_Z) is obtained. In other words,

$$\cos\theta_w = \frac{M_W}{M_Z} = \sqrt{1 - \sin^2\theta_w} \quad (2.3)$$

Another important implication of parity violation in weak interactions is the emergence of chiral fermion states, or fields, in the electroweak theory. As a result of this non-conservation of parity (mirror symmetry), a “*left-handed*” fermion field, represented by the “*left-handed*” spinor u_L , is different from a “*right-handed*” fermion field, represented by the “*right-handed*” spinor u_R . A “*left-handed*” fermion ψ_L and a “*right-handed*” fermion ψ_R are defined as the following

$$\psi_L = \frac{1 - \gamma^5}{2}\psi; \psi_R = \frac{1 + \gamma^5}{2}\psi, \text{ where } \psi \text{ represents the fermion field} \quad (2.4)$$

³¹ Compared to a QED vertex: $ig_e\gamma^\mu$ which is purely vectorial

“*Right-handed*” fermion fields are represented as singlets, namely u_R, c_R, t_R (“up-type” quarks); d_R, s_R, b_R (“down-type” quarks); and ℓ^- (charged leptons) where $\ell = e, \mu$ and τ , while “*left-handed*” fermion states are represented as doublets [6], explicitly

$$\begin{aligned} \ell_L &= \begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L, \text{ where } \ell = e, \mu, \tau \\ q_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \end{aligned} \quad (2.5)$$

As previously mentioned in Eq (1.2), the third component of the weak isospin T^3 for a “*left-handed*” fermion field differs from that for a “*right-handed*” fermion field. Although T^3 in Eq (1.2) is in the form of a 2×2 matrix, we are only interested in its eigenvalues, which are 1 and -1 . Combining the eigenvalues with the electric charge Q for the different fermion species and using the relation in Eq (1.2), one can easily obtain the weak hypercharge for the different “*left-handed*” fermion fields. The same can be done for the “*right-handed*” case, except the only difference is that $T^3 = 0$ for the “*right-handed*” fermion fields.

$$Q = T^3 + \frac{1}{2}Y \quad (1.2)$$

$$\begin{aligned} &\left. \begin{aligned} Q = 0, T^3 = \frac{1}{2} &\quad (\text{Neutrinos } \nu_\ell) \\ Q = -1, T^3 = -\frac{1}{2} &\quad (\text{Charged leptons } \ell^-) \end{aligned} \right\} Y = -1 \text{ (“Left-handed” leptons } \ell_L) \\ &Q = -1, T^3 = 0 \implies Y = -2 \text{ (“Right-handed” charged leptons } \ell^-) \\ &\left. \begin{aligned} Q = \frac{2}{3}, T^3 = \frac{1}{2} &\quad (\text{“Up-type” quarks}) \\ Q = -\frac{1}{3}, T^3 = -\frac{1}{2} &\quad (\text{“Down-type” quarks}) \end{aligned} \right\} Y = \frac{1}{3} \text{ (“Left-handed” quarks } q_L) \\ &Q = \frac{2}{3}, T^3 = 0 \implies Y = \frac{2}{3} \quad (\text{“Right-handed up-type” quarks}) \\ &Q = -\frac{1}{3}, T^3 = 0 \implies Y = -\frac{1}{3} \text{ (“Right-handed down-type” quarks)} \end{aligned} \quad (2.6)$$

In the electroweak theory, the weak isospin T is coupled to the three gauge bosons W^i , where $i = 1, 2, 3$, in the $SU(2)$ gauge group while the weak hypercharge Y is coupled to the B gauge boson in the $U(1)$ gauge group. In total, the electroweak theory contains these 4 gauge bosons, but they are merely mathematical representations and do not have any physical significance at the moment. However, the mixing of these gauge fields produces the 4 physical gauge bosons in the electroweak theory, namely the $W^\pm$ gauge bosons from the mixing of the first two W^i (W^1 and W^2) gauge fields on one hand, and the photon γ and the Z^0 boson from the mixing of the W^3 and B gauge fields on the other. This mixing of gauge fields to produce the physical gauge bosons will be discussed in greater

detail in the following section on the [Higgs Mechanism](#). In addition, the subsequent process in which the weak intermediate vector bosons, $W^\pm$ and Z^0 , acquire mass from spontaneous breaking of the electroweak symmetry will also be discussed in a later section on [Spontaneous Symmetry Breaking \(SSB\)](#).

Before proceeding on to the following sections, we will first calculate the cross section for the electron-muon scattering process, as shown in Figure 1.7, but now with the Z^0 boson as the mediator instead of a photon γ , which was done in the previous section.

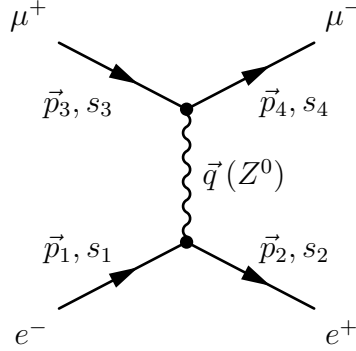


Figure 2.1: Electron-Muon scattering mediated by a Z^0 boson

Using the Feynman rules for the weak interactions discussed in the previous chapter and the same notations for the fermions, e^- and μ^- , and the anti-fermions, e^+ and μ^+ as that in the photon case, one can re-express the above Feynman diagram in the expression below

$$(2\pi)^4 \int \left[u(1) \left(-\frac{ig_z}{2} \right) \gamma^\mu (c_V^e - c_A^e \gamma^5) \bar{v}(2) \right] \left(\frac{i \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2} \right)}{q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z} \right) \\ \times \left[v(3) \left(-\frac{ig_z}{2} \right) \gamma^\nu (c_V^\mu - c_A^\mu \gamma^5) \bar{u}(4) \right] \times \delta^4(\vec{p}_1 + \vec{p}_2 - \vec{q}) \delta^4(\vec{q} - \vec{p}_3 - \vec{p}_4) \quad (2.7)$$

Note that the superscript μ in the $(c_V^\mu - c_A^\mu \gamma^5)$ is used to identify the upper Z^0 vertex in Figure 2.1 which shows the interaction involving the Z^0 propagator, the incoming and outgoing muons μ .

Extracting the probability amplitude $\mathcal{M}_Z$ from Eq (2.7) and letting $q = p_1 + p_2 = p_3 + p_4$,

$$\mathcal{M}_Z = \frac{g_z^2}{4(q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z)} \left[u(1) \gamma^\mu (c_V^e - c_A^e \gamma^5) \bar{v}(2) \right] \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2} \right) \\ \times \left[v(3) \gamma^\nu (c_V^\mu - c_A^\mu \gamma^5) \bar{u}(4) \right] \quad (2.8)$$

In the high-energy limit, $q^2 \gg (m_\mu c)^2 \gg (m_e c)^2 \implies m_\mu c = m_e c = 0$. Noting that since $\not{q} = \gamma^\mu q_\mu$ (using the Feynman-slash notation) and $\not{q} = \not{p}_1 + \not{p}_2$, $u(1) \not{q} (c_V^e - c_A^e \gamma^5) \bar{v}(2) = 0$. As a result, the $\frac{q_\mu q_\nu}{(M_Z c)^2}$ term in the $(g_{\mu\nu} - \frac{q_\mu q_\nu}{(M_Z c)^2})$ factor vanishes, leaving only the $g_{\mu\nu}$ term in the “sandwiched” factor. In addition, the vector c_V^ℓ and axial-vector c_A^ℓ couplings for all

charged leptons ℓ^- are the same, as shown in Table 2.1, in other words $c_V^e = c_V^\mu = c_V$ and $c_A^e = c_A^\mu = c_A$. Taking all these into considerations, Eq (2.8) simplifies to the following equation:

$$\mathcal{M}_Z = \frac{g_z^2}{4(q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z)} [u(1)\gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)] \times [v(3)\gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)] \quad (2.9)$$

Taking the absolute square of the probability amplitude $\mathcal{M}_Z$,

$$|\mathcal{M}_Z|^2 = \frac{g_z^4}{16[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \times [u(1)\gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)] [u(1)\gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)]^* \times [v(3)\gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)] [v(3)\gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)]^* \quad (2.10)$$

Using ‘‘Casimir’s trick’’²³,

$$\langle |\mathcal{M}_Z|^2 \rangle = \frac{g_z^4}{4\{16[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]\}} \times \text{Tr}\{\gamma^\mu (c_V - c_A \gamma^5) \not{p}_2 \gamma^\nu (c_V - c_A \gamma^5) \not{p}_1\} \times \text{Tr}\{\gamma_\mu (c_V - c_A \gamma^5) \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3\} \quad (2.11)$$

Using the relevant trace theorems in Appendix A and the relevant trace operations, the first trace, *i.e.* $\text{Tr}\{\gamma^\mu (c_V - c_A \gamma^5) \not{p}_2 \gamma^\nu (c_V - c_A \gamma^5) \not{p}_1\}$, can be re-written in the following manner

$$\begin{aligned} \text{Tr}\{\gamma^\mu (c_V - c_A \gamma^5) \not{p}_2 \gamma^\nu (c_V - c_A \gamma^5) \not{p}_1\} &= \text{Tr}\{\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1 (c_V - c_A \gamma^5)^2\} \\ &= \text{Tr}\left\{\gamma^\mu \gamma^\lambda (p_2)_\lambda \gamma^\nu \gamma^\sigma (p_1)_\sigma [c_V^2 - 2c_V c_A \gamma^5 + c_A^2 (\gamma^5)^2]\right\} \\ &\quad \text{since } (\gamma^5)^2 = \mathbb{1} \text{ (the identity matrix),} \\ &= (c_V^2 + c_A^2) \text{Tr}\{\gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma\} (p_2)_\lambda (p_1)_\sigma - 2c_V c_A \text{Tr}\{\gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma \gamma^5\} (p_2)_\lambda (p_1)_\sigma \\ &= 4\{(c_V^2 + c_A^2)[p_2^\mu p_1^\nu - g^{\mu\nu}(p_2 \cdot p_1) + p_1^\mu p_2^\nu] - 2i c_V c_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda}\} \end{aligned} \quad (2.12)$$

Similarly for the second trace, *i.e.* $\text{Tr}\{\gamma_\mu (c_V - c_A \gamma^5) \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3\}$, by replacing the upper indices with lower indices and interchanging the indices of the momenta ($1 \leftrightarrow 3$ and $2 \leftrightarrow 4$), it can be re-written in the following form

$$\begin{aligned} \text{Tr}\{\gamma_\mu (c_V - c_A \gamma^5) \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3\} \\ = 4\{(c_V^2 + c_A^2)[p_{4\mu} p_{3\nu} - g_{\mu\nu}(p_4 \cdot p_3) + p_{3\mu} p_{4\nu}] - 2i c_V c_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau\} \end{aligned} \quad (2.13)$$

Combining Eqs (2.12) and (2.13) and substituting them back into Eq (2.11), one will be

able to obtain the following expression

$$\begin{aligned} \langle |\mathcal{M}_Z|^2 \rangle &= \frac{g_z^4}{4[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \\ &\times \left\{ (c_V^2 + c_A^2)[p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)] - 2i c_V c_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda} \right\} \\ &\times \left\{ (c_V^2 + c_A^2)[p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g_{\mu\nu}(p_4 \cdot p_3)] - 2i c_V c_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau \right\} \end{aligned} \quad (2.14)$$

After expanding and simplifying the above expression, and using the following facts, $p^\mu p_\mu = p^\nu p_\nu = p \cdot p$, $\epsilon^{\mu\nu\sigma\lambda} \epsilon_{\mu\nu\kappa\tau} = -2(\delta_\kappa^\sigma \delta_\tau^\lambda - \delta_\tau^\sigma \delta_\kappa^\lambda)$, the above expression becomes

$$\begin{aligned} \langle |\mathcal{M}_Z|^2 \rangle &= \frac{1}{2} \left\{ \frac{g_z^4}{[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \right\} \\ &\times \left\{ (c_V^2 + c_A^2)^2 [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)] \right. \\ &\quad \left. + 4c_V^2 c_A^2 [(p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)] \right\} \end{aligned} \quad (2.15)$$

Expressing the momenta in the CM frames in terms of their four momenta, one will be able to obtain the following relations

$$\begin{aligned} (p_1 \cdot p_3)(p_2 \cdot p_4) &= \left[\left(\frac{E}{c} \right)^2 - (\vec{p}_1 \cdot \vec{p}_3) \right]^2 \\ (p_1 \cdot p_4)(p_2 \cdot p_3) &= \left[\left(\frac{E}{c} \right)^2 + (\vec{p}_1 \cdot \vec{p}_3) \right]^2 \end{aligned} \quad (2.16)$$

Using the relations above, in Eq (1.23) and the first line of Eq (1.24), the terms in the second curly brackets, excluding the pre-factors, in Eq (2.15) can be re-expressed in the following fashion

$$\begin{aligned} (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) &= 2 \left(\frac{E}{c} \right)^4 (1 + \cos^2 \theta) \\ (p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3) &= -4 \left(\frac{E}{c} \right)^4 \cos \theta \end{aligned} \quad (2.17)$$

Substituting both expressions above back into Eq (2.15) and letting $q = \frac{2E}{c}$ ³², Eq (2.15) becomes

$$\begin{aligned} \langle |\mathcal{M}_Z|^2 \rangle &= \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left\{ (c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] \right. \\ &\quad \left. - 8c_V^2 c_A^2 \cos \theta \right\} \end{aligned} \quad (2.18)$$

Substituting the above expression into Eq (1.10), and since the magnitude of the incoming and outgoing momenta are equal ($|\vec{p}_1| = |\vec{p}_3|$) and the spin configuration of the system has been already been accounted, one will obtain the differential cross-section for the

³² Using the approximations that were used to obtain Eq (1.24)

scattering process as shown below

$$\begin{aligned}
\left(\frac{d\sigma}{d\Omega}\right)_Z &= \left(\frac{\hbar c}{8\pi}\right)^2 \left(\frac{1}{2E}\right)^2 \left[\frac{g_z^4 E^4}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\
&\quad \times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \\
&= \left(\frac{\hbar c}{16\pi}\right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\
&\quad \times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\}
\end{aligned} \tag{2.19}$$

Integrating over all solid angles $d\Omega$, one will obtain the following expression for the total cross section σ

$$\begin{aligned}
\sigma_Z &= \int \left(\frac{\hbar c}{16\pi}\right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\
&\quad \times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} d\Omega \\
&= \left(\frac{\hbar c}{16\pi}\right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \\
&\quad \times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \\
&= \frac{[\hbar c E g_z^2 (c_V^2 + c_A^2)]^2}{48\pi \{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\}}
\end{aligned} \tag{2.20}$$

Expressing the neutral coupling constant g_z in terms of the EM coupling constant g_e using Eq (1.5) and Eq (1.6),

$$g_z = \frac{g_e}{\sin \theta_w \cos \theta_w} = \frac{\sqrt{4\pi\alpha_{\text{EM}}}}{\sin \theta_w \cos \theta_w} \implies g_z^4 = \frac{(4\pi\alpha_{\text{EM}})^2}{(\sin \theta_w \cos \theta_w)^4} \tag{2.21}$$

Substituting the vector c_V and axial-vector c_A couplings for the charged leptons from Table 2.1 and the expression above into Eq (2.20),

$$\sigma_Z = \frac{\pi}{3} \left[\frac{(\hbar c \alpha E)^2}{\{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\}} \right] \left[\frac{\frac{1}{2} - 2 \sin^2 \theta_w + 4 \sin^4 \theta_w}{\sin^2 \theta_w \cos^2 \theta_w} \right]^2 \tag{2.22}$$

Taking the ratio of the total cross section of the scattering process if it was mediated by a Z^0 boson, σ_Z , to that of the same scattering process if it was mediated by a photon γ , σ_γ ,

$$\frac{\sigma_Z}{\sigma_\gamma} = \frac{E^4 \left(\frac{1}{2} - 2 \sin^2 \theta_w + 4 \sin^4 \theta_w\right)^2}{\{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\} (\sin^2 \theta_w \cos^2 \theta_w)^2} \tag{2.23}$$

At the Z^0 pole (using the terminology that was used earlier in the chapter), $2E =$

$M_Z c^2 (E = \frac{1}{2} M_Z c^2) \implies [(2E)^2 - (M_Z c^2)^2] = 0$. Substituting the following values for the mass of the Z^0 boson $M_Z = 91.1876$ GeV, the decay width of the Z^0 boson $\Gamma_Z = 2.4952$ GeV, $\sin^2 \theta_w = 0.23122$ and $\cos^2 \theta_w = 1 - \sin^2 \theta_w$ into the above expression, and using natural units ($\hbar = 1, c = 1$),

$$\frac{\sigma_Z}{\sigma_\gamma} = \left[\frac{(M_Z c^2) \left(\frac{1}{2} - 2 \sin^2 \theta_w + 4 \sin^4 \theta_w \right)}{4(\hbar \Gamma_Z)(\sin^2 \theta_w \cos^2 \theta_w)} \right]^2 \approx 167 \quad (2.24)$$

Same as the cross section calculations for the electron-muon scattering process done in the previous section, the reader should refer to Appendix A for the full step-by-step derivation of the cross section calculation done above for the same process, including the cross section calculation that considers the mixing of the electromagnetic and weak interactions.

2.1 Higgs Mechanism

As mentioned earlier in the chapter, the Higgs Mechanism, followed by spontaneous breaking of the electroweak symmetry, is the crucial ingredient in the electroweak sector of the Standard Model in explaining why fermions, the $W^\pm$ and Z^0 bosons have mass while photons do not. Referring back to the simplified Lagrangian of the Standard Model (Eq (1.3)), the first term of the Lagrangian, *i.e.* $\mathcal{L}_{kin} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$, is the kinetic term, which is invariant under the following gauge transformation $A_\mu \rightarrow A_\mu + \partial_\mu \lambda$ irregardless of the choice of λ . However, adding a mass term, let's say $\frac{1}{2} m^2 A_\mu A^\mu$, in an attempt to transform the kinetic term to a form that is similar to that of the Klein-Gordon Lagrangian for a complex scalar field, $\mathcal{L}_{KG} = \frac{1}{2} (\partial_\mu \phi^\dagger) (\partial^\mu \phi) - \frac{1}{2} m^2 \phi^\dagger \phi$ (in natural units $\hbar = c = 1$), *i.e.*

$$\mathcal{L}_{kin} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \quad (2.25)$$

will result in a term that is no longer invariant under the above gauge transformation. In other words, the mass term “destroys” the $U(1)$ gauge symmetry, hence this enforces the fact that the photon needs to be massless under $U(1)$ gauge symmetry [6].

A plausible solution to this problem, instead of adding an explicit mass term, *i.e.* $\frac{1}{2} m^2 A_\mu A^\mu$, is to pair it up with the Higgs term $\mathcal{L}_{Higgs}$ in Standard Model Lagrangian. In other words,

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi) \quad (2.26)$$

where $D_\mu = \partial_\mu + ieA_\mu$ is known as the covariant derivative, where e is the electronic charge, and the $|D_\mu \phi|^2 = (D_\mu \phi)^\dagger (D^\mu \phi)$ term describes the coupling between the Higgs field ϕ and the massless gauge field A_μ . The last term in the expression above is the Higgs potential $V(\phi) = -\frac{1}{2} \mu^2 (\phi^\dagger \phi) + \frac{1}{4} \lambda (\phi^\dagger \phi)^2$, as mentioned in the description below Eq (1.3). It can be deduced rather easily that this new Lagrangian is invariant under the

gauge transformation of the gauge A_μ and Higgs ϕ fields, as shown below

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \lambda, \\ \phi &\rightarrow e^{-ie\lambda} \phi \end{aligned} \quad (2.27)$$

Before we start analysing this new Lagrangian in greater detail, we first discuss about the Higgs potential $V(\phi) = -\frac{1}{2}\mu^2(\phi^\dagger\phi) + \frac{1}{4}\lambda(\phi^\dagger\phi)^2$. Differentiating the Higgs potential $V(\phi)$ with respect to $(\phi^\dagger\phi)$ and equating the derivative $\frac{\partial V}{\partial(\phi^\dagger\phi)}$ to 0, one will observe that the minimum points of the potential are located at $\phi^\dagger\phi = \pm\sqrt{\frac{\mu^2}{\lambda}} = \pm\frac{\mu}{\sqrt{\lambda}}$. If $\mu^2 < 0$, *i.e.* $\phi^\dagger\phi$ acquires an imaginary value and hence the potential is no longer considered as a physical potential, the minimum point will be shifted to $\phi = 0$. However, in reality, in the case when $\mu^2 > 0$, *i.e.* $\phi^\dagger\phi$ is real, both minimum points of the Higgs potential lie equally far away from the vertical $V(\phi)$ axis along the complex plane³³. Note that the potentials for both minimum points are negative. Since the potential at the origin is 0 and the potentials for both minimum points are negative, the origin becomes a local maximum point between the two minimum points on opposite sides of the the vertical $V(\phi)$ axis. This results in the Higgs potential having a Mexican hat profile, as shown in the 3-Dimensional plot below:

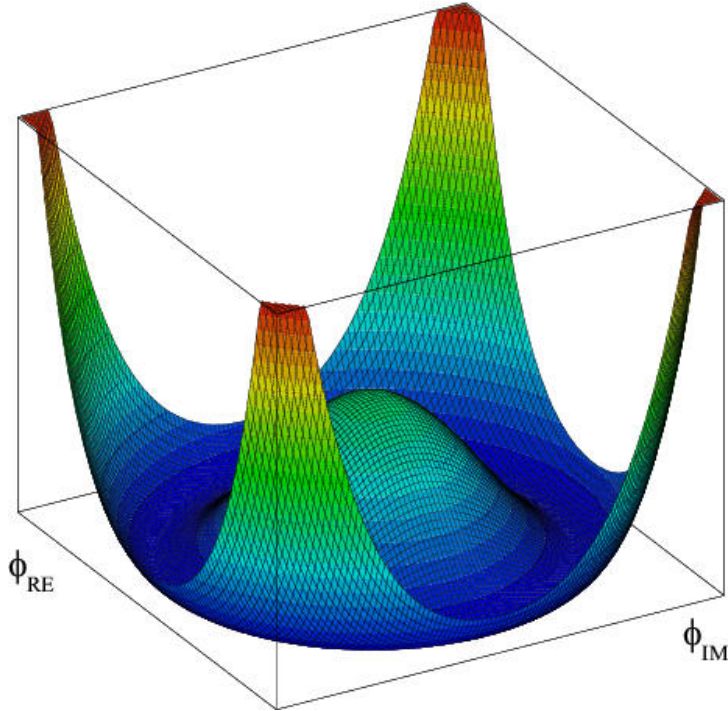


Figure 2.2: The Higgs potential $V(\phi)$ in the shape of a Mexican hat [7]

The minimum of the Higgs potential is known as the vacuum expectation value (VEV), which has a value of

$$\langle\phi\rangle = \frac{\mu}{\sqrt{\lambda}} \equiv v \quad (2.28)$$

³³ Since the Higgs field is a complex scalar field

Since the Higgs field is a 4-component spinor³⁴, the vacuum expectation value v can be re-expressed in the following form

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2.29)$$

After normalisation, the vacuum Higgs field ϕ_0 is obtained, which is a state of equilibrium that conserves $U(1)$ gauge symmetry prior to introducing fluctuations that causes the Higgs field to deviate away from this state.

$$\phi_0 = \langle\phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (2.30)$$

Introducing a scalar field h representing the Higgs boson into the vacuum Higgs field ϕ_0 as a form of fluctuation to ϕ_0 will result in a spontaneous breaking of the $U(1)$ gauge symmetry [8]. Hence, the resultant Higgs field, after introducing the fluctuations, can be expressed in the following manner

$$\phi = \phi_0 + h = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix} \quad (2.31)$$

Expressing the Higgs potential in terms of the vacuum expectation value v ,

$$V(\phi_0) = -\frac{1}{2}\mu^2 v^2 + \frac{1}{4}\lambda v^4 \quad (2.32)$$

Now, let's substitute the perturbed Higgs field ϕ in Eq (2.31) into the Higgs potential $V(\phi)$ and compare it with the unperturbed Higgs potential $V(\phi_0)$ above. Assuming h is a small perturbation applied to the vacuum Higgs field ϕ_0 ,

$$\begin{aligned} \phi^\dagger\phi &= \frac{1}{2}(v+h)^2 = \frac{1}{2}(v^2 + 2vh + h^2) \\ (\phi^\dagger\phi)^2 &= \frac{1}{4}(v^2 + 2vh + h^2)^2 = \frac{1}{4}(v^4 + 4v^3h + 6v^2h^2 + 4vh^3 + h^4) \\ &\approx \frac{1}{4}(v^4 + 4v^3h + 6v^2h^2) \\ V(\phi) &\approx V(\phi_0) - \frac{\mu^2}{2}(2vh + h^2) + \frac{\lambda}{4}(4v^3h + 6v^2h^2) \\ &\approx V(\phi_0) + \lambda v^2 h^2, \text{ where } \lambda v^2 h^2 = \mu^2 h^2 \end{aligned} \quad (2.33)$$

Substituting the above expression for the perturbed Higgs potential $V(\phi)$ into Eq (2.26) and comparing it with the Klein-Gordon Lagrangian for a complex scalar field, one can

³⁴ Also known as a doublet

identify the extra term that was produced as a result of the small perturbation h as the mass term. Comparing this extra term with the mass term in the Klein-Gordon Lagrangian $\mathcal{L}_{KG}$,

$$\frac{1}{2}m^2h^2 = \lambda v^2h^2 = \mu^2h^2 \Rightarrow m_H^2 = 2\lambda v^2 \equiv 2\mu^2 \Rightarrow m_H = \sqrt{2\lambda}v \equiv \sqrt{2}|\mu| \quad (2.34)$$

The implication of the above equation is that the small perturbation h , which causes the $U(1)$ gauge symmetry to be spontaneously broken, results in the Higgs boson, represented by h , acquiring a mass of m_H .

Returning back to the Standard Model Lagrangian in Eq (1.3), re-expressing the second line $\mathcal{L}_f$, which is the fermion term, in terms of the chiral fermion fields³⁵ [6],

$$\mathcal{L}_f = i\bar{\psi}_L \not{D} \psi_L + i\bar{\Psi}_R \not{D} \Psi_R \quad (2.35)$$

where $\not{D} = \gamma^\mu D_\mu$, using the Feynman-slash notation. The covariant derivatives D_μ acting on the “*left-handed*” ψ_L and the “*right-handed*” Ψ_R fermion fields here are not the same as that for the Higgs term in Eq (2.26), but they act on both fields as

$$\begin{aligned} D_\mu \psi_L &= (\partial_\mu + igW_\mu + ig'Y_L B_\mu)\psi_L, \\ D_\mu \Psi_R &= (\partial_\mu + ig'Y_R B_\mu)\Psi_R \end{aligned} \quad (2.36)$$

where g is the weak isospin coupling constant which is equal to the electromagnetic coupling constant g_e , g' is the weak hypercharge coupling constant which is equal to the weak coupling constant g_w , W_μ and B_μ are the gauge bosons that belongs to their respective gauge groups³⁶, and Y_L/Y_R are the Yukawa couplings for the “*left-handed*” and “*right-handed*” fermion fields respectively.

To ensure gauge invariance for both “*left-handed*” ψ_L and “*right-handed*” Ψ_R fermion fields, the following gauge transformations need to be applied to their respective fermion fields

$$\begin{aligned} \psi_L &\rightarrow \psi'_L = e^{iY_L\theta} U_L \psi_L, \\ \Psi_R &\rightarrow \Psi'_R = e^{iY_R\theta} \Psi_R \end{aligned} \quad (2.37)$$

where $U_L = \exp(iT^i\beta^i)$ is a unitary operator that transforms only the “*left-handed*” fermion fields ψ_L under the $SU(2)$ gauge group. $T^i = \frac{\tau^i}{2}$ and β^i in the exponential refer to the weak isospins along the three axes ($i = 1, 2, 3$), which are the generators of the $SU(2)$ gauge group, and the rotations along these three axes.

With a similar motivation, to conserve the gauge symmetry in $\mathcal{L}_f$ (Eq (2.35)), the

³⁵ Which are the *left-handed* and *right-handed* fermion fields, as mentioned earlier in the chapter

³⁶ As mentioned in the paragraph under Eq (2.6)

following gauge transformations need to be applied to the four gauge bosons in the $SU(2) \otimes U(1)$ gauge group, W_μ ³⁷ and B_μ ,

$$\begin{aligned} W_\mu &\rightarrow W'_\mu = U_L W_\mu U_L^\dagger + \frac{1}{g}(\partial_\mu U_L)U_L^\dagger, \\ B_\mu &\rightarrow B'_\mu = B_\mu - \frac{1}{g'}\partial_\mu \theta \end{aligned} \quad (2.38)$$

Note that the four additional parameters in the gauge transformations in Eq (2.37), three β^i 's and one θ , are connected to four gauge bosons in the electroweak theory ($SU(2) \otimes U(1)$). Hence, this is the reason why these four parameters also appear in Eq (2.38) for the gauge transformations of the four gauge bosons.

Returning back to the new Lagrangian in Eq (2.26), the kinetic term $\mathcal{L}_{kin}$ merely describes the $U(1)$ gauge symmetry. For a $SU(2) \otimes U(1)$ electroweak theory, this kinetic term $\mathcal{L}_{kin}$ needs to be transformed into a term, $\mathcal{L}_{gauge}$, that is invariant under the gauge transformations in Eq (2.38). Hence, the kinetic term $\mathcal{L}_{kin}$ needs to be modified in the following manner

$$\begin{aligned} \mathcal{L}_{gauge} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^i G^{\mu\nu i}, \text{ where} \\ G_{\mu\nu}^i &= \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - g\epsilon^{ijk}W_\mu^j W_\nu^k; \quad F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \end{aligned} \quad (2.39)$$

As seen in the equation above, the gauge term $\mathcal{L}_{gauge}$ contains 2 terms that have the same form as that of the original kinetic term $\mathcal{L}_{kin}$, except that the second quadratic term $G_{\mu\nu}^i G^{\mu\nu i}$ introduces additional cubic and quartic terms³⁸ which describes the self-interactions among the W^i gauge fields.

Bringing the discussion back to the fermion term $\mathcal{L}_f$ in Eq (2.35), just as how adding a mass term violates the $U(1)$ gauge symmetry in Eq (2.25), the gauge symmetry of the Standard Model lagrangian will again be broken with the introduction of a mass term for the fermions into the fermion term $\mathcal{L}_f$. Such a mass term in the Dirac Lagrangian, $\mathcal{L}_{Dirac} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$ (in natural units $\hbar = c = 1$) where

$$m\bar{\psi}\psi = m(\bar{\psi}_L + \bar{\psi}_R)(\psi_L + \psi_R) = m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L), \quad (2.40)$$

contains couplings from both “*left-handed*” and “*right-handed*” fermion fields, which have distinct gauge transformations from each other as shown in Eq (2.37). Hence, one can clearly see that this breaks the gauge symmetry of the Standard Model Lagrangian.

However, in the Standard Model, we do not observe massless fermions and gauge bosons, other than the photon. To explain why fermions and gauge bosons in the Standard

³⁷ Where $W_\mu = W_\mu^i T^i$

³⁸ After expanding the quadratic term

Model have masses, we will need to introduce the process in which electroweak symmetry is spontaneously broken and this will be the topic of our discussion in the following section.

2.2 Spontaneous Symmetry Breaking (SSB)

In the “pure” $U(1)$ gauge theory covered in the previous section, the vacuum Higgs field takes the form of that in Eq (2.30), and that in Eq (2.31) for the perturbed case. However, proceeding on to the complete $SU(2) \otimes U(1)$ electroweak sector of the Standard Model [6], the Higgs field is actually a weak isospin doublet having a weak hypercharge of $Y = +\frac{1}{2}$ and taking on a more general form of

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad (2.41)$$

where ϕ^0 can be interpreted as the neutral component of the doublet, since only a neutral scalar field can acquire the vacuum expectation value (VEV) v so as to conserve electric charge Q . In other words, $U(1)$ gauge symmetry in Quantum Electrodynamics (QED) is conserved by the VEV, which is a scalar value.

Similar to the pure $U(1)$ case in the previous section, the Higgs potential in the $SU(2) \otimes U(1)$ electroweak sector of the Standard Model has the form of $V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$, where the only difference from that in the pure $U(1)$ gauge theory is the prefactors at the front of both terms of the Higgs potential. In the electroweak sector of the Standard Model, the Higgs field also acquires a non-zero VEV in the case where $\mu^2 > 0$, which is exactly the case in the pure $U(1)$ gauge theory. Using the same procedure to obtain the minimum of the Higgs potential in the pure $U(1)$ case, one will be able to deduce that the minimum of the Higgs potential in the $SU(2) \otimes U(1)$ gauge theory occurs at $\phi^\dagger \phi = \frac{v^2}{2}$, where $v = \frac{\mu}{\sqrt{\lambda}}$ (Eq (2.28)). Since $|\phi|^2 = \phi^\dagger \phi$, taking the square root of this value, one can obtain the expectation value of the vacuum Higgs field ϕ_0 as the following

$$\langle \phi_0 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad (2.42)$$

which is exactly the same as the normalised vacuum Higgs field in Eq (2.30).

The covariant derivative in the Higgs term of the Lagrangian for the $SU(2) \otimes U(1)$ electroweak sector of the Standard Model no longer takes on the form of $D_\mu = \partial_\mu + ieA_\mu$. Instead, it now takes on the following form

$$D_\mu = \partial_\mu + igT^i W_\mu^i + \frac{i}{2}g'B_\mu = \partial_\mu + \frac{i}{2}g\tau^i W_\mu^i + \frac{i}{2}g'B_\mu \quad (2.43)$$

where the factors g , g' , T^i , W_μ^i and B_μ are exactly the factors that were used in the gauge transformations of four $SU(2) \otimes U(1)$ gauge bosons, W_μ and B_μ , in Eq (2.38). This transformation of the covariant derivative is of particular importance in the electroweak sector of the Standard Model because this is the term, after acting on the vacuum Higgs field ϕ_0 , which generates the masses for the 3 physical gauge bosons, $W^\pm$ and Z^0 , of the electroweak sector.

$$\begin{aligned}
 (D_\mu \phi_0)^\dagger (D^\mu \phi_0) &= \left| \left(\partial_\mu + \frac{i}{2} g \tau^i W_\mu^i + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\
 &\text{Factoring out } v \text{ and } \frac{1}{\sqrt{2}}, \\
 &= \frac{v^2}{8} \left| \left(g \tau^i W_\mu^i + g' B_\mu \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right|^2 \\
 &= \frac{v^2}{8} \left| \begin{pmatrix} g W_\mu^1 - i g W_\mu^2 \\ -g W_\mu^3 + g' B_\mu \end{pmatrix} \right|^2 \\
 &= \frac{v^2}{8} \left[g^2 \left\{ (W_\mu^1)^2 + (W_\mu^2)^2 \right\} + (g W_\mu^3 - g' B_\mu)^2 \right] \quad (2.44)
 \end{aligned}$$

The first term in the last line of the above equation excluding the prefactor, *i.e.* $(W_\mu^1)^2 + (W_\mu^2)^2$, is product of the two $W^\pm$ gauge bosons, which are complex conjugates of each other. In other words, after normalization, the two $W^\pm$ gauge bosons are defined as the following

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2) \quad (2.45)$$

Including the prefactor $\frac{g^2 v^2}{8}$ into the first term in the last line of Eq (2.44), one will observe that the first term is actually the mass term for both charged weak bosons $W^\pm$. Re-expressing the first term together with its prefactor and dropping the superscripts i that are used to identify the gauge bosons, the following expression is obtained

$$\frac{g^2 v^2}{8} [(W_\mu^1)^2 + (W_\mu^2)^2] = \frac{1}{2} \left(\frac{g v}{2} \right)^2 W_\mu^\dagger W^\mu, \text{ where } M_W = \frac{g v}{2} \quad (2.46)$$

Similarly, the second term in the last line of Eq (2.44) is the mass term for the neutral Z^0 boson. Re-expressing the second term together with the prefactor $\frac{v^2}{8}$,

$$\frac{v^2}{8} (g W_\mu^3 - g' B_\mu)^2 = \frac{1}{2} \left(\frac{v}{2} \sqrt{g^2 + g'^2} \right)^2 \left[\frac{1}{\sqrt{g^2 + g'^2}} (g W_\mu^3 - g' B_\mu) \right]^2$$

$$\text{where } Z_\mu^0 = \frac{1}{\sqrt{g^2 + g'^2}}(gW_\mu^3 - g'B_\mu), \quad M_Z = \frac{v}{2}\sqrt{g^2 + g'^2} \quad (2.47)$$

As mentioned earlier in the chapter³⁹, the neutral Z^0 boson and the photon are produced from the mixing of W_μ^3 and B_μ gauge fields. Since the photon does not interact with the vacuum Higgs field ϕ_0 , it is massless and hence does not appear in Eq (2.44). Since the photon γ is produced from the mixing of the W_μ^3 and B_μ gauge fields, it can be expressed as a linear combination of those gauge fields, just like for the neutral Z^0 boson in Eq (2.47). Hence, using a massless gauge field A_μ to represent a photon,

$$A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g'W_\mu^3 + gB_\mu) \quad (2.48)$$

Upon close inspection of the expressions for Z_μ^0 and A_μ , one will observe that the neutral Z^0 boson and the photon are related just by a 2D rotation. In other words, acting a $SO(2)$ rotation on the $B_\mu - W_\mu^3$ eigenstate will generate the A_μ and Z_μ^0 mixed states, *i.e.*

$$\begin{pmatrix} A_\mu \\ Z_\mu^0 \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} B_\mu \\ W_\mu^3 \end{pmatrix}, \quad \text{where } \cos \theta_w = \frac{g}{\sqrt{g^2 + g'^2}},$$

$$\sin \theta_w = \frac{g'}{\sqrt{g^2 + g'^2}} \quad (2.49)$$

where θ_w is the weak mixing angle. Taking the ratio of M_W in Eq (2.46) to M_Z in Eq (2.47), one will observe that this mass ratio is actually equal to $\cos \theta_w$, which confirms Eq (2.3).

Looking back at the general form of the Higgs field with a weak hypercharge of $Y = +\frac{1}{2}$ in Eq (2.41), a Higgs field with such a form can only couple to “*right-handed* down-type” quarks and “*right-handed*” charged leptons ℓ^- . For the “*right-handed* up-type” quarks, it will only couple to a Higgs field that has a weak hypercharge of $Y = -\frac{1}{2}$. Labelling such a field as $\tilde{\phi}$ and representing it with a different general form,

$$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}, \quad \text{where } \tilde{\phi}_i = \epsilon_{ij}\phi_j^* \quad (2.50)$$

Returning back to the Standard Model Lagrangian in Eq (1.3), the Yukawa term $\mathcal{L}_{Yuk}$ is the term responsible for generating the masses for the fermion fields. Using the above couplings together with the Yukawa couplings f_i for the different fermion families and inserting them into the Yukawa term $\mathcal{L}_{Yuk}$, the mass terms for the corresponding fermion fields can then be obtained. In explicit terms, the Yukawa term takes on the following

³⁹ In the paragraph right below Eq (2.6)

form [6]:

$$\begin{aligned}
\mathcal{L}_{Yuk} = & f_e \bar{\ell}_L \phi e_R + f_u \bar{q}_L \tilde{\phi} u_R + f_d \bar{q}_L \phi d_R \\
& + f_\mu \bar{\ell}_L \phi \mu_R + f_c \bar{q}_L \tilde{\phi} c_R + f_s \bar{q}_L \phi s_R \\
& + f_\tau \bar{\ell}_L \phi \tau_R + f_t \bar{q}_L \tilde{\phi} t_R + f_b \bar{q}_L \phi b_R + h.c.
\end{aligned} \tag{2.51}$$

For simplicity, we will use the vacuum Higgs field for both ϕ and $\tilde{\phi}$, instead of the perturbed Higgs field in Eq (2.31), which are represented using the expressions as shown below

$$\begin{aligned}
Y(\phi_0) = +\frac{1}{2} \Rightarrow \phi_0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \\
Y(\tilde{\phi}_0) = -\frac{1}{2} \Rightarrow \tilde{\phi}_0 &= \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \end{pmatrix}
\end{aligned} \tag{2.52}$$

Substituting the above expressions for the different representations of the vacuum Higgs field into the Yukawa term in Eq (2.51), the following expression is obtained

$$\begin{aligned}
\mathcal{L}_{Yuk} = & \frac{f_e v}{\sqrt{2}} (\bar{e}_L e_R + \bar{e}_R e_L) + \frac{f_u v}{\sqrt{2}} (\bar{u}_L u_R + \bar{u}_R u_L) + \frac{f_d v}{\sqrt{2}} (\bar{d}_L d_R + \bar{d}_R d_L) \\
& + \frac{f_\mu v}{\sqrt{2}} (\bar{\mu}_L \mu_R + \bar{\mu}_R \mu_L) + \frac{f_c v}{\sqrt{2}} (\bar{c}_L c_R + \bar{c}_R c_L) + \frac{f_s v}{\sqrt{2}} (\bar{s}_L s_R + \bar{s}_R s_L) \\
& + \frac{f_\tau v}{\sqrt{2}} (\bar{\tau}_L \tau_R + \bar{\tau}_R \tau_L) + \frac{f_t v}{\sqrt{2}} (\bar{t}_L t_R + \bar{t}_R t_L) + \frac{f_b v}{\sqrt{2}} (\bar{b}_L b_R + \bar{b}_R b_L)
\end{aligned} \tag{2.53}$$

Extracting the prefactors from all the terms, one will obtain the masses for the fermions. In other words, the mass of the fermion that are of interest is

$$m_i = \frac{f_i v}{\sqrt{2}}, \quad \text{where } i \text{ identifies the fermion} \tag{2.54}$$

To summarise this section, we have shown how the vacuum Higgs field ϕ_0 couples to the four gauge fields, W_μ^i and B_μ , to generate the physical $W^\pm$ and Z^0 gauge bosons, together with their masses. In addition, we have also shown how the vacuum Higgs field, in the two different representations: ϕ_0 and $\tilde{\phi}_0$, couples to the chiral fermion fields in the Yukawa term $\mathcal{L}_{Yuk}$ to generate the masses for the fermions.

2.3 Sequential Standard Model (SSM)

In the Standard Model, we have seen that the masses of the $W^\pm$ and Z^0 gauge bosons, which are the mediators of the weak interaction, are constant parameters in the theory. However, in many Beyond Standard Model (BSM) theories, these masses are no longer constant but are variables instead. One of such theories is the Sequential Standard Model (SSM), which focuses on the Z boson. Since the mass of the Z boson in the Sequential

Standard Model (SSM) varies, we will from now onwards refer to the Z boson as Z'_{SSM} , instead of Z^0 in the Standard Model, to avoid confusion. In this model, the decay width of Z'_{SSM} , $\Gamma_{Z'_{\text{SSM}}}$, is now proportional to its mass, $m_{Z'_{\text{SSM}}}$. In the case where Z'_{SSM} decays into a fermion-antifermion pair ($Z'_{\text{SSM}} \rightarrow f\bar{f}$), the decay width is related to the mass of Z'_{SSM} in the following equation [9]:

$$\begin{aligned} \Gamma(Z'_{\text{SSM}} \rightarrow f\bar{f}) &= \frac{N_c^f g_{Z'_{\text{SSM}}}^2 m_{Z'_{\text{SSM}}}}{24\pi \cos^2 \theta_w} \left[1 - 4 \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right]^{1/2} \\ &\quad \times \left\{ (a_f^2 + b_f^2) \left[1 - \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right] + 6a_f b_f \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right\} \\ \text{Since } g_{Z'_{\text{SSM}}}^2 &= \frac{g_e'^2}{\cos^2 \theta_w \sin^2 \theta_w} = \frac{4\pi \alpha'_{\text{EM}}}{\cos^2 \theta_w \sin^2 \theta_w}, \\ &= \frac{N_c^f \alpha'_{\text{EM}} m_{Z'_{\text{SSM}}}}{6 \sin^2 \theta_w \cos^4 \theta_w} \left[1 - 4 \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right]^{1/2} \\ &\quad \times \left\{ (a_f^2 + b_f^2) \left[1 - \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right] + 6a_f b_f \left(\frac{m_f}{m_{Z'_{\text{SSM}}}} \right)^2 \right\} \end{aligned} \quad (2.55)$$

where N_c^f refers to the color factor ($N_c^f = 1$ for leptons and $N_c^f = 3$ for quarks); $\alpha'_{\text{EM}} = \frac{1}{127.944}$ refers to the electromagnetic (EM) fine structure constant in the Sequential Standard Model (SSM) which accounts for $m_{Z'_{\text{SSM}}}$; and m_f is the mass of the fermion in question. For convenience, the following factors are used

$$b_f = -Q_f \sin^2 \theta_w, \quad a_f = T_f^3 + b_f \quad (2.56)$$

where Q_f is the electric charge of the fermion and T_f^3 is the third component of the weak isospin for the “*left-handed*” fermion field that is of interest.

