

Measurement of CP Violation in $B \rightarrow DK^*$ Decays with the LHCb Experiment

Edmund Robert Henry Smith
Linacre College, University of Oxford



Thesis submitted in fulfilment of the requirements for the degree of Doctor of
Philosophy at the University of Oxford

Trinity Term, 2014

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Abstract

This thesis reports an analysis of 3.0 fb^{-1} of pp collision data collected by the LHCb experiment in 2011 and 2012. Decays of neutral B mesons to neutral D mesons and excited neutral kaons are reconstructed, because of their sensitivity to the CP -violating phase of the CKM matrix, γ . The neutral D meson is reconstructed in four separate final states. These are $K^-\pi^+$, $K^+\pi^-$, K^+K^- and $\pi^+\pi^-$.

Several observables are defined, in terms of the partial widths of $B \rightarrow DK^*$ decays, that are sensitive to γ and other, CP -conserving, parameters of the decays of the B and D mesons. The measured results include the CP asymmetry between the partial widths of $\bar{B}^0 \rightarrow D(K^+K^-)\bar{K}^{*0}$ and $B^0 \rightarrow D(K^+K^-)K^{*0}$ decays, which is found to be

$$\mathcal{A}_d^{KK} = -0.198^{+0.144}_{-0.145} {}^{+0.019}_{-0.020}.$$

In addition, the Cabibbo-suppressed $B^0 \rightarrow D(K^-\pi^+)K^{*0}$ and $\bar{B}^0 \rightarrow D(K^+\pi^-)\bar{K}^{*0}$ decays are analysed for the first time with data collected by LHCb. The ratio of the partial widths of the Cabibbo-suppressed $B^0 \rightarrow D(K^-\pi^+)K^{*0}$ decay to the Cabibbo-favoured $B^0 \rightarrow D(K^+\pi^-)K^{*0}$ decay is measured, as well as the analogous quantity corresponding to $\bar{B}^0$ decays. These are found to be

$$\mathcal{R}_d^+ = 0.057^{+0.029}_{-0.027} {}^{+0.009}_{-0.012} \quad \text{and} \quad \mathcal{R}_d^- = 0.056^{+0.032}_{-0.030} {}^{+0.009}_{-0.012},$$

respectively.

Finally, the ratio of the magnitudes of the amplitudes corresponding to $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays is found to be

$$r_B = 0.230^{+0.063}_{-0.045}$$

at a confidence level of 68.3% and the first constraints on γ from $B_d \rightarrow DK^*$ decays are made.

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Introduction

The Standard Model of particle physics is a quantum field theory that seeks to explain the behaviour of the most fundamental constituents of the Universe. Over the course of its development it has been very successful in predicting or explaining the outcome of experiments. The Standard Model describes three out of the four known forces of nature. These are the strong and weak nuclear forces and the electromagnetic force, but not gravity. CP symmetry violation is, in general, expected by the Standard Model and is a well established experimental observation and the central topic of this thesis.

LHCb is one of four main experiments at the CERN research facility near Geneva, Switzerland. LHCb is designed to test the Standard Model with great precision, an approach that is complementary to that used by the general purpose experiments, CMS and ATLAS. CMS and ATLAS search for direct evidence of new particles, whereas LHCb is sensitive to their indirect effects on processes that are precisely predicted by the Standard Model.

The accurate determination of the angle γ is one of the main physics objectives of LHCb because it is one of the least well known parameters of the quark mixing matrix. It is also unique among CP -violating parameters in that it is measurable from purely tree-level processes, rendering it free from theoretical uncertainty and a useful reference point, against which the effects of new physics might be discerned. The analysis of $B \rightarrow DK^*$ decays¹ is foreseen as an important contribution to the measurement of γ [1].

Preliminary analysis of $B \rightarrow DK^*$ and related decays in LHCb started with the first data, collected in 2010. The author contributed to the first observation of the $B_s \rightarrow DK^*$

¹Here and after, B signifies B^0 , $\bar{B}^0$, B_s^0 and $\bar{B}_s^0$ mesons, D signifies an admixture of D^0 and $\bar{D}^0$ mesons and K^* signifies K^{*0} and $\bar{K}^{*0}$ mesons, unless otherwise stated.

decay², and the measurement of its branching fraction relative to the $B_d \rightarrow D\rho^0$ decay [2]. A subsequent analysis of the data collected in 2011 represented the first observation of the $B_d \rightarrow D(K^+K^-)K^*$ decay and the first measurement of CP violation in $B \rightarrow DK^*$ decays [3]. A related decay that will become important in the measurement of γ in the future is $B_s \rightarrow D\phi$, the first observation of this decay was made with the same dataset from 2011 [4]. The analysis presented in this thesis extends and updates Ref. [3], using three times more data. Additional final states of $B \rightarrow DK^*$ decays are analysed and the most accurate measurement to date of r_B , a key parameter in the determination of γ , is reported.

Chapter 1 gives a brief introduction to the Standard Model of particle physics. Emphasis is placed on topics that relate to CP violation and the current experimental situation with regard to γ is reviewed. Experimentally robust observables are defined that enable the measurement of γ and other interesting parameters of $B \rightarrow DK^*$ decays. Chapter 2 gives a detailed report of the LHCb experiment and methods used to calibrate the response of the RICH detectors in LHCb. The author contributed to the development of these methods.

The analysis of $B \rightarrow DK^*$ decays presented in this thesis has been conducted within the LHCb collaboration and in close collaboration with a small group based at the Laboratoire de l'Accélérateur Linéaire in Orsay, France. Documentation of the analysis starts in Chapter 3, where the strategy for identifying signal decays amongst the background is motivated and its performance evaluated. Chapter 4 continues with an account of the model of B meson invariant mass used to determine the observed numbers of signal $B \rightarrow DK^*$ decays. There is also a detailed discussion of all other contributing factors to the observed numbers of $B \rightarrow DK^*$ decays and their influence on the best-fit values and uncertainties in the observables defined in Chapter 1. Finally, the interpretation of these results is given in Chapter 5. The limits that the results are able to place on γ , δ_B and r_B are presented. Here r_B is the ratio of the magnitudes of the amplitudes corresponding to $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays and δ_B is the strong phase difference between these amplitudes. The future prospects for analyses of $B \rightarrow DK^*$ decays at LHCb are also discussed.

²Here and after, B_s signifies B_s^0 and $\bar{B}_s^0$ mesons. Similarly, B_d signifies B^0 and $\bar{B}^0$ mesons, unless otherwise stated.

Chapter 1

Theory

A brief introduction to the Standard Model of particle physics is given. The mixing of the quarks via interaction with the $W^\pm$ boson is explained. The CP symmetry-violating implications of complex elements of the quark mixing matrix are discussed, followed by the possible methods for observation of this CP violation. The current experimental constraints on the quark mixing matrix are presented and it is shown how these can be tightened by an analysis of $B \rightarrow DK^*$ decays.

1.1 The Standard Model

1.1.1 Introduction

The Standard Model (SM) of particle physics is a non-abelian, locally gauge-invariant theory of the interactions of the known fundamental particles. It describes three out of the four known forces of nature in the same quantum field theoretical framework. These are the strong and weak nuclear forces and the electromagnetic force, which are understood to arise from the exchange of particles of integer spin (bosons) between particles of half-integer spin (fermions). So far there is no successful or verifiable quantum field theory of the fourth force of nature, gravity.

In a quantum field theory (QFT) such as the SM, each particle is expressed as a field that pervades the whole of spacetime. A QFT is defined by the types of transformations it can undergo and remain invariant, these are the symmetry properties of the theory. Symmetry

properties of a theory are postulated in the form of Lie groups and are typically motivated by experimental observation or theoretical arguments. The interactions that can take place between the fields within a QFT are typically expressed in the Lagrangian formalism by the most general Lagrangian density that observes the symmetry of the theory. The SM Lagrangian density can be found in Ref. [5].

The symmetry of the Standard Model is defined by the direct product of Lie groups given in Equation 1.1.

$$SU_C(3) \otimes SU_L(2) \otimes U_Y(1) \quad (1.1)$$

The $SU_C(3)$ part describes the symmetry of the theory of the strong force (Quantum Chromodynamics or QCD) and the subscript “C” refers to the “colour” charge (or quantum number) of the fields under transformations of this group.

The $SU_L(2) \otimes U_Y(1)$ part describes the symmetry of the theory of electroweak interactions and is spontaneously broken to the $U_{em}(1)$ symmetry of Quantum Electrodynamics (QED) by the Higgs mechanism. The subscript “L” refers to left-handedness since only left-handed fields carry $SU_L(2)$ charge. The subscript “Y” denotes the weak hypercharge, which is the charge of the fields under $U_Y(1)$ transformations.

The $SU_L(2)$ charge is the weak isospin (T), the third component of which (T_3) is important in defining the electric charge of the field (Q), or the charge under $U_{em}(1)$ transformations. Q is related to T_3 and y , the eigenvalue of the generator of $U_Y(1)$, according to Equation 1.2. It is noted that the electric charge, Q , is not a quantum number carried by the SM fields under $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ transformations, but a derived quantity, hence the important distinction between $U_Y(1)$ and $U_{em}(1)$.

$$Q = T_3 + y \quad (1.2)$$

1.1.2 Matter content

The matter content of the SM is described by particles of half-integer spin, the fermions. The fundamental fermions are quarks, which interact via the strong force, and leptons, which do not. There are three generations of matter, with two quarks and two leptons in each generation. The particles in each generation differ from each other in mass but otherwise have the same properties and interactions. The quarks can be classified as “up-type” (u , c and t) and “down-type” (d , s and b). Up-type quarks have an electric charge of $+\frac{2}{3}$ whereas down-type quarks have an electric charge of $-\frac{1}{3}$. The known leptons are the electron (e), muon (μ) and the tau (τ) and their corresponding neutrinos, ν_e , ν_μ and ν_τ . The measured masses of the SM fermions are given in Table 1.1.

Table 1.1: The measured masses of the Standard Model fermions [6]. It is noted that neutrinos are massless in the Standard Model. This is contradicted by the observation of neutrino mixing, however no theoretical explanation of non-zero neutrino masses has yet been experimentally proven.

Quark	Mass
up (u)	$2.3^{+0.7}_{-0.5} \text{ MeV}/c^2$
down (d)	$4.8^{+0.5}_{-0.3} \text{ MeV}/c^2$
charm (c)	$1.275 \pm 0.025 \text{ GeV}/c^2$
strange (s)	$95 \pm 5 \text{ MeV}/c^2$
truth (t)	$173.07 \pm 0.52 \pm 0.72 \text{ GeV}/c^2$
beauty (b)	$4.18 \pm 0.03 \text{ GeV}/c^2$
Lepton	Mass
electron (e^-)	$0.511 \pm 10^{-8} \text{ MeV}/c^2$
electron neutrino (ν_e)	—
muon (μ^-)	$105.66 \pm 3.5 \times 10^{-8} \text{ MeV}/c^2$
muon neutrino (ν_μ)	—
tau (τ^-)	$1776.82 \pm 0.16 \text{ MeV}/c^2$
tau neutrino (ν_τ)	—

Each SM particle has an antiparticle partner, which arise from the interpretation of the negative energy solutions of the Dirac equation [7]. Antiparticles couple with identical strength to their corresponding particles, a phenomenon known as “crossing symmetry”.

The behaviour of particle and antiparticle fields under transformations of the SM symmetry groups is also related. In particular, charged antiparticles have opposite electric charge to their corresponding particles.

A standard way to indicate the transformation properties of fields under the symmetry groups of the SM uses the dimensions of their representations under those groups [5]. Any field can be expressed as $(\mathbf{a}, \mathbf{b}, y)$, where $\mathbf{a}$ and $\mathbf{b}$ are the dimensions of the representations of the field under the $SU_C(3)$ and $SU_L(2)$ groups, respectively. If the representation of the field has one dimension, it is invariant under transformations of the relevant group and is referred to as a “singlet”, signified by $\mathbf{1}$. Dimensions of two and three are signified by $\mathbf{2}$ (“doublet”) and $\mathbf{3}$ (“triplet”), respectively.

The left-handed and right-handed parts of the fermion fields are known to transform under the symmetry groups in different ways. The left-handed fields transform as doublets under $SU_L(2)$ and the right-handed fields as singlets. The transformation properties, weak isospins and electric charges of the fermions are summarised in Table 1.2

Table 1.2: The $SU_L(2)$ representations of the fermion fields of the SM, their transformation properties with respect to all SM symmetry groups, third component of weak isospin (T_3) and their electric charge (Q). $\mathcal{U}$ and $\mathcal{D}$ signify any up- or down-type quark and $\ell = e, \mu$ or τ . The notation is explained in the text.

Field	Transformation	T_3	Q
<i>Quarks</i>			
$\mathcal{U}_R$	$(\mathbf{3}, \mathbf{1}, \frac{2}{3})$	0	$\frac{2}{3}$
$\mathcal{D}_R$	$(\mathbf{3}, \mathbf{1}, -\frac{1}{3})$	0	$-\frac{1}{3}$
$\begin{pmatrix} \mathcal{U} \\ \mathcal{D} \end{pmatrix}_L$	$(\mathbf{3}, \mathbf{2}, \frac{1}{6})$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{2}{3}$ $-\frac{1}{3}$
<i>Leptons</i>			
ℓ_R^-	$(\mathbf{1}, \mathbf{1}, -1)$	0	-1
$\begin{pmatrix} \nu_\ell \\ \ell^- \end{pmatrix}_L$	$(\mathbf{1}, \mathbf{2}, -\frac{1}{2})$	$\frac{1}{2}$ $-\frac{1}{2}$	0 -1

1.1.3 Bosons

The particles that carry the forces of the SM are the bosons, which all have integer spin. The fundamental bosons are mostly described by vector fields that are introduced to maintain the local gauge-invariance of the SM Lagrangian¹ [5], hence the name “gauge bosons”.

It is found that vector fields must be introduced for every generator of the symmetry groups of the SM to achieve local gauge-invariance. This means that there are 8 gluon fields (G_μ^α where $\alpha = 1, \dots, 8$), which carry the strong force, three $SU_L(2)$ boson fields (W_μ^a where $a = 1, 2, 3$) and one $U_Y(1)$ boson field (B_μ).

The W_μ^a and B_μ fields do not correspond to particles that are observed today. This is because of the phase transition that occurred when the energy of the universe decreased below the electroweak scale, causing the spontaneous breaking of the $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ symmetry to $SU_C(3) \otimes U_{em}(1)$. At this phase transition, the Higgs field, ϕ , the last fundamental, and only non-vector, boson field of the SM acquired a vacuum expectation value. This process is known as the Higgs mechanism.

The Higgs mechanism is responsible for the formation of the electroweak gauge boson fields of the SM, the $W_\mu^\pm$, Z_μ and the A_μ (the photon), from the massless W_μ^a and B_μ fields of the unbroken SM and for giving the $W^\pm$ and Z^0 bosons their masses [5]. The Higgs boson, H^0 , was discovered in 2012 [8,9]. The relations between the gauge boson fields of the pre- and post-symmetry breaking SM are given in Equations 1.3 to 1.5, where θ_W is the Weinberg angle. The measured masses of the SM bosons are given in Table 1.3.

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp W_\mu^2) \quad (1.3)$$

$$Z_\mu = W_\mu^3 \cos \theta_W - B_\mu \sin \theta_W \quad (1.4)$$

$$A_\mu = W_\mu^3 \sin \theta_W + B_\mu \cos \theta_W \quad (1.5)$$

¹Here and after, “Lagrangian” and “Lagrangian density” are used interchangeably.

Table 1.3: The measured masses of the Standard Model bosons [6].

Boson	Mass
$W^\pm (W_\mu^\pm)$	$80.385 \pm 0.015 \text{ GeV}/c^2$
$Z^0 (Z_\mu)$	$91.1876 \pm 0.0021 \text{ GeV}/c^2$
Higgs, $H^0 (\phi)$	$125.9 \pm 0.4 \text{ GeV}/c^2$
photon, $\gamma (A_\mu)$	—
gluon, $g (G_\mu)$	—

1.1.4 Discrete symmetries

There are three discrete symmetries that could naturally be expected of a relativistic QFT, as well as the continuous gauge symmetries of the SM. These are the invariance to *charge conjugation* denoted by C , *parity inversion* denoted by P and *time reversal* denoted by T [10].

The effect of C is to change particles into their antiparticles. The effect of P is to change the sign of all Cartesian space coordinates simultaneously, *i.e.* $x \rightarrow -x$, $y \rightarrow -y$ and $z \rightarrow -z$, or $\mathbf{x} \rightarrow -\mathbf{x}$. This changes any left-handed field to its right-handed counterpart and vice versa. Similarly the action of T is to change the sign of the time coordinate, $t \rightarrow -t$. The operations of C , P and T on a Dirac spinor field are shown explicitly in Equations 1.6 to 1.8 [10].

$$C\psi(\mathbf{x}, t)C = -i(\bar{\psi}(\mathbf{x}, t)\gamma^0\gamma^2)^T \quad (1.6)$$

$$P\psi(\mathbf{x}, t)P = \gamma^0\psi(-\mathbf{x}, t) \quad (1.7)$$

$$T\psi(\mathbf{x}, t)T = -\gamma^1\gamma^3\psi(\mathbf{x}, -t) \quad (1.8)$$

C , P and T symmetries were thought to be individually obeyed by nature. However, parity violation was postulated in 1956 as an explanation of two different decay modes of what appeared to be the same particle [11]². In 1957 parity violation was observed in the beta decay of cold cobalt nuclei (a weak interaction). The spins of the cobalt nuclei were aligned by an external magnetic field and the electrons from the beta decay were emitted

²This was the “ τ - θ puzzle”. The τ and θ particles indeed transpired to be the same particle, the charged kaon, and one of the decay modes violated parity.

preferentially in the opposite direction to the nuclear spin. This was clear proof for the violation of P symmetry [12].

After the discovery of parity violation it was suggested that nature was instead invariant under the combined operation of C and P . However, CP violation was discovered in 1964 [13], in the decays of long-lived neutral kaons to two pions, which is a CP -violating process that arises because the mass eigenstates of neutral kaons are not exact CP eigenstates. CP violation in weak interactions is discussed in more detail in Section 1.2.

So far, no violation of C , P or CP symmetry has been observed in electromagnetic or strong interactions.

The invariance under the combined operation of CPT for a local, relativistic QFT is a proved theorem. Therefore, the observation of CP violation in many weak processes implies that T symmetry must be broken to preserve overall CPT invariance. Indeed the first direct evidence for T violation has recently been observed in $B_d \rightarrow c\bar{c}K_S^0$ and $B_d \rightarrow J/\psi K_L^0$ decays [14]. The analysis utilises pairs of entangled B_d mesons to compare the rates of pairs of T -conjugated processes and measures a departure from T invariance with a significance equivalent to 14σ .

1.1.5 The CKM matrix

The Higgs field gives mass to the gauge bosons but is also motivated by the need to explain the observed masses of the fermions. A fermion spinor field, ψ , can be expressed as a sum of left- and right-handed parts, $\psi = \psi_L + \psi_R$, where ψ_L and ψ_R are given by Equation 1.9 and the Dirac mass term, which mixes these two chiralities, is given by Equation 1.10.

$$\psi_L = P_L \psi = \frac{1}{2} (1 - \gamma^5) \psi \quad \psi_R = P_R \psi = \frac{1}{2} (1 + \gamma^5) \psi \quad (1.9)$$

$$m\bar{\psi}\psi = m\bar{\psi}_L\psi_R + m\bar{\psi}_R\psi_L \quad (1.10)$$

Therefore, a Dirac mass for an up-type quark would include a term of the form $\bar{U}_R Q_L$,

where $\mathcal{U}_R$ is the right-handed $SU_L(2)$ up-type quark singlet and $\mathcal{Q}_L$ is the left-handed $SU_L(2)$ quark doublet. $\overline{\mathcal{U}}_R$ and $\mathcal{Q}_L$ transform in an $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ theory according to $(\overline{\mathbf{3}}, \mathbf{1}, -\frac{2}{3})$ and $(\mathbf{3}, \mathbf{2}, \frac{1}{6})$, respectively. Therefore the Dirac mass term is not gauge-invariant unless an extra field is added. The extra field, ϕ , should have $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ transformation properties of $(\mathbf{1}, \mathbf{2}, \frac{1}{2})$ to restore gauge-invariance [5].

The mass term $\lambda^u \overline{\mathcal{U}}_R \mathcal{Q}_L \phi$ is gauge-invariant. The introduced field, ϕ , is the Higgs $SU_L(2)$ doublet and λ^u is the coupling of the Higgs to the up-type quark in question, called the ‘‘Yukawa coupling’’.

Once ϕ acquires a vacuum expectation value, which it does so through the spontaneous breaking of the $SU_L(2) \otimes U_Y(1)$ symmetry and the Higgs mechanism, the mass term for the fermion can be thought of as an interaction with the Higgs field that reverses the chirality of the fermion. The vacuum expectation value of ϕ is given by Equation 1.11.

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \quad (1.11)$$

The three generations of quarks have identical $SU_C(3) \otimes SU_L(2) \otimes U_Y(1)$ charges, so there should be mass terms of identical structure in the SM Lagrangian for each. The quark mass terms are given in Equation 1.12 [6].

$$\mathcal{L}_{\text{quark mass}} = \sum_{I,J} \lambda_1^{IJ} \overline{\mathcal{D}}_R^I \phi \mathcal{Q}_L^J + \lambda_2^{IJ} \overline{\mathcal{U}}_R^I \tilde{\phi} \mathcal{Q}_L^J + \text{h.c.} \quad (1.12)$$

In Equation 1.12, I and J run over quark generation, $\tilde{\phi}$ is the conjugate of the Higgs field, where $\tilde{\phi} = \epsilon \phi^*$ and ϵ is the antisymmetric tensor of $SU_L(2)$ and h.c. stands for hermitian conjugate. From Equations 1.11 and 1.12 the quark mass matrix can be deduced,

$$\mathcal{M}_{\text{quarks}} = \frac{v}{\sqrt{2}} (\overline{\mathcal{D}}_R \lambda_1 \mathcal{D}_L + \overline{\mathcal{U}}_R \lambda_2 \mathcal{U}_L + \text{h.c.}) . \quad (1.13)$$

In $\mathcal{M}_{\text{quarks}}$, the generation indices have been suppressed, the Yukawas, $\lambda_{1,2}$, are now

recognised as 3×3 hermitian matrices and $\overline{\mathcal{D}}(\overline{\mathcal{U}})$ and $\mathcal{D}(\mathcal{U})$ are row and column vectors of the three down(up)-type quarks, respectively.

An arbitrary field redefinition, to ensure that the Lagrangian is expressed in the physical basis in terms of mass eigenstates, is introduced. $\mathcal{U}$ and $\mathcal{D}$ are transformed with unitary matrices, V_U and V_D , as shown.

$$\mathcal{U} \rightarrow V_U \mathcal{U} \qquad \mathcal{D} \rightarrow V_D \mathcal{D} \qquad (1.14)$$

$$\overline{\mathcal{U}} \rightarrow \overline{\mathcal{U}} V_U^\dagger \qquad \overline{\mathcal{D}} \rightarrow \overline{\mathcal{D}} V_D^\dagger \qquad (1.15)$$

After this transformation the mass matrix becomes

$$\mathcal{M}'_{quarks} = \frac{v}{\sqrt{2}} \left(\overline{\mathcal{D}}_R \left(V_D^\dagger \lambda_1 V_D \right) \mathcal{D}_L + \overline{\mathcal{U}}_R \left(V_U^\dagger \lambda_2 V_U \right) \mathcal{U}_L + \text{h.c.} \right), \qquad (1.16)$$

and V_U and V_D can be chosen such that $V_D^\dagger \lambda_1 V_D$ and $V_U^\dagger \lambda_2 V_U$ are diagonal and are given by

$$V_D^\dagger \lambda_1 V_D = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \qquad V_U^\dagger \lambda_2 V_U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}. \qquad (1.17)$$

Choosing V_U and V_D such that the mass matrices are diagonal is equivalent to choosing to write the Lagrangian in the mass basis. There is no effect on most of the terms in the SM Lagrangian as a result of this redefinition of the quark fields. Factors of $V_U^\dagger$ and V_U perfectly cancel in the kinetic terms of the up-type quarks, for example. However, the $W^\pm$ interaction terms are affected. An example is given by the following expression, which contains a factor of $V_U^\dagger V_D$.

$$\overline{\mathcal{U}}_L \gamma^\mu W_\mu^- (1 - \gamma^5) \left(V_U^\dagger V_D \right) \mathcal{D}_L \qquad (1.18)$$

$V_U^\dagger V_D$ is a unitary matrix, which represents a rotation of the quark states from the mass basis to the $W^\pm$ interaction basis. This means that the $W^\pm$ interaction states of the quarks are not necessarily the mass eigenstates. Diagonal $V_U^\dagger V_D$ is allowed in the SM but

non-diagonality is a well established observation.

The first effective evidence for the non-diagonal nature of $V_U^\dagger V_D$ came before quarks were theorised. Nicola Cabibbo introduced the Cabibbo angle, θ_c , to preserve universality of the weak interaction and explain observed leptonic decay rates of hadrons [15]. Although quarks were not yet theorised, this amounted to a statement that the object that interacted with the u quark via a $W^\pm$ boson was not the d quark, but a superposition of the d and s quarks given by Equation 1.19. This effectively states that the d and s quarks mix via the $W^\pm$ interaction and θ_c gives the extent of the mixing.

$$d' = \cos\theta_c d + \sin\theta_c s \quad (1.19)$$

Shortly after this, the quark model, containing u , d and s quarks, was proposed to explain the multitude of new particle discoveries at the time [16]. Inconsistencies between Cabibbo's theory and some observed kaon decay rates led to the prediction of a fourth quark, the charm quark [17].

However, Kobayashi and Maskawa made the observation that CP violation of the weak interaction could not be explained with only four quarks. To remedy this they postulated a third generation of quarks [18], increasing the dimension of $V_U^\dagger V_D$ to three.

$V_U^\dagger V_D$ is now referred to as the Cabibbo-Kobayashi-Maskawa matrix or CKM matrix, V_{CKM} . By convention, the CKM matrix is defined such that it acts on the down-type quark fields. The CKM matrix is given by Equation 1.20, where V_{ij} represents the transition amplitude of a quark of species j to a quark of species i , or, in other words, the coupling of a $q_j \rightarrow W q_i$ vertex.

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (1.20)$$

The magnitudes of the CKM elements give information about the coupling of a $q_j \rightarrow W q_i$ vertex and therefore the nature of the quark mixing. The current best experimental

determinations of the sizes of the elements, assuming unitarity of the CKM matrix, are given in Equation 1.21 [6]. The CKM matrix is quite diagonal, meaning that intra-generation mixing occurs most frequently, followed by mixing that spans one generation, followed by mixing that spans two.

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.9743 \pm 0.0002 & 0.2252 \pm 0.0009 & 0.0042 \pm 0.0005 \\ 0.230 \pm 0.011 & 1.006 \pm 0.023 & 0.0409 \pm 0.0011 \\ 0.0084 \pm 0.0006 & 0.0429 \pm 0.0026 & 0.89 \pm 0.07 \end{pmatrix} \quad (1.21)$$

1.2 *CP* violation

The elements of the CKM matrix are coupling constants, which are complex numbers with arbitrary phases. In general, the phases in a Lagrangian involving complex fields can be redefined such that a coupling constant becomes real because of the freedom to redefine the fields. As a result, the only convention independent, and therefore physically meaningful, complex phases are those that cannot be absorbed by a redefinition. If no phase redefinition of the Lagrangian will allow exclusively real coupling constants then *CP* is not a good symmetry of the theory [19].

This behaviour can be understood by examining the coupling of the $W^\pm$ to the quarks, which is given by the following expression.

$$V_{ij}\bar{q}_i\gamma_\mu W^{+\mu}(1-\gamma_5)q_j + V_{ij}^*\bar{q}_j\gamma_\mu W^{-\mu}(1-\gamma_5)q_i \quad (1.22)$$

The effects of a *CP* operation on the various parts of the $W^\pm$ coupling are as follows:

$$\begin{aligned} \bar{\psi}_i\gamma^\mu\psi_j &\rightarrow -(-1)^\mu\bar{\psi}_j\gamma^\mu\psi_i, \\ \bar{\psi}_i\gamma^\mu\gamma^5\psi_j &\rightarrow -(-1)^\mu\bar{\psi}_j\gamma^\mu\gamma^5\psi_i, \\ W^{\pm\mu} &\rightarrow -(-1)^\mu W^{\mp\mu}, \end{aligned}$$

where $(-1)^\mu a^\mu = a_\mu$ [19]. Therefore the CP conjugate of the $W^\pm$ coupling to the quarks is given by

$$V_{ij}\bar{q}_j\gamma_\mu W^{-\mu}(1-\gamma_5)q_i + V_{ij}^*\bar{q}_i\gamma_\mu W^{+\mu}(1-\gamma_5)q_j. \quad (1.23)$$

This shows that CP is conserved if all the coupling constants, V_{ij} , are real. The observation made in Ref. [18] was that, with the addition of a third generation of quarks, the CKM matrix, and therefore the SM Lagrangian, could not be defined such that all V_{ij} were necessarily real. This accommodated the CP violation of the weak interaction in the SM and will be discussed at greater length in Section 1.2.4.

The CP violation arising from complex coupling constants can be grouped into three categories: “ CP violation in mixing”, “ CP violation in decay” and “ CP violation in the interference between decays, with and without mixing”, which is abbreviated to “ CP violation in the interference between mixing and decay”.

1.2.1 CP violation in mixing

CP violation in the mixing of neutral mesons is the form in which CP violation was initially discovered, in the decays of neutral kaons [13]. CP violation occurs when the two neutral mass eigenstates are not CP eigenstates. A B_d meson mass eigenstate is defined as an arbitrary linear combination of flavour eigenstates. A state produced as B^0 propagates as a mass eigenstate and therefore can, in general, transform into a $\bar{B}^0$ and back again any number of times before decaying. This is known as “mixing”. CP symmetry is violated in the mixing of B_d mesons if the amplitude corresponding to the $B^0 \rightarrow \bar{B}^0$ process is unequal in magnitude to that corresponding to the reverse process.

A linear combination of B_d meson flavour eigenstates is governed by the time-dependent Schrödinger equation:

$$i\frac{\partial}{\partial t} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix} = \mathbf{H} \begin{pmatrix} |B^0\rangle \\ |\bar{B}^0\rangle \end{pmatrix}. \quad (1.24)$$

The Hamiltonian of this system, $\mathbf{H}$, can be written as the sum of two Hermitian matrices, $\mathbf{M}$ and $\mathbf{\Gamma}$.

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} \quad (1.25)$$

The Hamiltonian is diagonalised to express Equation 1.24 in the mass basis. The light and heavy mass eigenstates, B_L and B_H , are given by Equation 1.26.

$$|B_L\rangle = p|B^0\rangle + q|\bar{B}^0\rangle \quad |B_H\rangle = p|B^0\rangle - q|\bar{B}^0\rangle \quad (1.26)$$

where the complex coefficients, p and q , satisfy the normalisation condition, $|p|^2 + |q|^2 = 1$.

CPT invariance implies that the diagonal elements of the original Hamiltonian, before diagonalisation, are equal, $H_{11} = H_{22}$. The off-diagonal elements of $\mathbf{H}$ give the values of p and q , which give the extent of the mixing and *CP* violation. If $|q/p|^2$, given by Equation 1.27, is not equal to one then *CP* symmetry is violated [19].

$$\left|\frac{q}{p}\right|^2 = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| \quad (1.27)$$

The off-diagonal elements of $\mathbf{M}$ and $\mathbf{\Gamma}$ also give the differences in mass, Δm_d , and width, $\Delta\Gamma_d$, of the light and heavy mass eigenstates:

$$\Delta m_d = 2|M_{12}|, \quad \Delta\Gamma_d = 2\frac{\Re(M_{12}\Gamma_{12}^*)}{|M_{12}|}. \quad (1.28)$$

1.2.2 *CP* violation in decay

CP violation in decay occurs when the amplitudes corresponding to a decay and its *CP* conjugate have different magnitudes. This form of *CP* violation is sometimes called “direct *CP* violation” and is the main topic of this thesis.

It is clear from Equations 1.22 and 1.23 that a coupling constant will appear in its complex conjugate form in any term in the Lagrangian that corresponds to the *CP*-conjugated process.

This means that their phases will have the opposite signs in the CP -conjugated amplitude. The phases that change sign under CP -conjugation are designated “weak phases”, since in the SM they only occur for the coupling constants of the weak interaction.

There is a second type of phase difference that can be present in an amplitude even when the Lagrangian is real. The origin of such phase differences is the possible contribution from intermediate on-shell states in the decay process. The dominant cause of such states is strong interactions, hence the designation “strong phases” for these phase differences [19].

The mechanism by which a weak phase difference between two amplitudes can violate CP symmetry can be understood if the interference of two decays is considered. In this case, the measured overall amplitude of the decay is the vector sum of the interfering amplitudes.

In general the amplitude of a process can be split into three parts: its complex coupling constant, g_i , with weak phase ϕ_i , its size, A_i , and its strong phase part, $e^{i\delta_i}$. A_f is defined as the amplitude of a decay from some initial state, X , to some final state, f , where there are two contributing processes to the overall decay. $\bar{A}_{\bar{f}}$ is the CP conjugate amplitude. A_f and $\bar{A}_{\bar{f}}$ are given by Equations 1.29 and 1.30.

$$A_f = A(X \rightarrow f) = g_1 A_1 e^{i\delta_1} + g_2 A_2 e^{i\delta_2} \quad (1.29)$$

$$\bar{A}_{\bar{f}} = A(\bar{X} \rightarrow \bar{f}) = g_1^* A_1 e^{i\delta_1} + g_2^* A_2 e^{i\delta_2} \quad (1.30)$$

The relevant quantity is the ratio of magnitudes of A_f and $\bar{A}_{\bar{f}}$, since if this is not equal to one then the process in question does not happen at the same rate as its CP conjugate and CP symmetry is violated. Clearly, for real g_i , CP symmetry holds.

It can be seen graphically in Figure 1.1 that CP violation of this type will not occur unless A_f and $\bar{A}_{\bar{f}}$ have different weak and strong phases. This is also shown by the expression for the difference in squared amplitudes, which is proportional to the difference in decay rates, given by Equation 1.31.

$$|A_f|^2 - |\bar{A}_{\bar{f}}|^2 = 2A_i A_j \Im(g_1 g_2^*) \sin(\delta_1 - \delta_2) \quad (1.31)$$

The potential observable difference from the weak phase part ($\Im(g_1 g_2^*) \propto \sin(\phi_1 - \phi_2)$) is thus entirely dependent on the strong phase difference in the amplitudes, $\delta = \delta_1 - \delta_2$, and is maximised for $\delta = n\pi/2$.

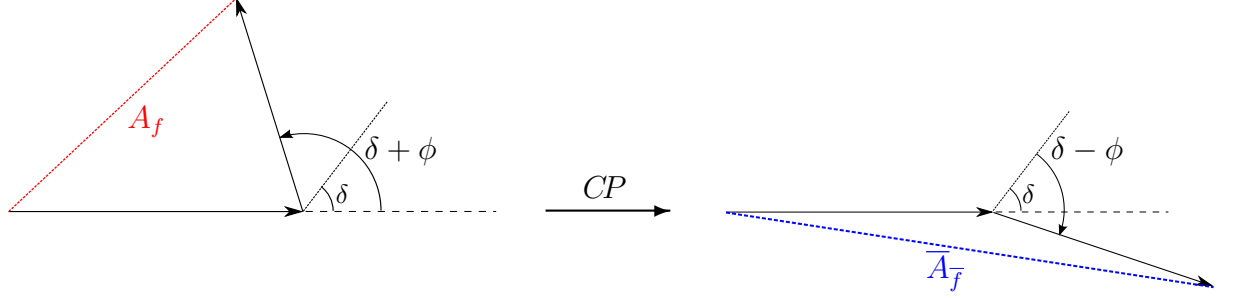


Figure 1.1: An illustration of how a complex phase that changes sign under CP -conjugation violates CP symmetry in the interference between two decays. δ is the strong phase difference that does not change sign under CP and ϕ is the weak phase difference. The difference in magnitude of A_f and $\bar{A}_{\bar{f}}$ results in different rates of a decay and its CP conjugate, and thus violates CP symmetry.

It is also clear from Figure 1.1 that in the case where there are two interfering amplitudes, the potential observable asymmetry (the difference divided by the sum of the amplitudes) is maximised when the ratio of the magnitudes of the interfering amplitudes is equal to one.

1.2.3 CP violation in the interference between mixing and decay

The final type of CP violation is manifest in decays of neutral mesons to a CP eigenstate, f_{CP} . A typical example of a decay that is utilised to measure this type of CP violation is $B_d \rightarrow J/\psi K_S^0$ [20]. The relevant quantity is given by Equation 1.32.

$$\lambda_{f_{CP}} = \eta_{f_{CP}} \frac{q}{p} \frac{\bar{A}_{\bar{f}_{CP}}}{A_{f_{CP}}} \quad (1.32)$$

Here, $\eta_{f_{CP}}$ is the CP eigenvalue of the state f_{CP} , the q/p factor relates to the mixing of the neutral mesons and the $\bar{A}_{\bar{f}_{CP}}/A_{f_{CP}}$ factor to their subsequent decay. It is recalled that when CP is conserved, $|q/p| = 1$ and $|\bar{A}_{\bar{f}_{CP}}/A_{f_{CP}}| = 1$. This implies that if $\lambda_{f_{CP}} \neq \pm 1$ then there is CP violation. However, it is possible that CP is separately conserved in mixing and decay and violated in their interference if $|\lambda_{f_{CP}}| = 1$ and $\Im(\lambda_{f_{CP}}) \neq 0$ [19].

To measure this kind of CP violation in B_d decays, the rate of decays into f_{CP} of a state produced as B^0 is compared to the rate of decays into f_{CP} of a state produced as $\bar{B}^0$. The time-dependent asymmetry is measured, which is given by

$$a_{f_{CP}} = \frac{\Gamma(B_{\text{init}}^0(t) \rightarrow f_{CP}) - \Gamma(\bar{B}_{\text{init}}^0(t) \rightarrow f_{CP})}{\Gamma(B_{\text{init}}^0(t) \rightarrow f_{CP}) + \Gamma(\bar{B}_{\text{init}}^0(t) \rightarrow f_{CP})}, \quad (1.33)$$

where t is the time between production and decay of the state.

This time-dependent asymmetry can be shown to have the following dependence on $\lambda_{f_{CP}}$ and the mass difference, Δm_d .

$$a_{f_{CP}} = \frac{(1 - |\lambda_{f_{CP}}|^2) \cos(\Delta m_d t) - 2\Im(\lambda_{f_{CP}}) \sin(\Delta m_d t)}{1 + |\lambda_{f_{CP}}|^2} \quad (1.34)$$

The expression for $a_{f_{CP}}$ simplifies if CP symmetry is, to a good approximation, conserved separately in mixing and decay [19]:

$$a_{f_{CP}} = -\Im(\lambda_{f_{CP}}) \sin(\Delta m_d t). \quad (1.35)$$

1.2.4 Unitarity

The CKM matrix is unitary by construction, to preserve probability and (d', s', b') as a valid basis. If the CKM matrix were found not to be unitary this would be evidence for new physics, for example, a fourth generation of quarks mixing with the known three.

The unitarity of the CKM matrix gives rise to 9 relations between the elements.

$$V_{CKM}^\dagger V_{CKM} = \mathbb{1} \quad \implies \quad \sum_k V_{ik} V_{jk}^* = \begin{cases} 1, & \text{if } i = j \\ 0, & \text{if } i \neq j \end{cases} \quad (1.36)$$

In Equation 1.36, i and j run over up-type quark flavour and k runs over down-type quark flavour. There are 6 unitarity relations pertaining to the off-diagonal elements (when $i \neq j$).

Three of these relations are independent and the other three are their complex conjugate.

An $N \times N$ unitary matrix has N^2 real parameters: $N(N-1)/2$ moduli and $N(N+1)/2$ phases. However, in the case of the CKM matrix, many of these parameters become irrelevant because of the freedom to redefine quark fields. The $2N$ quark fields in the SM have $2N-1$ relative phases that are rendered unobservable by this freedom [21]. The number of free parameters is reduced to $(N-1)^2$: $N(N-1)/2$ moduli and $(N-1)(N-2)/2$ phases.

It is noted that if $N = 2$, then the number of phases is zero. This is why the CKM matrix was extended to three generations, to accommodate CP violation [18]. Since $N = 3$, the CKM matrix has four independent parameters, which in the standard parameterisation are expressed as three real angles and a complex phase. The standard parameterisation of the CKM matrix is given in Equation 1.37.

$$V_{CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \quad (1.37)$$

In Equation 1.37, $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ and θ_{12} , θ_{13} and θ_{23} are the three real angles. δ_{13} is the only complex phase and therefore the only source of CP violation in the CKM matrix. Indeed it is the only non-absorbable complex phase in the whole SM Lagrangian [22].

There are many other parameterisations of the CKM matrix. The Wolfenstein parameterisation [23] is an expansion in a small parameter, λ , which elucidates the relative sizes of the real and imaginary parts of the CKM elements. From this it is possible to discern how readily a transition occurs and how much CP violation is expected in a process involving that transition. The four Wolfenstein parameters are A , λ , ρ and η and the CKM matrix,

expanded to $\mathcal{O}(\lambda^3)$, is given by Equation 1.38.

$$V_{CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4) \quad (1.38)$$

It is noted that, to $\mathcal{O}(\lambda^3)$, the only CKM elements that have a non-zero imaginary part are V_{ub} and V_{td} . This means that, to this order, only $b \rightarrow u$ and $t \rightarrow d$ transitions exhibit CP violation.

The relations pertaining to the off-diagonal elements encapsulated in Equation 1.36 are written out explicitly in Equations 1.39 to 1.41. Since all these relations require three complex quantities to sum to zero, they can be represented, using the Wolfenstein parameters, as triangles in the complex ρ - η plane. The order of the terms in the Wolfenstein parameter, λ , is proportional to the lengths of the sides of these triangles. Figure 1.2 illustrates this graphically.

The unitarity triangles can be experimentally constrained by making measurements of physical processes that are sensitive to their sides or angles. The first two triangles can be constrained by measuring kaon decays and B_s decays, respectively. The shapes of these two triangles explain why CP violation in the kaon and B_s systems is challenging to measure. The short sides have significantly different phases to the two long sides, meaning that the processes related to them (*e.g.* $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the kaon triangle) could exhibit a large amount of CP violation. However, because of the short length of these sides, these processes are extremely rare [19].

The openness of the third unitarity triangle, which relates to processes involving $B^\pm$ and B_d mesons, means that much larger CP -violating effects are expected and hence Equation 1.41 is the most commonly tested³.

³The term “Unitarity Triangle” will refer to Equation 1.41 from now on.

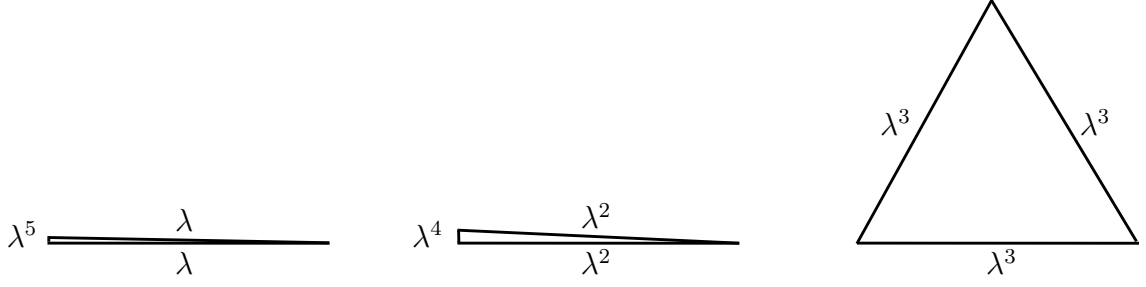


Figure 1.2: A graphical demonstration of the relative size of the sides of the triangles represented by the unitarity relations of the CKM matrix.

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \quad (1.39)$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \quad (1.40)$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (1.41)$$

Typically the Unitarity Triangle is plotted in the complex plane in coordinates normalised to $|V_{cd}V_{cb}^*|$, $(\bar{\rho}, \bar{\eta})$. A diagram of the Unitarity Triangle is shown in Figure 1.3, where the α , β and γ angles have been defined, $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ and $\alpha = \pi - \beta - \gamma$.

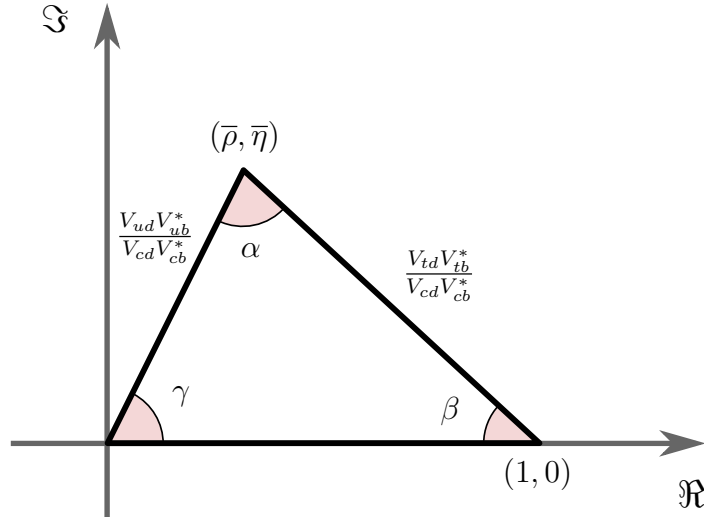


Figure 1.3: A diagram of the Unitarity Triangle.

The current experimental constraints on the Unitarity Triangle are shown in Figure 1.4. The red hatched region represents the region allowed, by the global fit to all measurements, for the apex of the triangle at a confidence level of 68.3%.

The constraints on the position of the apex of the triangle come from many different sources. The orange annulus represents the constraint from measurements of $B^0 - \bar{B}^0$ and $B_s^0 - \bar{B}_s^0$ mixing. The yellow ring is the constraint from $B^0 - \bar{B}^0$ mixing alone. The constraint on β comes largely from time-dependent analyses of $B_d \rightarrow J/\psi K_s^0$ decays that measure $\sin 2\beta$ from CP violation in the interference between mixing and decay. ϵ_K is a parameter measured in kaon mixing. α is measured in charmless B meson decays such as $B \rightarrow \rho\rho$, $B \rightarrow \pi\pi$ and $B \rightarrow \rho\pi^4$. The green annulus represents the constraint on $|V_{ub}|$. This region is split into two parts: that from semileptonic B_d decays (dark green hatched) and that from measurements of $|V_{ub}|$ from $B^\pm \rightarrow \tau^\pm \nu$ decays (light green hatched).

γ is the least well known of the angles. The uncertainty in γ is represented by the beige region in Figure 1.4. This large uncertainty motivates an accurate measurement of γ as it presents the most potential to detect a deviation from unitarity.

1.3 The Unitarity Triangle angle γ

1.3.1 Introduction

It is possible to measure all the angles of the Unitarity Triangle through analysing second order, or “loop”, processes. However, γ is unique in that it is the only angle that can be measured from a first order, or “tree”, process. A measurement of γ from a tree process provides the SM value of the parameter subject to very little theoretical uncertainty. This is because no heavy, undiscovered particles can propagate internally in a first order diagram. It is important to provide an SM measurement of γ to give a reference, against which effects from new physics in other measurements can be distinguished.

$X_b \rightarrow X_c X$ decays present a family of tree processes from which γ can be measured. Here, X_b represents a hadron containing a b quark, X_c a hadron containing a c quark and a u quark and X is another hadron or combination of hadrons. The crucial quality of these decays is

⁴Here B refers to either $B^\pm$ or B_d and all charge combinations of the ρ and π mesons are considered.

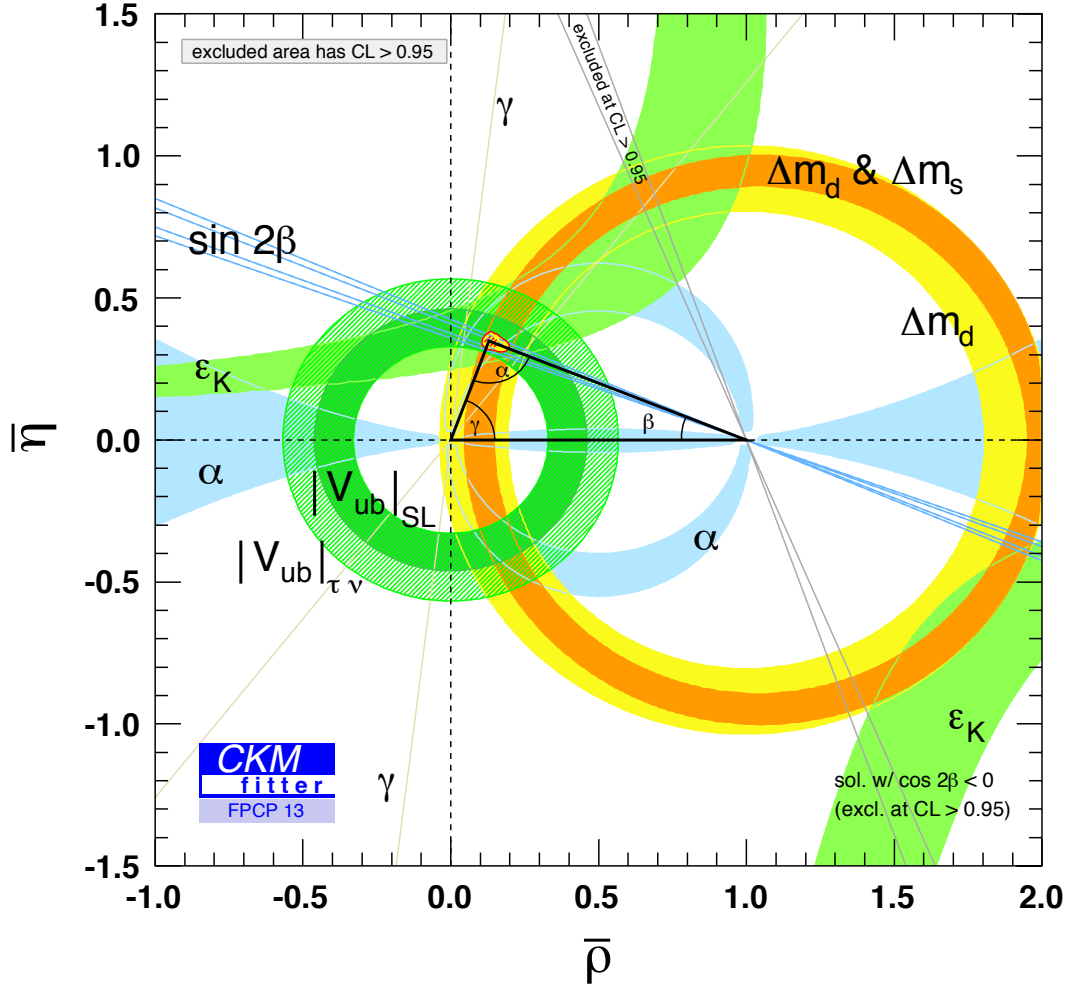


Figure 1.4: Constraints on the unitarity of the CKM matrix as of Summer 2013. The different contributions are explained in the text. Figure taken from Ref. [24].

that they can contain a $b \rightarrow u$ transition or a $b \rightarrow c$ transition, which have a weak phase difference of γ .

The type of CP violation that is used to measure γ in $X_b \rightarrow X_c X$ processes is CP violation in decay. The measurement of γ from tree decays presents several challenges. One is that $|V_{ub}|$ is small ($\mathcal{O}(\lambda^3)$) meaning that rare processes are needed. Also, the ratio of moduli of the interfering amplitudes can be small, which restricts the sensitivity to CP violation.

One decay that can be used is $B^\pm \rightarrow D(\rightarrow f_D) K^\pm$, where the neutral D meson is observed in a certain hadronic final state. The interference arises because the decay can

proceed via a D^0 or a $\bar{D}^0$ meson that both decay to the same final state, f_D . It is not possible to determine the D meson decay flavour, hence why ‘ D ’ is used to signify an admixture of D^0 and $\bar{D}^0$ mesons. The measurement of γ from $B^\pm \rightarrow D(\rightarrow f_D) K^\pm$ was first proposed in Ref. [25] and later in Refs. [26, 27]. Feynman diagrams of the two interfering decays are shown in Figure 1.5.

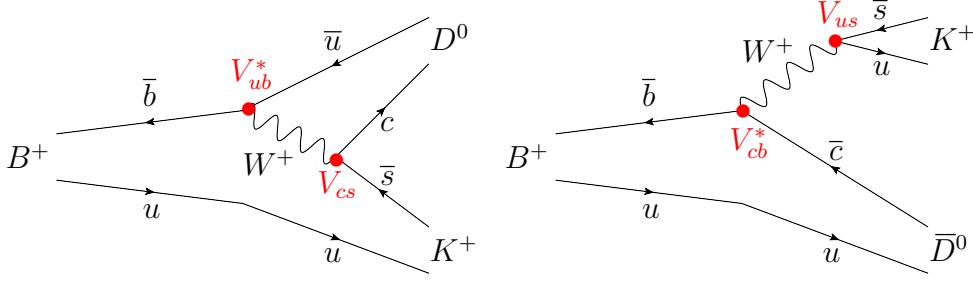


Figure 1.5: Feynman diagrams of $B^+ \rightarrow D^0 K^+$ (left) and $B^+ \rightarrow \bar{D}^0 K^+$ (right).

The Feynman diagrams in Figure 1.5 represent amplitudes that contain various CKM matrix elements as factors. The diagram on the left of Figure 1.5 has a $b \rightarrow u$ transition and the diagram on the right has a $b \rightarrow c$ transition. This means that the relative weak phase is γ and the measurement of, for example, a CP -violating asymmetry in the rates of $B^+ \rightarrow DK^+$ and $B^- \rightarrow DK^-$ decays is sensitive to this phase.

The CKM factors in the amplitudes also have differing sizes, which affect their ratio. As well as the relative Cabibbo-suppression, the diagram on the left of Figure 1.5 is colour-suppressed relative to the favoured decay because the W^+ is internal. This results in a ratio of moduli of the amplitudes corresponding to $B^\pm \rightarrow D^0 K^\pm$ and $B^\pm \rightarrow \bar{D}^0 K^\pm$ decays of $r_{B^\pm} = 0.089^{+0.008}_{-0.009}$ [28].

Measurements can be made with various D decay final states. CP eigenstates, *e.g.* $K^+ K^-$ or $\pi^+ \pi^-$, were first suggested in Ref. [25] and this was extended in Refs. [26, 27] to CP non-eigenstates, *e.g.* $K^\pm \pi^\mp$. In the case of a CP non-eigenstate, the D^0 and $\bar{D}^0$ decays do not have the same amplitude. Typically one is Cabibbo-suppressed relative to the other. The relative Cabibbo-suppression of the neutral D meson decays counteracts the relative colour-suppression of the $B^\pm$ decays and the total interfering amplitudes are brought closer in magnitude, enhancing the potential sensitivity.

It is also possible to perform an analysis of $B^\pm \rightarrow DK^\pm$ with multi-body D decays, for example, $D \rightarrow K_S^0 \pi^+ \pi^-$ [29]. The amount of CP violation varies across to the kinematic parameter space of the multi-body decay, due to a varying value of the strong phase difference and, in general, an analysis of multi-body decays must take this variation into account.