The decay width expression in Eq (2.55) will be used rather frequently in the following chapters, especially in Chapter 5 where we will be doing various branching ratio calculations for various Z'_{SSM} decay modes based on the expression in Eq (2.55). Hence, as an easy reference for the subsequent chapters, the table on the following page explicitly shows all the variables in Eqs (2.55) and (2.56) that vary across different species of fermions f :

fermion f	N_c^f	Q_f	T_f^3
ℓ^- (Charged leptons), where $\ell = e, \mu, \tau$	1	-1	$-\frac{1}{2}$
ν_ℓ (Neutrinos), where $\ell = e, \mu, \tau$	1	0	$\frac{1}{2}$
u, c, t (Up-type quarks)	3	$\frac{2}{3}$	$\frac{1}{2}$
d, s, b (Down-type quarks)	3	$-\frac{1}{3}$	$-\frac{1}{2}$

Table 2.2: Color factors, electric charges and eigenvalues of the third component of the weak isospin for the different species of fermions f

Chapter 3

DMSMs With Leptophobic Spin-One Mediators

In the previous two chapters, we have been discussing about the Standard Model, which is a highly established theory in the field of particle physics, and one of the three fundamental interactions in the theory, the weak interaction. Although the Standard Model is arguably one of the most successful theories in particle physics, its strength mainly draws on its ability to explain interactions within ordinary visible matter which only makes up a small proportion ($\sim 15\%$) of all matter in the universe. On the other hand, cosmological and astrophysical observations have indirectly indicated the existence of other exotic forms of matter, known as Dark Matter⁴⁰ (DM), which contributes to the vast majority ($\sim 85\%$) of all matter in the universe. One such evidence is gravitational effects observed in galaxies that cannot be explained by ordinary visible matter, whose mutual interactions are within the realm of the Standard Model.

Since Dark Matter has not been detected directly *yet*, it means that it *almost* does not interact with ordinary visible matter, or at least its interaction with ordinary matter is very weak. This would imply that Dark Matter is well beyond the current reach of the Standard Model and thus theories beyond the Standard Model (BSM) are needed to aid in our understanding on their behaviours and interactions, be it with ordinary matter or among themselves. Examples of such Beyond Standard Model (BSM) theories that explore the nature of Dark Matter include SuperSymmetry (SUSY) and Dark Matter Simplified Models (DMSMs). However, we will only be discussing about DMSMs in this chapter and in Chapter 5.

There are generally two types of Dark Matter: a) hot Dark Matter and; b) cold Dark Matter. In the context here, the terms “*hot*” and “*cold*” are not used to describe the temperature of the Dark Matter, but rather to classify their velocities, whether their motions are relativistic in nature or not. Observations have shown strong evidence for mainly cold Dark Matter (CDM) [1, 10, 11], where Dark Matter has been observed to be non-relativistic since at least the era of cosmological recombination⁴¹ in the early Universe. As a result of indirect observations of the existence of Dark Matter and our ignorance about them, candidates for Dark Matter can range from primordial black holes, whose mass are heavier than the sun, to weakly interacting massive particles (WIMPs), such as massive neutrinos and exotic particles like axions and magnetic monopoles, to

⁴⁰ Since they do not interact with the electromagnetic (EM) interaction and hence is invisible in the EM spectrum

⁴¹ The period in the early part of our Universe history where it is cool enough for neutral atoms to form

very light particles with large occupation numbers. In the case of WIMPs that are in thermal equilibrium with Standard Model (SM) particles in the early Universe, freeze-out relic density calculations suggest that WIMPs likely have masses in orders of magnitude of TeV, $\mathcal{O}(\text{TeV})$, which could be produced in colliders such as the LHC in CERN. Hence, the search for Dark Matter (DM) WIMPs has been one of the primary research objectives for LHC experiments, in particular the ATLAS and CMS experiments [12–14].

In experiments at the LHC, several approaches have been taken towards DM searches in terms of both the predictions for possible signals and the interpretations of search limits. Initially, many analyses were based on specific models that predict WIMPs capable of contributing to CDM, such as SUSY [15, 16] and a few other models with new physics at the TeV scale [17, 18]. These searches were focused typically on either the production of heavier new particles followed by cascade decays of the DM particle, or the direct production of DM particles in association with a single SM particle that is used for tagging purposes, also known as the ‘mono-X’ signature.

In addition to the initial searches presented briefly in the previous paragraph, there have been a growing interest recently towards more general model frameworks and analyses that were motivated by more generic experimental signatures. One of such initial approaches was to construct an effective field theory (EFT) for the interactions between DM and SM particles [19–23], focusing on the mono-X signatures. A weakness of this EFT approach is that, to obtain the appropriate cosmological DM relic density, the DM/SM interactions are likely to be mediated by particles in the TeV mass range, for which the approximation of a contact interaction may not be adequate. In particular, the LHC might also have the capability of producing the mediator particle directly, contributing to another signature that is beyond the EFT framework [24, 25]. As a result, this has spurred interest in DMSMs [26], which postulate possible effective Lagrangians that include explicitly the mediator particle and its interaction with both DM and SM particles, and hence this is why we will be focusing purely on DMSMs [1] in this thesis.

To give an overview on what we will be covering in this chapter, we will first discuss about the restrictions on the set of models suggested by the DMWG, and analysed by ATLAS and CMS experiments [12–14, 26], the effective Lagrangians for this set of models, and how the global likelihood analyses was done within **MasterCode** framework. It will then be followed by a discussion on the constraints that were used in the analyses. Lastly, we will wrap up the chapter with a discussion on the results of the analyses.

3.1 Analyses Restrictions and Framework

For the set of DMSMs that was used for the global likelihood analyses, interactions between mediator and the DM/SM particles were considered to be leptophobic [1]. In other

words, any LHC constraints on mediators interacting with charged leptons would not be considered, but we will discuss these constraints later in Chapter 5 for the leptophilic case. In addition, s -channel spin-one mediators would only be considered and the DM particle was assumed to be a neutral Dirac fermion, denoted by the greek symbol χ . The Lagrangian describing the interactions between the spin-one mediator Y and the DM particle χ is

$$\mathcal{L}_\chi = \bar{\chi}\gamma^\mu(g_V^\chi + g_A^\chi\gamma^5)\chi Y_\mu \quad (3.1)$$

and that describing the interactions between Y and the SM quarks q is

$$\mathcal{L}_q = \sum_i \bar{q}_i\gamma^\mu(g_V^{q_i} + g_A^{q_i}\gamma^5)q_i Y_\mu \quad \text{where } i \text{ identifies the quark} \quad (3.2)$$

For simplicity, adhering to the convention used by the LHC Dark Matter Working Group (DMWG)⁴² [14], the interactions between the mediator boson Y and quarks were assumed to be generation-independent, and that they were also assumed to be the same for both “up-” and “down-” types of quarks, *i.e.* $g_{V,A}^q = g_{V,A}^{q_i} \forall i$. The interactions assumed to be leptophobic in this chapter would, in other words, mean that the mediator Y boson would not have any interactions with leptons. For both the DM (g_{DM}) and SM (g_{SM}) couplings to the mediator Y , only two scenarios were considered, one being purely vectorial:

$$\begin{aligned} g_V^\chi &\equiv g_{\text{DM}} \neq 0, & g_A^\chi &= 0; \\ g_V^q &\equiv g_{\text{SM}} \neq 0, & g_A^q &= 0, \end{aligned} \quad (3.3)$$

and the other being purely axial:

$$\begin{aligned} g_A^\chi &\equiv g_{\text{DM}} \neq 0, & g_V^\chi &= 0; \\ g_A^q &\equiv g_{\text{SM}} \neq 0, & g_V^q &= 0. \end{aligned} \quad (3.4)$$

In other words, scenarios where the mediator Y has a combination of vector and axial couplings to the DM χ and SM q particles would not be considered. Note that the pure axial-vector model could in theory violate unitarity at high energies. However, the axial vector couplings needed for this to happen were beyond the range of the scans that were done in `MasterCode`, and would even bring the particle interpretation of the mediator into question [27, 28].

Moving on to the analyses framework, each model has four free parameters, two of which are the masses of the DM particle χ and of the mediator Y , m_χ and m_Y , and the other two are the mediator couplings to the DM and SM particles, g_{DM} and g_{SM} . The sampling over these 4-dimension parameter spaces could be controlled computationally. However, the couplings were often fixed by experimental studies and hence previous

⁴² The DMWG makes recommendations on the formulation of DMSMs for use by LHC experiments

analyses would in general have fixed two out of the four supposedly free parameters. For the global likelihood analyses, a more general approach would be used where the direct DM constraints was combined with constraints from the LHC experiments. In particular, such an approach is desirable since obtaining the preferred value of cosmological DM relic density $\Omega_\chi h^2$ would require the values of g_{DM} and g_{SM} which were also dependent on m_χ and m_Y . Hence, without fixing any parameter of the models, the model predictions from `MG5_aMC@NLO 2.4.3`⁴³ [29], using `DMSIMP` model files [30, 31], and `MicrOMEGAs 5.0.2`⁴³ [32–34] were implemented to build a likelihood function for each of the astrophysical and collider constraints. Using the `MasterCode` framework, a global likelihood was then be obtained by combining all the likelihood functions from the different constraints. In addition, with the help of `MultiNest 2.18` [35–37], regions where the global likelihood was minimised could be identified.

Since DMSMs are incomplete models, it would not be meaningful to compute p -values for these models. Instead, the `MasterCode` framework here was used to correlate the impacts of constraints from different sectors, explore correlations and identify regions of the model parameter space that were still allowed [1].

As mentioned in an earlier paragraph, each model has four free parameters, two of which are mass parameters, m_Y and m_χ , and the other two are DM and SM couplings to the mediator Y , g_{DM} and g_{SM} . The ranges over which these parameters were studied are shown in the table on following page, together with the number of segments/boxes which had aided in the efficient sampling of the whole parameter space. The range of m_Y was chosen in a way so as to avoid the low-mass region, close to the Z^0 pole, where mixing with the Z^0 boson could interfere with important constraints from electroweak precision data [38] and indirect searches for astrophysical DM annihilations⁴⁴, but instead include all masses where experimental searches at the LHC were sensitive. For the DM and SM couplings to the mediator Y g_{DM} and g_{SM} , they were limited to perturbative ranges below $\sqrt{4\pi}$.

On top of the generic sampling ranges shown in the table at the top of the following page, dedicated scans of specific regions were done to get a better understanding of some interesting features in the models. In particular, to get sufficient samples of annihilations close to the the mediator Y pole⁴⁵ where $\Gamma_Y/m_Y \ll 1$, a dedicated sampling was done in regions where $2(m_\chi/m_Y)$ has deviated from unity by less than factor of 10^{-3} . A similar sampling procedure was conducted for both the pure vector and the pure axial-vector models (Eqs (3.3) and (3.4), with a total number of ~ 100 million parameter sets generated for each case [1].

⁴³ These are external packages which had been interfaced and implemented into `MasterCode` framework. These packages will be discussed in greater detail in the following chapter which we will be discussing about the `MasterCode Framework`

⁴⁴ This will be elaborated further in the following section

⁴⁵ Using the same terminology for that of Z^0 boson in the previous chapter

Parameter	Range	Number of Segments
m_Y	(0.1, 5.0) TeV	10
m_χ	(0.0, 2.5) TeV	8
g_{SM}	$(0.0, \sqrt{4\pi})$	2
g_{DM}	$(0.0, \sqrt{4\pi})$	2
Total Number of Segments		320

Table 3.1: Sampling ranges of the 4 free parameters in each model, together with the number of segments into which they were divided during the sampling

3.2 Constraints

The constraints that were being implemented in the global likelihood analyses using the **MasterCode** framework can be classified as astrophysical or collider constraints. The astrophysical constraints were mainly based on observations from various DM indirect detection experiments around the world, such as the PANDAX-II⁴⁶ experiment at China JinPing Underground Laboratory (CJPL) and XENON1T experiment at Italy's Laboratori Nazionali del Gran Sasso (LNGS). The collider constraints, on the other hand, were based on the analyses of experimental signatures from LHC experiments, in particular the ATLAS and CMS experiments.

3.2.1 Astrophysical DM Constraints

3.2.1.1 DM Relic Density

Using the cosmic microwave background (CMB) measurements and other observations from the Planck satellite [39], the average density of cold DM (CDM) in the Universe was well estimated to be"

$$\Omega_{\text{CDM}} h^2 = 0.120 \pm 0.001 \quad (3.5)$$

It was assumed here that the WIMP DM χ , together with the interactions described by the Lagrangians in Eqs (3.3) and (3.4), provides the dominant contribution to CDM, hence $\Omega_\chi h^2 \approx \Omega_{\text{CDM}} h^2$. The relic density is approximately proportional to the inverse of the annihilation cross section, *i.e.* $\Omega_\chi h^2 \propto \langle \sigma v \rangle^{-1}$. Assuming the annihilation is dominated by the s -channel process: $\chi \bar{\chi} \rightarrow Y^* \rightarrow f \bar{f}$ where f and $\bar{f}$ are SM particles and their anti-particles, the expression for cross-section up to leading-order (LO) is given

⁴⁶ Particle and Astrophysical Xenon Detector

by [40, 41]:

$$\langle \sigma v \rangle_s \approx \left\langle \frac{c_Y}{32\pi} \left[\frac{g_{\text{SM}}^2 g_{\text{DM}}^2 m_\chi^2}{\{(m_\chi v)^2 + 4m_\chi^2 - m_Y^2\}^2 + (m_Y \Gamma_Y)^2} \right] \right\rangle, \quad c_Y \sim a \left(\frac{m_q}{m_\chi} \right)^2 + b v^2 \quad (3.6)$$

where v refers to the relative velocity of the annihilating DM particles, and c_Y is an $\mathcal{O}(1)$ constant for the pure vector mediator while for the pure axial-vector mediator it is suppressed by the quark mass m_q . The total width of the mediator Y is given by the following expression

$$\Gamma_Y = \Gamma_Y^{\chi\bar{\chi}} + N_c^f \sum_q \Gamma_Y^{q\bar{q}} \quad (3.7)$$

where $N_c^f = 3$ is the color factor⁴⁷, and

$$\Gamma_Y^{X\bar{X}} = \frac{g_X^2 m_Y}{12\pi} \cdot f\left(\frac{m_X^2}{m_Y^2}\right), \text{ where } X = \chi \text{ (DM) or } q \text{ (SM), and letting } x = \left(\frac{m_X}{m_Y}\right)^2, \quad (3.8)$$

$$f(x) = \begin{cases} (1+2x)(1-4x)^{1/2} & \text{(vector)} \\ (1-4x)^{3/2} & \text{(axial-vector)} \end{cases}$$

As a result of phase-space suppression, the mediator width Γ_Y becomes particularly crucial in the resonant region, where $m_Y \approx 2m_\chi$ and $\Gamma_Y^{\chi\bar{\chi}} \ll \Gamma_Y$.

In the axial-vector case, since c_Y is suppressed by the quark mass m_q as mentioned earlier in the section, c_Y and $\langle \sigma v \rangle_s$ (Eq (3.6)) is now approximately proportional to v^2 . Furthermore, if g_{SM} is very small, the velocity term becomes the dominant term in the denominator of Eq (3.6) at the resonance, which leads to the following approximate scaling

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle_s} \propto \frac{m_Y^2}{g_{\text{SM}}^2 g_{\text{DM}}^2}, \quad (3.9)$$

where $\propto$ denotes approximate proportionality. In the vector case where it is a s -wave annihilation, the narrow-width approximation is appropriate for very small g_{SM} . In the limit as g_{SM} approaches 0, one will observe that

$$\frac{1}{(m_Y^2 - 4m_\chi^2)^2 + (m_Y \Gamma_Y)^2} \rightarrow \frac{\delta(m_Y^2 - 4m_\chi^2)}{m_Y \Gamma_Y}, \quad (3.10)$$

hence the approximate scaling of the relic density $\Omega_\chi h^2$ is predicted to be

$$\Omega_\chi h^2 \propto \frac{1}{\langle \sigma v \rangle_s} \propto \frac{m_Y^2}{g_{\text{DM}}^2} \quad (3.11)$$

⁴⁷ Same as that in Table 2.2

near the mediator Y pole⁴⁵. However, the DM $\chi\bar{\chi}$ annihilation occurs in a thermal bath, with collisions occurring at a range of centre-of-mass (CM) energies $\geq 2m_\chi$. Hence, annihilations with effective CM energies within $\mathcal{O}(\Gamma_Y)$ of the mediator mass m_Y are not dominant when the mediator width Γ_Y is very small. Numerically, one can clearly see that Eq (3.11) reflects the dependence of $\Omega_\chi h^2$ on g_{DM} and its weak dependence on g_{SM} over an intermediate range of g_{SM} . However, $\Omega_\chi h^2$ does increase at both large and small values of g_{SM} :

$$\begin{aligned}\Omega_\chi h^2 &\propto \frac{m_Y^2 g_{\text{SM}}^2}{g_{\text{DM}}^2} && \text{for } g_{\text{SM}} \gg g_{\text{DM}} \\ \Omega_\chi h^2 &\propto \frac{m_Y^2}{g_{\text{SM}}^2} && \text{for } g_{\text{SM}} \ll g_{\text{DM}}\end{aligned}\quad (3.12)$$

In this case, the dependence of $\Omega_\chi h^2$ will be related by the relative sizes of the mediator width, Γ_Y , and the mass difference between m_Y and $2m_\chi$ [41], $m_Y - 2m_\chi$ ⁴⁸. On one hand, assuming that $\Gamma_Y/m_Y > \left[1 - 4(m_\chi/m_Y)^2\right]$ and $m_Y > 2m_\chi$, $\Omega_\chi h^2$ is proportional to g_{DM}^2 . On the other hand, assuming a very small mediator width⁴⁹ Γ_Y , the mass difference in the denominator of Eq (3.6) dominates and $\Omega_\chi h^2$ is inversely proportional to g_{DM}^2 , which is exactly the same as that in Eq (3.9) for the axial-vector case.

Proceeding on to another important process which is the t -channel process: $\chi\bar{\chi} \rightarrow YY$, the leading contribution to the cross section for this particular process is given by the following expression [40]

$$\langle\sigma v\rangle_t \approx \frac{g_{\text{DM}}^4}{32\pi} \left[\frac{(m_\chi^2 - m_Y^2)^{3/2}}{m_\chi(2m_\chi^2 - m_Y^2)^2} \right] \quad (3.13)$$

Note that the t -channel process is kinematically forbidden when the mass of the mediator, m_Y , is greater than the mass of the DM, m_χ .

In the parameter set for each model, the DM relic density $\Omega_\chi h^2$ was evaluated numerically using **MicrOMEGAS 5.0.2** [32–34], including an uncertainty of 10% around Eq (3.5). With this, a substantial but unbiased sample of model points was obtained.

In the figure at the top of the following page, as a cross-check, a comparison was made between the $\Omega_\chi h^2$ obtained using **MicrOMEGAS 5.0.2**, represented by the coloured regions and solid blue line, and the $\Omega_\chi h^2 = 0.12$ curve provided by the CMS Collaboration [42], represented by the dashed line, for the case of a spin-one mediator having the following vector-like SM and DM couplings:

$$g_{\text{SM}}^V = 0.25; \quad g_{\text{DM}}^V = 1.0 \quad (3.14)$$

⁴⁸ Regardless whether it is positive (> 0) or negative (< 0)

⁴⁹ Taking the case where $g_{\text{SM}} \ll 1$ and $m_Y < 2m_\chi$ as an example

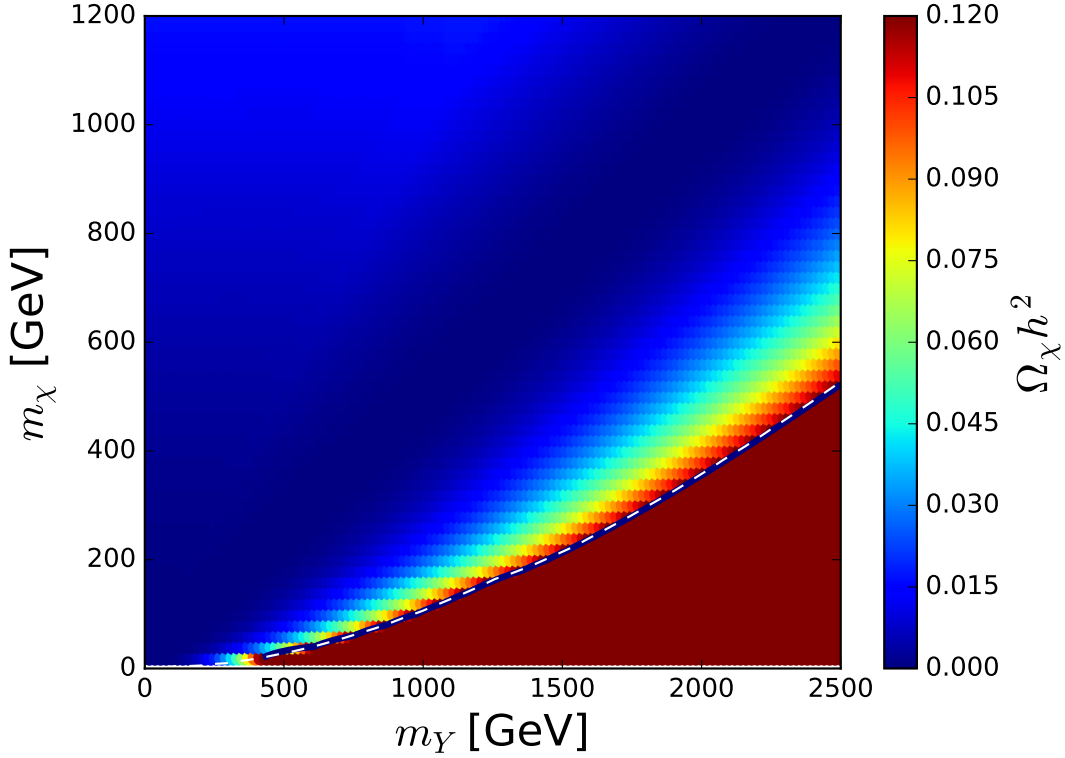


Figure 3.1: Comparison of $\Omega_\chi h^2$ calculated using MicrOMEGAs 5.0.2 (coloured regions and solid blue line) and the $\Omega_\chi h^2 = 0.12$ curve (dashed white line) provided by the CMS Collaboration for the DMSM with a spin-one mediator having vector-like SM and DM couplings $g_{\text{SM}}^V = 0.25$ and $g_{\text{DM}}^V = 1.0$

As seen in the figure above, the agreement was excellent, confirming that the values of $\Omega_\chi h^2$ obtained from MicrOMEGAs 5.0.2 were correct. Note that the red shaded region below the $\Omega_\chi h^2 = 0.12$ dashed line was excluded since $\Omega_\chi h^2 > 0.12$, and that the blue region shows that the DM particle χ was underdense.

3.2.1.2 Spin-Independent (SI) DM Scattering

By combining the spin-independent (SI) DM scattering cross-sections σ_p^{SI} from the LUX⁵⁰ [43], PANDAX-II⁴⁶ [44] and XENON1T [45] experiments, a constraint with the combined σ_p^{SI} was obtained using a joint likelihood function, where the theoretical prediction for σ_p^{SI} was obtained using MicrOMEGAs 5.0.2. In the case for the vector mediator, the SI cross section has a scaling [13] and approximate numerical value of the following forms:

$$\sigma_p^{\text{SI}} \propto \frac{g_{\text{DM}}^2 g_{\text{SM}}^2}{m_Y^4}; \quad \sigma_p^{\text{SI}} \approx \left(\frac{g_{\text{DM}}}{0.1} \right)^2 \left(\frac{g_{\text{SM}}}{0.1} \right)^2 \left(\frac{m_Y}{1 \text{ TeV}} \right)^{-4} \times 10^{-43} \text{ cm}^2 \quad (3.15)$$

in the s -channel region where the mediator mass m_Y is approximately twice of the DM mass m_χ . From the combined σ_p^{SI} constraint, crucial limits on the vector mediator mass, m_Y , and the product of the DM and SM couplings to the vector mediator, $g_{\text{DM}} g_{\text{SM}}$, can be obtained. Note that a 10% overall uncertainty in σ_p^{SI} was permitted.

⁵⁰ Large Underground Xenon experiment at Sanford Underground Research Facility (SURF) in South Dakota, US

3.2.1.3 Spin-Dependent (SD) DM Scattering

Using the upper limits on the spin-dependent (SD) DM-proton scattering cross-section σ_p^{SD} from the PICO-60⁵¹ experiment [46] and the SD DM-neutron scattering cross-section σ_n^{SD} from the XENON1T experiment [47], a constraint for both σ_p^{SD} and σ_n^{SD} was also obtained. Using a similar procedure as that for σ_p^{SI} , the theoretical predictions for σ_p^{SD} and σ_n^{SD} was also obtained using MicrOMEGAs 5.0.2, which used the estimated values of the elements in the spin-dependent scattering matrix consistent with [48]. Same as the case for σ_p^{SI} , a 10% overall uncertainty in the spin-dependent cross sections, both σ_p^{SD} and σ_n^{SD} , was also allowed.

3.2.1.4 Standard Halo Model (SHM)

Most direct searches for DM scattering assumed the so-called Standard Halo Model (SHM), where the model assumes that the local CDM density ρ_0 is 0.30 GeV/cm³ with negligible uncertainty. However, some recent studies seemed to favour a higher ρ_0 , in particular $\rho_0 = (0.55 \pm 0.17)$ GeV/cm³ as mentioned in [49]. Hence, the direct DM scattering cross-section limits, estimated future sensitivity curves and “neutrino floors” used in the analyses were rescaled to the curves published in previous analyses.

A recent analysis of the data from the Gaia satellite seemed to show evidence of a local debris flow that might affect the local velocity distribution, causing it to differ from that given by SHM [50]. However, this change in the local velocity distribution was understood to have minimal impact on the interpretation of the DM mass, m_χ , data from the XENON1T experiment in the range, given in Table 3.1. A similar conclusion was also suggested in [49]. Upon implementing the experimental constraints for both the SI and SD cross sections (σ_p^{SI} , σ_p^{SD} and σ_n^{SD}), a 30% overall uncertainty in the local DM density [49, 51] was imposed.

3.2.2 LHC Experimental Constraints

3.2.2.1 Monojet Constraints

Investigations conducted by the ATLAS [52] and CMS [42] Collaborations focusing on searches for monojet final states have placed constraints on spin-one mediators decaying to missing transverse momentum. In particular, results published by the CMS Collaboration [42], using the data from proton-proton (pp) collisions corresponding to an integrated luminosity of 35.9 fb⁻¹ at a centre-of-mass energy $\sqrt{s} = 13$ TeV, was used in the analyses. The analysis published by the ATLAS collaboration [52] for the case where the couplings are axial-vector in nature has a similar sensitivity. Note that the 95% Confidence Level (CL) upper limit $R_i^{\text{UL}}(\mathbf{m})$, where i identifies the type of mediator whether it is vector and axial-vector, which was imposed on the $\sigma(\mathbf{m})/\sigma^{\text{fix}}(\mathbf{m})$, where σ^{fix} refers to the reference

⁵¹ A C₃F₈ DM detector at Canada’s Surbury Neutrino Observatory underground Laboratory (SNOLAB)

cross section calculated using $(g_{\text{SM}}, g_{\text{DM}}) = (0.25, 1)$, evaluated at each and every single point in the $\mathbf{m} = (m_Y, m_\chi)$ plane was used in the analysis of this constraint. Using Figure 10 from [42], the plots of the expected and observed 95% CL upper limits for both the vector and axial-vector mediators are shown below:

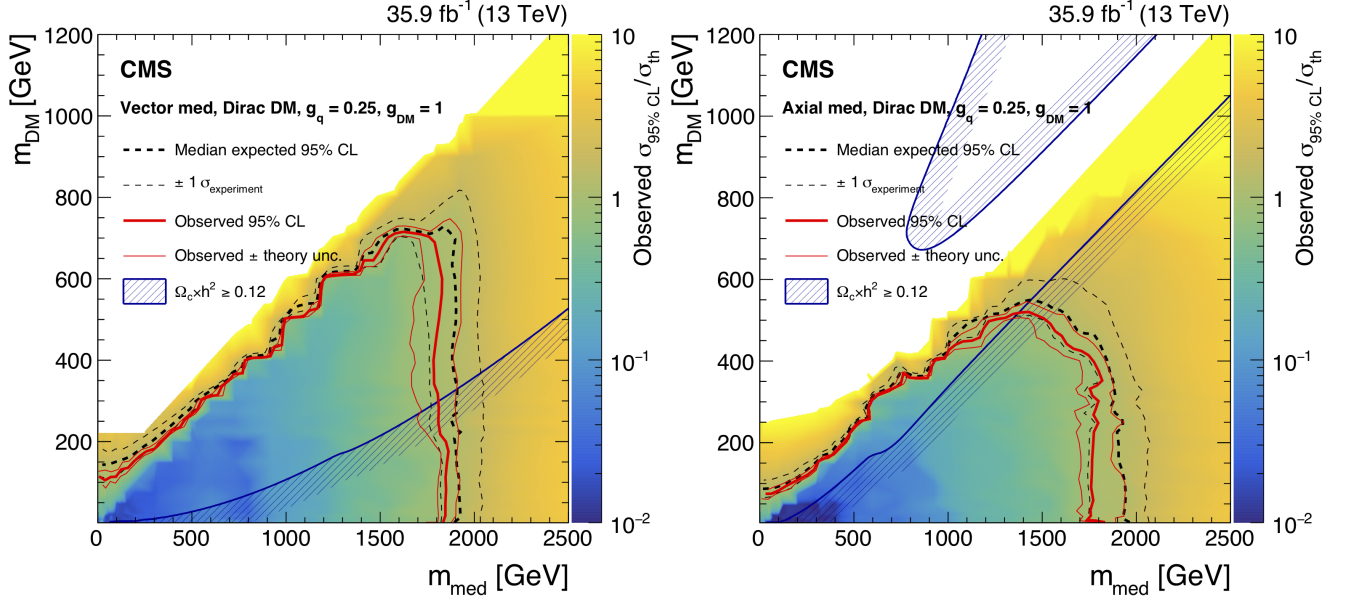


Figure 3.2: 95% CL exclusion limits imposed on $\sigma(\mathbf{m})/\sigma^{\text{fix}}(\mathbf{m})$ in the $m_{\text{med}} - m_{\text{DM}}$ ($m_Y - m_\chi$) plane for both cases where the mediator is either a vector (on the left) or an axial-vector (on the right)

The likelihood function based on these results was modelled using a procedure similar to that used in Fastlim [53] as shown below:

$$\Delta\chi^2 = 5.99 \times \left[\frac{1}{R_i^{\text{UL}}(\mathbf{m})} \left(\frac{\sigma_{\text{MG}}(\mathbf{m})}{\sigma_{\text{MG}}^{\text{fix}}(\mathbf{m})} \right) \right]^2, \quad (3.16)$$

where the subscript MG indicates that the monojet cross-section was evaluated using MG5_aMC@NLO 2.4.3 [29].

Performing another set of cross-checks, the implementations of the CMS monojet constraints, as shown by the limits in Figure 3.2, in MasterCode, represented by the coloured contours and the solid red lines, together with the corresponding $\Delta\chi^2$ calculated directly from the CMS limits [42], represented by the dashed green lines, are shown in the figure at the top of the following page. As one can see, the agreement was excellent, confirming that the monojet constraints from CMS limits were correctly implemented in MasterCode.

3.2.2.2 Dijet Constraints

Both ATLAS and CMS Collaborations have published results on their respective searches for Z' resonances in dijet invariant-mass distributions, using data from pp collisions at

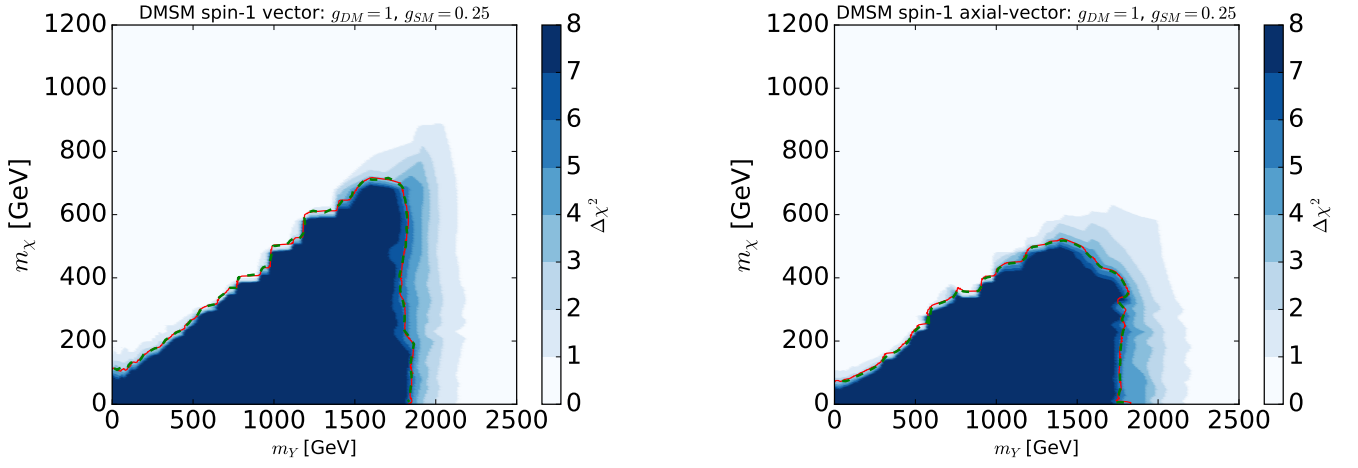


Figure 3.3: The likelihood plots of **MasterCode** implementation (coloured contours and solid red lines) of the CMS monojet constraints (dashed green lines) for a spin-one mediator with vector (on the left) and axial-vector (on the right) DM/SM couplings

$\sqrt{s} = 13$ TeV corresponding to an integrated luminosity of 139 fb^{-1} . A mixture of limits in the $m_{Z'} - g_q$ plane, as studied by a number of papers from both collaborations [54–60], was used in the analyses of the dijet constraints. Focusing on the $pp \rightarrow Y \rightarrow q\bar{q}$ channel in the models under these analyses, the dijet constraints were recasted accordingly and it was required that

$$\sigma_{Y \rightarrow q\bar{q}} \leq \sigma(g_q^*), \quad \text{for } m_Y = m_{Z'} \quad (3.17)$$

where g_q^* refers to the upper limit imposed on the quark coupling to Z' boson for a given $m_{Z'}$. Note that $\sigma(g_q^*)$ refers to the limit of the cross-section assuming a finite Γ_q^* ⁵². Also, note that both expressions on the LHS and RHS of Eq (3.17) can be expressed in the following form

$$\text{Using } \Gamma_x \equiv \Gamma(Y \rightarrow xx) : \sigma_{Y \rightarrow q\bar{q}} = c \left(\frac{g_{SM}^4}{\Gamma_q + \Gamma_\chi} \right); \quad \sigma(g_q^*) = c \left[\frac{(g_q^*)^4}{\Gamma_q^*} \right] \quad (3.18)$$

for $m_Y = m_{Z'}$ and $\Gamma_q = \left(\frac{g_{SM}}{g_q^*} \right)^2 \Gamma_q^*$, where c is some arbitrary constant. Using the relations above, Eq (3.17) attains equality when g_{SM} is equal to

$$g_{SM}^* = \sqrt{\frac{(g_q^*)^2}{2} \left[1 + \sqrt{1 + 4 \left(\frac{\Gamma_\chi}{\Gamma_q^*} \right)} \right]}, \quad (3.19)$$

which would become the upper limit on g_{SM} for a given set of m_Y , Γ_q and Γ_χ . The

⁵² Since the mediator might decay into DM particles, *i.e.* $Y \rightarrow \chi\bar{\chi}$, $\Gamma_Y \neq \Gamma_{Z'}$. However, in the case when this limit is crucial, the branching ratio for $Y \rightarrow q\bar{q}$ channel, *i.e.* $\Gamma_{Y \rightarrow q\bar{q}} / \sum \text{all possible decay channels } \Gamma_{Y \rightarrow \dots}$, becomes significant, justifying the approximation that $\Gamma_Y \sim \Gamma_{Z'}$ in this regime

corresponding likelihood function was modelled using the following equation:

$$\Delta\chi^2 = 4 \times \left(\left[\frac{g_{\text{SM}}^4}{\Gamma_q + \Gamma_\chi} \right] \middle/ \left[\frac{(g_{\text{SM}}^*)^4}{\Gamma_q(g_{\text{SM}}^*) + \Gamma_\chi} \right] \right)^2, \quad (3.20)$$

where the cross sections Γ_q and Γ_χ were evaluated using `MG5_aMC@NLO 2.4.3`.

Performing the final set of cross-checks, the recasted likelihoods for the CMS and ATLAS limits using the procedure above together with the corresponding likelihoods that were calculated directly from the limits provided by CMS and ATLAS [54–60] are shown in the figure⁵³ on the following page. The different coloured dashed lines represent different experimental analyses from the various papers [54–60] published by both the CMS and ATLAS collaborations. The coloured regions in the plane, on the other hand, represent the recasted likelihoods, $\Delta\chi^2$, obtained from `MasterCode`. Note that only the most sensitive upper limit for each value of m_Y was used and no attempt was made to combine the different experimental analyses. As seen in the figure below, the agreement was excellent, confirming that the dijet invariant-mass constraint was correctly implemented in `MasterCode`.

Note that for both the monojet and dijet constraints, the decay widths and the corresponding cross-sections were evaluated only to leading-order (LO) in QCD, as a result of the absence of a comprehensive set of model-dependent next to leading-order (NLO) corrections. Since these corrections are positive in general, the current treatment is expected to be minimal, hence the upper limits on the couplings that were obtained from the analyses of both constraints would become more robust if these NLO corrections were included in the calculations.

3.3 Results

In this section, the results of the sampling for both the vector and axial-vector models (Eqs (3.3) and (3.4)) are presented separately in 2 different sub-sections, the first for [Vectorial Couplings](#) and the second for [Axial Couplings](#). In this section, 2-Dimensional (2D) plots for the different parameter planes are being presented, where their z -axes correspond to the likelihood functions $\Delta\chi^2$ for their respective planes. In these planes, the red contours represent parameter regions where $\Delta\chi^2 < 2.30$ while the blue contours represent parameter regions where $\Delta\chi^2 < 5.99$. In other words, these regions are favoured at 68% and 95% CL, therefore are labelled as 1σ - and 2σ - regions, respectively. In addition, the dominant DM mechanisms driving the DM relic density into the allowed range at the point where the likelihood χ^2 is minimized will be illustrated in these plots

⁵³ The discontinuity in the limit at $m_Y = 2$ TeV is the result of an improvement in sensitivity of a recent ATLAS limit using data from pp collisions at $\sqrt{s} = 13$ TeV corresponding to an integrated luminosity of 139 fb^{-1}

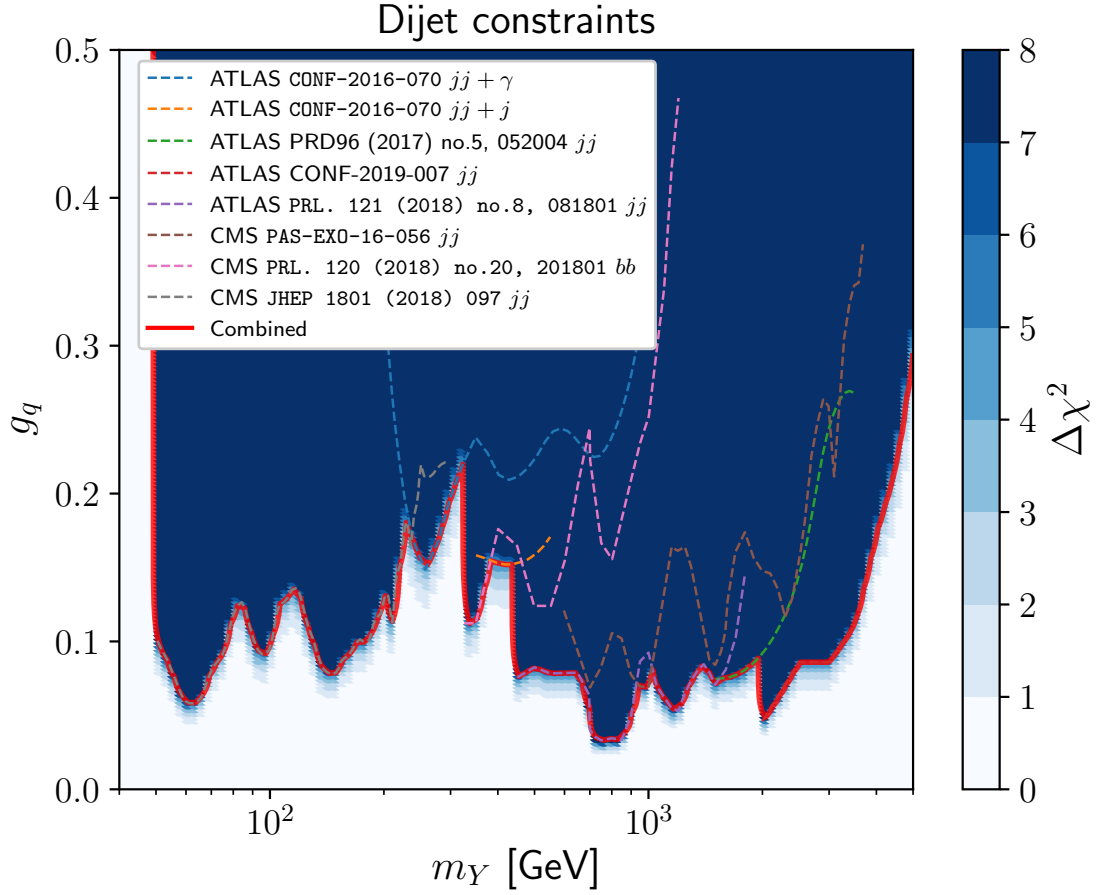


Figure 3.4: Comparison of `MasterCode` implementation (indicated by the colour coding) of the dijet invariant-mass constraint, where the mediator was assumed to be a Z' boson, based on the limits from the ATLAS and CMS Collaborations [54–60] with the corresponding likelihoods that were evaluated directly from these limits in the plane containing the Z' mass and its coupling to quarks

using the following colour coding schemes:

- 1) Green: t -channel annihilation via $\chi\bar{\chi}$ exchange into pairs of mediator particles Y which would then decay into SM particles in the region where $m_\chi \geq m_Y$, let's call this the $\chi\bar{\chi} \rightarrow YY$ channel
- 2) Yellow: Rapid and direct annihilation into SM particles via the s -channel Y resonance, in the region where $0.9 < m_Y/2m_\chi < 1.1$ for the vector case and $0.6 < m_Y/2m_\chi < 2.0$ for the axial case

3.3.1 Vectorial Couplings

The figure at the top of the following page shows the $m_Y - m_\chi$ plane in the model where the couplings are purely vectorial after the implementation of the constraints that were discussed in the previous section. As one can see in the figure, there are two distinctly separated regions in the plane that are favoured, one in the region of $m_Y \approx 2m_\chi$ where rapid annihilation via the Y funnel is the dominant DM mechanism, and the other in the region of $m_\chi > m_Y$ where annihilation via the $\chi\bar{\chi} \rightarrow YY$ channel is the dominant DM

mechanism. Both allowed regions have very sharp boundaries, which shows the steepness in the resonant peak for the s -channel case and the phase-space limit for the t -channel case⁵⁴.