The current, world's most precise, measurement of γ from a single experiment comes from a combination of analyses of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays, that use many different final states of the D meson, at LHCb [28]:

$$\gamma = (72.6^{+9.7}_{-17.6})^\circ \text{ at } 68.3\% \text{ C.L.}$$

This is combined with results from the BaBar and Belle experiments to give a world average compatible with the LHCb result [24]:

$$\gamma = (68^{+8.0}_{-8.5})^\circ \text{ at } 68.3\% \text{ C.L.}$$

The constraints on γ from BaBar, Belle, LHCb and their combination are summarised in Figure 1.6.

1.3.2 Measurement using $B \rightarrow DK^*(892)$ decays

The same methods used with $B^\pm$ meson decays can be employed using appropriate neutral B meson decays, such as $B \rightarrow DK^*(892)$ decays, which are the main topic of this thesis. $B \rightarrow DK^*$ decays⁵ have previously been studied in Refs. [30] and [31] and by the author in Ref. [3]. In contrast to $B^\pm$ meson decays, $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays are both colour-suppressed, which means the corresponding interfering amplitudes are more similar in magnitude, thus increasing the potential size of the CP -violating rate asymmetry.

$B \rightarrow DK^*$ decays differ from many other neutral B meson decays in that it is possible

⁵Here and after, K^{*0} refers to $K^*(892)^0$, $\bar{K}^{*0}$ to $\bar{K}^*(892)^0$ and K^* refers to both $K^*(892)^0$ and $\bar{K}^*(892)^0$ mesons, unless otherwise stated.

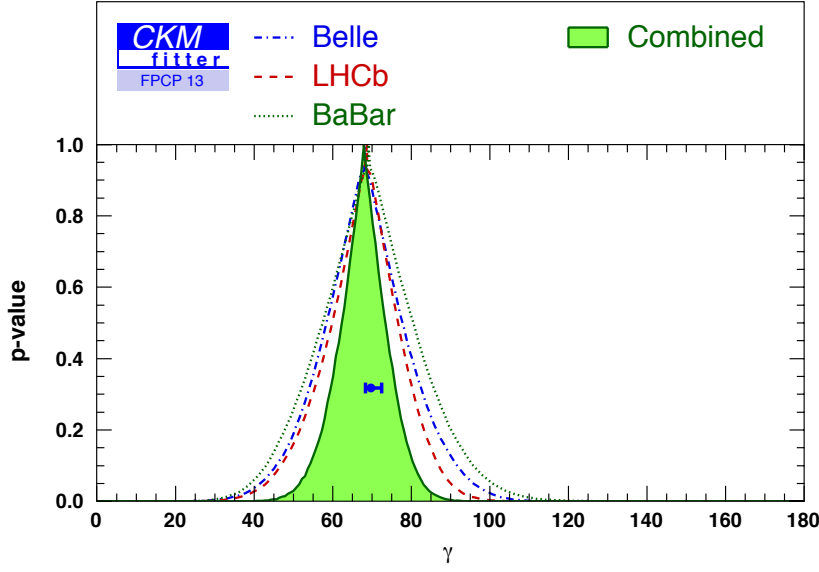


Figure 1.6: The current constraints on γ made by analyses at BaBar, Belle, LHCb and the constraint made by the combination of all these results. The point is the result of a global fit to all measurements. Figure taken from Ref. [24].

to unequivocally determine the decay flavour of the B meson, when it decays to D and K^* mesons. This is possible when the K^* meson is reconstructed as $K^* \rightarrow K^\pm \pi^\mp$. The charge of the kaon in the final state of the K^* is directly related to the charge of the b quark in the initial state. This “self-tagging” quality of the $B \rightarrow DK^*$ decay means that a time-integrated analysis is possible. This is in contrast to most analyses of neutral mesons, where production flavour, mixing and decay time must be taken into account to determine decay flavour. The Feynman diagrams of $B^0 \rightarrow D^0(\bar{D}^0)K^{*0}$ decays are shown in Figure 1.7. The focus of the following discussion is on B_d decays, however much of it is applicable to B_s decays and these are revisited later.

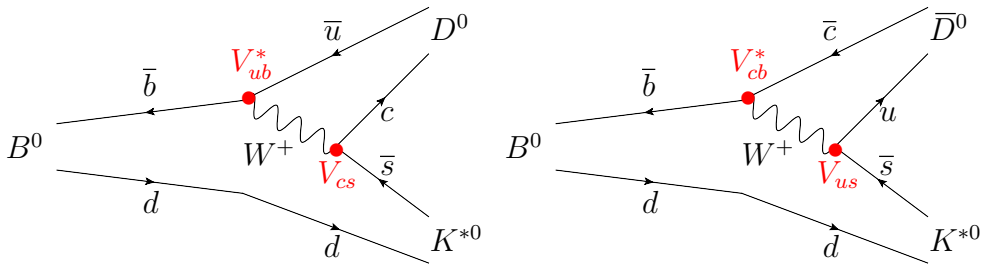


Figure 1.7: Feynman diagrams of $B^0 \rightarrow D^0 K^{*0}$ (left) and $B^0 \rightarrow \bar{D}^0 K^{*0}$ (right) decays.

In this thesis the two-body D decays, $D \rightarrow K^\pm \pi^\mp$, $D \rightarrow K^+ K^-$ and $D \rightarrow \pi^+ \pi^-$, are considered. As in the case of $B^\pm$ mesons, the analysis can be performed with many different final states of the D meson. However, these four final states are the only ones that it is practical to analyse, given the current size of the dataset and the capabilities of the detector. The Feynman diagrams of D^0 and $\bar{D}^0$ decays to $K^\pm \pi^\mp$, $K^+ K^-$ and $\pi^+ \pi^-$ are shown in Figure 1.8.

Formalism

No explicit calculation of the amplitudes corresponding to the decays being studied is needed as only relative properties of the amplitudes are of interest. The amplitudes of $B^0 \rightarrow DK^{*0}$ decays are expressed in terms of their magnitude, weak and strong phase differences:

$$A(B^0 \rightarrow \bar{D}^0 K^{*0}) = A_B, \quad A(B^0 \rightarrow D^0 K^{*0}) = A_B r_B e^{i(\delta_B + \gamma)}. \quad (1.42)$$

In Equation 1.42, A_B is the amplitude of the $B^0 \rightarrow \bar{D}^0 K^{*0}$ decay, r_B is the ratio of the magnitudes of the $B^0 \rightarrow D^0 K^{*0}$ to the $B^0 \rightarrow \bar{D}^0 K^{*0}$ amplitudes, δ_B is their relative strong phase and γ is their relative weak phase arising from the different CKM elements indicated in Figure 1.7. A naive prediction can be made about the value of r_B . Given the colour-suppression of both the $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decay amplitudes and the absence of colour-suppression in one of the $B^\pm \rightarrow DK^\pm$ decay amplitudes, the value of r_B is expected to be three times larger than the value of $r_{B^\pm}$. This argument leads to a prediction of $r_B \approx 0.27$. The only existing measurement of this ratio, apart from the one presented in Chapter 5, is $r_B \in [0.07, 0.41]$ at a confidence level of 95% [31].

The following amplitudes of the D^0 and $\bar{D}^0$ mesons decaying to two-body hadronic final states are also defined.

$$A(\bar{D}^0 \rightarrow f_D) = A(D^0 \rightarrow \bar{f}_D) = A_D \quad (1.43)$$

$$A(D^0 \rightarrow f_D) = A(\bar{D}^0 \rightarrow \bar{f}_D) = A_D r_D e^{i\delta_D} \quad (1.44)$$

In Equations 1.43 and 1.44, f_D represents one of $K^+\pi^-$, $K^-\pi^+$, K^+K^- and $\pi^+\pi^-$, and $\overline{f_D}$ is the CP conjugate of f_D . The parameters r_D and δ_D are the relative magnitude and strong phase of the suppressed and favoured amplitudes. r_D and δ_D depend on the final state, f_D .

The amplitude of the $B^0 \rightarrow f_D K^{*0}$ decay has two terms and is given by

$$\begin{aligned} A(B^0 \rightarrow f_D K^{*0}) &= A(B^0 \rightarrow D^0 K^{*0}) A(D^0 \rightarrow f_D) + \\ &A(B^0 \rightarrow \overline{D}^0 K^{*0}) A(\overline{D}^0 \rightarrow f_D). \end{aligned} \quad (1.45)$$

Using Equations 1.42, 1.43 and 1.44, this can be rewritten as follows,

$$A(B^0 \rightarrow f_D K^{*0}) = A_{Bf} = A_B A_D [r_D e^{i(\delta_B + \gamma)} r_D e^{i\delta_D} + 1]. \quad (1.46)$$

With two possible initial states (B^0 or $\overline{B}^0$) and two possible D final states (f_D and $\overline{f_D}$) there are three more decay chains to consider with the following amplitudes:

$$A_{B\overline{f}} = A(B^0 \rightarrow \overline{f_D} K^{*0}) = A_B A_D [r_D e^{i\delta_D} + r_B e^{i(\delta_B + \gamma)}], \quad (1.47)$$

$$\overline{A}_{Bf} = A(\overline{B}^0 \rightarrow f_D \overline{K}^{*0}) = A_B A_D [r_D e^{i\delta_D} + r_B e^{i(\delta_B - \gamma)}], \quad (1.48)$$

$$\overline{A}_{B\overline{f}} = A(\overline{B}^0 \rightarrow \overline{f_D} \overline{K}^{*0}) = A_B A_D [r_B e^{i(\delta_B - \gamma)} r_D e^{i\delta_D} + 1]. \quad (1.49)$$

Mixing and CP violation in the decay of the D^0 and $\overline{D}^0$ mesons have been neglected in these equations. This is reasonable since there is no conclusive evidence for CP violation in $D \rightarrow K^\pm \pi^\mp$, $D \rightarrow K^+ K^-$ or $D \rightarrow \pi^+ \pi^-$ decays and $D^0 - \overline{D}^0$ mixing occurs with a period of oscillation much longer than the D^0 meson lifetime [32, 33].

When the D meson final state is $K^+ K^-$ or $\pi^+ \pi^-$ ⁶, Equations 1.46 and 1.48 become degenerate with 1.47 and 1.49, respectively, since f_D is a CP -even eigenstate. This means that $f_D = \overline{f_D}$, $r_D = 1$ and $\delta_D = 0$.

⁶Needless indicators of charge on the $K^+ K^-$ and $\pi^+ \pi^-$ final states are henceforth omitted.

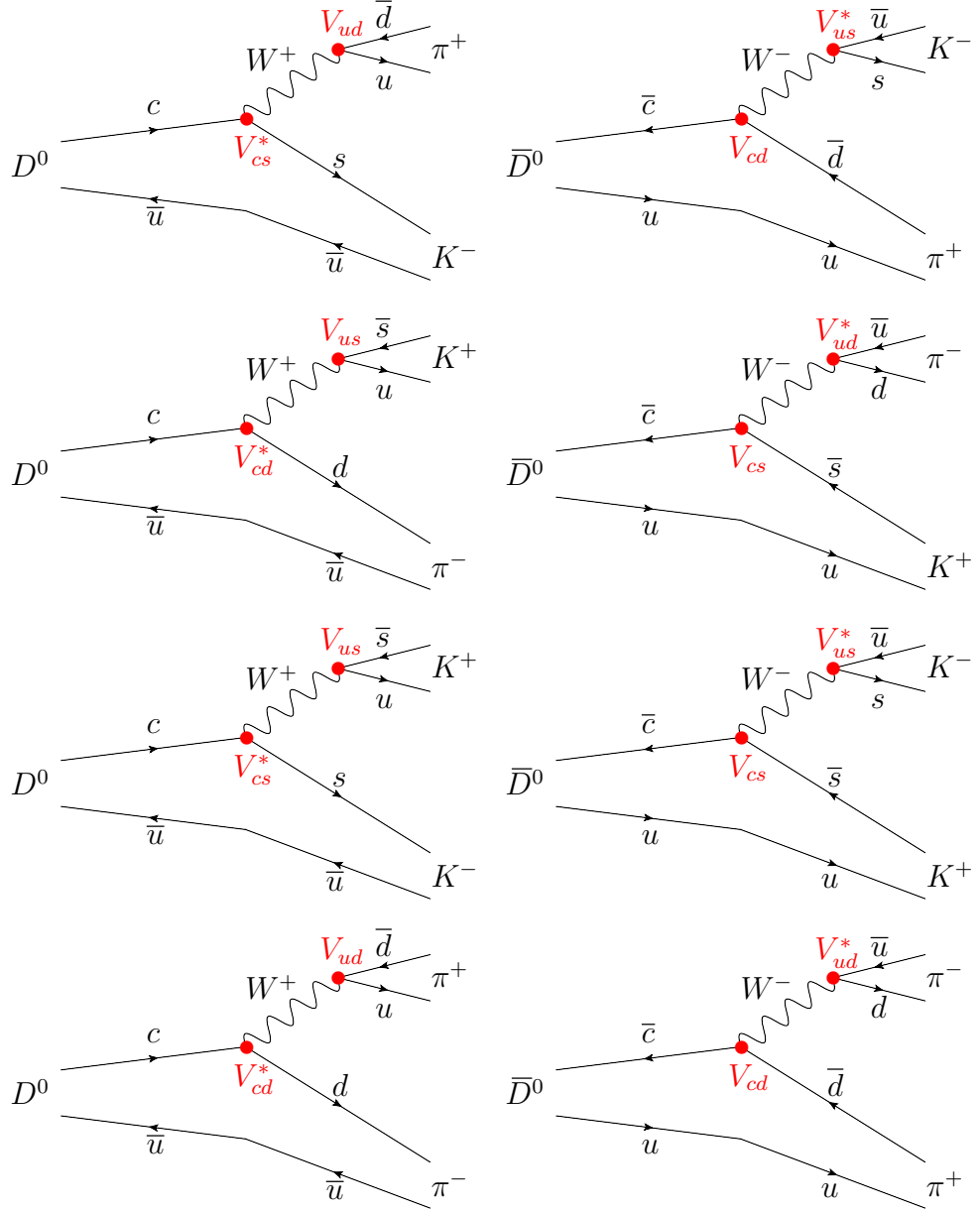


Figure 1.8: Feynman diagrams of (from top to bottom) $D^0(\bar{D}^0) \rightarrow K^- \pi^+$, $D^0(\bar{D}^0) \rightarrow K^+ \pi^-$, $D^0(\bar{D}^0) \rightarrow K^+ K^-$ and $D^0(\bar{D}^0) \rightarrow \pi^+ \pi^-$ decays.

Also, it is noted that $K^+\pi^-$ is the CP conjugate of $K^-\pi^+$, so when considering these final states, only four separate relations are gleaned from Equations 1.46 to 1.49 because of further degeneracies. Two amplitudes, corresponding to $D \rightarrow K^\pm\pi^\mp$ decays, given by Equations 1.46 and 1.49, are favoured. These amplitudes are characterised by kaons (from the D and the K^*) of the same electric charge in the final state and, for brevity, are henceforth indicated by $B^0 \rightarrow D(K\pi)K^{*0}$ and $\bar{B}^0 \rightarrow D(K\pi)\bar{K}^{*0}$. Two amplitudes, corresponding to $D \rightarrow K^\pm\pi^\mp$ decays, given by Equations 1.47 and 1.48, are suppressed. These amplitudes are characterised by kaons of opposite charge in the final state and are henceforth indicated by $B^0 \rightarrow D(\pi K)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$.

The parameters relating to the $D \rightarrow K^\pm\pi^\mp$ decay have been measured. R_D is the ratio of partial widths of the $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ decays and is approximately the square of r_D . $R_D = (3.568 \pm 0.066) \times 10^{-3}$ [32], which implies that $r_D \approx 0.06$. The strong phase difference is also known, $\delta_D = (-130^{+38}_{-28})^\circ$ [34].

In summary, there are 8 separate amplitudes, corresponding to B_d decays, with the four D final states. The time-integrated decay rates are proportional to the squared magnitudes of the amplitudes, which are given by

$$|A_{Bf}|^2 = |A_B|^2 |A_D|^2 \times [1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B + \delta_D + \gamma)], \quad (1.50)$$

$$|A_{B\bar{f}}|^2 = |A_B|^2 |A_D|^2 \times [r_B^2 + r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D + \gamma)], \quad (1.51)$$

$$|\bar{A}_{Bf}|^2 = |A_B|^2 |A_D|^2 \times [r_B^2 + r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D - \gamma)], \quad (1.52)$$

$$|\bar{A}_{B\bar{f}}|^2 = |A_B|^2 |A_D|^2 \times [1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B + \delta_D - \gamma)]. \quad (1.53)$$

where a coherence factor, κ , has been introduced. This factor arises because the magnitudes and complex phases of the two interfering amplitudes depend on the resonance structure of the three-body $B_d \rightarrow DK\pi$ decay, which varies from one point in the kinematic parameter space of the decay to another [35]. This causes a loss in sensitivity to the interference term containing the interesting weak phase, γ .

The coherence factor, κ , is given by Equation 1.54, where $A(p)$ and $\bar{A}(p)$ are the amplitudes

of the $B_d \rightarrow D^0 K \pi$ and $B_d \rightarrow \bar{D}^0 K \pi$ decays as a function of the point, p , in the parameter space, respectively. It is clear from Equation 1.54 that κ is maximised when $A(p)$ and $\bar{A}(p)$ are parallel to, or coherent with, each other.

$$\kappa e^{i\delta_B} \equiv \frac{\int dp A(p) \bar{A}(p)^*}{\sqrt{\int dp |A(p)|^2 \int dp |\bar{A}(p)|^2}} \quad (1.54)$$

The region of parameter space that is analysed can be chosen such that the variation in A and $\bar{A}$ is small and the coherence high. This is expected to be the case in the region of the $K^*(892)$ resonance. The coherence factor in this region has been determined using a Monte Carlo simulation method to be 0.95 ± 0.03 [36]. Therefore the sensitivity that is lost, due to $\kappa < 1$, is small compared with the statistical precision gained by integrating over the parameter space of the multi-body decay.

The 8 squared amplitudes encapsulated in Equations 1.50 to 1.53 correspond to 8 B^0 or $\bar{B}^0$ decays, the rates of which can be measured. These rates are related to the partial widths of the associated decays. Interesting and experimentally robust observables can be composed from the 8 partial widths, that do not depend on absolute efficiencies and are simply related to the parameters of interest (r_B , δ_B , γ , r_D , δ_D and κ).

The two asymmetries between the partial widths of $\bar{B}^0 \rightarrow D(hh)\bar{K}^{*0}$ and $B^0 \rightarrow D(hh)K^{*0}$ decays (the “ CP asymmetry”), $\mathcal{A}_d^{hh}$, where $hh = KK$ or $\pi\pi$, are considered.

$$\mathcal{A}_d^{hh} \equiv \frac{\Gamma(\bar{B}^0 \rightarrow D(hh)\bar{K}^{*0}) - \Gamma(B^0 \rightarrow D(hh)K^{*0})}{\Gamma(\bar{B}^0 \rightarrow D(hh)\bar{K}^{*0}) + \Gamma(B^0 \rightarrow D(hh)K^{*0})} \quad (1.55)$$

The flavour-averaged partial rate of the $B_d \rightarrow D_{CP+} K^*$ decay, $\mathcal{R}_{CP+}$, is also considered, where D_{CP+} means any CP -even final state of the D meson. An inclusive measurement is not possible, so this observable is approximated using the partial widths of the favoured $B_d \rightarrow D(K\pi)K^*$ decays and the $B_d \rightarrow D(hh)K^*$ decays. The approximation is due to the contributions of doubly Cabibbo-suppressed decays of $D^0 \rightarrow K^+\pi^-$ and $\bar{D}^0 \rightarrow K^-\pi^+$ in the

denominator that cannot be distinguished experimentally.

$$\mathcal{R}_{CP+} \equiv 2 \times \frac{\Gamma(B^0 \rightarrow D_{CP+} K^{*0}) + \Gamma(\bar{B}^0 \rightarrow D_{CP+} \bar{K}^{*0})}{\Gamma(B^0 \rightarrow \bar{D}^0 K^{*0}) + \Gamma(\bar{B}^0 \rightarrow D^0 \bar{K}^{*0})} \approx \mathcal{R}_d^{hh} \quad (1.56)$$

$$\mathcal{R}_d^{hh} \equiv \frac{\Gamma(B^0 \rightarrow D(hh) K^{*0}) + \Gamma(\bar{B}^0 \rightarrow D(hh) \bar{K}^{*0})}{\Gamma(B^0 \rightarrow D(K\pi) K^{*0}) + \Gamma(\bar{B}^0 \rightarrow D(K\pi) \bar{K}^{*0})} \times \frac{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}{\mathcal{B}(D \rightarrow hh)} \quad (1.57)$$

The CP asymmetry between the favoured $\bar{B}^0 \rightarrow D(K\pi) \bar{K}^{*0}$ and $B^0 \rightarrow D(K\pi) K^{*0}$ decays, $\mathcal{A}_d^{K\pi}$, is also sensitive to the parameters of interest and is given by

$$\mathcal{A}_d^{K\pi} \equiv \frac{\Gamma(\bar{B}^0 \rightarrow D(K\pi) \bar{K}^{*0}) - \Gamma(B^0 \rightarrow D(K\pi) K^{*0})}{\Gamma(\bar{B}^0 \rightarrow D(K\pi) \bar{K}^{*0}) + \Gamma(B^0 \rightarrow D(K\pi) K^{*0})}. \quad (1.58)$$

Two other observables can be defined, which are systematically robust (free from correction factors from external inputs) and statistically independent. These are $\mathcal{R}_d^+$ and $\mathcal{R}_d^-$, which are given by Equations 1.59 and 1.60.

$$\mathcal{R}_d^+ \equiv \frac{\Gamma(B^0 \rightarrow D(\pi K) K^{*0})}{\Gamma(B^0 \rightarrow D(K\pi) K^{*0})} \quad (1.59)$$

$$\mathcal{R}_d^- \equiv \frac{\Gamma(\bar{B}^0 \rightarrow D(\pi K) \bar{K}^{*0})}{\Gamma(\bar{B}^0 \rightarrow D(K\pi) \bar{K}^{*0})} \quad (1.60)$$

An analogous asymmetry and flavour-averaged partial rate using suppressed $B_d \rightarrow D(\pi K) K^*$ decays, to those constructed for the KK , $\pi\pi$ and favoured $K\pi$ final states of the D , could be constructed. However, $\mathcal{R}_d^+$ and $\mathcal{R}_d^-$ are preferred because of their independence of each other and lower correlation to the $\mathcal{A}_d^{hh}$, $\mathcal{R}_d^{hh}$ and $\mathcal{A}_d^{K\pi}$ observables.

Using Equations 1.50 to 1.53, the following relations between the observables and the parameters of interest can be derived.

$$\mathcal{A}_d^{hh} = \frac{2r_B \kappa \sin \delta_B \sin \gamma}{1 + r_B^2 + 2r_B \kappa \cos \delta_B \cos \gamma} \quad (1.61)$$

$$\mathcal{R}_d^{hh} = \frac{1 + r_B^2 + 2r_B \kappa \cos \delta_B \cos \gamma}{1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D) \cos \gamma} \quad (1.62)$$

$$\mathcal{A}_d^{K\pi} = \frac{2r_B r_D \kappa \sin(\delta_B - \delta_D) \sin \gamma}{1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D) \cos \gamma} \quad (1.63)$$

$$\mathcal{R}_d^\pm = \frac{r_B^2 + r_D^2 + 2r_B r_D \kappa \cos(\delta_B + \delta_D \pm \gamma)}{1 + r_B^2 r_D^2 + 2r_B r_D \kappa \cos(\delta_B - \delta_D \pm \gamma)} \quad (1.64)$$

From these observables, the measurements of parameters relating to the D meson decays and the determination of κ in Ref. [36], γ , r_B and δ_B can be determined.

In addition to the observables related to B_d decays, some more are defined involving $B_s \rightarrow DK^*$ decays, that are not expected to be sensitive to CP violation with the current size of the dataset but will become important in the future.

The analog of $\mathcal{A}_d^{hh}$, with $B_s \rightarrow DK^*$ partial widths, $\mathcal{A}_s^{hh}$, is considered.

$$\mathcal{A}_s^{hh} = \frac{\Gamma(\bar{B}_s^0 \rightarrow D(hh)K^{*0}) - \Gamma(B_s^0 \rightarrow D(hh)\bar{K}^{*0})}{\Gamma(\bar{B}_s^0 \rightarrow D(hh)K^{*0}) + \Gamma(B_s^0 \rightarrow D(hh)\bar{K}^{*0})} \quad (1.65)$$

The CP asymmetry between the $\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}$ and $B_s^0 \rightarrow D(\pi K)\bar{K}^{*0}$ decays, $\mathcal{A}_s^{\pi K}$, is also defined. This is the favoured decay mode of the B_s meson.

$$\mathcal{A}_s^{\pi K} = \frac{\Gamma(\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}) - \Gamma(B_s^0 \rightarrow D(\pi K)\bar{K}^{*0})}{\Gamma(\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}) + \Gamma(B_s^0 \rightarrow D(\pi K)\bar{K}^{*0})} \quad (1.66)$$

Finally, the ratio of the flavour-averaged partial rates of the $B_d \rightarrow D(hh)K^*$ and $B_s \rightarrow$

$D(hh)K^*$ decays is also considered.

$$\mathcal{R}_{ds}^{hh} = \frac{\Gamma(B^0 \rightarrow D(hh)K^{*0}) + \Gamma(\bar{B}^0 \rightarrow D(hh)\bar{K}^{*0})}{\Gamma(B_s^0 \rightarrow D(hh)\bar{K}^{*0}) + \Gamma(\bar{B}_s^0 \rightarrow D(hh)K^{*0})} \quad (1.67)$$

$\mathcal{A}_s^{hh}$ and $\mathcal{A}_s^{\pi K}$ are related to the parameters of interest in a similar way to $\mathcal{A}_d^{hh}$ and $\mathcal{A}_d^{K\pi}$, as in Equations 1.55 and 1.58. However, the parameters relating to the B_d decay are substituted for parameters of the B_s decay. No measurement of r_{B_s} has been made, but an estimate can be made from the CKM matrix elements in the B_s decay amplitudes of $r_{B_s} \approx 0.02$. This small value is the reason for these quantities' lack of sensitivity with the current dataset.

Measurements of $\mathcal{R}_{ds}^{hh}$ represent a possible route to a determination of $\mathcal{R}_d^{hh}$, where $\mathcal{R}_d^{hh}$ is approximated as $\mathcal{R}_{ds}^{hh}/\mathcal{R}_{ds}^{K\pi}$, that could have smaller systematic uncertainty. However, this is not investigated in this thesis.

1.4 Theoretical summary

The Standard Model of particle physics is a non-abelian, locally gauge-invariant theory of the interactions of the known fundamental particles. The CKM matrix is the part of the Standard Model Lagrangian that describes the mixing of the quarks via interaction with the $W^\pm$ boson. Complex elements of the CKM matrix, indeed any complex coupling constants, violate CP symmetry and the study of one of the relative complex phases of the CKM elements, γ , is a central topic of this thesis.

The angle γ is the least well known phase in the CKM matrix and from an analysis of $B \rightarrow DK^*$ decays its measurement is possible. $B_d \rightarrow DK^*$ decays are particularly suited to the study of γ because of the large predicted ratio of interfering amplitudes involved in the decay. This ratio, r_B , is predicted to be approximately 0.27. Several quantities are defined in terms of partial widths of $B \rightarrow DK^*$ decays that are expected to be sensitive now, or in the future, to γ , r_B and δ_B and these quantities are measured in Chapter 4.

Chapter 2

The LHC and LHCb

The Large Hadron Collider is described. LHCb is a particle physics experiment, designed primarily for measurements involving b and c hadrons, and is situated at one of the interaction points on the Large Hadron Collider. A technical overview of the LHCb detector is given, with a detailed description of the ring imaging Cherenkov subdetectors and the calibration of their response.

2.1 The Large Hadron Collider

The Large Hadron Collider (LHC) is primarily a proton-proton collider and is the main experimental facility at the headquarters of the European Organisation for Nuclear Research, or “Conseil Européen pour la Recherche Nucléaire” (CERN). The LHC measures 27km in circumference and is situated 100m below the Swiss-French border, in the vicinity of Geneva, Switzerland. The main components of the LHC are 1232 superconducting dipole magnets, which guide the protons around the curved sections of the collider, and 16 radiofrequency cavities, which accelerate the protons to close to the speed of light. The design centre-of-mass energy and luminosity of the LHC are 14 TeV and $10^{34} \text{ cm}^{-2}\text{s}^{-1}$, respectively.

The protons are accelerated via a series of different systems before being injected into the LHC. First hydrogen atoms are stripped of their electrons and are accelerated by Linac2, at the end of which they are injected into the Proton Synchrotron Booster (PSB) at an energy of 50 MeV. The PSB then accelerates the protons to 1.4 GeV before they are injected into the larger Proton Synchrotron (PS), where they are accelerated to 26 GeV. The last

stage in the accelerator complex is the SPS, which accelerates the protons to 450 GeV before injecting them into the LHC (in both directions). The LHC then accelerates the protons to collision energy before stable beams are established and collisions and data collection begin. A diagram of the accelerator complex is shown in Figure 2.1.

Protons were first successfully circulated in the LHC on 10th September 2008. However, 9 days later, an electrical fault led to a quench of one of the superconducting magnets, causing severe damage to the accelerator complex and contamination of the vacuum in the beampipe. This delayed operation by 14 months. Proton beams were successfully circulated again on November 20th 2009 with the first proton-proton collisions taking place three days later at a centre-of-mass energy of 900 GeV (450 GeV per beam). On March 30th 2010, the first collisions at a centre-of-mass energy of 7 TeV were recorded.

The LHC operated at 7 TeV in 2010 and 2011 and 8 TeV in 2012 before shutting down in 2013 to allow upgrade work necessary for a centre-of-mass energy of 13 TeV when it is switched on again in 2015.

The LHC exceeded luminosity delivery targets for the 2012 run, achieving this by spending over 36% of the time with stable beams, an improvement on 32% in 2011 [37]. Details of the LHC running time and the delivered luminosity to the experiments are given in Table 2.1.

The beams are collided at four different points on the LHC ring. The LHCb experiment is situated at one of these interaction points, between the CMS and ATLAS experiments.

2.2 The LHCb detector

2.2.1 Introduction

LHCb is a single-arm forward spectrometer designed to study physics in the forward region where the $b\bar{b}$ production cross-section is high. The design of LHCb is primarily motivated by the desire to study the physics of b quarks, however the objectives of LHCb also include charm physics and electroweak measurements. LHCb has a forward angular coverage from

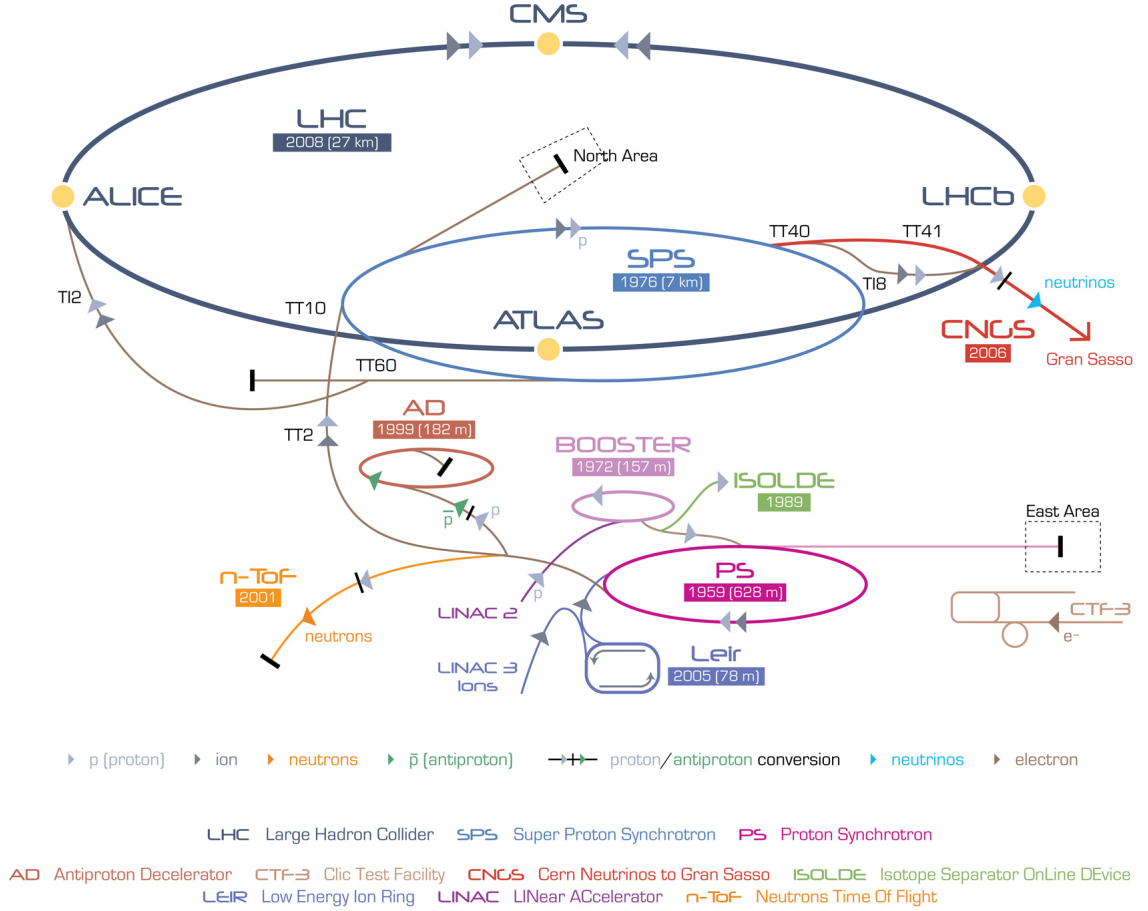


Figure 2.1: A diagram of the CERN accelerator complex. Figure taken from Ref. [38].

Table 2.1: The percentage of LHC running time spent in the various states of the LHC (*top*). “Access” refers to any period when repairs or similar are being carried out. “Setup” refers to configurations of the accelerator prior to protons being accelerated. “Ramp” refers to the period when the beam energy is being slowly increased from injection to collision energy. “Squeeze” refers to the period when beam parameters are finely tuned prior to collisions. “Injection” and “Physics” are self explanatory. The integrated luminosity delivered to each experiment is also shown (*bottom*) [37].

LHC running time						
LHC state	Access	Setup	Injection	Ramp	Squeeze	Physics
% running time	13.8%	27.6%	15.0%	2.1%	5.0%	36.5%
Stable beam time = 73.2 days. Total time = 200.5 days.						
Delivered Luminosity						
Experiment	ATLAS	CMS	LHCb	ALICE		
Integrated Luminosity	23.27 fb ⁻¹	23.27 fb ⁻¹	2.19 fb ⁻¹	9.96 pb ⁻¹		

approximately 10 mrad to 300(250) mrad in the bending (non-bending) plane of its dipole magnet [39].

The design luminosity of LHCb is $2 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ [39], two orders of magnitude lower than the maximum capacity of the LHC. The maximum instantaneous luminosity recorded in LHCb was in fact $1.005 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}$, late in the 2012 data collection period. However the instantaneous luminosity during 2012 mostly resided between 4×10^{32} and $5 \times 10^{32} \text{ cm}^{-2}\text{s}^{-1}$ [40].

Lower luminosity than the other LHC experiments eases the reconstruction of tracks in the high multiplicity forward region and is advantageous for B physics [41]. This lower luminosity is achieved by separating the beams in the interaction region. The luminosity is kept constant over an entire LHC fill by steadily moving the beams closer to one another as the instantaneous luminosity of the beam decreases due to other losses, a process known as “luminosity levelling”. The instantaneous luminosity as a function of time during a single LHC fill is shown in Figure 2.2.

The main subdetectors of LHCb include the silicon vertex locator (VELO), which is closest to the interaction point. The other subdetectors situated between the interaction point and the magnet are the first Ring Imaging Cherenkov detector (RICH1) and the TT tracking detectors.

LHCb has a warm dipole magnet with reversible polarity. Any potential bias due to detection asymmetries is mitigated by averaging the results obtained from data collected with different magnet polarities. Therefore, the polarity is regularly reversed during data collection periods.

Downstream of the magnet are the Inner Tracker (IT), Outer Tracker (OT), the second RICH detector (RICH2), the calorimeters and the muon detectors. A diagram of the detector is shown in Figure 2.3.

LHCb uses a right-handed Cartesian coordinate system. The origin of the coordinate system is at the nominal interaction point, inside the VELO. The z -axis runs through the beampipe, pointing towards the muon detectors. The y -axis points vertically upwards and the x -axis points away from the centre of the LHC ring.

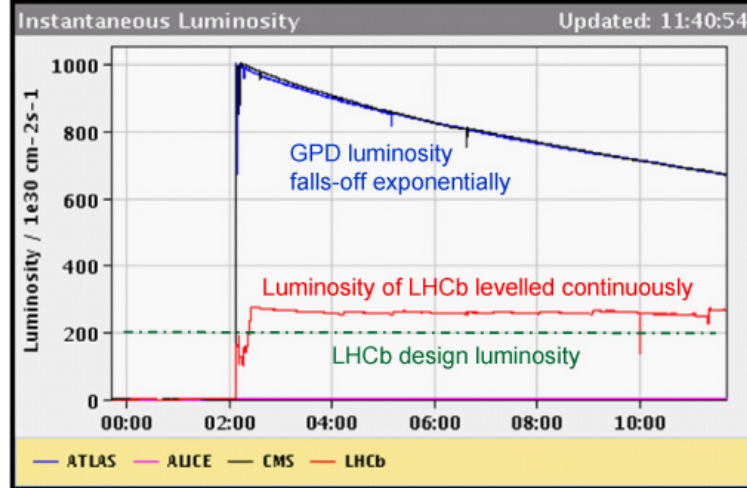


Figure 2.2: Instantaneous luminosity as a function of time during a single LHC fill. Figure taken from Ref. [42].

2.2.2 The VELO

The VErteX LOcator is the subdetector that is positioned closest to the interaction region in LHCb. It provides precise measurements of the displaced decay vertices of D and B mesons, which are crucial for their identification. The VELO consists of a series of semicircular modules of silicon detectors. Each module consists of two layers of silicon microstrips: The “R-sensor” reads the r coordinate (the strips are arranged concentrically) and the “ ϕ -sensor” reads the orthogonal ϕ coordinate (the strips are arranged radially). Where r and ϕ are related to the LHCb Cartesian coordinates by Equation 2.1.

$$r = \sqrt{x^2 + y^2} \qquad \phi = \arctan\left(\frac{x}{y}\right) \qquad (2.1)$$

There are 2048 active channels per sensor. Each half of the VELO consists of 21 two-layer modules. There are also two extra R-sensors in each half that provide a measurement of the number of primary vertices in an event and aid the LHCb data acquisition system in vetoing against those with more than one [39]. These are “pileup” events. A diagram of the VELO is shown in Figure 2.4 with a plot of the spatial positions of material interaction vertices in the x - z plane, which illustrates the position of the VELO when taking data.

The two halves of the VELO are separated from the vacuum of the beampipe by an

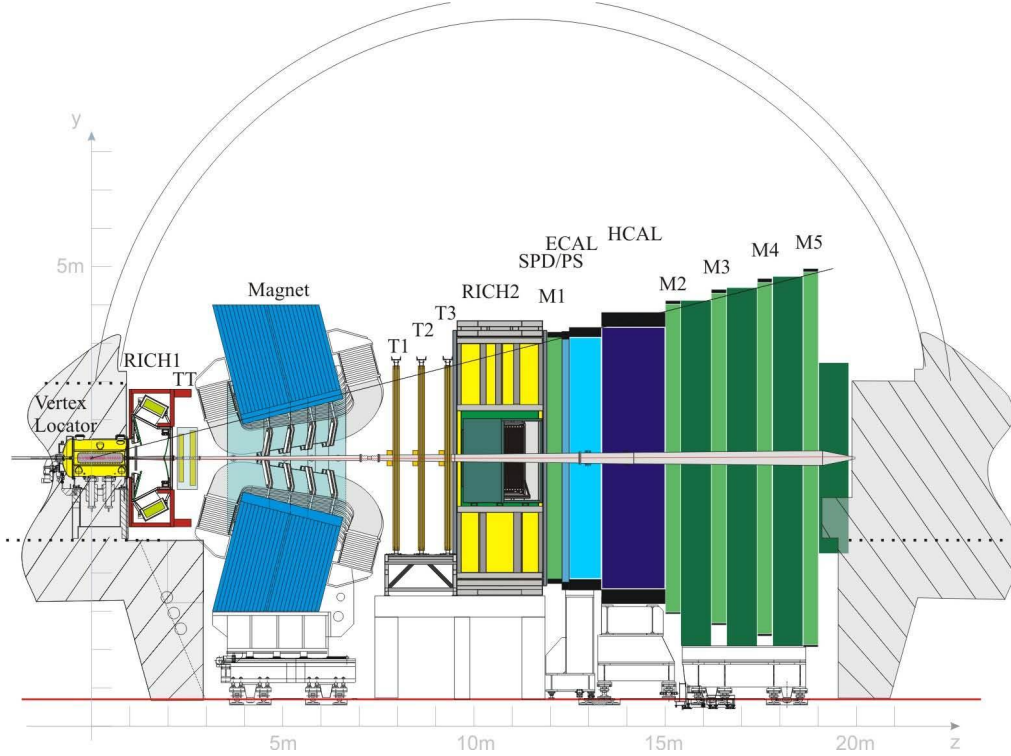


Figure 2.3: A diagram of the LHCb detector in the non-bending plane. Figure taken from Ref. [43].

aluminium alloy foil because the beampipe is kept at a higher quality of vacuum than the rest of the detector. The shape and position of this foil is evident from the plot of interaction vertices in Figure 2.4. This shows that a particle traversing the VELO in a direction approximately parallel to the beampipe is likely to pass through the foil several times. However, the foil is just 0.5 mm thick, which reduces, as far as possible, the amount of material that particles must pass through before reaching sensitive detector elements.

The VELO needs to be in close proximity to the interaction region to achieve the required displaced vertex resolution. However, during injection and ramping phases of the LHC the sensitive detector elements and the interaction region have to be separated by 30 mm to allow for beam excursions. Once the beams are stable the two halves are moved inwards to an optimal position. The optimal position varies according to the beam parameters of that particular fill, but typically involves situating the two halves of the VELO approximately 6 mm from the interaction region in the x - y plane [39].

The VELO achieves spatial resolutions on the reconstruction of the primary vertex (PV) of the order of $10\text{ }\mu\text{m}$ in the x and y directions and $100\text{ }\mu\text{m}$ in the z direction, as shown

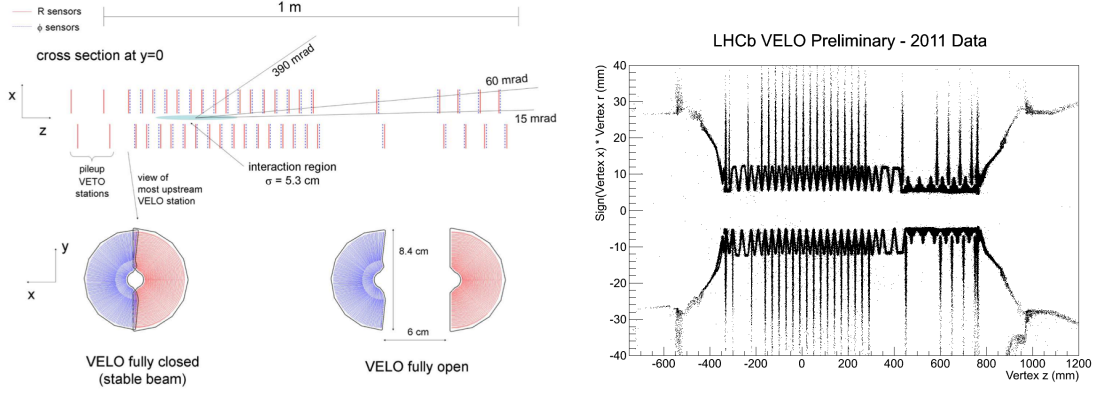


Figure 2.4: A schematic diagram showing the arrangement of sensors in the VELO when open and closed (*left*) and a plot of interaction vertices in the material in the VELO from beam gas events (*right*), which shows the VELO modules as straight vertical lines and the foil as irregular lines enclosing the modules. Figures taken from Ref. [39] and Ref. [44], respectively.

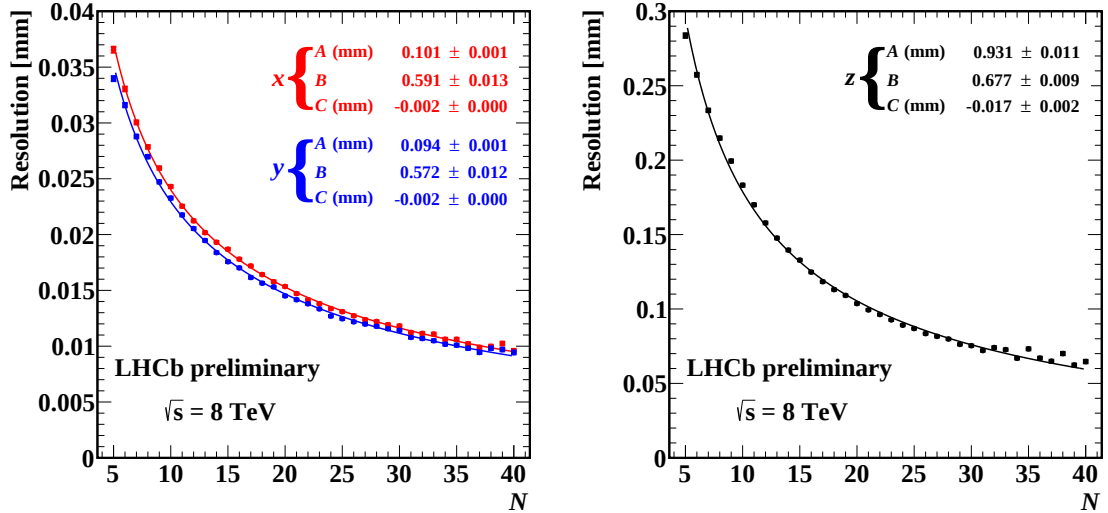


Figure 2.5: The primary vertex resolutions in x and y (*left*) and z (*right*) achieved by the VELO detector in 2012 data in events with one PV. The resolution is plotted as a function of number of tracks used to identify the vertex, N , and a reciprocal function is fit to the data, *i.e.* $\text{Resolution} = A/N^B + C$. The results of the fit are given on the plot. Figures taken from Ref. [44].

by Figure 2.5. This spatial resolution is given as a function of the number of tracks, N , used to identify the vertex. A greater number of tracks used will result in a more accurate determination of the position of the vertex, hence the reciprocal form of the dependence. The curves in Figure 2.5 start at $N = 5$ because this is the minimum number of tracks used to find a vertex. The resolution is estimated by splitting the tracks used to reconstruct the vertex in half, then vertices are remade using groups of tracks in the two halves. The spread of these remade vertices in x , y and z gives an estimate of the resolution.

The VELO also provides a precise measurement of the decay time of B and D mesons. The decay time resolution achieved by the VELO is 45 fs, as measured in $B_s \rightarrow J/\psi \phi$ and $B_s \rightarrow D_s^\pm \pi^\mp$ decays [43].

2.2.3 The RICH detectors

When charged particles traverse a medium at a speed, v , that is faster than the speed of light in that medium they emit an “electromagnetic shock wave” analogous to that of objects travelling faster than the speed of sound in a medium. This is called Cherenkov radiation. The Cherenkov photons are emitted at an angle relative to the direction of motion of the charged particle, which is proportional to v . This angle is given in terms of β and n in Equation 2.2. Here $\beta = v/c$, where c is the speed of light in a vacuum and n is the refractive index of the medium.

$$\cos\theta = \frac{1}{n\beta} \quad (2.2)$$

When a particle traverses the material inside the RICH detectors the Cherenkov radiation is reflected by spherical mirrors and focussed into rings projected onto the detection plane, hence the name “Ring Imaging Cherenkov” detector. The radius of the ring is proportional to the Cherenkov angle. The path of the photons from a charged particle track to the photon detectors is shown in the schematic diagram of RICH1 in Figure 2.6.

Based on a measurement of the momentum of the track by the tracking detectors, three hypotheses are formed by the RICH system; the track is a kaon, the track is a pion or the track is a proton. Each hypothesis corresponds to a ring of given radius determined by the mass of a pion, kaon or proton. These hypotheses are then compared to the photon hits on the detection plane and a maximum-likelihood fit of each ring to the hit positions is performed. A quantity commonly used for particle identification (PID) is the “Delta-Log-Likelihood”. This is the difference between the maximums of the logarithms of the likelihoods of the fits using different mass hypotheses. For example, $DLL_{K\pi}$ corresponds to the difference between

the maximums of the likelihoods of the kaon fit and the pion fit. Other quantities, such as multivariate classifiers that use information from the RICH as well as other subdetectors, are also used for particle identification in LHCb.

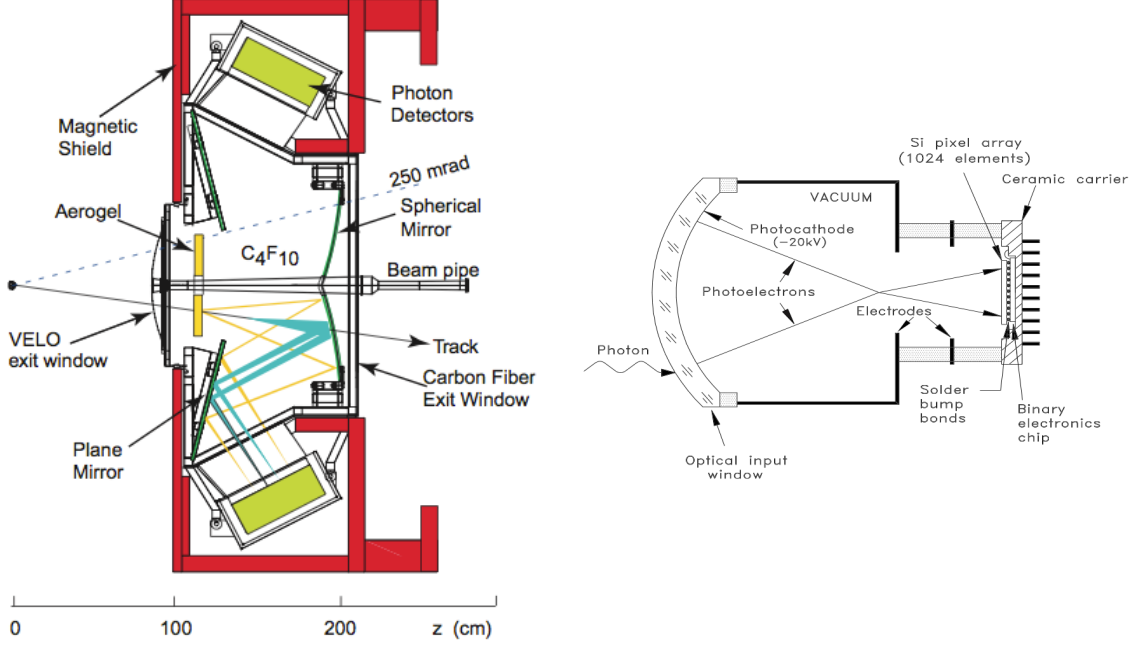


Figure 2.6: Schematic diagrams of the RICH1 detector (*left*) and of a Hybrid Photon Detector (*right*). Figures taken from Ref. [39].

The RICH system in LHCb consists of two detector volumes: RICH1, which is upstream of the magnet and RICH2, which is downstream of the magnet. RICH1 is intended to identify low momentum tracks that have hits in the VELO but will be swept out of the acceptance by the magnet and RICH2 provides particle identification for high momentum tracks. There are three different radiating materials used in the RICH with different refractive indices. These are silica aerogel ($n = 1.03$) and two fluorocarbon gases, C₄F₁₀ ($n = 1.0014$) and CF₄ ($n = 1.0005$) [45]. RICH1 utilises the aerogel and C₄F₁₀ radiators that have higher refractive indices and are therefore more sensitive to low momentum tracks. RICH1 provides particle identification for tracks in the 1 GeV/ c to 60 GeV/ c range. RICH2, which contains the CF₄ radiator, provides particle identification to tracks with momenta between 15 GeV/ c and 100 GeV/ c [39].

The RICH system as a whole achieves particle identification with a high efficiency over

the momentum range $2 \text{ GeV}/c$ to $100 \text{ GeV}/c$ as shown by Figure 2.7. The performance of the RICH deteriorates with increasing momentum, p , and with increasing pseudorapidity, η . This is because the rings corresponding to pion, kaon and proton mass hypotheses become increasingly difficult to resolve at high p , and track multiplicity increases with η , making pattern recognition more difficult.

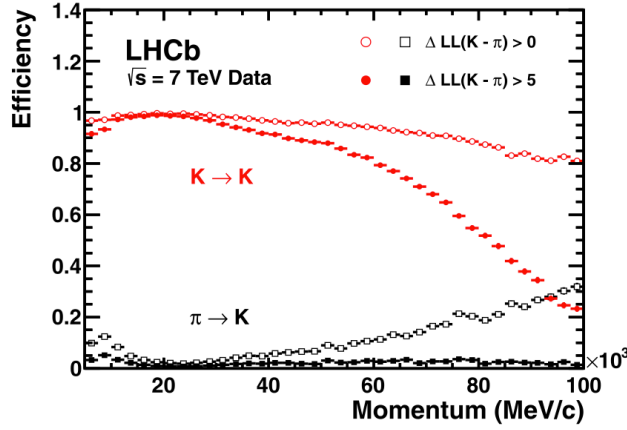


Figure 2.7: Kaon identification efficiency and pion misidentification rate as a function of track momentum. The efficiency and misidentification rate of two different requirements on $\Delta LL_{K\pi}$ are shown. Figure taken from Ref. [46].

Photons are detected by arrays of Hybrid Photon Detectors (HPDs) on the detection planes in both RICH1 and RICH2. The HPDs consist of a photocathode window, where incident photons cause the release of photoelectrons. The photoelectrons are accelerated across a vacuum chamber by a large electric field (typically 10 - 20kV) onto a silicon pixel detector. One electron-hole pair is created per 3.6 eV of energy deposited in the silicon making the HPDs very efficient at detecting single photoelectrons. There are 2×7 columns of HPDs in RICH1 with 14 HPDs per column and 2×9 columns in RICH2 with 16 HPDs per column [39]. A diagram of an HPD is shown in Figure 2.6. There are 1024 silicon pixels in each HPD, which means that there are approximately 2×10^5 detection channels in RICH1 and 3×10^5 in RICH2.

Many analyses including the one reported in this thesis would be impossible without the RICH detectors because particle identification is used to distinguish decays that have identical topologies but different flavour content. These decays present dangerous backgrounds in most analyses of CP violation because they will typically have different properties, under

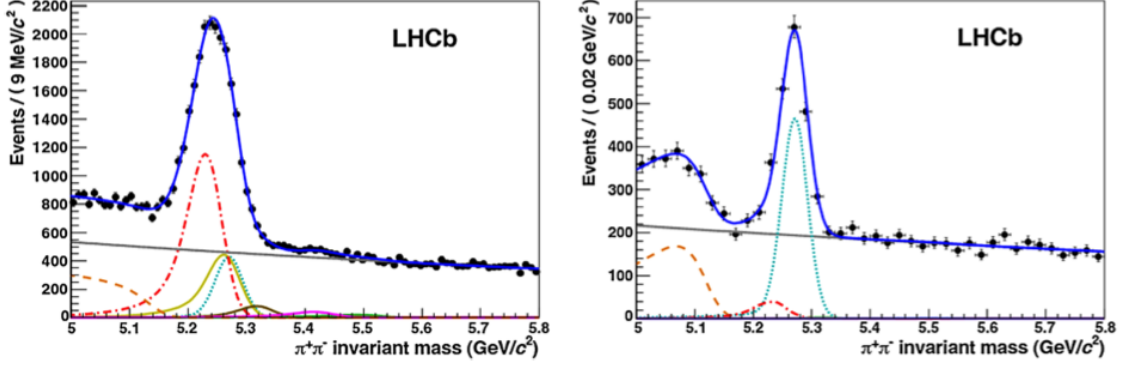


Figure 2.8: The invariant mass distribution of $B \rightarrow h^+ h^-$ candidates in LHCb data before (*left*) and after (*right*) applying requirements on RICH particle identification. The signal under study is the $B_d \rightarrow \pi^+ \pi^-$ decay (*turquoise dotted line*). The backgrounds from the different b -hadron decay modes, $B_d \rightarrow K^\pm \pi^\mp$ (*red dashed-dotted line*), $B_d \rightarrow$ three-body (*orange dashed-dashed line*), $B_s \rightarrow K^+ K^-$ (*yellow line*), $B_s \rightarrow K^\pm \pi^\mp$ (*brown line*), $\Lambda_b \rightarrow p^\pm K^\mp$ (*purple line*) and $\Lambda_b \rightarrow p^\pm \pi^\mp$ (*green line*)¹ are very much reduced by applying requirements on RICH particle identification, after which only two remain visible. Figure taken from Ref. [46].

CP operations, to the signal. An example is the $B_d \rightarrow D\rho^0$ background to the $B \rightarrow DK^*$ decay, discussed in Chapters 3 and 4. Another example is given by $B \rightarrow h^+ h^-$ decays and is illustrated in Figure 2.8. Here there are many contributions to the reconstructed two-hadron invariant mass and the signal under study, $B_d \rightarrow \pi^+ \pi^-$, is engulfed by backgrounds. Requiring that the RICH determines the final state tracks as pions results in a very different invariant mass distribution and the signal becomes very apparent.

2.2.4 Calibration of the RICH response

Particle identification is typically performed by placing a requirement, or set of requirements, on the response of the RICH detectors. This response is described by various PID quantities, for example, $DLL_{K\pi}$, which is discussed in the previous section. The efficiency of any requirements, on the decay being studied, must usually be determined. Instead of relying on simulated data, empirical methods that utilise real data have been developed to calibrate the response of the RICH detectors and determine this efficiency.

Control channels exist, where high purity samples of specific particles can be isolated without the use of the RICH. The channel most commonly used to isolate pure samples

¹Here and after, Λ_b signifies Λ_b^0 and $\bar{\Lambda}_b^0$ baryons, unless otherwise stated.

of kaons and pions is $D^{*\pm} \rightarrow D(K^\pm\pi^\mp)\pi^\pm$. The large charm cross-section at the LHC, together with this decay's unique kinematic properties arising from the low energy of the $\pi^\pm$ in the $D^{*\pm}$ rest frame, allow large samples of kaons and pions to be unambiguously identified [47]. Following Run I of the LHC, samples of $\mathcal{O}(50M)$ kaon and pion candidates have been collected. Residual backgrounds in the samples are discriminated against with the “sPlot” technique [48]. With all backgrounds removed, the response of the RICH (in the form of distributions of PID quantities) to pure samples of kaons and pions can be studied.

It has been checked that the response of the RICH depends on a restricted set of kinematic properties of the track or features of the containing event. Various quantities are considered, for example, the p , transverse momentum (p_T) and η of the track and also the number of tracks in the event. By comparison of these features of the kaons and pions in $D^{*\pm} \rightarrow D(K^\pm\pi^\mp)\pi^\pm$ decays with those of the tracks in the decay being studied, the response of the RICH to the decay being studied can be deduced.

Two different calibration methods exist for historical reasons. The first is applicable to problems where the true response of the RICH to a single track is needed, termed “single-track calibration”. However, it is not applicable to use the single-track calibration method multiple times when studying the response of the RICH to a multi-body decay. This is because the kinematic correlations between the final state tracks are not adequately taken into account. Therefore a method applicable to multi-body decays has subsequently been developed, termed “N-track calibration”.

The single-track calibration, although superseded by the N-track calibration, still has use in single-track problems. This is because the single-track calibration provides the distributions of PID quantities relating to the signal track prior to any requirements being made. The N-track calibration necessarily sacrifices information about the distributions of PID quantities to make an accurate determination of the efficiency of a set of requirements on the tracks in a multi-body decay.

Single-track calibration

To perform the single-track calibration procedure a pure sample of signal candidates must be obtained. Ideally a pure signal sample is obtained from real data, without making selection requirements on PID quantities, using the sPlot technique [48]. However, in many cases, including that of $B \rightarrow DK^*$ decays, this is not possible, therefore larger samples of simulated data are used as reference samples. This limits the quantities that can be compared with the pure calibration samples to those that are well simulated. Of these, p and η are assumed to be well simulated, however the number of tracks in the event is not.

The calibration and signal samples are binned in the chosen variables. Any binning scheme can be used although a sufficient number of candidates must be present in the calibration samples in each bin for the result to be reliable. In the hypothetical scenario where the sample size is sufficient to allow arbitrarily small bins with finite populations and the assumption that the RICH response depends on only the variables considered is correct, then the distribution of a given PID quantity in the calibration sample in a particular bin should be single-valued. In reality the bins cannot be arbitrarily small. However this is the core principle that underpins the calibration procedure and an uncertainty is evaluated due to deviation from this hypothetical scenario.

The population of each bin in the calibration and signal samples is compared and the ratio of these is assigned as the weight to that particular bin. The true distribution of the PID quantity of the signal track is then given by the distribution of the calibration sample weighted by this ratio. The efficiency on the signal of any requirement made on that PID quantity can be determined from the weighted calibration sample.

Figure 2.9 illustrates this calibration procedure graphically and Figure 2.10 shows the effects. The red distribution is the $DLL_{K\pi}$ distribution of the weighted calibration sample. The reference sample used is of kaons from the D meson, K_D , in simulated $B_d \rightarrow D(K\pi)K^*$ decays. Figure 2.10 is applicable if information about only the K_D is needed. However, this is typically not the case, for example when PID requirements are placed on multiple tracks in the

final state of a multi-body decay, this method cannot be applied due to kinematic correlations between the tracks. The calibration philosophy and method are therefore extended to multi-body final states.

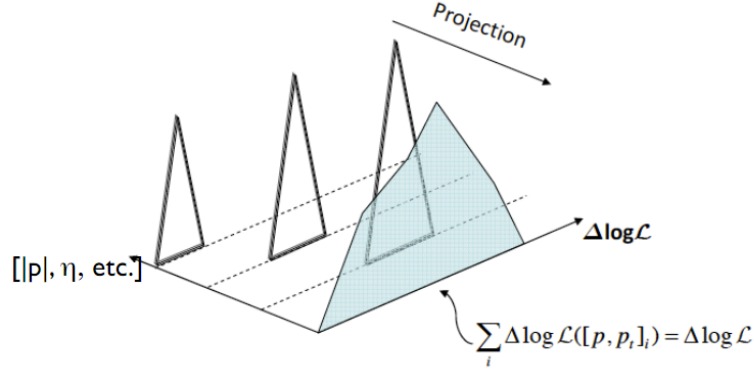


Figure 2.9: A graphical representation of the single-track calibration of the RICH response. The three black spikes are the distributions of the PID quantity in different kinematic bins of the calibration sample, weighted by the relative population of that bin in the calibration and the signal samples. The projection of all these distributions is the distribution of the PID quantity in the weighted calibration sample.

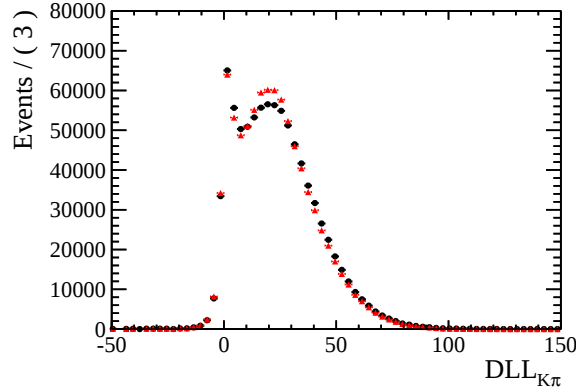


Figure 2.10: Example of the single-track calibration results for the K_D in $B_d \rightarrow D(K\pi)K^*$ decays. Unweighted (*black circles*) and weighted (*red triangles*) $DLL_{K\pi}$ distributions.

N-track calibration

Instead of exactly replicating the distributions of PID quantities, the end result of the N-track calibration, where $N \geq 1$, is purely the efficiency of a certain set of PID requirements placed on the tracks in the multi-body decay.

Histograms are produced from the calibration samples where the bin content is equal to the efficiency of the requirement used on the tracks in that particular bin. For example, if a

threshold of $DLL_{K\pi} > 0$ is set on a particular kaon track in the decay being analysed, then a histogram of the efficiency of this requirement on the kaons in the calibration sample is produced. Analogous histograms for all requirements placed on the decay being studied are also produced.

To account for variation in performance of the RICH over time, different calibration histograms are produced for different periods of data collection. The data collected in 2012 is split into 16 different periods (9 with magnet polarity down, 7 with magnet polarity up). The differing performance in these periods can be seen by comparing the histograms in Figure 2.11. The histograms used in the calibration procedure subsequently described are a weighted average over the applicable data collection periods, which is dependent on the analysis. An example of a weighted average over 2012 is shown in Figure 2.12.

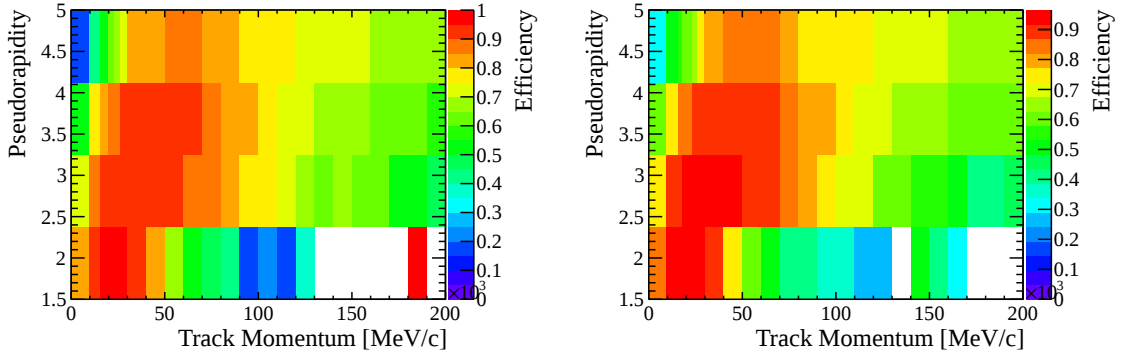


Figure 2.11: The efficiency of a requirement of $DLL_{K\pi} > 0$ on kaons from $D^{*\pm} \rightarrow D(K^\pm \pi^\mp) \pi^\pm$ decays in data collected in periods with the magnet polarity down (*left*) and up (*right*).

A reference sample of pure signal decays without any requirements on PID quantities is obtained. This signal sample is typically obtained in the same way as before, either by applying the sPlot technique [48] to real data, upon which no PID requirements have been made, or by using simulated data.

For each signal decay in the reference sample an individual efficiency is assigned based on that decay's particular kinematic properties. This is taken as the product of all the efficiencies on the tracks, which are determined by taking the efficiency from the relevant calibration histogram for the kinematic bin in which that reference track resides. The overall efficiency for that decay mode is taken as the mean of all individual decay efficiencies. Therefore, the

efficiencies of sets of PID requirements on multi-body decays are defined as overall efficiencies rather than efficiencies of individual requirements. The efficiency of a given set of PID requirements is given by Equation 2.3, where N_{ref} is the number of decays in the reference sample, k runs over reference decays, i runs over tracks in the final state and ϵ_{ik} is the efficiency of the requirement on that track as a function of the considered kinematic or event properties, taken from the relevant efficiency histogram.

$$\epsilon_{\text{PID}} = \frac{1}{N_{\text{ref}}} \sum_k \prod_i \epsilon_{ik} \quad (2.3)$$

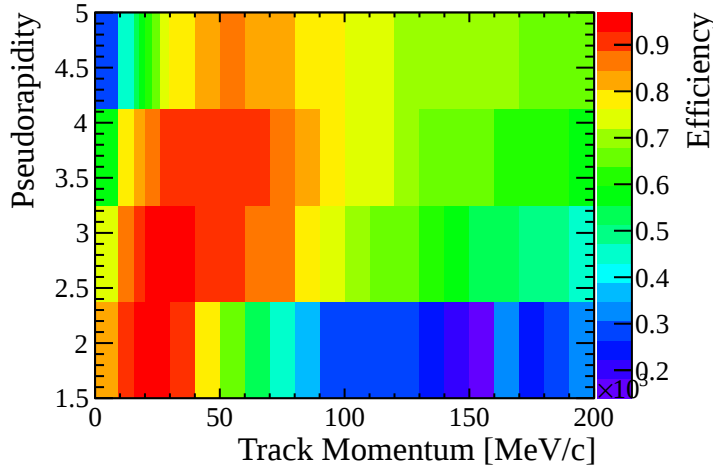


Figure 2.12: The weighted average efficiency of a requirement of $\text{DLL}_{K\pi} > 0$ on kaons from $D^{*\pm} \rightarrow D(K^\pm \pi^\mp) \pi^\pm$ decays over the whole 2012 data collection period. The same dependence of efficiency on momentum as in Figure 2.7 is evident as well as a deterioration in efficiency towards the edges of the acceptance, in η , of LHCb.

If real data are not available, simulated data are used as the reference. As stated, it is assumed that the p and η distributions of tracks are well represented by the simulated data. This assumption is justified, for the $B_d \rightarrow D(K\pi)K^*$ decay, by Figure 2.13, which compares the distributions of p and η in simulated $B_d \rightarrow D(K\pi)K^*$ decays with those of signal candidates in real data.

Systematic uncertainty

Two sources of systematic uncertainty are considered. One contribution comes from the calibration procedure itself. There are two indistinguishable components to this contribution.

The first component comes from necessarily using bins with a finite size. This causes a loss of information, and therefore introduces some uncertainty, since the appropriate weight or efficiency is averaged over the whole bin. The second component comes from the assumption that the RICH response depends only on the kinematic or event properties considered, while it could have a dependence on others.

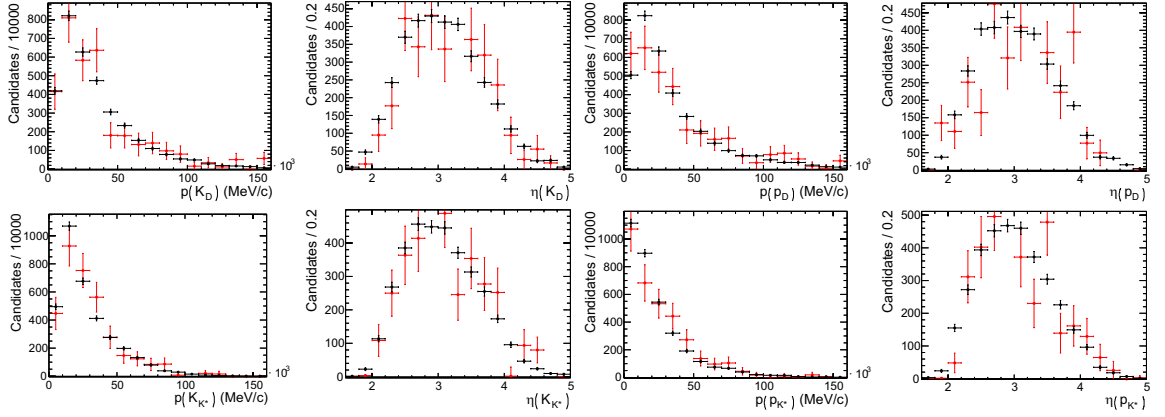


Figure 2.13: Comparison of the momentum and pseudorapidity distributions of the tracks in $B_d \rightarrow D(K\pi)K^*$ decays in real data (*red*) and simulated data (*black*). Here, “ $p_{D(K^*)}$ ” refers to the pion originating from the $D(K^*)$ meson.

The second contribution to the systematic uncertainty is not present in all analyses and comes from the need to use simulated rather than real data for the reference sample. This is assumed to be negligible as long as kinematic or event properties used for calibration are sensibly chosen.

The systematic uncertainty inherent in the calibration procedure is determined using simulated data for both the single-track and N-track calibration methods. Essentially, the calibration procedure is repeated using purely simulated data. This requires simulated versions of the calibration samples. The reference sample used is of simulated signal decays, whether this was used for the efficiency determination or not. The efficiency of a given PID requirement or set of PID requirements obtained from the calibration procedure, ϵ_{MCcal} , is compared with that evaluated directly on simulated signal decays, ϵ_{MCsig} . The systematic uncertainty is given by Equation 2.4 and is the asymmetry between ϵ_{MCsig} and ϵ_{MCcal} multiplied by the

scale factor of the efficiency in real data, ϵ_{PID} .

$$\Delta_{\text{PID}} = \left| \frac{\epsilon_{\text{MCsig}} - \epsilon_{\text{MCcal}}}{\epsilon_{\text{MCsig}} + \epsilon_{\text{MCcal}}} \times \epsilon_{\text{PID}} \right|. \quad (2.4)$$

Δ_{PID} is typically measured for a variety of requirements in the region of the working point of the analysis and the maximum in that region is conservatively assigned as a systematic uncertainty.

2.2.5 The tracking detectors

LHCb uses a combination of silicon and drift-tube tracking detectors. The tracking detectors are used to resolve the trajectories of charged particles and reconstruct vertices and therefore identify the topology of decays. They also make accurate measurements of the momentum of charged particles based on the curvature of their tracks when they pass through the dipole magnetic field. The momentum is given by Equation 2.5, where q is the electric charge of the particle in units of e , B is the magnetic flux density and r is the radius of curvature.

$$p = |q| B r \quad (2.5)$$

The tracking detectors in LHCb comprise the VELO, which has already been described in Section 2.2.2, the Tracker Turencis (TT) situated upstream of the magnet and the Inner Tracker (IT) and Outer Tracker (OT) situated downstream. The sensor technology used in the TT and IT are silicon microstrips whereas the OT uses drift-tubes. These choices in sensor technology are motivated by track multiplicity. Silicon microstrips are used where the occupancy is highest and therefore the finest spatial resolution is required. In the OT, where coarser spatial resolution is adequate, cheaper drift-tubes are used to reduce the monetary cost of the detector.

The TT is comprised of four detection layers and covers the whole acceptance of LHCb. The TT is situated between RICH1 and the dipole magnet as shown in Figure 2.3. Each

detection layer is made up of an array of individual silicon sensors. The silicon sensors in the TT are $500\text{ }\mu\text{m}$ thick, single sided p^+ -on- n sensors. They are 9.64 cm wide and 9.44 cm long and have 512 silicon microstrips with a pitch of $183\text{ }\mu\text{m}$. In each layer the silicon sensors are arranged in columns of 14 with readout electronics at each end. The central column is split into two half-columns to make space for the beampipe. Each detection layer consists of 15 (first 2 layers) or 17 (last 2 layers) columns. A schematic diagram of one detection layer of the TT is shown in Figure 2.14. The detection layers have a y - u - v - y orientation, meaning that the first and last layers are orientated vertically and the second and third layers are rotated by an angle, relative to the y -axis, of -5° and $+5^\circ$, respectively. The columns in each layer are staggered in z and overlap in x , this enables individual alignment of the columns and also avoids any acceptance gaps. All readout electronics, cooling and support infrastructure for the TT is situated outside of the acceptance of LHCb, minimising the material budget of the detector [39].

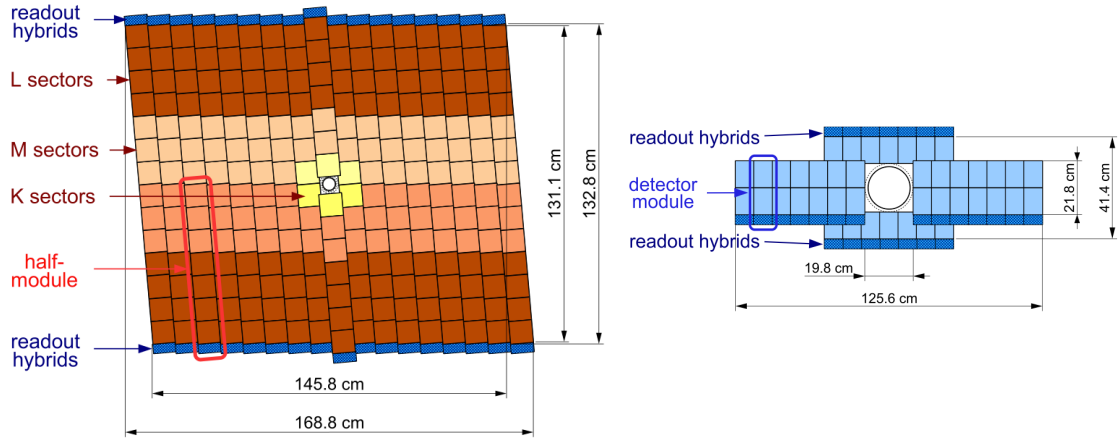


Figure 2.14: Schematic diagrams showing the layout of the silicon sensors in the third TT detection layer (*left*) and the layout of the detector boxes, and contained silicon sensors, in an IT station (*right*). Figures taken from Ref. [39].

The IT has 3 tracking stations, marked by ‘T1’, ‘T2’ and ‘T3’ in Figure 2.3, and is situated downstream of the magnet, before RICH2. Each station comprises four electrically and optically tight detector boxes arranged in a cross shape around the beampipe. In each box there are four detection layers, with seven modules of silicon sensors in each layer. The modules in the side boxes (to the right and left of the beampipe) contain two silicon sensors, while the modules in the top and bottom boxes have one sensor. A diagram of one IT station

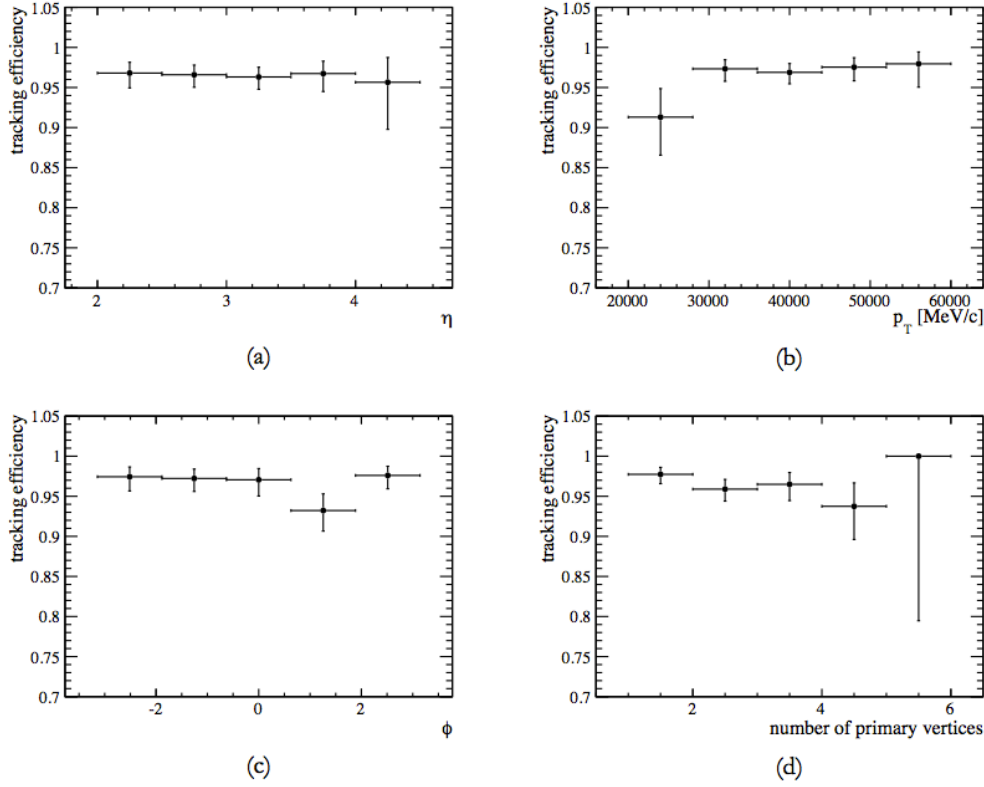


Figure 2.15: Track finding efficiency for high transverse momentum tracks, determined with $Z^0 \rightarrow \mu^+\mu^-$ decays in data collected in 2010. The efficiency is binned in the pseudorapidity (a), the transverse momentum (b) and the polar angle (c) of the track and the number of reconstructed primary vertices in the event (d). Figure taken from Ref. [49].

is shown in Figure 2.14.

The silicon microstrip technology used in the IT is essentially the same as that in the TT, but the sensors differ slightly in specification. Two types of silicon sensors of different thickness, but otherwise identical in design, are used in the IT. They are single-sided p^+ -on- n sensors, which are 7.6 cm wide and 11 cm long. Each sensor has 384 silicon microstrips with a pitch of $198 \mu\text{m}^2$. The sensors in the top and bottom boxes are $320 \mu\text{m}$ thick, whereas those in the left and right boxes are $410 \mu\text{m}$ thick. These thicknesses were chosen to ensure sufficiently high signal-to-noise ratios for each module type while minimising the material budget of the detector [39].

The OT covers the remainder of the acceptance around the IT in tracking stations T1, T2 and T3. The OT is an array of drift-tubes with inner diameters of 4.9 mm, containing

²The required single hit resolution of $50 \mu\text{m}$ in both the TT and the IT motivates a strip pitch of approximately $200 \mu\text{m}$ in both detectors.

70% Argon and 30% CO₂ as a drift gas. This enables a fast drift time (below 50 ns) and a drift-coordinate resolution of 200 μm . The drift-tubes are arranged in modules with each module containing two staggered layers of tubes. The design specifications of the drift-tube modules ensure rigidity to guarantee drift-tube position and alignment, low material budget to minimise multiple scattering before the calorimeters, electrical shielding to avoid cross-talk and noise and radiation hardness to withstand 10 years' running at nominal luminosity. Each OT station contains four modules in a y - u - v - y arrangement like that of the TT and the IT. This design ensures an excellent momentum resolution, of $\delta p/p \approx 0.4\%$, and therefore an excellent invariant mass resolution of reconstructed b -hadrons [39].

A full discussion of the offline track finding algorithms used in LHCb can be found in Ref. [49]. The average track finding efficiency of these algorithms is $(96.5 \pm 0.8)\%$ per track, as measured with 37.5 pb^{-1} of data collected in 2010 [49]. The dependence of the track finding efficiency on various properties of the track and the event is shown in Figure 2.15.

2.2.6 The calorimeters

The calorimeters provide an important input to the particle identification capabilities of LHCb in addition to the RICH detectors. They are primarily used for identifying electrons, photons and hadrons and supply the trigger with a measurement of the transverse energy, E_{T} , of these particles. Another key role of the calorimeters is the reconstruction of π^0 mesons.

The calorimeters in LHCb are arranged in a manner typical to particle physics experiments. The electromagnetic calorimeter (ECAL) is situated upstream of the hadronic calorimeter (HCAL). The calorimeters reside between the first two muon stations as shown in Figure 2.3. The most demanding particle identification requirement of the calorimeters is to identify electrons. The first level of triggering is required to reject 99% of inelastic pp collision events whilst enriching the data sample with events containing b quarks, with electrons in the final state, by a factor of 15 and this is accomplished by the selection of electrons with large E_{T} [39].

One of the main backgrounds to high E_T electrons is charged pions. These are rejected with the aid of longitudinal segmentation (in z) of the shower, which is helped by a preshower detector (PS). Another background to electrons are high E_T π^0 mesons. These are rejected with a scintillating pad detector (SPD) plane before the PS, which identifies charged particles.

The SPD and PS detectors are situated in front of the ECAL and constitute two rectangular layers of scintillating material, in between which there is a 15 mm lead converter, which corresponds to 2.5 radiation lengths, X_0 . The scintillating layers are split into cells with high granularity. There are 12032 detection channels in total and the active area of the detector is 7.6 m wide and 6.2 m high [39].

The ECAL is a sampling calorimeter with scintillator and lead layers. The scintillation light is read out by wavelength shifting (WLS) fibres, which feed into multi-anode photo multiplier tubes (MAPMTs). The amount of light incident on the MAPMTs is proportional to the energy of the particles that traverse the scintillation layers and is hence proportional to the energy of the particle initially incident on the ECAL.

The ECAL has a modular construction with a single ECAL module being built from alternating 2 mm lead and 4 mm scintillator layers. Sixty-six layers of lead and scintillator make up a 42 cm stack, which corresponds to $25X_0$. The ECAL is situated 12.5 m away from the interaction region and covers the acceptance up to 300 mrad in x and 250 mrad in y . The inner acceptance limit is 25 mrad due to the charged particle multiplicity in this region close to the beampipe being too high [39]. The design energy resolution of the ECAL is $\sigma_E/E = 10\%$. This resolution results in a mass resolution of 65 MeV/ c^2 for the $B_d \rightarrow K^*\gamma$ decay with a high- E_T photon and of 75 MeV/ c^2 for the $B \rightarrow \rho\pi^0$ decay³ with a π^0 mass resolution of ≈ 8 MeV/ c^2 [39]. This can be compared to the achieved mass resolution for the $B_d \rightarrow K^*\gamma$ decay of 100 MeV/ c^2 in 1 fb⁻¹ of data collected in 2011. This mass resolution is dominated by the ECAL energy resolution [50].

The LHCb HCAL is also a sampling calorimeter, using iron, not lead, as the absorber. The HCAL has a novel design of scintillating and absorbing tiles orientated parallel to the

³Here B refers to $B^\pm$ or B_d and ρ to $\rho^\pm$ or ρ^0 .

beam axis. Each layer is 1 cm thick, meaning that the scintillating tiles are separated laterally (in x) by 1 cm of iron. The length of tiles and iron spacers in z corresponds to the hadronic interaction length, λ_I . The light from the scintillating tiles is collected by WLS fibres that run between the layers. The fibres transmit the scintillation light to the back of the HCAL where it is collected by photo-multiplier tubes. A schematic diagram showing the structure of a module of the HCAL is shown in Figure 2.16.

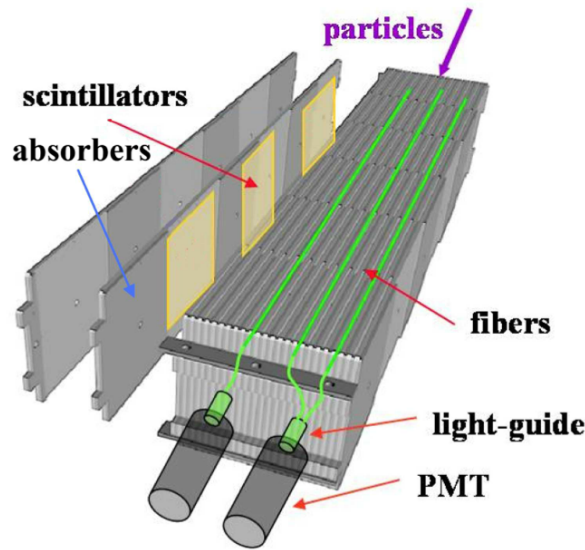


Figure 2.16: Schematic diagram showing the design and longitudinal orientation of sampling and absorbing layers in an HCAL module. Figure taken from Ref. [39].

The HCAL is split into inner and outer sections, closer and further away from the beampipe, respectively. In the inner region, readout cells are arranged with higher granularity to cope with the greater charged particle multiplicity. The HCAL is a wall of the modules shown in Figure 2.16 situated at $z = 13.33$ m from the interaction region, with dimensions of 8.4 m in height, 6.8 m in width and only 1.65 m in depth. This relatively modest depth corresponds to only $5.6\lambda_I$ and the ECAL and HCAL combined correspond to $6.35\lambda_I$. This is because accurate energy resolution is not required for hadronic triggering, which is the main purpose of the HCAL, and hence saves space for the muon detectors.

2.2.7 The muon detectors

The muon detection system in LHCb comprises five detector stations, designated M1 - M5. M1 is in front of the calorimeters and M2 - M5 are behind, as shown in Figure 2.3. The muon detectors play a very important role in the physics programme of LHCb since many important decays have muons in the final state. For example, $B_d \rightarrow J/\psi (\mu^+ \mu^-) K_s^0$ and $B_s \rightarrow J/\psi (\mu^+ \mu^-) \phi$ are very important decays for the measurement of time-dependent CP violation and mixing and $B_s \rightarrow \mu^+ \mu^-$ can powerfully constrain new physics models [51, 52]. The muon system is also vital for reconstruction of semileptonic decays of charged and neutral B mesons. Such decays are interesting to study in their own right and are also valuable because of their utility in tagging the initial flavour of the corresponding B meson in the event. The importance of reconstructing muons warrants the inclusion of information from the muon system at the first stage of data acquisition.

The first level of triggering requires a measurement of high p_T of the muon track. The muon system as a whole achieves a p_T resolution of 20%. The position of M1 (upstream of the calorimeters) and the high spatial resolution of M1, M2 and M3 in the bending plane contribute the most to this performance. M4 and M5 do not have such good resolution and mainly serve to identify penetrating particles and differentiate them from HCAL punch-through.

The five detection stations of the muon system are arranged as shown in Figure 2.17, where the iron absorbers placed in between stations M2 - M5 are introduced to aid identification of penetrating muons. In total the muon system comprises 1380 chambers and has an active area of 435 m². The chambers are arranged with varying granularity according to the specific station and the angular position of the chamber with respect to the beampipe. The granularity is governed by the expected occupancy and the purpose of the station. For example, M2 and M3 have four times the granularity, in x , of M4 and M5 because of their purpose in accurate p_T measurement. The granularity of the chambers and the detection areas, or “logical pads”, that they are split into is shown in Figure 2.17.

The dimensions of the logical pads define the spatial resolution of the detectors however

the readout electronics are separated by “physical pad”. Physical pads sometimes directly correspond to logical pads. In some cases the readout electronics perform the logical OR of several physical pads to construct a logical pad. In other cases the physical pads are larger than the size of logical pads needed for the required spatial resolution. In this case a mixed readout system is adopted that effectively creates logical pads of the required size [39].

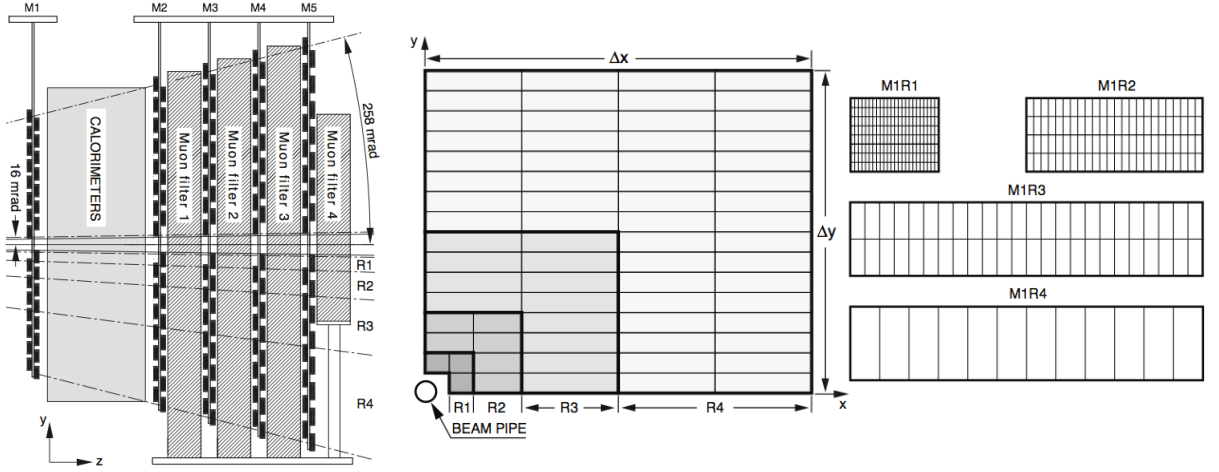


Figure 2.17: A diagram showing the arrangement of the muon stations and “muon filters”, or iron absorbers (*left*). The front view of a quadrant of a muon station (*middle*), each rectangle represents one chamber and each station contains 276 chambers. The division, into logical pads, of four chambers belonging to the four regions of station M1 (*right*). In each region of stations M2-M3(M4-M5) the number of pad columns per chamber is double(half) the number in the corresponding region of station M1, while the number of pad rows per chamber is the same. Figures taken from Ref. [39].

The chambers in the muon system are mostly traditional multi-wire proportional chambers (MWPCs), which contain a gas mixture of $\text{Ar}/\text{CO}_2/\text{CF}_4$ in the proportions 40/55/5. However, triple-GEM chambers are used in the central region of the M1 station. Triple-GEM detectors consist of three gas electron multiplier (GEM) foils sandwiched between anode and cathode planes and can effectively be used as tracking detectors with good time and position resolution. They are used in the central region of M1 because of their ability to cope with the higher occupancies in that region. More details of the GEM detectors can be found in Ref. [53].

The muon identification algorithms attain an average efficiency of approximately 98% for high p_T muons, while keeping pion and kaon misidentification rates below 1% [54].

2.2.8 Triggering and data acquisition

The rate of bunch crossings at the LHC and the LHCb event size of approximately 35kB [55] do not permit the readout and retention of data from every bunch crossing in LHCb. Therefore a triggering system must be employed to decide on the interesting events and reduce the rate. The LHCb trigger is split into two main stages: the Level-0 trigger (L0) which is implemented in hardware, and the High-Level trigger (HLT) which is implemented in software. The events that pass the HLT are retained. An effective offline trigger, termed the ‘Stripping’, is then applied to the reconstructed candidates before the data are analysed.

Level-0 trigger

The Level-0 trigger is implemented by custom designed hardware and makes use of parallelism and pipelining to perform the necessary operations in less than the maximum latency time of 4 μ s. The L0 is divided into three independent triggers. These are the L0-Calorimeter, -Muon and -Pileup triggers. The L0-Pileup trigger selects events that aid the calculation of the luminosity.

The L0-Calorimeter trigger makes use of information from the SPD, PS, ECAL and HCAL subdetectors. These detectors are divided into cells in the x and y directions and are clearly separated in z . The E_T deposited in zones of 2×2 cells is computed using Equation 2.6, where E_i is the energy in cell i and θ_i is the angle between the z -axis and a straight line from the mean position of the interaction region to the centre of the cell. Other deductions are made from the shower profile and three types of L0 candidate are formed by the L0-Calorimeter trigger.

$$E_T = \sum_{i=0}^4 E_i \sin \theta_i \quad (2.6)$$

L0 hadron candidates are defined by the highest E_T HCAL cluster. If the highest E_T ECAL cluster is in front of that then the E_T of the candidate is the sum of the two. L0 photon candidates are defined by the highest E_T ECAL cluster with hits in the PS and no hits in the SPD. The final type of candidate formed by the L0-Calorimeter trigger is the L0

electron candidate; the requirements are exactly the same as for L0 photon candidates but with the addition of hits in the SPD [56].

The L0-Muon trigger splits the muon system into quadrants and a processor is attached to each quadrant. As explained in Section 2.2.7 the muon stations are further split into physical pads. By reconstructing hits in the pads, each processor finds muons with the highest and second highest p_T in their quadrant. This is done by requiring hits in all 5 stations that define a straight line that points to the interaction envelope in the y - z plane. The first two muon stations provide a measurement of the p_T with a precision of approximately 25% of the full offline muon reconstruction. The L0-Muon trigger sets a threshold on the highest p_T of a single muon or on the product of the transverse momenta of the highest and second highest p_T muons and forms two types of candidates: L0 muon and L0 dimuon candidates.

All the L0 triggers are combined in a single L0 decision, which is transferred to the Readout Supervisor board (RS). The RS has information about the state of all the readout electronics of the different subdetectors and the availability of computers in the farm that implement the HLT. This enables the RS to retain or override a L0 decision as necessary. The total effect of the Level-0 trigger is a rate reduction from 40 MHz to approximately 1 MHz.

High-Level trigger

The HLT is a program written in C++, based on the same software that is used throughout LHCb data processing and simulation. Approximately 26000 copies of this program are run on a farm of multiprocessor PCs. The output rate of the L0 and the available computing resources mean that the approximate limit to the processing time of a single event in the HLT is 30 ms. The full event reconstruction takes approximately 2 s per event, therefore the HLT can only perform partial event reconstruction. The HLT is split into two stages: HLT1 performs partial event reconstruction and reduces the rate from 1 MHz to 43 kHz, HLT2 performs a more complete event reconstruction and is therefore able to make more complex requirements. HLT2 reduces the rate from 43 kHz to 3 kHz. There are many different series of requirements or “lines” in the HLT, which have been developed with different analyses

in mind. Therefore it is more appropriate to describe the information available to these algorithms as a result of the reconstruction steps that can be performed at this level of triggering, rather than detail the requirements as in the case of the L0.

In HLT1 a fast pattern recognition algorithm is run on the hits in the VELO to find primary vertices that are associated with at least 5 tracks. If the event contains either of the L0-Muon candidates, VELO tracks are also associated with muon candidates by extrapolating a search window in the muon stations based on multiple scattering and momentum requirements. A linear χ^2 fit to the track using the VELO track and the hits found in the muon chambers is performed. A requirement is placed on the maximum value of the χ^2/ndf of this fit. Once the first candidate track is accepted by this procedure, the algorithm is stopped and that VELO track is tagged as a muon candidate.

For tracks selected either because they come from the primary vertex or because they are muon candidates, track segments in the IT and OT are also constructed. The search windows in which to look for track segments are defined by a minimum momentum. This procedure is known as “forward tracking” and is used to measure the tracks’ momenta. Each reconstructed “forward track” is fitted using a simplified Kalman filter based track fit. From the information gained in HLT1 it has been shown that it is possible to make effective selection requirements on impact parameter, momentum and invariant mass [56]. The offline muon identification algorithms are also run on tracks tagged as muon candidates to enable improvement of the purity of the muon sample.

Events that pass HLT1 trigger lines are then passed to the second stage of the HLT. In HLT2, forward tracking of all the VELO tracks is performed, where momentum cuts of $p > 5 \text{ GeV}/c$ and $p_T > 0.5 \text{ GeV}/c$ are used to define limited search windows in the IT and OT and therefore reduce the processing time. The full offline muon identification algorithm is run on all the tracks from the forward tracking and association is made between the tracks and ECAL clusters to identify electrons.

There are also “topological” and “exclusive” trigger lines run in HLT2. The topological trigger lines are designed to select fully or partially reconstructed decays of b -hadrons and

their decisions are based on the properties of combinations of two or more “Topo-Tracks”. Topo-Tracks are a subset of HLT2 tracks that are subject to additional requirements on their track fit quality and impact parameter. Candidates for multi-body decays are formed from Topo-Tracks that have a distance of closest approach (DOCA) of less than 0.2 mm. If a candidate formed by the requirements on the DOCA only contains a subset of the daughter particles of a b -hadron, its invariant mass, m , will be less than the mass of a b -hadron. Thus, any requirement on m designed to select b -hadrons would need to be very loose if the trigger is to be inclusive of partially reconstructed decays. Instead a corrected mass, m_{corr} , is used, which is given by

$$m_{\text{corr}} = \sqrt{m^2 + |p_{\text{Tmiss}}'|^2} + |p_{\text{Tmiss}}'| \quad (2.7)$$

where p_{Tmiss}' is the missing momentum transverse to the direction of flight, as defined by the positions of the primary vertex and the displaced vertex of the b -hadron decay. This compensates for the loss of neutral particles in the final state. Candidates for b -hadron decays are then subject to a multivariate selection that uses m_{corr} and DOCA along with other variables to construct a classifier, upon which a requirement is made. Prompt c -hadrons are also vetoed with invariant mass requirements [56].

The exclusive trigger lines target specific decays and therefore require all the particles to be reconstructed in HLT2. Strict requirements are made on quantities such as invariant mass to reduce the acceptance of these lines. The exclusive lines are used to select prompt c -hadrons or to reconstruct b -hadron decays without using requirements that affect the decay time distribution [56].

The LHCb trigger has an average efficiency of approximately 30% on multi-body hadronic final states such as those studied in this thesis [43]. More information on the efficiency of the trigger on $B \rightarrow DK^*$ decays is given in Chapter 3.

Stripping

All events that pass HLT2 are retained in storage. These events are processed by the Stripping before being used in an analysis. The Stripping makes use of the fully reconstructed event and the same data processing application as in offline analysis. “Stripping lines” are analogous to trigger lines and constitute series of selections made to identify candidates for particular decays. Stripping lines are typically designed with particular analyses, collections of analyses or calibration tasks in mind. The candidates selected by the Stripping are saved in separate, smaller, datasets that make analysis of the data a computationally manageable task for the analyst. However, as the full dataset that passes HLT2 is saved to storage, the Stripping may be run multiple times. This typically happens a couple of times per year, because of the large amount of computing resources required.

The Stripping selection requirements made on the candidates used in the analysis in this thesis are listed in Chapter 3. The code used to configure these selections is written in PYTHON and is designed to configure selections for $X_b \rightarrow X_c + X$ decays, where X_b and X_c are any beauty or charm hadron and X is any other hadron or combination of hadrons. The code is arranged in a modular fashion, meaning that selections are defined that select specific X_c and X decays. These selections can then be combined in any permutation to produce a series of selections to reconstruct any given decay, for example, $B \rightarrow DK^*$.

2.2.9 LHCb and other analysis software

The LHCb software comprises many different applications. These applications are all built within the GAUDI framework. The GAUDI framework, which was originally developed for LHCb, is an object-oriented framework designed to provide a common infrastructure and environment for all the different software applications of the experiment [57]. This enables software projects to be developed separately, as the interfaces through which they interact are independent of the implementation of the software. The GAUDI framework provides the interfaces between different components of the LHCb software.

The LHCb event model is another key part of the software framework. It is a description of all the data objects that are handled by the algorithms in the different software applications. This includes Monte Carlo truth (*e.g.* generation and decay vertices), reconstruction (*e.g.* ECAL clusters) and physics objects (*e.g.* particles and vertices). A schematic diagram of the LHCb software applications and data flow is shown in Figure 2.18.

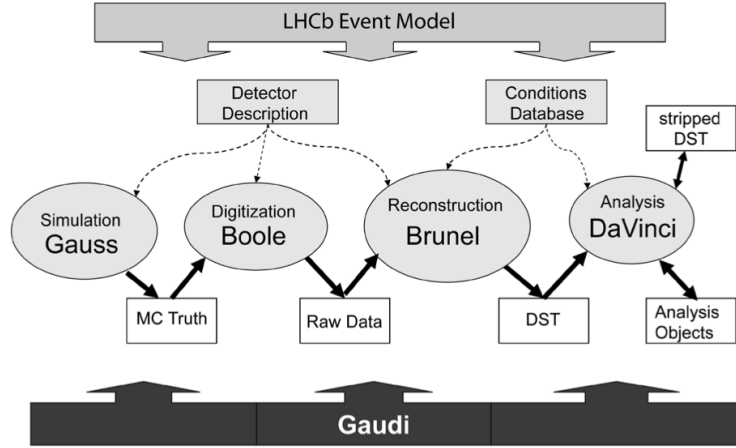


Figure 2.18: A schematic diagram showing the arrangement of the LHCb software applications, their interactions with other components and the flow of data between them. Figure taken from Ref. [57].

GAUSS is the application that handles the Monte Carlo generation of pp collisions and subsequent simulation of the passage of particles through the detector. GAUSS uses a PYTHIA [58,59] based decay package and the GEANT4 toolkit [60,61] to perform the generation and simulation phases, respectively. The final step in producing simulated data is the imitation of the detector response and the digitisation of the data, this task is performed by the BOOLE application.

BRUNEL is the application responsible for reconstruction. BRUNEL can be run on the digitised, simulated data output from BOOLE or on real data and as such is independent of the simulation process. This means that subsequent analysis steps can treat simulated data in exactly the same way as real data.

DAVINCI is the physics analysis application, it provides general utilities to perform analysis and manipulation (*e.g.* vertexing of tracks) of the reconstructed objects output from BRUNEL, and subsequent selection of candidates. The algorithms for the HLT and the

Stripping are developed and implemented using DAVINCI.

Much of the analysis in this thesis is performed using ROOT and its associated fitting library, RooFIT [62]. ROOT provides a set of object oriented frameworks with all the functionality needed to handle and analyse large amounts of data in an efficient way. The data are defined as a set of objects and specialized storage methods are used to get direct access to the separate attributes of the selected objects, without having to access the bulk of the data [63].

2.3 Experimental summary

The LHCb experiment is a single-arm forward spectrometer designed primarily to study the physics of b quarks by reconstructing particles produced in the proton-proton collisions in the Large Hadron Collider. LHCb occupies the forward region, where the $b\bar{b}$ production cross-section is high.

The main elements of LHCb are a silicon vertex locator, two ring imaging Cherenkov detectors that serve to identify different particle types, a combination of silicon and drift-tube tracking detectors, a warm dipole magnet, sampling electromagnetic and hadronic calorimeters and the muon detectors. A method, utilising real data, for the calibration of the response of the RICH detectors is presented in Section 2.2.4.

The first stage of the LHCb data acquisition system, the L0 trigger, is implemented in hardware. Further stages: HLT1, HLT2 and the Stripping, are implemented using the same software that is used for simulation, reconstruction and analysis of LHCb data.

Chapter 3

Signal and Background

The requirements made to distinguish $B \rightarrow DK^*$ signal candidates from background, and their motivation, are discussed. These include requirements made on the trigger decision, those made by the Stripping and a multivariate classifier. Studies of specific backgrounds are presented and these are removed with specially tailored requirements, where necessary. Finally, the efficiencies of all the requirements, on signal $B \rightarrow DK^*$ decays, are evaluated with a combination of real data and simulation based methods.

3.1 The data

The analysis described in this thesis uses $1.02 \pm 0.04 \text{ fb}^{-1}$ of data recorded at the LHCb experiment in pp collisions at a centre-of-mass energy of $\sqrt{s} = 7 \text{ TeV}$ during the year 2011 (60% with the magnet polarity down and 40% with the magnet polarity up). In addition, $2.06 \pm 0.10 \text{ fb}^{-1}$ of data recorded at a centre-of-mass energy of $\sqrt{s} = 8 \text{ TeV}$ during the year 2012 (50% with polarity down and 50% with polarity up) are used. These data were taken in stable running conditions, with a mean number of visible interactions per bunch-crossing of 1.5 in 2011 and 1.7 in 2012¹.

Simulated data are used throughout the analysis, to evaluate efficiencies and to model invariant mass distributions. The simulation software package, GAUSS, generates pp collisions using PYTHIA [58, 59] with a specific LHCb configuration [64]. Decays of hadrons are

¹An interaction is defined to be visible if it produces at least two charged particles with sufficient hits in the VELO and tracking stations T1-T3 to allow them to be reconstructable.

described by EVTGEN [65], in which final state radiation is generated using PHOTOS [66]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [60, 61] as described in Ref. [67]. All simulated data assume a centre-of-mass energy of $\sqrt{s} = 8 \text{ TeV}$ and that the differences in $B \rightarrow DK^*$ decays occurring in pp collisions at centre-of-mass energies of 8 TeV and 7 TeV are negligible, since the mass of the B mesons that are studied is negligible compared to the centre-of-mass energy.

3.2 Candidate selection

The $B \rightarrow DK^*$ candidates used to measure CP violation in Chapter 4 are selected from combinations of charged hadrons reconstructed in the detector. Two different methods have been developed to identify $B \rightarrow DK^*$ candidates. The first method, involving purely univariate selection requirements, was developed for the first analysis of CP violation in $B \rightarrow DK^*$ decays at LHCb [3]. This analysis used the data collected by LHCb in 2011. The updated analysis of $B \rightarrow DK^*$ decays presented here utilises three times the amount of data (everything collected in Run I of the LHC) and a different, multivariate, selection strategy. All of the measurements made in Ref. [3] are updated in this thesis. Therefore, the candidate selection designed for the latter, more complete, analysis is documented.

3.2.1 Trigger and Stripping

Three categories of $B \rightarrow DK^*$ candidates are defined, regarding the reasons that the event containing the candidate passed the L0 trigger. The categories are named “TOS”, which is an abbreviation of “Trigger On Signal”, “TIS”, which is an abbreviation of “Trigger Independent of Signal” and “TOB”, which is an abbreviation of “Trigger On Both”. The categories are defined below.

- The “TOS” category contains candidates from events that passed the L0 hadron trigger.

Furthermore, the calorimeter cluster associated with the L0 hadron decision is required

to be matched to one of the final state tracks of the $B \rightarrow DK^*$ candidate.

- The “TIS” category contains candidates from events that passed any L0 trigger (L0 hadron, electron, photon, muon or dimuon). However, it is required that the detector object associated with this L0 decision is not matched to any of the final state tracks of the $B \rightarrow DK^*$ candidate.
- The “TOB” category contains candidates from events that passed the L0 trigger because of a combination of the signal candidate and the rest of the event. For example, a final state track of the $B \rightarrow DK^*$ candidate combined with another random track that together produced a calorimeter cluster that passed the L0 hadron trigger, while neither individual track would have produced a calorimeter cluster of sufficient energy.

Candidates in the TOS and TIS categories are retained and candidates in the TOB category are excluded.

Requirements are also made on the reasons that the event containing the candidate passed the HLT. The event is required to have passed HLT1 by virtue of containing a track with a transverse momentum above a certain threshold and a large impact parameter (IP) with respect to the primary vertex (PV). The event is also required to have passed the “topological” trigger line in HLT2. This trigger line accepts events based on the value of a multivariate classifier designed to identify two, three or four track combinations with an invariant mass that is close to the mass of a B meson.

In the first stage of offline processing, the candidates from events retained by the trigger are subject to the Stripping, which is described in Section 2.2.8. The requirements made by the Stripping on $B \rightarrow DK^*$ candidates are detailed in Table 3.1, where some of the variables need to be explained.

- θ_{dira} is the angle between the momentum of the particle and its direction of flight, determined from the vector joining its production and decay vertices,
- **Min IP** χ^2 is the minimum χ^2 calculated from the distance between the trajectory of

the particle and any PV,

- **χ^2 distance to PV** is the χ^2 calculated from the distance between the decay vertex of the particle and the PV of the signal candidate,
- **$M_0(D^0)$** signifies the current average of all measurements of the D^0 meson mass, taken from Ref. [6], and
- **Stripping BDT output** refers to the value of a multivariate classifier that is designed to enrich the data with B mesons and is independent of the algorithm described in the next section.

Table 3.1: Requirements imposed by the Stripping on $B \rightarrow DK^*$ candidates.