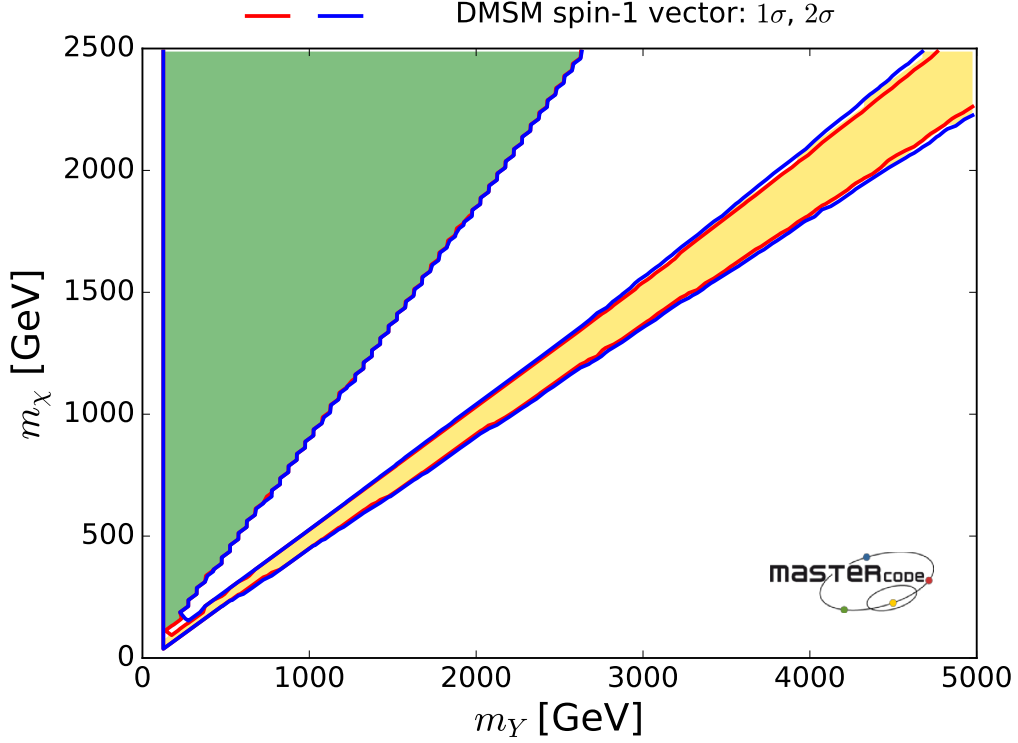


Figure 3.5: Preferred regions at 68% (1σ -) and 95% (2σ -) CL in the (m_Y, m_χ) plane for the vector model. The colour coding denotes the dominant DM mechanisms in the plane

One would observe that for any value of m_χ , the t -channel region is found at lower value of m_Y as compared to that for the s -channel region, as a result of the kinematic constraint $m_Y < m_\chi$. In this region, the following relation was observed:

$$\sigma_{\chi\bar{\chi} \rightarrow YY} \propto g_{\text{DM}}^4 \quad (3.21)$$

and $\sigma_{\chi\bar{\chi} \rightarrow YY}$ independent of g_{SM} . Hence, by tuning the DM coupling g_{DM} , the DM relic density can be driven to the observed value, other than for the case where $m_\chi \lesssim 100$ GeV. On the other hand, by suppressing g_{SM} , direct DM detection and collider constraints from the LHC experiments can be avoided altogether.

As seen in Figure 3.5 above, the vertical line at $m_Y = 100$ GeV representing the lower limit of the sampling range for m_Y enforces the requirement that $m_\chi \lesssim 100$ GeV in the t -channel region. Since the annihilation $\chi\bar{\chi} \rightarrow YY$ channel is kinematically forbidden where $m_Y > m_\chi$, the cross-section for this particular annihilation channel is suppressed, leading to a DM overdensity. In the s -channel region, on the other hand, the DM coupling

⁵⁴ Note that in other planes, the s - and t -channel regions might overlap, with χ^2 values very close to one another. Hence, there might be scattering of points with different colours

g_{DM} is suppressed by the DM relic density constraint (3.11) in the region where $m_Y \sim 100$ GeV, which would in turn mean $\Gamma_Y/m_Y \ll 1$. In addition, the rapid annihilations via the Y resonance are possible only if the following condition is satisfied:

$$m_\chi \approx m_Y/2 \gtrsim 50 \text{ GeV, since } m_Y \lesssim 100 \text{ GeV} \quad (3.22)$$

This sampling restriction on the range of m_Y can be seen as a uncoloured vertical band at very low masses for the rest of the plots.

Throughout the whole range of m_Y beyond the 100 GeV cutoff, $\Delta\chi^2 \approx 0$. For the range of m_χ above the 50 GeV cutoff, a similar trend was also observed where $\Delta\chi^2 \ll 1$ and the likelihood function is basically featureless. In addition, no upper limits were found for the mass ranges that were sampled. For intermediate ranges of the SM couplings g_{SM} , the numerical results from the sampling had suggested that

$$\Omega_\chi h^2 \sim 0.1 \left(\frac{m_Y}{2 \text{ TeV}} \right)^2 \left(\frac{g_{\text{DM}}}{0.01} \right)^{-2} \quad (3.23)$$

Outside these intermediate ranges of g_{SM} , larger value of the DM relic density $\Omega_\chi h^2$ were obtained. Extrapolating the above numerical approximation would suggest that the upper limit for m_Y in the s -channel region would in the order of magnitude of ~ 700 TeV for $g_{\text{DM}} \lesssim \sqrt{4\pi}$.

The figure on the following page shows the $g_{\text{SM}} - g_{\text{DM}}$ plane in logarithmic scales for the model where the g_{SM} and g_{DM} couplings are both purely vectorial. Similar to the $m_Y - m_\chi$ plane in Figure 3.5, one will again observe that there are two clearly distinct preferred regions, which are bordered by the boundaries where $g_{\text{SM}} \sim 3 \times 10^{-4}$ and $g_{\text{DM}} \sim 3 \times 10^{-1}$. Comparing the colour coding in Figure 3.6 and that in Figure 3.5, one will see that the green region at the top left corner of the plot in Figure 3.6 where $g_{\text{SM}} \lesssim 3 \times 10^{-4}$ and $g_{\text{DM}} \gtrsim 3 \times 10^{-1}$ corresponds to triangle-shaped t -channel region on the left half of Figure 3.5. On the other hand, outside this region where $g_{\text{SM}} \gtrsim 3 \times 10^{-4}$ corresponds to the s -channel funnel region⁵⁵ in Figure 3.5, where rapid annihilations via the Y resonance occur. At large values of the product $g_{\text{SM}}g_{\text{DM}}$, this yellow shaded region is bounded by the upper limit on the SI DM scattering cross-section σ_p^{SI} (Eq (3.15)) and the sampling range for m_Y , which is less than 5 TeV. In the region where $3 \times 10^{-4} < g_{\text{DM}} < 7 \times 10^{-2}$, the DM relic density $\Omega_\chi h^2$ is proportional to the ratio m_Y^2/g_{DM}^2 and can be small enough if $m_Y \approx 2m_\chi$ and m_Y is less than 5 TeV. Using the Eqs (3.15) and (3.23), the upper limit on g_{SM} can be approximated using the following expression

$$g_{\text{SM}} \lesssim 0.1 \left(\frac{m_\chi}{1 \text{ TeV}} \right) \left[\frac{(\sigma_p^{\text{SI}})^{\text{UL}}(m_\chi)}{10^{-45} \text{ cm}^2} \right]^{1/2}, \quad (3.24)$$

⁵⁵ The uncoloured area close to the boundary between the 2 coloured regions is the result of incomplete sampling in that region

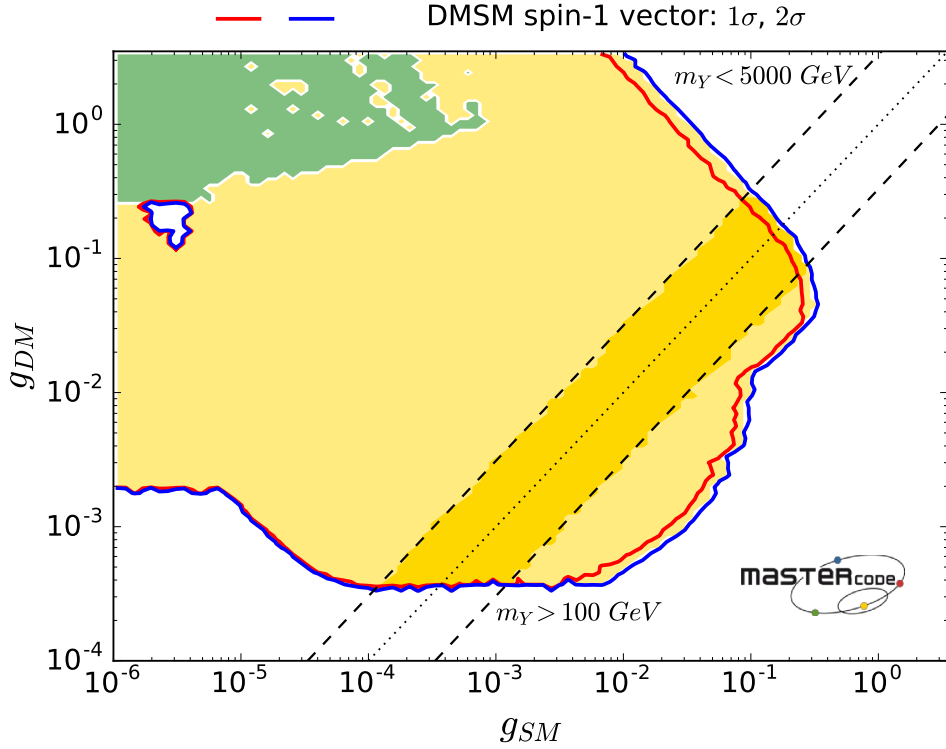


Figure 3.6: Preferred regions in the $(g_{\text{SM}}, g_{\text{DM}})$ plane (in logarithmic scales) for the vector model where the ranges of m_χ and m_Y were to those that were sampled. Again, the colour coding denotes the dominant DM mechanisms in the plane. The central diagonal dotted line denotes the region in the plane where the DM and SM couplings are equal, while the diagonal dashed lines above and below it are used to bound the region where $\frac{1}{3} < g_{\text{DM}}/g_{\text{SM}} < 3$ and this region is shaded with a darker yellow

where $(\sigma_p^{\text{SI}})^{\text{UL}}(m_\chi)$ refers to the experimental upper limit on σ_p^{SI} which is dependent on the DM mass, m_χ . Note that the dijet constraint contributes most significantly at the maximum value of the SM coupling g_{SM} of roughly 3×10^{-1} . Therefore, the lower right corner of Figure 3.6 at large values of g_{SM} was excluded by upper limit on the SI DM scattering cross section, $(\sigma_p^{\text{SI}})^{\text{UL}}$. The exclusion region in the bottom part of the figure where $g_{\text{DM}} \lesssim 3 \times 10^{-4}$, increasing at small values of g_{SM} ($g_{\text{SM}} \lesssim 6 \times 10^{-4}$), is the result of a combination of the lower limit on the sampling range for m_Y , where $m_Y > 100$ GeV, together with the DM relic density $\Omega_\chi h^2$ constraint.

With regards to the 1-Dimensional plot of the likelihood function χ^2 against g_{SM} , it would not be useful to present it here since the likelihood function varies very minimally other than a steep increase when g_{SM} is roughly greater than 3×10^{-1} . The same could be said for the 1D plot of the likelihood function χ^2 against g_{DM} , where the likelihood function in the g_{DM} case also has very little variations other than a significant increase when g_{DM} is roughly less than 3×10^{-4} which is due to the lower limit on the sampling range for m_Y .

The figure below shows the planes of the 2D likelihood functions $\Delta\chi^2$ for the SM and DM couplings, g_{SM} and g_{DM} , as functions of the mass of the mediator, m_Y . On the left, the green-coloured t -channel region in the lower left “quadrant” of the $m_Y - g_{\text{SM}}$ plane shows a strict upper limit on the SM coupling g_{SM} , where $g_{\text{SM}} \lesssim 10^{-3}$. This was the result of a combination of the upper limit on the SI DM scattering cross section, $(\sigma_p^{\text{SI}})^{\text{UL}}$, which bounded the product $g_{\text{SM}}g_{\text{DM}}$, with the lower limit on the DM coupling g_{DM} , as shown by the green-coloured t -channel region at the top left corner of the $m_Y - g_{\text{DM}}$ plane where $g_{\text{DM}} \gtrsim 4 \times 10^{-1}$, as a result of the lower limit on the sampling range for m_Y . Recalling Eq (3.15), since the SI DM scattering cross section σ_p^{SI} has the following scaling

$$\sigma_p^{\text{SI}} \propto \frac{g_{\text{DM}}^2 g_{\text{SM}}^2}{m_Y^4}, \quad (3.25)$$

the upper limit on g_{SM} would be stricter for smaller values of m_Y , where $g_{\text{SM}} < 10^{-4}$ if $m_Y \lesssim 100$ GeV. For small values of g_{SM} , the upper limit on m_Y towards the right of the green-coloured t -channel region is a result of the upper limit on the sampling range for m_χ , where $m_\chi < 2.5$ TeV. The main contribution to the upper limit on g_{SM} in the yellow-coloured s -channel region is from the dijet constraint, with an additional contribution from the SI DM scattering cross section, σ_p^{SI} , constraint where $m_Y \lesssim 500$ GeV.

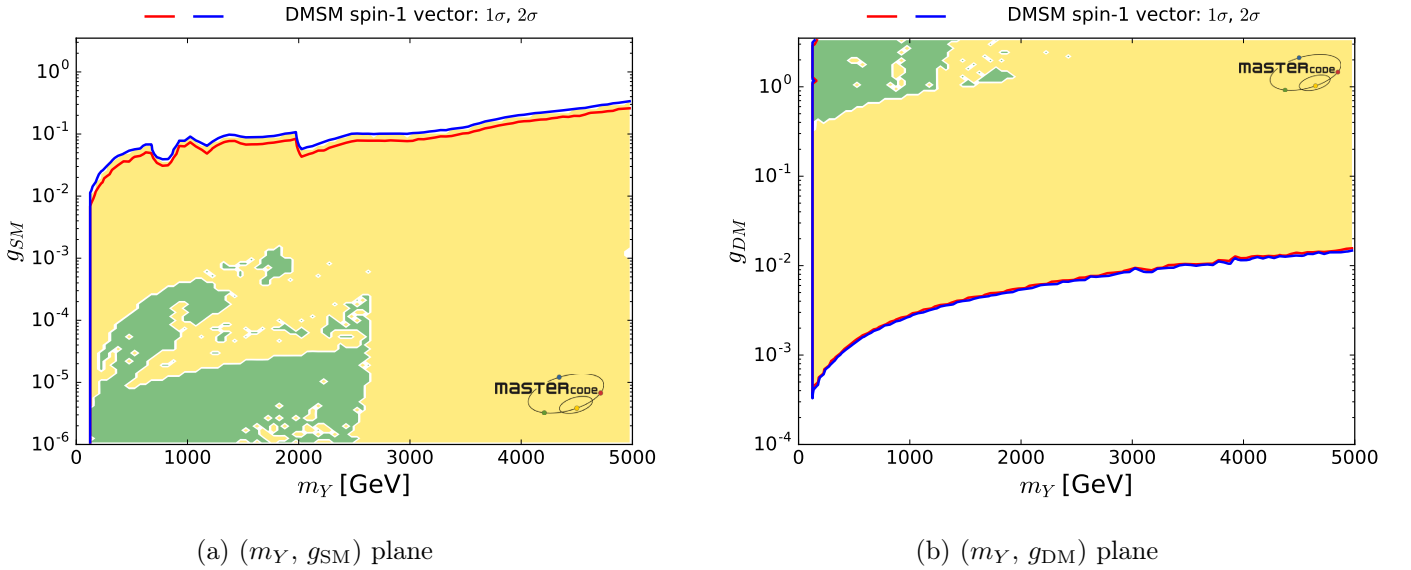


Figure 3.7: The 2D likelihood functions χ^2 for the SM coupling g_{SM} (on the left) and the DM coupling g_{DM} (on the right) as functions of the mediator mass, m_Y , in the vector model. Again, the colour coding denotes the dominant DM mechanisms in the 2 planes.

Note that when regions in which both the s - and t -channel mechanisms are dominant were to be projected in some specific region of the $m_Y - g_{\text{SM}}$ plane, these regions would overlap as a result of the very similar likelihoods χ^2 and as a result of statistical fluctuations one mechanism would be chosen over the other.

On the right of Figure 3.7, the lower limit on g_{DM} , which is the DM coupling, in the

green-coloured t -channel region at the top left corner of the $m_Y - g_{\text{DM}}$ plane was a result of the DM relic density $\Omega_\chi h^2$ constraint. On the other hand, the $\Omega_\chi h^2$ constraint had less of an impact on the limit on the DM coupling in the yellow-coloured s -channel region, which is given by the approximate inequality below:

$$\left(\frac{m_Y}{2\text{TeV}}\right)^2 \left(\frac{g_{\text{DM}}}{0.01}\right)^2 \lesssim 1 \quad (3.26)$$

following from Eq (3.23) when $m_Y \sim 2m_\chi$. Note that, similar to what was observed for the $m_Y - g_{\text{SM}}$ plane, the scattered coloured islands in the region at the top left corner of the $m_Y - g_{\text{DM}}$ plane were due to the overlapping of the regions where both the s - and t -channel mechanisms dominate as a result of the very similar likelihoods χ^2 .

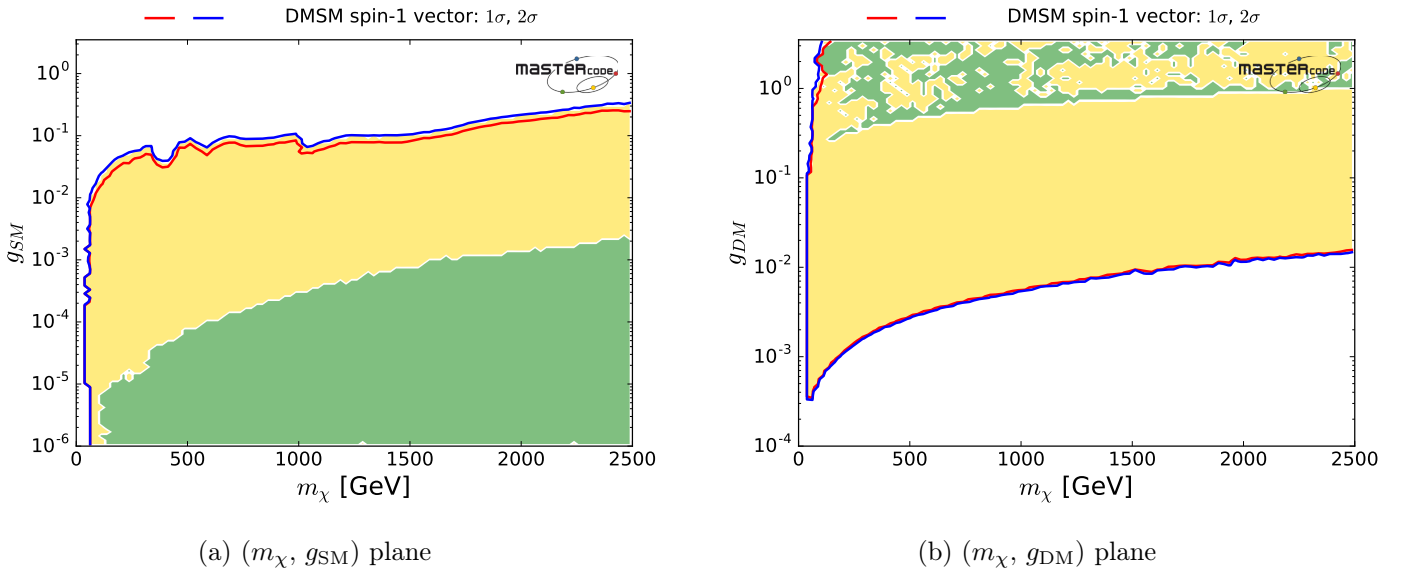


Figure 3.8: The 2D likelihood functions χ^2 for the SM coupling g_{SM} (on the left) and the DM coupling g_{DM} (on the right) as functions of the DM mass m_χ , in the vector model. Again, the colour coding denotes the dominant DM mechanisms in the 2 planes.

The above figure shows the planes of the 2D likelihood functions $\Delta\chi^2$ for g_{SM} , on the left, and g_{DM} , on the right, as functions of the DM mass, m_χ . The upper limit on g_{SM} and the lower limit on g_{DM} are identical to that seen in Figure 3.7, other than noting that now $m_\chi = m_Y/2$, and one would observe that both limits are increasing with increasing m_χ .

The figure on the following page shows the 2D likelihood function χ^2 in the $(m_\chi, \sigma_p^{\text{SI}})$ plane for the vector model. As seen on the left of Figure 3.8, only values of $m_\chi \gtrsim 50$ GeV were permitted due to the combination of the lower limit on the sampling range for m_Y , where $m_Y > 100$ GeV, together with the DM relic density $\Omega_\chi h^2$ constraint, as mentioned earlier in the paragraph below Eq 3.24. Beyond this cutoff, a wide range of σ_p^{SI} values is permitted in both the green-coloured t - and the yellow-coloured s -channel regions. The upper limits on σ_p^{SI} from the different experiments, denoted by the different-coloured solid contours, were obtained from the combined experimental likelihood for these experiments,

which were rescaled to account for the different local DM relic density that was assumed by the $\Omega_\chi h^2$ constraint. In the yellow-coloured s -channel region, small values of σ_p^{SI} would be permitted for small values of the DM/SM couplings, g_{DM} or g_{SM} , and for values of m_χ satisfying $m_Y \sim 2m_\chi$ that were favoured by the $\Omega_\chi h^2$ constraint. In the green-coloured t -channel region, small values of σ_p^{SI} would also be permitted as a result of the small values of g_{SM} that are permitted in the $m_\chi - g_{\text{SM}}$ plane on the left of Figure 3.8. As one can see in the figure below, the range of σ_p^{SI} in the s - and t -channel regions are well within the sensitivities of future experiments, such as the LUX-ZEPLIN (LZ)⁵⁶ [61] and XENONnT⁵⁷ [62] experiments which are denoted by dashed purple and blue contours respectively, although it might be significantly lower than the sensitivities of current experiments. There might even be a possibility of σ_p^{SI} being below the “neutrino floor” which is denoted by the light yellow region bounded by the dashed orange contour, as suggested by [49, 63, 64].

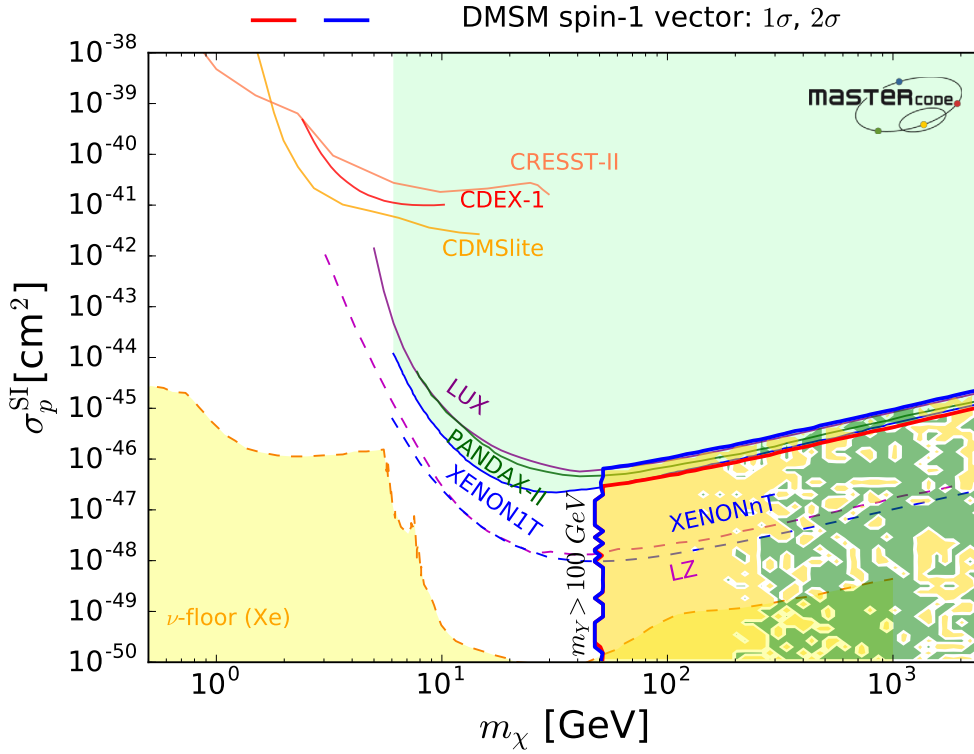


Figure 3.9: The likelihood contours in the $(m_\chi, \sigma_p^{\text{SI}})$ plane for the vector model. The plane includes the current upper limits from the LUX [43] (solid purple contour), PANDAX-II [44] (solid green contour) and XENON1T [45] (solid blue contour) experiments, the “neutrino floor” [63, 64] (the light yellow region bounded by the dashed orange contour), and the possible sensitivities of the future LUX-ZEPLIN [61] (dashed purple contour) and XENONnT [62] (dashed blue contour) experiments. Again, the colour coding denotes the dominant DM mechanisms in the plane. The effective lower limit on m_χ , where $m_\chi > 50$ GeV, which was imposed by the lower limit of the sampling range for m_Y is also shown.

⁵⁶ An upcoming experiment which supercedes the LUX experiment, decommissioned in 2016, and is expected to start data collection from 2020 onwards

⁵⁷ An upgrade of the XENON1T experiment which has stopped data-collection since the start of 2019

3.3.2 Axial Couplings

Now moving on from the analysis of the vector model to the analysis of the model where the couplings are purely axial after the constraints discussed in the previous section were implemented. Following the order of the previous subsection for the model with [Vectorial Couplings](#), the first figure in this subsection, as shown below, shows the $m_Y - m_\chi$ plane for the case where both the SM and DM couplings, g_{SM} and g_{DM} , are purely axial.

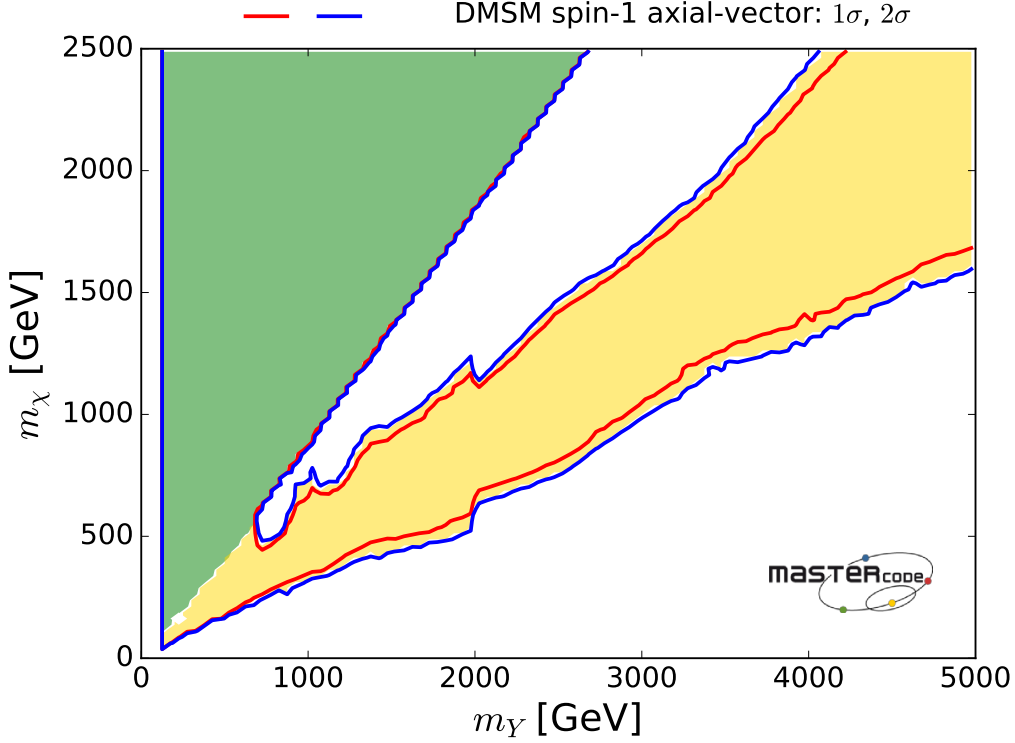


Figure 3.10: Preferred regions at 68% (1σ -) and 95% (2σ -) CL in the (m_Y, m_χ) plane for the axial-vector model. Exactly the same as that in the vector case, the colour coding denotes the dominant DM mechanisms in the plane.

If one compares the $m_Y - m_\chi$ plane in the plot above with the same plot for the vector case in Figure 3.5, one would observe that the yellow-coloured s -channel funnel is wider for the axial case compared to that for the vector case. In the case where the mediator is a vector as discussed in the previous subsection, the product $g_{\text{SM}}g_{\text{DM}}$ was suppressed as a result of the upper limit on the SI DM scattering cross section $(\sigma_p^{\text{SI}})^{\text{UL}}$. In addition, for the sufficient annihilation of DM particles⁵⁸, certain parameter ranges were favoured near the resonance peak where $m_Y \approx 2m_\chi$. However, in the case where the mediator is an axial-vector, since the σ_p^{SI} constraint does not apply, off-resonance regions with larger values of the product $g_{\text{SM}}g_{\text{DM}}$ are permitted, where the quark mass m_q is suppressed. This results in the widening of the yellow-coloured s -channel region in the $m_Y - m_\chi$ plane to include regions where there are deviations from the condition set by the dijet constraint which requires $m_Y \sim 2m_\chi$. Note that for regions where $m_Y > 2m_\chi$, this constraint is weaker since there would be a possibility that the mediator particle Y might decay back

⁵⁸ So as to reduce the DM relic density $\Omega_\chi h^2$ to the allowed range

into the DM particle-anti-particle $\chi\bar{\chi}$ pair. In addition, note that at low masses, same as that seen in Figure 3.5 for the vector case, the yellow-coloured s -channel funnel also merges with the green-coloured triangular t -channel region, where $m_Y < m_\chi$, in the axial case.

Again, exactly the same as the vector case, the lower limit on DM mass, where $m_\chi \gtrsim 50$ GeV, is due to the combination of the lower limit on the sampling range for m_Y , *i.e.* $m_Y > 100$ GeV, and the DM relic density $\Omega_\chi h^2$ constraint. Just as that observed in the vector case, beyond the above thresholds for both m_Y and m_χ , the 1D $\Delta\chi^2$ functions are featureless.

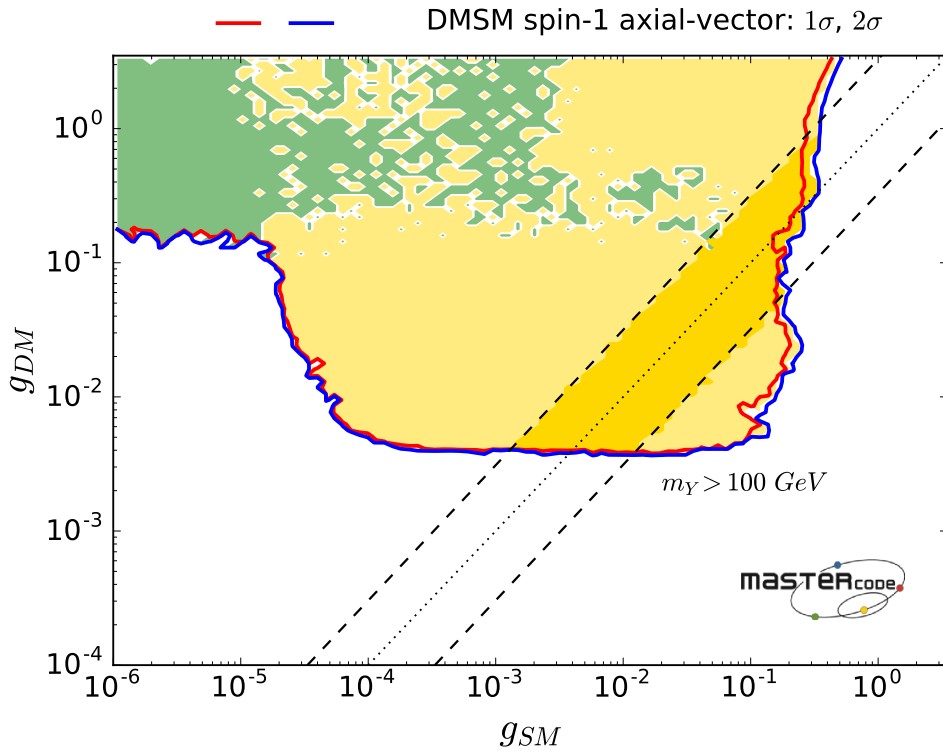


Figure 3.11: Preferred regions in the $(g_{\text{SM}}, g_{\text{DM}})$ plane (in logarithmic scales) for the axial-vector model. Again, same as the plot in the vector case, the colour coding denotes the dominant DM mechanisms in the plane. Just as the case for vector model, the central diagonal dotted line denotes the region in the plane where the DM and SM couplings are equal, while the diagonal dashed lines above and below it are used to bound the region where $\frac{1}{3} < g_{\text{DM}}/g_{\text{SM}} < 3$ and this region is shaded with a darker yellow

The figure overhead shows the $g_{\text{SM}} - g_{\text{DM}}$ plane in logarithmic scales for the model where both couplings are purely axial. Since the values of the product $g_{\text{SM}}g_{\text{DM}}$ are not suppressed for the axial case as compared to that for vector case, larger values of the SM and DM couplings, g_{SM} and g_{DM} , are allowed in the yellow-coloured s -channel region. Even values as large as the upper limit on the sampling range for g_{DM} , $g_{\text{DM}} \sim \sqrt{4\pi}$, are permitted. In the region of the plane where $g_{\text{SM}} \sim 10^{-2}$ and $g_{\text{DM}} \sim 10^{-1}$, one would observe that there seems to be a separation between the regions where the s -, shaded in yellow, and

t -channel, shaded in green, mechanisms dominate. Just like for the vector case, the upper limit on g_{SM} is a result of the dijet constraint. In addition, just like in the vector case, there is a region at $g_{\text{DM}} \sim 7 \times 10^{-2}$, increasing at small values of g_{SM} ($g_{\text{SM}} \lesssim 10^{-4}$), whose exclusion by the DM relic density $\Omega_\chi h^2$ constraint is the consequence of the lower limit set on the sampling range for m_Y , where $m_Y > 100$ GeV.

The figure on the below shows the $m_Y - g_{\text{SM}}$, on the left, and $m_Y - g_{\text{DM}}$, on the right, planes for the model where the couplings are purely axial. On the left, the t -channel DM mechanism, denoted by the green region in the lower left “quadrant” of the $m_Y - g_{\text{SM}}$ plane, is dominant for values of $m_Y \lesssim 2.6$ TeV. The limit at the middle of the horizontal m_Y axis was the result of the upper limit set on the sampling range for m_χ , where $m_\chi < 2.5$ TeV. As seen on the left of Figure 3.12, with increasing values of m_Y and g_{SM} , the s -channel DM mechanism becomes more dominant and is the only DM mechanism for ranges of m_Y above 2.6 TeV, but up to the upper bound of the sampling range which is 5 TeV. As mentioned in the previous paragraph, the upper bound on g_{SM} where $m_Y < 5$ TeV is the result of the dijet constraint. Unlike the vector case as seen on the left of Figure 3.7, the s -channel region in the axial case was excluded for small values of g_{SM} . This is to prevent the DM relic density $\Omega_\chi h^2$ from exceeding the observed range as a consequence of the approximate scaling mentioned in Eq 3.9.

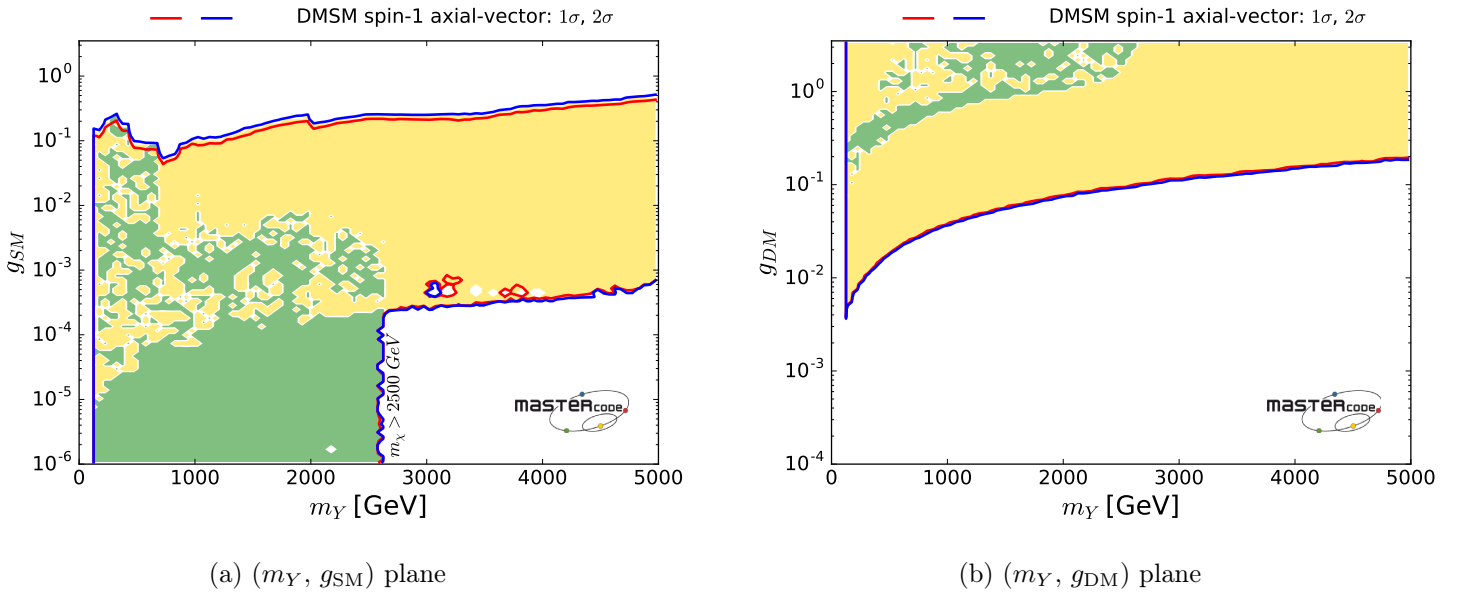


Figure 3.12: The 2D likelihood functions χ^2 for the SM coupling g_{SM} (on the left) and the DM coupling g_{DM} (on the right) as functions of the mass of the mediator, m_Y , in the axial-vector model. Again, the colour coding denotes the dominant DM mechanisms in the 2 planes.

Similarly for the $m_Y - g_{\text{DM}}$ plane on the right, the t -channel mechanism, denoted by the green regions in the top left corner of the plane, on one hand is pertinent for values of $m_Y \lesssim 3$ TeV and large values of g_{DM} , $g_{\text{DM}} \gtrsim 10^{-1}$. On the other hand, the s -channel mechanism, denoted by the yellow-coloured region, dominates for larger values of m_Y or

smaller values of g_{DM} above 8×10^{-2} . Just as in the vector case, the lower limit on g_{DM} in the axial case is also the consequence of the DM relic density $\Omega_\chi h^2$ constraint. Just as in the case for the $m_Y - g_{\text{SM}}$ plane, for small values of g_{DM} and when $m_Y = 2m_\chi$, the DM relic density $\Omega_\chi h^2$ exceeds the observed range due to the approximate scaling mentioned in Eq 3.9.

The figure on the following page shows the $m_\chi - g_{\text{SM}}$, on the left, and the $m_\chi - g_{\text{DM}}$, on the right, planes for the model where the couplings are purely axial. In the $m_\chi - g_{\text{SM}}$ plane on the left, values of m_χ , up to 2.5 TeV which is the upper limit on its sampling range, for both smaller values, where the t -channel DM mechanism dominates, and larger values of g_{DM} , where the s -channel DM mechanism dominates, are allowed. For very low values of the DM mass m_χ ⁵⁹, there is a region where the s -channel mechanism, coloured in yellow, is the dominant DM mechanism. The lower bound on g_{DM} is constrained by the DM relic density $\Omega_\chi h^2$ as described by Eqs (3.12) and (3.13).

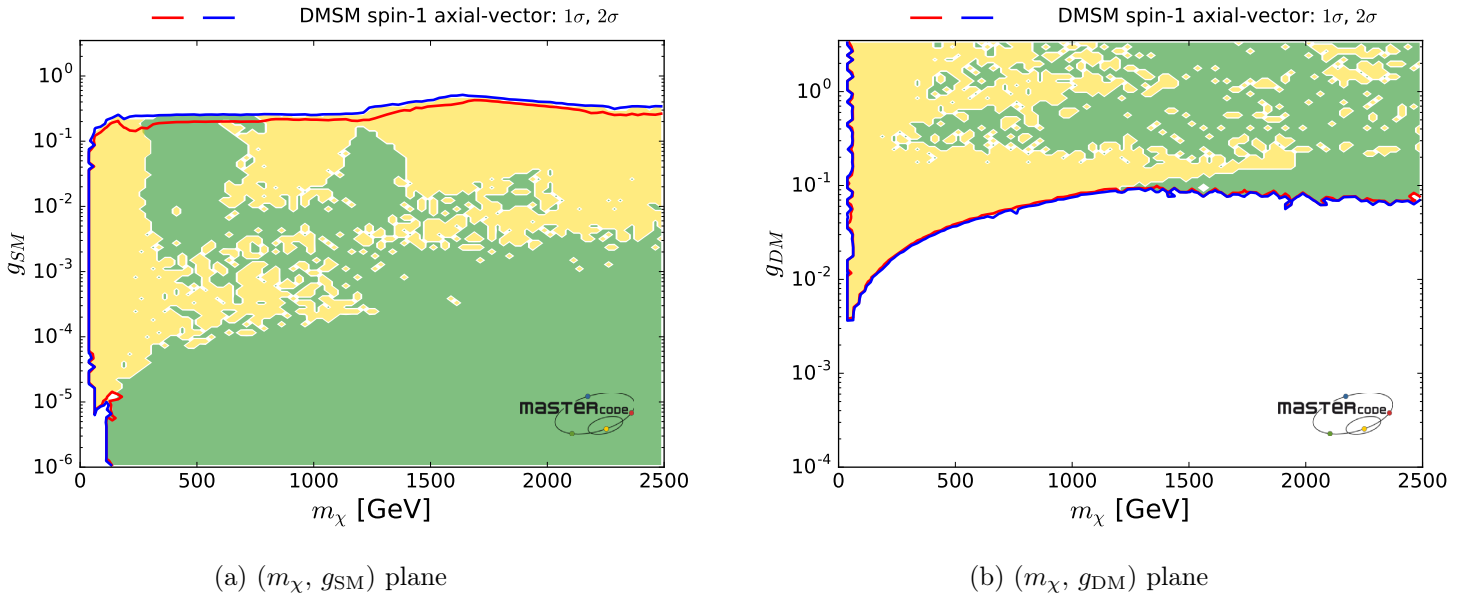


Figure 3.13: The 2D likelihood functions χ^2 for the SM coupling g_{SM} (on the left) and the DM coupling g_{DM} (on the right) as functions of the DM mass, m_χ , in the axial-vector model. Again, the colour coding denotes the dominant DM mechanisms in the 2 planes.

Note that in the axial-vector model, the 1D $\Delta\chi^2$ functions for both g_{SM} and g_{DM} increase drastically for values of $g_{\text{SM}} \gtrsim 3 \times 10^{-1}$ and $g_{\text{DM}} \lesssim 4 \times 10^{-3}$ respectively. Just as that observed in the $m_Y - m_\chi$ plane for the axial case, the 1D $\Delta\chi^2$ functions for both m_Y and m_χ are featureless for values beyond 100 GeV and 50 GeV respectively.

Lastly, the figure on the subsequent page shows the current limits on the spin-dependent (SD) DM-proton, σ_p^{SD} [46] (on the left), and DM-neutron, σ_n^{SD} [47] (on the right), scattering cross sections from direct DM detection experiments, the potential sensitivities of

⁵⁹ Towards the extreme left of the $m_\chi - g_{\text{SM}}$ plane which is still within the red (1 σ -) and blue (2 σ -) contours

future experiments on σ_p^{SD} and σ_n^{SD} , and the regions in both the $m_\chi - \sigma_p^{\text{SD}}$ and $m_\chi - \sigma_n^{\text{SD}}$ planes that are favoured by the axial-vector model. Just as in the vector model, the current experimental limits and potential experimental sensitivities in the axial-vector model were also rescaled to account for the different local DM relic density that was assumed by the $\Omega_\chi h^2$ constraint. Constraints on σ_p^{SD} based on solar $\chi\bar{\chi} \rightarrow \tau^+\tau^-$ annihilation⁶⁰ limits provided by the Super-Kamiokande⁶¹ (Super-K) [65] and IceCube experiments [66] were irrelevant since only leptophobic mediators were considered in the analyses. In addition, hadronic annihilation limits were not as competitive as the limits from the direct constraints on σ_p^{SD} , hence they were also not considered in the analyses.