Meson	Variable	Threshold
K, π	p_T	$> 100 \text{ MeV}/c$
	p	$> 1000 \text{ MeV}/c$
	Track χ^2/ndf	< 3
	Min IP χ^2	> 4
D	$p_T(h) + p_T(h')$	$> 1.8 \text{ GeV}/c$
	DOCA(h, h')	$< 0.5 \text{ mm}$
	Vertex χ^2/ndf	< 10
	χ^2 distance to PV	> 36
	$\cos(\theta_{\text{dira}})$	> 0
	$ M(hh') - M_0(D^0) $	$< 100 \text{ MeV}/c^2$
K^*	$p(K_{K^*})$	$> 2000 \text{ MeV}/c$
	$p(\pi_{K^*})$	$> 2000 \text{ MeV}/c$
	DOCA(K, π)	$< 0.5 \text{ mm}$
	Vertex χ^2/ndf	< 16
	χ^2 distance to PV	> 16
	$\cos(\theta_{\text{dira}})$	> 0
	$p_T(K) + p_T(\pi)$	$> 1000 \text{ MeV}/c$
	$M(K\pi)$	$< 5.2 \text{ GeV}/c^2$
B	DOCA(D, K^*)	$< 1 \text{ mm}$
	$p_T(D) + p_T(K^*)$	$> 5 \text{ GeV}/c$
	Vertex χ^2/ndf	< 10
	Min IP χ^2	< 25
	τ	$> 0.2 \text{ ps}$
	$\cos(\theta_{\text{dira}})$	> 0.999
	$M(DK^*)$	$\in [4.75, 6] \text{ GeV}/c^2$
	Stripping BDT output	> 0.05

3.2.2 Boosted decision tree

Candidates that are retained by the Stripping are processed by a Boosted Decision Tree (BDT) [68], used with the Gradient Boost algorithm [69], to further differentiate signal $B \rightarrow DK^*$ decays from background. Separate BDTs are developed for $B \rightarrow D(K^\pm\pi^\mp)K^*$, $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ decays, where $B \rightarrow D(K^\pm\pi^\mp)K^*$ signifies both $B \rightarrow D(K\pi)K^*$ and $B \rightarrow D(\pi K)K^*$ decays.

A BDT is an example of a supervised machine learning algorithm. The data samples used for training the BDTs are simulated signal decays and candidates from the upper “sideband” of the B meson invariant mass distribution in real data. The latter is intended to represent combinatorial background in the signal region. The upper sideband is defined as candidates that have an invariant mass outside the range that is used for the invariant mass fit described in Chapter 4, namely $M(B) > 5.8 \text{ GeV}/c^2$.

In summary, there are three independent datasets. These are the signal and background training samples and the sample of $B \rightarrow DK^*$ candidates used in the measurement (henceforth known as “the analysis dataset”). Selection requirements are applied to these data before training, to make them more representative of the $B \rightarrow DK^*$ decays that the BDTs are designed to identify.

The invariant masses of the D and K^* mesons are required to be within $20 \text{ MeV}/c^2$ and $50 \text{ MeV}/c^2$ of the measured masses, respectively, where $M_0(D^0) = 1864.86 \pm 0.13 \text{ MeV}/c^2$ and $M_0(K^{*0}) = 895.81 \pm 0.19 \text{ MeV}/c^2$ [6]. Also, the modulus of the cosine of the helicity angle, θ^* , is required to be greater than 0.4. The helicity angle is defined as the angle between the direction of the daughter kaon of the K^* in the K^* rest frame, and the direction of the K^* in the B rest frame. The $|\cos(\theta^*)|$ distribution is peaked at one (the kaon is preferentially emitted forward or backward) because of angular momentum conservation in the decay of the K^* meson, a vector particle.

Many configurations of the BDTs have been tested. For example, different boosting algorithms, such as AdaBoost [70], and many different combinations of input variables

have been investigated. The configuration and input variables with the most power in discriminating signal from combinatorial background is used. The properties of candidates used by the BDTs are listed in Table 3.2: the transverse momentum of the particles in the final state, the D and B vertex fit χ^2/ndf , the $\text{IP}\chi^2$ of the K^* , D and B , the cosine of the θ_{dira} of the B and the sum of the square roots of the $\text{IP}\chi^2$ of the four charged tracks with respect to the PV. Despite their importance for background reduction, particle identification quantities are not used, due to the knowledge that their distributions are inadequately described by simulated data.

The distributions of the BDT input variables in the signal and background samples used for training the $B \rightarrow D(K^\pm\pi^\mp)K^*$ BDT are shown in Figure 3.1, where K_D , for example, signifies the kaon originating from the D meson and similar notation is used for the other final state particles. The correlations between these variables are shown in Figure 3.2, for both signal and background training samples. As expected, large correlations exist between the impact parameter variables, namely between the sum of the square roots of the $\text{IP}\chi^2$ of the four charged tracks and the $\text{IP}\chi^2$ of the D and K^* mesons. Correlations between the input variables give a multivariate selection algorithm an advantage over a conventional univariate selection as it is able to take these correlations into account and exclude regions of the parameter space with non-linear boundaries.

The analysis dataset is split into four subsamples, corresponding to the final state of the $B \rightarrow DK^*$ decay. Explicitly, there are subsamples containing $B \rightarrow D(K\pi)K^*$, $B \rightarrow D(\pi K)K^*$, $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ candidates. Each subsample is processed by the relevant BDT, which assigns each candidate a classifier value.

The suppressed $B_d \rightarrow D(\pi K)K^*$ decay is thus far unobserved with LHCb data, therefore, to reduce the risk of bias, a blind analysis has been performed. The invariant mass distribution in the B_d signal region and the number signal candidates in this channel has not been inspected until the final stages of the analysis. A lack of access to this information renders the determination of an optimal value of a threshold on the classifier of this BDT more difficult. However, an estimation is made using the information available.

Table 3.2: Information available to the BDTs.

Meson	Variable
K, π	p_T
D	Vertex fit χ^2/ndf $\text{IP}\chi^2$
K^*	$\text{IP}\chi^2$
B	Vertex fit χ^2/ndf $\text{IP}\chi^2$ $\sum_{\text{tracks}} \sqrt{\text{IP}\chi^2}$ $\cos(\theta_{\text{dira}})$

To determine the optimal classifier threshold, the expected numbers of signal candidates, S , and background candidates, B , in the $B_d \rightarrow D(\pi K)K^*$ signal region are estimated. The background level is estimated by extrapolating the distribution in the range $[5.5, 5.8] \text{ GeV}/c^2$. It is assumed that the only contribution in this region is combinatorial background and that it is described by a first order polynomial function. S is estimated from the total number of candidates, in the region of the B_s^0 mass, for $B_s \rightarrow D(\pi K)K^*$ decays minus the number of extrapolated background candidates. The total minus the expected background, $N(B_s \rightarrow D(\pi K)K^*)$, is scaled according to Equation 3.1.

$$N(B_d \rightarrow D(\pi K)K^*) = S = N(B_s \rightarrow D(\pi K)K^*) \times \frac{f_d}{f_s} \times \frac{|V_{cb}^* V_{us}|^2}{|V_{cb}^* V_{ud}|^2} \times r_B^2 \quad (3.1)$$

The ratio of hadronisation fractions of b quarks, f_d/f_s , is $0.267^{+0.021}_{-0.020}$ [71], the squared ratio of CKM matrix elements is approximately 0.05 and r_B is approximately 0.27. It is recalled that the last estimate comes from the measurement of the equivalent parameter in $B^\pm \rightarrow DK^\pm$ decays and the colour-suppression of both $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays.

Two figures of merit are defined to assess the optimal threshold on this classifier. These are $S/\sqrt{S+B}$ and $S/(\sqrt{B} + a/2)$, where S and B correspond to the numbers of signal and background candidates expected in 3 fb^{-1} of data and a is the expected significance of the signal, in terms of standard deviations, which is set equal to 5. $S/\sqrt{S+B}$ is a standard

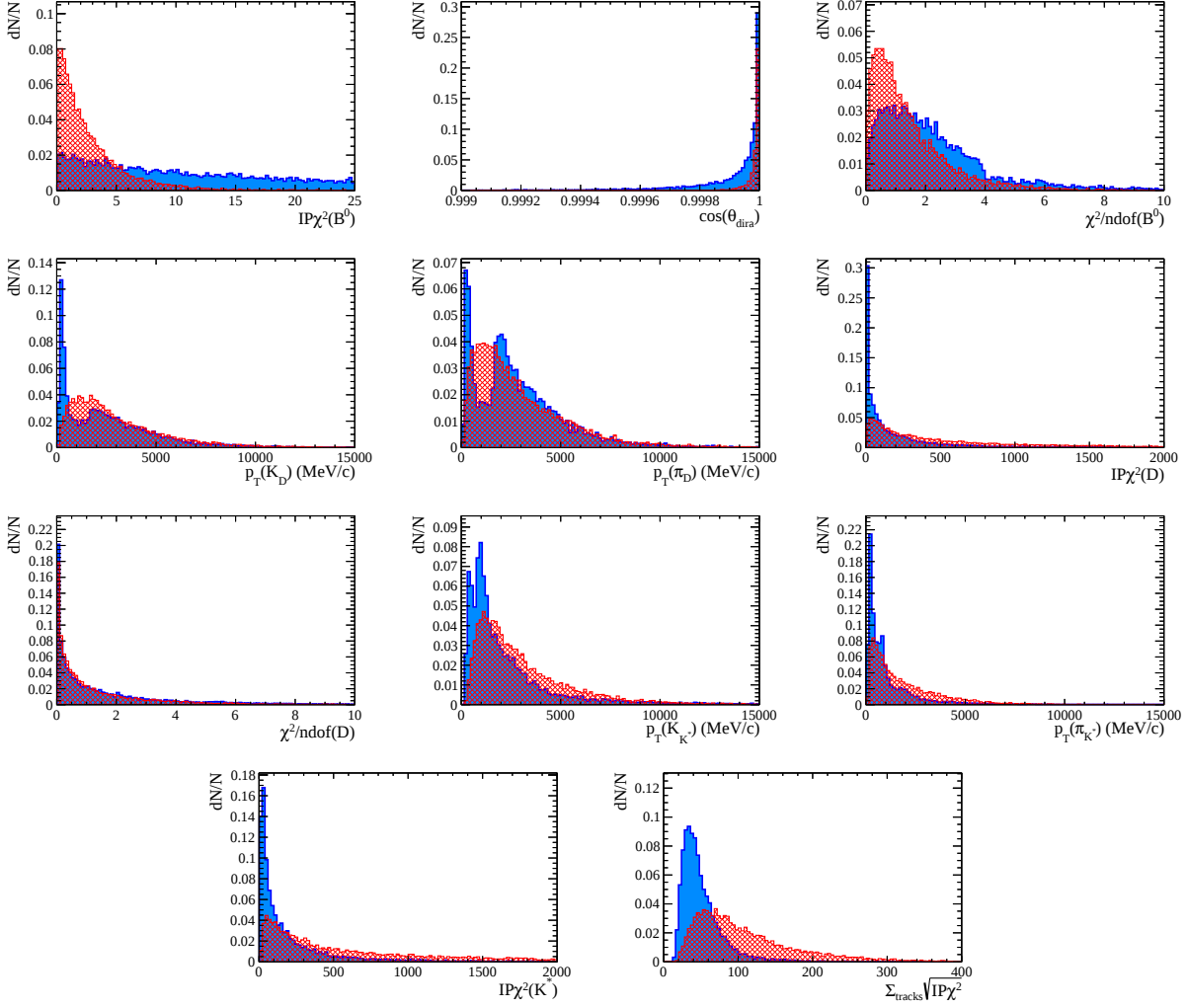


Figure 3.1: The distributions of the variables input to the $B \rightarrow D(K^\pm \pi^\mp) K^*$ BDT in the signal training sample (*red*) and background training sample (*blue*). The histograms are normalised to one. Analogous variables are input to the BDTs used to identify $B \rightarrow D(KK) K^*$ and $B \rightarrow D(\pi\pi) K^*$ decays.

measure of signal significance and $S/(\sqrt{B} + a/2)$ is the Punzi figure of merit [72]. Both figures of merit are shown as a function of the threshold on the BDT classifier in Figure 3.3, where additional requirements, that are described in subsequent sections, are applied to the data prior to estimating S and B . A threshold on the BDT classifier value of greater than 0.6 is chosen since this is favoured by both figures of merit.

The BDTs used to identify $B \rightarrow D(KK) K^*$ and $B \rightarrow D(\pi\pi) K^*$ decays are trained in a similar way, using analogous input variables. The figure of merit, $S/\sqrt{S+B}$, with respect to the $B_d \rightarrow D(KK) K^*$ and $B_d \rightarrow D(\pi\pi) K^*$ signals, is computed as a function of the requirement placed on the relevant BDT classifier. S and B are determined using the same

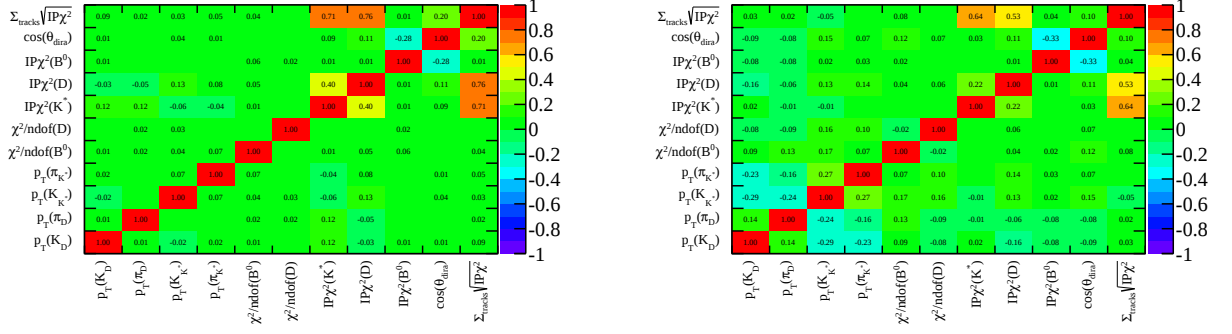


Figure 3.2: The correlations of the variables input to the $B \rightarrow D(K^\pm\pi^\mp)K^*$ BDT in the signal training sample (*left*) and the background training sample (*right*).

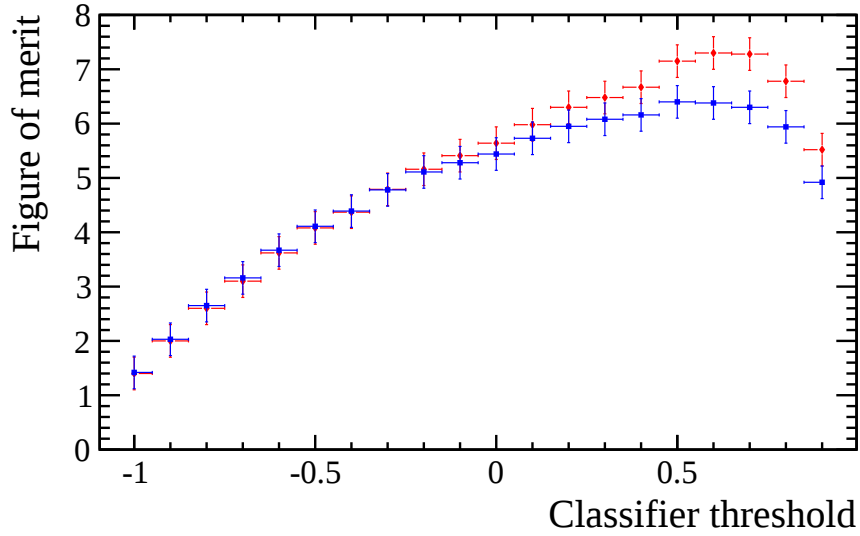


Figure 3.3: Two figures of merit, $S/\sqrt{S+B}$ (*blue squares*) and $S/(\sqrt{B} + a/2)$ (*red diamonds*), as a function of the threshold on the BDT classifier. S and B are the expected numbers of signal and background candidates for the $B_d \rightarrow D(\pi K)K^*$ decay, respectively, in 3fb^{-1} of data.

procedure as for the $B_d \rightarrow D(\pi K)K^*$ signal, except that S is taken directly from the number of candidates in the region of the B^0 meson mass, instead of scaling the number of events in the B_s^0 mass region. This is because the analysis is not blind with respect to these channels as they have been observed before. Requirements on the classifier of greater than 0.4 for $B \rightarrow D(KK)K^*$ decays and greater than 0.5 for $B \rightarrow D(\pi\pi)K^*$ decays are chosen as a result of this study.

Additional requirements on signal candidates are made to reject specific backgrounds or to further enrich the analysis dataset with signal candidates. These requirements and the studies relating to them are described in the following sections.

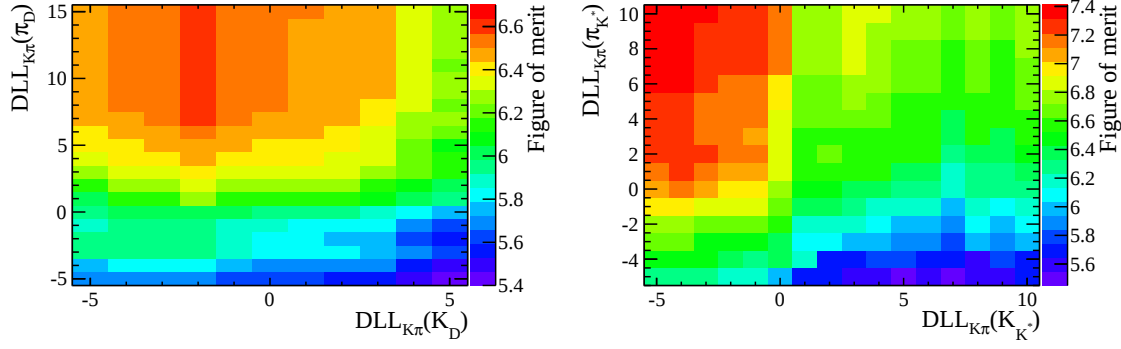


Figure 3.4: The standard figure of merit, $S/\sqrt{S+B}$, as a function of the requirements on the $DLL_{K\pi}$ of the D meson daughters (*left*) and the K^* meson daughters (*right*). S and B are the expected numbers of signal and background candidates for the $B_d \rightarrow D(\pi K)K^*$ decay, respectively, in 3 fb^{-1} of data.

3.2.3 Particle identification

Particle identification criteria are applied to all the final state tracks to distinguish charged pions from charged kaons. To determine the optimal particle identification criteria on the daughter tracks of the D meson in $B_d \rightarrow D(\pi K)K^*$ decays, the standard figure of merit, $S/\sqrt{S+B}$, as a function of the requirements on the $DLL_{K\pi}$ of the tracks is examined in Figure 3.4. Since the daughter tracks are assumed to be kinematically correlated and the response of the RICH detectors has a dependence on the kinematic properties of the tracks, the optimal requirements on each track are assumed correlated and this analysis is performed in two dimensions.

The signal yield, S , of $B_d \rightarrow D(\pi K)K^*$ candidates is estimated using the same procedure as described in Section 3.2.2. The optimal requirement on the kaon track is $DLL_{K\pi}(K_D) > -2$ and the optimal requirement on the pion track is $DLL_{K\pi}(\pi_D) < 8$.

The same study is performed for the $B_d \rightarrow D(KK)K^*$ and $B_d \rightarrow D(\pi\pi)K^*$ decays, where the problem is reduced to one dimension as the D meson decay products are of the same type and mass. It is deemed logical that these tracks have identical PID requirements since, by symmetry arguments, their kinematic properties are the same. The requirements favoured by this study are the same as those used for the $B_d \rightarrow D(\pi K)K^*$ decay, namely $DLL_{K\pi}(K_D) > -2$ and $DLL_{K\pi}(\pi_D) < 8$.

K^* mesons are reconstructed in the $K^* \rightarrow K^\pm \pi^\mp$ decay mode and the optimal PID criteria on the daughter tracks of the K^* are also studied. The study is conducted using only the $B_d \rightarrow D(\pi K)K^*$ channel, since the kinematic properties of the K^* meson are expected to be identical across all the channels analysed. The standard figure of merit, with respect to the $B_d \rightarrow D(\pi K)K^*$ signal, as a function of the requirements on the $\text{DLL}_{K\pi}$ of the K^* meson daughter tracks is shown in Figure 3.4.

The PID requirements favoured by this study are loose: $\text{DLL}_{K\pi} > -4$ on the kaon and $\text{DLL}_{K\pi} < 8$ on the pion. When these requirements are imposed, the expected number of background candidates in the signal region is larger than the expected number of signal candidates. This is undesirable due to the expectation of larger uncertainty in the number of signal candidates when that signal is dominated by background.

Also, there is a specific background to $B \rightarrow DK^*(K^\pm \pi^\mp)$ decays that has been identified from $B_d \rightarrow D\rho^0(\pi^+\pi^-)$ decays. The branching fraction of the $B_d \rightarrow D\rho^0$ decay is larger and the kinematic properties of it are very similar to those of the $B \rightarrow DK^*$ decay as a result of the similar mass of the ρ^0 and K^* mesons. In light of this, the only way to discriminate against this background is through a PID requirement on the kaon from the K^* meson.

Since the optimisation procedure is not sensitive to the background from $B_d \rightarrow D\rho^0$, the conservative requirements on the daughter tracks of the K^* are taken from Ref. [3]. The requirements are $\text{DLL}_{K\pi} > 3$ and $\text{DLL}_{K\pi} < 3$ on the kaon and pion, respectively.

The studies presented in this and the previous section find the optimal selection requirements for $B_d \rightarrow DK^*$ decays. The same requirements are assumed valid for $B_s \rightarrow DK^*$ decays due to their similar kinematic properties.

3.3 Specific backgrounds

3.3.1 Charmless $B \rightarrow hh'K^*$ decays

Peaking backgrounds are anticipated from charmless $B \rightarrow hh'K^*$ decays, where h and h' are kaons or pions. These decays do not proceed via a D meson so involve different quark transitions to signal decays, which means that the associated amplitudes have different weak phases. The D^0 meson has a shorter lifetime than those of B mesons, but it is non-negligible and the accuracy of the VELO allows the flight-distance of the D meson to be resolved. Therefore, to discriminate against charmless B meson decays, candidates with D mesons that have flown in the detector are selected. The D daughters are required to come from a vertex that is displaced from the B decay vertex. The “flight-distance significance” is used to characterise this displacement. The flight-distance significance of the D meson, FDS_D , is given by Equation 3.2, where z_D and z_B are the z positions of the D and B decay vertices, respectively. Their difference is divided by their uncertainties added in quadrature to take the uncertainty in the measurement of the flight-distance into account.

$$\text{FDS}_D = \frac{z_D - z_B}{\sqrt{\sigma(z_D)^2 + \sigma(z_B)^2}} \quad (3.2)$$

To study charmless background and determine the optimal requirement on FDS_D , the requirement on the invariant mass of the D meson is relaxed. A simple model is fit to the D meson invariant mass distribution. The results of this fit are shown in Figure 3.5, which allows the identification of candidates that do not contain real D mesons. These candidates are in the sidebands of the D meson invariant mass, defined as $M(K^\pm\pi^\mp) > 1915 \text{ MeV}/c^2$ for $B \rightarrow D(K^\pm\pi^\mp)K^*$ candidates, $M(KK) < 1835 \text{ MeV}/c^2$ for $B \rightarrow D(KK)K^*$ candidates and $M(\pi\pi) > 1895 \text{ MeV}/c^2$ for $B \rightarrow D(\pi\pi)K^*$ candidates.

The B meson invariant mass distribution of the candidates in the sidebands of D meson invariant mass is examined. A simplified model, compared with that described in Chapter 4 is used to determine yields of decays of B mesons that do not involve D mesons. The model is

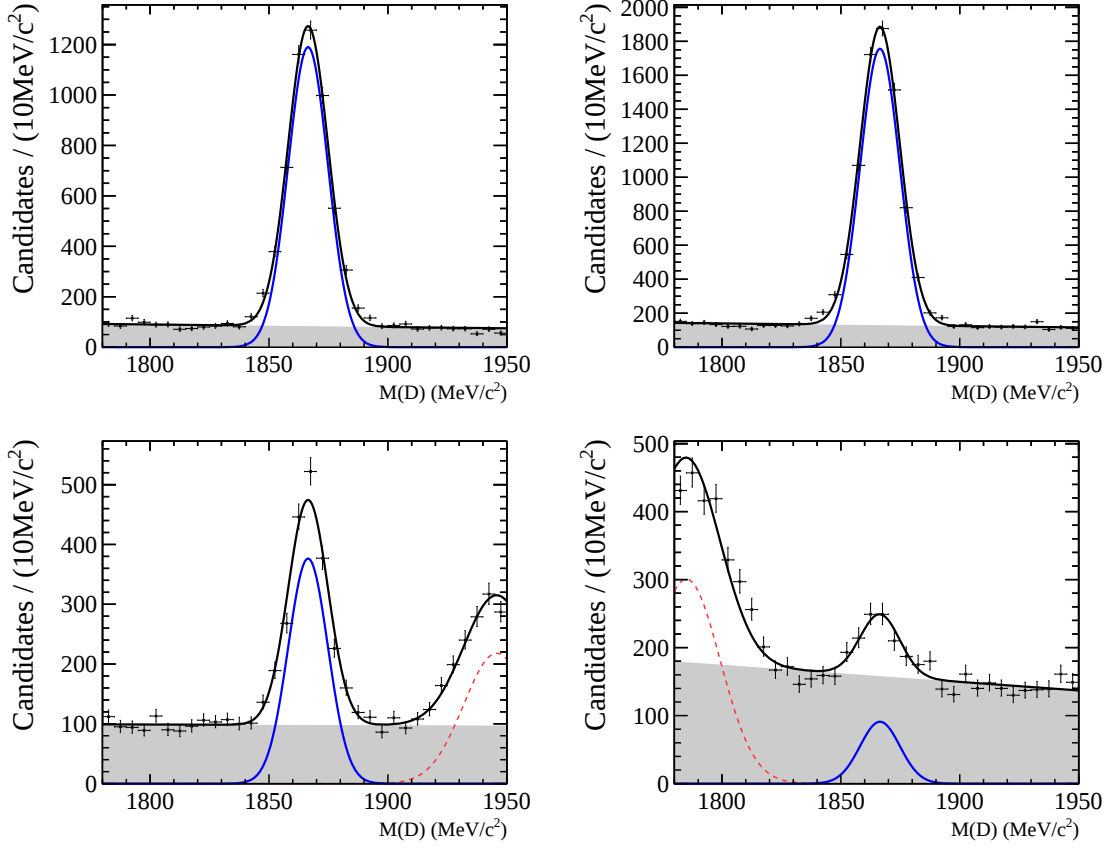


Figure 3.5: The reconstructed D meson invariant mass distributions of candidates subject to no requirement on invariant mass or FDS_D . *Clockwise from top left:* $B \rightarrow D(K\pi)K^*$ candidates, $B \rightarrow D(\pi K)K^*$ candidates, $B \rightarrow D(\pi\pi)K^*$ candidates and $B \rightarrow D(KK)K^*$ candidates. The black line is the result of a maximum-likelihood fit to the distributions, using a linear function to model the combinatorial background (*grey fill*), a Gaussian function to model the signal (*blue line*) and Crystal Ball functions to model the misidentification of $B \rightarrow D(K^\pm\pi^\mp)K^*$ decays as $B \rightarrow D(KK)K^*$ or $B \rightarrow D(\pi\pi)K^*$ candidates (*red dashed line*).

fit to the data with different thresholds placed on FDS_D . Examples of the B meson invariant mass distributions of charmless decay candidates subject to a requirement of $\text{FDS}_D > 0$ and $\text{FDS}_D > 3$ are shown in Figure 3.6 and Figure 3.7, respectively.

It is assumed that the reconstructed D meson invariant mass distribution is uniform for $B \rightarrow hh'K^*$ decays. Therefore the yields of $B \rightarrow hh'K^*$ candidates obtained from the fit to the B meson invariant mass distribution are linearly scaled with the ratio of sizes of the sideband studied to the region allowed by the D meson invariant mass requirement. This allows an estimate of the number of charmless background decays that are expected in the analysis dataset. This scaled yield is shown as a function of the threshold imposed on FDS_D in Figure 3.8 for $B_d \rightarrow hh'K^*$ decays and Figure 3.9 for $B_s \rightarrow hh'K^*$ decays.

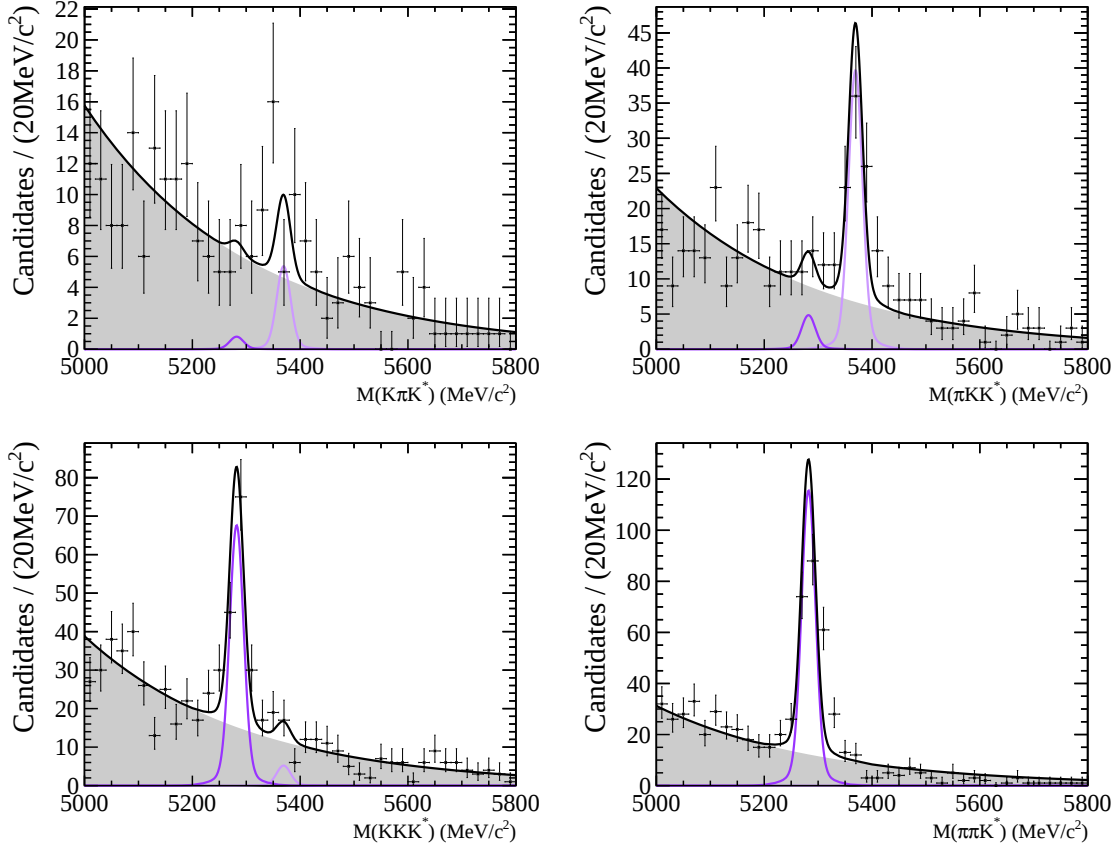


Figure 3.6: The B meson invariant mass distributions of charmless B decay candidates subject to a requirement of $FDS_D > 0$. *Clockwise from top left:* $B \rightarrow K\pi K^*$ candidates, $B \rightarrow \pi K K^*$ candidates, $B \rightarrow \pi\pi K^*$ candidates and $B \rightarrow K K K^*$ candidates. The black line is the result of a maximum-likelihood fit to the distributions, using a exponential function to model the combinatorial background (*grey fill*) and a double Gaussian function to model both the B_d signal (*purple line*) and B_s signal (*light purple line*).

The background from charmless B decays exists in different amounts in subsamples corresponding to different $B \rightarrow DK^*$ signal candidates. In the case of B_d decays, the backgrounds from $B_d \rightarrow \pi\pi K^*$ and $B_d \rightarrow K K K^*$ are the largest and a small amount of background from $B_d \rightarrow \pi K K^*$ also exists. This is explained by the branching fractions of charmless B^0 decays.

Of the relevant, known, charmless B^0 decays, the one with the largest branching fraction is $B^0 \rightarrow K^*(892)^0 \pi^+ \pi^-$, with $\mathcal{B}(B^0 \rightarrow K^*(892)^0 \pi^+ \pi^-) = (5.5 \pm 0.5) \times 10^{-5}$. The second largest is $\mathcal{B}(B^0 \rightarrow K^*(892)^0 K^+ K^-) = (2.75 \pm 0.26) \times 10^{-5}$, followed by the $B^0 \rightarrow K^*(892)^0 K^- \pi^+$ decay, with $\mathcal{B}(B^0 \rightarrow K^*(892)^0 K^- \pi^+) = (4.5 \pm 1.3) \times 10^{-6}$, which is a background to the $B_d \rightarrow D(\pi K) K^*$ signal, recalling that the $K^*(892)^0$ decays to $K^+ \pi^-$. The charmless B^0

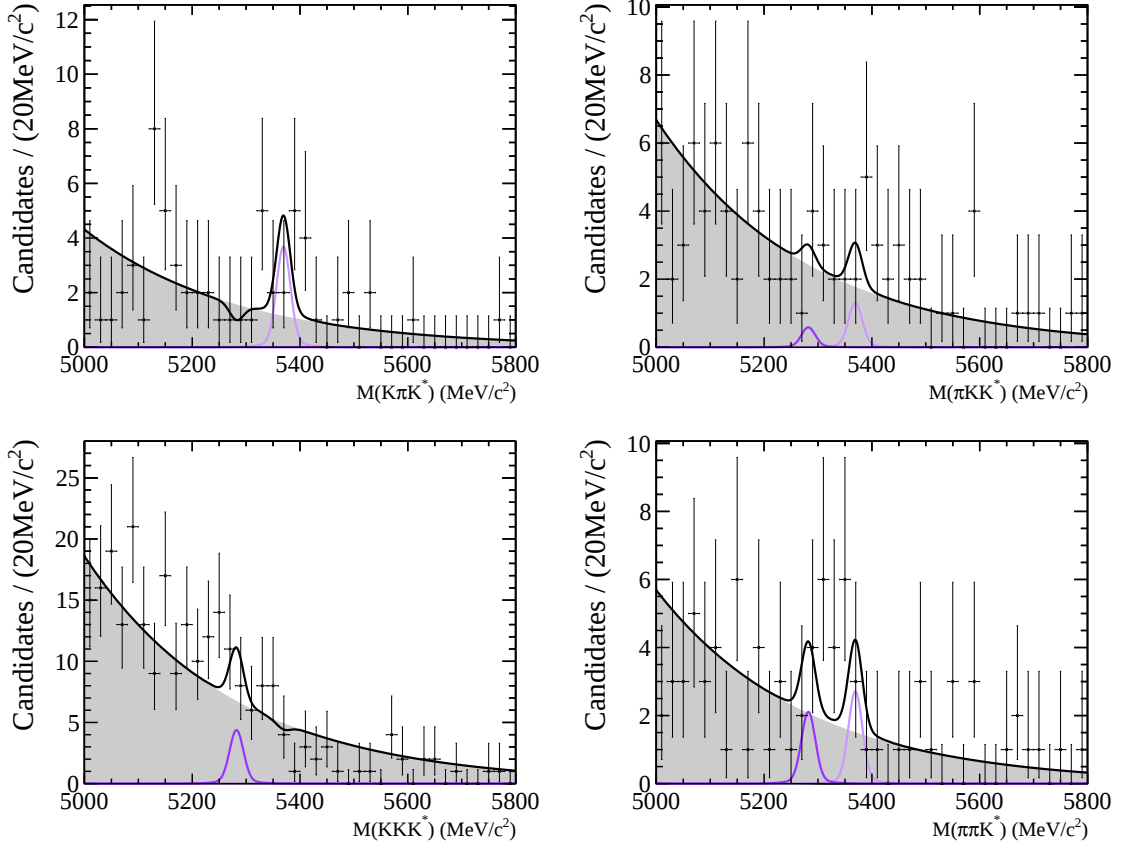


Figure 3.7: The B meson invariant mass distributions of charmless B decay candidates subject to a requirement of $\text{FDS}_D > 3$. *Clockwise from top left:* $B \rightarrow K\pi K^*$ candidates, $B \rightarrow \pi K K^*$ candidates, $B \rightarrow \pi\pi K^*$ candidates and $B \rightarrow K K K^*$ candidates. The black line is the result of a maximum-likelihood fit to the distributions, using a exponential function to model the combinatorial background (grey fill) and a double Gaussian function to model both the B_d signal (purple line) and B_s signal (light purple line).

decay with kaons of the same sign in the final state has not been observed and an upper limit is set of $\mathcal{B}(B^0 \rightarrow K^*(892)^0 K^+ \pi^-) < 2.2 \times 10^{-6}$ at 90% C.L. [6], which explains its absence.

In the case of B_s decays, the only appreciable background from charmless decays is observed in the $B \rightarrow D(\pi K) K^*$ subsample. It is assumed that this contamination is dominated by $B_s \rightarrow \bar{K}^{*0} K^{*0}$, as this decay has the largest branching fraction of all suitable, measured, charmless B_s^0 decays. The branching fraction of this decay is $\mathcal{B}(B_s^0 \rightarrow \bar{K}^{*0}(892)^0 K^{*0}(892)^0) = (2.8 \pm 0.7) \times 10^{-5}$ [6]. The expected number of charmless background decays goes to zero for a stringent requirement on FDS_D . A threshold of greater than three is chosen to remove the background from all types of charmless B decays. This requirement also serves to reduce combinatorial background, as shown by the much lower level of combinatorial background in

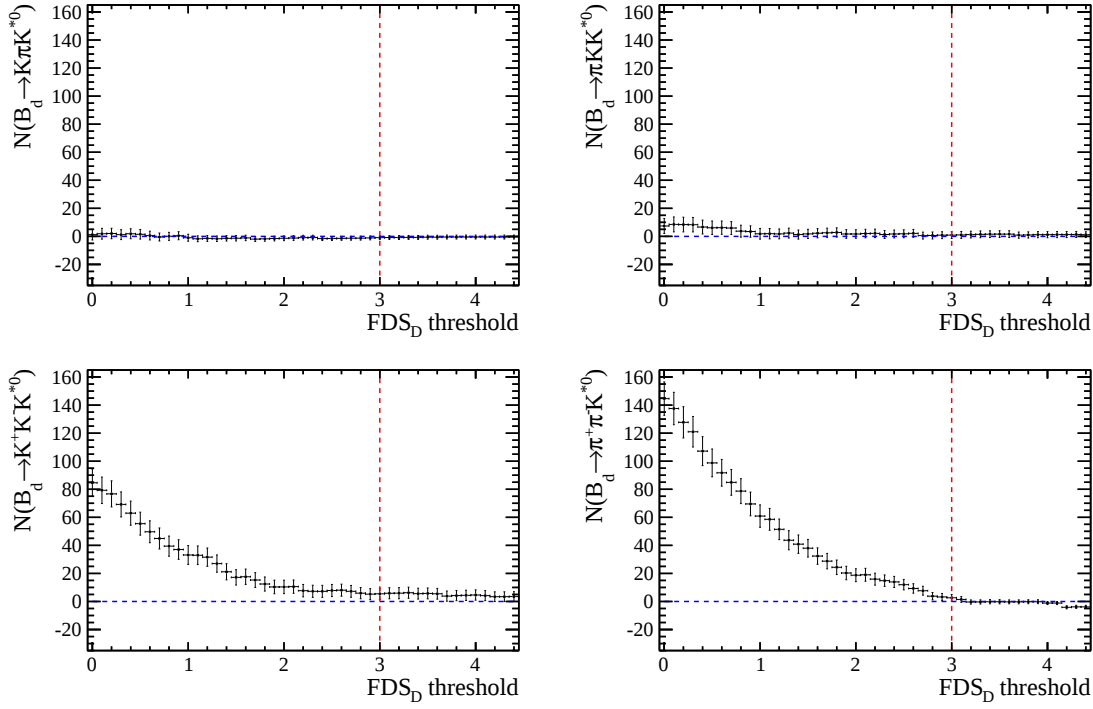


Figure 3.8: Yields of charmless (*clockwise from top left*) $B_d \rightarrow K\pi K^*$, $B_d \rightarrow \pi K K^*$, $B_d \rightarrow \pi\pi K^*$ and $B_d \rightarrow K K K^*$ decays scaled to the D meson invariant mass region as a function of the requirement on FDS_D . The red dashed line indicates the threshold placed on the FDS_D of the $B \rightarrow DK^*$ candidates in the analysis dataset.

Figure 3.10 compared to Figure 3.5.

3.3.2 Misidentification of D meson decays

The PID requirements discussed in Section 3.2.3 do, in principle, allow for the same decay to be reconstructed as more than one of the different signal candidates, since the regions of parameter space allowed by the requirements on each type of candidate overlap. This means that the same decay could be present in more than one subsample. The misidentification of $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ decays as $B \rightarrow D(K^\pm\pi^\mp)K^*$ candidates and vice versa is assessed using the D meson invariant mass distributions in Figure 3.10.

It is clear that no sizeable cross-feed from the $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ subsamples into the $B \rightarrow D(K^\pm\pi^\mp)K^*$ subsamples occurs since no other structures, apart from the signal peak, are present in the reconstructed $D \rightarrow K\pi$ or $D \rightarrow \pi K$ invariant mass distributions.

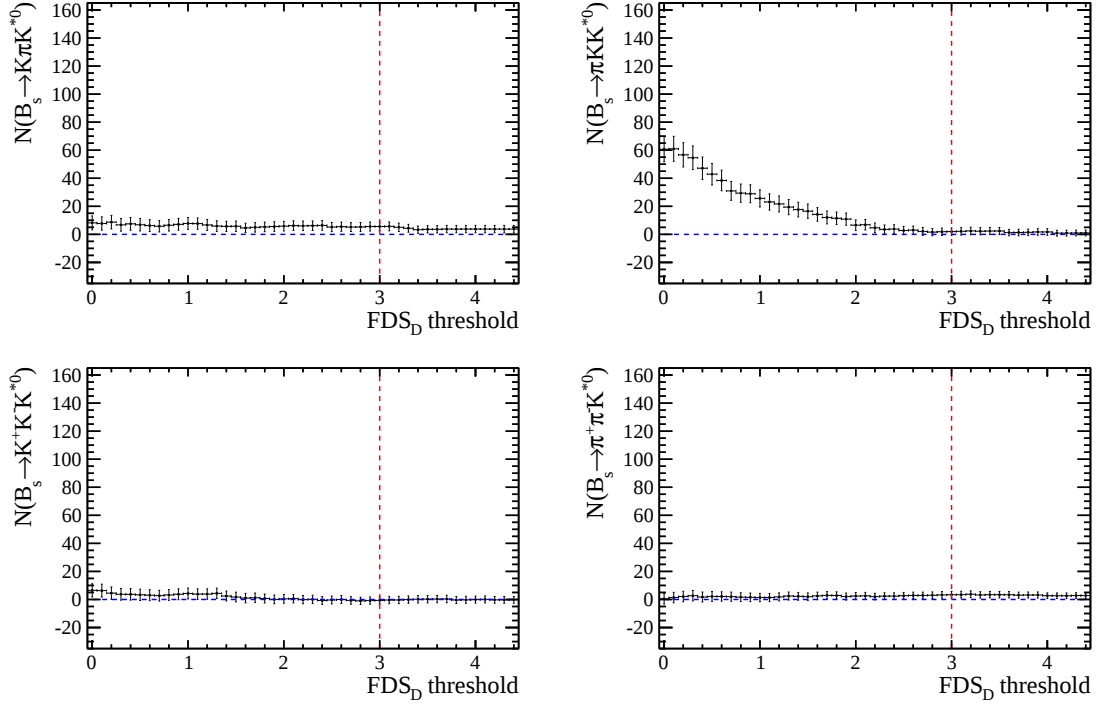


Figure 3.9: Yields of charmless (*clockwise from top left*) $B_s \rightarrow K\pi K^*$, $B_s \rightarrow \pi K K^*$, $B_s \rightarrow \pi\pi K^*$ and $B_s \rightarrow K K K^*$ decays scaled to the D meson invariant mass region as a function of the requirement on FDS_D . The red dashed line indicates the threshold placed on the FDS_D of the $B \rightarrow DK^*$ candidates in the analysis dataset.

There is clearly a non-negligible amount of misidentification of $B \rightarrow D(K^\pm\pi^\mp)K^*$ decays as $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ candidates because of the large structures from misidentified decays occupying the high invariant mass region in the $D \rightarrow KK$ distribution and the low invariant mass region in the $D \rightarrow \pi\pi$ distribution. However, the yield of the misidentified candidates inside the region allowed by the requirement on the D meson invariant mass, $|M(hh') - M_0(D^0)| < 20 \text{ MeV}/c^2$, is of the order of 0.1% of the yield of signal candidates. It is therefore considered negligible.

A potentially dangerous background from the simultaneous misidentification of both the D meson daughters of the $B_d \rightarrow D(K\pi)K^*$ decay exists, because this causes a contamination from the favoured B_d decay in the subsample of $B \rightarrow D(\pi K)K^*$ candidates, which contains candidates for the suppressed B_d decay. To assess the contamination from decays where the kaon from $D \rightarrow K\pi$ is misidentified as a pion and the pion is simultaneously misidentified as a kaon, simulated data are used. Figure 3.11 shows the reconstructed D meson invariant mass distribution of simulated $B_d \rightarrow D(K^\pm\pi^\mp)K^*$ decays when the mass hypotheses of

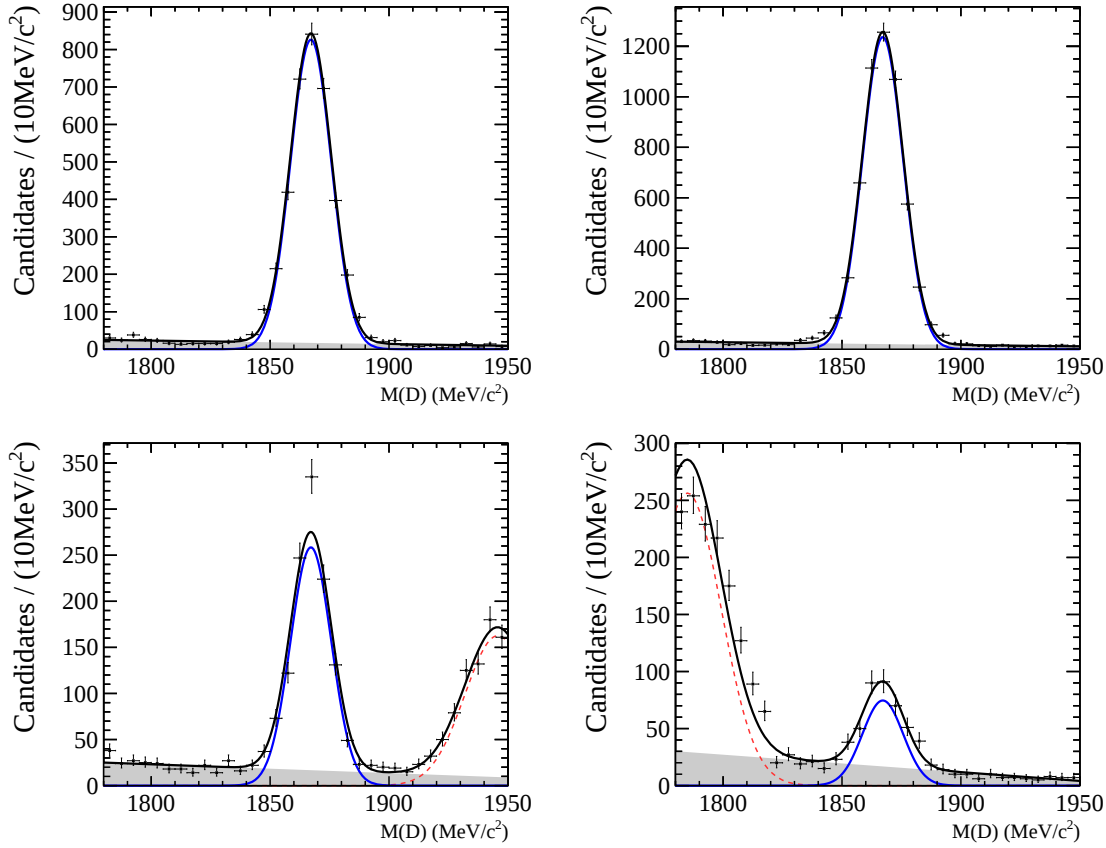


Figure 3.10: The reconstructed D meson invariant mass distributions of candidates subject to a requirement of $FDS_D > 3$ but no requirement on invariant mass. *Clockwise from top left:* $B \rightarrow D(K\pi)K^*$ candidates, $B \rightarrow D(\pi K)K^*$ candidates, $B \rightarrow D(\pi\pi)K^*$ candidates and $B \rightarrow D(KK)K^*$ candidates. The black line is the result of a maximum-likelihood fit to the distributions, using a linear function to model the combinatorial background (*grey fill*), a Gaussian function to model the signal (*blue line*) and Crystal Ball functions to model the misidentification of $B \rightarrow D(K^\pm\pi^\mp)K^*$ decays as $B \rightarrow D(KK)K^*$ or $B \rightarrow D(\pi\pi)K^*$ candidates (*red dashed line*).

the daughter tracks are interchanged, $M_{\text{swap}}(K\pi)$. The distribution is wider than when the daughters have the correct mass hypotheses. However, a non-negligible fraction of candidates still lie within the region allowed by the D meson invariant mass requirement, indicated by the red lines.

The particle identification requirements on the D meson daughter tracks and a requirement of $|M_{\text{swap}}(K\pi) - M_0(D^0)| > 7 \text{ MeV}/c^2$ reduce this background to $(0.80 \pm 0.02)\%$ of the favoured $B_d \rightarrow D(K\pi)K^*$ signal yield. This small amount of contamination is taken into account later.

Misidentification of $B \rightarrow D(KK)K^*$ decays as $B \rightarrow D(\pi\pi)K^*$ candidates, and vice versa, is excluded by the D meson invariant mass requirement.

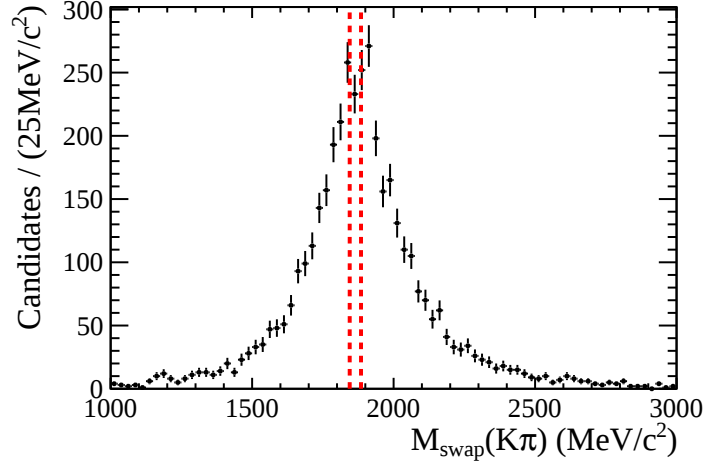


Figure 3.11: The D meson invariant mass distribution of simulated $B_d \rightarrow D(K^\pm\pi^\mp)K^*$ decays, when the pion and kaon mass hypotheses are interchanged. The red dashed lines show the region allowed by the D meson invariant mass requirement imposed on the analysis dataset.

3.3.3 Λ_b decays

Two backgrounds originating from decays of Λ_b baryons are considered. Contamination from $\Lambda_b \rightarrow Dp^\pm\pi^\mp$, where the proton is misidentified as the K_{K^*} , is assessed by examining the invariant mass distribution of $B \rightarrow DK^*$ candidates when the mass hypothesis of the K_{K^*} is changed to that of a proton. These invariant mass distributions are shown in Figure 3.12. It is clear from the excess in the region of the Λ_b^0 mass² in the relevant distribution, and the lack in any other, that the background from $\Lambda_b \rightarrow Dp^\pm\pi^\mp$ decays is only present in a significant amount in the $B \rightarrow D(\pi K)K^*$ subsample. This is expected since the $\Lambda_b \rightarrow D(K^\mp\pi^\pm)p^\pm\pi^\mp$ decay is favoured relative to the $\Lambda_b \rightarrow D(K^\pm\pi^\mp)p^\pm\pi^\mp$ decay. This background is removed by requiring that $\text{DLL}_{pK}(K_{K^*}) < 10$, as shown in Figure 3.12. There is no excess in the region of the Λ_b^0 mass after making this requirement.

The second background considered comes from $\Lambda_b \rightarrow \Lambda_c^\pm(p^\pm K^\mp\pi^\pm)\pi^\mp$ decays, when the proton from the $\Lambda_c^\pm$ decay is misidentified as a kaon. This background can clearly only be present in the $B \rightarrow D(\pi K)K^*$ subsample since the proton and kaon in the final state of the $\Lambda_c^\pm$ decay are of opposite electric charge. Therefore, to assess this background the invariant mass distribution of $B \rightarrow D(\pi K)K^*$ candidates is inspected, changing the mass hypothesis of each kaon to a proton in turn. These distributions are shown in Figure 3.13.

² $M_0(\Lambda_b^0) = 5619.4 \pm 0.6 \text{ MeV}/c^2$ [6].

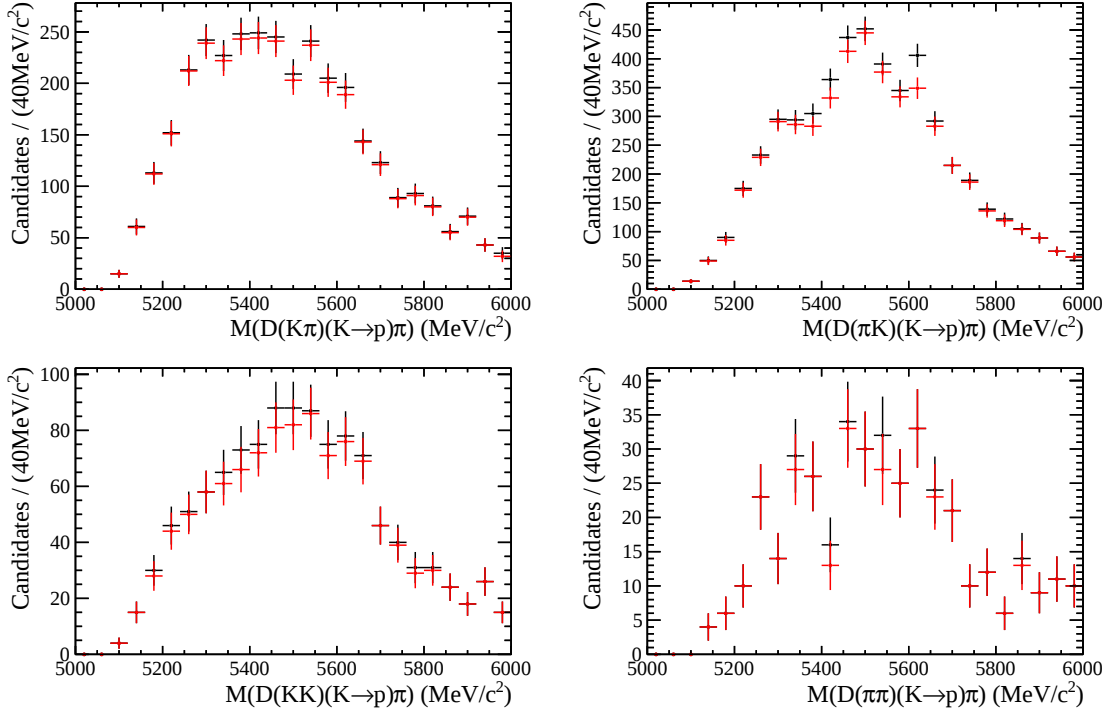


Figure 3.12: The invariant mass distribution of (*clockwise from top left*) $B \rightarrow D(K\pi)K^*$, $B \rightarrow D(\pi K)K^*$, $B \rightarrow D(\pi\pi)K^*$ and $B \rightarrow D(KK)K^*$ candidates, when the mass hypothesis of the daughter kaon of the K^* is changed to that of a proton. The black points are the data with all selection requirements applied apart from $DLL_{pK}(K_{K^*}) < 10$ and the red points are the data with this requirement applied. The disappearance of a small excess in the region of Λ_b^0 mass is observed in the distribution of $B \rightarrow D(\pi K)K^*$ candidates (*top right*) after applying this requirement.

The information given by Figure 3.13 is inconclusive with regard to the presence of this background. Therefore, all combinations of the three-hadron invariant mass of these candidates are inspected in Figure 3.14. The invariant mass distributions of three-hadron combinations that involve the daughter pion of the D meson in Figure 3.14 reside well away from the Λ_c^+ mass. Therefore these combinations cannot be the $\Lambda_c^\pm$ decay products. The distribution of $M(K_D(K_{K^*} \rightarrow p)\pi_{K^*})^3$ is populated in the region of the Λ_c^+ mass, however the lack of any structure in this distribution confirms that this is not the route by which this background is misidentified as the signal.

It is clear from the excess in the region of the Λ_c^+ mass, which is $M_0(\Lambda_c^+) = 2286.46 \pm 0.14 \text{ MeV}/c^2$ [6], in the distribution of $M((K_D \rightarrow p)K_{K^*}\pi_{K^*})$ in Figure 3.14 that this background is present in a small amount in the $B \rightarrow D(\pi K)K^*$ subsample and arises when the

³This notation is used to mean the invariant mass of the K_D , π_{K^*} and K_{K^*} when the mass hypothesis of the K_{K^*} is changed to that of a proton.

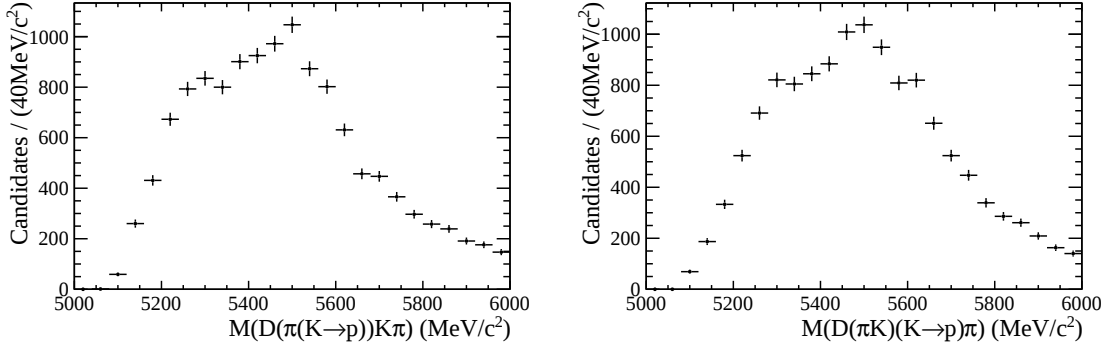


Figure 3.13: The invariant mass distribution of $B \rightarrow D(\pi K)K^*$ candidates, when the mass hypothesis of the daughter kaon of the D meson (*left*) and the daughter kaon of the K^* meson (*right*) is changed to that of a proton. All selection requirements have been applied to the data, apart from the $\text{FDS}_D > 3$ requirement.

proton from the $\Lambda_c^\pm$ decay is misidentified as the kaon from the D meson and the pion from the $\Lambda_c^\pm$ decay is misidentified as the pion from the K^* decay.

The background from $\Lambda_b \rightarrow \Lambda_c^\pm (p^\pm K^\mp \pi^\pm) \pi^\mp$ is eliminated by the requirement on FDS_D , as shown by the absence of the excess around $M_0(\Lambda_c^+)$ in the distribution of $M((K_D \rightarrow p)K_{K^*}\pi_{K^*})$ in Figure 3.14 after applying this requirement. This is expected because the $\Lambda_c^\pm \rightarrow p^\pm K^\mp \pi^\pm$ decay has a different topology to the signal, with three charged tracks originating from the $\Lambda_c^\pm$ decay vertex.

3.3.4 $B \rightarrow D_{(s)}^{(*)\pm} h^\mp$ decays

Specific backgrounds involving charged $D_{(s)}^{(*)}$ mesons, in particular from $B \rightarrow D_{(s)}^{(*)\pm} \pi^\mp$ or $B \rightarrow D_{(s)}^{(*)\pm} K^\mp$ decays, are assessed by studying the invariant mass of three out of the four charged tracks of $B \rightarrow DK^*$ candidates. The measured masses of the D mesons involved in these decays are $M_0(D^+) = 1869.62 \pm 0.15 \text{ MeV}/c^2$, $M_0(D_s^+) = 1968.50 \pm 0.32 \text{ MeV}/c^2$, $M_0(D^{*+}) = 2010.29 \pm 0.13 \text{ MeV}/c^2$ and $M_0(D_s^{*+}) = 2112.3 \pm 0.5 \text{ MeV}/c^2$ [6].

The large majority of these backgrounds are found to be negligible, as shown by the lack of structure in the distributions in Figures 3.15 to 3.17. The only indication of contamination from this source is from $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$ decays in the $B \rightarrow D(\pi\pi)K^*$ subsample, as shown by the small excesses at the D^+ meson mass in Figure 3.17.

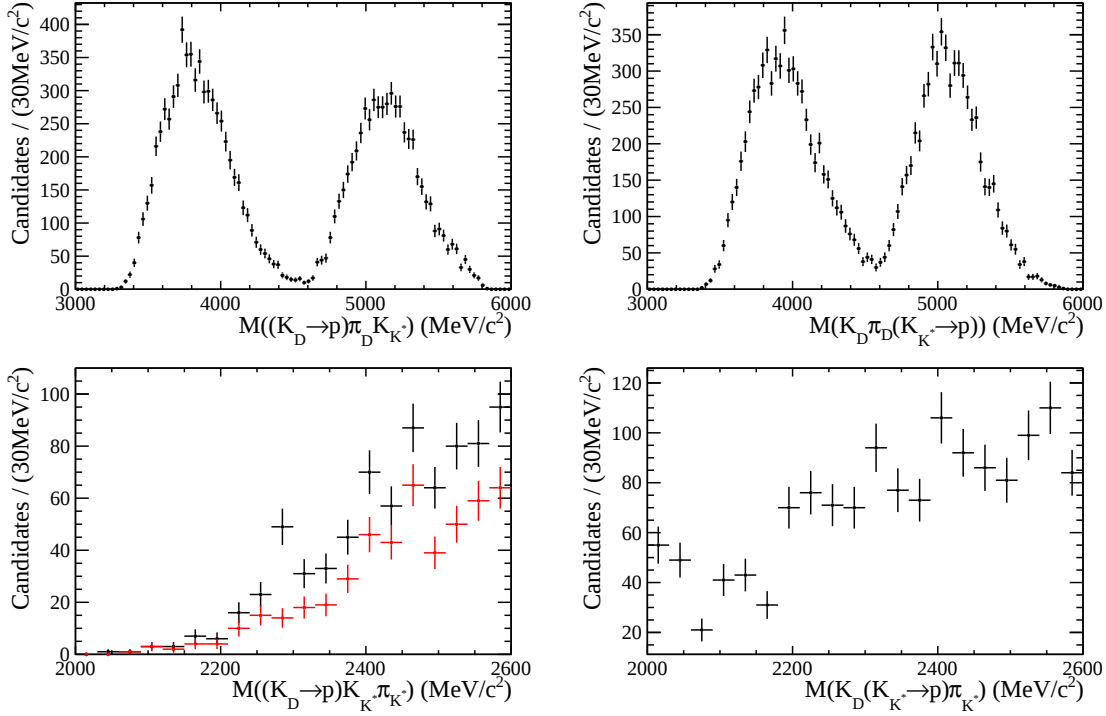


Figure 3.14: The three-hadron invariant mass distributions of $B \rightarrow D(\pi K)K^*$ candidates, when the mass hypothesis of the daughter kaon of the D meson (*left*) and the daughter kaon of the K^* (*right*) is changed to that of a proton. The pions from the D (*top*) and K^* (*bottom*) mesons are used to form the three-hadron combinations with the two kaons. All selection requirements have been applied to the data in the black distributions, apart from the $\text{FDS}_D > 3$ requirement. The red distribution (*bottom left*) has the $\text{FDS}_D > 3$ requirement applied.

Despite the indication that these decays are not a significant background, requirements are still placed on the three-hadron invariant masses that correspond to all Cabibbo-favoured decays of the $D^\pm$ and $D_s^\pm$ that could be involved in $B \rightarrow D_{(s)}^\pm h^\mp$ backgrounds to $B \rightarrow DK^*$ decays, since the efficiency of these requirements on $B \rightarrow DK^*$ decays is approximately 100%. Namely, candidates are rejected when the invariant masses $M(K^\pm \pi^\mp \pi^\mp)$ or $M(K^\mp K^\pm \pi^\mp)$ are within $\pm 15 \text{ MeV}/c^2$ of the measured mass of the $D^\pm$ or $D_s^\pm$ mesons.

3.4 Selection summary

The selection requirements applied to the $B \rightarrow DK^*$ candidates in the analysis dataset are summarised in Table 3.3.

After all the selection requirements are applied to the analysis dataset, 0.9% of candidates

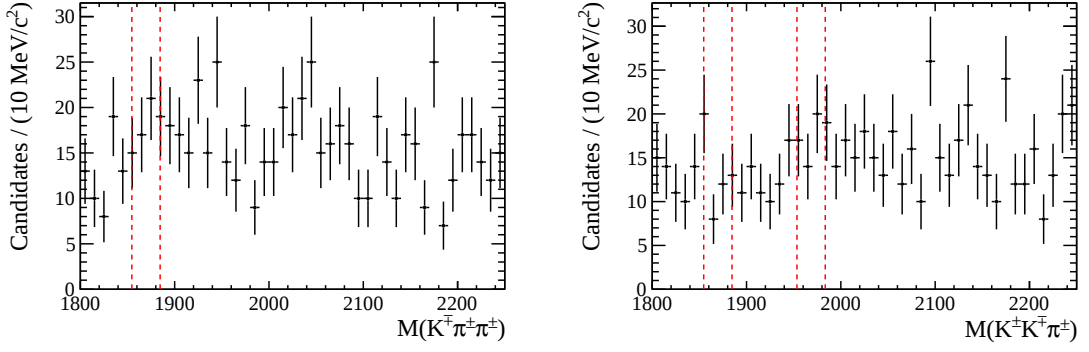


Figure 3.15: Three-hadron invariant mass distributions of $B \rightarrow D(K^\pm \pi^\mp) K^*$ candidates. *Left:* The $K^\mp \pi^\pm \pi^\pm$ invariant mass distribution. *Right:* The $K^\pm K^\mp \pi^\pm$ invariant mass distribution. The regions inside the red dashed lines enclose those candidates that are excluded in order to eliminate backgrounds involving $D^\pm \rightarrow K^\mp \pi^\pm \pi^\pm$, $D^\pm \rightarrow K^\pm K^\mp \pi^\pm$ and $D_s^\pm \rightarrow K^\pm K^\mp \pi^\pm$ decays.

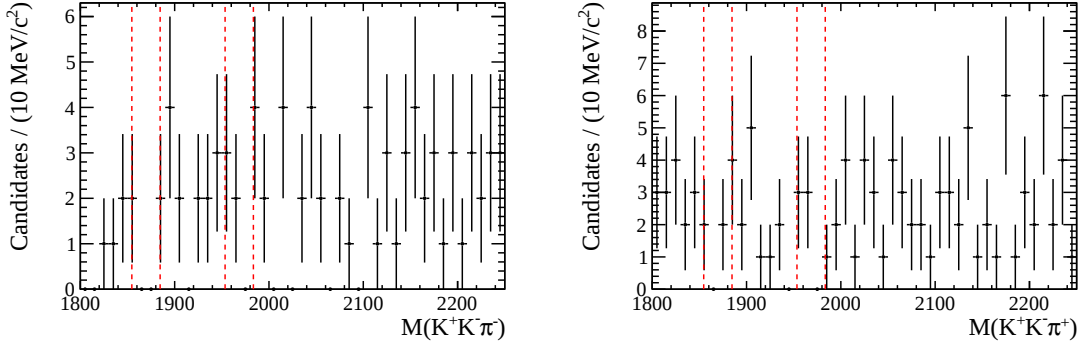


Figure 3.16: Three-hadron invariant mass distributions of $B \rightarrow D(KK) K^*$ candidates. *Left:* The $K^+ K^- \pi^-$ invariant mass distribution. *Right:* The $K^+ K^- \pi^+$ invariant mass distribution. The regions inside the red dashed lines enclose those candidates that are excluded in order to eliminate backgrounds involving $D_{(s)}^- \rightarrow K^+ K^- \pi^-$ and $D_{(s)}^+ \rightarrow K^+ K^- \pi^+$ decays.

are found to have occurred in the same event as another signal candidate. Only one candidate per event is retained. This is because the branching fractions of $B \rightarrow DK^*$ decays [6] are such that the probability of two signal decays in one event is negligible. Therefore, the additional candidates in the event are assumed to be background. In the events with more than one candidate, the candidate with the largest B meson flight-distance significance with respect to the PV is retained. If several PVs are reconstructed, the PV that is used in the calculation of the flight-distance significance is that to which the B meson has the smallest $\text{IP}\chi^2$.

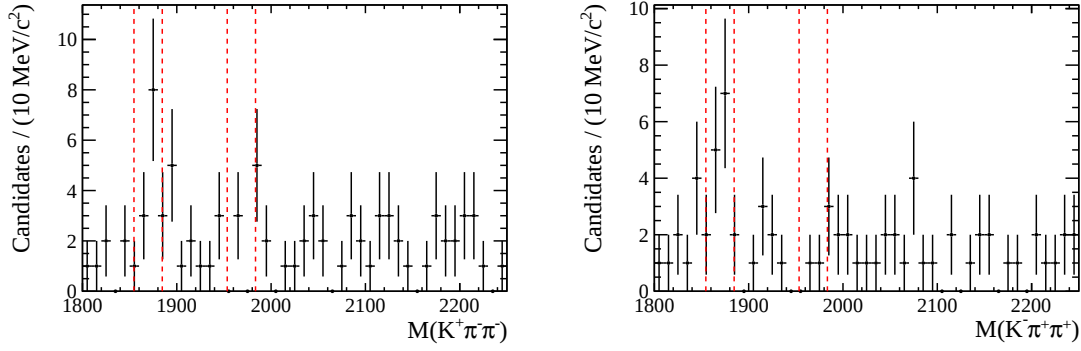


Figure 3.17: Three-hadron invariant mass distributions of $B \rightarrow D(\pi\pi)K^*$ candidates. *Left*: The $K^+\pi^-\pi^-$ invariant mass distribution. *Right*: The $K^-\pi^+\pi^+$ invariant mass distribution. The regions inside the red dashed lines enclose those candidates that are excluded in order to eliminate backgrounds involving $D_{(s)}^- \rightarrow K^+\pi^-\pi^-$ and $D_{(s)}^+ \rightarrow K^-\pi^+\pi^+$ decays.

Table 3.3: Selection criteria for $B \rightarrow D(K\pi)K^*$, $B \rightarrow D(\pi K)K^*$, $B \rightarrow D(KK)K^*$ and $B \rightarrow D(\pi\pi)K^*$ candidates.

Meson	Variable	Threshold
D	$\text{DLL}_{K\pi}(K)$	> -2
	$\text{DLL}_{K\pi}(\pi)$	< 8
	FDS_D	> 3
	$ M(hh') - M_0(D^0) $	$< 20 \text{ MeV}/c^2$
	$ M_{\text{swap}}(K\pi) - M_0(D^0) $ (only $D \rightarrow K^\pm\pi^\mp$)	$> 7 \text{ MeV}/c^2$
K^*	$\text{DLL}_{K\pi}(K)$	> 3
	$\text{DLL}_{K\pi}(\pi)$	< 3
	$\text{DLL}_{pK}(K)$	< 10
	$ M(K\pi) - M_0(K^{*0}) $	$< 50 \text{ MeV}/c^2$
B	$ \cos\theta^* $	> 0.4
	$ M(K^\mp\pi^\pm\pi^\pm) - M_0(D^\pm, D_s^\pm) $	$> 15 \text{ MeV}/c^2$
	$ M(K^\pm K^\mp\pi^\pm) - M_0(D^\pm, D_s^\pm) $	$> 15 \text{ MeV}/c^2$
All	$B \rightarrow D(K^\pm\pi^\mp)K^*$ BDT classifier	> 0.6
	$B \rightarrow D(KK)K^*$ BDT classifier	> 0.4
	$B \rightarrow D(\pi\pi)K^*$ BDT classifier	> 0.5

3.5 Efficiencies

The observables defined in Chapter 1 are measured in Chapter 4 by relating the observed number of signal candidates to the partial width of that decay. The efficiency with which the presented selection requirements reconstruct $B \rightarrow DK^*$ decays directly affects the

number observed. Therefore the efficiencies of the selection requirements on signal decays are evaluated.

The efficiencies on $B \rightarrow DK^*$ signal decays of the selection requirements are split into three parts: the efficiency of kinematic selections, ϵ_{sel} , the efficiency of the PID requirements, ϵ_{PID} , and the efficiency of the requirements on the L0 trigger decisions, ϵ_{L0} . The kinematic selections include the geometric acceptance of LHCb, the HLT, the Stripping, the BDT, the requirements placed on three-hadron invariant mass to remove $B \rightarrow D_{(s)}^{(*)\pm} h^\mp$ backgrounds and the requirement on FDS_D . The distributions of these variables are accurately represented by the simulation software, so ϵ_{sel} is determined using simulated data. The response of the RICH and the behaviour of the L0 trigger are not accurately represented by the simulation software. Therefore, ϵ_{PID} and ϵ_{L0} are determined using real data.

The total efficiency is the product of the three separate efficiencies, $\epsilon_{\text{tot}} = \epsilon_{\text{sel}} \times \epsilon_{\text{PID}} \times \epsilon_{\text{L0}}$.

3.5.1 Kinematic requirements

The efficiencies of the kinematic selection requirements are computed using simulated pp collisions containing signal decays.

The efficiency of a kinematic selection requirement is computed as the ratio of the number of true signal decays in the simulated data sample before and after applying this requirement. The results are shown in Tables 3.4 to 3.7, where the calculation is repeated sequentially for every kinematic selection requirement, meaning that each efficiency is given relative to the preceding row. The efficiencies are determined separately for B and $\bar{B}$ decays.

3.5.2 Particle identification requirements

The efficiencies of the PID requirements are evaluated using the N-track calibration method described in Section 2.2.4. The reference sample used for calibration is simulated signal decays that are subject to all requirements in Table 3.3 apart from those related to PID. The

Table 3.4: Efficiencies of the kinematic selection requirements on $B^0 \rightarrow D(K^\pm\pi^\mp)K^{*0}$ and $\bar{B}^0 \rightarrow D(K^\pm\pi^\mp)\bar{K}^{*0}$ decays. The total number of candidates for $B \rightarrow D(K^\pm\pi^\mp)K^*$ decays in the data sample after the requirement is also shown.

Requirement	B^0 Efficiency	$\bar{B}^0$ Efficiency	Sample size
LHCb acceptance	0.1640 ± 0.0008	0.1639 ± 0.0008	
Reconstruction and Stripping	0.0277 ± 0.0003	0.0268 ± 0.0003	16861179
$ M(K\pi)_D - M_0(D^0) < 20 \text{ MeV}/c^2$	0.949 ± 0.003	0.946 ± 0.003	5610995
$ M(K\pi)_{K^*} - M_0(K^*(892)^0) < 50 \text{ MeV}/c^2$	0.770 ± 0.005	0.767 ± 0.005	1520002
$ \cos\theta^* > 0.4$	0.926 ± 0.004	0.920 ± 0.004	1088667
BDT classifier	0.690 ± 0.006	0.691 ± 0.007	263649
$\text{FDS}_D > 3$	0.705 ± 0.008	0.708 ± 0.008	165516
$ M(h^\pm h'^\mp h''^\pm) - M_0(D^\pm, D_s^\pm) > 15 \text{ MeV}/c^2$	0.987 ± 0.002	0.983 ± 0.003	164128
$ M_{\text{swap}}(K\pi) - M_0(D^0) > 7 \text{ MeV}/c^2$	0.968 ± 0.004	0.969 ± 0.004	149722
Kinematic selection efficiency (ϵ_{sel})	0.00143 ± 0.00003	0.00137 ± 0.00003	

Table 3.5: Efficiencies of the kinematic selection requirements on $B_s^0 \rightarrow D(\pi K)\bar{K}^{*0}$ and $\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}$ decays. The total number of candidates for $B \rightarrow D(K^\pm\pi^\mp)K^*$ decays in the data sample after the requirement is also shown.

Requirement	B_s^0 Efficiency	$\bar{B}_s^0$ Efficiency	Sample size
LHCb acceptance	0.1645 ± 0.0008	0.1647 ± 0.0008	
Reconstruction and Stripping	0.0289 ± 0.0003	0.0290 ± 0.0003	16861179
$ M(\pi K)_D - M_0(D^0) < 20 \text{ MeV}/c^2$	0.934 ± 0.003	0.939 ± 0.003	5610995
$ M(K\pi)_{K^*} - M_0(K^*(892)^0) < 50 \text{ MeV}/c^2$	0.761 ± 0.005	0.760 ± 0.005	1520002
$ \cos\theta^* > 0.4$	0.930 ± 0.003	0.928 ± 0.003	1088667
BDT classifier	0.699 ± 0.006	0.712 ± 0.006	263649
$\text{FDS}_D > 3$	0.704 ± 0.008	0.701 ± 0.008	165516
$ M(h^\pm h'^\mp h''^\pm) - M_0(D^\pm, D_s^\pm) > 15 \text{ MeV}/c^2$	0.987 ± 0.002	0.986 ± 0.002	164128
$ M_{\text{swap}}(K\pi) - M_0(D^0) > 7 \text{ MeV}/c^2$	0.962 ± 0.004	0.968 ± 0.004	149722
Kinematic selection efficiency (ϵ_{sel})	0.00147 ± 0.00003	0.00151 ± 0.00003	

Table 3.6: Efficiencies of the kinematic selection requirements on $B^0 \rightarrow D(KK)K^{*0}$ and $\bar{B}^0 \rightarrow D(KK)\bar{K}^{*0}$ decays. The total number of candidates for $B \rightarrow D(KK)K^*$ decays in the data sample after the requirement is also shown.

Requirement	B^0 Efficiency	$\bar{B}^0$ Efficiency	Sample size
LHCb acceptance	0.1709 ± 0.0008	0.1716 ± 0.0008	
Reconstruction and Stripping	0.0267 ± 0.0003	0.0262 ± 0.0003	10655504
$ M(KK)_D - M_0(D^0) < 20 \text{ MeV}/c^2$	0.976 ± 0.002	0.974 ± 0.002	2115972
$ M(K\pi)_{K^*} - M_0(K^*(892)^0) < 50 \text{ MeV}/c^2$	0.772 ± 0.005	0.770 ± 0.005	571295
$ \cos\theta^* > 0.4$	0.922 ± 0.004	0.932 ± 0.003	403798
BDT classifier	0.714 ± 0.006	0.708 ± 0.006	63336
$\text{FDS}_D > 3$	0.674 ± 0.008	0.695 ± 0.008	28617
$ M(h^\pm h'^\mp h''^\pm) - M_0(D^\pm, D_s^\pm) > 15 \text{ MeV}/c^2$	0.979 ± 0.003	0.976 ± 0.003	28298
Kinematic selection efficiency (ϵ_{sel})	0.00149 ± 0.00003	0.00151 ± 0.00003	

Table 3.7: Efficiencies of the kinematic selection requirements on $B^0 \rightarrow D(\pi\pi)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ decays. The total number of candidates for $B \rightarrow D(\pi\pi)K^*$ decays in the data sample after the requirement is also shown.

Requirement	B^0 Efficiency	$\bar{B}^0$ Efficiency	Sample size
LHCb acceptance	0.1592 ± 0.0008	0.1572 ± 0.0008	
Reconstruction and Stripping	0.0280 ± 0.0003	0.0282 ± 0.0003	5095055
$ M(\pi\pi)_D - M_0(D^0) < 20 \text{ MeV}/c^2$	0.923 ± 0.003	0.921 ± 0.003	895083
$ M(K\pi)_{K^*} - M_0(K^*(892)^0) < 50 \text{ MeV}/c^2$	0.766 ± 0.005	0.771 ± 0.005	235747
$ \cos \theta^* > 0.4$	0.925 ± 0.004	0.925 ± 0.004	164579
BDT classifier	0.752 ± 0.006	0.750 ± 0.006	22046
$\text{FDS}_D > 3$	0.731 ± 0.007	0.733 ± 0.007	7923
$ M(h^\pm h'^\mp h''^\pm) - M_0(D^\pm, D_s^\pm) > 15 \text{ MeV}/c^2$	0.991 ± 0.002	0.987 ± 0.002	7716
Kinematic selection efficiency (ϵ_{sel})	0.00159 ± 0.00003	0.00158 ± 0.00003	

Table 3.8: Efficiencies of the particle identification requirements, ϵ_{PID} .

Decay	B efficiency	$\bar{B}$ efficiency
$B_d \rightarrow D(K^\pm \pi^\mp)K^*$	0.7530 ± 0.0044	0.7519 ± 0.0045
$B_s \rightarrow D(\pi K)K^*$	0.7532 ± 0.0043	0.7463 ± 0.0044
$B_d \rightarrow D(KK)K^*$	0.7592 ± 0.0041	0.7516 ± 0.0042
$B_d \rightarrow D(\pi\pi)K^*$	0.7595 ± 0.0042	0.7625 ± 0.0040

effect of the momentum and the pseudorapidity of the tracks on the response of the RICH detectors is considered. The variation in efficiency over 2011 and 2012 is accounted for and the weighted average efficiencies over this period are shown in Table 3.8.

3.5.3 L0 trigger requirements

The requirements placed on the signal candidates based on the decisions of the L0 trigger form three categories, as described in Section 3.2.1. 64.7% of candidates are in the TOS category, 54.8% of candidates are in the TIS category and 23.4% of candidates are in both these categories. Approximately 3.9% of candidates are in the TOB category and are excluded from the analysis dataset.

The efficiency, on signal decays, of the TOS requirement is evaluated using a similar method to that used to evaluate the efficiencies of PID requirements described in Section 2.2.4.

Table 3.9: Efficiencies of the requirement that signal decays are in the TOS trigger category, ϵ_{TOS} .

Decay	B Efficiency	$\overline{B}$ Efficiency
$B_d \rightarrow D(K^\pm \pi^\mp) K^*$	0.477 ± 0.002	0.490 ± 0.002
$B_s \rightarrow D(\pi K) K^*$	0.488 ± 0.002	0.492 ± 0.002
$B_d \rightarrow D(KK) K^*$	0.492 ± 0.003	0.496 ± 0.003
$B_d \rightarrow D(\pi\pi) K^*$	0.512 ± 0.003	0.502 ± 0.003

Pure samples of kaons and pions are obtained from similar $D^{*\pm} \rightarrow D(K^\pm \pi^\mp) \pi^\pm$ decays. However, the events that contain these decays are required to have passed the L0 muon or dimuon triggers. This ensures that the sample is unbiased by the L0 trigger, since the muon or muons that pass the L0 trigger are not associated with the $D^{*\pm}$ decay.

The efficiency of the L0 hadron trigger on the kaons and pions is then trivially calculated by observing the proportion that have also passed the L0 hadron trigger. This efficiency is assumed to depend on the p_T , type (kaon or pion) and electric charge of the track [56, 73]. In analogy to the method of Section 2.2.4, the distributions of these quantities in the calibration sample and sample of signal $B \rightarrow DK^*$ decays are compared to evaluate the efficiency. The efficiencies on signal decays of the TOS requirement are summarised in Table 3.9.

The efficiency of the overall requirement on the trigger decisions, ϵ_{L0} , is related to the efficiency of the TOS requirement, the efficiency of the TIS requirement, ϵ_{TIS} , and the fraction of events that belong to the TOS category but not to the TIS category, f , by Equation 3.3.

$$\epsilon_{\text{L0}} = (1 - f)\epsilon_{\text{TIS}} + f\epsilon_{\text{TOS}} \quad (3.3)$$

The efficiency of the TIS requirement is assumed to be equal for all signal channels since it does not depend on the tracks that form the signal candidate. The fraction, f , and ϵ_{TIS} are computed using the signal candidates for the favoured $B_d \rightarrow D(K\pi)K^*$ and $B_s \rightarrow D(\pi K)K^*$ decays in the analysis dataset. The fraction is found to be $f = 0.466 \pm 0.008$ and $\epsilon_{\text{TIS}} = 0.376 \pm 0.013$. The resultant values of ϵ_{L0} for each signal channel are given in

Table 3.10.

Table 3.10: Efficiencies of the requirements on L0 trigger decision, ϵ_{L0} .

Decay	B Efficiency	$\bar{B}$ Efficiency
$B_d \rightarrow D(K^\pm \pi^\mp) K^*$	0.423 ± 0.007	0.429 ± 0.007
$B_s \rightarrow D(\pi K) K^*$	0.428 ± 0.007	0.430 ± 0.007
$B_d \rightarrow D(KK) K^*$	0.430 ± 0.007	0.432 ± 0.007
$B_d \rightarrow D(\pi\pi) K^*$	0.439 ± 0.007	0.435 ± 0.007

3.5.4 Total efficiencies

The total selection efficiencies, ϵ_{tot} , equal to the product of the kinematic, PID and trigger selection efficiencies given in the previous sections are shown in Table 3.11, for each signal decay. The differences between the rows in Table 3.11 presumably originate from the different kinematic properties of the decays or a detection asymmetry between kaons and pions, since there are different numbers of each particle in the different final states. No attempt is made to disentangle these effects. Charge detection asymmetry is the assumed cause of the difference between the efficiencies on B and $\bar{B}$ decays.

Table 3.11: Overall efficiencies of the selection requirements, ϵ_{tot} .

Decay	B Efficiency ($\times 10^3$)	$\bar{B}$ Efficiency ($\times 10^3$)
$B_d \rightarrow D(K^\pm \pi^\mp) K^*$	0.457 ± 0.013	0.440 ± 0.012
$B_s \rightarrow D(\pi K) K^*$	0.462 ± 0.013	0.480 ± 0.013
$B_d \rightarrow D(KK) K^*$	0.487 ± 0.013	0.490 ± 0.013
$B_d \rightarrow D(\pi\pi) K^*$	0.530 ± 0.014	0.523 ± 0.014

Chapter 4

Measurement

A maximum-likelihood fit to the B meson invariant mass distributions of $B \rightarrow DK^*$ candidates is discussed. The choices that are made in designing the model are explained and motivated. This model is fit to the data to determine the observed numbers of signal $B \rightarrow DK^*$ decays, from which various CP violation sensitive quantities are measured. The measurement of these quantities also involves several corrections for efficiencies, known branching ratios and other considerations. The studies or origins of these corrections are reported. Finally, a detailed account of the uncertainties assigned to the measurements is given.

4.1 Invariant mass model

The numbers of signal decays necessary to evaluate the CP violation sensitive quantities defined in Chapter 1 are determined by performing a simultaneous maximum-likelihood fit to the B meson invariant mass distributions of the reconstructed $B \rightarrow DK^*$ candidates.

The four subsamples of selected $B \rightarrow DK^*$ candidates are each split into two categories, according to the decay flavour of the B meson. The defined categories and the signal decays that each category contains are given in Table 4.1.

The invariant mass distributions of $B \rightarrow DK^*$ candidates are modelled with a function, which is dependent on the category and is a sum of probability density functions (PDFs) describing the different species (signal and various backgrounds) in that category.

4.1.1 Signal

Several different functions for describing the invariant mass distribution of signal $B \rightarrow DK^*$ candidates have been investigated. Various functions are fit to the B meson invariant mass distribution of simulated signal decays to decide on the most suitable.

The PDF that is chosen, as a result of these studies, is a sum of two Gaussian functions with a common mean value, or “double Gaussian”. This function is given by Equation 4.1. Other investigated functions include the Cruijff and the Apollonios functions, given by Equations 4.2 and 4.3, respectively. The Cruijff function is similar to a Gaussian in the central region but has tails resembling a power law as the α^2 terms start to dominate further away from the mean. In Equation 4.3, $B = \frac{n\sqrt{1+a^2}}{ba} - a$ and $A = e^{-b\sqrt{1+a^2}}(B+a)^n$. The double Gaussian is selected because the χ^2/ndf of the fit to the simulated data is the lowest, when using this PDF.

$$f_{\text{signal}}(M; N_{\text{sig}}, \mu, \sigma, r_W, f_{\text{core}}) = N_{\text{sig}} \left[\frac{f_{\text{core}}}{\sigma\sqrt{2\pi}} e^{-\frac{(M-\mu)^2}{2\sigma^2}} + \frac{1-f_{\text{core}}}{r_W\sigma\sqrt{2\pi}} e^{-\frac{(M-\mu)^2}{2r_W^2\sigma^2}} \right] \quad (4.1)$$

Table 4.1: The categories that $B \rightarrow DK^*$ candidates are placed in and the signal decays that each category corresponds to.

Category	Signal decays
$B (\pi K)$	$B^0 \rightarrow D(\pi K)K^{*0}$ and $\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}$
$\bar{B} (\pi K)$	$\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$ and $B_s^0 \rightarrow D(\pi K)\bar{K}^{*0}$
$B (K\pi)$	$B^0 \rightarrow D(K\pi)K^{*0}$
$\bar{B} (K\pi)$	$\bar{B}^0 \rightarrow D(K\pi)\bar{K}^{*0}$
$B (KK)$	$B^0 \rightarrow D(KK)K^{*0}$ and $\bar{B}_s^0 \rightarrow D(KK)K^{*0}$
$\bar{B} (KK)$	$\bar{B}^0 \rightarrow D(KK)\bar{K}^{*0}$ and $B_s^0 \rightarrow D(KK)\bar{K}^{*0}$
$B (\pi\pi)$	$B^0 \rightarrow D(\pi\pi)K^{*0}$ and $\bar{B}_s^0 \rightarrow D(\pi\pi)K^{*0}$
$\bar{B} (\pi\pi)$	$\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ and $B_s^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$

$$f(M; N_{\text{sig}}, \mu, \sigma_L, \sigma_R, \alpha_L, \alpha_R) = \begin{cases} N_{\text{sig}} e^{-\frac{(M-\mu)^2}{2\sigma_L^2 + \alpha_L^2(M-\mu)^2}}, & \text{if } M - \mu < 0 \\ N_{\text{sig}} e^{-\frac{(M-\mu)^2}{2\sigma_R^2 + \alpha_R^2(M-\mu)^2}}, & \text{otherwise} \end{cases} \quad (4.2)$$

$$f(M; N_{\text{sig}}, \mu, \sigma, b, a, n) = \begin{cases} N_{\text{sig}} e^{-b\sqrt{1 + \frac{(M-\mu)^2}{\sigma^2}}}, & \text{if } \frac{M-\mu}{\sigma} > -a \\ N_{\text{sig}} A(B - \frac{M-\mu}{\sigma})^{-n}, & \text{otherwise.} \end{cases} \quad (4.3)$$

In Equation 4.1, M is the dependent variable (B meson invariant mass), N_{sig} is the number of signal decays, μ is the mean, σ is the standard deviation of the narrower Gaussian, r_W is the ratio of the standard deviation of the wider Gaussian to that of the narrower Gaussian and f_{core} is the ratio of normalisations of the narrower to the wider Gaussian.

The studies performed on simulated data also inform choices regarding constraints to make on parameters of the signal PDF. The parameters of the double Gaussians fit to the invariant mass distributions of simulated $B_d \rightarrow D(K^\pm \pi^\mp)K^*$, $B_d \rightarrow D(KK)K^*$, $B_d \rightarrow D(\pi\pi)K^*$ and $B_s \rightarrow D(\pi K)K^*$ decays are of compatible values. Therefore, to simplify the model, various parameters are common to all PDFs used to describe the signal decays.

The f_{core} and r_W parameters are fixed to the values found when fitting the double Gaussian PDF to simulated data; $f_{\text{core}} = 0.88$ and $r_W = 2.67$. Clearly, the signal PDFs describing B_d and B_s decays will have different means, therefore this parameter is not common to all signals. However, the mean of the B_d signal is common to all appropriate PDFs and ΔM , the difference in mass of the B_s^0 and B^0 mesons, is fixed to the measured value, $\Delta M = 87.19 \text{ MeV}/c^2$ [6]. Therefore there is one free parameter of the model related to the means of the signal PDFs. The width, σ , is also common to all signal PDFs and is a free parameter.

There is one more free parameter of the model for every signal listed in Table 4.1, which is the yield of candidates for that signal decay. The yields of $\bar{B}_s^0 \rightarrow D(K\pi)K^{*0}$ and $B_s^0 \rightarrow D(K\pi)\bar{K}^{*0}$ decays are fixed to zero, since these decays are highly Cabibbo-suppressed and no signal is expected with the current size of the dataset.

4.1.2 Combinatorial background

Combinatorial background exists because random combinations of tracks are sometimes reconstructed as $B \rightarrow DK^*$ decays. The reconstructed invariant mass distribution of combinatorial background is described by a decreasing exponential function, f_{comb} , with slope c .

$$f_{\text{comb}}(M; N_{\text{comb}}, c) = N_{\text{comb}} \frac{e^{-M/c}}{c} \quad (4.4)$$

In Equation 4.4, N_{comb} is the number of combinatorial background candidates. Three different functions with independent free parameters for their slopes are defined. One is used to model the background to $B \rightarrow D(K^\pm \pi^\mp)K^*$ decays, one for the background to $B \rightarrow D(KK)K^*$ decays and one for the background to $B \rightarrow D(\pi\pi)K^*$ decays. The numbers of combinatorial background candidates are constrained to be equal in the B and $\bar{B}$ categories, since no CP violation is expected in a background that originates from random combinations of tracks.

4.1.3 Partially reconstructed $B \rightarrow D^*K^*$ background

A background from $B \rightarrow D^*K^*$ decays exists because the D^* meson decays to $D\gamma$ or $D\pi^0$ and the γ or π^0 is not reconstructed. As a result, the momentum of the lost particle in the final state does not contribute to the reconstructed invariant mass of the B meson and the invariant mass of such candidates is lower than the B meson mass.

The $B \rightarrow D^*K^*$ decay is the decay of a pseudo-scalar particle into two vector particles. This means that the final state particles can be in several different helicity states. The amplitude corresponding to this decay is expressed as a linear combination of amplitudes that correspond to different helicities of the D^* and K^* mesons.

Angular momentum conservation in $B \rightarrow D^*K^*$ decays implies that there are three terms in the total amplitude corresponding to the three possible, orthogonal, helicity states of the final state particles. These are the amplitude corresponding to the decay when the D^* and K^*

are in the -1 helicity state ($\lambda(D^*) = \lambda(K^*) = -1$), $\mathcal{A}_{001}$, the amplitude corresponding to the decay when the D^* and K^* are in the 0 helicity state, $\mathcal{A}_{010}$, and the amplitude corresponding to the decay when the D^* and K^* are in the $+1$ helicity state, $\mathcal{A}_{100}$.

The angular distribution of the reconstructed particles in the $B \rightarrow D^* K^*$ decay and therefore the reconstructed B meson invariant mass depends on one angle, θ' . This is the angle between the D momentum in the D^* meson rest frame and the D^* momentum in the B meson rest frame. The distributions of this angle, when the D^* decays to $D\pi^0$ and $D\gamma$, are given by $I_{D\pi^0}(\theta')$ and $I_{D\gamma}(\theta')$, respectively.

$$I_{D\pi^0}(\theta') \propto \frac{1}{2} |\mathcal{A}_{001}|^2 \sin^2 \theta' + |\mathcal{A}_{010}|^2 \cos^2 \theta' + \frac{1}{2} |\mathcal{A}_{100}|^2 \sin^2 \theta' \quad (4.5)$$

$$\begin{aligned} I_{D\gamma}(\theta') \propto & \frac{1}{4} |\mathcal{A}_{001}|^2 [|B_-|^2 (1 - \cos \theta')^2 + |B_+|^2 (1 + \cos \theta')^2] \\ & + \frac{1}{2} |\mathcal{A}_{010}|^2 \sin^2 \theta' + \frac{1}{4} |\mathcal{A}_{100}|^2 [|B_-|^2 (1 + \cos \theta')^2 + |B_+|^2 (1 - \cos \theta')^2] \end{aligned} \quad (4.6)$$

The $D^* \rightarrow D\gamma$ decay has a vector particle, the photon, in the final state so also has contributions to the total amplitude from different helicity states of the photon. B_- and B_+ are the two terms in the total amplitude of the $D^* \rightarrow D\gamma$ decay. However, because $D^* \rightarrow D\gamma$ is an electromagnetic process, parity is conserved. Parity conservation in the $D^* \rightarrow D\gamma$ decay implies that $B_- = -B_+$ and therefore Equation 4.6 simplifies to Equation 4.7.

$$I_{D\gamma}(\theta') \propto |B_+|^2 (|\mathcal{A}_{001}|^2 + |\mathcal{A}_{100}|^2) (1 + \cos^2 \theta') + |\mathcal{A}_{010}|^2 \sin^2 \theta' \quad (4.7)$$

The terms proportional to $|\mathcal{A}_{100}|^2$ and $|\mathcal{A}_{001}|^2$ in Equations 4.5 and 4.7 have the same angular dependence. Therefore it follows that the reconstructed B meson invariant mass distribution of $B \rightarrow D^* K^*$ decays in the corresponding helicity states will be the same. Therefore, in the model, they can be treated as one species. The description of the partially reconstructed $B \rightarrow D^* K^*$ background is then simplified and has only two contributions: $B \rightarrow D^* K^*$ with $\lambda(D^*) = \lambda(K^*) = 0$ and $B \rightarrow D^* K^*$ with $\lambda(D^*) = \lambda(K^*) = \pm 1$. The

reconstructed invariant mass distribution of $B \rightarrow D^*(D\gamma)K^*$ decays is different to the distribution from $B \rightarrow D^*(D\pi^0)K^*$ decays, because of the different angular dependence and mass of the lost particle. The PDFs describing the partially reconstructed $B_s \rightarrow D^*K^*$ and

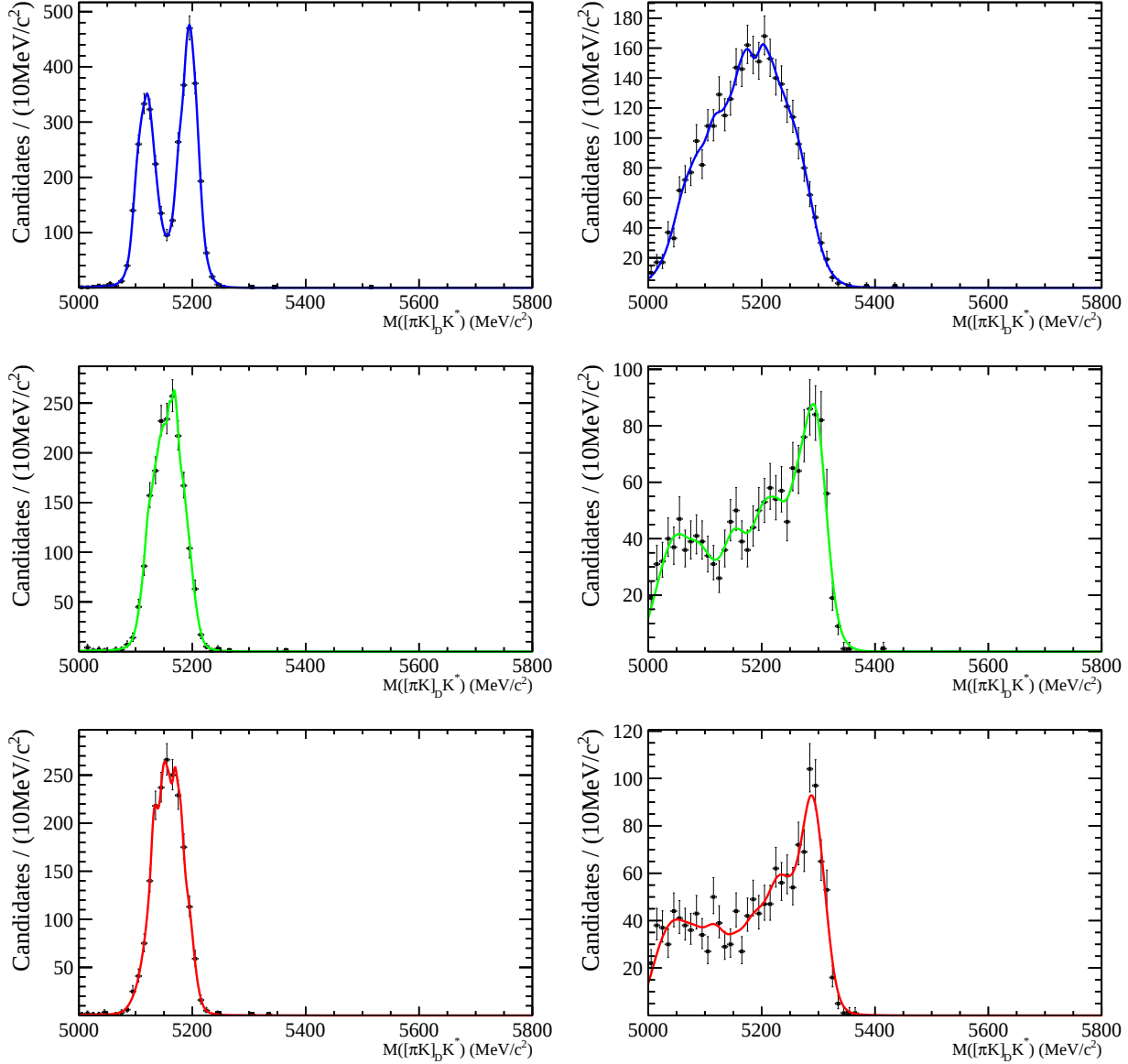


Figure 4.1: The B meson invariant mass distributions of $B \rightarrow D(\pi K)K^*$ candidates reconstructed from simulated $B_s \rightarrow D^*(D(\pi K)\pi^0)K^*$ (left column) and $B_s \rightarrow D^*(D(\pi K)\gamma)K^*$ decays (right column). The helicity states of the D^* and K^* mesons in the simulated decays are $\lambda(D^*) = \lambda(K^*) = 0$ (top row), $\lambda(D^*) = \lambda(K^*) = +1$ (middle row) and $\lambda(D^*) = \lambda(K^*) = -1$ (bottom row).

$B_d \rightarrow DK^*$ backgrounds are given by Equations 4.8 and 4.9, respectively.

$$f_{\text{pb}}^{B_s}(M) = N_{\text{pb}}^{B_s} \left\{ \alpha_0 \left[\frac{G_0}{G_0 + P_0} f_0^\gamma(M) + \frac{P_0}{G_0 + P_0} f_0^{\pi^0}(M) \right] + (1 - \alpha_0) \left[\frac{G_\pm}{G_\pm + P_\pm} f_\pm^\gamma(M) + \frac{P_\pm}{G_\pm + P_\pm} f_\pm^{\pi^0}(M) \right] \right\} \quad (4.8)$$

$$f_{\text{pb}}^{B_d}(M) = N_{\text{pb}}^{B_d} \left\{ \beta_0 \left[\frac{G_0}{G_0 + P_0} f_0^\gamma(M + \Delta M) + \frac{P_0}{G_0 + P_0} f_0^{\pi^0}(M + \Delta M) \right] + (1 - \beta_0) \left[\frac{G_\pm}{G_\pm + P_\pm} f_\pm^\gamma(M + \Delta M) + \frac{P_\pm}{G_\pm + P_\pm} f_\pm^{\pi^0}(M + \Delta M) \right] \right\} \quad (4.9)$$

In Equations 4.8 and 4.9, f_X^γ and $f_X^{\pi^0}$ are non-parametric functions [74] that are modelled on the invariant mass distributions of $B \rightarrow DK^*$ candidates reconstructed from simulated $B_s \rightarrow D^*(D(\pi K)\gamma)K^*$ and $B_s \rightarrow D^*(D(\pi K)\pi^0)K^*$ decays, respectively. $X = 0$ corresponds to the decay with the D^* and K^* in the 0 helicity state and $X = \pm$ corresponds to the decay with the D^* and K^* in the ± 1 helicity state. The same functions are used to model the background from $B_d \rightarrow D^*K^*$ decays, though displaced in invariant mass by ΔM . The invariant mass distribution of these backgrounds is also assumed independent of the D meson decay, in line with this assumption about the signal.

Figure 4.1 shows the reconstructed B meson invariant mass distributions and confirms that these are identical when the D^* and K^* mesons in $B \rightarrow D^*(D\gamma)K^*$ decay have helicities of $+1$ and -1 . For this reason, there is only one function, $f_\pm^\gamma$, describing the invariant mass distribution of these decays. The simulated data, upon which f_X^γ and $f_X^{\pi^0}$ are modelled, are smeared and shifted to take into account known differences in the invariant mass resolution and alignment of the detector in the simulation software and in real data.

The coefficients, G_X and P_X , in Equation 4.8 are products of the branching fractions of $D^{*0} \rightarrow D^0\gamma$ or $D^{*0} \rightarrow D^0\pi^0$ decays and efficiencies with which $B \rightarrow D^*K^*$ decays are reconstructed as $B \rightarrow DK^*$ candidates. They serve to correctly normalise the species corresponding to different decays of the D^* meson relative to each other. G_X and P_X are given by Equations 4.10 and 4.11, where ϵ_{acc} is the geometric acceptance efficiency of the

LHCb detector and ϵ_{tot}^X is the efficiency to reconstruct that decay as a $B \rightarrow DK^*$ candidate. ϵ_{tot}^X depends on the helicity states of the D^* and K^* mesons because of the different angular distribution of the reconstructed particles. Both ϵ_{acc} and ϵ_{tot}^X are determined from the same simulated data that are used to model the functions in Figure 4.1.

$$G_X = \mathcal{B}(D^{*0} \rightarrow D^0 \gamma) \epsilon_{\text{acc}}(D\gamma) \epsilon_{\text{tot}}^X(D\gamma) \quad (4.10)$$

$$P_X = \mathcal{B}(D^{*0} \rightarrow D^0 \pi^0) \epsilon_{\text{acc}}(D\pi^0) \epsilon_{\text{tot}}^X(D\pi^0) \quad (4.11)$$

Since Equations 4.8 and 4.9 can be expressed purely in terms of P_X/G_X ratios, the measurement of the ratio of branching fractions, $\frac{\mathcal{B}(D^{*0} \rightarrow D^0 \pi^0)}{\mathcal{B}(D^{*0} \rightarrow D^0 \gamma)} = 1.74 \pm 0.13$ [6], is used. The values of the efficiencies are given in Table 4.2. The same G_X and P_X coefficients are used to describe the $B \rightarrow D^* K^*$ backgrounds in all categories, because the ratios of the efficiencies in Equations 4.10 and 4.11 are assumed independent of the D meson decay.

The α_0 and β_0 parameters give a measure of the size of the contribution from the 0 helicity state of the D^* and K^* mesons to the overall $B_s \rightarrow D^* K^*$ and $B_d \rightarrow D^* K^*$ decay amplitude, respectively. These parameters are common to the model in all categories since the helicity states of the D^* and K^* mesons are independent of B meson flavour and D meson decay.

The numbers of $B_s^0 \rightarrow D^*(D \rightarrow hh') \bar{K}^{*0}$ and $\bar{B}_s^0 \rightarrow D^*(D \rightarrow hh') K^{*0}$ background candidates are constrained to the same value for each D meson final state, hh' . This is because CP violation in this decay mode is expected to be small because of similar arguments to those put forward with regard to the $B_s \rightarrow DK^*$ signal in Section 1.3.2. However, this constraint cannot be applied to the yields of $B^0 \rightarrow D^* K^{*0}$ and $\bar{B}^0 \rightarrow D^* \bar{K}^{*0}$ background candidates, because of possible detectable CP violation in this decay. The

Table 4.2: The efficiencies with which $B \rightarrow D^* K^*$ decays are reconstructed as $B \rightarrow DK^*$ decays.

Decay	ϵ_{acc}	$\epsilon_{\text{tot}}^\pm$	ϵ_{tot}^0
$D^* \rightarrow D\pi^0$	$(14.5 \pm 0.1) \%$	$(0.338 \pm 0.008) \%$	$(0.646 \pm 0.011) \%$
$D^* \rightarrow D\gamma$	$(16.4 \pm 0.1) \%$	$(0.287 \pm 0.007) \%$	$(0.558 \pm 0.010) \%$

amplitudes corresponding to $B_d \rightarrow D^* K^*$ decays contain the same CKM elements as factors as $B_d \rightarrow DK^*$ decays, therefore $B_d \rightarrow D^* K^*$ decays could exhibit similar CP violation. However, the strong phase differences between the amplitudes are not necessarily the same. It is recalled from Section 1.2.2 that the CP -violating asymmetry is dependent on the strong phase difference. Therefore different CP -violating asymmetries in the numbers of $B_d \rightarrow D^* K^*$ to those exhibited by the signal are allowed for by the model.

The numbers of partially reconstructed $B_s \rightarrow D^*(D \rightarrow KK)K^*$ and $B_s \rightarrow D^*(D \rightarrow \pi\pi)K^*$ candidates, $N_{\text{pb}}^{B_s hh}$, are constrained from the number of $B_s \rightarrow D^*(D \rightarrow \pi K)K^*$ candidates, $N_{\text{pb}}^{B_s \pi K}$, according to Equation 4.12, using the measured values of D^0 meson branching fractions and the selection efficiencies, ϵ_{tot} , in Table 3.11. It is assumed that the ratio of selection efficiencies on $B \rightarrow DK^*$ decays with different final states is independent of the initial state. This is why the efficiencies on B_d decays, not B_s decays, are used in Equation 4.12. The D^0 decay branching fractions are $\mathcal{B}(D^0 \rightarrow K^-\pi^+) = (3.88 \pm 0.05)\%$, $\mathcal{B}(D^0 \rightarrow K^+K^-) = (0.396 \pm 0.008)\%$ and $\mathcal{B}(D^0 \rightarrow \pi^+\pi^-) = (0.140 \pm 0.003)\%$ [6].

$$N_{\text{pb}}^{B_s hh} = N_{\text{pb}}^{B_s \pi K} \times \frac{\mathcal{B}(D^0 \rightarrow hh)}{\mathcal{B}(D^0 \rightarrow K^-\pi^+)} \times \frac{\epsilon_{\text{tot}}(B_d \rightarrow D(hh)K^*)}{\epsilon_{\text{tot}}(B_d \rightarrow D(K^\pm\pi^\mp)K^*)} \quad (4.12)$$

Analogous constraints are not applied to the numbers of $B^0 \rightarrow D^* K^{*0}$ and $\bar{B}^0 \rightarrow D^* \bar{K}^{*0}$ background candidates and therefore these are free parameters of the model. The numbers of $B_s^0 \rightarrow D^*(D \rightarrow K\pi)\bar{K}^{*0}$ and $\bar{B}_s^0 \rightarrow D^*(D \rightarrow K\pi)K^{*0}$ background candidates are fixed to zero since these decays are highly Cabibbo-suppressed, much like the $B_s \rightarrow DK^*$ signal decays with kaons of the same charge in the final state.