On the left of Figure 3.14, the estimated “neutrino floor” applicable only to experiments that use C_3F_8 in their detectors was denoted in the $m_\chi - \sigma_p^{\text{SD}}$ plane by the light blue region bounded by the dashed blue contour. As one can see in the plot, the upper 95% CL limit on σ_p^{SD} , denoted by the blue 2σ - contour, for the axial-vector model is closest to the current limit from the PICO-60 experiment, denoted by the solid brown contour, within the range of $90 \text{ GeV} \lesssim m_\chi \lesssim 200 \text{ GeV}$. This upper limit on σ_p^{SD} might even be accessible to the upcoming LZ [61] and PICO-500 experiments within the range of $50 \text{ GeV} \lesssim m_\chi \lesssim 2 \text{ TeV}$. On the other hand, as one can see in the plot of the $m_\chi - \sigma_n^{\text{SD}}$ plane on the right of Figure 3.14, the upper 95% CL upper limit on σ_n^{SD} , denoted by the blue 2σ - contour, for the axial-vector model is sensitive to the limits from XENON1T experiment, which are currently the most sensitive limits to SD DM-neutron scattering. Furthermore, these limits might improve drastically based on the proposed potential sensitivity of the upcoming LZ experiment. Similar to the plot for the $m_\chi - \sigma_p^{\text{SD}}$ plane, the “neutrino floor” applicable only to experiments that use Xenon (Xe) in their detectors was also plotted in the $m_\chi - \sigma_n^{\text{SD}}$ plane as the blue-coloured region bounded by the dashed blue contour. Note that there are quite a few regions in both the $m_\chi - \sigma_p^{\text{SD}}$ and $m_\chi - \sigma_n^{\text{SD}}$ planes where allowed values of σ_p^{SD} and σ_n^{SD} in both the s - and t -channel regions are accessible by experiments above their corresponding “neutrino floors”.

3.4 Summary

To summarise what was discussed this chapter, we have motivated the need for the development of Dark Matter Simplified Models (DMSMs) as a possible Beyond Standard Model (BSM) theory. It was then followed by a brief introduction of these models, including the Lagrangians describing the interactions in the theory, the restrictions on the analyses of these models, and the analyses framework in **MasterCode**. The constraints considered in the analyses, which were categorised into astrophysical DM constraints and collider constraints from the experiments at the LHC, was then discussed in the following section. The results of the analyses, for the model where the couplings were purely

⁶⁰ Self-annihilation of DM particles trapped in the sun

⁶¹ Japan’s Super-Kamioka Neutrino Detection Experiment in the Mozumi Mine under Mount Keno, near the city of Hida

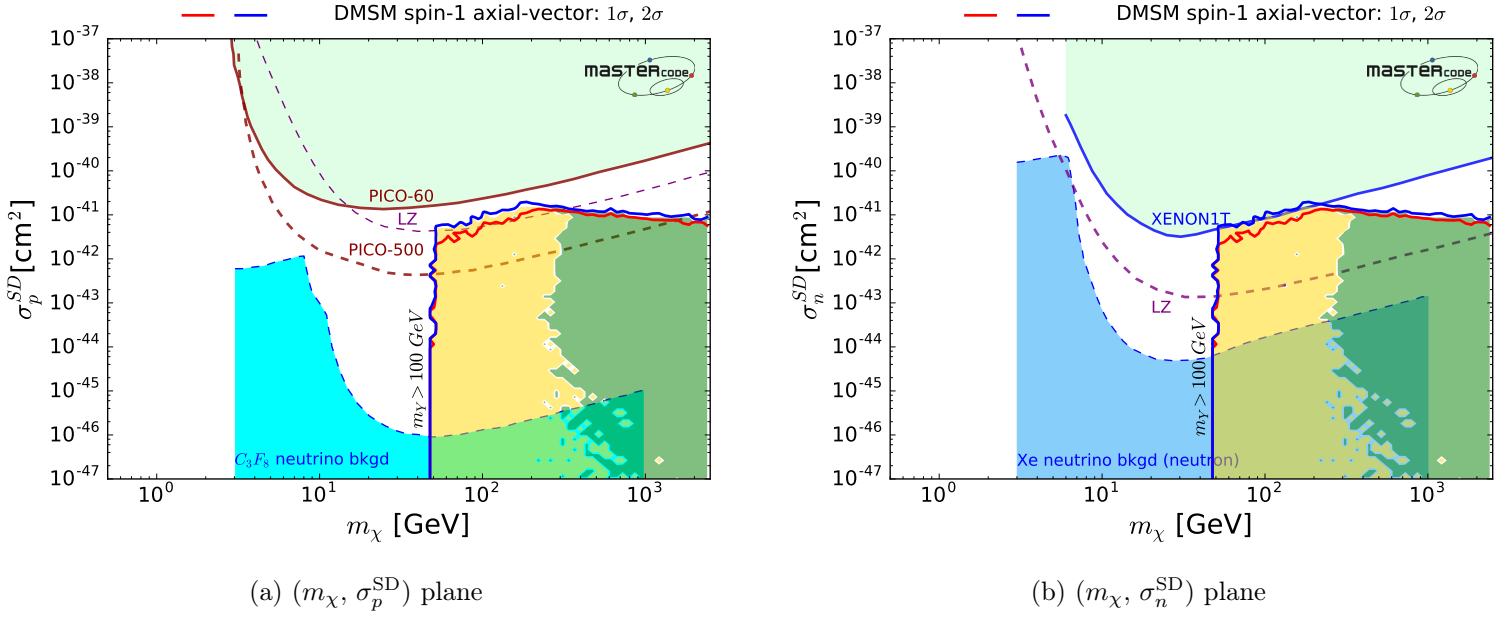


Figure 3.14: The likelihood contours in the (m_χ, σ_p^{SD}) , on the left, and (m_χ, σ_n^{SD}) , on the right, planes for the axial-vector model. On the left, the $m_\chi - \sigma_p^{SD}$ plane contains the current rescaled upper limit on σ_p^{SD} from the PICO-60 experiment (solid brown contour), the potential sensitivities of the future PICO-500 (dashed brown contour) and LZ (dashed purple contour) experiments to σ_p^{SD} , and the “neutrino floor” applicable only to the PICO-60 and PICO-500 experiments which use C₃F₈ in their detectors (the light blue region bounded by the dashed blue contour). On the right, the $m_\chi - \sigma_n^{SD}$ plane contains the current upper rescaled upper limit on σ_n^{SD} from the XENON1T experiment (solid blue contour), the potential sensitivity of the future LZ (dashed purple contour) experiment to σ_n^{SD} , and the “neutrino floor” applicable only to experiments, such as the XENON1T and LZ experiments, that use Xenon (Xe) in their detectors (the blue region bounded by the dashed blue contour). Again, the colour coding denotes the dominant DM mechanisms in both planes. Same as that in the vector case, the effective lower limit on m_χ , where $m_\chi > 50$ GeV, which was imposed by the lower limit of the sampling range for m_Y is also shown.

vectorial and the other where the couplings were purely axial, was then discussed in the following section. Note that only DMSMs with leptophobic spin-one mediators was only considered and discussed in this chapter. For DMSMs where leptonic couplings were considered, such models will be discussed in Chapter 5.

Chapter 4



MasterCode Framework

In the previous chapter, we have discussed Dark Matter Simplified Models (DMSMs) with leptophobic spin-one mediators. In addition, we have also discussed very briefly how **MasterCode** was implemented in the analyses of these models and how the various external packages, namely **MG5_aMC@NLO** 2.4.3 and **MicrOMEGAs** 5.0.2, implemented in **MasterCode** were used to calculate various observables, such as cross sections and the DM relic density $\Omega_\chi h^2$. Hence, in this chapter, we will discuss in greater detail about the **MasterCode** framework [1], in particular with regards to the layout of the package and the external packages that were used in the analyses of DMSMs. We will also have more in-depth discussion on how the sampling was conducted within **MasterCode** in addition to what was discussed (very briefly) in the previous chapter.

Before we start discussing in detail about **MasterCode** let's do brief introduction about what the framework is about. The **MasterCode** framework is a package that interfaces various public and private code packages, and uses these packages to conduct global fits based on experimental constraints that act as initial sampling conditions. The package is mainly written in **Python**, **Cython**⁶², **C++** and **Fortran**⁶³.

Previously, **MasterCode** was mainly used for the analysis of supersymmetric (SUSY) models, such as the Constrained *Minimal Supersymmetric Standard Model* (CMSSM) [67] and the phenomenological Minimal Supersymmetric Standard Model with 11 parameters (pMSSM11) [68]. However, the framework can more generally be used to analyse various other possible Beyond Standard Model (BSM) theories, in particular DMSMs [1].

As mentioned in the previous chapter, the main objectives of the **MasterCode** framework are to combine constraints from different sectors, understand their relationships to one another and locate allowed regions in the parameter spaces that are of interest to models based on these constraints. The package aims to achieve the above objectives using the following methodology:

- A) First, the individual likelihood functions χ^2 are calculated for each of the constraints;

⁶² Some of the codes in **MasterCode** were written in **Cython** to improve their performance to something similar to that if these codes were written in **C/C++**. This will immensely improve the efficiency of computationally intensive jobs during the sampling phase

⁶³ Most of the external packages in **MasterCode** were written in **Fortran** for historical and performance reasons

- B) It is then followed by the evaluation of the combined likelihood function based on all the constraints, which is merely the sum of all individual likelihood functions that were calculated for each of the constraints.

As seen in the previous chapter, the likelihood functions for the individual constraints require the outputs of observables from the external packages. However, most of the external packages used by **MasterCode** require input files that adhere to the SUSY Les Houches Accord (SLHA)⁶⁴ [69, 70] for computation of these observables. Although this convention for input files is mainly designed for SUSY or SUSY-related computations, it can also be used for the computation of other models since these files are organised in a way where model related parameters are stored as **BLOCK**'s so that they can be easily extracted for the computation of observables. An example of a SLHA file will be shown later in the chapter as an illustration of how such a file will look like.

4.1 External Packages

As mentioned earlier in the chapter, **MasterCode** is a package that interfaces various external packages and uses these packages for the computation of various observables relevant to the theory or model under analysis. Since **MasterCode** was mainly used to study supersymmetric (SUSY) models [67, 68], most of the external packages in **MasterCode** are used to calculate SUSY observables, such as the decay widths of SUSY particles. However, for this thesis, we will only be focusing on DMSMs, hence we will only be looking at external packages that are relevant to the computation of observables associated with this set of models.

External Package	References	Observables
MG5_aMC@NLO 2.4.3	[29]	Decay Widths Γ Cross-sections σ
MicrOMEGAs 5.0.2	[32–34]	DM relic density $\Omega_\chi h^2$ SI cross-section σ_p^{SI} SD cross-sections σ_p^{SD} and σ_n^{SD}

Table 4.1: External packages used by **MasterCode** to compute the DMSM observables

The first external package **MG5_aMC@NLO 2.4.3** is actually a combination of two different packages, **MADGRAPH5** and **AMC@NLO** [29], which incorporates all features from both packages into a single package. The package is mainly designed to automate Quantum Chromodynamics (QCD) calculations at leading order (LO) and next to leading-order (NLO) corrections, which would be highly non-trivial if done by hand. The package can be used to carry out calculations at tree- or loop-level for a multitude of processes based

⁶⁴ Hence they are known as SLHA files

on any arbitrary user-defined Lagrangian. The package was constructed using several high-level programming languages, including **Python**, **C++** and **Fortran**, which carries out the computation of process-specific observables. Prior to the computation of these observables, the following ingredients are required:

- A model of the theory: The Lagrangian of the theory including its parameters, such as couplings and masses. From the Lagrangian provided by the user, the Feynman rules can be derived directly from **FEYNRULES**⁶⁵ at LO. However, at NLO, it is not so straightforward and we will not discuss about this here.⁶⁶
- A set of process-independent building blocks

Using the ingredients above and a given process, **MG5_aMC@NLO 2.4.3** will be able to build a code that is specific to the model and process in question. From this model- and process-specific code, **MG5_aMC@NLO 2.4.3** will proceed to generate possible events relevant to the process using Monte Carlo (MC) simulations. From the Feynman diagrams of the events that were generated, **MG5_aMC@NLO 2.4.3** will then compute a set of relevant observables, including the decay widths of unstable particles and the cross-section of the process. For the computation of decay widths, an additional program, called **MADWIDTH**, was introduced specifically to calculate the partial width of N -body decays, for any values of N , at tree-level using the Narrow-Width Approximation (NWA) method. This additional module was designed in a way so as to improve the computational efficiency of the package significantly by ignoring certain classes of Feynman diagrams that are irrelevant.

Moving on to the second external package, **MicrOMEGAs 5.0.2** is a package dedicated to the calculations of DM observables. The package is able to carry out these calculations based on the data and results from both DM direct and indirect detection experiments using a set of routines which were written in **Fortran** and **C**. By passing input parameters from a SLHA file through different routines in the package, different DM observables, including the DM relic density, $\Omega_\chi h^2$, and the DM scattering cross-sections, σ_p^{SI} , σ_p^{SD} and σ_n^{SD} , can be obtained. We have seen in the previous chapter that the package was used to calculate various DM observables for two-body DM $\chi\bar{\chi}$ annihilations. In addition to these DM annihilation processes, the package is also able to handle other DM processes, including semi-annihilations⁶⁷ and annihilations to three or four bodies⁶⁸ [34]. Furthermore, the package is also able to deal with scenarios where there is an asymmetry in the DM χ and $\bar{\chi}$ anti-DM abundances, Y_+ and Y_- .

Lets consider the case where there is an asymmetry between the the relative abundances of DM and anti-DM, *i.e.* $Y_+ \neq Y_-$, when the Universe is hotter in the past but an

⁶⁵ A Mathematica® package developed for the calculation of Feynman rules

⁶⁶ As mentioned in the previous chapter, observables will only be calculated to LO due to the absence of a complete set of model-dependent NLO corrections

⁶⁷ $\chi\bar{\chi} \rightarrow \bar{\chi}X$ where X can be any Standard Model (SM) particle

⁶⁸ $\chi\bar{\chi} \rightarrow Xff$ where X can be any SM particle, $f\bar{f}$ can be any SM fermion-antifermion pair

equilibrium has been attained later in the history of the Universe. The thermal evolution equation for DM expressed in terms of the relative abundances Y_+ and Y_- is shown below:

$$\frac{dY_{\pm}}{ds} = \frac{\langle\sigma v^{+-}\rangle}{3H} (Y_+ Y_- - Y_+^{eq} Y_-^{eq}) \quad (4.1)$$

where s is the entropy density, H is the Hubble constant and the superscript eq indicates that the relative abundances are at thermal equilibrium. Note that $Y_+^{eq} = Y_-^{eq}$ and $\langle\sigma v^{+-}\rangle = \langle\sigma v^{-+}\rangle$, which is the relativistic thermally averaged annihilation cross-section. Let's define the following quantities to simplify the subsequent set of calculations:

$$\begin{aligned} \Delta Y &= Y_+ - Y_- \\ Y &= 2\sqrt{Y_+ Y_-} \Rightarrow Y_{\pm} = \frac{\pm Y + \sqrt{Y^2 + (\Delta Y)^2}}{2} \end{aligned} \quad (4.2)$$

Using the quantities above, the thermal evolution equation for Y is defined as the following:

$$\begin{aligned} \frac{dY}{ds} &= \frac{\langle\sigma v\rangle}{3H} \left[\frac{Y_+ + Y_-}{Y} \right] (Y^2 - Y_{eq}^2) \\ &= \frac{\langle\sigma v\rangle}{3H} (Y^2 - Y_{eq}^2) \sqrt{1 + \left(\frac{\Delta Y}{Y} \right)^2} \end{aligned} \quad (4.3)$$

The equation above can be solved numerically, using the procedure outlined in , to obtain today's relative abundances, $Y_0 = Y(T = T_0)$. The total DM relic density is given by the following expression:

$$\Omega h^2 = \frac{8\pi}{3H_{100}^2} \left(\frac{m_{\chi}}{M_{Planck}} \right) \left[\frac{\sqrt{Y_0^2 + (\Delta Y)^2}}{s_0} \right] \quad (4.4)$$

where $H_{100} = 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ⁶⁹, M_{Planck} is the Planck's mass and s_0 is today's entropy. Expressing the relic density for each species, DM or anti-DM, in terms of the asymmetry parameter δ_{DM} :

$$\Omega_{\pm} h^2 = \frac{\Omega h^2}{1 + e^{\mp \delta_{DM}}}, \quad \text{where } \delta_{DM} = \log \left[\frac{\sqrt{Y_0^2 + (\Delta Y)^2} + \Delta Y}{\sqrt{Y_0^2 + (\Delta Y)^2} - \Delta Y} \right] \quad (4.5)$$

In the **MicrOMEGAs 5.0.2** package, the DM asymmetry parameters, ΔY and δ_{DM} , are global variables⁷⁰, `deltaY` and `DMasymm`, that can be used anywhere in the code. Note that δ_{DM} (`DMasymm`) is obtained using the expression in (4.5). The computation of the DM relic density for each species, $\Omega_{\pm} h^2$, in **MicrOMEGAs 5.0.2** automatically takes into account the total DM relic density, which is obtained using Eq (4.4), and δ_{DM} in Eq (4.5).

⁶⁹ $H_0 = H_{100} h \approx 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is today's Hubble constant

⁷⁰ Note that these variables are set to 0 by default

4.2 Interface Files

In the previous section, we have briefly discussed how some of the external packages in `MasterCode` were used in the calculation of the DMSM observables. In this section, we will discuss more in detail on how these external packages were interfaced and implemented in `MasterCode`.⁷¹

As mentioned earlier in the chapter, most of the external packages used by `MasterCode`, including `MG5_aMC@NLO 2.4.3` and `MicrOMEGAs 5.0.2`, require SLHA files as inputs in order to proceed on with the computation. As an illustration, an example of a simplified SLHA file is shown on the following page. As one can see in Figure 4.1, the SLHA file acts just like a database storing all relevant model-dependent parameters, such as couplings, masses and decays, necessary for the calculation of observables. In order for the external packages to carry out their respective calculations, the SLHA file has to be passed through first so that the relevant parameters can be extracted. Once these parameters have been extracted from the SLHA file, they will then be assigned as inputs into the functions and routines used by these external packages to carry out the relevant calculations of the observables.

Moving on to the first external package, `MG5_aMC@NLO 2.4.3`, as discussed in the previous section, is a dedicated package specially designed for automated QCD calculations at LO and NLO [29]. To carry out these calculations, `MG5_aMC@NLO 2.4.3` will take three step approach, which is process-generation $\rightarrow$ output $\rightarrow$ execution. These three steps are normally executed as commands in the `MG5_aMC@NLO 2.4.3` shell, which is initialised from `MG5_aMC@NLO 2.4.3` home directory using the command “`./bin/mg5_aMC`”. However, in `MasterCode`, the whole process is completely automated using a `Cython` script, which executes all the required commands on the command line using `Python`’s built-in `os` and `subprocess` modules.

As mentioned in the previous section, a model of the theory is required by `MG5_aMC@NLO 2.4.3` before the computation can be carried out. For convenience and ease of access, some models, such as `MSSM` and `DMsimp_s_spin1` (DM simplified models with s -channel spin-one mediators), have already stored in a library within `MG5_aMC@NLO 2.4.3` home directory. To import a model, the following command needs to be executed: `import model_name`. In the case of DMSM, this command will take on the following form: `import DMsimp_s_spin1`, which will then be stored in a `.dat` file, which acts as a command file. Once the model has been imported, `MG5_aMC@NLO 2.4.3` will proceed to the next two phases, which are the process-generation and output phases, before the package starts running the computation.

⁷¹ In the sense of how these packages were run/executed so as to generate the outputs for the observables

```

BLOCK DMINPUTS
    2      {gVDM}      # gVXd
    3      {gADM}      # gAXd
    4      {gVSM}      # gVd11
    5      {gVSM}      # gVu11
    6      {gVSM}      # gVd22
    7      {gVSM}      # gVu22
    8      {gVSM}      # gVu33
    9      {gVSM}      # gVd33
   10      {gVLSM}     # gVl11
   11      {gVLSM}     # gVl22
   12      {gVLSM}     # gVl33
   13      {gASM}      # gAd11
   14      {gASM}      # gAu11
   15      {gASM}      # gAd22
   16      {gASM}      # gAu22
   17      {gASM}      # gAd33
   18      {gASM}      # gAu33
   19      {gAlSM}     # gAl11
   20      {gAlSM}     # gAl22
   21      {gAlSM}     # gAl33
   22      {gnu}       # gnu11
   23      {gnu}       # gnu22
   24      {gnu}       # gnu33
BLOCK MASS
    23      9.11876000E+01  # MZ
    24      7.98243600E+01  # MW
    25      1.25000000E+02  # MH
5000001    {mY1}      # MY1
5000511    1.00000000E+09  # MXr
5000512    1.00000000E+09  # MXc
5000521    {mXd}      # MXd
BLOCK SMINPUTS
    1      1.27900000E+02  # EWM1
    2      1.16637000E-05  # GF
    3      1.18400000E-01  # aS
BLOCK YUKAWA
    6      1.72000000E+02  # ymt
   15      1.77700000E+00  # ymtau
DECAY      6      1.50833600E+00  # Gamma(top)
DECAY      23     2.49520000E+00  # Gamma(Z)
DECAY      24     2.08500000E+00  # Gamma(W)
DECAY      25     4.07000000E-03  # Gamma(h0)
DECAY 5000001    {wY1}      # Gamma(y1)
DECAY 5000521    1.00000000E-12  # Gamma(xd)
DECAY 9000002    2.49520000E+00  # Gamma(ghz)
DECAY 9000003    2.08500000E+00  # Gamma(ghwp)
DECAY 9000004    2.08500000E+00  # Gamma(ghwm)

```

Figure 4.1: An example of a simplified SLHA file with some of the BLOCK entries displayed. Note that `gVDM`, `gVSM` and `gVLSM` are the DM, quark and leptonic vector couplings; `gADM`, `gASM` and `gAlSM` are the DM, quark and leptonic axial couplings; `mY1` and `mXd` are the mediator and DM masses; and `wY1` is the decay width of the mediator

Just like in the model importing phase, the commands for the execution of these two phases are also stored in the same `.dat` file, below the `import DMSimp_s_spin1` command. Since the DMSM observables that MasterCode obtains from MG5_aMC@NLO 2.4.3 are mainly from [LHC Experimental Constraints](#), the relevant process is the $\chi\bar{\chi}$ production from pp collisions at the LHC mediated by a jet, which follows that of the [Monojet Constraints](#). Generating this process as a command into MG5_aMC@NLO 2.4.3: `generate p p > xd xd~ j` where $xd = \chi$, $xd\sim = \bar{\chi}$, and j indicates that the process is mediated by a jet. After the process has been generated, MG5_aMC@NLO 2.4.3 needs to store the relevant information into a directory. For the case of MasterCode, all information regarding the above process will be stored in the `dmsimp` directory. Executing this step as a command: `output dmsimp -nojpeg`.

Once the above steps have been completed, MG5_aMC@NLO 2.4.3 will be able to initialise the computation, starting from the extraction of the relevant inputs from an SLHA file, storing these inputs in the `param_card.dat` file and generating the possible events relevant to the process, which in MasterCode's case is 100 events. After everything has been completed, MG5_aMC@NLO 2.4.3 will store the outputs in a file from which observables, in particular the cross section, can easily be extracted.

Following a similar set of steps used by MG5_aMC@NLO 2.4.3 in handling input parameters, MicrOMEGAs 5.0.2 also expects to receive an SLHA file first before proceeding to extract the relevant inputs from the SLHA file. However, the way MicrOMEGAs 5.0.2 stores the input parameters differs from that of MG5_aMC@NLO 2.4.3. In MicrOMEGAs 5.0.2, individual input parameters are stored as user-defined variables using a function called `assignval(variable_name, value)` [32]. After every single input parameter has been assigned to a variable, MicrOMEGAs 5.0.2 will then initialise the computation by passing these variables into different routines and functions. As mentioned in Table 4.1, the observables that MasterCode extracts from MicrOMEGAs 5.0.2 are mainly the DM relic density, the SI and SD DM scattering cross sections.

In MicrOMEGAs 5.0.2, the DM relic density $\Omega_\chi h^2$ is primarily calculated using a routine called `darkOmega` [32] which takes in three arguments, namely `Xf`, `fast` and `Beps`. The input `Xf`, defined as the ratio of the DM mass to the freeze-out temperature $X_f = m_\chi/T_f$, is used as parameter that characterises the freeze-out temperature T_f . Note that `Xf` can also be used as an input into a routine called `printchannels` which returns the relative contribution of each decay channel to the total DM relic density Ωh^2 . The `fast = 1` flag instructs MicrOMEGAs 5.0.2 to execute fast calculation to an accuracy of $\sim 1\%$. Finally, the input `Beps` defines the criteria which determines whether a given decay channel will be considered in the computation of the thermally averaged annihilation cross-section $\langle\sigma v\rangle$. This criteria is defined using the following threshold:

$$B_\epsilon < B \approx \exp\left(X \left[\frac{m_i + m_j - 2m_\chi}{m_\chi} \right]\right) \quad (4.6)$$

where B is known as the Boltzmann suppression factor, m_i and m_j are the masses of the incoming particles, and $X = m_\chi/T$. Above this value B_ϵ , the given decay process will be included in the $\langle\sigma v\rangle$ calculation. In **MasterCode**, this threshold was set to **Beps**= 10^{-5} .

As mentioned in the previous section, in addition to the usual DM annihilations to two-body final states, **MicrOMEGAs 5.0.2** is also able to execute computations for DM annihilation mechanisms involving three- or four-body final states. In **MicrOMEGAs 5.0.2**, the **VWdecay** or **VZdecay** flag gives the user the option on which type of DM process he/she wants to include in the computation. In the case where the user chooses to set **VWdecay/VZdecay**=1, **MicrOMEGAs 5.0.2** will then proceed on to include three-body final states in the computation of observables for the DM annihilation processes. On the other hand, if the user chooses to set **VWdecay/VZdecay**=2 instead, three-body final states will also be considered in the same computation for the coannihilation processes. In the case of **MasterCode** and DMSMs, both flags were set to 1, *i.e.* **VWdecay**=1 and **VZdecay**=1, since coannihilation processes were not considered in DMSMs.

Lastly, for the remaining set of DM observables, the SI and SD DM scattering cross sections are obtained using the following formulae [33]:

$$\begin{aligned} \sigma_p^{\text{SI}} &= \frac{4\mu_\chi^2}{\pi} |A_p^{\text{SI}}|^2; & \sigma_n^{\text{SI}} &= \frac{4\mu_\chi^2}{\pi} |A_n^{\text{SI}}|^2 \\ \sigma_p^{\text{SD}} &= \frac{4\mu_\chi^2}{\pi} \left(3|A_p^{\text{SD}}|^2\right); & \sigma_n^{\text{SD}} &= \frac{4\mu_\chi^2}{\pi} \left(3|A_n^{\text{SD}}|^2\right), \end{aligned} \quad (4.7)$$

where $\mu_\chi = \frac{M_\chi M_{\text{nucleon}}}{M_\chi + M_{\text{nucleon}}}$, and A^{SI} and A^{SD} are the SI and SD DM scattering amplitudes obtained using a routine in **MicrOMEGAs 5.0.2** called **nucleonAmplitudes**(**pA0**, **pA5**, **nA0**, **nA5**). The four arguments in the **nucleonAmplitudes** routine are the respective individual SI and SD scattering amplitudes for both the protons, **pA0**⁷² and **pA5**⁷², and the neutrons, **nA0**⁷² and **nA5**⁷², and are in the form of 2-Dimensional arrays, *i.e.* matrices.

4.3 Source Files

In the previous chapter and at the start of this chapter, we have discussed about the main objectives of **MasterCode** and how these objectives can be achieved using a two-step process. In the previous two sections, we have shown how the necessary observables required to construct the individual likelihood functions for each of the constraints are obtained from the external packages **MG5_aMC@NLO 2.4.3** and **MicrOMEGAs 5.0.2**. With all the necessary ingredients in place, we can proceed on to discuss how these observables are used to construct the individual likelihood functions for the different constraints.

⁷² Following the naming convention used in **MasterCode**

In addition, we will also discuss in this section how the raw data for these constraints are stored and how they contribute to the calculation of the likelihood functions for the individual constraints.

In favour for computational efficiency, the source codes used to compute the likelihood functions for the individual constraints are all written in C++. As seen under the [LHC Experimental Constraints](#) subsection in the previous chapter, the pre-factor in the likelihood function for the [Monojet Constraints](#), as shown in Eq (3.16), differs from that for the [Dijet Constraints](#), as shown in Eq (3.20). This is the result of the 2-Dimensional (2D) likelihood function for the case of the monojet constraint instead of the usual 1-Dimensional (1D) likelihood function in the context of the other constraints. Hence to keep the package in order and for the ease of maintenance, the source codes used to compute the 2D likelihood functions for some of the constraints are stored in a separate C++ file from those used to calculate the usual 1D likelihood functions for the rest of the constraints.

To further improve computational efficiency, the calculation of the likelihood function is reformulated from a direct method, which is very slow computationally, to a method that uses a reference grid R , which is the same as the 95% CL upper limit R_i^{UL} used in Eq 3.16. Using the reference grid in the computation of the likelihood function outside of the MasterCode package has shown a drastic reduction in computation time compared to the direct calculation of the likelihood function⁷³. To generate this reference grid R , a reference, *i.e.* fixed, set of parameters, such as the couplings and the DM mass m_χ , were used. In cases where R is dependent on the mediator mass m_Y or DM mass m_χ , either one will be the only varying parameter while the rest of parameters are kept constant. Note that this reference grid is generated externally outside of the MasterCode package. Once after the reference grids for the different constraints have been generated, they will be imported into MasterCode and stored inside a single folder. Since points would be randomly sampled during the sampling phase, interpolation of the reference grids R within the sampling ranges of the DM m_χ and mediator m_Y masses would be required before the likelihood function could be evaluated for each individual point. Note that if the likelihood function is 1-dimensional, a 1-dimensional interpolation will be executed in the case. On the other hand, if the likelihood function is 2-dimensional, a 2-dimensional interpolation will then be conducted.

4.4 Sampling Algorithm

In the previous few sections, we have been discussing how observables in DMSMs were being computed using different external packages. In this section, we will proceed to discuss how the sampling was conducted within MasterCode. In particular, the discussion

⁷³ This will be presented more clearly in the comparison between the 2 methods used in the [External Implementations](#) of the dilepton constraint

will be mainly focused on **MultiNest** 2.18, the external package that was mainly used by **MasterCode** to conduct the sampling, and its algorithm.

The name of the **MultiNest** 2.18 package is actually a shorthand for Multimodal Nested sampling algorithm [36]. This algorithm has been specially designed for statistical analysis in various fields of high-energy physics, such as astrophysics, cosmology and particle physics. In these fields, statistical analyses, in particular Bayesian analyses, are usually employed to achieve 2 main objectives: a) model selection and b) parameter estimation. However, achieving these objectives might not always be straight-forward, especially in cases where the distribution under analysis contains multiple modes⁷⁴ and/or pronounced (curving) degeneracies in higher dimensions [36]. Using traditional Markov Chain Monte Carlo (MCMC) techniques to achieve these objectives could also be extremely computational intensive and might even be inaccurate. In light of these potential challenges which **MasterCode** would very likely face during the sampling, the Multimodal Nested sampling algorithm was thus chosen.

The idea behind the **MultiNest** 2.18 algorithm is focused mainly on a concept known as the *simultaneous ellipsoidal nested sampling*. The simultaneous ellipsoidal nested sampling works by first ensuring that the set of live points, also known as active points, is uniformly distributed throughout the parameter space within a D -dimensional unit hypercube⁷⁵, where D is the dimensionality of the parameter space. After this set of live points have uniformly distributed over the D -dimensional unit hypercube, the set can then be partitioned into a number of clusters, simultaneously constructing an ellipsoidal bound for each cluster. The optimal partitioning and ellipsoidal decomposition of this set of live points can be achieved using an approach known the “expectation-maximization” (EM) algorithm. A detailed discussion on this algorithm can be found in Section 5.2 of [36], in particular Algorithm 1 on page 5 of [36]. This algorithm is repeated over and over again at the start of each iteration i .

Once the optimal ellipsoidal bounds have been constructed using the EM algorithm at the start each iteration i , a set of ellipsoids $\{E_1, E_2, E_3, \dots, E_k\}$ is then obtained. Before random samples can be drawn from this set, an ellipsoid, say E_1 , needs to be chosen with a probability of

$$p_1 = \frac{V(E_1)}{V_{tot}}, \text{ where } V_{tot} = \sum_{k=1}^K V(E_k) \quad (4.8)$$

and $V(E_k)$ is the volume of k^{th} ellipsoid. Once the first ellipsoid has been chosen based on the probability above, samples are then drawn uniformly from this ellipsoid until a sample whose likelihood $\mathcal{L}$ is greater than the value of the lowest likelihood $\mathcal{L}_i$ in the

⁷⁴ Peaks/Maximum points in the distribution

⁷⁵ The exact procedure on how this is done is described in Section 5.1 of [36]

whole set of live points at the iteration i , *i.e.* $\mathcal{L} > \mathcal{L}_i$. To account for the possibility of overlapping ellipsoids, this sample is to be accepted with a probability of $1/n_e$ where n_e is the number of ellipsoids in which this sample lies. The above procedure is then repeated for the remaining ellipsoids $E_2, E_3, \dots, E_k$.

In cases where distributions have multiple modes, it is particularly useful to identify which samples belongs to which mode [36]. At each iteration i , after performing the mode identification procedure as outlined in Section 5.6 of [36], 2 separate sets of “active” and “inactive” groups are obtained, partitioning the complete set of generated points (both active/live and inactive points). As MultiNest 2.18 approaches regions with higher likelihoods, it is expected that the number of live points in some of the active groups will decrease and even tend to 0. This may be a result of 2 possibilities:

- i) First, these active groups may lie in regions of the distribution where the likelihood $\mathcal{L}$ is low.
- ii) Second, assuming that these groups remain active at the end of the sampling, these groups could possibly be lying in regions where the likelihood is maximized and hence promoted to modes, denoted by the set $\{M_1, M_2, \dots, M_q\}$.

Note that with each subsequent iteration i , the total prior volume becomes diminishingly small. To give a clearer picture of how the mode identification procedure works, an illustration is depicted in a diagram below (Figure 4 in [36]).

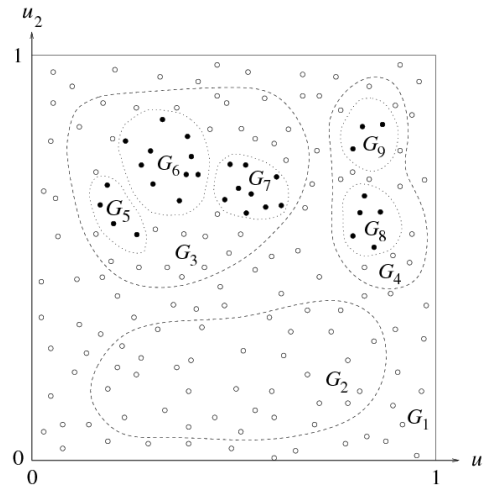


Figure 4.2: An illustration showing how points are assigned to different groups [36], where the solid circles denote active points at iteration $i = i_2$ and the open circles are inactive points at this iteration. At iteration $i = i_2$, the inactive groups are G_1, G_3 and G_4 , while the active groups are G_2 and G_5 to G_9 . At iteration $i = i_1$, the active group G_1 splits into G_2 to G_4 . At the next iteration $i = i_2$, G_3 splits into active groups G_5 to G_7 and G_4 splits into active groups G_8 and G_9 . G_2 remains unsplit at the end of iteration $i = i_2$ but the number of active points within it has fallen to 0 as a result of possibility i)

Once the modes $\{M_1, M_2, M_3, \dots, M_q\}$ have been correctly identified, the local evidence $\mathcal{Z}_q$ for each mode M_q can be evaluated using the following equation:

$$\mathcal{Z}_q = \sum_{j=1}^{n_q} \mathcal{L}_j w_j + \sum_g \mathcal{L}_g w_g \alpha_g^{(q)} \quad (4.9)$$

The first term in the above equation represents the contributions from the active groups to the local evidence and the second term represents the contributions from the inactive groups. In the first term, $w_j = X_M / N_{\text{live}}$ where X_M is the prior volume occupied by the M^{th} point, whether it is active or inactive, in mode M_q and N_{live} is the total number of active points in M_q . In the second term, $w_g = \frac{1}{2}(X_{i-1} - X_{i+1})$ where i refers to the iteration in which point g was discarded and X_{i-1} (X_{i+1}) refers to the prior volume at the end of the iteration prior to (after) the discarding of the g^{th} point. The factors $\alpha_g^{(q)}$ in the second term can be determined by reversing the process illustrated in Figure 4.2. The procedure on how this is done can be found on page 8 of [36]. It can be easily checked that Eq (4.9) is valid by checking that the sum of the local evidences $\mathcal{Z}_q$ for each mode M_q over all modes is equal to the global evidence $\mathcal{Z}$, *i.e.*

$$\sum_{q=1}^Q \mathcal{Z}_q = \mathcal{Z}, \text{ where } Q \text{ is the total number of modes} \quad (4.10)$$

MultiNest 2.18 will stop sampling once the stopping criterion, $\Delta \mathcal{Z}_i = \mathcal{L}_{\text{max}} X_i < \mathcal{Z}_{\text{tol}}$ where $\mathcal{L}_{\text{max}}$ is the maximum likelihood in the current set of live points and X_i is the expected remaining prior volume in the i^{th} iteration, is met. In other words, **MultiNest 2.18** terminates when the largest possible contribution to the global evidence from the remaining set of live points in i^{th} iteration, $\mathcal{Z}_i$ is less than the user-defined tolerance $\mathcal{Z}_{\text{tol}}$ [37].

In the current sampling run for the leptophilic model, which will be discussed in greater detail in the following chapter, a total of 1000 live points and a maximum number of 200 iterations were provided as inputs into **MultiNest 2.18**. In addition, a user-defined evidence tolerance of $\mathcal{Z}_{\text{tol}} = 1 \times 10^{-6}$ and a sampling efficiency of $f = 0.1$ was set as required by **MultiNest 2.18**. Additional definitions were also introduced to “control” how **MultiNest 2.18** will sample the priors, which are basically the input parameters shown in Tables 3.1 and 5.1. Since the ranges over which these priors would be sampled were split into segments to ensure the efficient sampling of the whole parameter space, it would be useful to define how **MultiNest 2.18** should conduct the sampling within and outside these segments by introducing different types of distributions into the definition of the priors. These definitions include a **soft** prior which is a flat prior with gaussian tails outside the priors, a **hardsoft** prior which is a flat prior with a upper gaussian tail (on the right hand side of the prior), and a **softhard** prior which is a flat prior with a lower gaussian tail (on the left hand side of the prior). The **hardsoft** and **softhard** priors were introduced on top of the **soft** prior to prevent **MultiNest 2.18** from sampling into

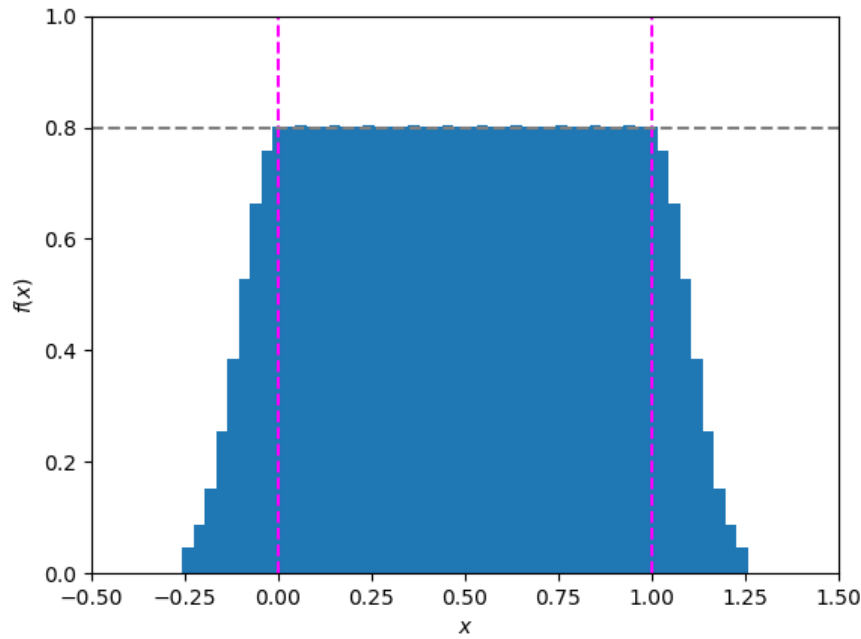


Figure 4.3: An illustration of a **soft** prior on $[0, 1]$ interval with a flatness of 80%. 20% of the prior distribution lies outside of the interval, acting as gaussian tails on both the left hand and right hand sides of the prior distribution (Figure 7.1 on page 176 of [71]).

unphysical regions, especially regions with negative priors. An illustration of a **soft** prior is shown in the diagram above.

Once each point has been sampled, the input parameters, the value of the likelihood function χ^2 and the observables for that sample will be stored in SQLite database (.db file). In the SQLite database file, each point is expected to have a total of 80 rows, where each row corresponds to each of the above quantities, associated with it. At the end of each round of sampling, all relevant information of all individual samples will be stored inside the SQLite database file, which can be easily extracted for further analysis.

Chapter 5

DMSMs With Leptophilic Spin-One Mediators

In Chapter 3, we were focusing with Dark Matter Simplified Models (DMSMs) with spin-one mediators that are leptophobic. In other words, the mediator particles considered in such models were assumed to couple to only quarks and the DM particles, and couplings to leptons were ignored. This can be clearly seen in the Lagrangians shown in both Eqs (3.1) and (3.2).

Moving on to this chapter, the couplings between the mediator particles and leptons will now be included and hence these models will now be called as DMSMs with leptophilic spin-one mediators. On top of the two Lagrangians in both Eqs (3.1) and (3.2), a new Lagrangian describing the interaction between the mediator particle Y and the SM leptons needs to be included and it takes on the following form:

$$\mathcal{L}_\ell = \sum_j \bar{\ell}_j \gamma^\mu (g_V^{\ell_j} + g_A^{\ell_j} \gamma^5) \ell_j Y_\mu \quad \text{where } j \text{ identifies the lepton } \ell \quad (5.1)$$

Combining all three terms together will produce an effective Lagrangian describing all possible interactions in DMSMs which can be expressed in the form:

$$\mathcal{L}_{eff} = \mathcal{L}_\chi [\text{From Eq (3.1)}] + \mathcal{L}_q [\text{From Eq (3.2)}] + \mathcal{L}_\ell [\text{From Eq (5.1)}] \quad (5.2)$$

Since the effective Lagrangian has been modified with the addition of a new term $\mathcal{L}_\ell$, additional constraints need to be included to expand and improve the analyses of DMSMs. Hence, in the first section, we will discuss these additional constraints, which are mainly from the LHC experiments. In particular, we will be focusing on the dilepton constraints, which also includes the external implementation and verification of the constraint before it was implemented in **MasterCode** together with the implementation of the constraint in **MasterCode**. We will discuss results of selected 1-Dimensional and 2-Dimensional plots in the following section. Note that Chapter 3 was based on a full analysis of DMSMs with leptophobic spin-one mediator in a paper previously published by the **MasterCode** Collaboration. However, the analyses of DMSMs with leptophilic spin-one mediators, which will be discussed in the chapter, are still on-going and have yet to be completed. Hence, the results will not be as comprehensive as that in [DMSMs With Leptophobic Spin-One Mediators](#). In addition, the analyses in this chapter will be focusing on couplings which are purely vectorial, *i.e.* g_V^χ , g_V^q and g_V^ℓ . With regards to the analysis for axial-vector model, it will be elaborated further in the following chapter.

5.1 LHC Experimental Constraints

Since the astrophysical constraints used in the current analysis of the pure vectorial model is the same as those described in [Astrophysical DM Constraints](#), we will not be covering it here. Possible astrophysical constraints which will be included in future analyses will be further elaborated in the next chapter.

Returning back to the LHC experimental constraints, as highlighted earlier in the chapter, since couplings between the mediator and leptons are now being included into the analyses, an additional constraint needs to be introduced, namely the dilepton constraints. In addition, the monojet constraints need to be modified since the cross sections for the production of neutrinos need to be considered also on top of the cross section for DM production. For the dijet constraints, the CMS Collaboration has released a paper on high-mass dijet resonances using data from pp collisions at $\sqrt{s} = 13$ TeV [72], providing new experimental constraints for future analyses. Since these new set of dijet constraints have yet to be implemented in **MasterCode**, they will be further elaborated in the following chapter.

5.1.1 Monojet Constraints (Modified)

Earlier in Chapter 3, since the discussion and analyses were focused on [DMSMs With Leptophobic Spin-One Mediators](#), couplings between the mediator particles Y and leptons were ignored. The same applies for decay processes involving neutrinos, ν_ℓ , and charged leptons, ℓ^- . Hence, previously in Chapter 3, only processes involving decays of the mediator particles into DM particles were considered. These decay processes were simulated using **MG5_aMC@NLO 2.4.3**. The cross sections for DM production were then calculated based on the procedure outlined under the [Interface Files](#) section in the previous chapter. Following Eq 3.16, the likelihood function for the [Monojet Constraints](#) in the leptophobic model was then obtained.

In this chapter, the focus of the analyses will now be expanded to include couplings between the mediator particles and leptons. As highlighted by the CMS Collaboration in [42], the decays of the Z' boson into neutrinos, ν_ℓ , were identified as the largest contributions to the background signal. Hence, these processes need to be considered, together with the evaluation of the cross sections for such processes.

However, during the process generation phase in **MG5_aMC@NLO 2.4.3**, only processes involving DM ($\chi\bar{\chi}$ production from pp collisions at the LHC via a jet, *i.e.* $p p \rightarrow \chi\bar{\chi} + j$), were considered. Hence, only the cross sections for such DM production processes were computed. To include $Z' \rightarrow \nu_\ell \nu_\ell$ decay processes, a new command was added to generate these processes and this command takes on the following form: $p p \rightarrow \nu_l \nu_l + j$

$j/\bar{z}$, where $\nu_l = \nu_\ell$, $\nu_l \sim \bar{\nu}_\ell$ ⁷⁶ and $j/\bar{z}$ indicates that the process is mediated by a jet and/or a Z boson. Note that this command was stored inside a separate command file (`dmsimp_nu.dat`) within a completely separate directory labelled as `dmsimp_nu` to distinguish it from the `dmsimp` directory. After the $Z' \rightarrow \nu_\ell \nu_\ell$ process has been generated, all relevant information regarding the process needs to be stored and this is done with the help of the following command: `output dmsimp_nu -nojpeg`. Upon the execution of the above commands, `MG5_aMC@NLO 2.4.3` will then proceed to compute the cross section for the $Z' \rightarrow \nu_\ell \nu_\ell$ process, following the same exact procedure as that outlined in the previous chapter under the [Interface Files](#) section for the $pp \rightarrow \chi\bar{\chi}$ process. To prevent any conflicts when storing the cross sections for the 2 different processes, the cross sections for the DM ($\chi\bar{\chi}$) and neutrino ($\nu_\ell \nu_\ell$) productions are stored separately as `xsec_DM` and `xsec_nu`.

As mentioned earlier in the section, the 2-Dimensional likelihood function for the [Monojet Constraints](#) in the leptophobic model was obtained using Eq (3.16). However, as identified in [42] by the CMS Collaboration, the $Z' \rightarrow \nu_\ell \nu_\ell$ decay contributes to the majority of the background signal. As a result, the cross section for this process needs to be included into the computation of the 2-Dimensional likelihood function for the monojet constraints for the leptophilic case. For the $Z' \rightarrow \nu_\ell \nu_\ell$ decay process, the 95% CL upper limit R_i^{UL} is no longer a function of $\mathbf{m} = (m_Y, m_\chi)$, as that in the leptophobic case for the DM production, but a function of m_Y alone with $m_\chi = 0$. To avoid any confusion, the 95% CL upper limit R_i^{UL} for these process will now be written as a function of $\mathbf{m}'$, where $\mathbf{m}' = (m_Y, 0)$. To obtain the likelihood function for the monojet constraint in the leptophilic case, the expression within the brackets in Eq (3.16) will now be replaced with the ratio $\sigma_{\text{MG}}/R_i^{\text{UL}}$. Once the cross sections σ_{MG} for both processes have been obtained using `MG5_aMC@NLO 2.4.3` together with their corresponding upper limits R_i^{UL} at each individual point, the ratio $\sigma_{\text{MG}}/R_i^{\text{UL}}$ is then calculated for both processes. The maximum of the 2 ratios will afterwards be used to obtain the likelihood function for the modified monojet constraint in the leptophilic model. In other words,

$$\Delta\chi^2 = 5.99 \times \left[\max \left(\frac{\sigma_{\text{MG}}}{R_i^{\text{UL}}(\mathbf{m}/\mathbf{m}')} \right) \right]^2 \quad (5.3)$$

As a quick and basic form of cross-check, a few points on the (m_Y, m_χ) plane were randomly selected. At these randomly selected points, the ratios for both processes, $\sigma_{\text{MG}}/R_i^{\text{UL}}(\mathbf{m})$ in the DM production case and $\sigma_{\text{MG}}/R_i^{\text{UL}}(\mathbf{m}')$ in the neutrino production case, were then computed and printed out, followed by the evaluation of the likelihood function by hand using the equation above. Comparing these values with those obtained from `MasterCode` confirms that the modified monojet constraints were correctly implemented in `MasterCode`.

⁷⁶ Although it was mentioned earlier in the first chapter under the [Standard Model \(SM\)](#) section that neutrinos are Majorana fermions in which they are their own anti-particles, $\bar{\nu}_\ell$ needs to be included in the command as required by the convention outlined in `MG5_aMC@NLO 2.4.3` documentation

5.1.2 Dilepton Constraints

The ATLAS Collaboration has published results on their search for high-mass dilepton resonances based on data from pp collision at $\sqrt{s} = 13$ TeV corresponding to an integrated luminosity of 139 fb^{-1} [73]. This search for high-mass dilepton resonances also includes dielectron, ee , and dimuon, $\mu\mu$, final states. Although these results are from searches for three different types of high-mass resonances, only results on the search for high mass spin-one resonances targeting the Sequential Standard Model (SSM) Z'_{SSM} boson would be used here.

Searches for spin-one dilepton resonances involving the Z'_{SSM} boson are mainly focused on Drell-Yan processes which are of the form: $q\bar{q} \rightarrow \gamma/Z \rightarrow \ell\bar{\ell}$ where $\ell = e^-$ or μ^- and $\bar{\ell} = e^+$ or μ^+ . However, in the LHC, protons are collided with each other at a center-of-mass energy of $\sqrt{s} = 13$ TeV. Although protons were previously thought to be composite structures containing 2 up (u) and 1 down (d) quarks held together by gluons, QCD studies from pp collisions have shown that the structure of protons are much more complicated than what was previously perceived⁷⁷. To avoid any complications in other non-QCD studies, the structure of protons are now simply described to be made up of partons. Since pp collisions do not merely involve the interactions between only 2 quarks coupled with the complicated structure of the proton, the Drell-Yan process will need to be modified to avoid “meddling around” with the complicated structure of the proton. An example of the modified Drell-Yan process is shown in the Feynman diagram below.

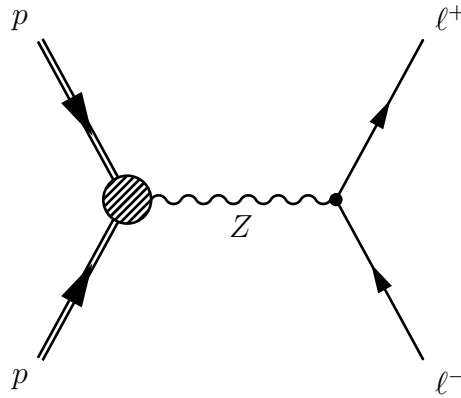


Figure 5.1: Modified Drell-Yan process from pp collisions at the LHC, where $\ell = e/\mu$

Although Drell-Yan events were the focus of the search for spin-one dilepton resonances in [73], not all of these events were selected for the analysis. The fraction of events being selected depends on the *selection efficiency*. Not only does the selection efficiency depend from one species of leptons to another in the final state, it also varies with the pole mass of the Z' boson. A plot of how the selection efficiency depends on the lepton species and how it varies with the Z' pole mass is shown in a diagram on the following page.

⁷⁷ Which I will not discuss in detail here. Please refer to Sections 8.5 and 8.6 in [4] for further details on this discussion

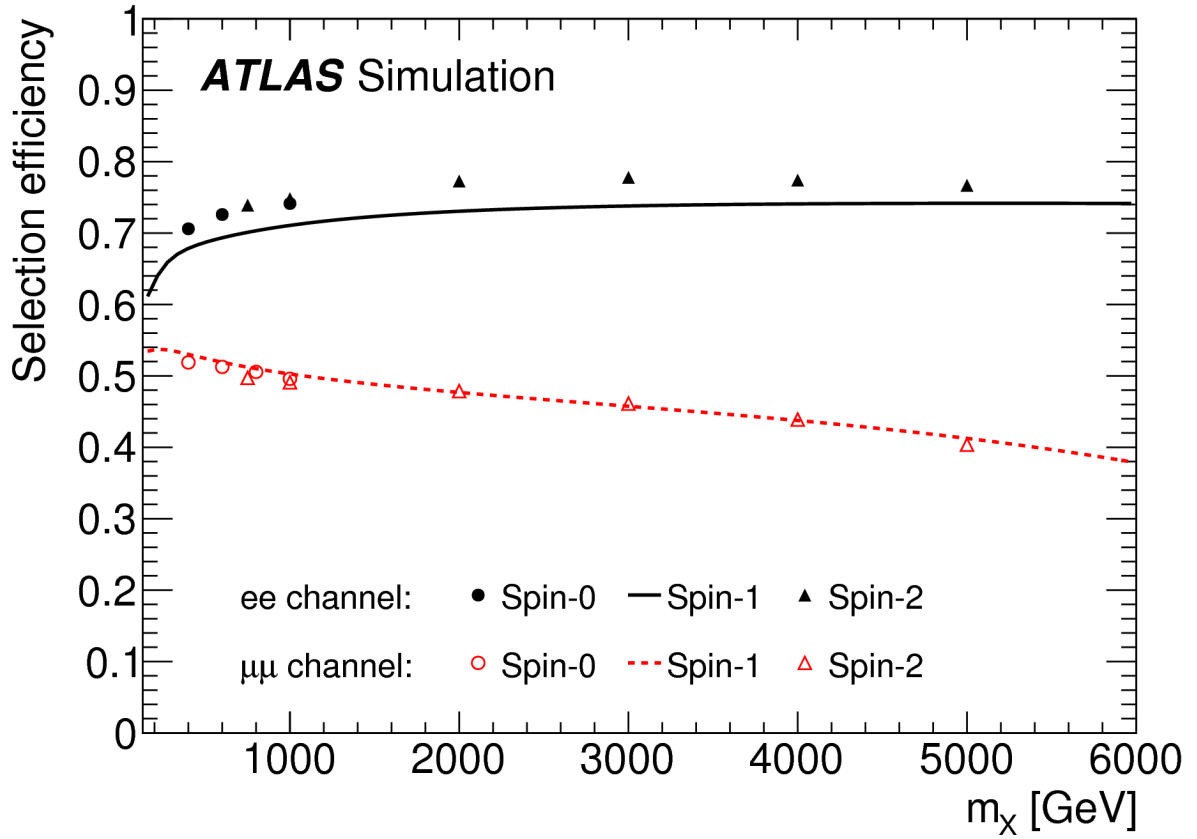


Figure 5.2: Selection efficiencies of Drell-Yan events for the dielectron, ee , and the dimuon, $\mu\mu$, channels for all 3 different type of resonances. However, for the analysis of the dilepton constraints, only the case for spin-one resonances, solid black line for the ee channel and red dashed line for the $\mu\mu$ channel, were considered.

The 95% Confidence Level (CL) upper limits on the “*weighted*” fiducial cross section as a function of the pole mass of the Z' boson for a range of relative widths $\Gamma_{Z'}/m_{Z'}$ ⁷⁸ are shown in Fig 5.3 on the following page. The “*weighted*” fiducial cross section here refers to the inclusive cross section multiplied by the branching ratio of the decay channel and divided by the selection efficiency for that decay channel. In other words,

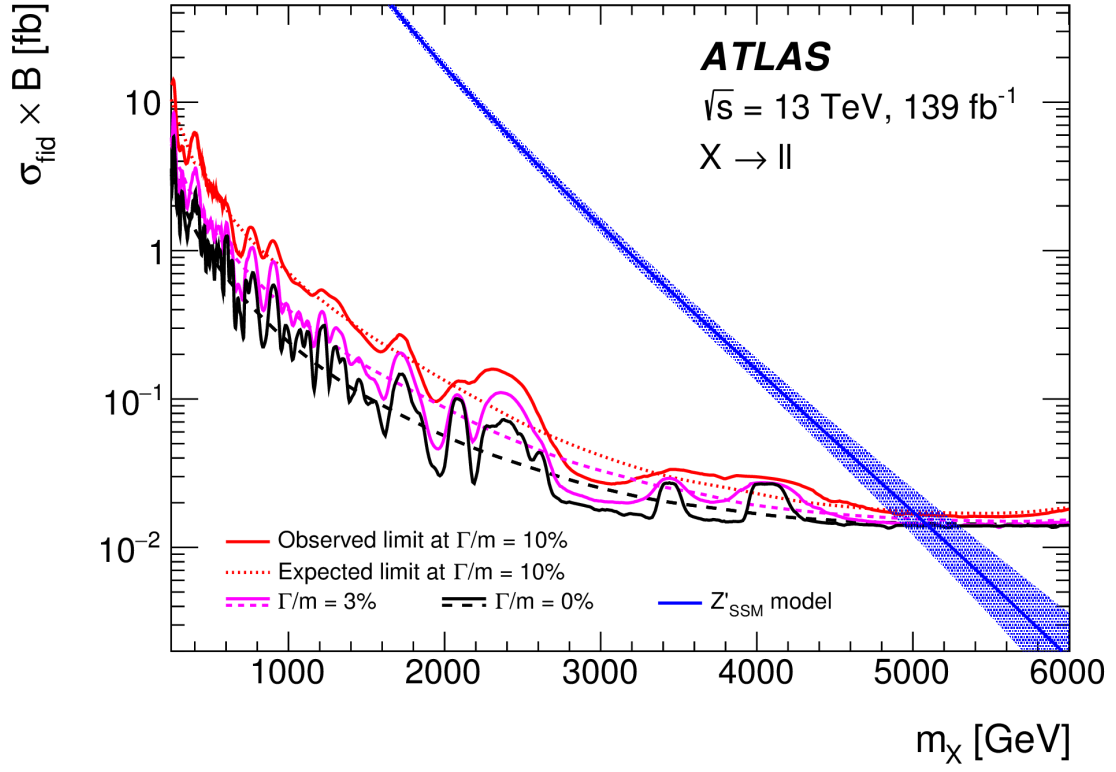
$$\sigma_{\text{inc}} = \frac{\sigma_{\text{fid}} \times \text{Br}}{\text{Selection Efficiency}} \quad (5.4)$$

where σ_{inc} and σ_{fid} refers to the inclusive and fiducial cross sections respectively, and Br refers to the branching ratio. The theoretical upper limit on the fiducial cross section for a relative width of $\Gamma_{Z'}/m_{Z'} = 3.0\%$ as predicted by the Sequential Standard Model (SSM)⁷⁹ is also shown on the upper panel of the figure.

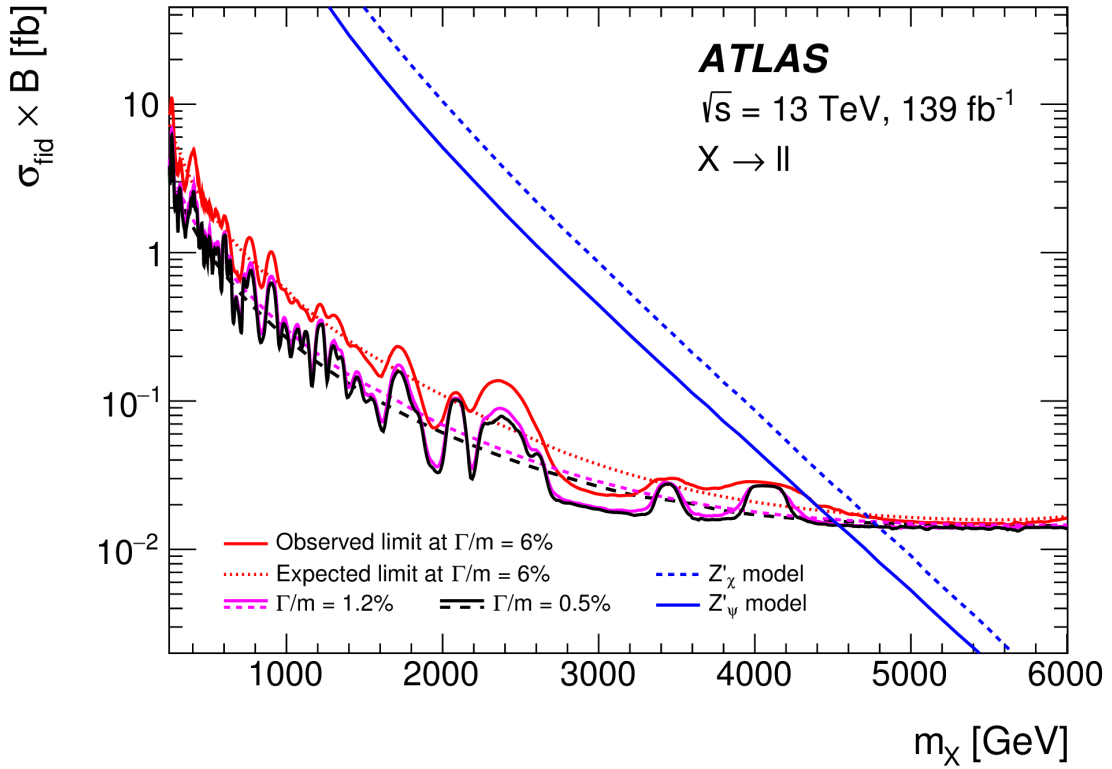
The ATLAS Collaboration has also published the full set of data points for the cross section limits that were used in the plots in Fig 5.3, together with the selection efficiency

⁷⁸ The relative width $\Gamma_{Z'}/m_{Z'}$ here refers to the ratio of the decay width of the Z' boson to the pole mass of the Z' boson

⁷⁹ The decay width $\Gamma_{Z'}$ can be obtained directly from Eq (2.55)



(a)



(b)

Figure 5.3: The 95% CL upper limits on the “weighted” fiducial cross sections as a function of the pole mass of the Z boson for $\Gamma_{Z'}/m_{Z'} =$ (a) 0%, 3%, 10%; and (b) 0.5%, 1.2%, 6%. The theoretical fiducial cross section as predicted by SSM for $\Gamma_{Z'}/m_{Z'} = 3.0\%$ is shown by the blue shaded region in (a)

data in Fig 5.2 for both channels. However, the Collaboration did not published the theoretical upper limit on the fiducial cross section as predicted by the Sequential Standard Model (SSM), represented by the shaded blue band in Fig 5.3(a). Despite this set of data points not being available, it was understood that this upper limit was obtained directly from the fiducial partonic cross sections, based on data from the pp collisions at the LHC⁸⁰, multiplied by the branching ratios for the different channels, obtained using Eq (5.5) below. Note that a 1-Dimensional interpolation was done for all data points regardless whether they were published by the ATLAS Collaboration or obtained directly from the fiducial partonic cross sections.

The branching ratios for the dielectron ee and dimuon $\mu\mu$ channels are obtained using the following formula:

$$\text{Br}(Z' \rightarrow \ell\ell) = \frac{\Gamma_{Z'}^{\ell\ell}}{\Gamma_{Z'}}, \quad \text{where } \ell = e/\mu. \quad (5.5)$$

Note that $\Gamma_{Z'}$ here refers to the total width of the Z' boson obtained using a slightly modified version of Eq (3.7), which takes on the following form

$$\Gamma_{Z'} = \Gamma_{Z'}^{\chi\bar{\chi}} + N_c^f \sum_f \Gamma_{Z'}^{f\bar{f}} \quad (5.6)$$

where N_c^f for the different species of fermions can be obtained from Table 2.2, and Eq (3.8), where X is now either χ (DM) or f (SM fermion).

5.1.2.1 External Implementations

After the 95% CL upper limits on the “weighted” fiducial cross sections for all 6 relative widths $\Gamma_{Z'}/m_{Z'}$ and the theoretical upper limit on the “weighted” fiducial cross section predicted by SSM have been obtained, the likelihood function for the dilepton constraints can now be modelled. The likelihood function for the dilepton constraints was modelled using 2 different approaches.

The first approach involved directly calculating the ratio of the experimental upper limits to the theoretical upper limit on the “weighted” fiducial cross section. Using this approach, the likelihood function is modelled in the following fashion:

$$\Delta\chi^2 = 4 \times \left(\frac{\sigma_{\text{fid}}^{\text{SSM}}}{\sigma_{\text{fid}}^{\text{exp}}} \right)^2, \quad (5.7)$$

where $\sigma_{\text{fid}}^{\text{SSM}}$ refers to the theoretical upper limit on the “weighted” fiducial cross section as predicted by the Sequential Standard Model (SSM), and $\sigma_{\text{fid}}^{\text{exp}}$ refers to the experimental 95% CL upper limits published by the ATLAS Collaboration.

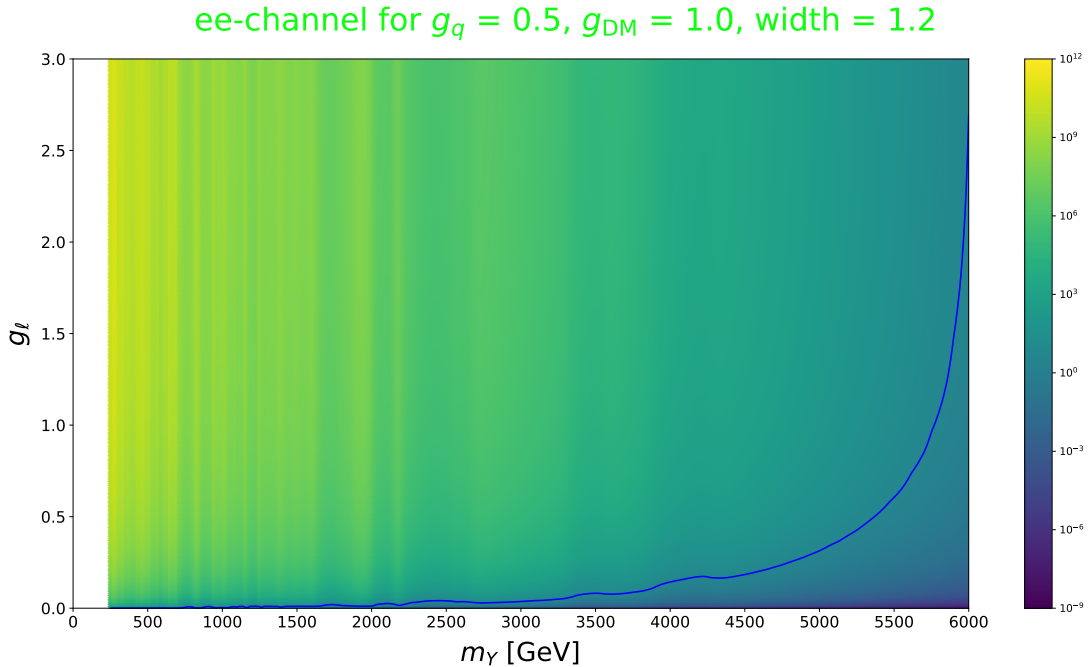
⁸⁰ As mentioned in 77, the physics inside pp collisions is very complicated and hence it will not be elaborated here

In the second approach, a reference grid R as a function of the mediator mass m_Y for each of the 6 relative widths Γ/m and each of the 3 channels (ee , $\mu\mu$ and $\ell\ell$ channels) was obtained using a reference set of couplings ($g_q = 0.1$, $g_\ell = 0.1$ and $g_{\text{DM}} = 0$) and DM mass ($m_\chi = 200$ GeV). Using these reference grids $R(m_Y)$, the likelihood function in Eq (5.7) is now recasted into the following form:

$$\Delta\chi^2 = \frac{1}{R(m_Y)} \times \left[\frac{(g_q \times g_\ell)_{\text{point}}}{(g_q \times g_\ell)_{\text{ref}}} \right]^2 \times \left(\frac{\Gamma_Y^{\text{ref}}}{\Gamma_Y^{\text{point}}} \right), \quad (5.8)$$

where the subscript ref in the product $g_q \times g_\ell$ indicates that the product was obtained using a reference set of couplings, and Γ_Y refers to the total width of the mediator Y obtained using Eq (5.6).

As a form of cross-check, the (m_Y, g_ℓ) planes for the dielectron ee , dimuon $\mu\mu$ and combined dilepton $\ell\ell$ channels were plotted using both approaches. As shown in the figures below, the agreement is excellent for all 3 channels confirming that both approaches has been implemented correctly. Although the agreement for both approaches is excellent, an important distinction was observed between the 2 approaches with regards to their computational times. It has been observed that the time taken to compute and plot the likelihood function for every single point on the (m_Y, g_ℓ) plane using the first approach was significantly longer than that using the second approach.



(a) First Approach

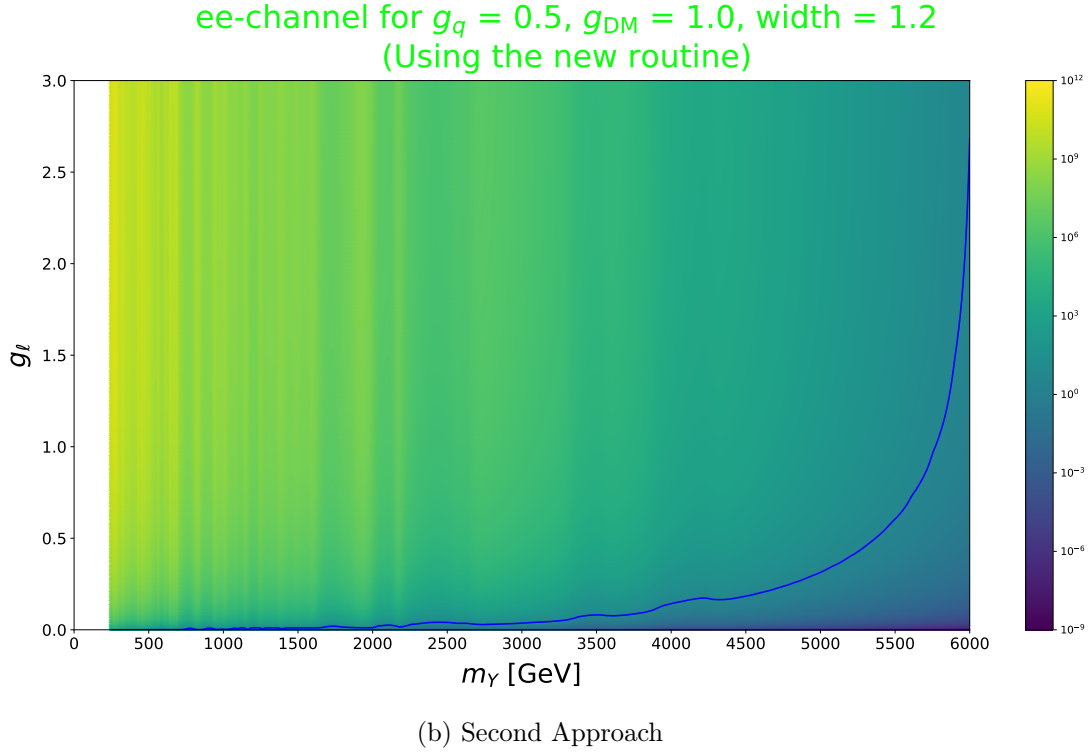
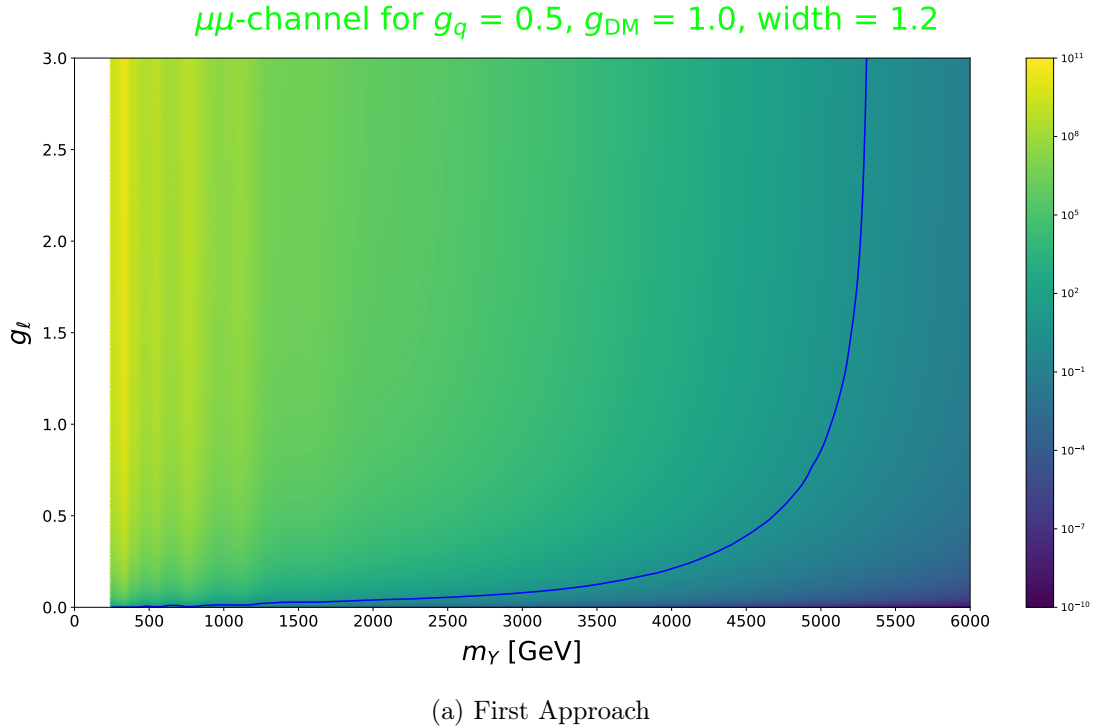


Figure 5.4: The (m_Y, g_l) planes for the dielectron ee channel and for the relative width $\Gamma_Y/m_Y = 1.2\%$ using both the (a) first and (b) second approach. A fixed set of DM and quark couplings ($g_q = 0.5$ and $g_{DM} = 1.0$) together with a fixed DM mass of $m_\chi = 400$ GeV were used to plot these planes. The colour bar on the right of the plot shows the variations of the likelihood function χ^2 . The blue $\chi^2 = 4$ contour in the plot represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.



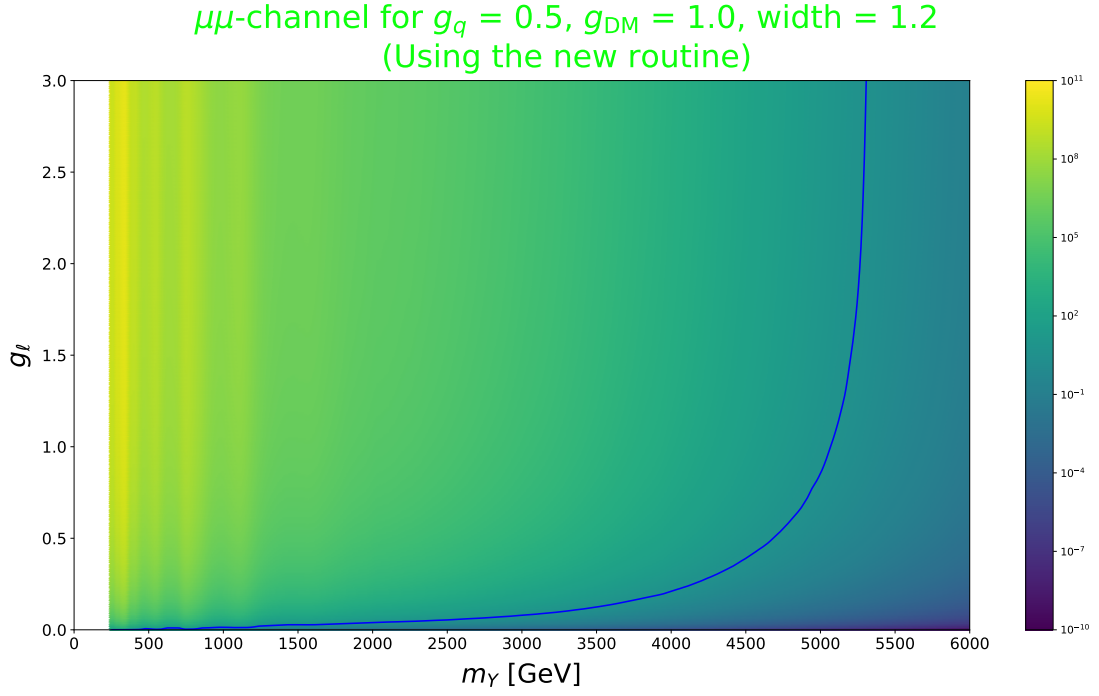
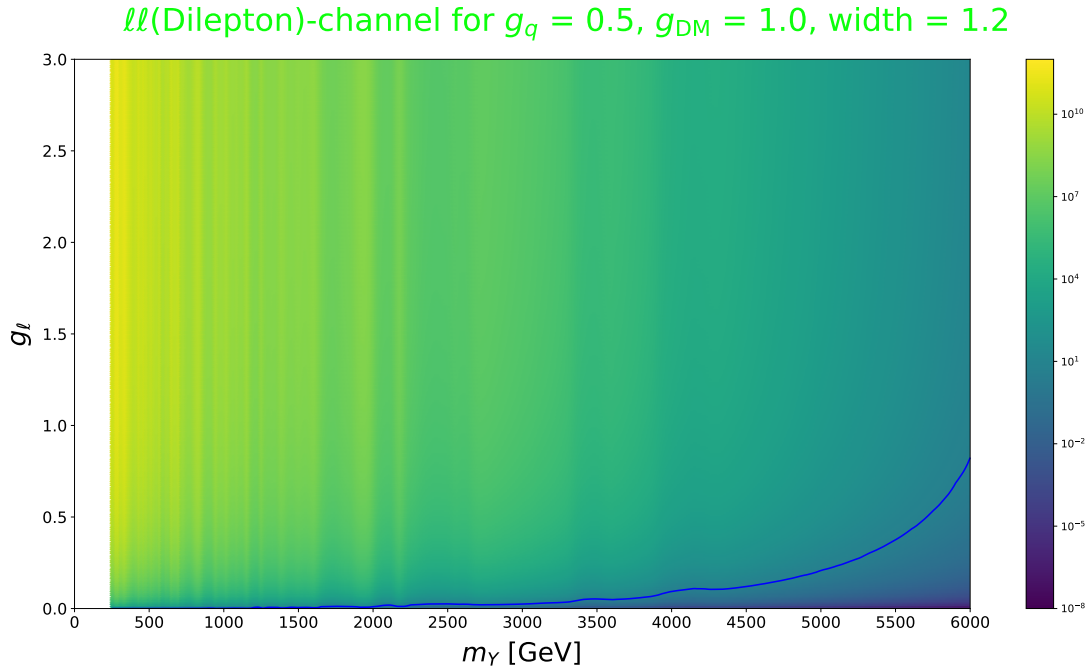


Figure 5.5: The (m_Y, g_ℓ) planes for the dimuon $\mu\mu$ channel and for the relative width $\Gamma_Y/m_Y = 1.2\%$ using both the (a) first and (b) second approach. Again, a fixed set of DM and quark couplings together with a fixed DM mass were used to plot these planes. The colour bar on the right of the plot shows the variations of the likelihood function χ^2 . The blue $\chi^2 = 4$ contour in the plot represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.



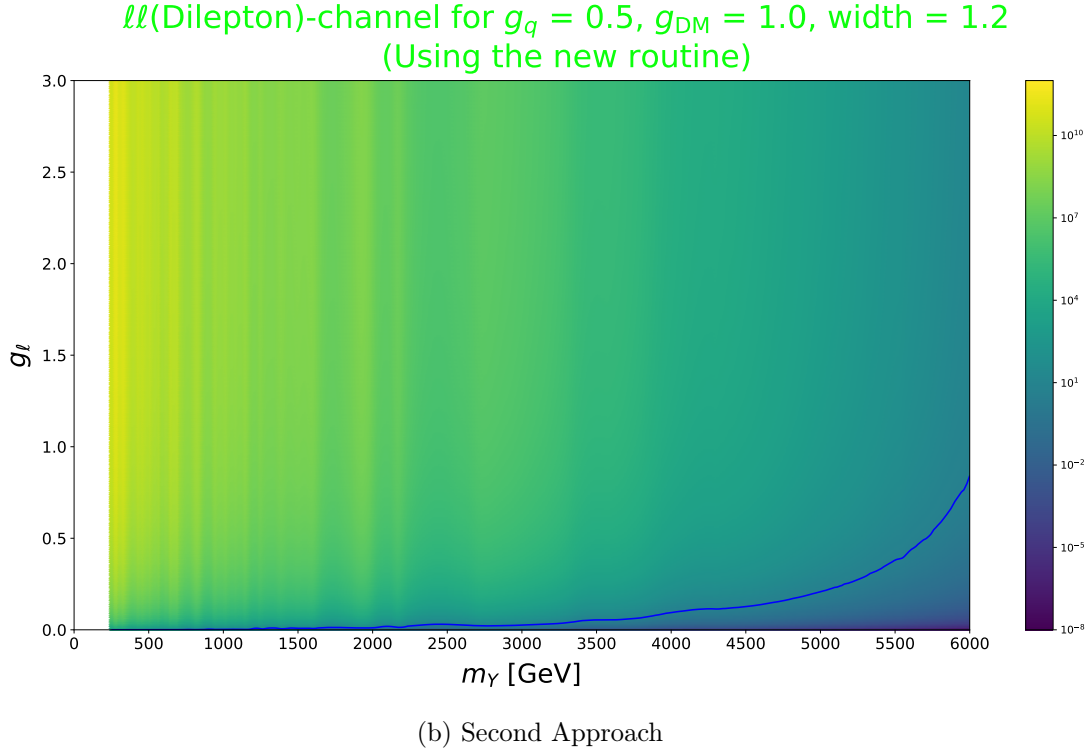


Figure 5.6: The (m_Y, g_ℓ) planes for the combined dilepton $\ell\ell$ channel and for the relative width $\Gamma_Y/m_Y = 1.2\%$ using both the (a) first and (b) second approach. Again, a fixed set of DM and quark couplings together with a fixed DM mass were used to plot these planes. The colour bar on the right of the plot shows the variations of the likelihood function χ^2 . The blue $\chi^2 = 4$ contour in the plot represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.

5.1.2.2 Implementation In MasterCode

After the external implementations for both approaches had been validated, they were then implemented into **MasterCode**. However, since the second approach which uses the recasted form of the likelihood function in Eq (5.8) was shown to be more computationally efficient, only the second approach was implemented in **MasterCode**. As mentioned in the previous chapter, to maximize computational efficiency, the recasted form of the likelihood function in Eq 5.8, *i.e.* the second approach, was written and stored in the same C++ file as that containing the source codes used to compute the 1-Dimensional likelihood functions for the rest of the constraints.

Just as what was done for the existing constraints, the reference grids for all 3 channels (ee , $\mu\mu$ and $\ell\ell$ channels) and all 6 relative widths Γ_Y/m_Y for each channel were imported into **MasterCode**. Instead of running the computation of the likelihood function over all 6 relative widths, the relative width for a particular point $(\Gamma_Y/m_Y)^{\text{point}}$ was first calculated. It was then compared with the 6 relative widths ($\Gamma_Y/m_Y = 0\%$, 0.5% , 1.2% , 3% , 6% , and 10%) to determine which of the 6 was closest to it. Once the closest of the 6 relative

widths has been determined, the reference grid for that chosen relative width would then be used for the computation of the likelihood function at that point.

As a final form of cross-check, the (m_Y, g_ℓ) planes for all 3 channels were plotted inside the `MasterCode` package. Even though the likelihood function was implemented slightly differently inside `MasterCode`, the general profile of the plots as shown in the figures below seem to match those in Figs 5.4 to 5.6. This confirms that the new dilepton constraints have been correctly implemented in `MasterCode`.

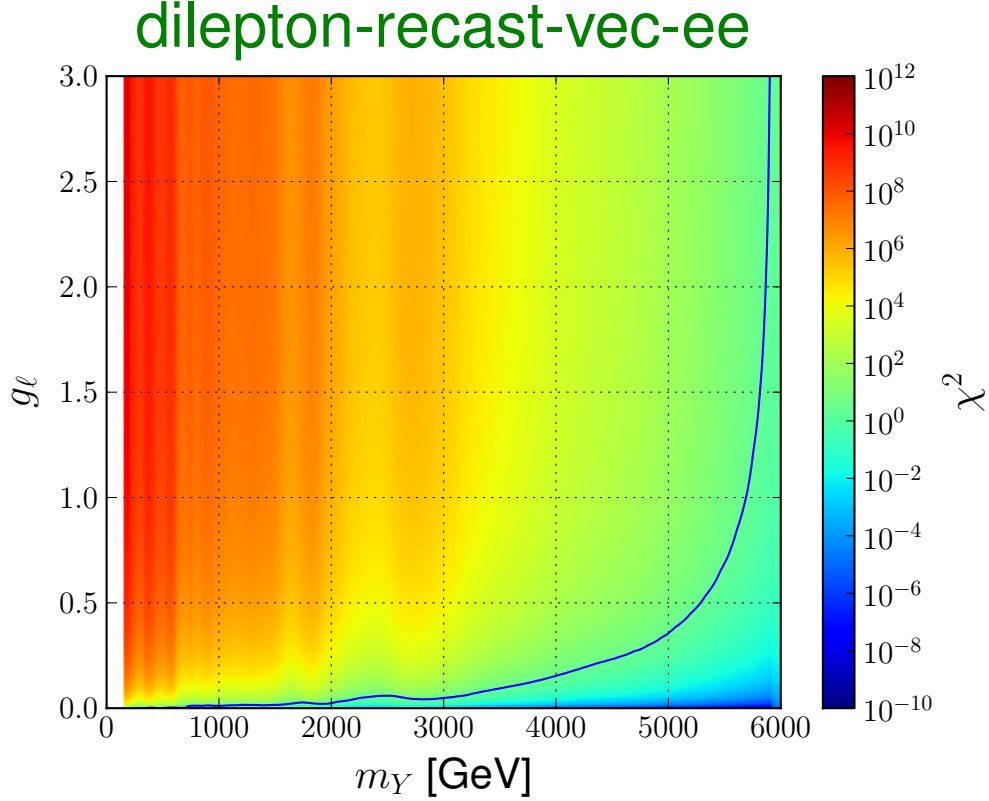


Figure 5.7: A plot of the (m_Y, g_ℓ) plane for the dielectron ee channel using the `MasterCode` package. The fixed set of quark and DM couplings together with the DM mass were the same as those used in the external implementations. Likewise, the colour bar on the right of the plot again shows how the likelihood function χ^2 varies throughout the plane. The thin blue contour in the plane represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.

As a result of the new dilepton constraints, an additional parameter which is the leptonic coupling g_ℓ needs to be included into the sampling. In addition, using the results published by the ATLAS Collaboration in [73], the range of the mediator mass m_Y needs to be modified, in which its lower limit was set as 250 GeV to avoid Z^0 pole region, and extended to an upper limit of 6 TeV. Including these modifications into Table 3.1, the new updated sampling ranges of the 5 free parameters for the leptophilic model is shown in the table on the following page.