The above considerations mean that the partially reconstructed background has 11 associated free parameters: the yield of $\bar{B}_s^0 \rightarrow D^*(D \rightarrow \pi K)K^{*0}$ candidates, $N_{\text{pb}}^{\bar{B}_s^0 \pi K}$, 8 yields of $B_d \rightarrow D^* K^*$ background candidates, $N_{\text{pb}}^{B_d hh'}$, α_0 and β_0 .

4.1.4 Misidentified $B_d \rightarrow D\rho^0$ background

The final modelled background is from the misidentification of $B_d \rightarrow D\rho^0(\pi^+\pi^-)$ decays as $B \rightarrow DK^*$ decays. One pion from the ρ^0 is misidentified as the kaon from the K^* so this background accumulates at higher invariant masses than the B^0 meson mass. This background is reduced by the PID requirements discussed in Section 3.2.3, but still exists in small quantities in all categories. Similar to the backgrounds from $B \rightarrow D^*K^*$ decays, the invariant mass distribution of this background is described by a non-parametric function, $f_{\text{cf}}(M)$, that is modelled on simulated $B_d \rightarrow D\rho^0$ decays that are reconstructed as $B \rightarrow DK^*$ decays. The simulated data are also smeared and shifted to account for the differences between them and real data. $f_{\text{cf}}(M)$ is shown in Figure 4.2.

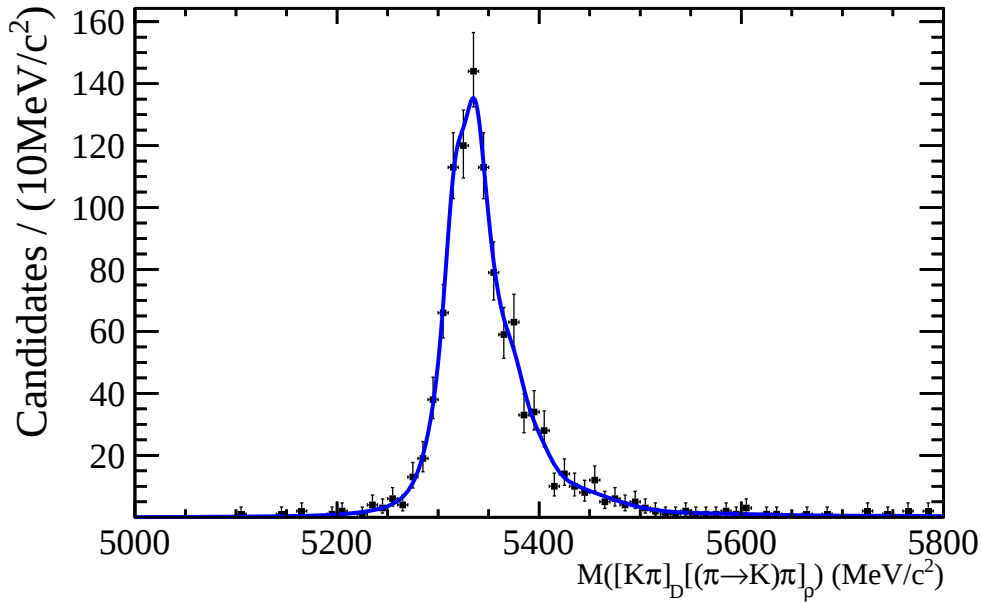


Figure 4.2: The reconstructed $B \rightarrow DK^*$ invariant mass distribution of simulated $B_d \rightarrow D(K^\pm \pi^\mp) \rho^0$ decays. The blue line is the function used in the invariant mass model to describe the $B_d \rightarrow D\rho^0$ background.

The numbers of $B_d \rightarrow D\rho^0$ background candidates are constrained to be equal in the $B(\pi K)$, $\bar{B}(\pi K)$, $B(K\pi)$ and $\bar{B}(K\pi)$ categories. This is because misidentification rate is assumed to be independent of the charge of the misidentified pion. Therefore, this background should contaminate B and $\bar{B}$ categories in equal amounts.

The numbers of $B_d \rightarrow D(hh)\rho^0$ background candidates, where $hh = KK$ or $\pi\pi$, are

constrained from the number of $B_d \rightarrow D(K^\pm\pi^\mp)\rho^0$ candidates in the B (πK) category with branching fractions and ratios of selection efficiencies, using an analogous relation to Equation 4.12.

4.1.5 Summary

The functions that are simultaneously fit to the invariant mass distributions of the 8 categories of $B \rightarrow DK^*$ candidates are sums of the PDFs described in the previous sections. The coefficients of the PDFs within each sum are, in some cases, free parameters and, in others, constrained according to physical arguments. Most categories are modelled with the same signal and background PDFs, namely, double Gaussian functions to describe the $B_d \rightarrow DK^*$ and $B_s \rightarrow DK^*$ signals, non-parametric functions to describe the backgrounds from $B_d \rightarrow D\rho^0$ and $B \rightarrow D^*K^*$ decays and an exponential function to describe the combinatorial background. There are two categories of $B \rightarrow DK^*$ candidates that are exceptions to this description. These are the $B(K\pi)$ and $\bar{B}(K\pi)$ categories, which do not include contributions from the $B_s \rightarrow DK^*$ signal or $B_s \rightarrow D^*K^*$ background due to the negligible branching fraction of these decays.

The 35 free parameters of the B meson invariant mass model are summarised in Tables 4.3 and 4.4, where each parameter is given an index for the purpose of labelling figures. The 16 fixed parameters are summarised in Table 4.5.

Table 4.3: The free parameters of the invariant mass model, that relate to signal $B \rightarrow DK^*$ decays, and the indices used for labelling figures.

Parameter	Description	Index
μ	B_s^0 meson mass	23
σ	Width of the signal PDF	24
$N^{B^0 \pi K}$	Number of $B^0 \rightarrow D(\pi K)K^{*0}$ candidates	9
$N^{\bar{B}^0 \pi K}$	Number of $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$ candidates	13
$N^{B^0 K \pi}$	Number of $B^0 \rightarrow D(K \pi)K^{*0}$ candidates	10
$N^{\bar{B}^0 K \pi}$	Number of $\bar{B}^0 \rightarrow D(K \pi)\bar{K}^{*0}$ candidates	14
$N^{\bar{B}_s^0 \pi K}$	Number of $\bar{B}_s^0 \rightarrow D(\pi K)K^{*0}$ candidates	18
$N^{B_s^0 \pi K}$	Number of $B_s^0 \rightarrow D(\pi K)\bar{K}^{*0}$ candidates	21
$N^{B^0 KK}$	Number of $B^0 \rightarrow D(K K)K^{*0}$ candidates	8
$N^{\bar{B}^0 KK}$	Number of $\bar{B}^0 \rightarrow D(K K)\bar{K}^{*0}$ candidates	12
$N^{\bar{B}_s^0 KK}$	Number of $\bar{B}_s^0 \rightarrow D(K K)K^{*0}$ candidates	17
$N^{B_s^0 KK}$	Number of $B_s^0 \rightarrow D(K K)\bar{K}^{*0}$ candidates	20
$N^{B^0 \pi \pi}$	Number of $B^0 \rightarrow D(\pi \pi)K^{*0}$ candidates	11
$N^{\bar{B}^0 \pi \pi}$	Number of $\bar{B}^0 \rightarrow D(\pi \pi)\bar{K}^{*0}$ candidates	15
$N^{\bar{B}_s^0 \pi \pi}$	Number of $\bar{B}_s^0 \rightarrow D(\pi \pi)K^{*0}$ candidates	19
$N^{B_s^0 \pi \pi}$	Number of $B_s^0 \rightarrow D(\pi \pi)\bar{K}^{*0}$ candidates	22

Table 4.4: The free parameters of the invariant mass model, that relate to backgrounds, and the indices used for labelling figures.

Parameter	Description	Index
$c_{K\pi}$	Slope of combinatorial background in $D \rightarrow K^\pm \pi^\mp$ categories	30
c_{KK}	Slope of combinatorial background in $D \rightarrow KK$ categories	27
$c_{\pi\pi}$	Slope of combinatorial background in $D \rightarrow \pi\pi$ categories	32
$N_{\text{comb}}^{\pi K}$	Number of combinatorial background candidates in $B (\pi K)$ category	28
$N_{\text{comb}}^{K\pi}$	Number of combinatorial background candidates in $B (K\pi)$ category	29
N_{comb}^{KK}	Number of combinatorial background candidates in $B (KK)$ category	26
$N_{\text{comb}}^{\pi\pi}$	Number of combinatorial background candidates in $B (\pi\pi)$ category	31
$N_{\text{pb}}^{\bar{B}_s^0 \pi K}$	Number of $\bar{B}_s^0 \rightarrow D^*(D \rightarrow \pi K)K^{*0}$ candidates	16
$N_{\text{pb}}^{B^0 \pi K}$	Number of $B^0 \rightarrow D^*(D \rightarrow \pi K)K^{*0}$ candidates	1
$N_{\text{pb}}^{\bar{B}^0 \pi K}$	Number of $\bar{B}^0 \rightarrow D^*(D \rightarrow \pi K)\bar{K}^{*0}$ candidates	5
$N_{\text{pb}}^{B^0 K\pi}$	Number of $B^0 \rightarrow D^*(D \rightarrow K\pi)K^{*0}$ candidates	2
$N_{\text{pb}}^{\bar{B}^0 K\pi}$	Number of $\bar{B}^0 \rightarrow D^*(D \rightarrow K\pi)\bar{K}^{*0}$ candidates	6
$N_{\text{pb}}^{B^0 KK}$	Number of $B^0 \rightarrow D^*(D \rightarrow KK)K^{*0}$ candidates	0
$N_{\text{pb}}^{\bar{B}^0 KK}$	Number of $\bar{B}^0 \rightarrow D^*(D \rightarrow KK)\bar{K}^{*0}$ candidates	4
$N_{\text{pb}}^{B^0 \pi\pi}$	Number of $B^0 \rightarrow D^*(D \rightarrow \pi\pi)K^{*0}$ candidates	3
$N_{\text{pb}}^{\bar{B}^0 \pi\pi}$	Number of $\bar{B}^0 \rightarrow D^*(D \rightarrow \pi\pi)\bar{K}^{*0}$ candidates	7
α_0	Coefficient of helicity 0 component in $B_s \rightarrow D^* K^*$ background	34
β_0	Coefficient of helicity 0 component in $B_d \rightarrow D^* K^*$ background	33
$N_{\text{cf}}^{B^0 \pi K}$	Number of $B_d \rightarrow D(K^\pm \pi^\mp)\rho^0$ candidates in $B (\pi K)$ category	25

Table 4.5: The fixed parameters of the invariant mass model.

Parameter	Description	Value
ΔM	Difference in B_s^0 and B^0 meson masses	$87.19 \pm 0.29 \text{ MeV}/c^2$
f_{core}	Fraction of signal yield in core Gaussian	0.88 ± 0.02
r_W	Width ratio	2.67 ± 0.17
$\frac{\mathcal{B}(D^{*0} \rightarrow D^0 \pi^0)}{\mathcal{B}(D^{*0} \rightarrow D^0 \gamma)}$	Ratio of D^{*0} decay branching fractions	1.74 ± 0.13
$\epsilon_{\text{acc}}(D\pi^0)$	Acceptance efficiency on $B \rightarrow D^*(D\pi^0)K^*$ decays	$(14.5 \pm 0.1) \%$
$\epsilon_{\text{acc}}(D\gamma)$	Acceptance efficiency on $B \rightarrow D^*(D\gamma)K^*$ decays	$(16.4 \pm 0.1) \%$
$\epsilon_{\text{tot}}^{\pm}(D\pi^0)$	Efficiency to reconstruct $B \rightarrow D^*(D\pi^0)K^*$ decays as $B \rightarrow DK^*$	$(0.338 \pm 0.008) \%$
$\epsilon_{\text{tot}}^0(D\pi^0)$	Efficiency to reconstruct $B \rightarrow D^*(D\pi^0)K^*$ decays as $B \rightarrow DK^*$	$(0.646 \pm 0.011) \%$
$\epsilon_{\text{tot}}^{\pm}(D\gamma)$	Efficiency to reconstruct $B \rightarrow D^*(D\gamma)K^*$ decays as $B \rightarrow DK^*$	$(0.287 \pm 0.007) \%$
$\epsilon_{\text{tot}}^0(D\gamma)$	Efficiency to reconstruct $B \rightarrow D^*(D\pi^0)K^*$ decays as $B \rightarrow DK^*$	$(0.558 \pm 0.010) \%$
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	D^0 decay branching fraction	$(3.88 \pm 0.05) \%$
$\mathcal{B}(D^0 \rightarrow K^+ K^-)$	D^0 decay branching fraction	$(0.396 \pm 0.008) \%$
$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$	D^0 decay branching fraction	$(0.140 \pm 0.003) \%$
$\epsilon_{\text{tot}}(B_d \rightarrow D(K^{\pm} \pi^{\mp})K^*)$	Overall reconstruction efficiency on $B_d \rightarrow D(K^{\pm} \pi^{\mp})K^*$ decays	$(0.0462 \pm 0.0013) \%$
$\epsilon_{\text{tot}}(B_d \rightarrow D(KK)K^*)$	Overall reconstruction efficiency on $B_d \rightarrow D(KK)K^*$ decays	$(0.0488 \pm 0.0013) \%$
$\epsilon_{\text{tot}}(B_d \rightarrow D(\pi\pi)K^*)$	Overall reconstruction efficiency on $B_d \rightarrow D(\pi\pi)K^*$ decays	$(0.0526 \pm 0.0014) \%$

4.2 Invariant mass fit results

The best-fit values and uncertainties in all the free parameters of the B meson invariant mass model, resulting from the simultaneous unbinned maximum-likelihood fit to the data, are given in Table 4.6. The data and the fitted model are shown in Figures 4.3 to 4.6. The correlation matrix of all the free parameters is shown in Figure 4.7.

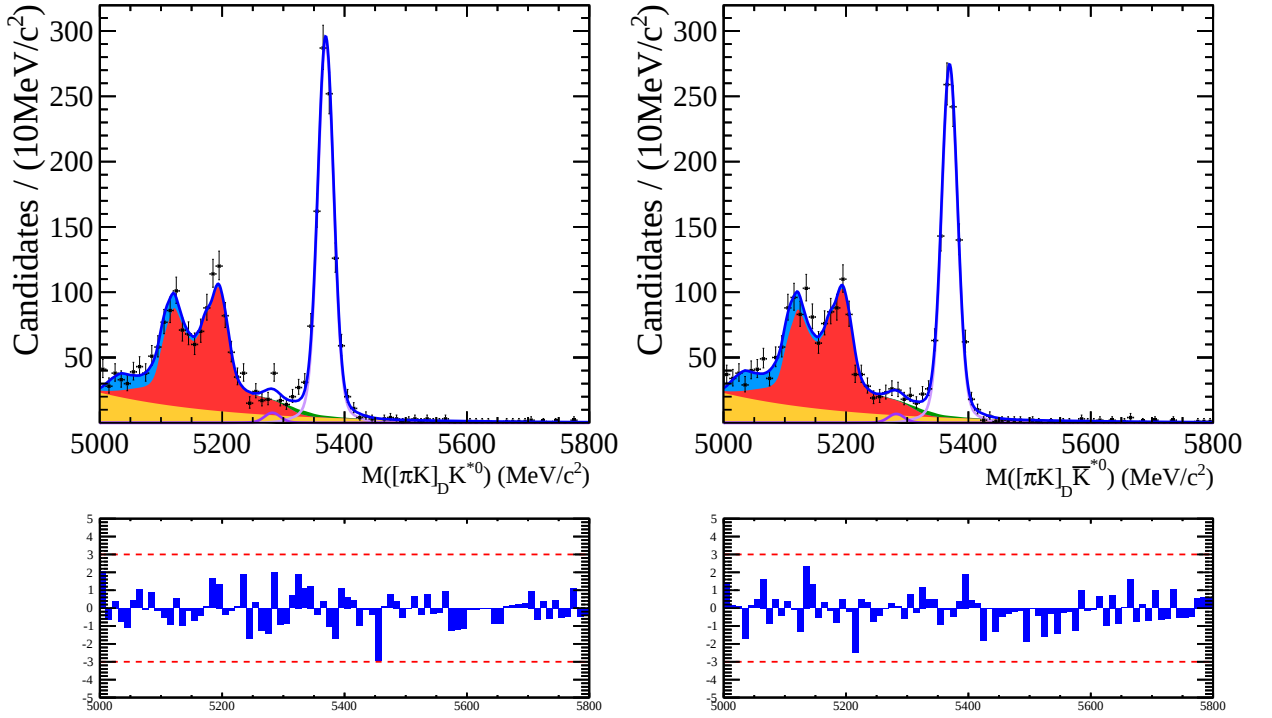


Figure 4.3: $D(\pi K)K^{*0}$ (left) and $D(\pi K)\bar{K}^{*0}$ (right) invariant mass distributions. The data (black points) and the fitted invariant mass model (blue line) are shown. The PDFs corresponding to the different species are also indicated: The B_d signal (purple line), B_s signal (lilac line), combinatorial background (yellow fill), $B_d \rightarrow D\rho^0$ background (green fill) and partially reconstructed $B_s \rightarrow D^* K^*$ (red fill) and $B_d \rightarrow D^* K^*$ (light blue fill) backgrounds. The pull of the model on the data, which is the difference in the value of the model and the data divided by the uncertainty in the data, in each bin is also shown.

Table 4.6: The best-fit values of the free parameters of the B meson invariant mass model.

Parameter	Result	Parameter	Result
μ	$5369.3^{+0.3}_{-0.3} \text{ MeV}/c^2$	$c_{K\pi}$	$0.0049^{+0.0002}_{-0.0002} \text{ MeV}/c^2$
σ	$12.7^{+0.3}_{-0.3} \text{ MeV}/c^2$	c_{KK}	$0.0044^{+0.0003}_{-0.0003} \text{ MeV}/c^2$
$N^{B^0 \pi K}$	$26.4^{+11.7}_{-11.0}$	$c_{\pi\pi}$	$0.0035^{+0.0006}_{-0.0006} \text{ MeV}/c^2$
$N^{\bar{B}^0 \pi K}$	$23.8^{+11.9}_{-11.2}$	$N_{\text{comb}}^{\pi K}$	$461.3^{+45.7}_{-43.6}$
$N^{B_s^0 \pi K}$	$932.7^{+32.6}_{-32.0}$	$N_{\text{comb}}^{K\pi}$	$885.0^{+48.8}_{-47.6}$
$N^{\bar{B}_s^0 \pi K}$	$993.3^{+33.8}_{-33.1}$	N_{comb}^{KK}	$278.5^{+27.3}_{-26.6}$
$N^{B^0 K\pi}$	$405.3^{+22.8}_{-22.2}$	$N_{\text{comb}}^{\pi\pi}$	$101.0^{+17.9}_{-17.0}$
$N^{\bar{B}^0 K\pi}$	$369.6^{+22.0}_{-21.3}$	$N_{\text{pb}}^{\bar{B}^0 \pi K}$	$1025.8^{+30.3}_{-30.2}$
$N^{B^0 KK}$	$53.0^{+9.9}_{-9.1}$	$N_{\text{pb}}^{B^0 \pi K}$	$170.7^{+41.9}_{-42.3}$
$N^{\bar{B}^0 KK}$	$35.9^{+8.8}_{-8.0}$	$N_{\text{pb}}^{\bar{B}^0 \pi K}$	$205.6^{+42.3}_{-42.7}$
$N^{B_s^0 KK}$	$114.7^{+12.2}_{-11.5}$	$N_{\text{pb}}^{B^0 K\pi}$	$504.6^{+47.0}_{-46.9}$
$N^{\bar{B}_s^0 KK}$	$124.7^{+12.8}_{-12.1}$	$N_{\text{pb}}^{\bar{B}^0 K\pi}$	$538.6^{+47.4}_{-47.4}$
$N^{B^0 \pi\pi}$	$21.2^{+6.5}_{-5.8}$	$N_{\text{pb}}^{B^0 KK}$	$53.5^{+24.7}_{-24.3}$
$N^{\bar{B}^0 \pi\pi}$	$17.6^{+6.0}_{-5.3}$	$N_{\text{pb}}^{\bar{B}^0 KK}$	$57.6^{+24.1}_{-23.7}$
$N^{B_s^0 \pi\pi}$	$39.0^{+7.3}_{-6.6}$	$N_{\text{pb}}^{B^0 \pi\pi}$	$24.1^{+15.9}_{-15.8}$
$N^{\bar{B}_s^0 \pi\pi}$	$34.5^{+6.9}_{-6.2}$	$N_{\text{pb}}^{\bar{B}^0 \pi\pi}$	$28.2^{+14.9}_{-14.8}$
		α_0	$0.73^{+0.04}_{-0.04}$
		β_0	$0.70^{+0.06}_{-0.06}$
		$N_{\text{cf}}^{B^0 \pi K}$	$24.6^{+9.6}_{-9.4}$

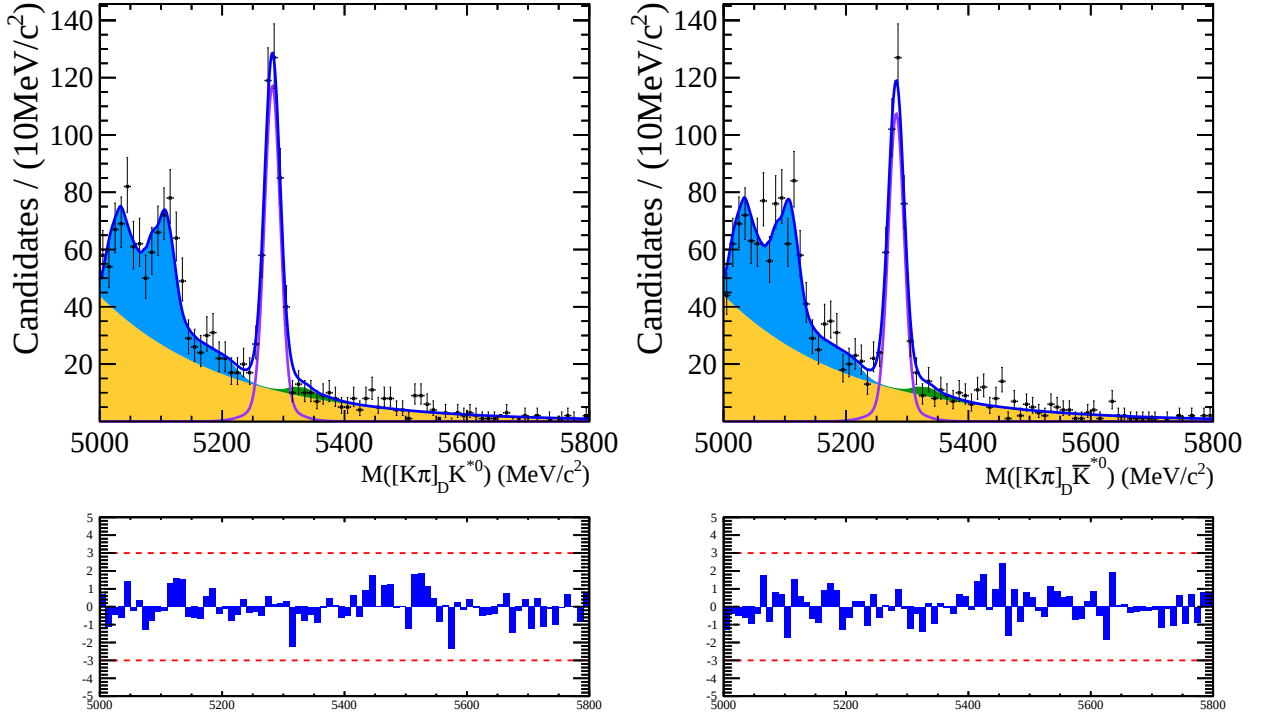


Figure 4.4: $D(K\pi)K^{*0}$ (left) and $D(K\pi)\bar{K}^{*0}$ (right) invariant mass distributions. The data (*black points*) and the fitted invariant mass model (*blue line*) are shown. The PDFs corresponding to the different species are also indicated: The B_d signal (*purple line*), combinatorial background (*yellow fill*), $B_d \rightarrow D\rho^0$ background (*green fill*) and partially reconstructed $B_d \rightarrow D^*K^*$ (*light blue fill*) background. The pull of the model on the data, which is the difference in the value of the model and the data divided by the uncertainty in the data, in each bin is also shown.

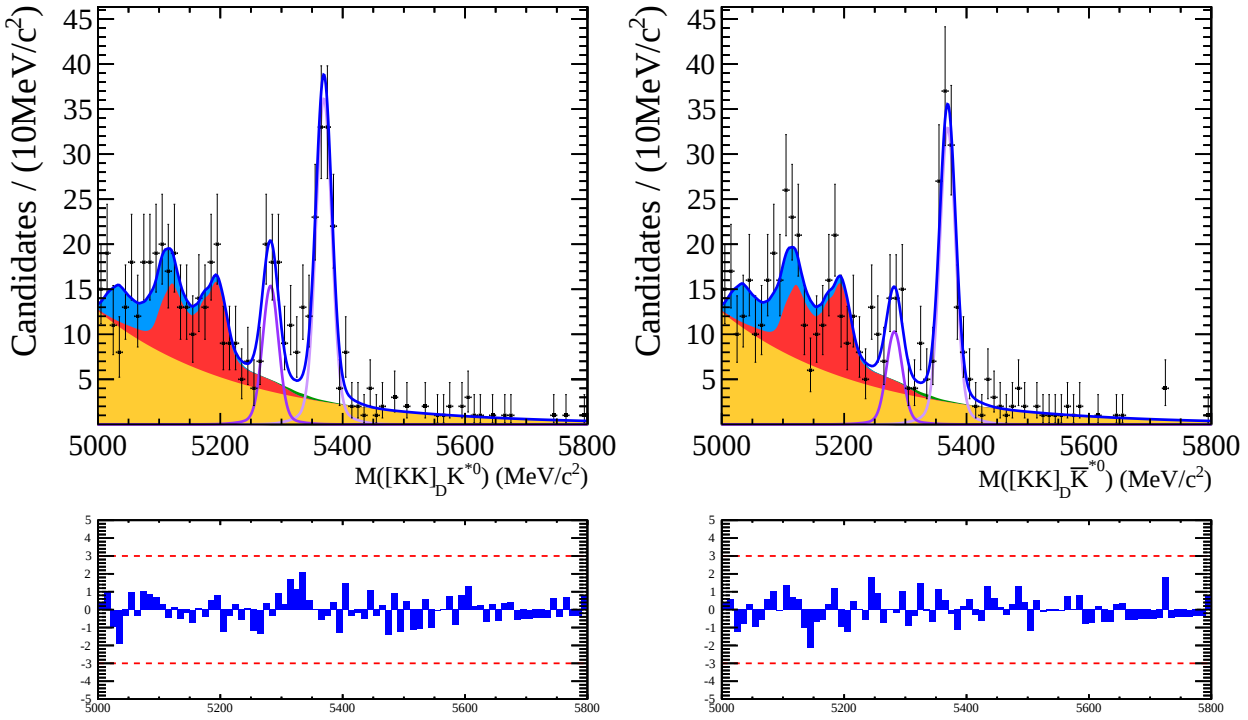


Figure 4.5: $D(KK)K^{*0}$ (left) and $D(KK)\bar{K}^{*0}$ (right) invariant mass distributions. The data (*black points*) and the fitted invariant mass model (*blue line*) are shown. The PDFs corresponding to the different species are also indicated: The B_d signal (*purple line*), B_s signal (*lilac line*), combinatorial background (*yellow fill*), $B_d \rightarrow D\rho^0$ background (*green fill*) and partially reconstructed $B_s \rightarrow D^*K^*$ (*red fill*) and $B_d \rightarrow D^*K^*$ (*light blue fill*) backgrounds. The pull of the model on the data, which is the difference in the value of the model and the data divided by the uncertainty in the data, in each bin is also shown.

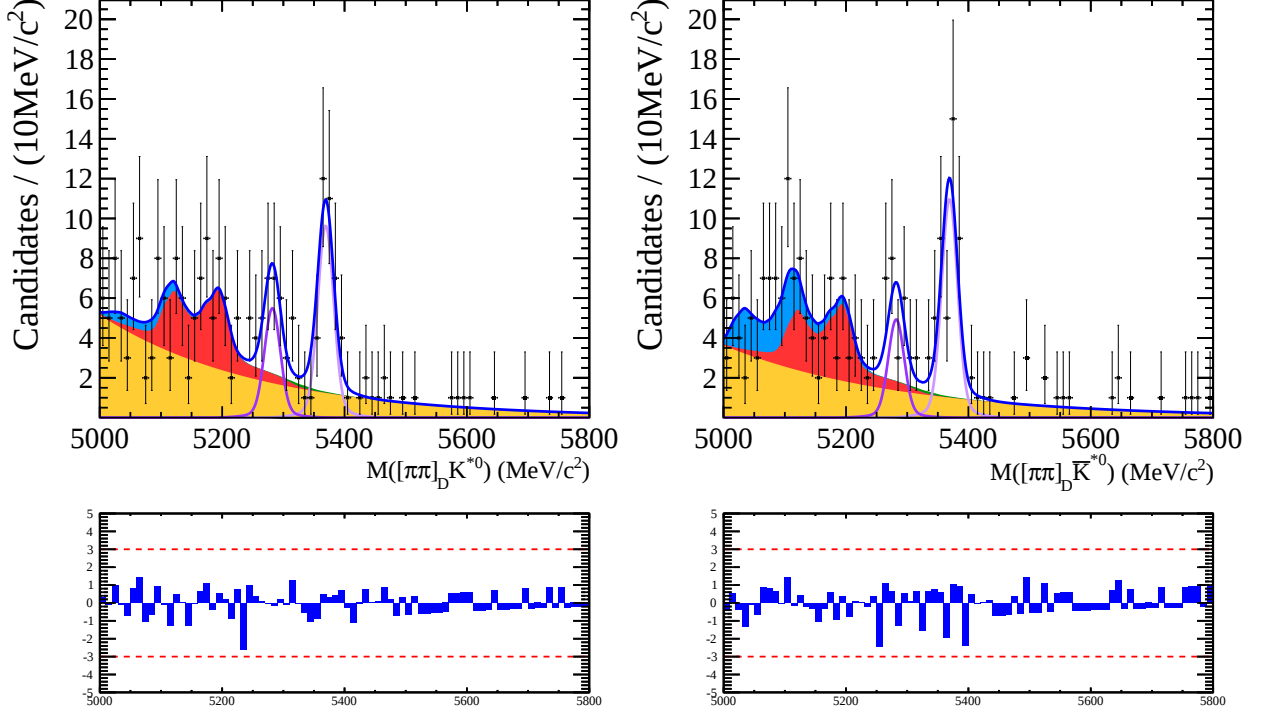


Figure 4.6: $D(\pi\pi)K^{*0}$ (left) and $D(\pi\pi)\bar{K}^{*0}$ (right) invariant mass distributions. The data (black points) and the fitted invariant mass model (blue line) are shown. The PDFs corresponding to the different species are also indicated: The B_d signal (purple line), B_s signal (lilac line), combinatorial background (yellow fill), $B_d \rightarrow D\rho^0$ background (green fill) and partially reconstructed $B_s \rightarrow D^*K^*$ (red fill) and $B_d \rightarrow D^*K^*$ (light blue fill) backgrounds. The pull of the model on the data, which is the difference in the value of the model and the data divided by the uncertainty in the data, in each bin is also shown.

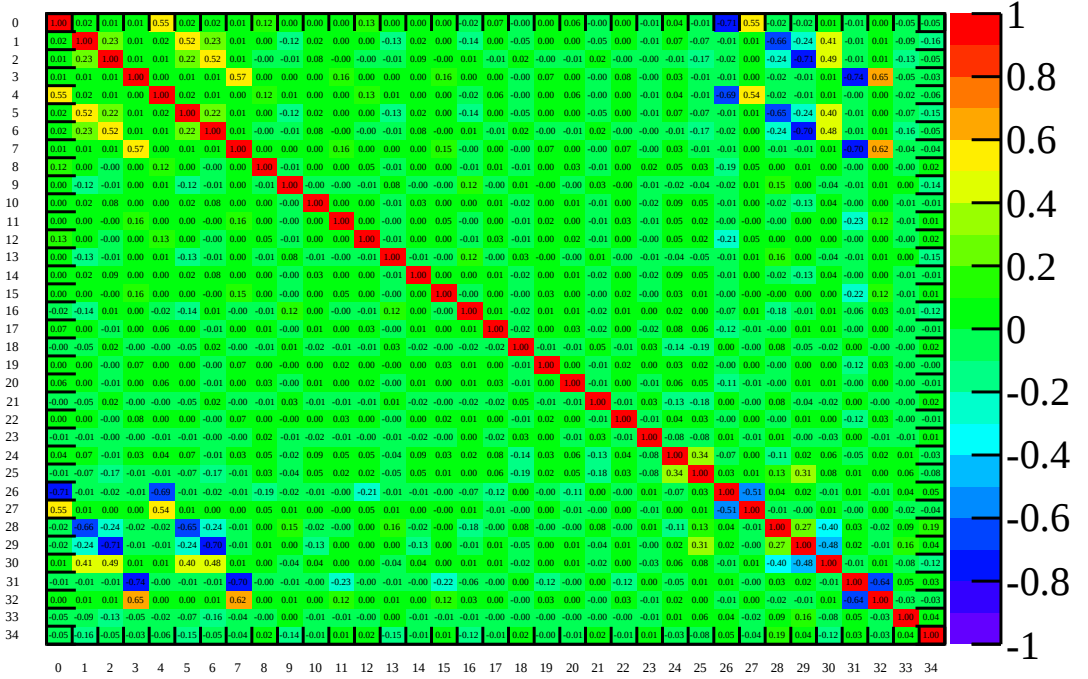


Figure 4.7: The correlation matrix of the free parameters of the invariant mass model. The parameters are labelled with their indices that are given in Tables 4.3 and 4.4.

4.3 Toy studies

4.3.1 Model validation

To validate the invariant mass model and evaluate biases in either the best-fit values of the free parameters or their uncertainties, pseudo-experiments are performed. This involves using the model to generate a comparable dataset to the real data, the “toy dataset”, and fitting the model to this toy dataset. This is performed many times and the parameters of the model used to generate the toy datasets are varied in a random fashion, according to their best-fit values on the real data and their uncertainties.

To discern any inherent bias in the determination of the free parameters of the model, δ is defined as $x_{\text{fit}} - x_{\text{gen}}$. Here x_{fit} is the best-fit value of a free parameter, x , when fitting the model to the toy dataset and x_{gen} is the value of that parameter used to generate the toy dataset. The pull of x is then defined as

$$P(x) = \begin{cases} \frac{\delta}{\sigma_x^+}, & \text{if } \delta < 0 \\ \frac{\delta}{\sigma_x^-}, & \text{otherwise} \end{cases} \quad (4.13)$$

where σ_x^+ and σ_x^- are the upper and lower uncertainties in x_{fit} .

The suppressed $B_d \rightarrow D(\pi K)K^*$ decays are thus far unobserved with LHCb data, therefore a blind analysis has been performed, as explained in Section 3.2. Therefore the toy studies are performed without knowledge of the best-fit values of the numbers of $B^0 \rightarrow D(\pi K)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$ candidates. Realistic estimates of these signal yields, of $N^{B^0 \pi K} = N^{\bar{B}^0 \pi K} = 25 \pm 5$, are made when defining the model with which to generate toy datasets.

Three thousand toy datasets are generated, fitted and the pull of each free parameter is calculated. The distribution of $P(x)$ is expected to be well described by a Gaussian with a mean of zero and a standard deviation of one. An example of the pull distribution of $N^{B^0 K \pi}$

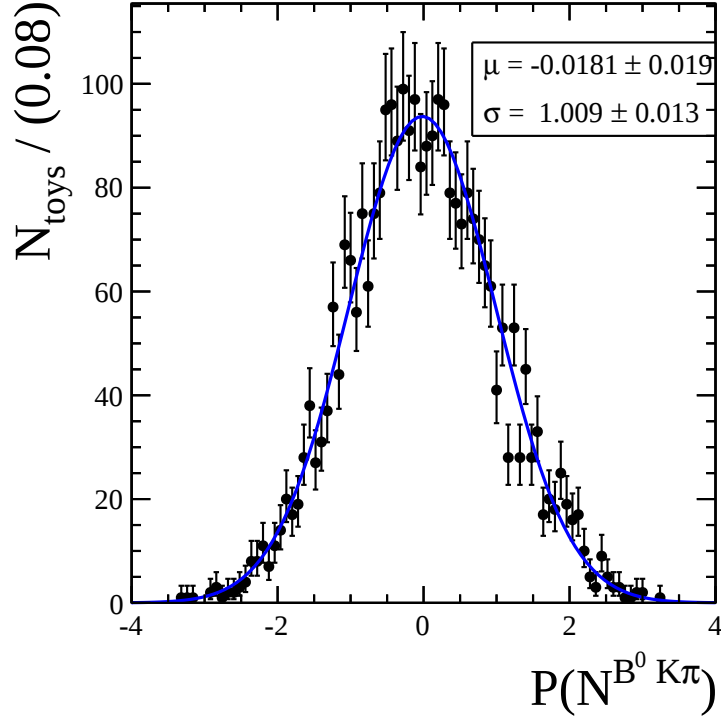


Figure 4.8: The distribution of the pull, $P(N^{B^0 K\pi})$. The result of a maximum-likelihood fit of a Gaussian function to the distribution is also shown.

is given in Figure 4.8 and the pull distributions of the other free parameters are presented in Appendix A. Summaries of the means and widths of the Gaussian functions fit to each pull distribution are given in Figure 4.9.

Since the numbers of $B^0 \rightarrow D(\pi K)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$ candidates were unknown at the time of the study, the study was repeated with different values, corresponding to when the CP -violating asymmetry in these yields is large. The estimates of the best-fit values of these yields used in this toy study were $N^{B^0 \pi K} = 12 \pm 3.5$ and $N^{\bar{B}^0 \pi K} = 25 \pm 5$. Negligible difference with respect to the results presented in Figure 4.9 was observed.

The mean of the Gaussian fit to the pull distribution of a free parameter is interpreted as a bias in the value determined by the model. Small but significant biases in the best-fit values of the free parameters of the model are observed. The biases in those parameters crucial to the measurement of the observables, the signal yields, are presented in Table 4.7.

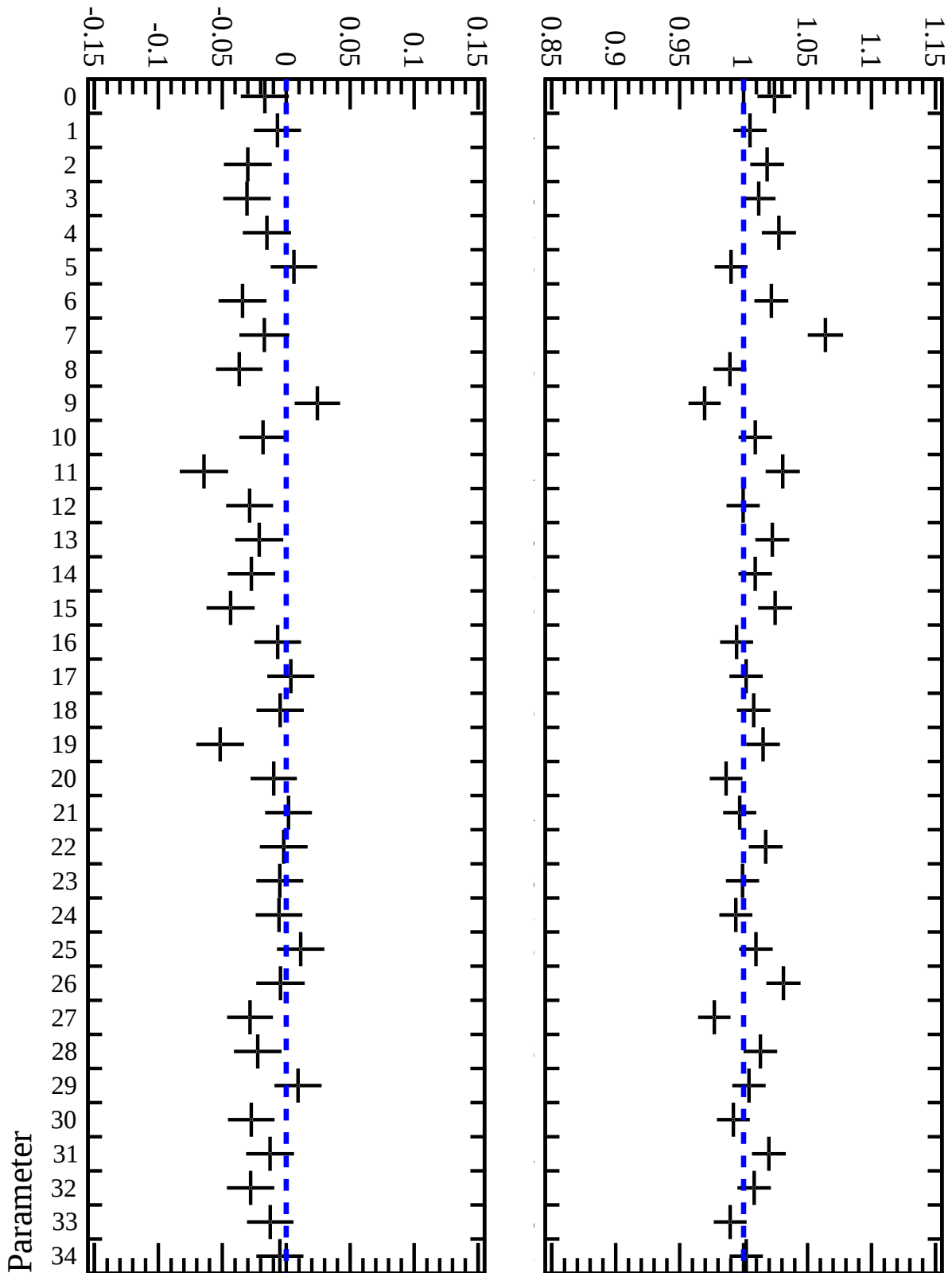


Figure 4.9: A summary of the means (*left*) and widths (*right*) of the Gaussian functions fit to the pull distributions of the free parameters of the invariant mass model.

Table 4.7: The means of the Gaussian functions fit to the pull distributions, or biases in the best-fit values, of the signal yields. The bias is expressed as a fraction of the statistical uncertainty in the yield, therefore the value of the bias multiplied by the statistical uncertainty is also given.

Parameter	$\mu(P)$	$\mu(P) \times \sigma(x)$
$N^{B^0 \pi K}$	0.024 ± 0.018	0.273 ± 0.204
$N^{\bar{B}^0 \pi K}$	-0.021 ± 0.019	-0.242 ± 0.219
$N^{B_s^0 \pi K}$	0.002 ± 0.018	0.065 ± 0.581
$N^{\bar{B}_s^0 \pi K}$	-0.005 ± 0.019	-0.167 ± 0.635
$N^{B^0 K \pi}$	-0.018 ± 0.019	-0.405 ± 0.428
$N^{\bar{B}^0 K \pi}$	-0.027 ± 0.019	-0.585 ± 0.412
$N^{B^0 K K}$	-0.037 ± 0.018	-0.351 ± 0.171
$N^{\bar{B}^0 K K}$	-0.029 ± 0.018	-0.244 ± 0.151
$N^{B_s^0 K K}$	-0.01 ± 0.018	-0.118 ± 0.213
$N^{\bar{B}_s^0 K K}$	0.004 ± 0.018	0.050 ± 0.224
$N^{B^0 \pi \pi}$	-0.064 ± 0.019	-0.392 ± 0.116
$N^{\bar{B}^0 \pi \pi}$	-0.044 ± 0.019	-0.247 ± 0.107
$N^{B_s^0 \pi \pi}$	-0.002 ± 0.019	-0.014 ± 0.133
$N^{\bar{B}_s^0 \pi \pi}$	-0.052 ± 0.019	-0.342 ± 0.125

4.3.2 Feldman-Cousins confidence intervals

The $B_d \rightarrow D(\pi K)K^{*0}$ and $B_d \rightarrow D(\pi\pi)K^{*0}$ decays are seen here for the first time. The yields of candidates for these decays are small and therefore the confidence level as a function of these yields is evaluated using a Feldman-Cousins-style method [75] to assess the reliability of the uncertainty determined from the invariant mass fit. The $\chi^2(\vec{x})$ function is defined, given by Equation 4.14, where $\vec{x}$ contains all the free parameters of the invariant mass model and $\mathcal{L}$ is the likelihood function.

$$\chi^2(\vec{x}) = -2 \ln \mathcal{L}(\vec{x}) \quad (4.14)$$

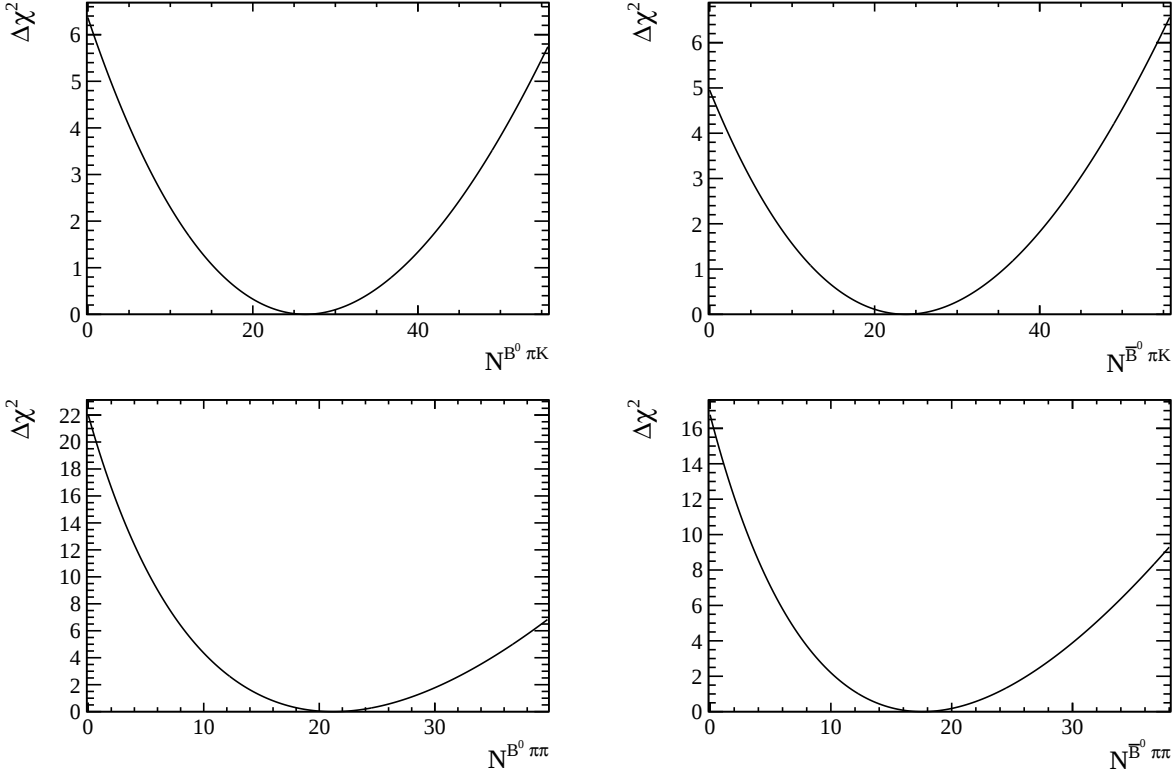


Figure 4.10: $\Delta\chi^2$, as defined in the text, as a function of the fixed value of the number of (*clockwise from top left*) $B^0 \rightarrow D(\pi K)K^{*0}$, $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$, $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ and $B^0 \rightarrow D(\pi\pi)K^{*0}$ candidates.

The χ^2 function has a minimum value, $\chi_{\min}^2$, where all the components of $\vec{x}$ are at their best-fit values. The $\Delta\chi^2$ is defined in Equation 4.15, where $\vec{x}'$ is $\vec{x}$ with the parameter of interest, x_i , fixed to a certain value, x'_i .

$$\Delta\chi^2 = \chi_{\min}^2(\vec{x}') - \chi_{\min}^2 \quad (4.15)$$

The $\Delta\chi^2$ is shown as a function of x'_i in Figure 4.10, where x_i is equal to the number of candidates for the four signals of interest.

A scan in discrete steps of x'_i is performed, where sensible ranges of x'_i are chosen according to the parameter being studied and the size of the step is 0.25. One thousand toy datasets are generated at each point in the scan. The model used for generation is the invariant mass model with $x_i = x'_i$ and all other parameters set equal to their best-fit values under this condition. Two models are fitted to each toy dataset, one with x_i as a free parameter and

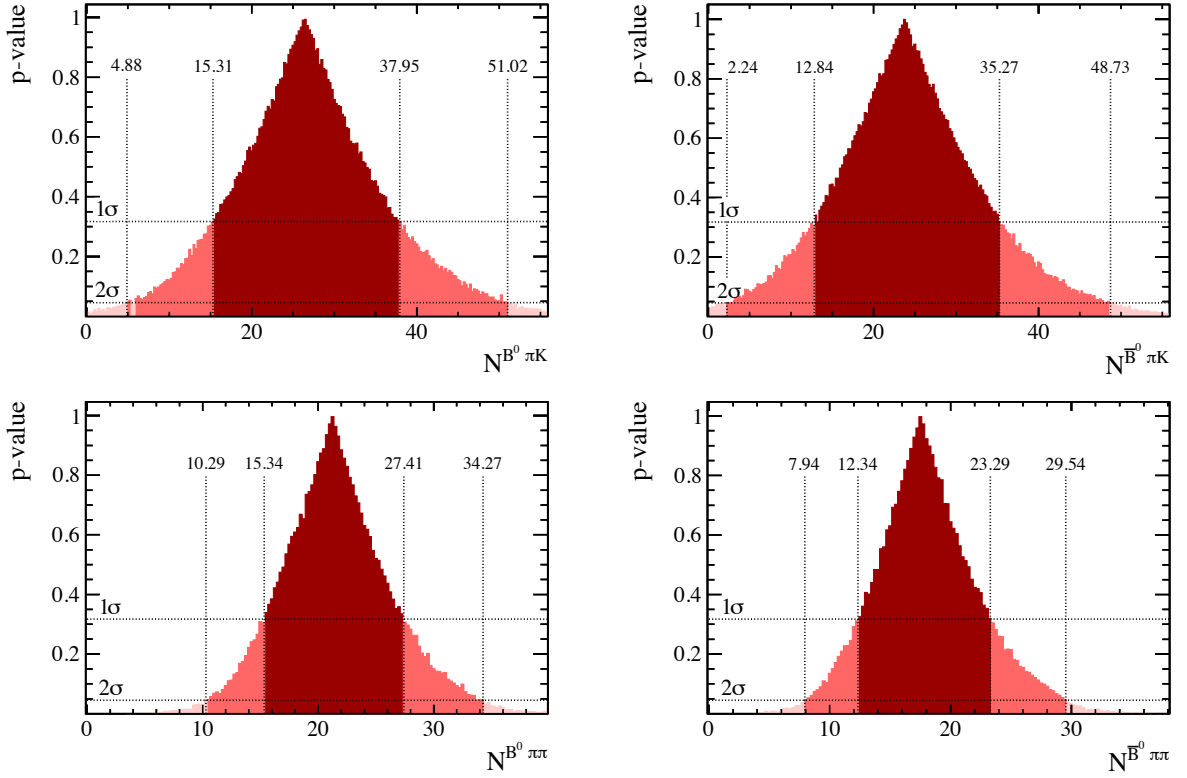


Figure 4.11: The p -value as a function of the yield of (clockwise from top left) $B^0 \rightarrow D(\pi K)K^{*0}$, $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$, $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ and $B^0 \rightarrow D(\pi\pi)K^{*0}$ candidates.

one with it fixed to x'_i , and the $\Delta\chi^2$ is computed. The p -value, at a particular value of x'_i , is given by the fraction of the toys that have a $\Delta\chi^2$ larger than the $\Delta\chi^2$ computed using the analysis dataset, as shown in Figure 4.10.

$$p\text{-value} = 1 - \text{C.L.} = \frac{N(\Delta\chi^2_{\text{toy}} > \Delta\chi^2_{\text{data}})}{N_{\text{toys}}} \quad (4.16)$$

The p -values as functions of the four yields are shown in Figure 4.11. In general, the boundaries of the regions allowed at a confidence level, C.L., of 68.3% and 95%, which correspond to approximately one and two standard deviations of a Gaussian, do not occur at precise points in the scan. Linear interpolation between the two data points on either side of the boundary is used to determine its position. The boundaries of the regions allowed at a confidence level of 68.3% and 95% are shown in Figure 4.11 and Table 4.8. The boundaries of the regions allowed at a confidence level of 68.3% are in good agreement with the corresponding statistical uncertainties in the best-fit values of these yields determined

from the invariant mass fit.

Table 4.8: Confidence intervals in the numbers of $B_d \rightarrow DK^*$ candidates.

Parameter	Confidence Interval	
	68.3%	95%
$N^{B^0 \pi K}$	[15.31, 37.95]	[4.88, 51.02]
$N^{\bar{B}^0 \pi K}$	[12.84, 35.27]	[2.24, 48.73]
$N^{B^0 \pi \pi}$	[15.34, 27.41]	[10.29, 34.27]
$N^{\bar{B}^0 \pi \pi}$	[12.34, 23.29]	[7.94, 29.54]

4.4 Corrections and calculations

The 12 CP violation sensitive observables defined in Section 1.3.2 are calculated using the best-fit values of the numbers of signal candidates given in Table 4.6. However, a number of considerations must also be made. These account for differences in efficiency of the selection requirements, production asymmetry, B meson mixing, misidentification of D meson decays, bias introduced by the invariant mass model, hadronisation fractions, D^0 decay branching fractions and the difference in B^0 and B_s^0 meson lifetimes.