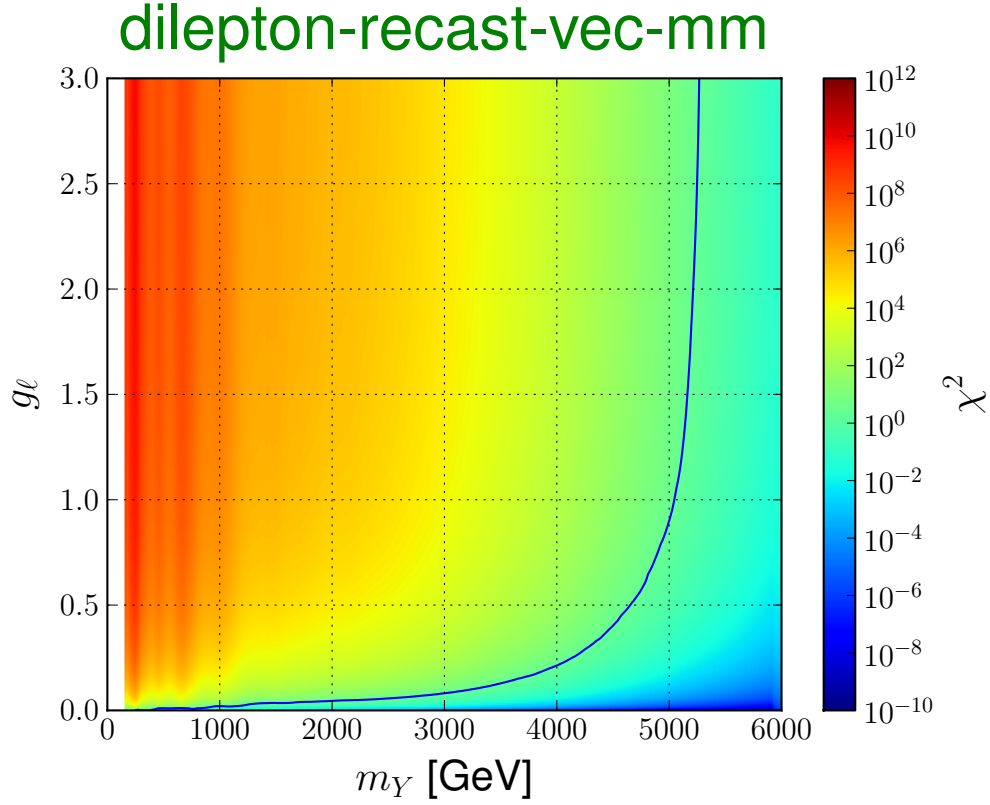


Figure 5.8: A plot of the (m_Y, g_ℓ) plane for the dimuon $\mu\mu$ channel using the **MasterCode** package. The fixed set of quark and DM couplings together with the DM mass were the same as those used in the external implementations. Likewise, the colour bar on the right of the plot again shows how the likelihood function χ^2 varies throughout the plane. The thin blue contour in the plane represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.

Parameter	Range	Number of Segments
m_Y	(0.25, 6.0) TeV	10
m_χ	(0.0, 2.5) TeV	8
g_q	$(0.0, \sqrt{4\pi})$	2
g_ℓ	$(0.0, \sqrt{4\pi})$	2
g_{DM}	$(0.0, \sqrt{4\pi})$	2
Total Number of Segments		640

Table 5.1: Sampling ranges of the 5 free parameters in the leptophilic model, together with the number of segments into which they were divided during the sampling

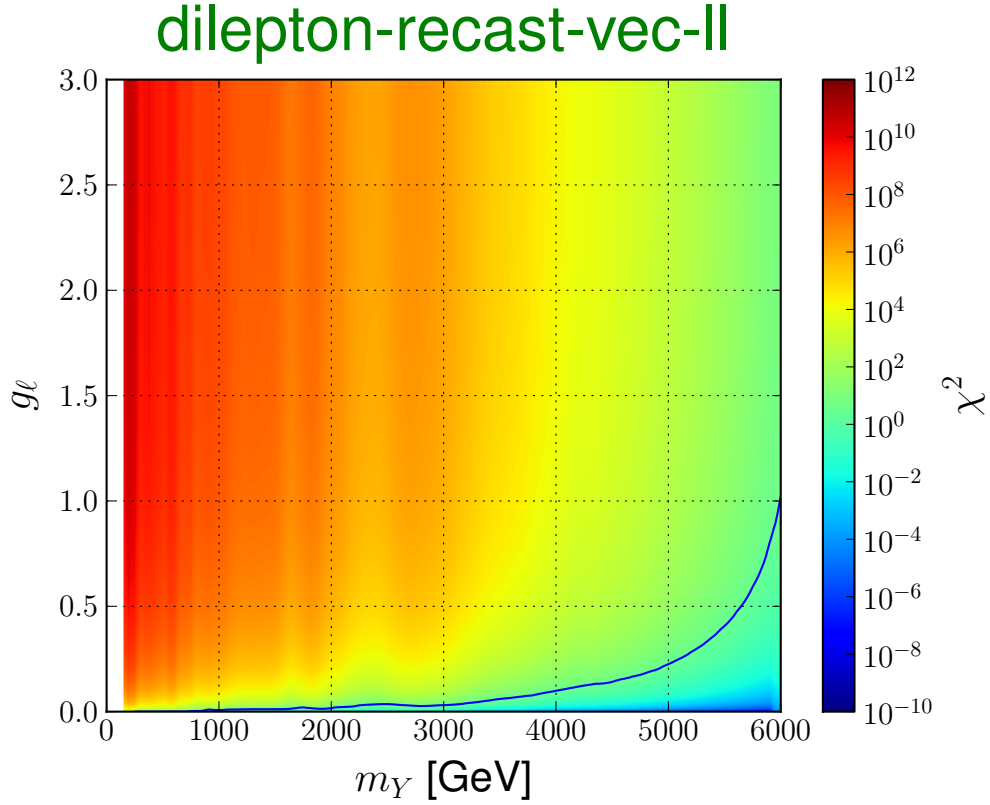


Figure 5.9: A plot of the (m_Y, g_ℓ) plane for the combined dilepton $\ell\ell$ channel using the **MasterCode** package. The fixed set of quark and DM couplings together with the DM mass were the same as those used in the external implementations. Likewise, the colour bar on the right of the plot again shows how the likelihood function χ^2 varies throughout the plane. The thin blue contour in the plane represents the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.

5.2 Additional Constraints

On top of the experimental constraints from astrophysical observations and the LHC, an additional constraint was included to restrict the range of allowed relative widths of the mediator $\Gamma_Y/m_Y \leq 0.3$. Since the search in [73] focuses on high-mass dilepton resonances, the resonance condition determined by this range of allowed relative widths was required. The effect of this additional constraint can be clearly seen in the [Selected Results](#) section, especially in the 1-Dimensional plot of the likelihood function as function of the leptonic coupling g_ℓ in Fig 5.11.

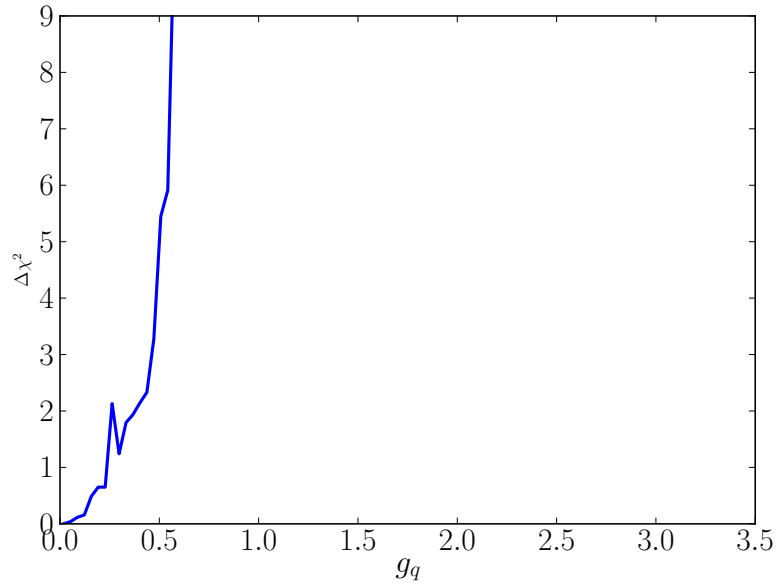
5.3 Selected Results

In this section, only a set of selected plots are presented. The 1- and 2-Dimensional plots are presented separately in 2 different subsections. Note that especially for the 2-Dimensional plots, some of the plots to be presented later are chosen as a form of

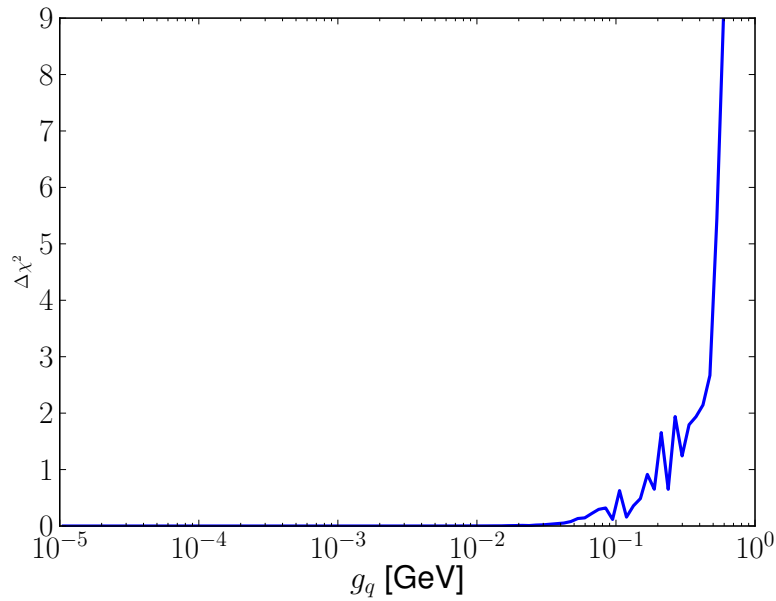
comparison with the plots of the same plane in the leptophobic model, and the rest are chosen because they have well-defined features.

5.3.1 1-Dimensional Plots

The 1-Dimensional plots presented in this section are designed to show how likelihood function $\Delta\chi^2$ varies with different parameters which are the 3 couplings, q_q , g_ℓ and g_χ . Note that for the quark coupling g_q , 2 plots are presented where one is in linear scale and the other is in logarithmic (log) scale.



(a) Linear scale



(b) Logarithmic scale

Figure 5.10: The relative likelihood, $\Delta\chi^2$, as a function of the quark coupling g_q in both a) linear and b) logarithmic scales, for values of $\Delta\chi^2 \leq 9$

One would observe from the plots on the previous page that only small values of the quark coupling, $g_q \leq 0.6$, are allowed. In other words, only small values of g_q are accepted based on the allowed values for the relative likelihood, $\Delta\chi^2 \leq 9$. These allowed values of the quark coupling g_q are comparable to those allowed in the leptophobic model, as shown in Figures 3.6 and 3.7.

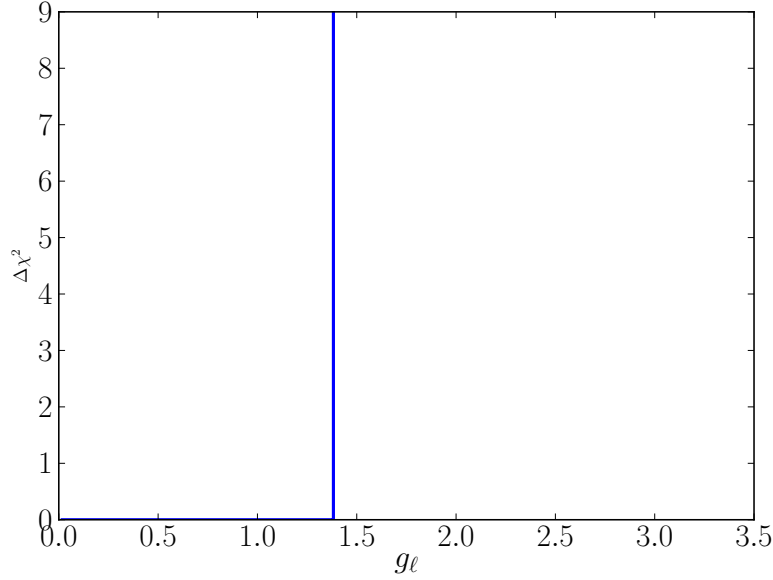


Figure 5.11: The relative likelihood, $\Delta\chi^2$, as a function of the leptonic coupling, g_ℓ , for values of $\Delta\chi^2 \leq 9$

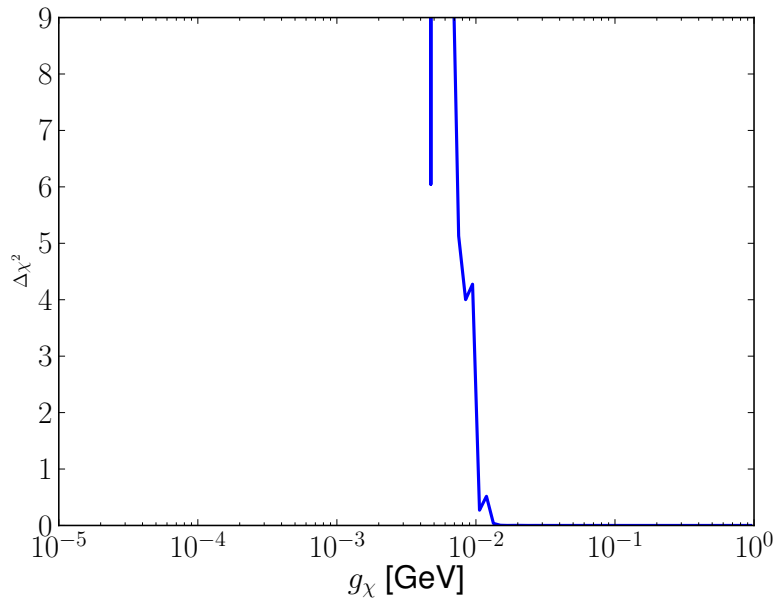


Figure 5.12: The relative likelihood, $\Delta\chi^2$, as a function of the logarithmic of the DM coupling, g_{DM} , for values $\Delta\chi^2 \leq 9$

In Figure 5.11, one would be able to see a clear boundary for the allowed values of the leptonic coupling g_ℓ . At $g_\ell \approx 1.4$, there seems to be a “wall” below which any values of g_ℓ are allowed but above which none is allowed. This “wall” will appear again as a distinct feature in both Figures 5.15 and 5.17. At the current stage of the analysis, with the

sampling of the vector model yet to be completed, the reason behind this "wall" has yet to be fully understood although it is believed that the additional constraint $\Gamma_Y/m_Y < 0.3$ has strictly restricted the values of g_ℓ to the allowed range.

On the contrary to the plot for the quark coupling g_q , the plot in Figure 5.12 for the logarithmic of the DM coupling $\log g_\chi$ shows a completely opposite story. One would observe from the plot that small values of $\log g_\chi$ are not allowed while larger values of $\log g_\chi$, $\log g_\chi \gtrsim 7 \times 10^{-3}$, becomes increasingly allowed.

5.3.2 2-Dimensional Plots

Same as the 2-Dimensional plots in the leptophobic case, the z-axes for all the 2-Dimensional plots, other than the plot of the (m_Y, m_χ) plane, correspond to the likelihood functions $\Delta\chi^2$ for their respective planes. Just as the plots in the leptophobic model, the red contours in the following plots, other than the plot in Figure 5.13, represent regions favoured at 68% (1σ -) Confidence Level (CL), where $\Delta\chi^2 < 2.30$, while the blue contours represent regions favoured at 95% (2σ -) CL, where $\Delta\chi^2 < 5.99$

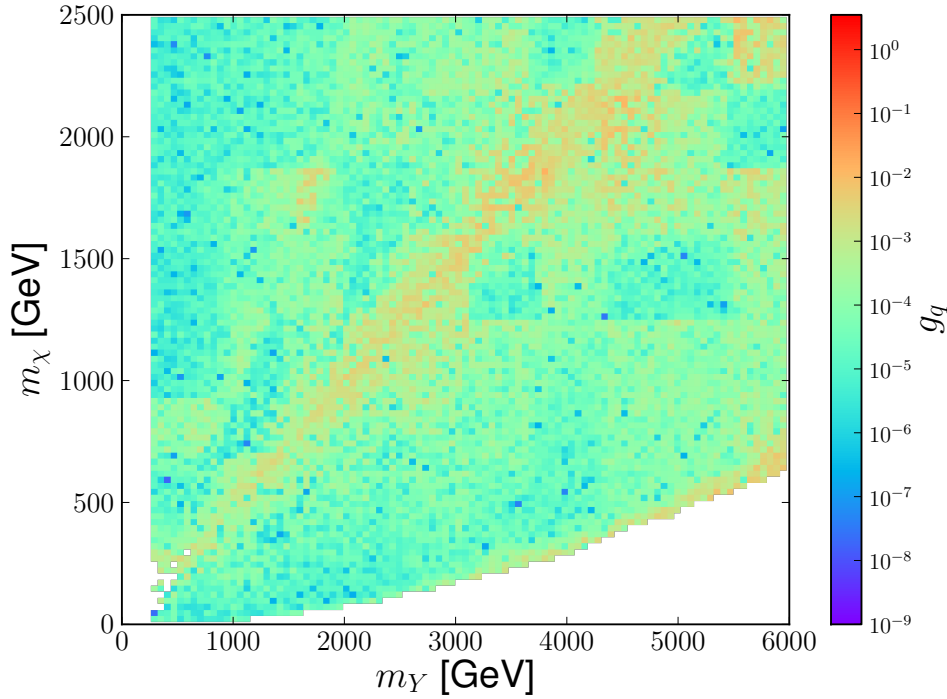
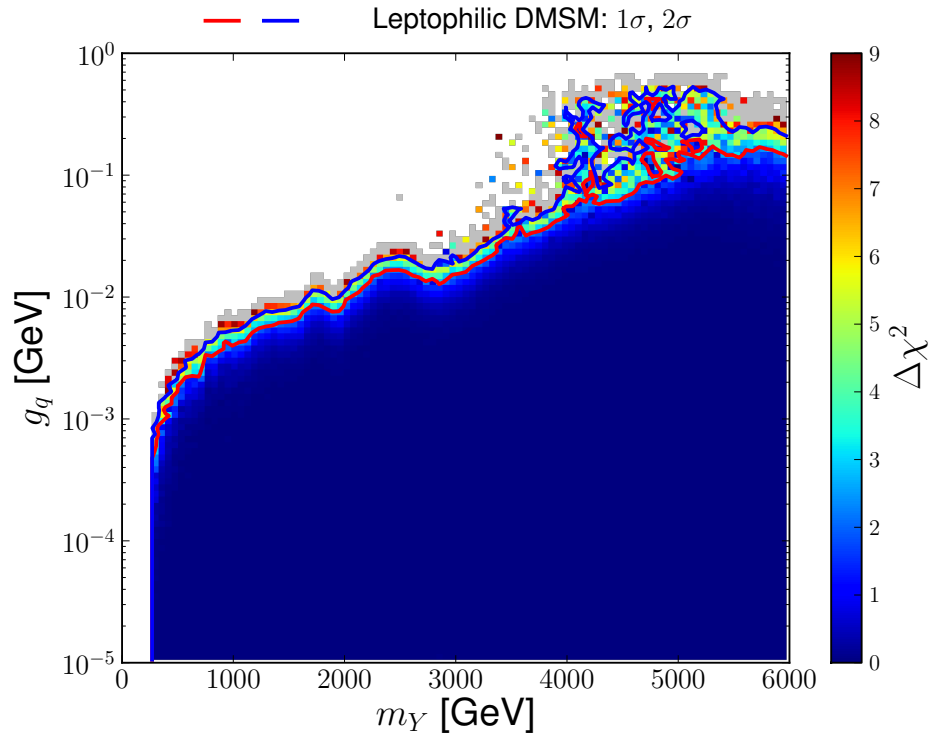


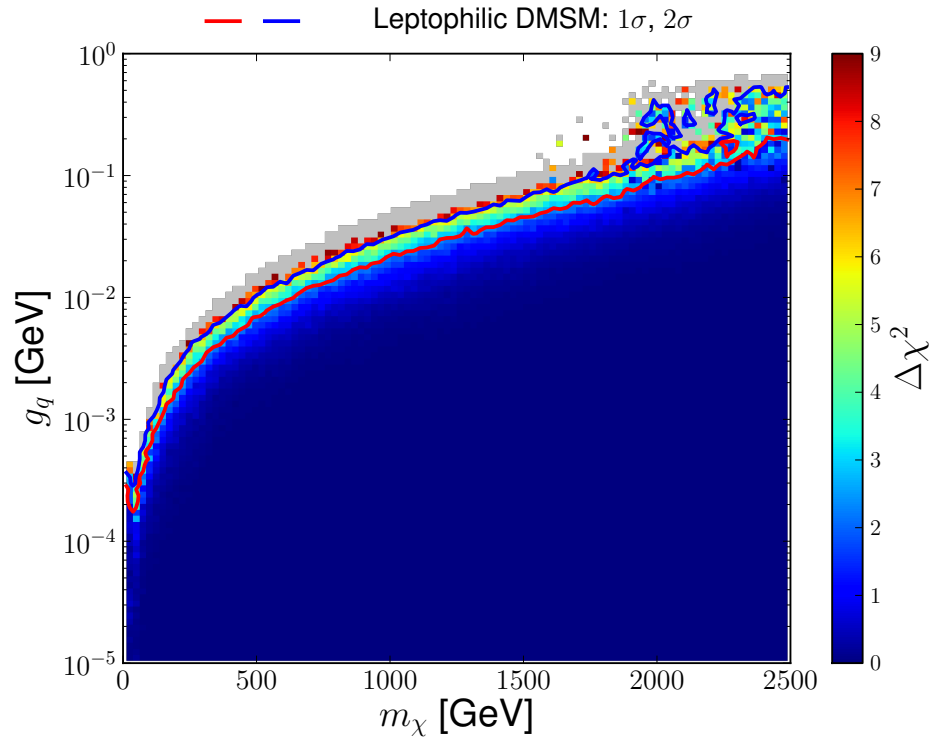
Figure 5.13: The (m_Y, m_χ) plane with the quark coupling, g_q , corresponding to the z-axis

The figure above shows the $m_Y - m_\chi$ plane with the quark coupling g_q corresponding to the z-axis. From the figure, upon close inspection, one would be able to identify an “orange” funnel, where larger values of g_q are allowed, which looks very similar to the s -channel funnel in Figure 3.5. The reason why there is a presence of this orange funnel has yet to be understood. Further analysis will probably provide a clearer picture about the orange funnel and why it somehow looks like the s -channel funnel for the same plane

in the leptophobic model.



(a) $(m_Y, \log g_q)$ plane



(b) $(m_\chi, \log g_q)$ plane

Figure 5.14: The 2-Dimensional likelihood functions $\Delta\chi^2$ for the logarithmic of the quark coupling $\log g_q$ as functions of the mediator, m_Y , and DM, m_χ , masses

The figure on the previous page shows the 2-Dimensional likelihood functions for the logarithmic of the quark coupling $\log g_q$ as functions of the mass of the mediator, m_Y , and DM, m_χ . For both planes, one would observe that the upper limits on $\log g_q$ increase together with increasing mediator and DM masses. However, in the leptophobic model, for the same set of planes in Figures 3.7a and 3.8a, it was observed that the upper limits on SM coupling g_{SM} ⁸¹ does not vary significantly with increasing mediator and DM masses, unlike what was observed in the leptophilic model. The reason behind this difference in observations has yet to be understood, hence further analysis is required.

The rest of the plots in the following pages have yet to be analysed and understood, hence further analyses of these plots are required to better understand the features in these planes.

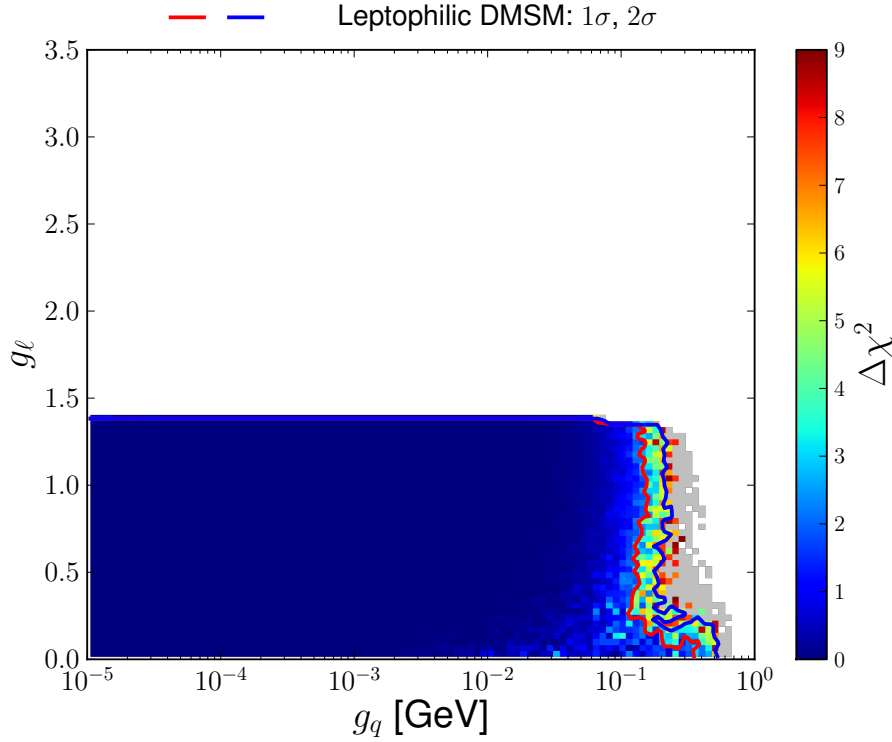


Figure 5.15: The preferred regions at 68% (1σ -) and 95% (2σ -) CL in the $(\log g_q, g_\ell)$ plane. Notice that there is a strict upper limit that seems to be imposed on the allowed values of g_ℓ . This is probably the result of the additional constraint $\Gamma_Y/m_Y < 0.3$ but the reason behind this strict upper limit has yet to be fully understood.

⁸¹ Equivalent to the quark coupling in the leptophilic model

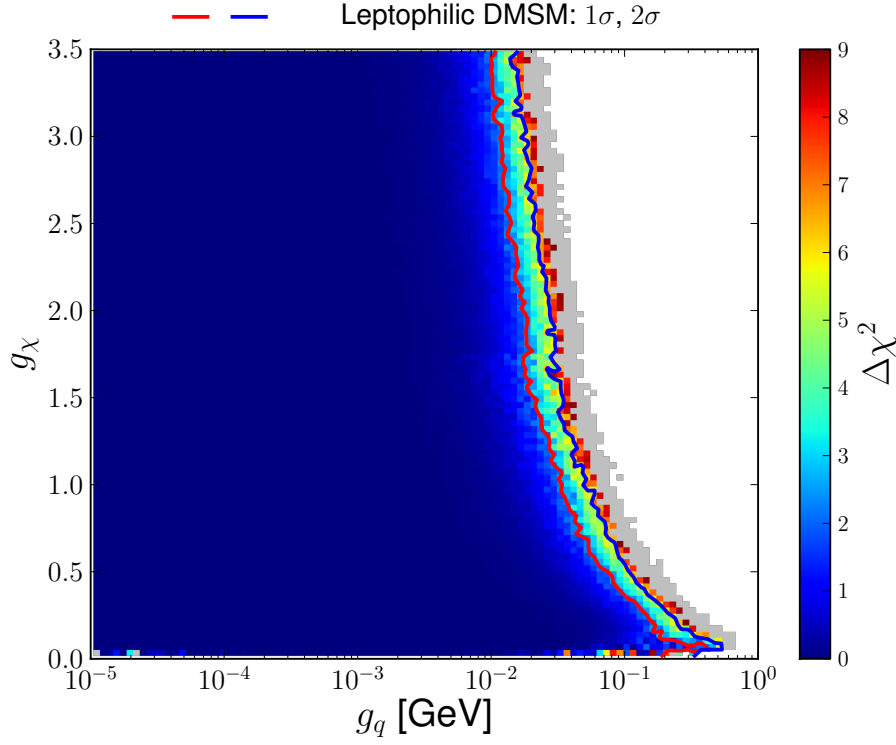


Figure 5.16: The preferred regions at 68% (1σ -) and 95% (2σ -) CL in the $(\log g_q, g_\chi)$ plane

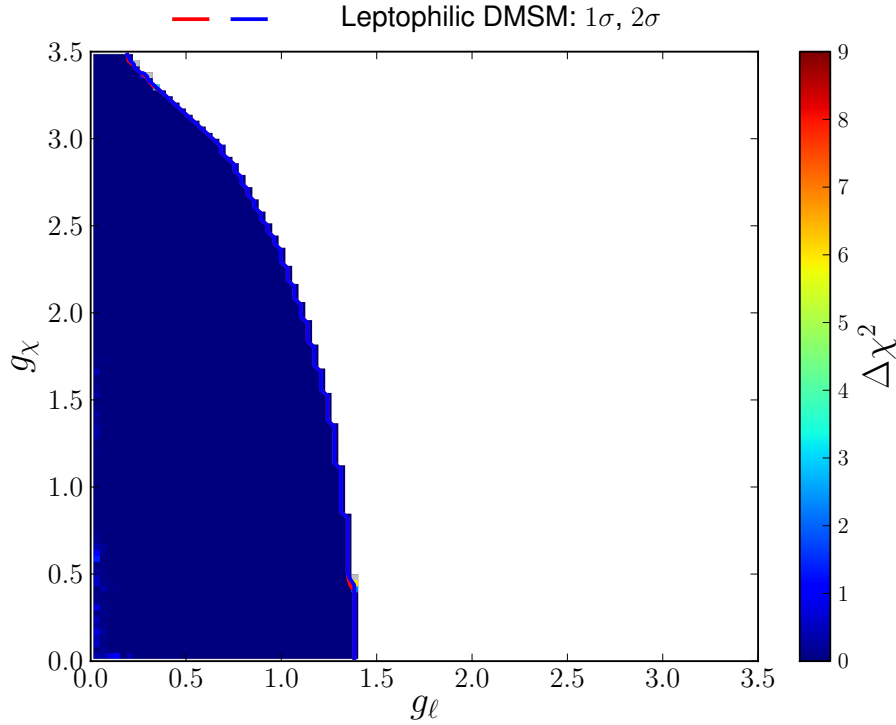


Figure 5.17: The preferred regions at 68% (1σ -) and 95% (2σ -) CL in the (g_ℓ, g_χ) plane. Notice that, just like in the $(\log g_q, g_\ell)$ plane, there seems to be a strict upper limit imposed on the allowed values of g_ℓ , which becomes tighter with increasing values of g_χ . This is probably the result of the additional constraint $\Gamma_Y/m_Y < 0.3$ but the reason behind this strict upper limit has yet to be fully understood.

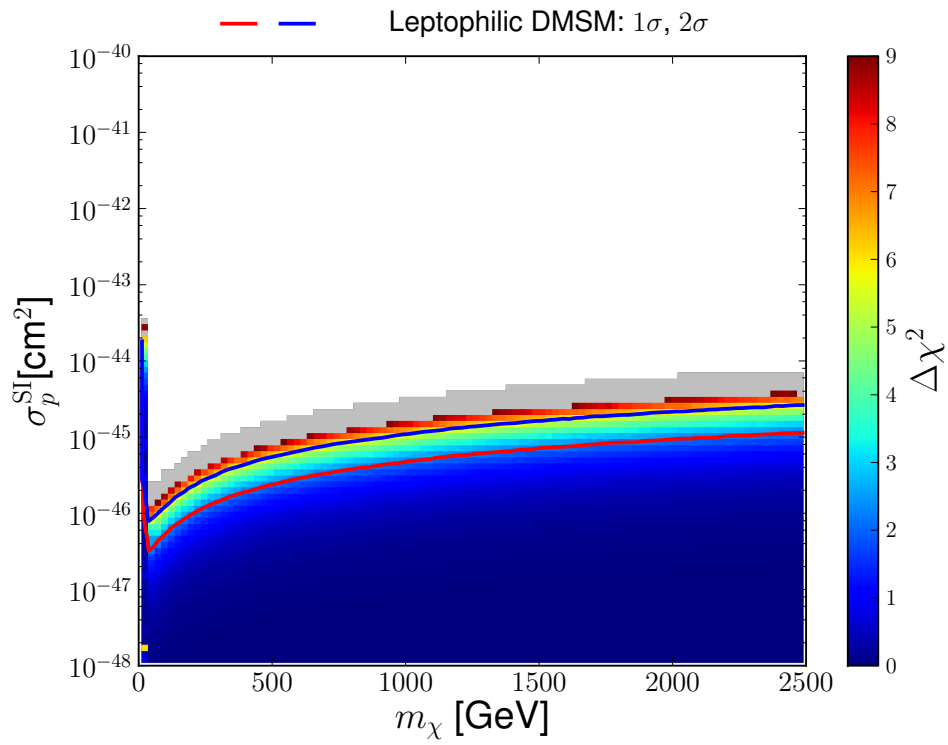


Figure 5.18: The preferred regions at 68% (1σ -) and 95% (2σ -) CL in the $(m_\chi, \sigma_p^{\text{SI}} [\text{cm}^2])$ plane

Chapter 6

Conclusion and Future Work

6.1 Conclusion

In this thesis, we have discussed Dark Matter Simplified Models (DMSMs) and the analyses of these models using the `MasterCode` framework. However, before proceeding straight into the discussion of these models, we first need to introduce the basics of particle physics where the discussion was done in the first 2 chapters. In the first chapter, the [Standard Model \(SM\)](#) was introduced, where we discussed the different representations of the Standard Model (SM). From there, focusing on a particular representation known as the Lagrangian formulation of the Standard Model, we proceeded on to discuss about [Feynman Calculus](#), where we discussed the mathematical formalism behind Feynman diagrams. In addition, an example on the cross section calculation for the electron-muon scattering process, mediated by a photon, was also introduced to put what was discussed in the section into perspective.

In the next chapter, the scope of the discussion was narrowed down to one of the fundamental forces in the Standard Model, the [Weak Interactions](#). The chapter started out from a general discussion of this fundamental force, which included the cross section calculation for the same scattering process towards the end of the previous chapter but the process was mediated by a Z boson instead. It was then followed by a discussion on the [Higgs Mechanism](#) in conjunction with a subsequent section on [Spontaneous Symmetry Breaking \(SSB\)](#), where both processes are essential in explaining how fermions and massive vector bosons in the Standard Model acquire mass but not the photon. Closing the chapter, the Sequential Standard Model (SSM), a Beyond Standard Model (BSM) theory, was briefly discussed in preparation for the discussion of the [Dilepton Constraints](#) which was done in Chapter 5.

After the basic theoretical background had been established, we could now proceed on with our discussion of DMSMs. We first started discussing about [DMSMs With Leptophobic Spin-One Mediators](#), where the interactions between the mediator particles and leptons was ignored. At the start of the chapter, the need to introduce DMSMs as a BSM theory was explained, together with a brief and general introduction about Dark Matter (DM). It was then followed by the [Analyses Restrictions and Framework](#), which included a brief discussion of the theoretical framework of the DMSMs that were being analysed. In the same section, the sampling ranges of the 4 free parameters in the leptophobic

model were also presented, together with the reasons why these ranges were chosen. The different experimental constraints, which was further subdivided into [Astrophysical DM Constraints](#) and [LHC Experimental Constraints](#), were presented in the following section. Finally, the results from the implementation of the experimental constraints were presented in the subsequent section. The results for models with purely vectorial couplings were presented separately from those with purely axial couplings.

In the following chapter, the [MasterCode Framework](#) was discussed in greater detail. The chapter started out by providing a brief introduction about the framework, including the layout, motivation and objectives of the package. It was then followed by a brief discussion of the external packages used by [MasterCode](#) for the computation of observables in DMSMs. In the subsequent section, we proceeded on to discuss about the [Source Files](#) in the [MasterCode](#) package that were used to calculate the individual likelihood functions for the different experimental constraints. Lastly, a discussion of the algorithm which [MasterCode](#) uses to conduct its sampling closes the chapter.

In the final chapter, the discussion was focused on [DMSMs With Leptophilic Spin-One Mediators](#), following up on the discussion in the previous 2 chapters. The chapter started out with a short discussion on the modified theoretical framework of the DMSMs under analysis after including the interactions between the mediator particles and the SM leptons. It was followed by the discussion of the modified [LHC Experimental Constraints](#), which included the modified version of the monojet constraints and a new constraint known as the [Dilepton Constraints](#). A discussion of how the new [Dilepton Constraints](#) was implemented both externally and in [MasterCode](#) was also included. The discussion continued on to a small section on [Additional Constraints](#) that were implemented on top of previously mentioned experimental constraints. Finally, ending the chapter with the presentation of the results for some selected 1-Dimensional and 2-Dimensional plots.

6.2 Potential Future Work

As mentioned in the previous chapter, the analyses of DMSMs with leptophilic spin-one mediators is still in progress. This is especially so since the analysis is still currently on the vector model and we have yet to proceed to the analysis for the axial-vector model. Hence, the next phase of the project would be focused on the analysis of the axial-vector model.

In addition, as mentioned in the previous chapter, additional constraints will be included as *a posteriori* once the sampling for both the vector and axial-vector models have been completed. These additional constraints include results from direct Dark Matter (DM) detection experiments, such as those published by the IceCube Collaboration [\[74\]](#), and a new set of dijet constraints published by the CMS Collaboration [\[72\]](#).

Appendix A

Complete Derivations

In this appendix, the complete derivations for the cross sections of the electron-muon scattering process is presented, in both cases where the process is either mediated by a photon γ or a Z^0 boson. In addition, the contribution from the mixing of the electromagnetic and weak interactions to the cross section is also included in this set of derivations.

A.1 Trace Theorems

Before proceeding on with the full derivations for the electron-muon scattering cross sections, we will first discuss trace algebra together with a set of “trace theorems”. These theorems will be heavily used throughout the derivations in the following section.

Starting off with some basic trace algebra. Let A , B and C be any arbitrary matrices, and α be any arbitrary constant, the following properties hold:

- I) $\text{Tr}(A + B) = \text{Tr}(A) + \text{Tr}(B)$
- II) $\text{Tr}(\alpha A) = \alpha \text{Tr}(A)$
- III) $\text{Tr}(AB) = \text{Tr}\{BA\} \iff \text{Tr}(ABC) = \text{Tr}(CAB) = \text{Tr}(BCA)$
(The cyclic property of traces)

where the first 2 properties are derived from the linearity of traces. The following results will prove to be useful:

- IV) $g_{\mu\nu}g^{\mu\nu} = 4$ (The product of the space-time metric with its inverse)
- V) $\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \implies \{\not{a}, \not{b}\} = \not{a}\not{b} + \not{b}\not{a} = 2(a \cdot b)$
(The anti-commutation relation for the γ matrices and the corresponding relation in terms of the Feynman-slash notation)

Following from these results, the set of “contraction theorems” are derived:

- VI) $\gamma_\mu\gamma^\mu = 4$
- VII) $\gamma_\mu\gamma^\nu\gamma^\mu = -2\gamma^\nu \implies \gamma_\mu\not{a}\gamma^\mu = -2\not{a}$
- VIII) $\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\mu = 4g^{\nu\lambda} \implies \gamma_\mu\not{a}\not{b}\gamma^\mu = 4(a \cdot b)$
- IX) $\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\gamma^\mu = -2\gamma^\sigma\gamma^\lambda\gamma^\nu \implies \gamma_\mu\not{a}\not{b}\not{c}\gamma^\mu = -2\not{c}\not{b}\not{a}$

together with the following set of “*trace theorems*”:

$$\text{X) } \text{Tr}(\mathbb{1}) = 4 \text{ (The trace of a } 4 \times 4 \text{ identity matrix)}$$

$$\text{XI) } \text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu} \implies \text{Tr}(\not{a} \not{b}) = 4(a \cdot b)$$

$$\begin{aligned} \text{XII) } \text{Tr}(\gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) &= 4(g^{\mu\nu} g^{\lambda\sigma} - g^{\mu\lambda} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\lambda}) \implies \\ \text{Tr}(\not{a} \not{b} \not{c} \not{d}) &= 4[(a \cdot b)(c \cdot d) - (a \cdot c)(b \cdot d) + (a \cdot d)(b \cdot c)] \end{aligned}$$

$$\text{XIII) } \text{Tr}(\text{Product of odd number of } \gamma \text{ matrices}) = 0$$

Including the fifth gamma matrix $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ into the above set of trace theorems,

$$\text{XIV) } \text{Tr}(\gamma^5 \gamma^\mu) = \text{Tr}(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\lambda) = 0 \text{ (From theorem [XIII](#))}$$

$$\text{XV) } \text{Tr}(\gamma^5) = 0$$

$$\text{XVI) } \text{Tr}(\gamma^5 \gamma^\mu \gamma^\nu) = 0 \implies \text{Tr}(\gamma^5 \not{a} \not{b}) = 0$$

$$\text{XVII) } \text{Tr}(\gamma^5 \gamma^\mu \gamma^\nu \gamma^\lambda \gamma^\sigma) = 4i\epsilon^{\mu\nu\lambda\sigma} \implies \text{Tr}(\gamma^5 \not{a} \not{b} \not{c} \not{d}) = 4i\epsilon^{\mu\nu\lambda\sigma} a_\mu b_\nu c_\lambda d_\sigma$$

where $\epsilon_{\mu\nu\lambda\sigma}$ is the 4-Dimensional Levi-Civita symbol.

A.2 Electron-Muon Scattering Cross Sections

A.2.1 Scattering Process Mediated By A Photon γ

After extracting expression (1.11) from the Feynman diagram in Figure 1.7 and the probability amplitude $\mathcal{M}_\gamma$ (1.12), taking the absolute square of the probability amplitude $\mathcal{M}_\gamma$ will yield the following expression:

$$|\mathcal{M}_\gamma|^2 = \frac{g_e^4}{(p_1 + p_2)^4} [u(1)\gamma^\mu \bar{v}(2)] [v(3)\gamma_\mu \bar{u}(4)] [u(1)\gamma^\mu \bar{v}(2)]^* [v(3)\gamma_\mu \bar{u}(4)]^* \quad (1.13)$$

Extracting the first and third terms from the above equation, relabelling γ^μ in the first term and γ_μ in the third term as Γ_1 and Γ_2 , and naming the resultant factor G_1 ,

$$\begin{aligned} G_1 &= [u(1)\Gamma_1 \bar{v}(2)] [u(1)\Gamma_2 \bar{v}(2)]^* \\ &= [u(1)\Gamma_1 \bar{v}(2)] [u(1)\Gamma_2 v(2)^\dagger \gamma^0]^\dagger \end{aligned}$$

Using the fact that $\gamma^0 \gamma^\alpha \gamma^0 = \gamma^\alpha$ where γ^α can be any arbitrary matrix and that $(\gamma^0)^2 = \mathbb{1}$,

$$\begin{aligned} &= [u(1)\Gamma_1 \bar{v}(2)] [\gamma^0 u(1) \gamma^0 \Gamma_2 \gamma^0 \gamma^0 v(2)^\dagger \gamma^0]^\dagger \\ &= [u(1)\Gamma_1 \bar{v}(2)] [(\gamma^0)^\dagger v(2) (\gamma^0)^\dagger (\gamma^0)^\dagger \Gamma_2^\dagger (\gamma^0)^\dagger u(1)^\dagger (\gamma^0)^\dagger] \end{aligned}$$

Since $(\gamma^0)^\dagger = \gamma^0$,

$$= [u(1)\Gamma_1 \bar{v}(2)] [\gamma^0 v(2) \gamma^0 \Gamma_2^\dagger \gamma^0 u(1)^\dagger \gamma^0]$$

$$= [u(1)\Gamma_1\bar{v}(2)][v(2)\bar{\Gamma}_2\bar{u}(1)] \text{ where } \bar{\Gamma}_2 = \gamma^0\Gamma_2^\dagger\gamma^0 \quad (\text{A.1})$$

Summing G_1 over all spin s_2 ,

$$\begin{aligned} \sum_{s_2} G_1 &= u(1)\Gamma_1 \left[\sum_{s_e=1,2} \bar{v}(2)v(2) \right] \bar{\Gamma}_2\bar{u}(1) \\ &= u(1)\Gamma_1 \left[\sum_{s_e=1,2} v(2)^\dagger\gamma^0v(2) \right] \bar{\Gamma}_2\bar{u}(1) \\ &= u(1)\Gamma_1(\not{p}_2 - m_2c)\bar{\Gamma}_2\bar{u}(1) \end{aligned} \quad (\text{A.2})$$

Let $Q = \Gamma_1(\not{p}_2 - m_2c)\bar{\Gamma}_2$ and summing the above expression over all spin s_1 ,

$$\begin{aligned} \sum_{s_1} \sum_{s_2} G_1 &= \sum_{s_1} u(1)_i Q_{ij} \bar{u}(1)_j \\ &= Q_{ij} \left[\sum_{s_1} u(1)_i \bar{u}(1)_j \right] \\ &= Q_{ij}(\not{p}_1 + m_1c)_{ij} \\ &= \text{Tr}[Q(\not{p}_1 + m_1c)] \end{aligned}$$

Substituting $Q = \Gamma_1(\not{p}_2 - m_2c)$ into the expression above

$$= \text{Tr}[\Gamma_1(\not{p}_2 - m_2c)\Gamma_2(\not{p}_1 + m_1c)] \quad (\text{A.3})$$

This is known as “*Casimir’s trick*” and this mathematical trick will be used again later in the calculation.