The formulae used to determine the observables from the signal yields and the various correction factors are given in Equations 4.17 to 4.22.

$$\mathcal{A}_d^{hh'} = \frac{a_P^d \hat{\epsilon}_{L0}^{hh'} \hat{\epsilon}_{PID}^{hh'} \hat{\epsilon}_{sel}^{hh'} N^{\bar{B}^0 hh'} - N^{B^0 hh'}}{a_P^d \hat{\epsilon}_{L0}^{hh'} \hat{\epsilon}_{PID}^{hh'} \hat{\epsilon}_{sel}^{hh'} N^{\bar{B}^0 hh'} + N^{B^0 hh'}} \quad (4.17)$$

$$\mathcal{A}_s^{hh'} = \frac{a_P^s \hat{\epsilon}_{L0}^{hh'} \hat{\epsilon}_{PID}^{hh'} \hat{\epsilon}_{sel}^{hh'} N^{\bar{B}_s^0 hh'} - N^{B_s^0 hh'}}{a_P^s \hat{\epsilon}_{L0}^{hh'} \hat{\epsilon}_{PID}^{hh'} \hat{\epsilon}_{sel}^{hh'} N^{\bar{B}_s^0 hh'} + N^{B_s^0 hh'}} \quad (4.18)$$

$$\mathcal{R}_d^{hh} = \frac{a_P^d \hat{\epsilon}_{L0}^{hh} \hat{\epsilon}_{PID}^{hh} \hat{\epsilon}_{sel}^{hh} N^{\bar{B}^0 hh} + N^{B^0 hh}}{a_P^d \hat{\epsilon}_{L0}^{K\pi} \hat{\epsilon}_{PID}^{K\pi} \hat{\epsilon}_{sel}^{K\pi} N^{\bar{B}^0 K\pi} + N^{B^0 K\pi}} \times r_{L0}(\mathcal{R}_d^{hh}) \times r_{PID}(\mathcal{R}_d^{hh}) \times r_{sel}(\mathcal{R}_d^{hh}) \times \frac{\mathcal{B}(D^0 \rightarrow K^- \pi^+)}{\mathcal{B}(D^0 \rightarrow hh)} \quad (4.19)$$

$$\mathcal{R}_{ds}^{hh} = \frac{a_P^d \hat{\epsilon}_{L0}^{hh} \hat{\epsilon}_{PID}^{hh} \hat{\epsilon}_{sel}^{hh} N^{\bar{B}^0 hh} + N^{B^0 hh}}{a_P^s \hat{\epsilon}_{L0}^{hh} \hat{\epsilon}_{PID}^{hh} \hat{\epsilon}_{sel}^{hh} N^{\bar{B}_s^0 hh} + N^{B_s^0 hh}} \times r_{sd} \times \frac{f_s}{f_d} \times \frac{\tau_s}{\tau_d} \quad (4.20)$$

$$\mathcal{R}_d^+ = \frac{N^{B^0 \pi K}}{N^{B^0 K\pi}} \quad (4.21)$$

$$\mathcal{R}_d^- = \frac{N^{\bar{B}^0 \pi K}}{N^{\bar{B}^0 K\pi}} \quad (4.22)$$

In Equation 4.17, $hh' = K\pi$, KK or $\pi\pi$, in Equation 4.18, $hh' = \pi K$, KK or $\pi\pi$, and in Equations 4.19 and 4.20, $hh = KK$ or $\pi\pi$. The symbols representing the signal yields have already been defined. All other symbols in these equations are correction factors. The symbols are defined and the determination of these corrections is discussed in Sections 4.4.1 to 4.4.5.

4.4.1 Production asymmetry

The difference between $\bar{B}^0$ and B^0 , or $\bar{B}_s^0$ and B_s^0 , production rates in pp collisions is accounted for by applying a correction factor, $a_P = \frac{1-\omega A_P}{1+\omega A_P}$, where A_P is the raw production asymmetry of the B_d or B_s mesons and ω is a dilution factor due to mixing.

A_P is defined as the asymmetry in the production rates of $\bar{B}$ and B mesons by Equation 4.23.

$$A_P(B) = \frac{R(\bar{B}) - R(B)}{R(\bar{B}) + R(B)} \quad (4.23)$$

In the case of B_d mesons, A_P has been measured, using $B_d \rightarrow J/\psi K^*$ decays, to be $A_P = 0.010 \pm 0.013$ [76].

The effect of the raw production asymmetry on the observed number of $\bar{B}^0$ or B^0 decays becomes less pronounced for larger decay times, where the decay time, t , is the time between production and decay. This is due to the mixing of B_d mesons. An efficiency dependent factor, ω , accounts for this dilution and is given, for B_d mesons, by Equation 4.24.

$$\omega = \frac{\int_0^{+\infty} e^{-\Gamma_d t} \cos(\Delta m_d t) \epsilon_{\text{tot}}(t) dt}{\int_0^{+\infty} e^{-\Gamma_d t} \cosh\left(\frac{\Delta \Gamma_d t}{2}\right) \epsilon_{\text{tot}}(t) dt}. \quad (4.24)$$

The efficiency of the selection requirements, $\epsilon_{\text{tot}}(t)$, is equal to the initial number of decays in the sample divided by the number after making the selection requirements. The number of $B_d \rightarrow DK^*$ decays before selection is assumed proportional to $e^{-\Gamma_d t}$, where Γ_d is the total width of the B^0 meson. Also, $\Delta \Gamma_d$, the difference in widths of the heavy and light B_d mass eigenstates, is approximately zero. Therefore, Equation 4.24 simplifies to Equation 4.25.

$$\omega = \frac{\int_0^{+\infty} \cos(\Delta m_d t) N_{\text{tot}}(t) dt}{\int_0^{+\infty} N_{\text{tot}}(t) dt} \quad (4.25)$$

In Equation 4.25, N_{tot} is the number of simulated $B_d \rightarrow DK^*$ decays that satisfy the selection requirements. This is evaluated in bins of t using the same methods that are used to evaluate the efficiencies in Section 3.5. Each bin is weighted by $\cos(\Delta m_d t)$, where t is taken from the centre of the bin, and a numerical integration is performed.

The dilution factor is evaluated for each $B_d \rightarrow DK^*$ decay since it is dependent on the separately designed and optimised selection requirements. Figure 4.12 shows an example of the integrand in the numerator of Equation 4.25. The resulting values of ω , for each $B_d \rightarrow DK^*$ decay, are given in Table 4.9. The different values for the dilution factor mean that the correction for production asymmetry is slightly different for different signal decays.

The dilution factor is not evaluated for the $B_s \rightarrow DK^*$ decays since the oscillation frequency of B_s mesons is assumed large enough to mean that ω is approximately zero.

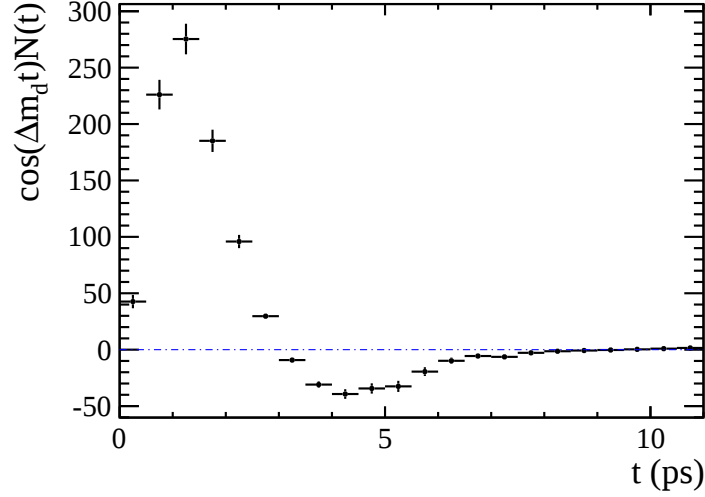


Figure 4.12: The number of selected simulated $B_d \rightarrow D(K^\pm \pi^\mp) K^*$ decays multiplied by the cosine of the product of the mass difference, Δm_d , and the decay time, plotted as a function of the decay time.

Table 4.9: The dilution factor, ω , due to $\bar{B}^0 - B^0$ mixing for each $B_d \rightarrow DK^*$ signal decay.

Decay	ω
$B_d \rightarrow D(K^\pm \pi^\mp) K^*$	$\omega^{K\pi} = 0.362 \pm 0.014$
$B_d \rightarrow D(KK) K^*$	$\omega^{KK} = 0.391 \pm 0.014$
$B_d \rightarrow D(\pi\pi) K^*$	$\omega^{\pi\pi} = 0.398 \pm 0.014$

Therefore the correction factor due to the $\bar{B}_s^0 - B_s^0$ production asymmetry, a_P^s , is one.

4.4.2 Efficiencies

The corrections applied to account for differing efficiencies of the requirements placed on L0 trigger decision are $\hat{\epsilon}_{L0}^{hh'}$ and $r_{L0}(\mathcal{R}_d^{hh})$. The former is the ratio of the efficiencies on $B^0 \rightarrow D(hh') K^{*0}$ and $\bar{B}^0 \rightarrow D(hh') \bar{K}^{*0}$ decays and the latter is the ratio of the efficiencies, averaged over B meson flavour, on $B_d \rightarrow D(K^\pm \pi^\mp) K^*$ and $B_d \rightarrow D(hh) K^*$ decays, where hh is KK or $\pi\pi$.

The corrections applied to account for differing efficiencies of the particle identification and kinematic selection requirements on $B \rightarrow DK^*$ decays follow the same notation convention

Table 4.10: The corrections applied to account for differences in efficiency.

L0 trigger	$\hat{\epsilon}_{\text{L0}}^{K\pi}$	0.99 ± 0.02
	$\hat{\epsilon}_{\text{L0}}^{KK}$	1.00 ± 0.02
	$\hat{\epsilon}_{\text{L0}}^{\pi\pi}$	1.01 ± 0.02
	$r_{\text{L0}}(\mathcal{R}_d^{KK})$	0.99 ± 0.02
	$r_{\text{L0}}(\mathcal{R}_d^{\pi\pi})$	0.97 ± 0.02
PID	$\hat{\epsilon}_{\text{PID}}^{K\pi}$	1.00 ± 0.01
	$\hat{\epsilon}_{\text{PID}}^{KK}$	1.01 ± 0.01
	$\hat{\epsilon}_{\text{PID}}^{\pi\pi}$	1.00 ± 0.01
	$r_{\text{PID}}(\mathcal{R}_d^{KK})$	1.00 ± 0.01
	$r_{\text{PID}}(\mathcal{R}_d^{\pi\pi})$	0.99 ± 0.01
Kinematic	$\hat{\epsilon}_{\text{sel}}^{K\pi}$	1.05 ± 0.03
	$\hat{\epsilon}_{\text{sel}}^{KK}$	0.99 ± 0.03
	$\hat{\epsilon}_{\text{sel}}^{\pi\pi}$	1.01 ± 0.03
	$r_{\text{sel}}(\mathcal{R}_d^{KK})$	0.96 ± 0.03
	$r_{\text{sel}}(\mathcal{R}_d^{\pi\pi})$	0.90 ± 0.03
All	r_{sd}	1.05 ± 0.02

as those relating to the L0 trigger and are given by $\hat{\epsilon}_{\text{PID}}^{hh'}$, $r_{\text{PID}}(\mathcal{R}_d^{hh})$, $\hat{\epsilon}_{\text{sel}}^{hh'}$ and $r_{\text{sel}}(\mathcal{R}_d^{hh})$.

Two more assumptions, aside from those made in the construction of the model of the partially reconstructed $B \rightarrow D^* K^*$ background, are made about the efficiency of the selection requirements. The efficiency on $B_d \rightarrow D(K\pi)K^*$ and $B_d \rightarrow D(\pi K)K^*$ decays is assumed to be equal, hence why only one result is given in Section 3.5, for the efficiency on $B_d \rightarrow D(K^\pm \pi^\mp)K^*$ decays. Also, the ratio of efficiencies on $B_d \rightarrow D(hh')K^*$ to $B_s \rightarrow D(hh')K^*$ is assumed to be equal, regardless of the D meson final state. Therefore, this value is calculated using the $B_d \rightarrow D(K^\pm \pi^\mp)K^*$ and $B_s \rightarrow D(\pi K)K^*$ decay modes.

The correction factor accounting for the difference in efficiency of the selection requirements

on $B_s \rightarrow DK^*$ and $B_d \rightarrow DK^*$ decays is denoted by r_{sd} and is understood to come about because of the different lifetimes of the B^0 and B_s^0 mesons. The difference in selection efficiencies arises from the Stripping, where there is a requirement on the vertex fit χ^2/ndf and the Min IP χ^2 of the B meson, and from the use of $\sum_{\text{tracks}} \sqrt{\text{IP}\chi^2}$ in training the BDTs. All these quantities are dependent on the B meson decay time.

These considerations and the results presented in Section 3.5 imply the values of the efficiency corrections given in Table 4.10.

4.4.3 Misidentification of D decays

The background from favoured $B_d \rightarrow D(K\pi)K^*$ decays being misidentified as suppressed $B_d \rightarrow D(\pi K)K^*$ decays is studied in Section 3.3.2, where it is found that the rate at which this happens is 0.0080 ± 0.0002 . Therefore the best-fit values of the numbers of $B_d \rightarrow D(\pi K)K^*$ decays are corrected to take this into account.

The size of the correction depends, according to Equation 4.26, on the best-fit value of the number of signal $B_d \rightarrow D(K\pi)K^*$ decays, where $N_{\text{fit}}^{B_d \pi K}$ and $N_{\text{fit}}^{B_d K \pi}$ are the best-fit values of the numbers of $B_d \rightarrow D(\pi K)K^*$ and $B_d \rightarrow D(K\pi)K^*$ decays, respectively, and $N_{\text{corr}}^{B_d \pi K}$ is the corrected value of the number of $B_d \rightarrow D(\pi K)K^*$ decays used in Equations 4.21 and 4.22.

$$N_{\text{corr}}^{B_d \pi K} = N_{\text{fit}}^{B_d \pi K} - 0.008 N_{\text{fit}}^{B_d K \pi} \quad (4.26)$$

4.4.4 Model bias correction

The invariant mass model exhibits some small but significant biases in the determination of the numbers of signal $B \rightarrow DK^*$ decays as shown by Table 4.7. The best-fit values of the affected numbers of signal candidates are therefore corrected for these biases. The bias is expressed in terms of a fraction of the statistical uncertainty, so the correction applied is the bias multiplied by the statistical uncertainty. Explicitly, the best-fit numbers of signal candidates, N_{fit} , are corrected according to Equation 4.27 and N_{corr} is used in Equations 4.17

to 4.22.

$$N_{\text{corr}} = N_{\text{fit}} - \mu(P) \times \sigma(N_{\text{fit}}) \quad (4.27)$$

4.4.5 Other corrections

Two ratios of D^0 meson decay branching fractions are needed to compute the final results because of the approximation made between $\mathcal{R}_{CP+}$ and $\mathcal{R}_d^{hh}$ in Equation 1.56. These are taken from Ref. [6], the results of which imply that the ratio of $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ to $\mathcal{B}(D^0 \rightarrow K^+ K^-)$ is 9.798 ± 0.2348 and the ratio of $\mathcal{B}(D^0 \rightarrow K^- \pi^+)$ to $\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$ is 27.695 ± 0.6421 .

The hadronisation fraction of b -quarks into B_s and B_d mesons has an effect on the number of B_s and B_d mesons produced in LHCb and therefore must be accounted for when relating the number of observed decays to the partial width of a decay. The $\mathcal{R}_{ds}^{hh}$ observables are ratios of B_d and B_s decay partial widths. Therefore they are corrected with the hadronisation fraction, $\frac{f_s}{f_d}$, which is 0.267 ± 0.021 [77].

The $\mathcal{R}_{ds}^{hh}$ observables also contain a factor of $\frac{\tau_s}{\tau_d}$, which arises because of the different total widths of the B^0 and B_s^0 mesons. This is taken from Ref. [6], the results of which imply that $\frac{\tau_s}{\tau_d}$ is 0.99 ± 0.01 .

4.5 Uncertainty

4.5.1 Statistical uncertainty

There is statistical uncertainty in the final results due to the finite size of the analysis dataset. This is evaluated, using a linear approximation, from the statistical uncertainties in the best-fit numbers of signal candidates in Table 4.6. The observables are functions of the best-fit numbers of signal $B \rightarrow DK^*$ decays, denoted by $f(a, \vec{x})$, where a is one of the numbers of signal decays and $\vec{x}$ is the rest. The statistical uncertainty in f , due to the statistical

uncertainty in a , is given, in the linear approximation, by Equation 4.28.

$$\sigma_a(f) = f(a + \sigma(a), \vec{x}) - f(a, \vec{x}) \quad (4.28)$$

Equation 4.28 is used to calculate the statistical uncertainty in f due to the statistical uncertainties in all the best-fit numbers of signal decays in $\vec{x}$, by setting each parameter in $\vec{x}$ equal to a in turn. These uncertainties form a column vector, denoted by $\vec{\sigma}$. The total statistical uncertainty in f , $\sigma(f)$, is given by Equation 4.29, where $\boldsymbol{\rho}$ is the correlation matrix of the parameters in $\vec{x}$, shown in Figure 4.7.

$$\sigma(f)^2 = \vec{\sigma}^T \boldsymbol{\rho} \vec{\sigma} \quad (4.29)$$

Asymmetry in the statistical uncertainties in the final results is possible. Therefore, upper and lower uncertainties are treated separately. Mean values are summarised in Tables 4.11 and 4.12.

4.5.2 Systematic uncertainty

Systematic uncertainty refers to any uncertainty in the final results that is not directly related to the size of the analysis dataset. Typically this originates from uncertainty in assumptions made in the analysis method. The emphasis of this section is placed on the uncertainty in the fixed parameters in Equations 4.17 to 4.22. However, there is also systematic uncertainty in the best-fit numbers of signal decays due to assumptions made in the design of the invariant mass model. This will be discussed in Section 4.5.3, although ultimately the treatment of these uncertainties is the same.

Individual sources of systematic uncertainty are identified that can affect one or more parameters. For example, the kinematic selection efficiencies are all determined with simulated data and their uncertainties are a result of the finite size of the simulated dataset. Therefore, the systematic uncertainties in the selection efficiency correction factors are considered to be

from the same source. Another assumption made is about the ratio of D decay branching fractions and this uses the results of Ref. [6], which is considered to be a different source.

Since the equivalent of the statistical correlation matrix with regard to systematic uncertainty is unknown, full correlation of systematic uncertainties from the same source and no correlation of those from different sources is assumed. Therefore, in the linear approximation, the systematic uncertainty in an observable, f , from a particular source is given by Equation 4.30. The vector of parameters that have systematic uncertainty from this source is given by $\vec{a}$, $\vec{x}$ are the unaffected parameters and $\vec{\sigma}(a)$ is a vector of the uncertainties in the parameters in $\vec{a}$.

$$\sigma_{\text{syst}}(f) = f(\vec{a} + \vec{\sigma}(a), \vec{x}) - f(\vec{a}, \vec{x}) \quad (4.30)$$

Asymmetry in the systematic uncertainties in the final results is also possible. Therefore, upper and lower uncertainties are treated separately. Mean values are summarised in Tables 4.11 and 4.12.

4.5.3 Model-related systematic uncertainty

There are various assumptions made when constructing the invariant mass model described in Section 4.1. These assumptions do not directly concern parameters in Equations 4.17 to 4.22. However, they are understood to have an effect on the best-fit values of the numbers of signal $B \rightarrow DK^*$ decays, which are parameters of Equations 4.17 to 4.22. Therefore the systematic uncertainty in the numbers of signal decays that these assumptions cause is evaluated.

Systematic uncertainties due to these assumptions are evaluated using the same toy datasets that are generated for the toy studies in Section 4.3.1. When studying the systematic uncertainty in a signal yield, x , due to the uncertainty in the value of a fixed parameter, y , this parameter is changed to the upper bound of its uncertainty, $y \rightarrow y' = y + \sigma(y)$, to produce an altered model with $y = y'$. The original and alternate models are fit to each toy dataset.

The distribution of ζ is studied, where $\zeta = x' - x$, x' is the best-fit value of a number of signal $B \rightarrow DK^*$ candidates when using the alternate model and x is the best-fit value when using the model with y at its original value. A Gaussian function is fit to the ζ distribution and the quantity Δ is defined. Δ is given by Equation 4.31, where μ and σ are the mean and width of the ζ distribution, respectively. The Δ of the studied ζ distribution gives one bound of the systematic uncertainty in x due to the uncertainty in the assumption about the parameter, y .

$$\Delta = \text{sgn}(\mu)\sqrt{\mu^2 + \sigma^2} \quad (4.31)$$

This process is repeated with y changed to the lower bound of its uncertainty, $y \rightarrow y'' = y - \sigma(y)$. The Δ of this ζ distribution gives the other bound of the systematic uncertainty in x from the uncertainty in the assumption about y .

The fixed parameters of the invariant mass model are summarised in Table 4.5. The systematic uncertainty in each signal yield, x , arising from the uncertainties in each of these fixed parameters is determined. In addition to this, a systematic uncertainty due to the assumption that the signal is well described by a double Gaussian function is estimated.

Clearly the method described here cannot directly apply to the evaluation of the systematic uncertainty due to the assumption that the signal PDF is well described by a double Gaussian function. In this case, the toy datasets are fit with three different PDFs: the usual model, with a Cruijff function, a constrained Cruijff function and an Apollonios function to describe the signal. The Cruijff and Apollonios functions are given by Equation 4.2 and 4.3, respectively. The constrained Cruijff function refers to when a constraint of $\sigma_L = \sigma_R$ is made on Equation 4.2.

Now x' is the best-fit value of x when using the model with the alternate signal PDF, in analogy to the described method. However, there are now three ζ distributions. The “envelope” of the three Δ values is taken as the bounds of the systematic uncertainty from assuming that the signal distribution is well described by a double Gaussian function. For example, say that the Δ values of the ζ distributions relating to a hypothetical x were +2.0,

-2.5 and -1.0 . The systematic uncertainty in x from assuming that the true signal PDF is a double Gaussian would be $x_{-2.5}^{+2.0}$.

The other, less obvious, exceptions to the described method are the assumptions about the parameters of the double Gaussian signal PDF, f_{core} and r_W . These parameters are fixed to the best-fit values found when fitting a double Gaussian function to simulated data. To remove dependence on the simulated data for the correct determination of the uncertainty in these parameters, a method, using the analysis dataset, to assign conservative uncertainties is devised.

The invariant mass fit is performed with the constraint on f_{core} or r_W relaxed. A symmetric error band, around the original value of the constraint, is defined that encompasses the best-fit value and the uncertainty when the constraint is relaxed. To be explicit, f_{core} is usually fixed to 0.88. When this constraint is relaxed, the best-fit value is $f_{\text{core}} = 0.85 \pm 0.04$. Therefore, for the purposes of evaluating systematic uncertainty in the signal yields from the uncertainty in the value of f_{core} , $f_{\text{core}} = 0.88 \pm 0.07$ is assumed. This is because $0.85 - 0.04 = 0.81$ and $0.88 - 0.81 = 0.07$.

The same procedure to conservatively estimate the uncertainty in the value of r_W is followed. Then the described method of fitting Gaussian functions to ζ distributions is used to determine the systematic uncertainties in the numbers of signal decays.

Examples of the ζ distributions, concerning the systematic uncertainty in the number of $B^0 \rightarrow D(K\pi)K^{*0}$ signal candidates from fixing f_{core} , are shown in Figure 4.13. When f_{core} is changed to 0.95, $\Delta = -8.763$. When f_{core} is changed to 0.81, $\Delta = 7.557$. Therefore the best-fit value and systematic uncertainty in $N^{B^0 K\pi}$ introduced by fixing $f_{\text{core}} = 0.88$ is $N^{B^0 K\pi} = 405.3_{-8.763}^{+7.557}$.

Analogous ζ distributions are studied to assess the systematic uncertainty in each number of signal decays from fixing f_{core} and every other parameter in Table 4.5. The systematic uncertainty resulting from the choice of signal PDF is evaluated as described. More detailed results of the studies of model-related systematic uncertainties are presented in Appendix B.

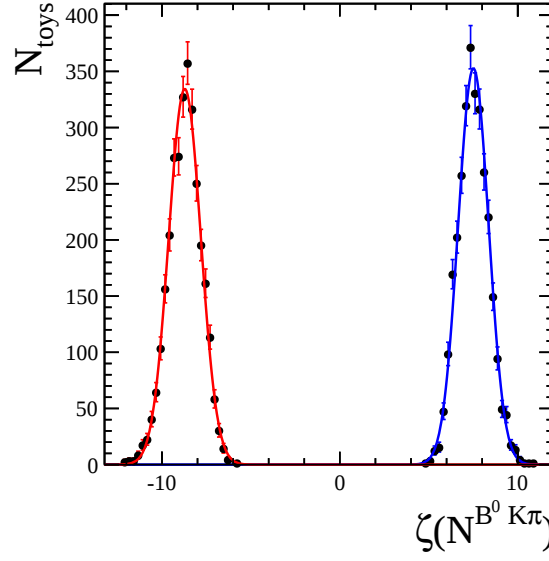


Figure 4.13: The distributions of $\zeta(N^{B^0 K\pi})$ arising when varying f_{core} to its upper bound (*red*) and lower bound (*blue*).

The numbers of signal decays are parameters in Equations 4.17 to 4.22. Therefore the systematic uncertainties in the signal yields resulting from the assumptions about the model, evaluated using the ζ distributions, are treated in the way described in Section 4.5.2 using Equation 4.30.

Model bias

A final source of systematic uncertainty in the numbers of signal decays, resulting from assumptions made about the fit model, is considered. The numbers of signal decays are corrected for the bias observed in the toy studies in Section 4.3.1. The uncertainties in $\mu(P) \times \sigma(N)$, given in Table 4.7, equate to the systematic uncertainties in the corrected values of the signal yields due to the uncertainties in these corrections. These uncertainties are treated as described in Section 4.5.2.

4.5.4 Summary

Tables 4.11 and 4.12 show statistical and systematic uncertainties in the observables, where the values given are the mean of the upper and lower uncertainties. In total, 28 separate sources

of systematic uncertainty are considered. All 18 model-related systematic uncertainties have been added in quadrature and listed in one row for illustrative purposes. The full results are presented in Appendix C¹.

Observables are not affected by certain sources of systematic uncertainty if the correction or consideration defining that source is not involved in the determination of the observable. For example, observables that only concern B_s mesons incur no systematic uncertainty due to correction for production asymmetry.

In general, the dominant sources of systematic uncertainty are the uncertainties in the correction factors for efficiency and D decay branching fractions. Moreover, the systematic uncertainty incurred by the $\mathcal{R}_d^{hh}$ observables from these sources is larger than that incurred by the others. This is recognised as an effect due to the direct dependence of $\mathcal{R}_d^{hh}$, given by Equation 4.19, on corrections for efficiency and D decay branching fractions. These corrections enter as overall multiplicative factors in $\mathcal{R}_d^{hh}$ observables, which are therefore more sensitive to their values than other observables where they do not.

It is noted that the $\mathcal{R}_d^\pm$ observables are affected by the least sources of systematic uncertainty because corrections, for efficiencies and such, cancel in the ratios of the concerned signal yields. Indeed this was an important reason for measuring these quantities. However, they do not have the smallest systematic uncertainty, owing to the relatively large model-related systematic uncertainty. This is understood to be because of the background to the $B_d \rightarrow D(\pi K)K^*$ signal from $B_s \rightarrow D^*(D\gamma)K^*$ decays. The number of candidates for this background is partially dictated by the fixed value of the ratio of branching fractions of $D^{*0} \rightarrow D^0\pi^0$ and $D^{*0} \rightarrow D^0\gamma$ decays and, since the background yield is large compared to the signal, the signal yield is sensitive to the value of this ratio. Indeed on inspection of Table C.1, it is found that this is the dominant source of model-related systematic uncertainty

¹It is noted that fixed parameters of the invariant mass model are also sometimes used in corrections applied externally to the numbers of signal decays in Equations 4.17 to 4.22. These have been treated as separate sources of systematic uncertainty, and therefore uncorrelated, which is not particularly defensible, given the rest of the logic of the evaluation of systematic uncertainty. The parameters for which this oversight has been committed are the total selection efficiencies, ϵ_{tot} , and the D^0 decay branching fractions. The systematic uncertainties caused by the uncertainties in these parameters are negligible and therefore it is assumed that this makes little difference to the total systematic uncertainty, which is smaller than the statistical uncertainty in all of the final results.

Table 4.11: A summary of the uncertainties in the first six observables. All model-related systematic uncertainties have been added in quadrature and the result is shown as one source of systematic uncertainty. The presence of ‘–’ indicates that the source of uncertainty does not affect the observable.

Source	Observable					
	$\mathcal{R}_d^+$	$\mathcal{R}_d^-$	$\mathcal{R}_d^{KK}$	$\mathcal{A}_s^{KK}$	$\mathcal{A}_d^{K\pi}$	$\mathcal{R}_{ds}^{KK}$
Production asymmetry	–	–	0.001	–	0.005	$<10^{-3}$
ω (dilution)	–	–	$<10^{-3}$	–	$<10^{-3}$	$<10^{-3}$
f_s/f_d	–	–	–	–	–	0.008
τ_s/τ_d	–	–	–	–	–	0.001
D^0 decay BFs	–	–	0.025	–	–	–
Trigger efficiency	–	–	0.015	0.011	0.012	$<10^{-3}$
Lifetime difference	–	–	–	–	–	0.002
PID efficiency	–	–	0.010	0.005	0.005	$<10^{-3}$
Selection efficiency	–	–	0.029	0.015	0.014	$<10^{-3}$
D decay misID	$<10^{-3}$	0.001	–	–	–	–
Model-related	0.009	0.010	0.011	0.001	0.001	0.001
Total systematic	0.010	0.011	0.044	0.020	0.020	0.009
Statistical	0.028	0.031	0.159	0.073	0.041	0.017

in the $\mathcal{R}_d^\pm$ observables. A related observation is that the overall trend with regard to the size of model-related systematic uncertainty is that observables that concern larger signals are less affected. This is presumably because, proportionally, the numbers of candidates for these signals are less dependent on the fixed parameters of the model that govern the yields of various backgrounds.

Finally, the most important observation to make about Tables 4.11 and 4.12 is that the statistical uncertainty in all observables is larger than the systematic uncertainty.

Table 4.12: A summary of the uncertainties in the second six observables. All model-related systematic uncertainties have been added in quadrature and the result is shown as one source of systematic uncertainty. The presence of ‘–’ indicates that the source of uncertainty does not affect the observable.

Source	Observable					
	$\mathcal{R}_d^{\pi\pi}$	$\mathcal{R}_{ds}^{\pi\pi}$	$\mathcal{A}_s^{\pi K}$	$\mathcal{A}_d^{\pi\pi}$	$\mathcal{A}_s^{\pi\pi}$	$\mathcal{A}_d^{KK}$
Production asymmetry	$<10^{-3}$	0.001	–	0.005	–	0.005
ω (dilution)	$<10^{-3}$	$<10^{-3}$	–	$<10^{-3}$	–	$<10^{-3}$
f_s/f_d	–	0.012	–	–	–	–
τ_s/τ_d	–	0.001	–	–	–	–
D^0 decay BFs	0.028	–	–	–	–	–
Trigger efficiency	0.019	$<10^{-3}$	0.012	0.011	0.011	0.011
Lifetime difference	–	0.003	–	–	–	–
PID efficiency	0.012	$<10^{-3}$	0.005	0.005	0.005	0.005
Selection efficiency	0.037	$<10^{-3}$	0.014	0.014	0.014	0.014
D decay misID	–	–	–	–	–	–
Model-related	0.012	0.001	$<10^{-3}$	0.001	0.002	0.002
Total systematic	0.053	0.012	0.019	0.019	0.019	0.020
Statistical	0.268	0.038	0.025	0.217	0.131	0.144

4.6 Final results

As a result of the analysis presented, the measurements of the observables are

$$\begin{aligned}
\mathcal{A}_d^{KK} &= -0.198^{+0.144}_{-0.145} {}^{+0.019}_{-0.020} & \mathcal{A}_d^{K\pi} &= -0.032^{+0.041}_{-0.041} {}^{+0.019}_{-0.020} \\
\mathcal{A}_s^{KK} &= -0.044^{+0.073}_{-0.073} {}^{+0.019}_{-0.020} & \mathcal{R}_d^{KK} &= 1.054^{+0.165}_{-0.153} {}^{+0.044}_{-0.044} \\
\mathcal{R}_{ds}^{KK} &= 0.103^{+0.018}_{-0.016} {}^{+0.009}_{-0.009} & \mathcal{A}_d^{\pi\pi} &= -0.092^{+0.217}_{-0.217} {}^{+0.019}_{-0.019} \\
\mathcal{A}_s^{\pi\pi} &= 0.064^{+0.130}_{-0.131} {}^{+0.018}_{-0.019} & \mathcal{R}_d^{\pi\pi} &= 1.214^{+0.283}_{-0.252} {}^{+0.053}_{-0.053} \\
\mathcal{R}_{ds}^{\pi\pi} &= 0.147^{+0.040}_{-0.036} {}^{+0.012}_{-0.012} & \mathcal{A}_s^{\pi K} &= -0.014^{+0.025}_{-0.025} {}^{+0.019}_{-0.019} \\
\mathcal{R}_d^+ &= 0.057^{+0.029}_{-0.027} {}^{+0.009}_{-0.012} & \mathcal{R}_d^- &= 0.056^{+0.032}_{-0.030} {}^{+0.009}_{-0.012}
\end{aligned}$$

where the first uncertainty is statistical and the second is systematic. The statistical and systematic correlation matrices of these observables are given in Figure 4.14 and Figure 4.15, respectively.

Some evidence for CP violation is found in $B_d \rightarrow D(KK)K^*$ decays, where the CP -violating asymmetry, $\mathcal{A}_d^{KK}$, deviates from zero by greater than one standard deviation. The measurements of $\mathcal{R}_d^{KK}$ and $\mathcal{A}_d^{KK}$ are consistent with the results of Ref. [3], although the best-fit value of $\mathcal{A}_d^{KK}$ is decreased in magnitude. In addition, these results are consistent with the best-fit

Table 4.13: The observables and their corresponding indices used for labelling figures.

Observable	$\mathcal{A}_d^{KK}$	$\mathcal{A}_d^{K\pi}$	$\mathcal{A}_d^{\pi\pi}$	$\mathcal{A}_s^{KK}$	$\mathcal{A}_s^{\pi K}$	$\mathcal{A}_s^{\pi\pi}$
Index	1	2	3	4	5	6
Observable	$\mathcal{R}_d^{KK}$	$\mathcal{R}_d^{\pi\pi}$	$\mathcal{R}_d^-$	$\mathcal{R}_d^+$	$\mathcal{R}_{ds}^{KK}$	$\mathcal{R}_{ds}^{\pi\pi}$
Index	7	8	9	10	11	12

values of $\mathcal{R}_d^{\pi\pi}$ and $\mathcal{A}_d^{\pi\pi}$, which is expected due to the fact that K^+K^- and $\pi^+\pi^-$ are both CP -even eigenstates.

The measurements of $\mathcal{R}_d^+$ and $\mathcal{R}_d^-$ represent the first observations of the suppressed $B^0 \rightarrow D(\pi K)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$ decays at a level of confidence corresponding to greater than one standard deviation. A more quantitative statement is made about the significance of the observations of these decays in Section 4.6.

Although accurate measurements of the observables relating to B_s mesons are made, no significant evidence for CP violation in $B_s \rightarrow DK^*$ decays is found. This is expected because of the small ratio of interfering amplitudes concerned and more data is needed to make a statement about CP violation in this decay.

The correlation between the observables, with regard to statistical uncertainties, is taken from the correlations of the free parameters of the invariant mass model. The correlation between the systematic uncertainties of observable A and observable B , ρ_{AB} , is calculated using Equation 4.32.

$$\rho_{AB} = \frac{\sum_{\alpha\beta} \rho_{\alpha\beta} \sigma_A^\alpha \sigma_B^\beta}{\sigma_A^{\text{tot}} \sigma_B^{\text{tot}}} \quad (4.32)$$

Where σ_A^α is the systematic uncertainty in A from source α , σ_A^{tot} is the total systematic uncertainty in A , $\rho_{\alpha\beta}$ is the correlation between sources α and β and α and β run over all sources of systematic uncertainty in Table C.1. In the simplified case where systematic uncertainties from the same source are assumed 100% correlated and systematic uncertainties from different sources are assumed uncorrelated, Equation 4.32 reduces to Equation 4.33.

$$\rho_{AB} = \frac{\sum_{\alpha} \sigma_A^\alpha \sigma_B^\alpha}{\sigma_A^{\text{tot}} \sigma_B^{\text{tot}}} \quad (4.33)$$

Signal significance

In addition to the main results, calculations are made of the significance of observations of the decays that have thus far not been observed. The significances of the $B^0 \rightarrow D(\pi K)K^{*0}$,

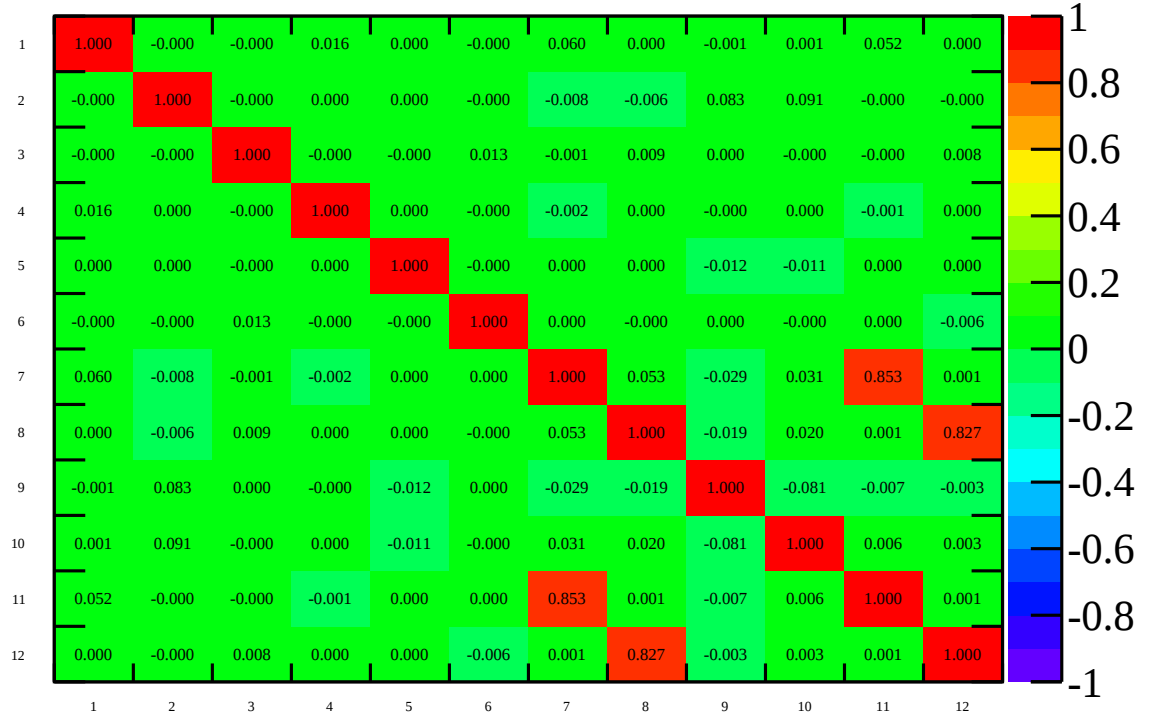


Figure 4.14: The statistical correlation matrix of the twelve observables. The observables are labelled with their indices given in Table 4.13.



Figure 4.15: The systematic correlation matrix of the twelve observables. The observables are labelled with their indices given in Tables 4.13.

$\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$, $B^0 \rightarrow D(\pi\pi)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ signals are determined. The significance of a signal is defined by Equation 4.34, where $\Delta \ln \mathcal{L}(\vec{x})$ is the difference in the natural logarithm of the likelihood relative to the global maximum as a function of the parameters of the model, $\vec{x}$. The $\Delta \ln \mathcal{L}$ is evaluated with the yield of the signal under study set to zero to determine the significance with which it is observed².

$$\text{Significance} = \sqrt{2\Delta \ln \mathcal{L}(\vec{x})} \quad (4.34)$$

In addition to the likelihood function of the invariant mass fit, systematic uncertainty must be taken into account when evaluating the significance. The systematic uncertainties in the yields of $B^0 \rightarrow D(\pi K)K^{*0}$, $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$, $B^0 \rightarrow D(\pi\pi)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ candidates from various sources are reported in Appendix B. These uncertainties are added in quadrature to define the total systematic uncertainty in that yield.

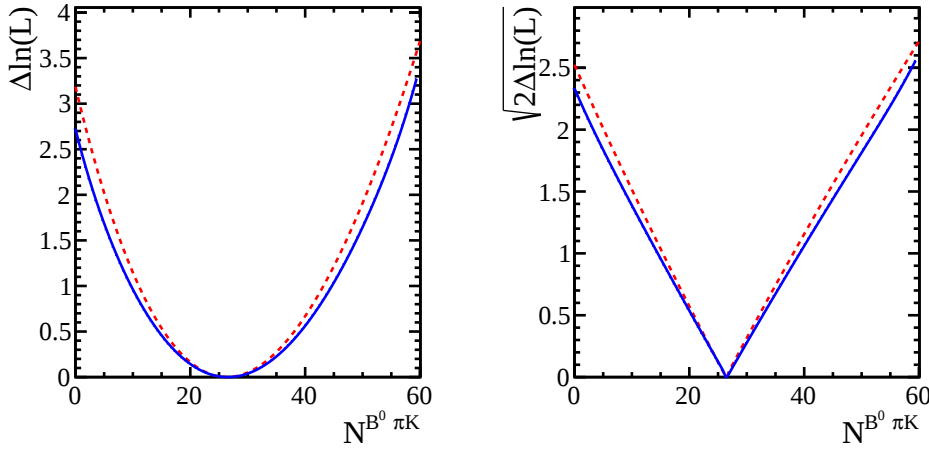


Figure 4.16: The $\Delta \ln(\mathcal{L})$ as a function of the number of $B^0 \rightarrow D(\pi K)K^{*0}$ candidates (*left*) and the square root of double this quantity (*right*). The likelihood function from the fit is used (*red dashed line*) as well as this function convolved with a Gaussian describing the systematic uncertainty (*blue line*). The significance of the observation of the $B^0 \rightarrow D(\pi K)K^{*0}$ decay can be determined from the right-hand graph as the intercept of the blue line with the y -axis.

For each signal yield, the likelihood as a function of that yield is convolved with a Gaussian function that has a width equal to the systematic uncertainty in that yield. This convolution is used as $\mathcal{L}$ in Equation 4.34. For example, the $\Delta \ln \mathcal{L}$ as a function of the yield of

²It is noted that the right-hand side of Equation 4.34 is equal to $\sqrt{\Delta\chi^2}$, where the $\Delta\chi^2$ is the same quantity defined in Section 4.3.2

$B^0 \rightarrow D(\pi K)K^{*0}$ candidates is shown in Figure 4.16. The effect of accounting for systematic uncertainty with the convolution can be seen and the significance of the $B^0 \rightarrow D(\pi K)K^{*0}$ signal is determined by inspection to be 2.3σ .

The significances of the $\bar{B}^0 \rightarrow D(\pi K)\bar{K}^{*0}$, $B^0 \rightarrow D(\pi\pi)K^{*0}$ and $\bar{B}^0 \rightarrow D(\pi\pi)\bar{K}^{*0}$ signals are evaluated in the same way and are found to be 2.1σ , 4.7σ and 4.1σ , respectively. The significances of the combined $B_d \rightarrow D(\pi K)K^*$ and $B_d \rightarrow D(\pi\pi)K^*$ signals are 2.9σ and 5.8σ , respectively.

Chapter 5

Interpretation and Conclusions

The primary goal of the analysis presented in this thesis is the measurement of the CKM angle, γ , the amplitude ratio, r_B , and the strong phase difference, δ_B . A subset of the measurements reported in Chapter 4 is used to determine these parameters. The future prospects for analyses of $B \rightarrow DK^*$ decays are also discussed.

5.1 Determination of r_B , δ_B and γ

To determine the best-fit values of the parameters, γ , r_B and δ_B , a multidimensional Gaussian PDF, f , is defined. The PDF is given by Equation 5.3, where $\vec{A}$ is a vector of observables,

$$\vec{A} = (\mathcal{A}_d^{KK}, \mathcal{A}_d^{\pi\pi}, \mathcal{R}_d^{KK}, \mathcal{R}_d^{\pi\pi}, \mathcal{A}_d^{K\pi}, \mathcal{R}_d^+, \mathcal{R}_d^-)^T \quad (5.1)$$

and $\vec{A}_{\text{obs}}$ contains their measured values. V is the experimental covariance matrix, which is given by Equation 5.2.

$$V = (\vec{\sigma}_{\text{stat}}(A) \cdot \vec{\sigma}_{\text{stat}}(A)^T) \circ \boldsymbol{\rho}_{\text{stat}} + (\vec{\sigma}_{\text{syst}}(A) \cdot \vec{\sigma}_{\text{syst}}(A)^T) \circ \boldsymbol{\rho}_{\text{syst}} \quad (5.2)$$

In Equation 5.2, $\vec{\sigma}_{\text{stat}}(A)$ and $\vec{\sigma}_{\text{syst}}(A)$ are column vectors of the statistical and systematic uncertainties in the observables in $\vec{A}$, respectively. The statistical and systematic correlation matrices, $\boldsymbol{\rho}_{\text{stat}}$ and $\boldsymbol{\rho}_{\text{syst}}$, are given in Figures 4.14 and 4.15, respectively, and ‘ $\circ$ ’ signifies

the element-wise product.

$$f \propto \exp(-(\vec{A} - \vec{A}_{\text{obs}})^T V^{-1} (\vec{A} - \vec{A}_{\text{obs}})) \quad (5.3)$$

This multidimensional Gaussian PDF is used to evaluate the likelihood as a function of the parameters of Equations 1.61 to 1.64, $\mathcal{L}(\vec{\alpha})$, where $\vec{\alpha}$ is given by

$$\vec{\alpha} = (\gamma, r_B, \delta_B, \kappa, r_D, \delta_D)^T. \quad (5.4)$$

The likelihood function of $\vec{\alpha}$ is also multiplied by two other Gaussian terms. One is a function of δ_D and r_D , defined by the measurements of Ref. [34] and the other, a function of the coherence factor, defined by the calculation of Ref. [36]. The coherence factor is 0.95 ± 0.03 , which is corroborated by an examination of the contribution from non- $K^*(892)$ $B_d \rightarrow DK^\pm \pi^\mp$ decays to the signal in Appendix D. The likelihood function is explored, to determine the location of the maximum and therefore the best-fit values of $\vec{\alpha}$, given the measurements presented in this thesis, Ref. [34] and Ref. [36].

An analogous $\Delta\chi^2$ quantity to that given by Equation 4.15, is defined, where the χ^2 is now a function of $\vec{\alpha}$ rather than $\vec{x}$, the free parameters of the invariant mass model. The $\Delta\chi^2$ is evaluated as a function of the parameter of interest, γ , r_B or δ_B or any combination thereof, and two methods are used to determine the p -value.

The p -value is given by the integral of the χ^2 distribution, with one degree of freedom, from $\Delta\chi^2$ to ∞ in Equation 5.5.

$$p\text{-value} = 1 - \text{C.L.} = \frac{1}{\sqrt{2}\Gamma(\frac{1}{2})} \int_{\Delta\chi^2}^{\infty} e^{-t/2} t^{-1/2} dt \quad (5.5)$$

Equation 5.5 represents a computationally cheap way to determine the p -value but carries with it an assumption that the uncertainties in the parameters of interest are Gaussian. Therefore a Feldman-Cousins-style method [75], similar to that described in Section 4.3.2 is

also used to calculate the p -value. The “toy datasets”, in this case, are randomly generated values of the observables in $\vec{A}$. The PDF from which the toy dataset is generated is that of Equation 5.3. However, $\vec{A}_{\text{obs}}$ is changed to $\vec{A}'_{\text{obs}}$, which contains the best-fit values of the observables in $\vec{A}$ under the condition that the parameters of interest are fixed to the particular set of values corresponding to the point in the parameter space being studied. The $\Delta\chi^2$ of the toy is found using the minimum of the χ^2 function with the parameters fixed and the minimum with the parameters free, and the p -value is given by Equation 4.16.

The p -value as a function of r_B , δ_B and γ is shown in Figures 5.1 and 5.2, which display approximate agreement between the two methods to determine the p -value. Equation 5.5 is used to determine the p -value in Figure 5.4, because the number of points in the scan renders the Feldman-Cousins method unusable due to computational expense.

Figure 5.1 shows that the ratio of the magnitudes of the amplitudes corresponding to $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decays is

$$r_B = 0.230^{+0.063}_{-0.045}$$

at a confidence level of 68.3%. This result is consistent with the prediction of $r_B \approx 0.27$, which arose from the value of the equivalent parameter of the closely related $B^\pm \rightarrow DK^\pm$ decays, $r_{B^\pm} = 0.089^{+0.008}_{-0.009}$ [28], and the colour-suppression of both $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ amplitudes. This result is also consistent with and more precise than that of Ref. [31].

The p -value as a function of γ found from a combination of analyses of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays is shown in Figure 5.3. The comparison of Figures 5.3 and 5.2 and the blue bands superimposed on Figure 5.4 show good compatibility between the results in this thesis and Ref. [28]. However, as shown by Figure 5.4, very little of the (δ_B, γ) parameter space is excluded at any reasonable level of confidence.

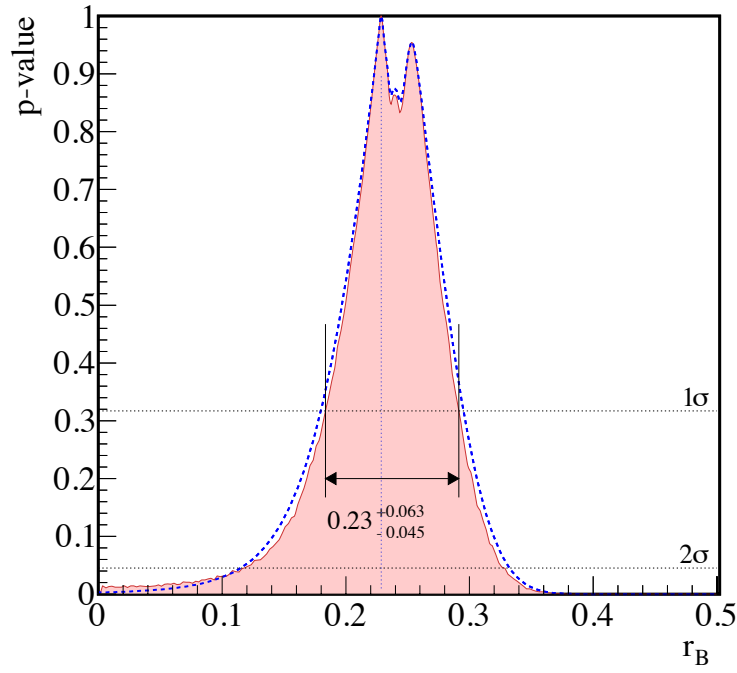


Figure 5.1: The p -value as a function of r_B , evaluated using Equation 5.5 (*blue dashed line*) and using the Feldman-Cousins method (*red fill*).

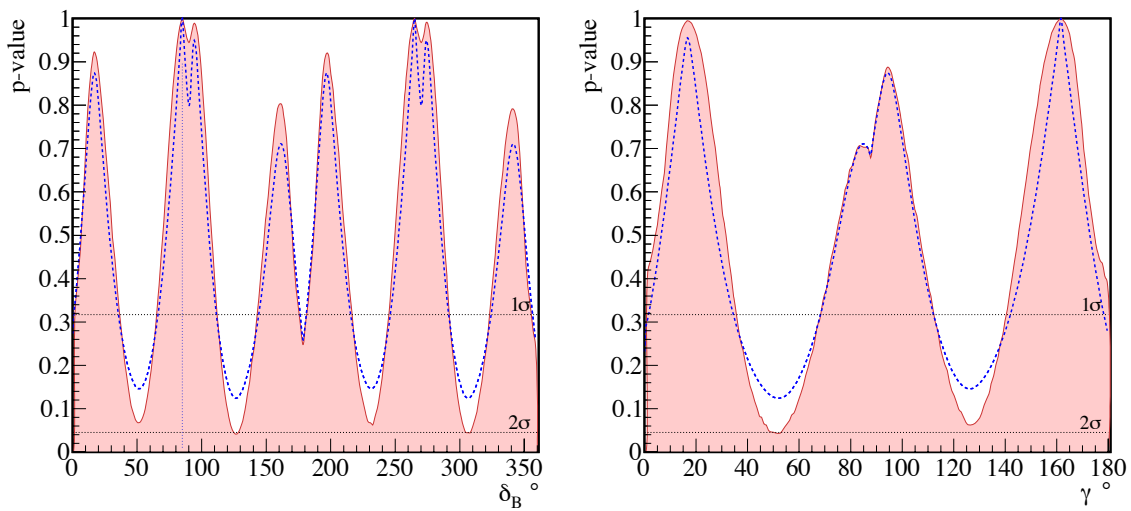


Figure 5.2: The p -value as a function of δ_B (*left*) and γ (*right*), evaluated using Equation 5.5 (*blue dashed line*) and using the Feldman-Cousins method (*red fill*).

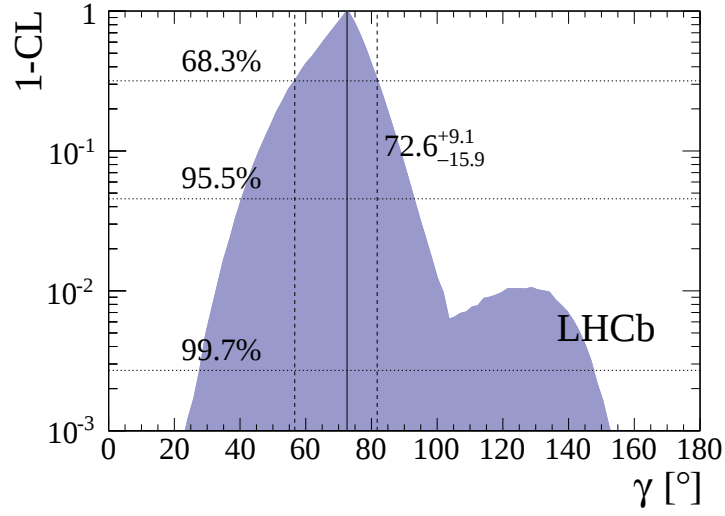


Figure 5.3: The p -value as a function of γ , found from a combination of analyses of $B^\pm \rightarrow DK^\pm$ and $B^\pm \rightarrow D\pi^\pm$ decays. Figure taken from Ref. [28].