Repeating the above procedure for the second and fourth terms and labelling the resultant factor G_2 ,

$$G_2 = [v(3)\Gamma_3\bar{u}(4)][v(3)\Gamma_4\bar{u}(4)]^* = [v(3)\Gamma_3\bar{u}(4)][u(4)\bar{\Gamma}_4\bar{v}(3)] \quad (\text{A.4})$$

Summing G_2 over all spins s_4 ,

$$\sum_{s_4} G_2 = v(3)\Gamma_3(\not{p}_4 + m_4c)\Gamma_4\bar{v}(3) \quad (\text{A.5})$$

Let $P = \Gamma_3(\not{p}_4 + m_4c)\bar{\Gamma}_4$ and summing the above expression over all spins s_3 ,

$$\begin{aligned} \sum_{s_3} \sum_{s_4} G_2 &= \sum_{s_3} v(3)_i P_{ij} \bar{v}(3)_j \\ &= P_{ij}(\not{p}_3 - m_3c)_{ij} \\ &= \text{Tr}[P(\not{p}_3 - m_3c)] \end{aligned}$$

Substituting $P = \Gamma_3(\not{p}_4 + m_4c)$ into the expression above

$$= \text{Tr}[\Gamma_3(\not{p}_4 + m_4 c)\bar{\Gamma}_4(\not{p}_3 - m_3 c)] \quad (\text{A.6})$$

Combining the results from Eqs (A.3) and (A.6), inserting them back into (1.11), $\Gamma_1 \leftrightarrow \gamma^\mu$, $\bar{\Gamma}_2 \leftrightarrow \gamma^\nu$, $\Gamma_3 \leftrightarrow \gamma_\mu$, $\bar{\Gamma}_4 \leftrightarrow \gamma_\nu$, and averaging over all possible spin orientations yields the following expression:

$$\begin{aligned} \langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{g_e^4}{4(p_1 + p_2)^4} \text{Tr}[\gamma^\mu(\not{p}_2 - m_e c)\gamma^\nu(\not{p}_1 + m_e c)] \\ &\quad \times \text{Tr}[\gamma_\mu(\not{p}_4 + m_\mu c)\gamma_\nu(\not{p}_3 - m_\mu c)] \end{aligned} \quad (\text{A.7})$$

Extracting the first term from the above equation and expanding the term out:

$$\begin{aligned} \text{Tr}[\gamma^\mu(\not{p}_2 - m_e c)\gamma^\nu(\not{p}_1 + m_e c)] &= \text{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1) + m_e c [\text{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu) - \text{Tr}(\gamma^\mu \gamma^\nu \not{p}_1)] \\ &\quad - (m_e c)^2 \text{Tr}(\gamma^\mu \gamma^\nu) \end{aligned} \quad (\text{A.8})$$

where properties I) and II) were used.

Computing the first term:

$$\text{Tr}(\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1) = (p_2)_\lambda (p_1)_\sigma \text{Tr}(\gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma)$$

Using Theorem XII),

$$\begin{aligned} &= (p_2)_\lambda (p_1)_\sigma [4(g^{\mu\lambda} g^{\nu\sigma} - g^{\mu\nu} g^{\lambda\sigma} + g^{\mu\sigma} g^{\lambda\nu})] \\ &= 4p_2^\mu p_1^\nu - g^{\mu\nu}(p_2 \cdot p_1) + p_1^\mu p_2^\nu, \text{ where } (p_2)^\sigma (p_1)_\sigma = p_2 \cdot p_1 \end{aligned} \quad (\text{A.9})$$

As a consequence of Theorem XIII), the terms in the square brackets in Eq (A.8) vanish. And as a consequence of Theorem XI), the last term becomes

$$(m_e c)^2 \text{Tr}(\gamma^\mu \gamma^\nu) = 4g^{\mu\nu}(m_e c)^2 \quad (\text{A.10})$$

Substituting the above results into Eq (A.8) and factoring out the common factor 4, the following expression is obtained:

$$\text{Tr}[\gamma^\mu(\not{p}_2 - m_e c)\gamma^\nu(\not{p}_1 + m_e c)] = 4[p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}\{(m_e c)^2 + (p_2 \cdot p_1)\}] \quad (\text{A.11})$$

Replacing the upper indices with lower indices, $m_e \leftrightarrow m_\mu$, $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$, the second term in Eq (A.7) transforms into the following form:

$$\text{Tr}[\gamma_\mu(\not{p}_4 + m_\mu c)\gamma_\nu(\not{p}_3 - m_\mu c)] = 4[p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g^{\mu\nu}\{(m_\mu c)^2 + (p_4 \cdot p_3)\}] \quad (\text{A.12})$$

Inserting Eqs (A.11) and (A.12) back into Eq (A.7), expanding and simplifying the re-

sultant expression:

$$\begin{aligned}
\langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{4g_e^4}{(p_1 + p_2)^4} [p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu} \{ (m_e c)^2 + (p_2 \cdot p_1) \}] \\
&\quad \times [p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g^{\mu\nu} \{ (m_\mu c)^2 + (p_4 \cdot p_3) \}] \\
&= \frac{4g_e^4}{(p_1 + p_2)^4} [(p_2 \cdot p_4)(p_1 \cdot p_3) + (p_1 \cdot p_4)(p_2 \cdot p_3) - p_{4\nu} p_{3\nu} \{ (m_e c)^2 + (p_2 \cdot p_1) \} \\
&\quad + (p_2 \cdot p_3)(p_1 \cdot p_4) + (p_1 \cdot p_3)(p_2 \cdot p_4) - p_{3\nu} p_{4\nu} \{ (m_e c)^2 + (p_2 \cdot p_1) \} \\
&\quad - p_{2\nu} p_{1\nu} \{ (m_\mu c)^2 + (p_4 \cdot p_3) \} - p_{1\nu} p_{2\nu} \{ (m_\mu c)^2 + (p_4 \cdot p_3) \} \\
&\quad + 4(m_e m_\mu c^2)^2 + 4(p_2 \cdot p_1)(m_\mu c)^2 + 4(p_4 \cdot p_3)(m_e c)^2 + 4(p_2 \cdot p_1)(p_4 \cdot p_3)] \\
&= \frac{8g_e^4}{(p_1 + p_2)^4} [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) + (p_3 \cdot p_4)(m_e c)^2 + (p_1 \cdot p_2)(m_\mu c)^2 \\
&\quad + 2(m_e m_\mu c^2)^2] \tag{A.13}
\end{aligned}$$

Expressing the momenta in terms of their 4-momenta in their centre-of-mass (CM) frame, $p_1 = (\frac{E_e}{c}, \vec{p}_1)$; $p_2 = (\frac{E_e}{c}, -\vec{p}_1)$; $p_3 = (\frac{E_\mu}{c}, \vec{p}_3)$; $p_4 = (\frac{E_\mu}{c}, -\vec{p}_3)$. Computing the dot products of the momenta, $(p_1 \cdot p_2) = (\frac{E_e}{c})^2 + |\vec{p}_1|^2$; $(p_3 \cdot p_4) = (\frac{E_\mu}{c})^2 + |\vec{p}_3|^2$; $(p_1 \cdot p_3) = (\frac{E_e}{c})(\frac{E_\mu}{c}) - (\vec{p}_1 \cdot \vec{p}_3) = (p_2 \cdot p_4)$; $(p_1 \cdot p_4) = (\frac{E_e}{c})(\frac{E_\mu}{c}) + (\vec{p}_1 \cdot \vec{p}_3) = (p_2 \cdot p_3)$. Using these results:

$$\begin{aligned}
(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) &= 2 \left[\left(\frac{E_e}{c} \right)^2 \left(\frac{E_\mu}{c} \right)^2 + (\vec{p}_1 \cdot \vec{p}_3)^2 \right] \\
&= 2 \left[\left(\frac{E_e}{c} \right)^2 \left(\frac{E_\mu}{c} \right)^2 + |\vec{p}_1|^2 |\vec{p}_3|^2 \cos^2 \theta \right] \tag{A.14}
\end{aligned}$$

In the high energy limit, the following approximations can be made: $E_e \approx E_\mu \approx E \approx pc$, $m_e c \approx m_\mu c \approx 0$, $\vec{p}_1 \approx \vec{p}_3$, $|\vec{p}_1| \approx |\vec{p}_3| \approx \frac{E}{c}$. Using these approximations, the last 3 terms in the final line of Eq (A.13) vanish and substituting the expression above back into Eq (A.13) will yield the following expression:

$$\begin{aligned}
\langle |\mathcal{M}_\gamma|^2 \rangle &= \frac{8g_e^4}{(\frac{2E}{c})^4} \left\{ 2 \left[\left(\frac{E}{c} \right)^4 + \left(\frac{E}{c} \right)^4 \cos^2 \theta \right] \right\} \\
&= g_e^4 (1 + \cos^2 \theta) \\
&= (\sqrt{4\pi\alpha_{\text{EM}}})^4 (1 + \cos^2 \theta) \tag{A.15}
\end{aligned}$$

Substituting the above expression into Eq (1.10), and since $|\mathbf{p}_i| = |\mathbf{p}_f|$ ($|\vec{p}_1| = |\vec{p}_3|$) and the above expression for the probability amplitude absolute squared has already accounted for the spin configuration of the system, the resultant expression for the differential cross section is:

$$\left(\frac{d\sigma}{d\Omega} \right)_\gamma = \left(\frac{\hbar c}{8\pi} \right)^2 \left[\frac{(\sqrt{4\pi\alpha})^4 (1 + \cos^2 \theta)}{(2E)^2} \right] = \frac{\hbar^2 c^2 \alpha^2}{16E^2} (1 + \cos^2 \theta) \tag{A.16}$$

Integrating the above expression over all solid angles $d\Omega$ yields the total cross section σ for the electron-muon scattering process when mediated by a photon γ :

$$\begin{aligned}
\sigma_\gamma &= \int \left[\frac{\hbar^2 c^2 \alpha^2}{16E^2} (1 + \cos^2 \theta) \right] d\Omega \\
&= \left(\frac{\hbar c \alpha}{4E} \right)^2 \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta (1 + \cos^2 \theta) \\
&= \frac{\pi}{8} \left(\frac{\hbar c \alpha}{E} \right)^2 \left[-\cos \theta - \frac{\cos^3 \theta}{3} \right]_0^\pi \\
&= \frac{\pi}{8} \left(\frac{\hbar c \alpha}{E} \right)^2 \left(\frac{8}{3} \right) \\
&= \frac{\pi}{3} \left[\frac{\hbar c \alpha}{E} \right]^2
\end{aligned} \tag{A.17}$$

A.2.2 Scattering Process Mediated By A Z^0 Boson

From the Feynman diagram in Figure 2.1, the expression in Eq (2.7) can be extracted. Extracting the probability amplitude $\mathcal{M}_Z$ (2.8) from Eq (2.7), taking whatever was discussed below Eq (2.8) into consideration, and taking the absolute square of the modified expression for $\mathcal{M}_Z$ (2.9) will yield the following expression:

$$\begin{aligned}
|\mathcal{M}_Z|^2 &= \frac{g_z^4}{16[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \\
&\times [u(1)\gamma^\mu(c_V - c_A\gamma^5)\bar{v}(2)] [u(1)\gamma^\mu(c_V - c_A\gamma^5)\bar{v}(2)]^* \\
&\times [v(3)\gamma_\mu(c_V - c_A\gamma^5)\bar{u}(4)] [v(3)\gamma_\mu(c_V - c_A\gamma^5)\bar{u}(4)]^*
\end{aligned} \tag{2.10}$$

Using Casimir's trick and averaging over all possible spin orientations, the following expression is obtained:

$$\begin{aligned}
\langle |\mathcal{M}_Z|^2 \rangle &= \frac{g_z^4}{4\{16[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]\}} \\
&\times \text{Tr}\{\gamma^\mu(c_V - c_A\gamma^5)\not{p}_2\gamma^\nu(c_V - c_A\gamma^5)\not{p}_1\} \\
&\times \text{Tr}\{\gamma_\mu(c_V - c_A\gamma^5)\not{p}_4\gamma_\nu(c_V - c_A\gamma^5)\not{p}_3\}
\end{aligned} \tag{2.11}$$

Extracting the first term from the equation above, expanding it out and then simplifying it:

$$\begin{aligned}
&\text{Tr}\{\gamma^\mu(c_V - c_A\gamma^5)\not{p}_2\gamma^\nu(c_V - c_A\gamma^5)\not{p}_1\} \\
&= \text{Tr}\{\gamma^\mu\not{p}_2\gamma^\nu\not{p}_1(c_V - c_A\gamma^5)^2\} \\
&= \text{Tr}\{\gamma^\mu\gamma^\lambda(p_2)_\lambda\gamma^\nu\gamma^\sigma(p_1)_\sigma[c_V^2 - 2c_Vc_A\gamma^5 + c_A^2(\gamma^5)^2]\} \\
&\text{since } (\gamma^5)^2 = \mathbb{1}, \\
&= (c_V^2 + c_A^2) \text{Tr}\{\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma\}(p_2)_\lambda(p_1)_\sigma - 2c_Vc_A \text{Tr}\{\gamma^\mu\gamma^\lambda\gamma^\nu\gamma^\sigma\gamma^5\}(p_2)_\lambda(p_1)_\sigma
\end{aligned}$$

Using Theorems XII) and XVII) and applying a change of indices twice,

$$\begin{aligned}
&= 4(c_V^2 + c_A^2) [g^{\mu\lambda} g^{\nu\sigma} - g^{\mu\nu} g^{\lambda\sigma} + g^{\mu\sigma} g^{\lambda\nu}] (p_2)_\lambda (p_1)_\sigma - 2c_V c_A [4i\epsilon^{\mu\nu\sigma\lambda}] (p_1)_\sigma (p_2)_\lambda \\
&= 4\{(c_V^2 + c_A^2)[p_2^\mu p_1^\nu - g^{\mu\nu}(p_2 \cdot p_1) + p_1^\mu p_2^\nu] - 2i c_V c_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda}\}, \tag{A.18}
\end{aligned}$$

where $(p_2)^\sigma (p_1)_\sigma = p_2 \cdot p_1$. Replacing the upper indices with the lower indices, $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$, the second term in Eq (2.11) transforms into the following form:

$$\begin{aligned}
&\text{Tr}\{\gamma_\mu (c_V - c_A \gamma^5) \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3\} \\
&= 4\{(c_V^2 + c_A^2)[p_{4\mu} p_{3\nu} - g_{\mu\nu}(p_4 \cdot p_3) + p_{3\mu} p_{4\nu}] - 2i c_V c_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau\} \tag{A.19}
\end{aligned}$$

Inserting Eqs (A.18) and (A.19) into Eq (2.11), expanding and simplifying the resultant expression:

$$\begin{aligned}
\langle |\mathcal{M}_Z|^2 \rangle &= \frac{g_z^4}{4[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \\
&\times \{(c_V^2 + c_A^2)[p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)] - 2i c_V c_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda}\} \\
&\times \{(c_V^2 + c_A^2)[p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g_{\mu\nu}(p_4 \cdot p_3)] - 2i c_V c_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau\} \\
&= \frac{g_z^4}{4[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \\
&\times \{(c_V^2 + c_A^2)^2 [(p_2 \cdot p_4)(p_1 \cdot p_3) + (p_1 \cdot p_4)(p_2 \cdot p_3) - (p_2 \cdot p_1)(p_4 \cdot p_3) \\
&\quad + (p_2 \cdot p_3)(p_1 \cdot p_4) + (p_1 \cdot p_3)(p_2 \cdot p_4) - (p_2 \cdot p_1)(p_4 \cdot p_3) \\
&\quad - (p_2 \cdot p_1)(p_4 \cdot p_3) - (p_2 \cdot p_1)(p_4 \cdot p_3) + 4(p_2 \cdot p_1)(p_4 \cdot p_3)] \\
&\quad - 4c_V^2 c_A^2 [-2(\delta_\kappa^\sigma \delta_\tau^\lambda - \delta_\tau^\sigma \delta_\kappa^\lambda)(p_1)_\sigma (p_2)_\lambda (p_3)^\kappa (p_4)^\tau]\}, \\
&\text{where the following fact was used: } \epsilon^{\mu\nu\sigma\lambda} \epsilon_{\mu\nu\kappa\tau} = -2(\delta_\kappa^\sigma \delta_\tau^\lambda - \delta_\tau^\sigma \delta_\kappa^\lambda) \\
&= \frac{g_z^4}{2[(q^2 - (M_Z c)^2)^2 + (\hbar M_Z \Gamma_Z)^2]} \{(c_V^2 + c_A^2)^2 [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)] \\
&\quad + 4c_V^2 c_A^2 [(p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3)]\} \tag{A.20}
\end{aligned}$$

Using the dot product results above Eq (A.14) and the approximations in the high-energy limit:

$$\begin{aligned}
(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) &= 2 \left[\left(\frac{E}{c} \right)^4 + \left(\frac{E}{c} \right)^4 \cos^2 \theta \right] \\
&= 2 \left(\frac{E}{c} \right)^4 [1 + \cos^2 \theta] \tag{A.14}
\end{aligned}$$

$$\begin{aligned}
(p_1 \cdot p_3)(p_2 \cdot p_4) - (p_1 \cdot p_4)(p_2 \cdot p_3) &= -4 \left(\frac{E}{c} \right)^2 (\vec{p}_1 \cdot \vec{p}_3) \\
&= -4 \left(\frac{E}{c} \right)^2 (|\vec{p}_1| |\vec{p}_3| \cos \theta) \\
&= -4 \left(\frac{E}{c} \right)^4 \cos \theta \tag{A.21}
\end{aligned}$$

Substituting both expressions (A.14) and (A.21) back into Eq (A.20), and letting $q = \frac{2E}{c}$:

$$\begin{aligned} \langle |\mathcal{M}_Z|^2 \rangle &= \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \\ &\times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \end{aligned} \quad (\text{A.22})$$

Substituting the above expression into Eq (1.10), since $|\mathbf{p}_i| = |\mathbf{p}_f|$ ($|\vec{p}_1| = |\vec{p}_3|$) and the spin configuration of the system has already been accounted for in the above expression, the resultant expression for the differential cross section is:

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right)_Z &= \left(\frac{\hbar c}{8\pi} \right)^2 \left(\frac{1}{2E} \right)^2 \left[\frac{g_z^4 E^4}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\ &\times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \\ &= \left(\frac{\hbar c}{16\pi} \right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\ &\times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \end{aligned} \quad (\text{A.23})$$

Integrating the above expression over all solid angles $d\Omega$ yields the total cross section σ for the electron-muon scattering process when mediated by a Z^0 boson:

$$\begin{aligned} \sigma_Z &= \int \left(\frac{\hbar c}{16\pi} \right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\ &\times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} d\Omega \\ &= \left(\frac{\hbar c}{16\pi} \right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin \theta \\ &\times \{(c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta\} \\ &= \frac{1}{2\pi} \left(\frac{\hbar c}{8\pi} \right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \\ &\times \left\{ (c_V^2 + c_A^2)^2 \left[-\cos \theta - \frac{\cos^3 \theta}{3} \right]_0^\pi - 8c_V^2 c_A^2 \left[-\frac{\cos 2\theta}{4} \right]_0^\pi \right\} \\ &= \frac{1}{2\pi} \left(\frac{\hbar c}{8\pi} \right)^2 \left[\frac{g_z^4 E^2}{\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2} \right] \left[(c_V^2 + c_A^2)^2 \left(\frac{8}{3} \right) - 0 \right] \\ &= \frac{[\hbar c E g_z^2 (c_V^2 + c_A^2)]^2}{48\pi \{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\}} \\ &= \frac{[\hbar c E (c_V^2 + c_A^2)]^2}{48\pi \{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\}} \left(\frac{g_e}{\sin \theta_w \cos \theta} \right)^4 \\ &= \frac{\pi}{3} \left\{ \frac{[\hbar c \alpha E (c_V^2 + c_A^2)]^2}{\{[(2E)^2 - (M_Z c^2)^2]^2 + (\hbar M_Z c^2 \Gamma_Z)^2\}} \right\} \left(\frac{1}{\sin \theta_w \cos \theta} \right)^4, \text{ where } g_e = \sqrt{4\pi \alpha_{\text{EM}}} \end{aligned} \quad (\text{A.24})$$

A.2.3 Scattering Process When Mixing of EM and Weak Interactions Is Taken Into Consideration

When the mixing of the electromagnetic (EM) and weak interactions is taken into account, the absolute square of the probability amplitude averaged over all spin orientations of the system takes on the following form:

$$\langle |\mathcal{M}|^2 \rangle = |\mathcal{M}_\gamma + \mathcal{M}_Z|^2 = \langle |\mathcal{M}_\gamma|^2 \rangle + \langle |\mathcal{M}_Z|^2 \rangle + (\mathcal{M}_\gamma \mathcal{M}_Z^* + \mathcal{M}_Z \mathcal{M}_\gamma^*), \quad (2.2)$$

where $\langle |\mathcal{M}_\gamma|^2 \rangle$ and $\langle |\mathcal{M}_Z|^2 \rangle$ have already been calculated in Eqs (A.15) and (A.22) respectively. The remaining terms that needs to be calculated are the mixing terms, $\mathcal{M}_\gamma \mathcal{M}_Z^*$ and $\mathcal{M}_Z \mathcal{M}_\gamma^*$.

Taking the complex conjugates of probability amplitudes $\mathcal{M}_\gamma$ and $\mathcal{M}_Z$:

$$\begin{aligned} \mathcal{M}_\gamma^* &= -\frac{g_e^2}{q^2} [u(1)\gamma^\mu \bar{v}(2)]^* [v(3)\gamma_\mu \bar{u}(4)]^* \\ \mathcal{M}_Z^* &= \frac{g_z^2}{4[q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} [u(1)\gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)]^* \\ &\quad \times [v(3)\gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)]^* \end{aligned} \quad (A.25)$$

Hence, the first mixing term $\mathcal{M}_\gamma \mathcal{M}_Z^*$ takes on the following form:

$$\begin{aligned} \mathcal{M}_\gamma \mathcal{M}_Z^* &= -\frac{g_e^2 g_z^2}{4q^2 [q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} [u(1)\gamma^\mu \bar{v}(2)] [u(1)\gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)]^* \\ &\quad \times [v(3)\gamma_\mu \bar{u}(4)] [v(3)\gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)]^* \end{aligned} \quad (A.26)$$

Using Casimir's trick and averaging all possible spin orientations yields the following expression:

$$\begin{aligned} \mathcal{M}_\gamma \mathcal{M}_Z^* &= -\frac{g_e^2 g_z^2}{16q^2 [q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} \\ &\quad \times \text{Tr}[\gamma^\mu \not{p}_2 \gamma^\nu (c_V - c_A \gamma^5) \not{p}_1] \text{Tr}[\gamma_\mu \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3] \end{aligned} \quad (A.27)$$

Extracting the first term from the above equation and evaluating it:

$$\begin{aligned} &\text{Tr}[\gamma^\mu \not{p}_2 \gamma^\nu (c_V - c_A \gamma^5) \not{p}_1] \\ &= \text{Tr}[\gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1 c_V - \gamma^\mu \not{p}_2 \gamma^\nu \not{p}_1 \gamma^5 c_A] \\ &= \text{Tr}[\gamma^\mu \gamma^\lambda (p_2)_\lambda \gamma^\nu \gamma^\sigma (p_1)_\sigma c_V - \gamma^\mu \gamma^\lambda (p_2)_\lambda \gamma^\nu \gamma^\sigma (p_1)_\sigma \gamma^5 c_A] \\ &= c_V \text{Tr}[\gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma] (p_2)_\lambda (p_1)_\sigma - c_A \text{Tr}[\gamma^5 \gamma^\mu \gamma^\lambda \gamma^\nu \gamma^\sigma] (p_2)_\lambda (p_1)_\sigma \end{aligned}$$

Using Theorems XII) and XVII) and applying a change of indices twice

$$= 4c_V [g^{\mu\lambda} g^{\nu\sigma} - g^{\mu\nu} g^{\lambda\sigma} + g^{\mu\sigma} g^{\lambda\nu}] (p_2)_\lambda (p_1)_\sigma - 4ic_A \epsilon^{\mu\nu\sigma\lambda} (p_1)_\sigma (p_2)_\lambda$$

$$= 4c_V [p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)] - 4ic_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda}, \text{ where } (p_2)^\sigma (p_1)_\sigma = p_2 \cdot p_1 \quad (\text{A.28})$$

Replacing the upper indices with lower indices, $1 \leftrightarrow 3$ and $2 \leftrightarrow 4$, the second term in Eq (A.27) transforms into the following form:

$$\text{Tr}[\gamma_\mu \not{p}_4 \gamma_\nu (c_V - c_A \gamma^5) \not{p}_3] = 4\{c_V [p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g_{\mu\nu}(p_4 \cdot p_3)] - ic_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau\} \quad (\text{A.29})$$

Inserting Eqs (A.28) and (A.29) back into Eq (A.27), expanding and simplifying the resultant expression:

$$\begin{aligned} \mathcal{M}_\gamma \mathcal{M}_Z^* &= -\frac{g_e^2 g_z^2}{q^2 [q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} \\ &\times \{c_V [p_2^\mu p_1^\nu + p_1^\mu p_2^\nu - g^{\mu\nu}(p_2 \cdot p_1)] - ic_A \epsilon^{\mu\nu\sigma\lambda} p_{1\sigma} p_{2\lambda}\} \\ &\times \{c_V [p_{4\mu} p_{3\nu} + p_{3\mu} p_{4\nu} - g_{\mu\nu}(p_4 \cdot p_3)] - ic_A \epsilon_{\mu\nu\kappa\tau} p_3^\kappa p_4^\tau\} \\ &= -\left(\frac{g_e g_z}{q}\right)^2 \left[\frac{1}{q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z} \right] \\ &\times \{c_V^2 [(p_2 \cdot p_4)(p_1 \cdot p_3) + (p_1 \cdot p_4)(p_2 \cdot p_3) - (p_4 \cdot p_3)(p_2 \cdot p_1) \\ &\quad + (p_2 \cdot p_3)(p_1 \cdot p_4) + (p_1 \cdot p_3)(p_2 \cdot p_4) - (p_3 \cdot p_4)(p_2 \cdot p_1) \\ &\quad - (p_2 \cdot p_1)(p_4 \cdot p_3) - (p_1 \cdot p_2)(p_4 \cdot p_3) + 4(p_2 \cdot p_1)(p_4 \cdot p_3)] \\ &\quad - c_A^2 [-2(\delta_\kappa^\sigma \delta_\tau^\lambda - \delta_\tau^\sigma \delta_\kappa^\lambda)(p_1)_\sigma (p_2)_\lambda (p_3)^\kappa (p_4)^\tau]\} \\ &= -\left(\frac{g_e g_z}{q}\right)^2 \left[\frac{2}{q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z} \right] \\ &\times \{c_V^2 [(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3)] \\ &\quad + c_A^2 [(p_1 \cdot p_3)(p_2 \cdot p_4 - (p_1 \cdot p_4)(p_2 \cdot p_3))]\} \end{aligned} \quad (\text{A.30})$$

Substituting both expressions (A.14) and (A.21) into the above equation and letting $q = \frac{2E}{c}$:

$$\mathcal{M}_\gamma \mathcal{M}_Z^* = -\frac{(g_e g_z E)^2}{(2E)^2 - (M_Z c)^2 + i\hbar M_Z c^2 \Gamma_Z} [c_V^2 (1 + \cos^2 \theta) - 2c_A^2 \cos \theta] \quad (\text{A.31})$$

Repeating the same procedure for the second mixing term $\mathcal{M}_Z \mathcal{M}_\gamma^*$:

$$\begin{aligned} \mathcal{M}_Z \mathcal{M}_\gamma^* &= -\frac{g_e^2 g_z^2}{4q^2 [q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} [u(1) \gamma^\mu (c_V - c_A \gamma^5) \bar{v}(2)] [u(1) \gamma^\mu \bar{v}(2)]^* \\ &\quad \times [v(3) \gamma_\mu (c_V - c_A \gamma^5) \bar{u}(4)] [v(3) \gamma_\mu \bar{u}(4)]^* \end{aligned} \quad (\text{A.32})$$

Using Casimir's trick one last time and averaging over all possible spin orientations yields

the following expression:

$$\begin{aligned} \mathcal{M}_Z \mathcal{M}_\gamma^* &= -\frac{g_e^2 g_z^2}{16q^2[q^2 - (M_Z c)^2 + i\hbar M_Z \Gamma_Z]} \\ &\times \text{Tr}[\gamma^\mu (c_V - c_A \gamma^5) \not{p}_2 \gamma^\nu \not{p}_1] \text{Tr}[\gamma_\mu (c_V - c_A \gamma^5) \not{p}_4 \gamma_\nu \not{p}_3] \end{aligned} \quad (\text{A.33})$$

Using Property III) which states that even permutations of the matrices within the trace leave the trace invariant, it can be concluded that:

$$\begin{aligned} \mathcal{M}_Z \mathcal{M}_\gamma^* &= \mathcal{M}_\gamma \mathcal{M}_Z^* = -\frac{(g_e g_Z E)^2}{(2E)^2 - (M_Z c^2)^2 + i\hbar M_Z c^2 \Gamma_Z} \\ &\times \left[c_V^2 (1 + \cos^2 \theta) - 2c_A^2 \cos \theta \right] \end{aligned} \quad (\text{A.34})$$

Combining both mixing terms, $\mathcal{M}_\gamma \mathcal{M}_Z^*$ and $\mathcal{M}_Z \mathcal{M}_\gamma^*$, with $\langle |\mathcal{M}_\gamma|^2 \rangle$ and $\langle |\mathcal{M}_Z|^2 \rangle$:

$$\begin{aligned} \langle |\mathcal{M}|^2 \rangle &= |\mathcal{M}_\gamma + \mathcal{M}_Z|^2 \\ &= g_e^4 (1 + \cos^2 \theta) \\ &+ \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left\{ (c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta \right\} \\ &- \text{Re} \left(\frac{2(g_e g_Z E)^2}{(2E)^2 - (M_Z c^2)^2 + i\hbar M_Z c^2 \Gamma_Z} \left[c_V^2 (1 + \cos^2 \theta) - 2c_A^2 \cos \theta \right] \right) \end{aligned} \quad (\text{A.35})$$

Substituting the expression above into Eq (1.10), since $|\mathbf{p}_i| = |\mathbf{p}_f|$ ($|\vec{p}_1| = |\vec{p}_3|$) and the spin configuration of the system has already been accounted for in the expression above, the combined expression for the differential cross section takes on the following form:

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega} \right)_{\gamma+Z} &= \left(\frac{\hbar c}{8\pi} \right)^2 \left(\frac{1}{2E} \right)^2 \left[g_e^4 (1 + \cos^2 \theta) \right. \\ &+ \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left\{ (c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta \right\} \\ &- \text{Re} \left(\frac{2(g_e g_Z E)^2}{(2E)^2 - (M_Z c^2)^2 + i\hbar M_Z c^2 \Gamma_Z} \left[c_V^2 (1 + \cos^2 \theta) - 2c_A^2 \cos \theta \right] \right) \left. \right] \end{aligned} \quad (\text{A.36})$$

Finally integrating the expression over all solid angles $d\Omega$ yields the total cross section σ for the electron-muon scattering process when the mixing of the electromagnetic (EM) and weak interactions is accounted for:

$$\begin{aligned} \sigma_{\gamma+Z} &= \int \left(\frac{\hbar c}{8\pi} \right)^2 \left(\frac{1}{2E} \right)^2 \left[g_e^4 (1 + \cos^2 \theta) \right. \\ &+ \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left\{ (c_V^2 + c_A^2)^2 [1 + \cos^2 \theta] - 8c_V^2 c_A^2 \cos \theta \right\} \\ &- \text{Re} \left(\frac{2(g_e g_Z E)^2}{(2E)^2 - (M_Z c^2)^2 + i\hbar M_Z c^2 \Gamma_Z} \left[c_V^2 (1 + \cos^2 \theta) - 2c_A^2 \cos \theta \right] \right) \left. \right] d\Omega \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{\hbar c}{8\pi}\right)^2 \left(\frac{1}{2E}\right)^2 \int_0^{2\pi} d\phi \int_0^\pi d\theta \sin\theta \left[g_e^4 (1 + \cos^2\theta) \right. \\
&\quad + \frac{g_z^4 E^4}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left\{ (c_V^2 + c_A^2)^2 [1 + \cos^2\theta] - 8c_V^2 c_A^2 \cos\theta \right\} \\
&\quad \left. - \operatorname{Re} \left(\frac{2(g_e g_Z E)^2}{(2E)^2 - (M_Z c^2)^2 + i\hbar M_Z c^2 \Gamma_Z} [c_V^2 (1 + \cos^2\theta) - 2c_A^2 \cos\theta] \right) \right] \\
&= \frac{1}{2\pi} \left(\frac{\hbar c}{8E}\right)^2 \left[\frac{8g_e^4}{3} + \frac{8g_e^4}{3[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \left(\frac{E}{\sin\theta_w \cos\theta_w}\right)^4 \right. \\
&\quad \times \left. \left(c_V^2 + c_A^2 \right)^2 - \left(\frac{8g_e^4 [(2E)^2 - (M_Z c^2)^2] (2c_V^2)}{3[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \right) \left(\frac{E}{\sin\theta_w \cos\theta_w}\right)^2 \right] \\
&= \frac{g_e^4}{48\pi} \left(\frac{\hbar c}{E}\right)^2 \left[1 + \left(\frac{1}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \right) \left(\frac{E}{\sin\theta_w \cos\theta_w}\right)^2 \right. \\
&\quad \times \left. \left\{ \left(\frac{E(c_V^2 + c_A^2)}{\sin\theta_w \cos\theta_w} \right)^2 - 2c_V^2 [(2E)^2 - (M_Z c^2)^2] \right\} \right] \\
&= \frac{\pi}{3} \left(\frac{\hbar c \alpha}{E}\right)^2 \left[1 + \left(\frac{1}{[\{(2E)^2 - (M_Z c^2)^2\}^2 + (\hbar M_Z c^2 \Gamma_Z)^2]} \right) \left(\frac{E}{\sin\theta_w \cos\theta_w}\right)^2 \right. \\
&\quad \times \left. \left\{ \left(\frac{E(c_V^2 + c_A^2)}{\sin\theta_w \cos\theta_w} \right)^2 - 2c_V^2 [(2E)^2 - (M_Z c^2)^2] \right\} \right], \text{ where } g_e = \sqrt{4\pi\alpha_{\text{EM}}} \quad (\text{A.37})
\end{aligned}$$

Appendix B

Additional Plots

In this appendix, the plots of $m_Y - g_\ell$ plane for all 6 relative widths Γ/m and from the external implementations of the dilepton constraints using the recasted likelihood function in Eq (5.8) are presented. The appendix is subdivided into 3 sections, one for each of the channels. Note that a fixed set of DM and quark couplings ($g_q = 0.5$ and $g_{\text{DM}} = 1.0$), and a fixed DM mass of $m_\chi = 400$ GeV were used to plot these planes. The colour bars on the right of all of the plots show the variations of the likelihood function χ^2 . And the blue $\chi^2 = 4$ contours in all of these plots represent the exclusion threshold, below which the region of the parameter space is accepted and above which that region is excluded.

B.1 Dielectron ee Channel

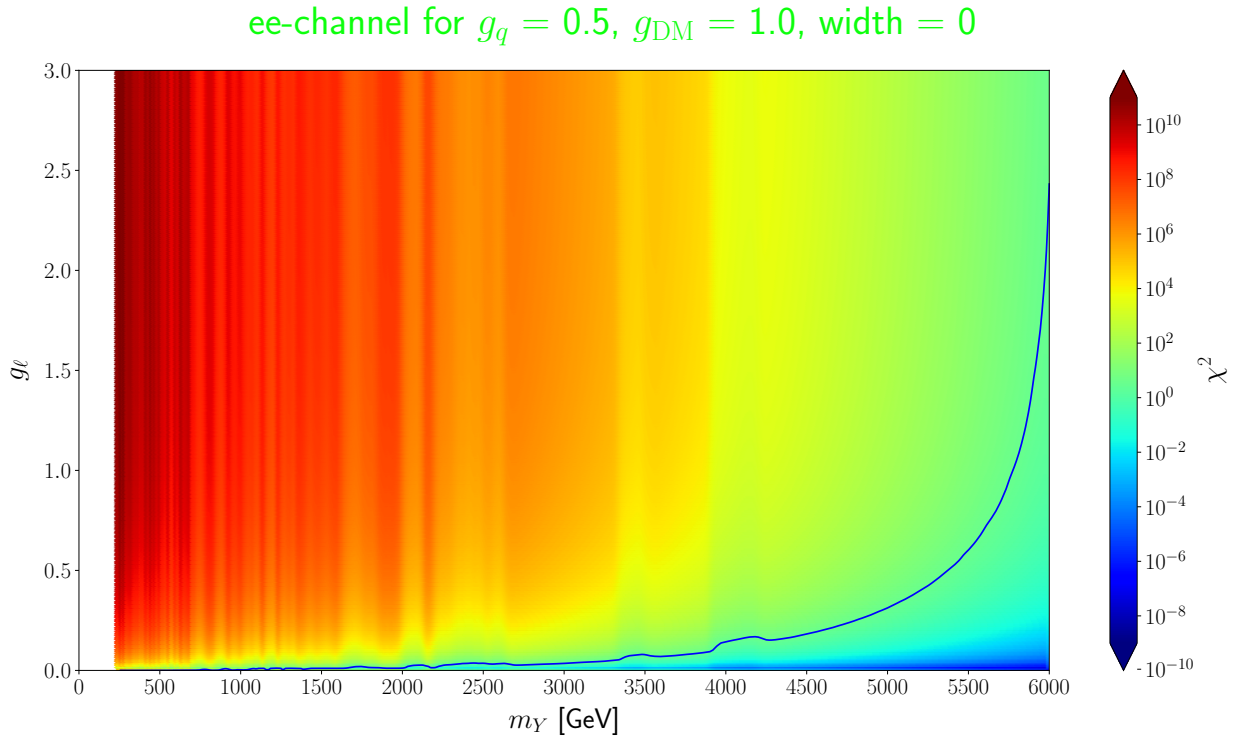
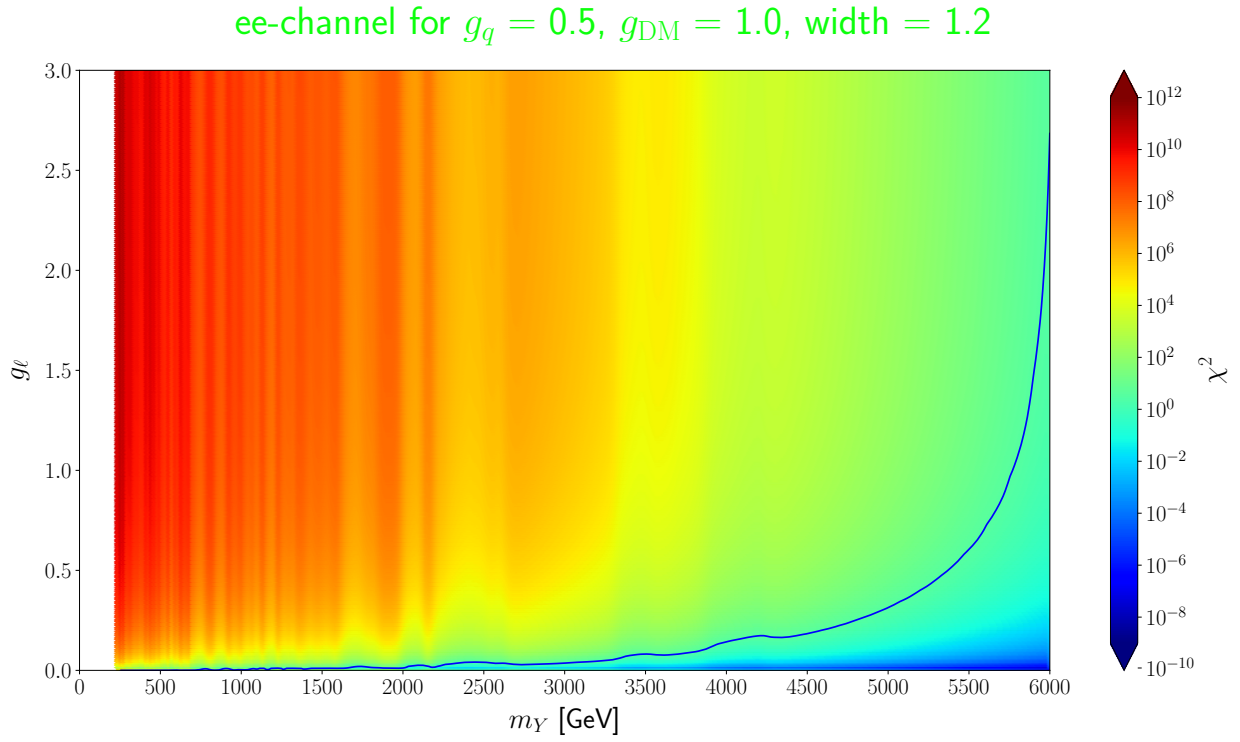
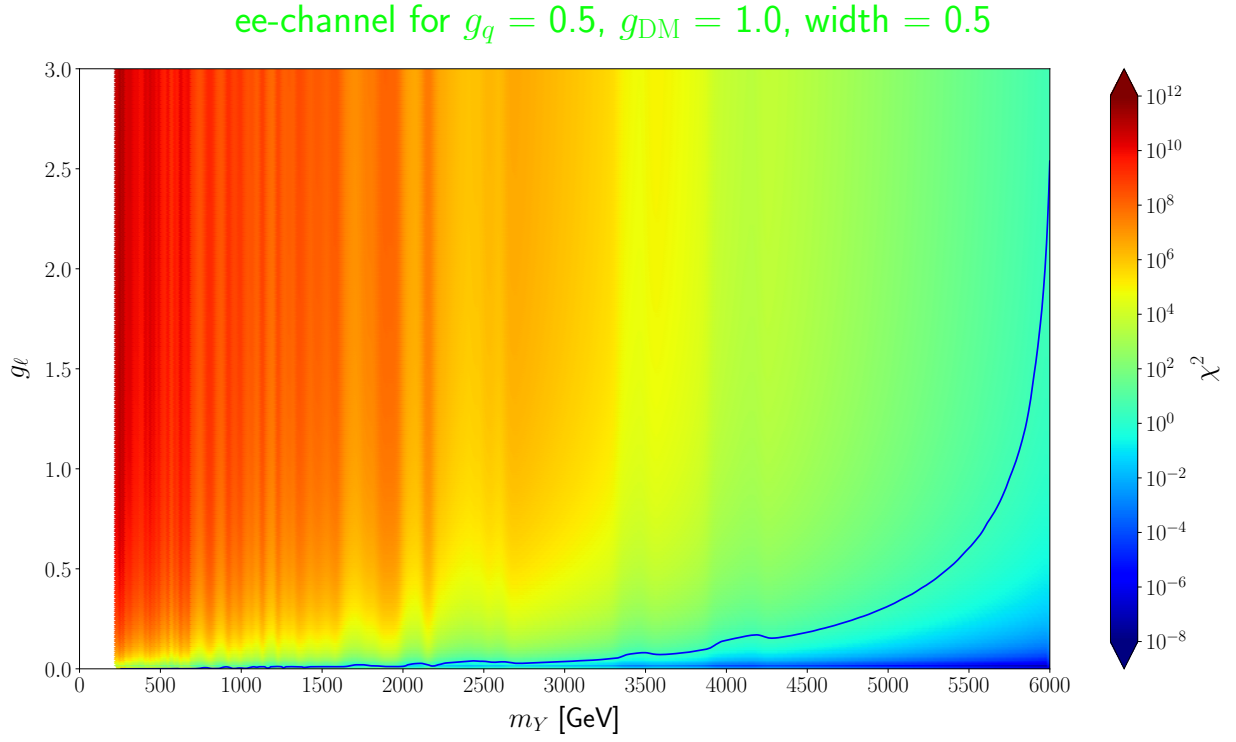
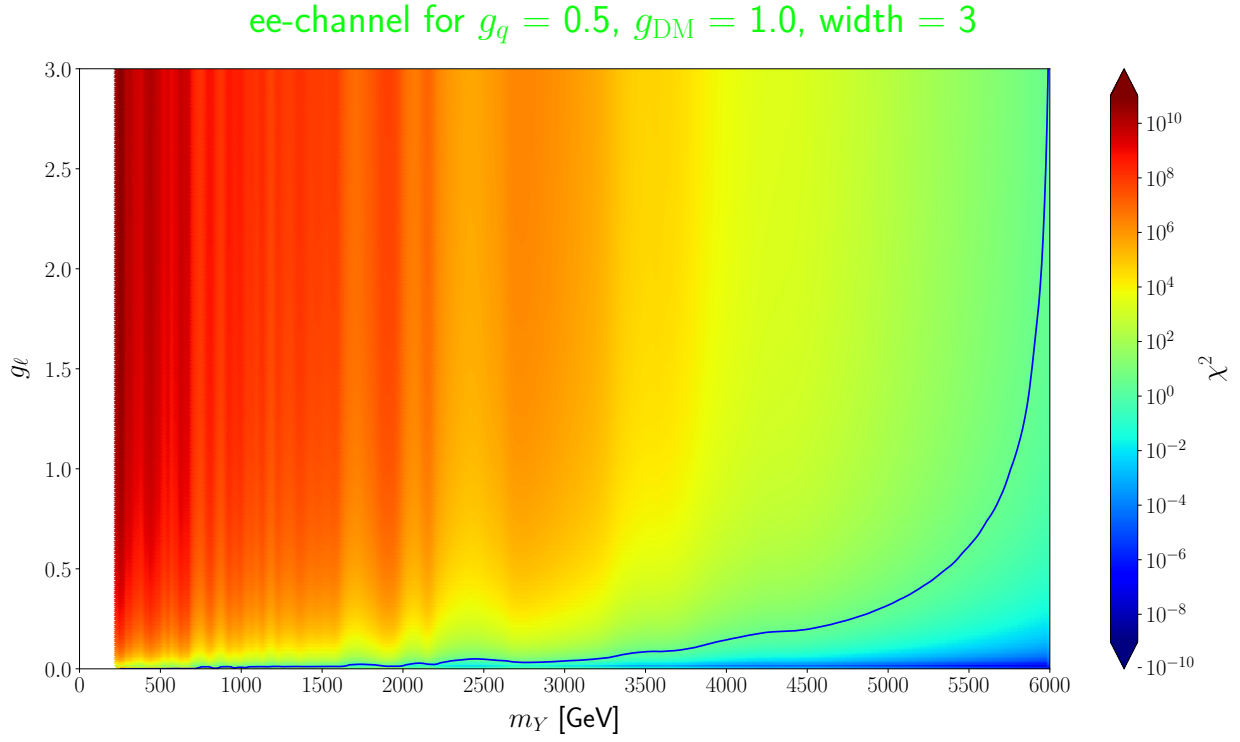
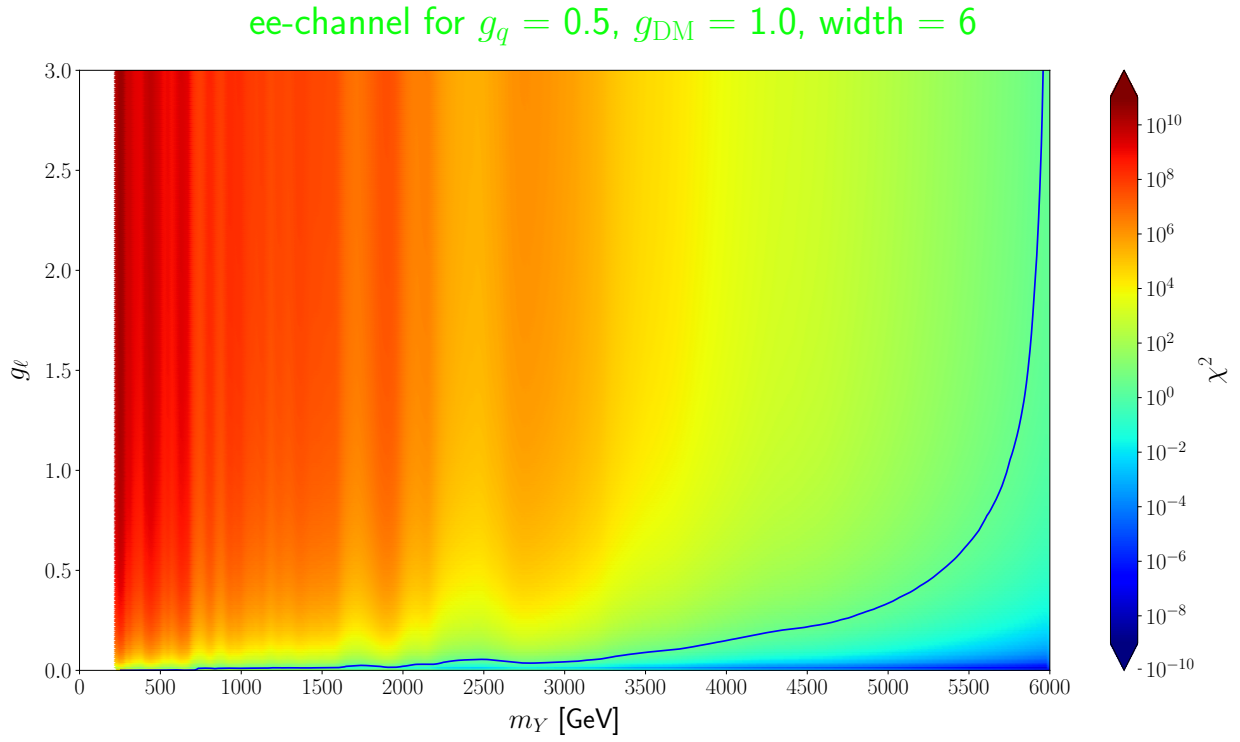


Figure B.1: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 0\%$



Figure B.4: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 3\%$ Figure B.5: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 6\%$

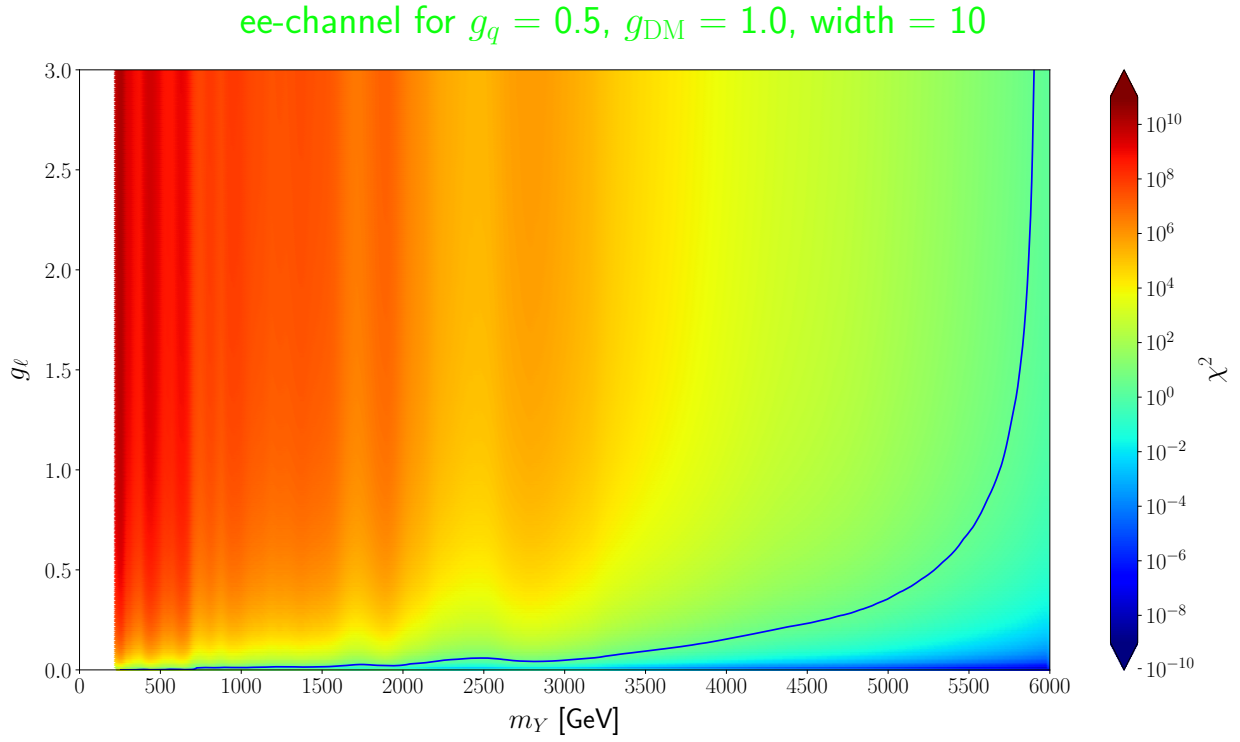


Figure B.6: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 10\%$

B.2 Dimuon $\mu\mu$ Channel

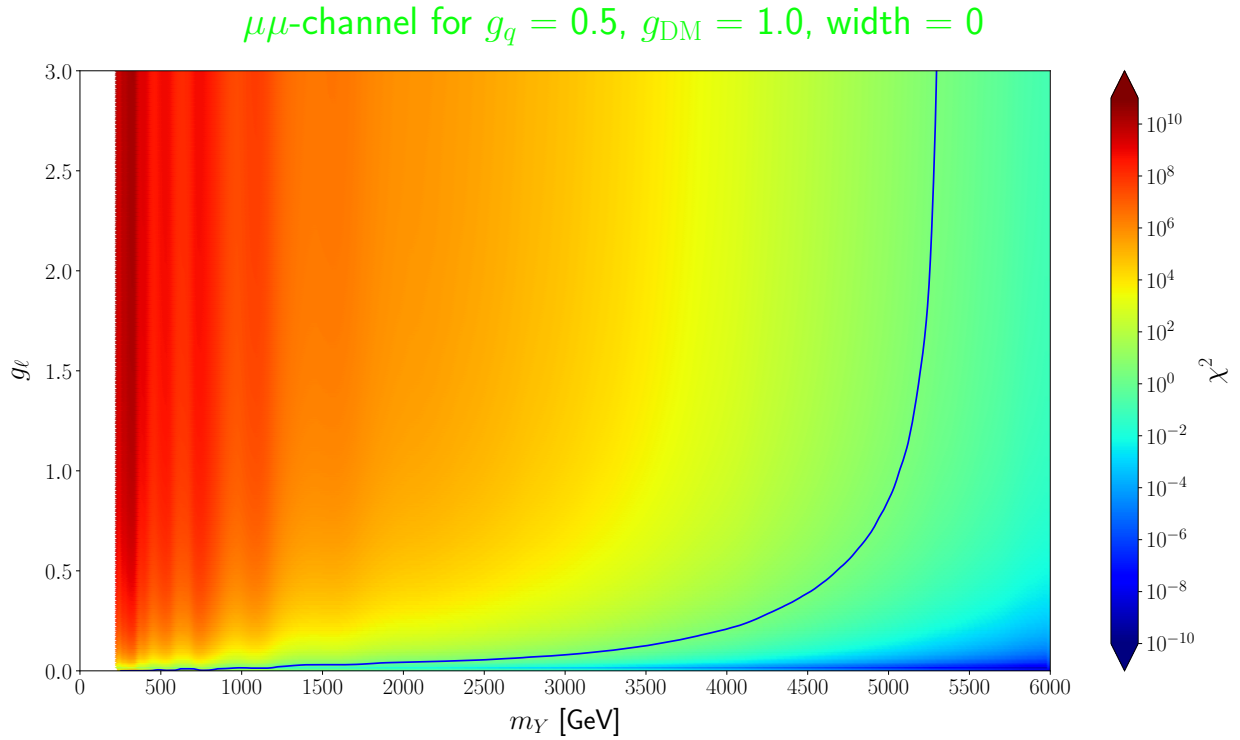
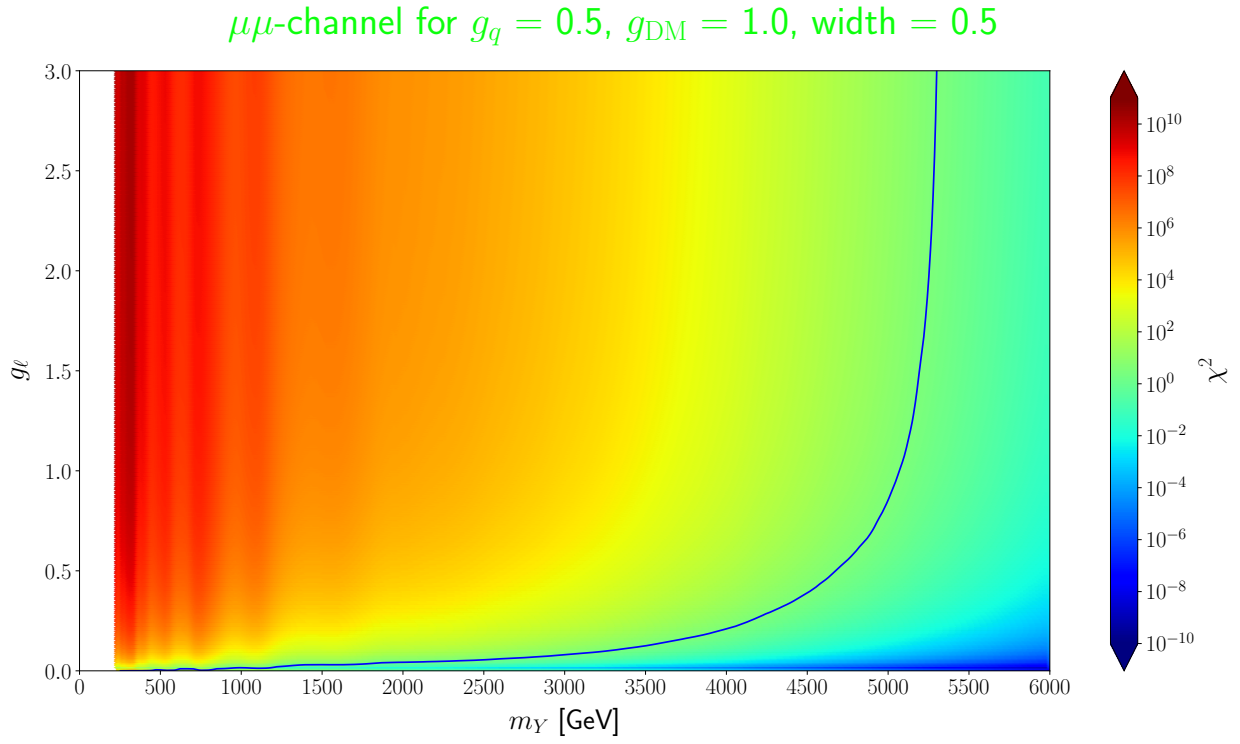
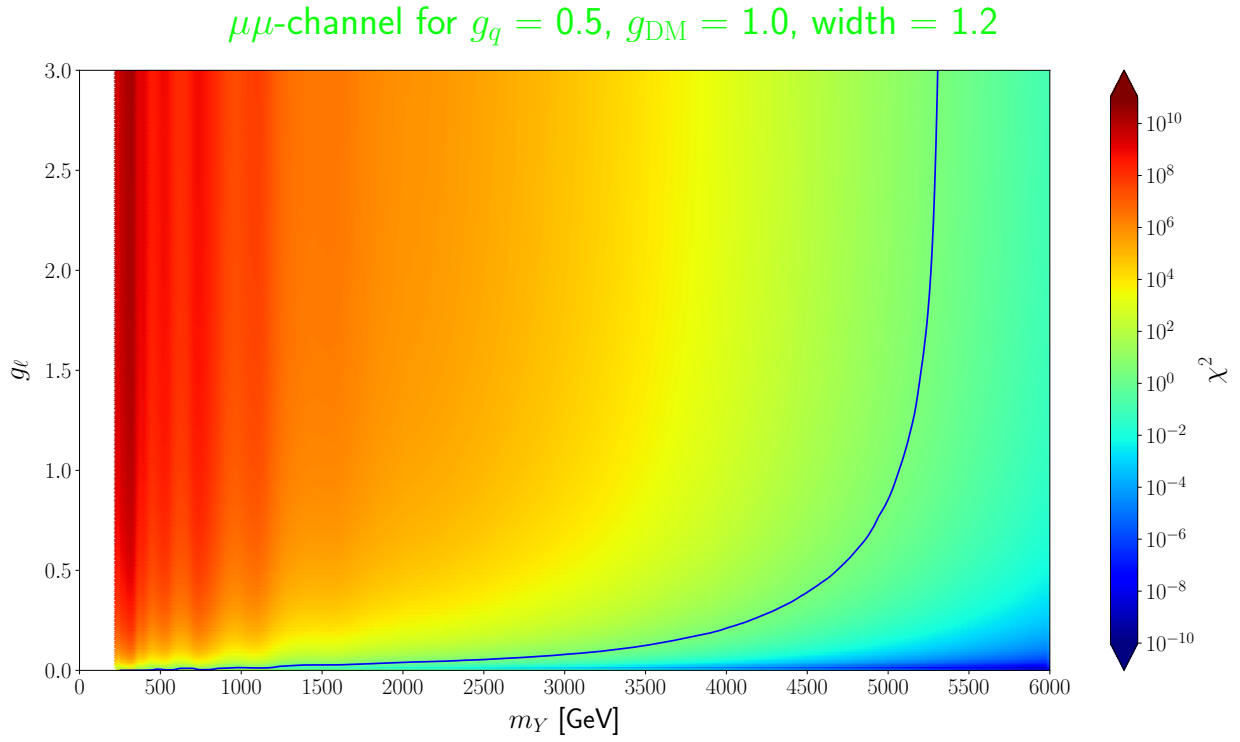
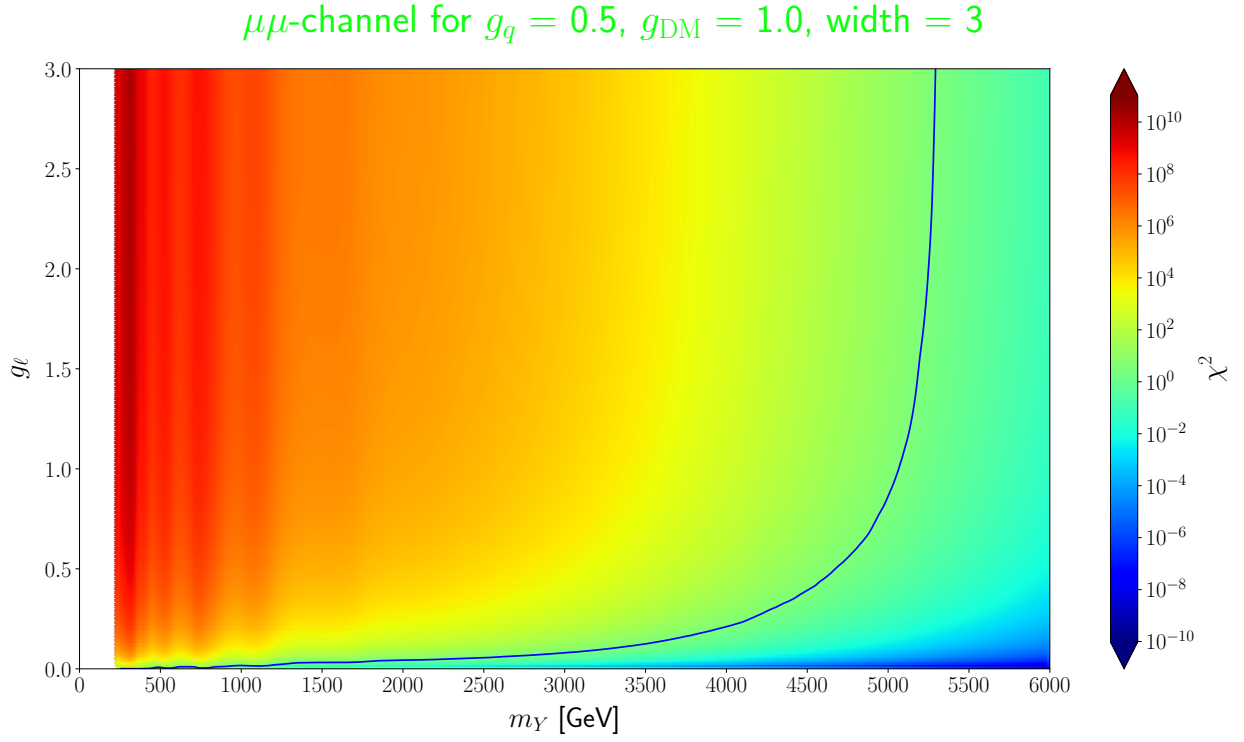
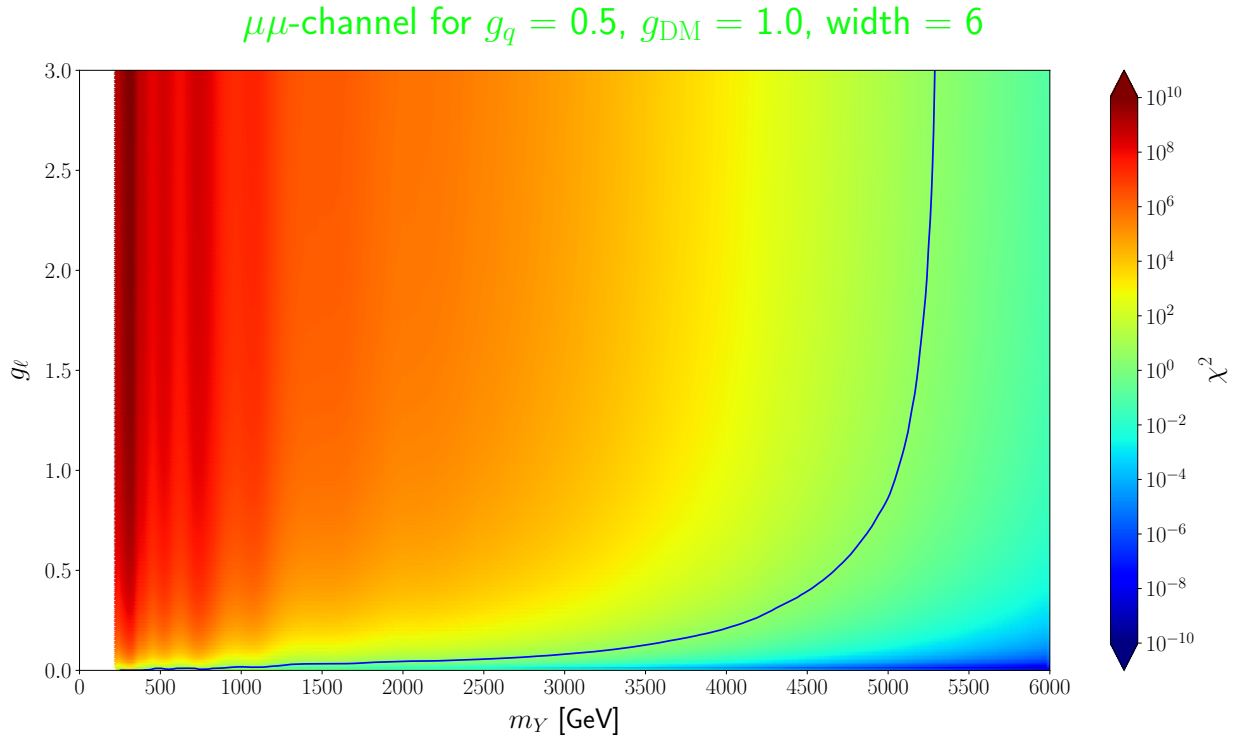


Figure B.7: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 0\%$

Figure B.8: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 0.5\%$ Figure B.9: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 1.2\%$

Figure B.10: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 3\%$ Figure B.11: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 6\%$

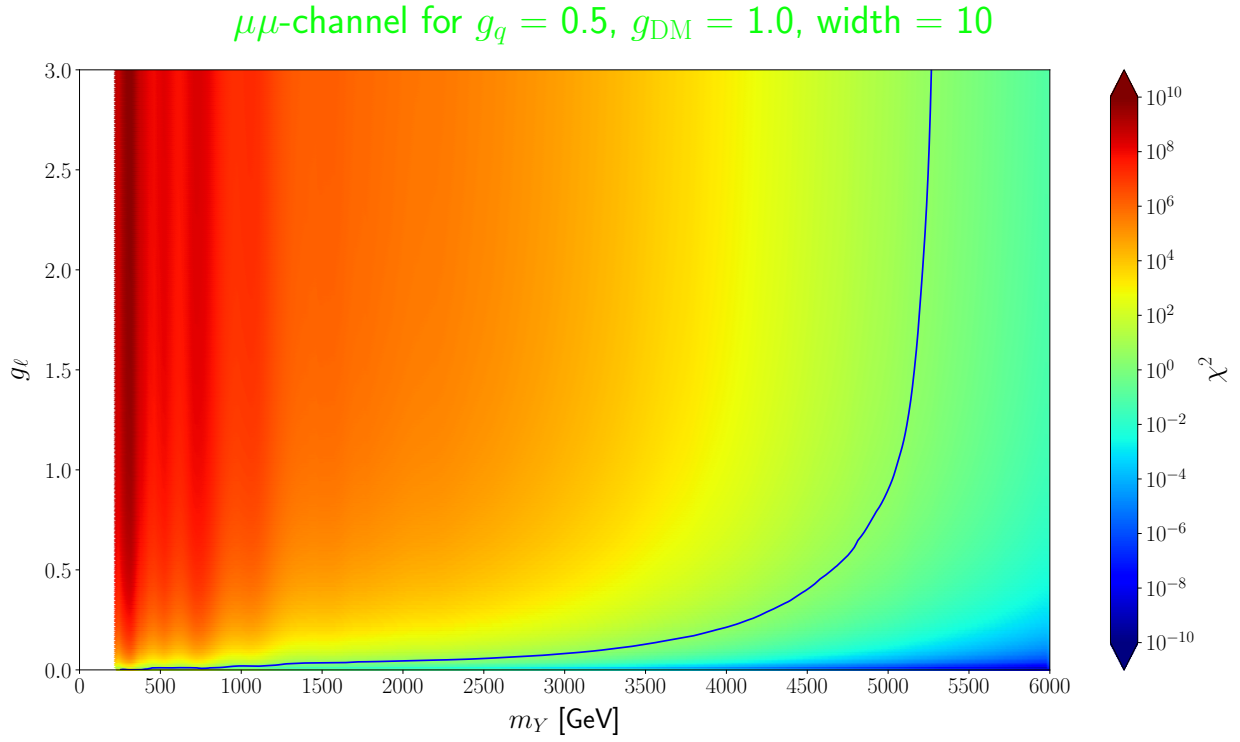


Figure B.12: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 10\%$

B.3 Combined Dilepton $\ell\ell$ channel

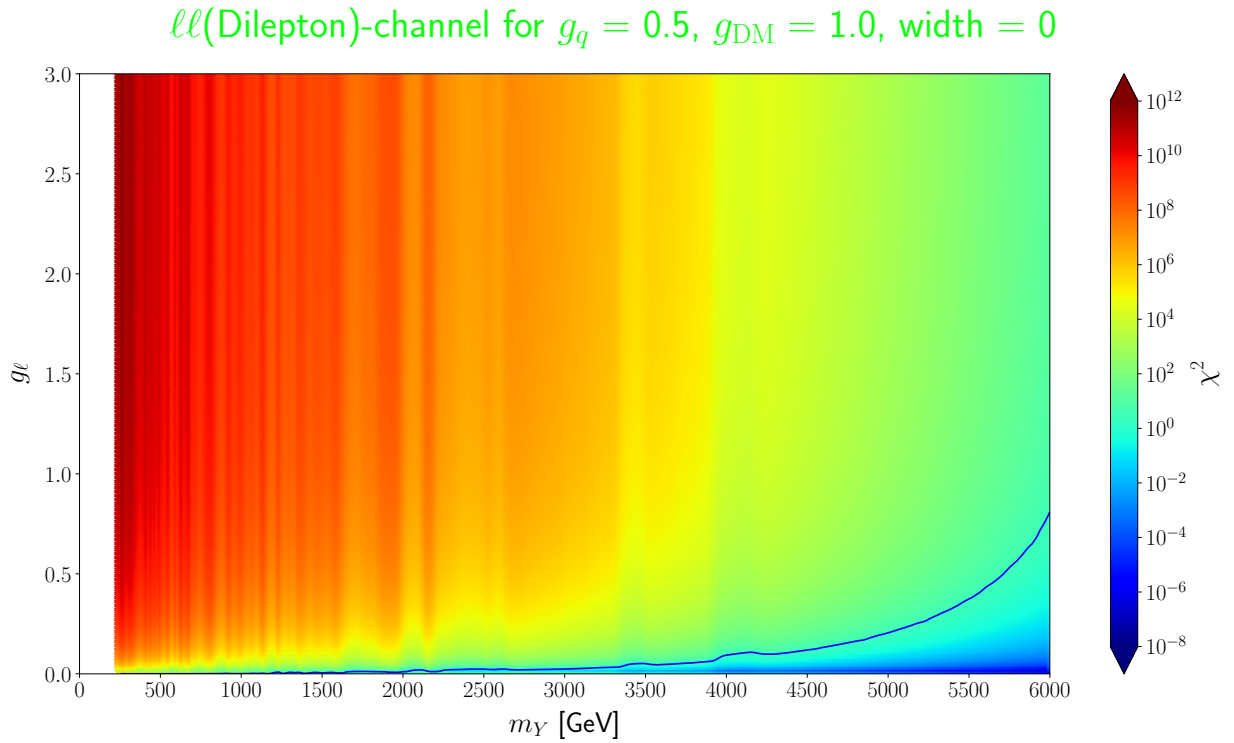
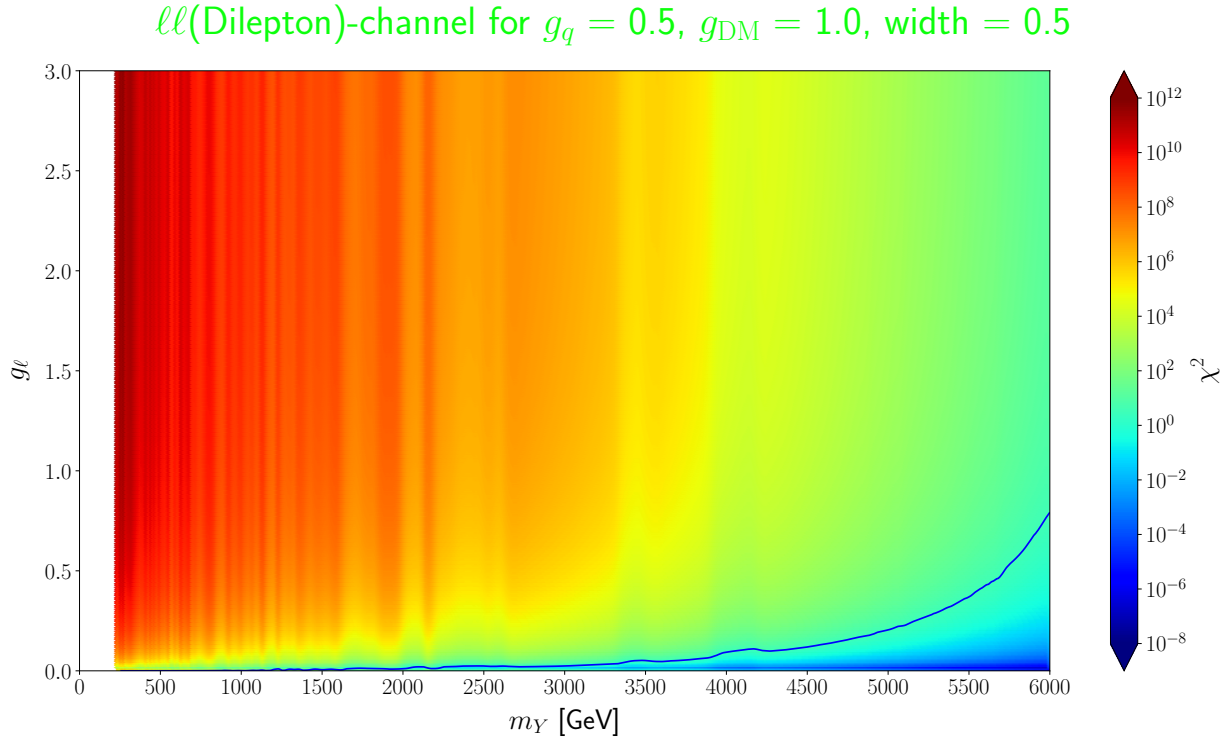
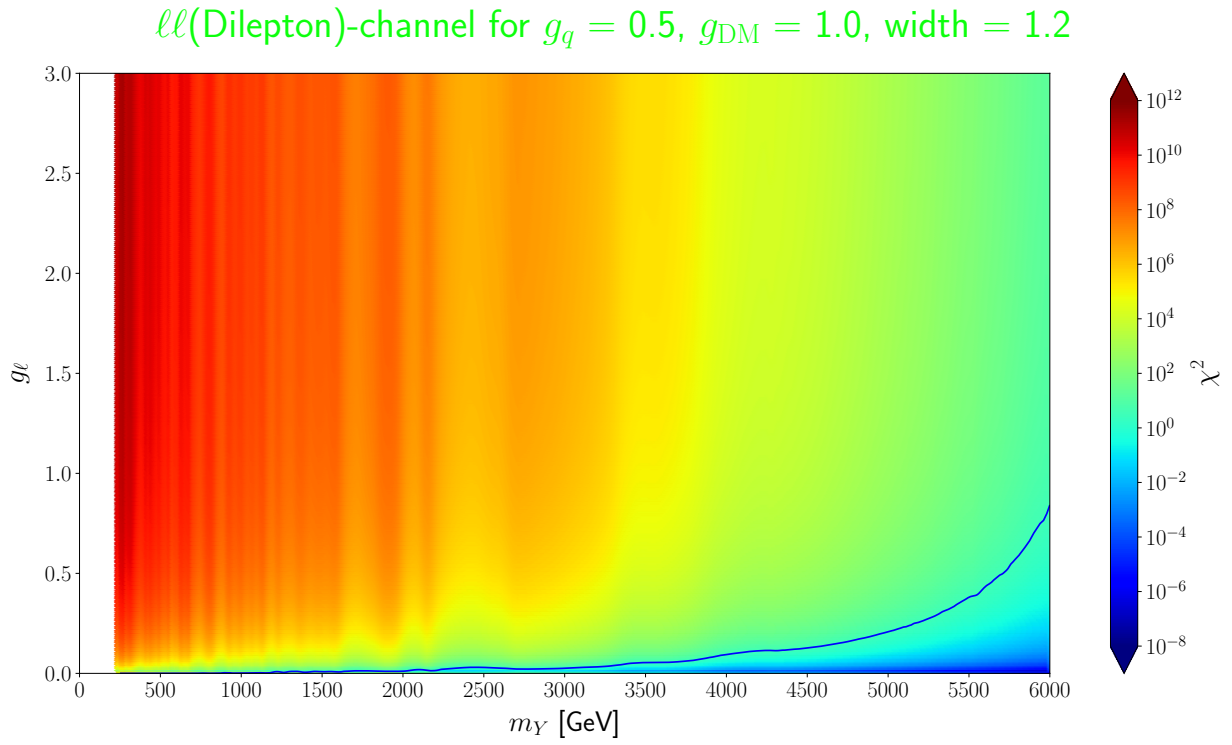
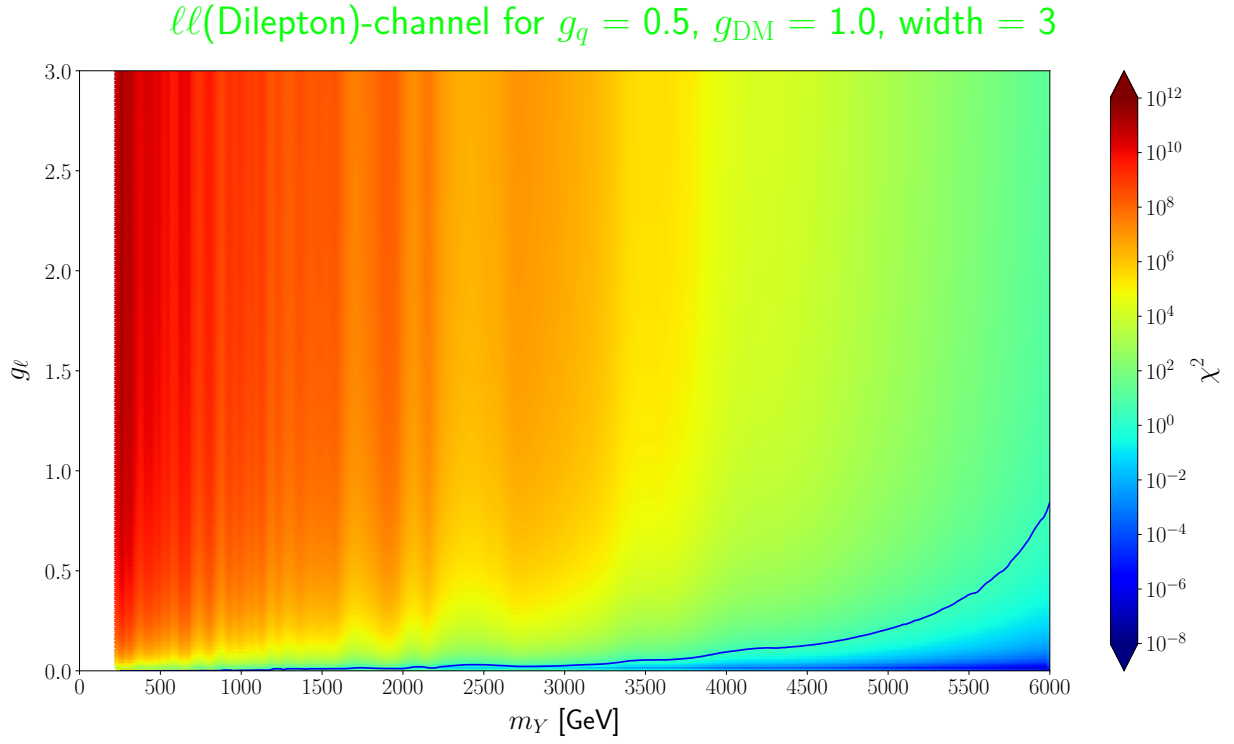
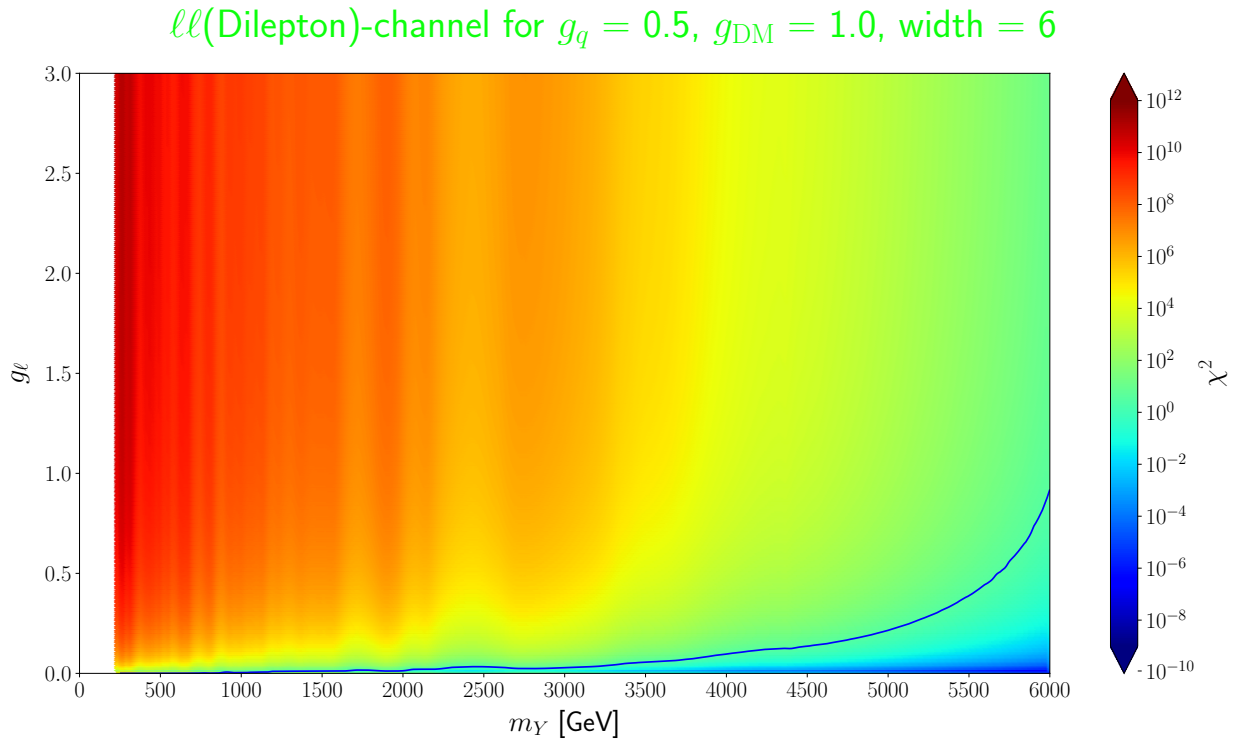


Figure B.13: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 0\%$

Figure B.14: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 0.5\%$ Figure B.15: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 1.2\%$

Figure B.16: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 3\%$ Figure B.17: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 6\%$

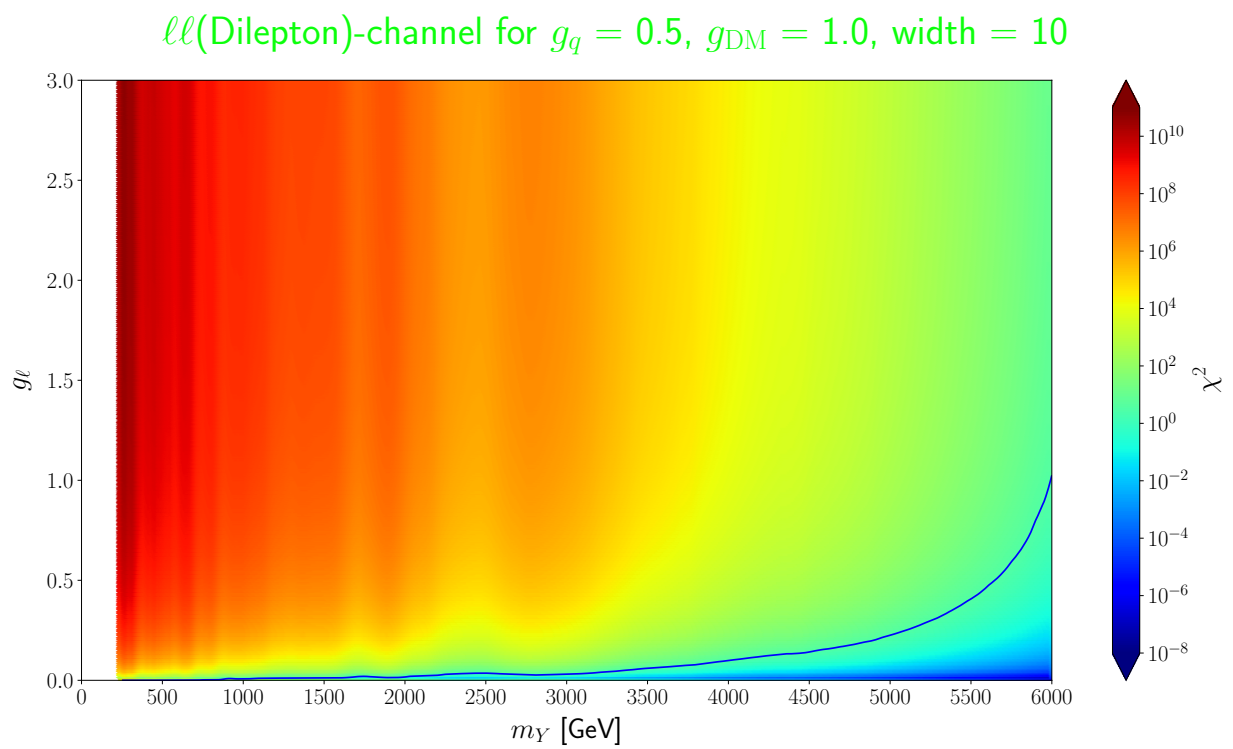


Figure B.18: The (m_Y, g_ℓ) plane for the relative width of $\Gamma_Y/m_Y = 10\%$

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