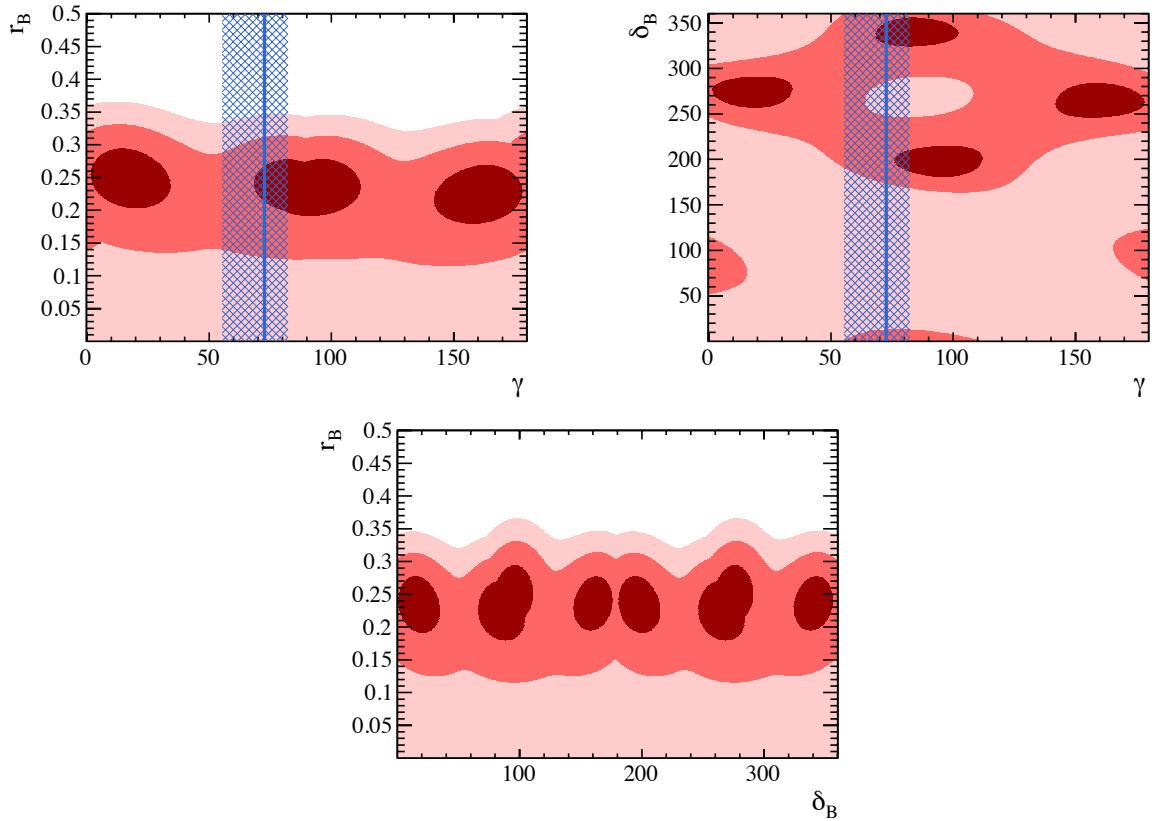


Figure 5.4: Two-dimensional projections of the p -value in (r_B, δ_B, γ) parameter space onto r_B and γ (left), δ_B and γ (right) and r_B and δ_B (bottom). The p -value is evaluated using Equation 5.5. The regions allowed at confidence levels of 68.3%, 95% and 99.7% are indicated by lightening shades of red. The blue line and band represents the best-fit value of γ and the region allowed at a confidence level of 68.3% by Ref. [28].

5.2 Summary and outlook

This thesis has reported an analysis of $B \rightarrow DK^*$ decays, using 3.0 fb^{-1} of pp collision data, recorded by the LHCb experiment at CERN. The primary goal of this analysis is to constrain the γ angle of the Unitarity Triangle and $B_d \rightarrow DK^*$ decays represent a promising route to the measurement of γ because of the large value of r_B .

Many important measurements are presented in this thesis. Hints of CP violation are seen in the $B_d \rightarrow D(KK)K^*$ decay and several important decays for the determination of γ are measured for the first time. The $B_d \rightarrow D(\pi\pi)K^*$ decay is observed at a 5.8σ level of confidence, which is the first observation of this decay at a confidence level of greater than 5σ . Most importantly, the most accurate determination, to date, of the ratio of $B^0 \rightarrow D^0 K^{*0}$ and $B^0 \rightarrow \bar{D}^0 K^{*0}$ decay amplitudes is made. This is measured to be $r_B = 0.230^{+0.063}_{-0.045}$. This is conclusive proof that this parameter is larger than the equivalent parameter of $B^\pm \rightarrow DK^\pm$ decays and therefore that the potential sensitivity to the relative weak phase of these amplitudes, γ , is enhanced.

Any future analysis of $B \rightarrow DK^*$ decays will clearly benefit from an increase in the size of the dataset. The dominance of the statistical over systematic uncertainty in all of the results shows that this is, in the near future, the foremost source of increased precision. Moreover, for this particular analysis of $B \rightarrow DK^*$ decays to make an accurate determination of γ or a significant contribution to a combined measurement, considerably greater precision is needed.

With decreasing statistical uncertainty, systematic uncertainty will, eventually, start to dominate. When this time comes, improvements to the analysis are foreseen from a more efficient selection strategy. Making particle identity information available to the BDT algorithms would surely be beneficial. This was not possible because of the need to use simulated data to train the BDTs and PID information not being accurately represented by simulated data at present. Perhaps future improvements to the simulation software or conception of a training method that utilised real data would enable this. Similar, and therefore kinematically representative, decays with higher branching fractions, such as

$B_d \rightarrow D\rho^0$, may be of use with the latter.

Further improvement to the selection strategy could also come from inclusion of the FDS_D variable in the BDT. The requirement on this variable is one of the least efficient. However, the background from charmless decays is a significant one, as shown in Section 3.3.1, so any alterations to this part of the selection strategy must be treated with caution.

The final suggestion for improvement that is made, although there are numerous that surely go unmentioned, is the reduction of the background from $B_s \rightarrow D^*K^*$ decays. This is the dominant background to the suppressed $B_d \rightarrow D(\pi K)K^*$ signal and its reduction will be important for accurate measurement of the B_d decay. However, the similarity of the signal and background render this a difficult task. Perhaps the angular distribution of reconstructed tracks could be used to differentiate between this background and the signal but no investigation is made in this thesis.

Additional constraints on γ would come from analysing $B \rightarrow DK^*$ decays in conjunction with more D meson decays. For example, $D^0 \rightarrow K^-2\pi^+\pi^-$ and $D^0 \rightarrow K^-\pi^+\pi^0$ both have larger branching fractions than any D meson decay studied in this thesis, however, these decays bring difficulty for a number of reasons. The reconstruction efficiency of a decay with a greater number of tracks or neutral tracks is significantly lower. Furthermore, the variation of the strong phase difference across the kinematic parameter space of the multi-body decay would need to be dealt with. To discern whether the optimal method is integration over a region of high coherence, modelling of the variation or the separation of the parameter space into regions of similar strong phase difference clearly requires detailed studies.

In a similar vein, the full parameter space of the $B \rightarrow DK\pi$ decay could be analysed and the presence of other intermediate resonances exploited. Indeed this is suggested in Ref. [78] and studies of the sensitivity of such analyses have shown a potential gain of 50% over the quasi-two-body analysis presented in this thesis [79]. Therefore the gain in statistical precision attained by integrating over the region of the $K^*(892)$ resonance could be surpassed by analyses of this kind in the future.

An upgrade of the LHCb detector is planned in 2018, for the start of Run III of the LHC. The upgraded detector and higher instantaneous luminosity mean that approximately 50 fb^{-1} of data are expected by the end of the 10 year running period [80]. The $b\bar{b}$ production cross-section also depends linearly on the centre-of-mass energy, which will be increased from 8 TeV to 14 TeV. In addition, the data acquisition system of LHCb will be upgraded and the detector will be read out at 40 MHz, this is expected to increase the efficiency of the trigger on hadronic decays by a factor of two [80]. All these considerations imply an increase in signal candidate yields by approximately a factor of 60. Therefore the answers to a lot of these questions will surely become clear and analyses of $B \rightarrow DK^*$ decays promise accurate constraints on γ and the unitarity of the CKM matrix.

Appendix A

Pull distributions

The distributions of the pulls of all the free parameters of the invariant mass model, evaluated as part of the toy study presented in Section 4.3.1, are shown in Figures A.1 to A.4. The mean and width of the Gaussian functions fit to each distribution are shown here and also summarised in Figure 4.9.

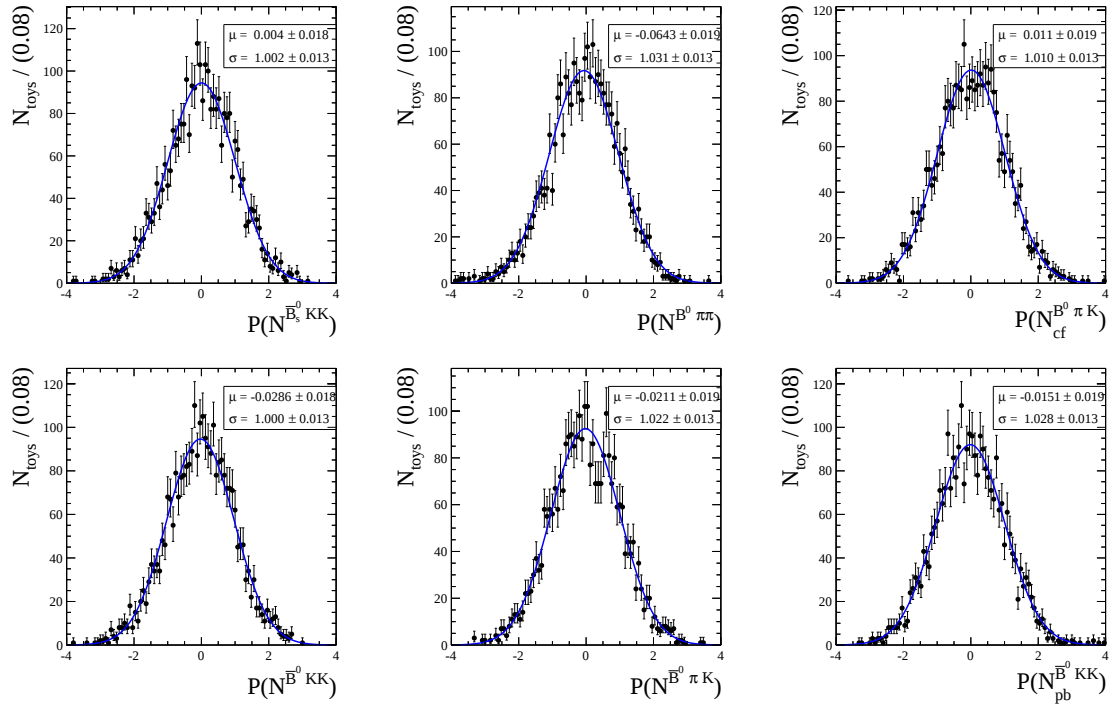


Figure A.1: Distributions of the pull of the free parameters of the invariant mass model. Each distribution is labelled with the parameters of a Gaussian function fit to the distribution.

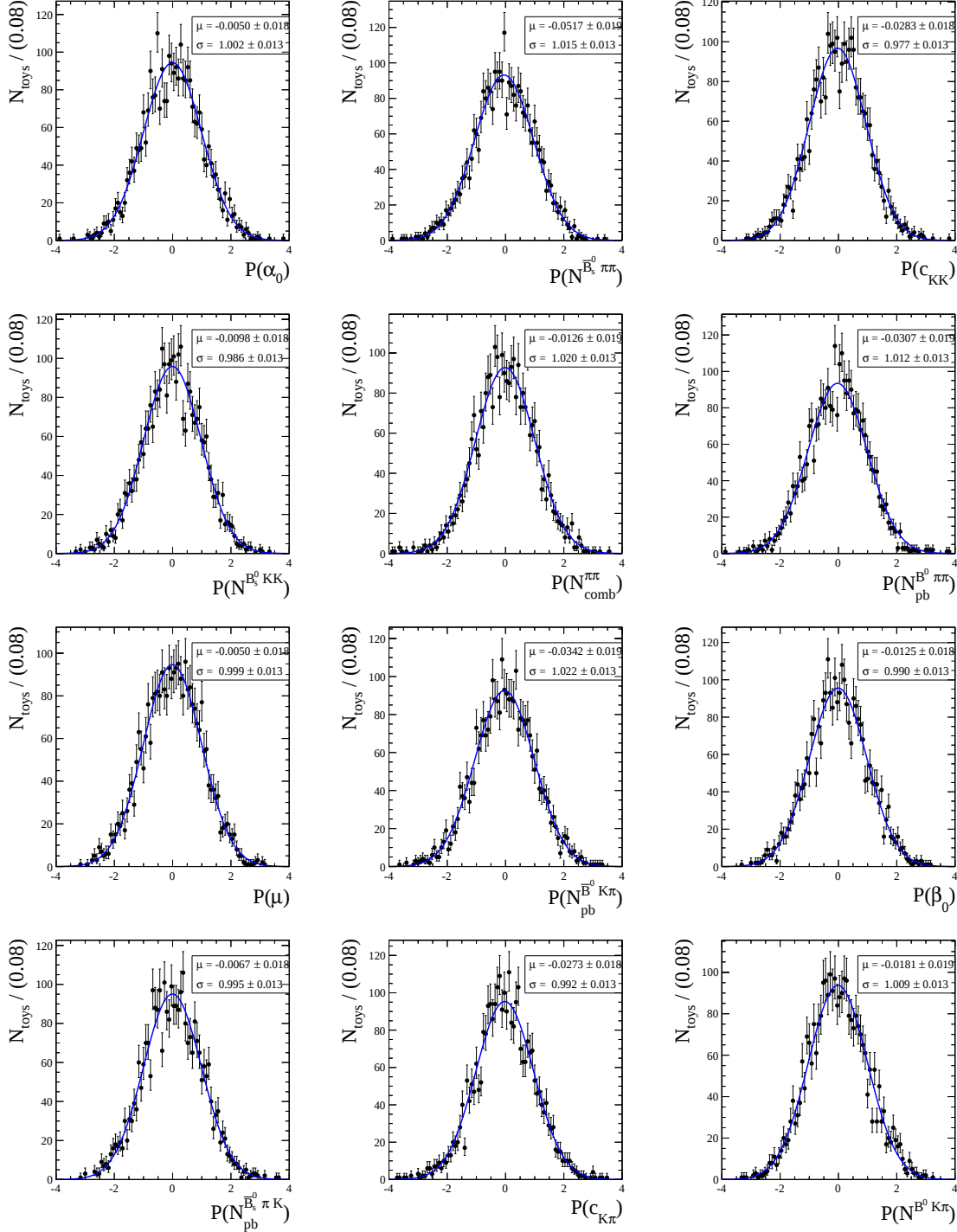


Figure A.2: Distributions of the pull of the free parameters of the invariant mass model. Each distribution is labelled with the parameters of a Gaussian function fit to the distribution.

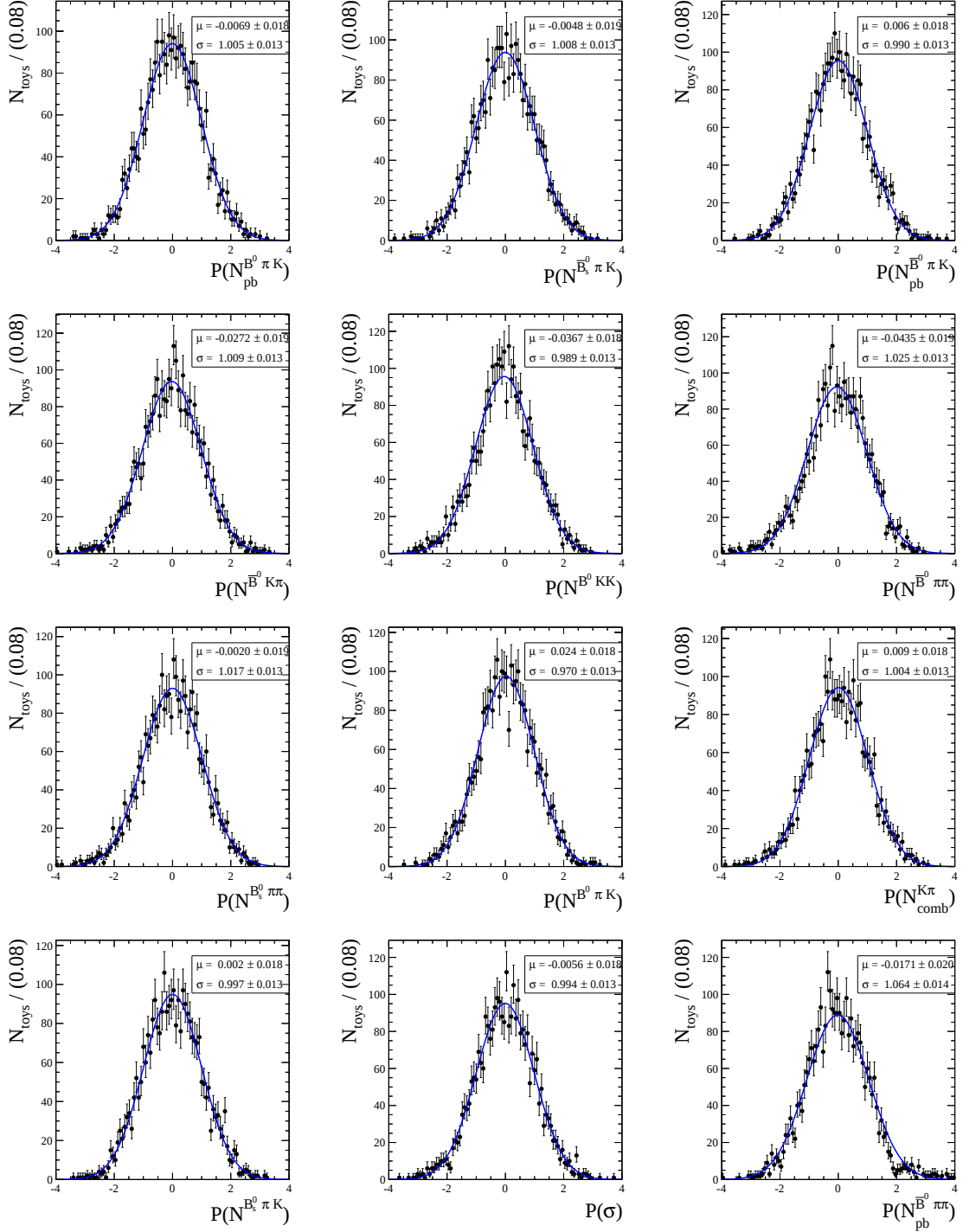


Figure A.3: Distributions of the pull of the free parameters of the invariant mass model. Each distribution is labelled with the parameters of a Gaussian function fit to the distribution.

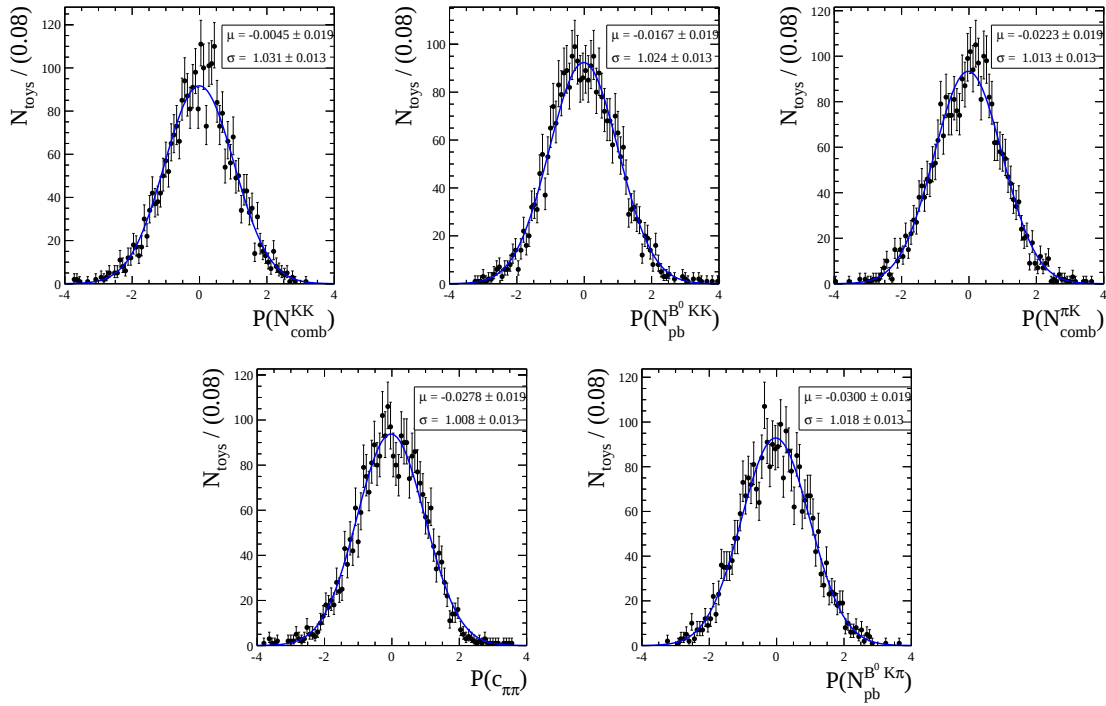


Figure A.4: Distributions of the pull of the free parameters of the invariant mass model. Each distribution is labelled with the parameters of a Gaussian function fit to the distribution.

Appendix B

Model-related systematic uncertainty

The conservative method to assign a systematic uncertainty in the f_{core} and r_W parameters is described in Section 4.5.3, where the uncertainty in the fixed value of f_{core} is also given. The same method results in assigning an uncertainty of ± 0.64 in r_W , so $r_W = 2.67 \pm 0.64$.

The ζ distributions relating to the systematic uncertainties resulting from the uncertainty in the assumptions made in the design of the invariant mass model about f_{core} , r_W , the difference in B^0 and B_s^0 meson masses and the choice of the signal parameterisation are shown in Figures B.1, B.2, B.3 and B.4, respectively. The Δ values of the relevant ζ distributions are given in Tables B.1 and B.2.

Thirteen branching ratios and efficiencies, given in Table 4.5, are used as fixed parameters of the invariant mass model. The uncertainties in these parameters also cause systematic uncertainty in the best-fit numbers of signal decays. The uncertainties in these fixed parameters are assumed reliable so are used, instead of conservatively assigning uncertainties as with f_{core} and r_W . The relevant ζ distributions are not shown for brevity but the associated Δ values are given in Tables B.3 to B.5.

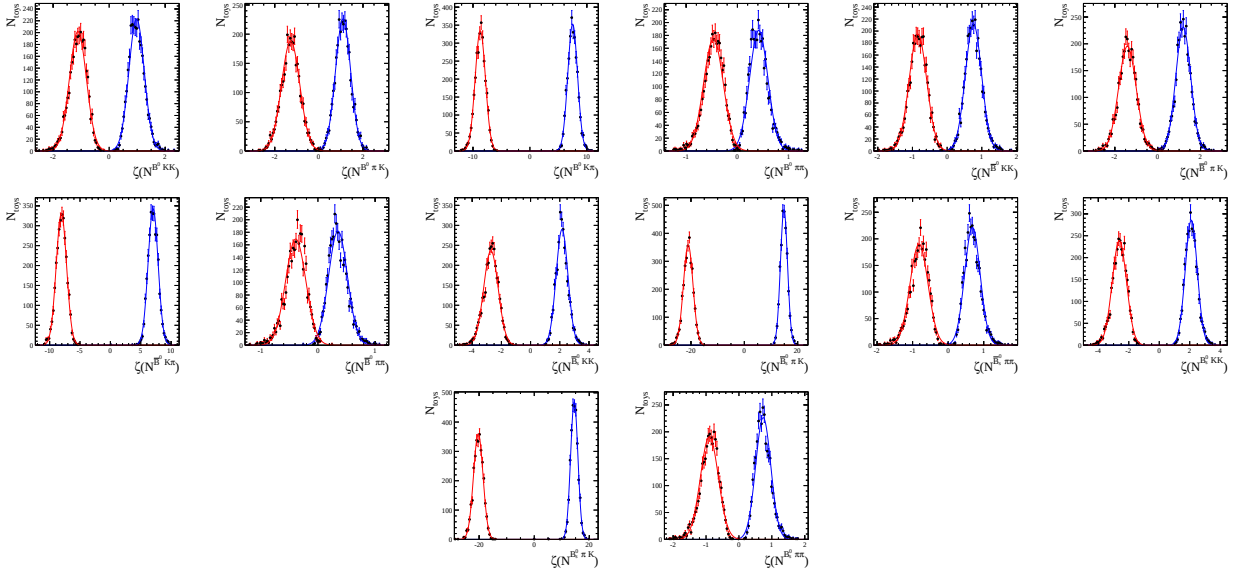


Figure B.1: The ζ distributions of the numbers of signal $B \rightarrow DK^*$ decays, when varying the f_{core} parameter to its upper bound (*red*) and lower bound (*blue*).

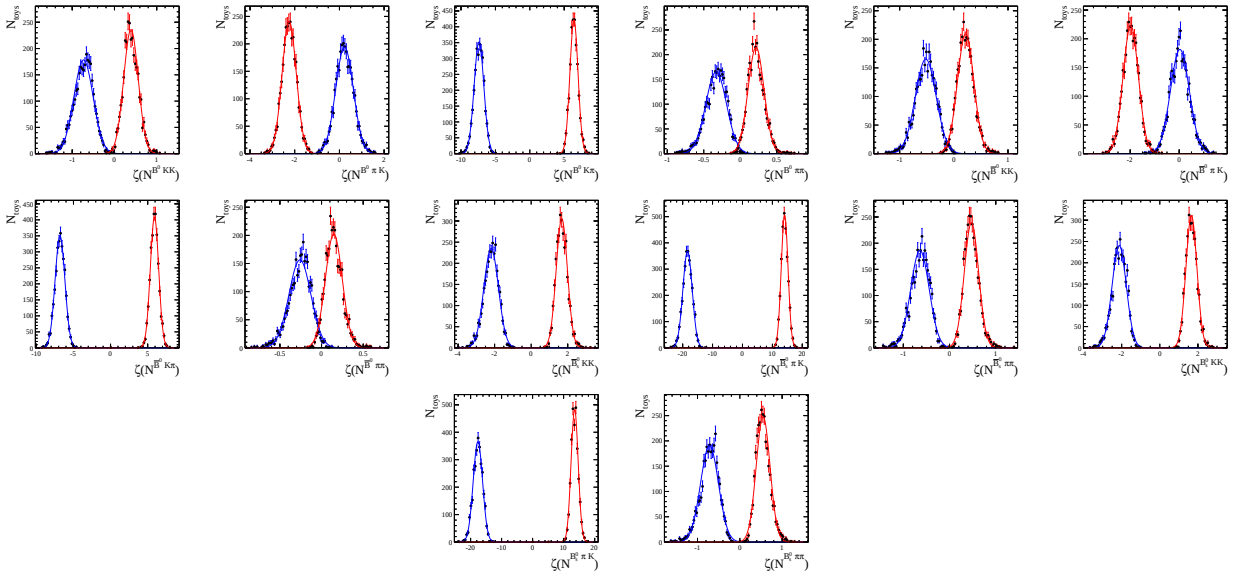


Figure B.2: The ζ distributions of the numbers of signal $B \rightarrow DK^*$ decays, when varying the r_W parameter to its upper bound (*red*) and lower bound (*blue*).

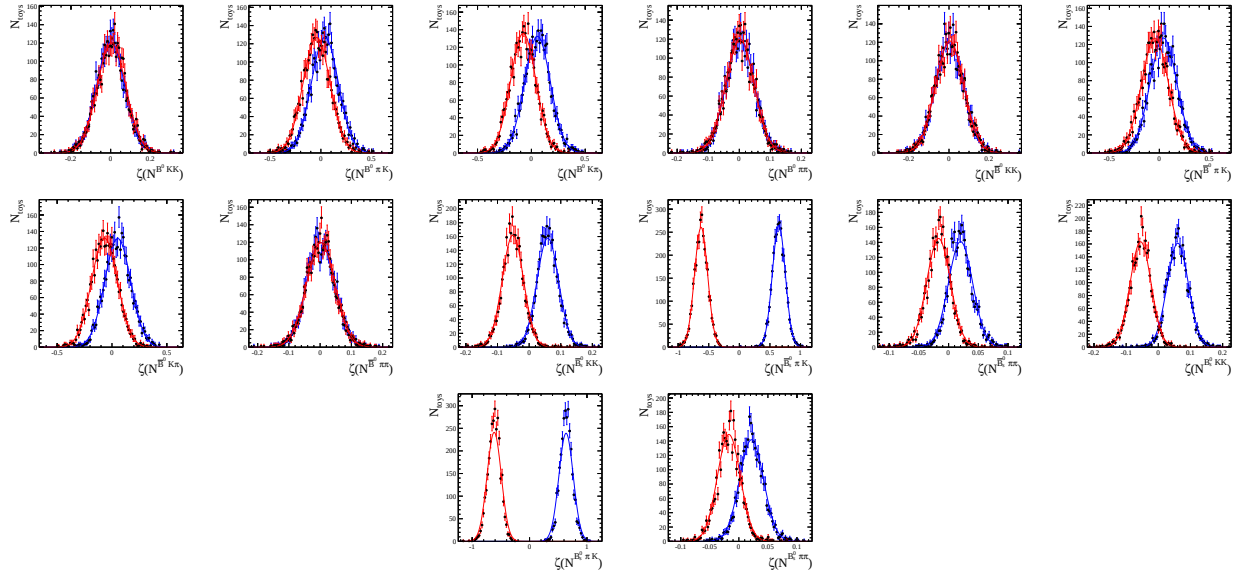


Figure B.3: The ζ distributions of the numbers of signal $B \rightarrow DK^*$ decays, when varying the difference in the B_s^0 and B^0 meson masses, ΔM , to its upper bound (*red*) and lower bound (*blue*).

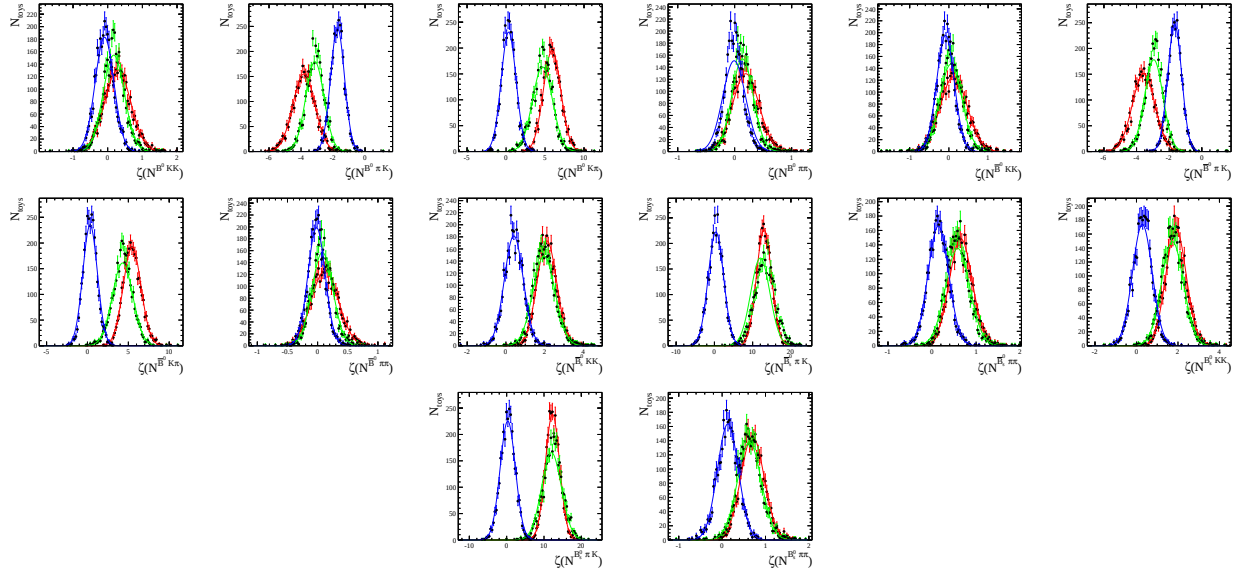


Figure B.4: The ζ distributions of the numbers of signal $B \rightarrow DK^*$ decays, when changing the signal parametrisation to an Apollonios function (*red*), a Cruijff function (*green*) and a constrained Cruijff function (*blue*).

Table B.1: Δ values corresponding to the ζ distributions obtained when fitting models with altered parameters to the toy datasets. The parameter and the sign of the variation are labelled.

Parameter	f_{core}		r_W		ΔM	
Sign of variation	+	-	+	-	+	-
Signal yield						
$N^{B^0 KK}$	-1.156	1.016	0.427	-0.767	0.072	-0.071
$N^{B^0 \pi K}$	-1.344	1.150	-2.270	0.457	-0.140	0.139
$N^{B^0 K\pi}$	-8.763	7.557	6.442	-7.422	-0.140	0.133
$N^{B^0 \pi\pi}$	-0.505	0.454	0.242	-0.360	0.046	0.046
$N^{\bar{B}^0 KK}$	-0.879	0.795	0.266	-0.552	0.071	0.072
$N^{\bar{B}^0 \pi K}$	-1.472	1.242	-2.003	0.387	-0.142	0.141
$N^{\bar{B}^0 K\pi}$	-8.107	7.078	5.950	-6.847	-0.137	0.130
$N^{\bar{B}^0 \pi\pi}$	-0.427	0.388	0.179	-0.291	0.046	0.046
$N^{\bar{B}_s^0 KK}$	-2.763	2.192	1.706	-2.227	-0.061	0.068
$N^{\bar{B}_s^0 \pi K}$	-21.108	14.937	14.137	-18.42	-0.635	0.654
$N^{\bar{B}_s^0 \pi\pi}$	-0.859	0.717	0.500	-0.671	-0.026	0.029
$N^{B_s^0 KK}$	-2.661	2.140	1.685	-2.164	-0.064	0.069
$N^{B_s^0 \pi K}$	-20.391	14.587	13.598	-17.703	-0.629	0.648
$N^{B_s^0 \pi\pi}$	-0.939	0.773	0.573	-0.748	-0.026	0.030

Table B.2: Δ values corresponding to the ζ distributions obtained when fitting models with different signal PDFs to the toy datasets. The “envelope” of these Δ values is used to determine the systematic uncertainty in the number of signal decays due to the uncertainty in the choice of signal PDF, as described in Section 4.5.3. The last column shows the systematic uncertainty assigned to that signal yield.

	Signal PDF			
Signal yield	Apollonios	Cruijff	Constrained Cruijff	Uncertainty
$N^{B^0 KK}$	0.474	0.352	-0.286	+0.474 -0.286
$N^{B^0 \pi K}$	-3.881	-3.187	-1.774	+0.000 -3.881
$N^{B^0 K\pi}$	6.007	4.850	1.010	+6.007 -0.000
$N^{B^0 \pi\pi}$	0.303	0.232	-0.200	+0.303 -0.200
$N^{\bar{B}^0 KK}$	0.366	0.289	-0.241	+0.366 -0.241
$N^{\bar{B}^0 \pi K}$	-3.631	-2.997	-1.715	+0.000 -3.631
$N^{\bar{B}^0 K\pi}$	5.568	4.469	0.952	+5.568 -0.000
$N^{\bar{B}^0 \pi\pi}$	0.255	0.190	-0.145	+0.255 -0.145
$N^{\bar{B}_s^0 KK}$	2.176	2.047	0.637	+2.176 -0.000
$N^{\bar{B}_s^0 \pi K}$	13.114	12.258	2.058	+13.114 -0.000
$N^{\bar{B}_s^0 \pi\pi}$	0.702	0.643	0.275	+0.702 -0.000
$N^{B_s^0 KK}$	1.968	1.814	0.554	+1.968 -0.000
$N^{B_s^0 \pi K}$	12.42	12.29	2.064	+12.42 -0.000
$N^{B_s^0 \pi\pi}$	0.749	0.685	0.276	+0.749 -0.000

Table B.3: Δ values corresponding to the ζ distributions obtained when fitting models with altered parameters to the toy datasets. The parameter and the sign of the variation are labelled.

Parameter	$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$		$\mathcal{B}(D^0 \rightarrow K^+ K^-)$		$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$		$\frac{\mathcal{B}(D^{*0} \rightarrow D^0 \pi^0)}{\mathcal{B}(D^{*0} \rightarrow D^0 \gamma)}$	
Sign of variation	+	-	+	-	+	-	+	-
Signal yield								
$N^{B^0 KK}$	0.045	-0.041	-0.059	0.064	0.025	0.025	0.280	-0.301
$N^{B^0 \pi K}$	-0.053	0.052	0.056	-0.057	0.036	-0.037	2.588	-2.845
$N^{B^0 K \pi}$	-0.053	-0.062	-0.062	-0.053	-0.052	-0.052	-0.494	0.509
$N^{B^0 \pi \pi}$	0.022	-0.020	0.019	0.018	-0.024	0.027	0.105	-0.109
$N^{\bar{B}^0 KK}$	0.043	-0.039	-0.056	0.061	0.024	0.023	0.271	-0.289
$N^{\bar{B}^0 \pi K}$	-0.052	0.052	0.056	-0.057	0.036	-0.037	2.589	-2.847
$N^{\bar{B}^0 K \pi}$	-0.053	-0.062	-0.062	-0.053	-0.052	-0.052	-0.484	0.523
$N^{\bar{B}^0 \pi \pi}$	0.021	-0.019	0.018	0.017	-0.023	0.026	0.103	-0.108
$N^{\bar{B}_s^0 KK}$	-0.039	0.044	0.060	-0.056	0.026	0.027	-0.061	0.072
$N^{\bar{B}_s^0 \pi K}$	0.077	-0.076	-0.078	0.077	-0.074	0.076	-0.173	0.195
$N^{\bar{B}_s^0 K \pi}$	-0.021	0.023	-0.016	0.016	0.032	-0.030	-0.033	0.038
$N^{B_s^0 KK}$	-0.040	0.041	0.061	-0.056	-0.025	0.025	-0.061	0.070
$N^{B_s^0 \pi K}$	0.100	-0.099	-0.083	0.124	-0.097	0.074	-0.183	0.228
$N^{B_s^0 \pi \pi}$	-0.021	0.023	-0.016	0.017	0.033	-0.031	-0.033	0.038

Table B.4: Δ values corresponding to the ζ distributions obtained when fitting models with altered parameters to the toy datasets. The parameter and the sign of the variation are labelled. Shorter notation has been introduced for the column headings to save space: $\epsilon_{\text{tot}}(K\pi) = \epsilon_{\text{tot}}(B \rightarrow D(K^\pm \pi^\mp) K^*)$, $\epsilon_{\text{tot}}(KK) = \epsilon_{\text{tot}}(B \rightarrow D(KK) K^*)$ and $\epsilon_{\text{tot}}(\pi\pi) = \epsilon_{\text{tot}}(B \rightarrow D(\pi\pi) K^*)$.

Parameter	$\epsilon_{\text{tot}}(K\pi)$		$\epsilon_{\text{tot}}(KK)$		$\epsilon_{\text{tot}}(\pi\pi)$		$\epsilon_{\text{acc}}(D\gamma)$		$\epsilon_{\text{acc}}(D\pi^0)$	
Sign of variation	+	-	+	-	+	-	+	-	+	-
Signal yield										
$N^{B^0 KK}$	0.149	0.025	-0.077	0.080	0.025	0.025	-0.032	0.036	0.039	-0.034
$N^{B^0 \pi K}$	-0.175	-0.031	0.070	-0.070	0.038	-0.040	-0.227	0.223	0.250	-0.255
$N^{B^0 K \pi}$	-0.054	-0.052	-0.053	-0.052	-0.052	-0.052	0.074	-0.068	-0.071	0.075
$N^{B^0 \pi \pi}$	0.047	0.019	0.021	0.018	-0.027	0.030	-0.019	0.022	0.022	-0.019
$N^{\bar{B}^0 KK}$	0.142	0.024	-0.073	0.076	0.024	0.023	-0.030	0.035	0.038	-0.032
$N^{\bar{B}^0 \pi K}$	-0.175	-0.031	0.070	-0.070	0.038	-0.040	-0.227	0.223	0.250	-0.255
$N^{\bar{B}^0 K \pi}$	-0.055	-0.052	-0.051	-0.051	-0.051	-0.052	0.076	-0.065	-0.069	0.076
$N^{\bar{B}^0 \pi \pi}$	0.046	0.017	0.019	0.018	-0.026	0.029	-0.018	0.021	0.021	-0.018
$N^{\bar{B}_s^0 KK}$	-0.142	0.027	0.075	-0.072	0.027	0.027	0.027	-0.027	-0.027	0.027
$N^{\bar{B}_s^0 \pi K}$	0.110	-0.291	-0.079	0.080	-0.075	0.076	0.084	-0.075	-0.075	0.077
$N^{\bar{B}_s^0 K \pi}$	-0.066	0.016	-0.017	0.016	0.037	-0.036	0.016	-0.016	-0.016	0.016
$N^{B_s^0 KK}$	-0.141	0.025	0.076	-0.072	-0.025	0.025	0.026	-0.026	-0.025	0.026
$N^{B_s^0 \pi K}$	0.127	0.110	-0.117	0.118	-0.077	0.118	0.105	-0.089	-0.088	0.099
$N^{B_s^0 \pi \pi}$	-0.067	0.016	-0.017	0.017	0.038	-0.036	0.017	-0.017	-0.016	0.017

Table B.5: Δ values corresponding to the ζ distributions obtained when fitting models with altered parameters to the toy datasets. The parameter and the sign of the variation are labelled.

Parameter	$\epsilon_{\text{sel}}^0(D\gamma)$		$\epsilon_{\text{sel}}^{\pm}(D\gamma)$		$\epsilon_{\text{sel}}^0(D\pi^0)$		$\epsilon_{\text{sel}}^{\pm}(D\pi^0)$	
Sign of variation	+	-	+	-	+	-	+	-
Signal yield								
$N^{B^0 KK}$	-0.035	0.040	-0.058	0.065	0.039	-0.035	0.062	-0.058
$N^{B^0 \pi K}$	-0.280	0.279	-0.513	0.514	0.261	-0.271	0.487	-0.510
$N^{B^0 K\pi}$	0.079	-0.084	0.091	-0.098	-0.082	0.078	-0.095	0.090
$N^{B^0 \pi\pi}$	-0.020	0.023	-0.026	0.030	0.023	-0.020	0.029	-0.026
$N^{\bar{B}^0 KK}$	-0.033	0.039	-0.056	0.063	0.037	-0.032	0.061	-0.056
$N^{\bar{B}^0 \pi K}$	-0.280	0.279	-0.514	0.515	0.261	-0.271	0.488	-0.510
$N^{\bar{B}^0 K\pi}$	0.078	-0.082	0.150	-0.095	-0.080	0.077	-0.091	0.091
$N^{\bar{B}^0 \pi\pi}$	-0.019	0.022	-0.027	0.029	0.021	-0.019	0.028	-0.025
$N^{\bar{B}_s^0 KK}$	0.029	-0.028	0.029	-0.028	-0.028	0.028	-0.027	0.029
$N^{\bar{B}_s^0 \pi K}$	0.079	-0.077	0.078	-0.079	-0.078	0.078	-0.078	0.079
$N^{\bar{B}_s^0 \pi\pi}$	0.017	-0.016	0.017	-0.016	-0.016	0.017	-0.016	0.017
$N^{B_s^0 KK}$	0.027	-0.026	0.027	-0.026	-0.027	0.027	-0.026	0.027
$N^{B_s^0 \pi K}$	0.100	-0.090	0.078	-0.102	-0.103	0.100	-0.101	0.097
$N^{B_s^0 \pi\pi}$	0.017	-0.017	0.017	-0.017	-0.017	0.017	-0.017	0.017

Appendix C

Full report of uncertainty

Table C.1 shows the averages of the upper and lower uncertainties on each observable from each source. Tables 4.11 and 4.12 are identical apart from the model-related systematic uncertainties, those in the second block of Table C.1, have been added in quadrature and the result is given in one row.

Table C.1: A summary of the uncertainties in each observable. The first block are systematic uncertainties that are not related to the invariant mass model. The second block are related to the model. The presence of ‘–’ indicates that the source of uncertainty does not affect the observable and ‘0.0’ indicates that the uncertainty is zero to three decimal places.

Source	Observable											
	$\mathcal{R}_d^+$	$\mathcal{R}_d^-$	$\mathcal{R}_d^{KK}$	$\mathcal{A}_s^{KK}$	$\mathcal{A}_d^{K\pi}$	$\mathcal{R}_{ds}^{KK}$	$\mathcal{R}_d^{\pi\pi}$	$\mathcal{R}_{ds}^{\pi\pi}$	$\mathcal{A}_s^{\pi K}$	$\mathcal{A}_d^{\pi\pi}$	$\mathcal{A}_s^{\pi\pi}$	$\mathcal{A}_d^{KK}$
Production Asymmetry	–	–	0.001	–	0.005	0.0	0.0	0.001	–	0.005	–	0.005
ω (dilution)	–	–	0.0	–	0.0	0.0	0.0	0.0	–	0.0	–	0.0
f_s/f_d	–	–	–	–	–	0.008	–	0.012	–	–	–	–
τ_s/τ_d	–	–	–	–	–	0.001	–	0.001	–	–	–	–
D^0 decay BFs	–	–	0.025	–	–	–	0.028	–	–	–	–	–
Trigger Efficiency	–	–	0.015	0.011	0.012	0.0	0.019	0.0	0.012	0.011	0.011	0.011
Lifetime Difference	–	–	–	–	–	0.002	–	0.003	–	–	–	–
PID Efficiency	–	–	0.010	0.005	0.005	0.0	0.012	0.0	0.005	0.005	0.005	0.005
Selection Efficiency	–	–	0.029	0.015	0.014	0.0	0.037	0.0	0.014	0.014	0.014	0.014
D decay mis-ID	0.0	0.001	–	–	–	–	–	–	–	–	–	–
Bias Correction	0.001	0.001	0.003	–	0.001	0.0	0.006	0.001	–	0.0	0.002	0.0
ΔM	0.0	0.0	0.001	0.0	0.0	0.0	0.002	0.0	0.0	0.0	0.0	0.0
Signal Param.	0.005	0.005	0.003	0.0	0.0	0.0	0.005	0.001	0.0	0.0	0.0	0.001
f_{core}	0.002	0.003	0.001	0.001	0.0	0.0	0.003	0.0	0.0	0.0	0.0	0.001
r_W	0.002	0.002	0.006	0.001	0.0	0.001	0.004	0.0	0.0	0.0	0.0	0.0
$\mathcal{B}(D^0 \rightarrow K^+ K^-)$	0.0	0.0	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	0.0	0.0	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^-)$	0.0	0.0	0.0	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\frac{\mathcal{B}(D^{*0} \rightarrow D^0 \pi^0)}{\mathcal{B}(D^{*0} \rightarrow D^0 \gamma)}$	0.007	0.007	0.005	0.0	0.0	0.001	0.005	0.001	0.0	0.0	0.0	0.001
$\epsilon_{\text{acc}}(D\gamma)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{acc}}(D\pi^0)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}(B_d \rightarrow D(KK)K^*)$	0.0	0.0	0.002	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}(B_d \rightarrow D(K^\pm \pi^\mp)K^*)$	0.0	0.0	0.003	0.0	0.0	0.0	0.003	0.0	0.0	0.0	0.0	0.001
$\epsilon_{\text{tot}}(B_d \rightarrow D(\pi\pi)K^*)$	0.0	0.0	0.0	0.0	0.0	0.0	0.002	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}^0(D\gamma)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}^\pm(D\gamma)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}^0(D\pi^0)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
$\epsilon_{\text{tot}}^\pm(D\pi^0)$	0.001	0.001	0.001	0.0	0.0	0.0	0.001	0.0	0.0	0.0	0.0	0.0
Total Systematic	0.010	0.011	0.044	0.020	0.020	0.009	0.053	0.012	0.019	0.019	0.019	0.020
Statistical	0.028	0.031	0.159	0.073	0.041	0.017	0.268	0.038	0.025	0.217	0.131	0.144

Appendix D

Non- $K^*(892)$ $B_d \rightarrow DK^\pm\pi^\mp$ decays

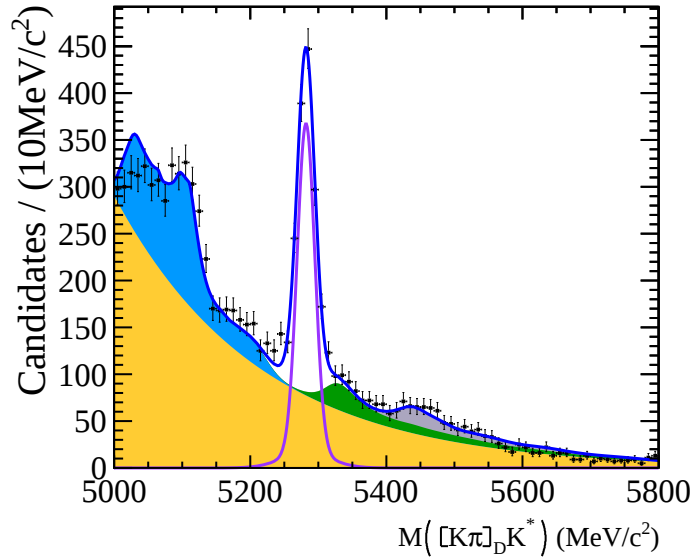


Figure D.1: The invariant mass distribution of $B \rightarrow D(K\pi)K^*$ candidates, when the requirement on the K^* meson invariant mass is relaxed. The combinatorial background (*yellow fill*) is modelled by an exponential function. The backgrounds from $B_d \rightarrow D\rho^0$ (*green fill*), $B_d \rightarrow D^*K^*$ (*blue fill*) and $B^\pm \rightarrow DK^\pm$ (*lilac fill*) are modelled by non-parametric functions. The B_d signal (*purple line*) is modelled by a double Gaussian function.

In this thesis the $K^\pm\pi^\mp$ part of the three-body $B_d \rightarrow DK^\pm\pi^\mp$ decay is reconstructed as the $K^*(892)$ resonance. However, the decay can proceed via many different intermediate resonances, for example, $K_0^*(1430)$, $K^*(1680)$ or indeed non-resonant $K^\pm\pi^\mp$. As explained in Chapter 1, the contribution of these other components of the resonance structure of the decay, within the analysed region of the parameter space, cause the coherence factor, κ , to

differ from one.

Table D.1: The parameters of the model that is fit to the reconstructed K^* invariant mass distribution of $B_d \rightarrow D(K\pi)K^*$ candidates. The numbers in parentheses are the yields of the two PDFs in the region allowed by the K^* invariant mass requirement in the analysis and σ_{res} is the width of the Gaussian resolution function.

Parameter	Value
Fixed Parameters	
Γ	$49.7 \text{ MeV}/c^2$
J	1
R	4
m_π	$140 \text{ MeV}/c^2$
m_K	$494 \text{ MeV}/c^2$
σ_{res}	$9 \text{ MeV}/c^2$
Free Parameters	
μ	$895.7 \pm 1.5 \text{ MeV}/c^2$
$N(K^*(892))$	1019 ± 97 (778 ± 74)
$N(\text{non-}K^*(892))$	258 ± 101 (65 ± 26)

The coherence factor for a very similar region of the parameter space to the one analysed in this thesis is determined, in Ref. [36], to be 0.95 ± 0.03 . However, the contribution from non- $K^*(892)$ decays to the $B_d \rightarrow DK^*$ signal in the analysis dataset is also studied, by inspecting the reconstructed K^* invariant mass distribution of $B \rightarrow D(K\pi)K^*$ candidates. In this study, all selection requirements are applied to the data except the requirement on the K^* meson invariant mass.

The model that is fit to the B meson invariant mass distribution of these candidates is based on the model presented in Chapter 4. The PDFs used are similar but they have been altered to take the relaxation of the requirement on the K^* invariant mass into account. This makes little difference to the form of the PDFs in most cases. The cross-feed background from $B_d \rightarrow D\rho^0$ has a larger tail at high mass and an additional background from $B^\pm \rightarrow DK^\pm$, which is usually eliminated by the K^* invariant mass requirement, is modelled. The B meson invariant mass distribution of these candidates and the fitted model are shown in Figure D.1.

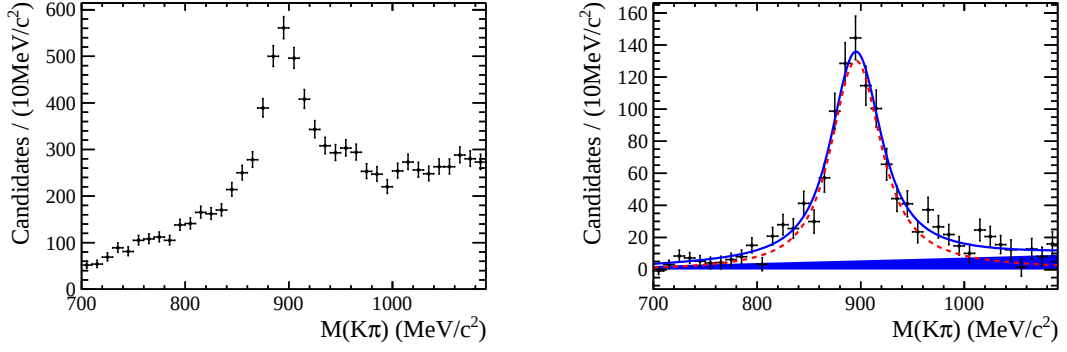


Figure D.2: The reconstructed K^* invariant mass distribution of all $B \rightarrow D(K\pi)K^\pm\pi^\mp$ candidates (*left*) and of $B_d \rightarrow D(K\pi)K^*$ candidates (*right*), where background has been subtracted using the sWeight technique [48]. All selection requirements have been applied to the data except the K^* invariant mass requirement. The result of a fit to the background subtracted distribution is shown. The $K^*(892)$ resonance (*red dashed line*) is modelled by a relativistic Breit-Wigner function and the contribution from non- $K^*(892)$ candidates (*blue fill*), which are assumed to be $K_0^*(1430)$, is modelled by the parametrisation of Ref. [81].

The signal component of the data that is modelled in Figure D.1 is isolated using the sPlot technique [48]. The reconstructed K^* invariant mass distribution of the signal component is examined and fit in Figure D.2. The $K^*(892)$ resonance is modelled using a relativistic Breit-Wigner function, given by Equation D.1, convolved with a Gaussian resolution function. In Equation D.1, M is the dependent variable ($K^\pm\pi^\mp$ invariant mass), μ is the mean and Γ_f is the mass-dependent width of the resonance. Γ_f is given as a function of M , μ , the width (Γ), the spin (J) and the “radius” (R) in Equation D.2. The non- $K^*(892)$ contribution to the $K^\pm\pi^\mp$ invariant mass is modelled using the parametrisation of the $K_0^*(1430)$ resonance of Ref. [81].

$$f_{\text{BW}}(M; \mu, \Gamma_f) = \frac{M\Gamma_f}{(\mu^2 - M^2)^2 + \mu^2\Gamma_f^2} \quad (\text{D.1})$$

$$\begin{aligned} \Gamma_f(M; \mu, \Gamma, J, R) = & \frac{\Gamma\mu}{M} \left(\frac{\mu}{M} \sqrt{\frac{(M^2 - m_K^2 - m_\pi^2)^2 - 4m_K^2m_\pi^2}{(\mu^2 - m_K^2 - m_\pi^2)^2 - 4m_K^2m_\pi^2}} \right)^{2J+1} \\ & \times \frac{1 + \frac{R^2}{4\mu^2} \left((\mu^2 - m_K^2 - m_\pi^2)^2 - 4m_K^2m_\pi^2 \right)}{1 + \frac{R^2}{4M^2} \left((M^2 - m_K^2 - m_\pi^2)^2 - 4m_K^2m_\pi^2 \right)} \end{aligned} \quad (\text{D.2})$$

The fixed parameters of the model of the $K^*(892)$ resonance, and the best-fit values of the free parameters, are given in Table D.1. The non- $K^*(892)$ contribution in the region

allowed by the K^* meson invariant mass requirement is found to be $(8.4 \pm 3.4)\%$. This result implies that the coherence factor, κ , is 0.916 ± 0.034 , which corroborates Ref. [36].

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